Computer Vision Assignment #1

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A. Introduction

In this assignment, we are going to implement camera calibration. We have several photos of a certain chessboard picture taken from different poses. Our goal is to find out the extrinsic matrix and intrinsic matrix. At the end, we will plot the extrinsic matrixes.

B. Implementation Procedure

1. First of all, we have multiple photos from different poses. Then, for each photo, we have (u, v) from image plane, and (U, V) from object plane. The relationship between these two planes can be described as following:

$$\begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \underset{\text{up to a scale}}{\underline{\rho}} H \begin{bmatrix} U \\ V \\ 1 \end{bmatrix}$$

(where *H* is Homography)

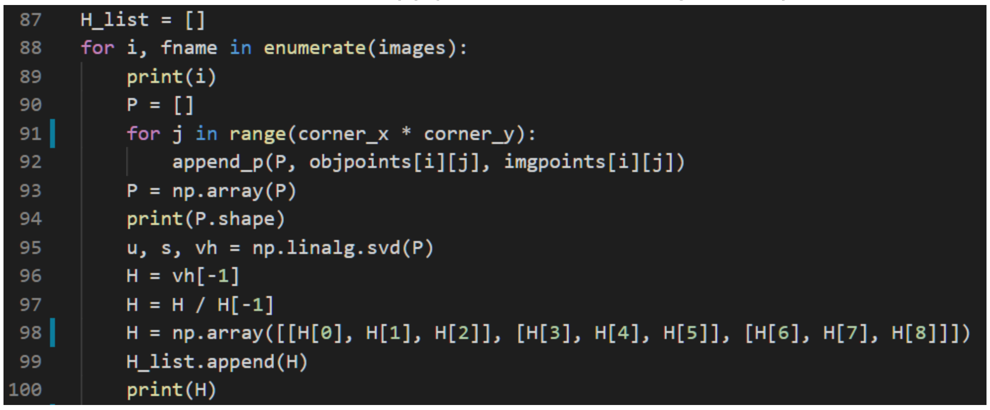
2. We can remodel the above equation in a simpler way.

$$\begin{pmatrix} -X & -Y & -1 & 0 & 0 & 0 & u.X & u.Y & u \\ 0 & 0 & 0 & -X & -Y & -1 & v.X & v.Y & v \end{pmatrix} \begin{pmatrix} h_{00} \\ h_{01} \\ h_{02} \\ h_{10} \\ h_{11} \\ h_{12} \\ h_{20} \\ h_{21} \\ h_{22} \end{pmatrix} = 0$$

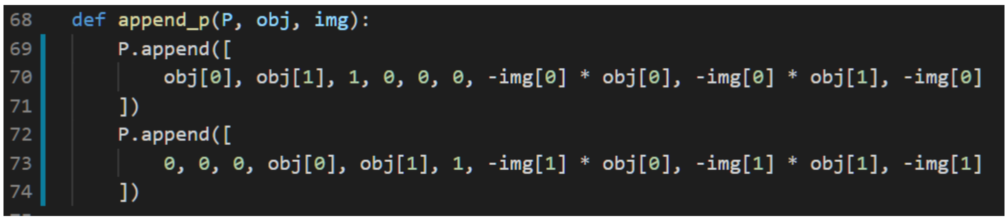
Now, we can find out *Hi* using the points in each images. Here, to find *H*, we first define *Matrix P* using the above equation:

$$P \cdot H = 0$$

Then, apply SVD of P: P = $U\Sigma V^T$. Thus, H can be found by the last column of V. Note that in Line97, we apply normalization to get an optimal solution.



```
87   H_list = []
88   for i, fname in enumerate(images):
89       print(i)
90       P = []
91       for j in range(corner_x * corner_y):
92           append_p(P, objpoints[i][j], imgpoints[i][j])
93       P = np.array(P)
94       print(P.shape)
95       u, s, vh = np.linalg.svd(P)
96       H = vh[-1]
97       H = H / H[-1]
98       H = np.array([[H[0], H[1], H[2]], [H[3], H[4], H[5]], [H[6], H[7], H[8]]])
99       H_list.append(H)
100      print(H)
```


```python
def append_p(P, obj, img):
    P.append([
        obj[0], obj[1], 1, 0, 0, 0, -img[0] * obj[0], -img[0] * obj[1], -img[0]
    ])
    P.append([
        0, 0, 0, obj[0], obj[1], 1, -img[1] * obj[0], -img[1] * obj[1], -img[1]
    ])
```

