

# Computer Vision Assignment #4

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## A. Introduction

In this assignment, we are going to implement SfM (Structure from Motion). We have two images with different perspective on the same object. By finding the matching point and calculate the fundamental matrix. Estimate 3D positions after recovering the canonical camera projection.

## B. Implementation Procedure

1. First of all, we use the SIFT algorithm as HW3 to find out keypoints, and pair the points to get the good matches.

```
kp_0, des_0 = find_img_keypoint(img_0)
kp_1, des_1 = find_img_keypoint(img_1)

good_matches, good_matches_for_img_show = find_good_matches(des_0, des_1)
```

- a. The matched points found by `knnmatch`, then are determined by a threshold that if it is a good match by calculating the distance of them.

```
21 matches = bf.knnMatch(des_0, des_1, k=GOOD_MATCH_K)
```

- b. In our case we set k at 2
- c. We use 0.8 as good ratio to detect significantly better match.

```
24 for m, n in matches:
25     if m.distance < GOOD_DISTANCE_RATIO * n.distance:
26         good_matches_for_img_show.append([m])
27         good_matches.append(m)
```

- d. Then, choose top 25% of matched points

```
good_matches = sorted(good_matches, key=lambda x: x.distance)
num_good_matches = int(len(good_matches) * 0.25)
good_matches = good_matches[:num_good_matches]
return good_matches, good_matches_for_img_show
```

2. Then, to estimate the fundamental matrix we use RANSAC method to repeating select points and solve them.

```
for i in range(1000):
    idx = np.random.randint(0, len(normal_p0), 60)

    f = compute_fundamental_normalized(match_points_0[idx], match_points_1[idx], shape)
```

- a. First we randomly select 60 points from the matched points list, then calculate the fundamental matrix by these random points.

b. To calculate the matrix we create the x matrix as the format below,

1. Solve a system of homogeneous linear equations

a. Write down the system of equations

$$X^T F X' = 0$$

$$x'x f_{11} + x'y f_{12} + x'f_{13} + y'x f_{21} + y'y f_{22} + y'f_{23} + x f_{31} + y f_{32} + f_{33} = 0$$

$$A\mathbf{f} = \begin{bmatrix} x'_1 x_1 & x'_1 y_1 & x'_1 & y'_1 x_1 & y'_1 y_1 & y'_1 & x_1 & y_1 & 1 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ x'_n x_n & x'_n y_n & x'_n & y'_n x_n & y'_n y_n & y'_n & x_n & y_n & 1 \end{bmatrix} \begin{bmatrix} f_{11} \\ f_{12} \\ f_{13} \\ f_{21} \\ f_{22} \\ f_{23} \\ f_{31} \\ f_{32} \\ f_{33} \end{bmatrix} = \mathbf{0}$$



```
for i in range(n):
    A[i] = [x1[0,i]*x2[0,i], x1[0,i]*x2[1,i], x1[0,i]*x2[2,i],
            x1[1,i]*x2[0,i], x1[1,i]*x2[1,i], x1[1,i]*x2[2,i],
            x1[2,i]*x2[0,i], x1[2,i]*x2[1,i], x1[2,i]*x2[2,i] ]
```

c. Then we solve the linear equations by using SVD from  $A\mathbf{f}=0$ , also resolve  $\det(F)=0$

1. Solve a system of homogeneous linear equations

a. Write down the system of equations

b. Solve  $\mathbf{f}$  from  $A\mathbf{f}=0$  using SVD

Matlab:

```
[U, S, V] = svd(A);
```

```
f = V(:, end);
```

```
F = reshape(f, [3 3])';
```