3. Once we get H, and we know that H is composed of *K(r1, t2, t)*, we can conduct the equation as below:

$$\mathbf{h}_1^\top \mathbf{K}^{-\top}\mathbf{K}^{-1}\mathbf{h}_2 = 0$$

$$\mathbf{h}_1^\top \mathbf{K}^{-\top}\mathbf{K}^{-1}\mathbf{h}_1 = \mathbf{h}_2^\top \mathbf{K}^{-\top}\mathbf{K}^{-1}\mathbf{h}_2$$

Now, we define B as:

$$\mathbf{B} := \mathbf{K}^{-\top}\mathbf{K}^{-1}$$

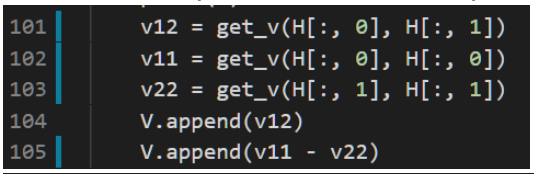
4. The next step is to build a matrix *v,* such that

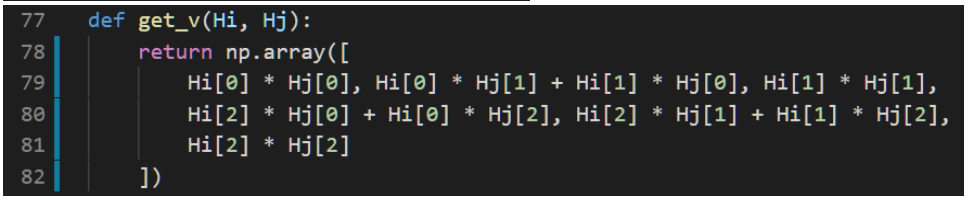
$$v_{ij} = \begin{bmatrix} h_{i0}.\,h_{j0} \\ h_{i0}.\,h_{j1} + h_{i1}.\,h_{j0} \\ h_{i1}.\,h_{j1} \\ h_{i2}.\,h_{j0} + h_{i0}.\,h_{j2} \\ h_{i2}.\,h_{j1} + h_{i1}.\,h_j 2 \\ h_i 2.\,h_{j2} \end{bmatrix}$$

Therefore, using the dot production constaint for B, we can get

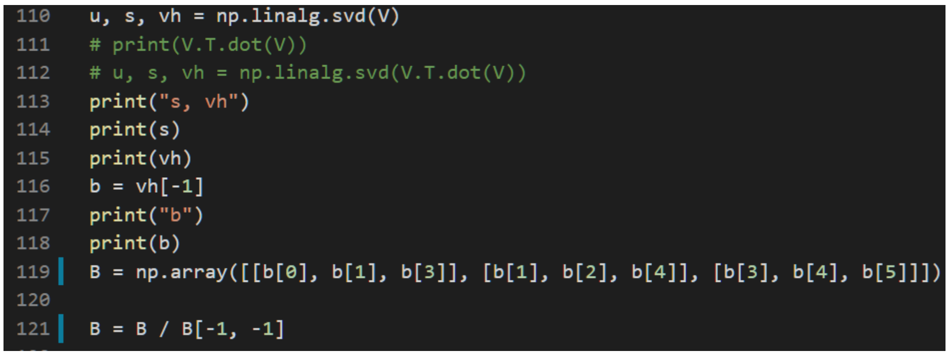
$$\begin{bmatrix} v_{12}^T \\ (v_{11} - v_{22}) \end{bmatrix}.\,b = V.\,b = 0$$

where *b* is a representation of *B* as (b11, b12, b13, b22, b23, b33) .


```python
        v12 = get_v(H[:, 0], H[:, 1])
        v11 = get_v(H[:, 0], H[:, 0])
        v22 = get_v(H[:, 1], H[:, 1])
        V.append(v12)
        V.append(v11 - v22)
```


```python
def get_v(Hi, Hj):
    return np.array([
        Hi[0] * Hj[0], Hi[0] * Hj[1] + Hi[1] * Hj[0], Hi[1] * Hj[1],
        Hi[2] * Hj[0] + Hi[0] * Hj[2], Hi[2] * Hj[1] + Hi[1] * Hj[2],
        Hi[2] * Hj[2]
    ])
```

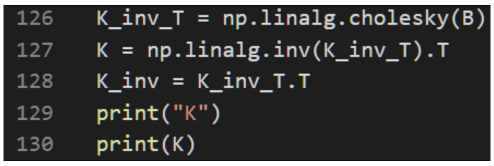
5. Again, the solution is computed using SVG of *V* which yields us *b,* and by extension *B*. Note that in Line121, we also apply normalization to make sure correction.



```
110    u, s, vh = np.linalg.svd(V)
111    # print(V.T.dot(V))
112    # u, s, vh = np.linalg.svd(V.T.dot(V))
113    print("s, vh")
114    print(s)
115    print(vh)
116    b = vh[-1]
117    print("b")
118    print(b)
119    B = np.array([[b[0], b[1], b[3]], [b[1], b[2], b[4]], [b[3], b[4], b[5]]])
120
121    B = B / B[-1, -1]
```

6. Once we get *B*, we apply Cholesky factorization to find *K*.

$$\mathbf{B} := \mathbf{K}^{-\top}\mathbf{K}^{-1}$$



```
126    K_inv_T = np.linalg.cholesky(B)
127    K = np.linalg.inv(K_inv_T).T
128    K_inv = K_inv_T.T
129    print("K")
130    print(K)
```

7. Now, we can easily find r1, r2, r3, t.

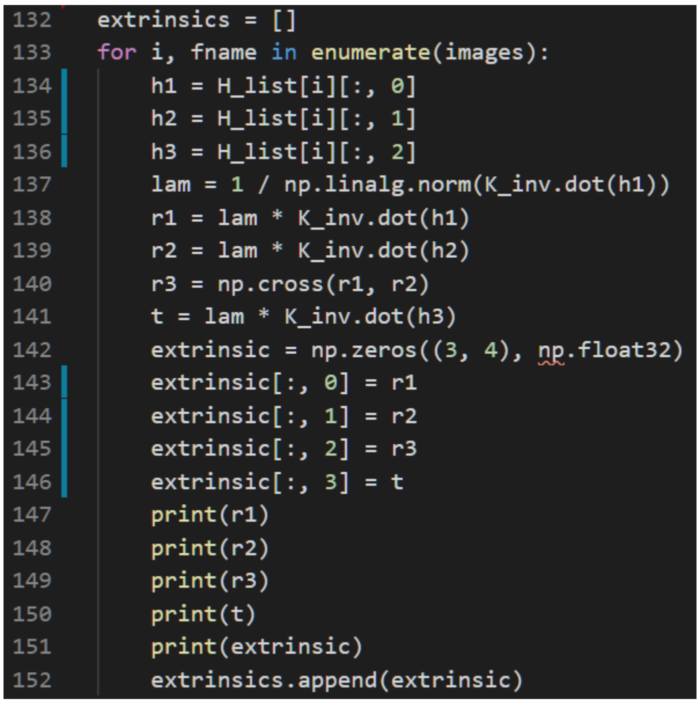
$$\mathbf{r}_1 = \lambda\mathbf{K}^{-1}\mathbf{h}_1$$

$$\mathbf{r}_2 = \lambda\mathbf{K}^{-1}\mathbf{h}_2$$

$$\mathbf{r}_3 = \mathbf{r}_1 \times \mathbf{r}_2$$

$$\mathbf{t} = \lambda\mathbf{K}^{-1}\mathbf{h}_3$$

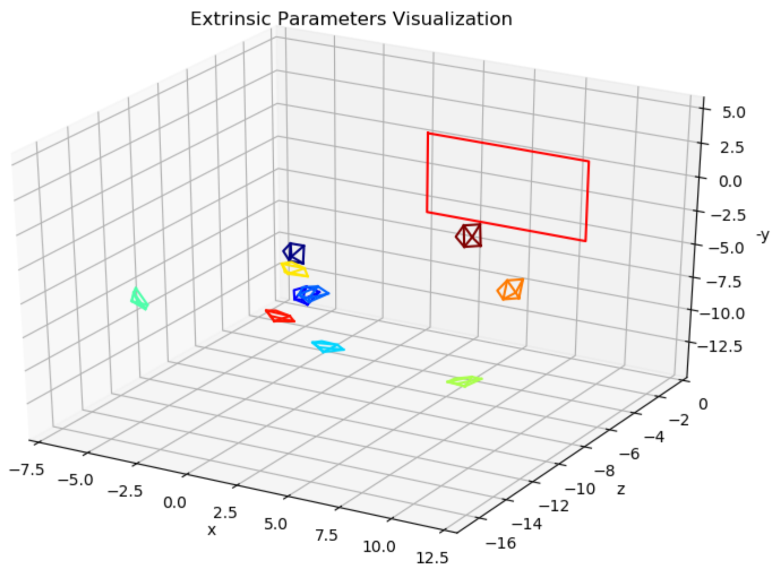
$$\lambda = 1/||\mathbf{K}^{-1}\mathbf{h}_1||$$



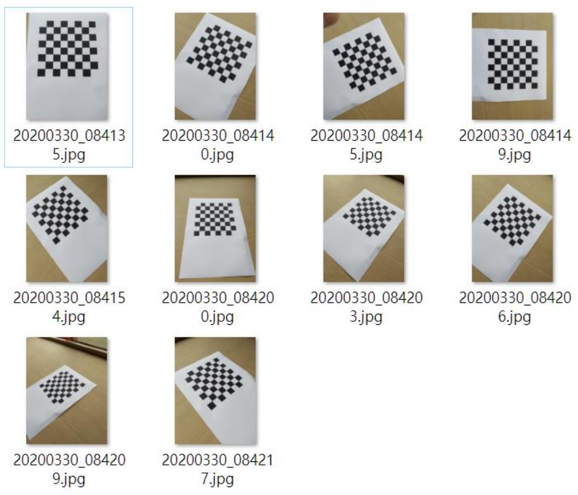
```
132    extrinsics = []
133    for i, fname in enumerate(images):
134        h1 = H_list[i][:, 0]
135        h2 = H_list[i][:, 1]
136        h3 = H_list[i][:, 2]
137        lam = 1 / np.linalg.norm(K_inv.dot(h1))
138        r1 = lam * K_inv.dot(h1)
139        r2 = lam * K_inv.dot(h2)
140        r3 = np.cross(r1, r2)
141        t = lam * K_inv.dot(h3)
142        extrinsic = np.zeros((3, 4), np.float32)
143        extrinsic[:, 0] = r1
144        extrinsic[:, 1] = r2
145        extrinsic[:, 2] = r3
146        extrinsic[:, 3] = t
147        print(r1)
148        print(r2)
149        print(r3)
150        print(t)
151        print(extrinsic)
152        extrinsics.append(extrinsic)
```

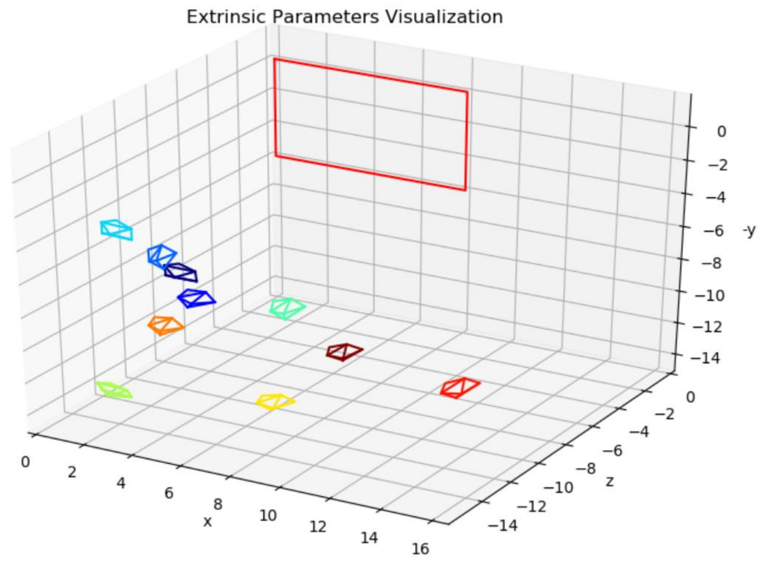
C. Experimental Result

1. Result of provided data:



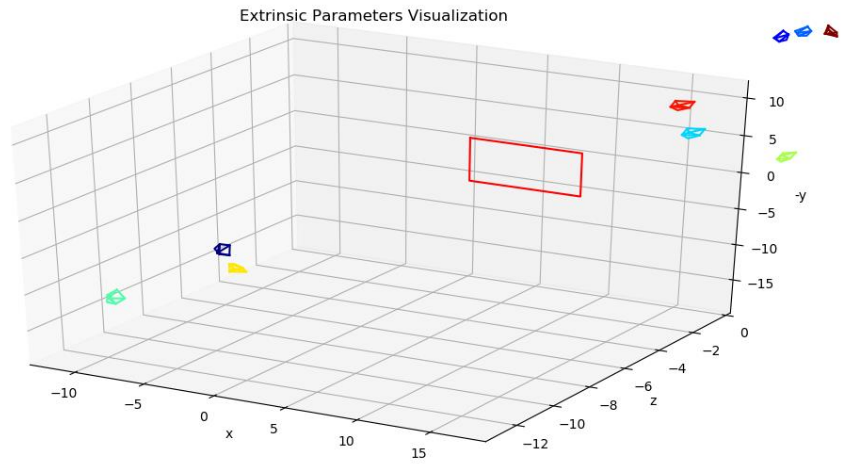
2. Result of our own data:





D. Discussion

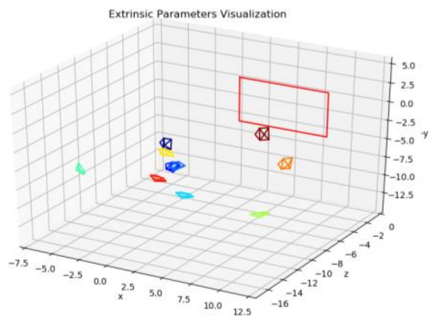
1. **The Normalization to keep accuracy:**
   Originally, we didn’t add normalization terms in our code. And the result of provided data looked like this:



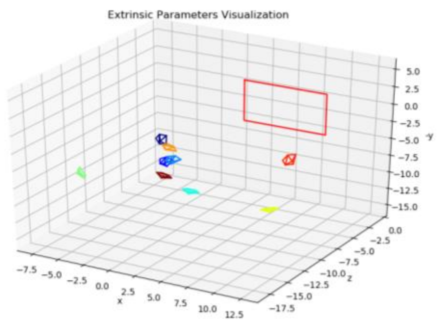
   The result was so weird that we googled for it and found out that the only part our code missing was the normalization term. After we added it, the result looked correct.

2. **How many images do we need to achieve stability?**
   In class, professor mentioned that in theory, we need at least 2 images to get enough information of homography. But in practice, we need more images to achieve better stability. Therefore, we do some experiments to check how many images in our case can we achieve stability. The results are shown below:

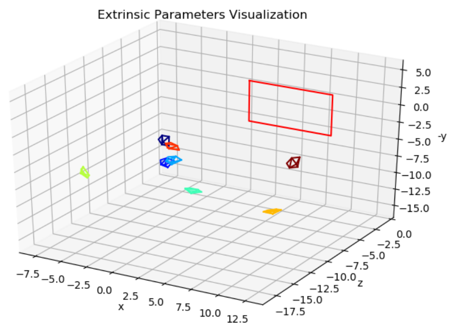
(M = 10)



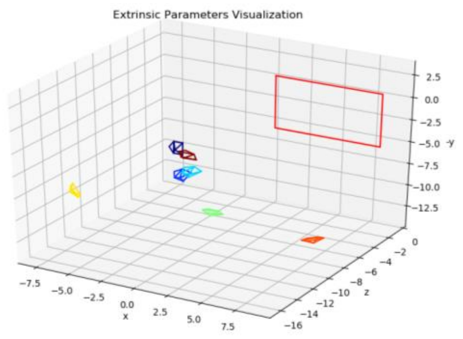
(M = 9)



(M = 8)

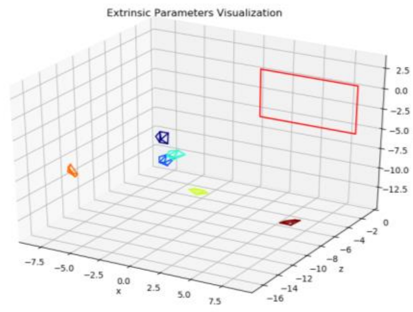


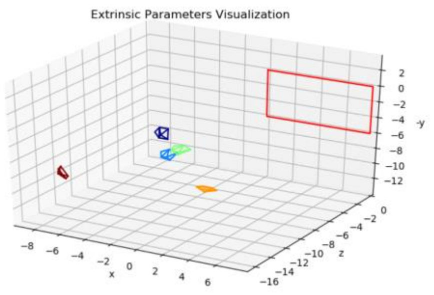
(M = 7)



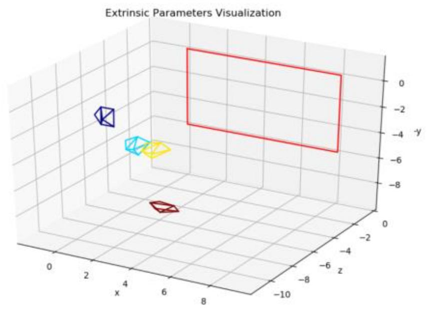
(M = 6)

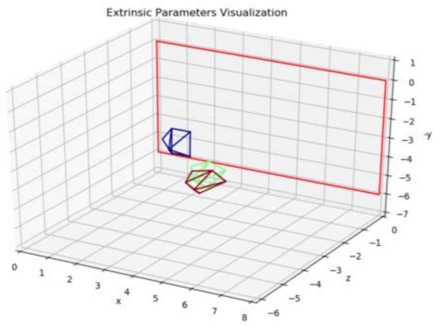
(M = 5)



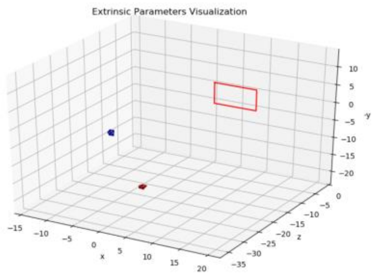


(M = 4)　　　　　　　　　　　　(M = 3)





(M = 2)



From the experiments results, we find out that when M>=5, the result start to be stable.

E. Conclusion

1. Normalization on H and B truly affects the accuracy.
2. In our case, at least 5 images are needed to get enough information.

F. Work Assignment Plan

1. 黃聖祺-Coding
2. 林宮白-Coding
3. 楊上萱-Report