3. Resolve  $\det(F)=0$  constraint using SVD

Matlab:

```
[U, S, V] = svd(F);
```

```
S(3,3) = 0;
```

```
F = U*S*V';
```

```
for i in range(n):
    A[i] = [x1[0,i]*x2[0,i], x1[0,i]*x2[1,i], x1[0,i]*x2[2,i],
            x1[1,i]*x2[0,i], x1[1,i]*x2[1,i], x1[1,i]*x2[2,i],
            x1[2,i]*x2[0,i], x1[2,i]*x2[1,i], x1[2,i]*x2[2,i] ]
```

d. Then get the distance by estimating the points by calculating  $|x'Fx|$ , the threshold is 0.01 in this case.

```

extend_p0 = np.array([p0[0], p0[1], 1])
extend_p1 = np.array([p1[0], p1[1], 1])
disparity = abs((extend_p1.T @ f) @ extend_p0)
# return np.linalg.norm(error)
return disparity

```

- e. Pick the Fundamental matrix with most inliers.

```

if len(inliers) > len(max_inliers):
    print("inlier length: {}".format(len(inliers)))
    max_inliers = inliers
    # max_inlier_h = h
    max_inlier_f = f

```

3. After getting the best Fundamental as we show the matching points, and try to draw the epipolar lines

```

good_matches, good_matches_for_img_show = find_good_matches(des_0, des_1)
img = cv2.drawMatchesKnn(img_0, kp_0, img_1, kp_1, good_matches_for_img_show, None, flags=2)
cv2_img_show(img)

```

4. We get 4 possible solution of essential Matrix from Fundamental matrix

```

205 def compute_essential_candidate(f, K):
206     E = K.T @ f @ K
207     U, S, V = np.linalg.svd(E)
208     S = np.diag(S)
209
210     m = (S[1, 1] + S[2, 2]) / 2
211     E = U @ np.array([[m, 0, 0], [0, m, 0], [0, 0, 0]]) @ V.T
212
213     U, S, V = np.linalg.svd(E)
214     S = np.diag(S)
215     W = np.array([[0, -1, 0], [1, 0, 0], [0, 0, 1]])
216
217     P1 = K @ np.array([[1, 0, 0, 0], [0, 1, 0, 0], [0, 0, 1, 0]])
218
219     # P2 chooses
220     P2_1 = np.zeros((3, 4))
221     P2_1[:, :-1] = U @ W @ V.T
222     P2_1[:, -1] = U[2]
223
224     P2_2 = np.zeros((3, 4))
225     P2_2[:, :-1] = U @ W @ V.T
226     P2_2[:, -1] = (-1)*U[2]
227
228     P2_3 = np.zeros((3, 4))
229     P2_3[:, :-1] = U @ W.T @ V.T
230     P2_3[:, -1] = U[2]
231
232     P2_4 = np.zeros((3, 4))
233     P2_4[:, :-1] = U @ W.T @ V.T
234     P2_4[:, -1] = (-1)*U[2]
235
236     P2_choices = np.array([P2_1, P2_2, P2_3, P2_4])
237
238     return P1, P2_choices

```

5. Calculate how many points are in front of camera with each essential matrix, and choose the one that has most points in front of camera as our best solution.

```
def find_best_solution(x1, x2, P1, P2_chooses):
    ### add 1 for homogeneous ###
    x1_extend = np.ones((x1.shape[0], 3))
    x1_extend[:, :-1] = x1
    x2_extend = np.ones((x1.shape[0], 3))
    x2_extend[:, :-1] = x2

    ### find best solution between each P2_chooses ###
    cnt_of_front_points = np.zeros(4)
    for i, P2 in enumerate(P2_chooses):
        R = P2[:, :-1]
        t = P2[:, -1].reshape(3,1)
        ### Create matrix A ###

        for point_idx in range(x1_extend.shape[0]): # for each [u,v] pair
            p1 = P1[0, :]
            p2 = P1[1, :]
            p3 = P1[2, :]
            p1p = P2[0, :]
            p2p = P2[1, :]
            p3p = P2[2, :]

            u = x1[point_idx, 0]
            v = x1[point_idx, 1]
            up = x2[point_idx, 0]
            vp = x2[point_idx, 1]
            A = np.array([u * p3.T - p1.T,
                          v * p3.T - p2.T,
                          up * p3p.T - p1p.T,
                          vp * p3p.T - p2p.T])

            # U, S, VH = np.linalg.svd(A)
            A = np.array(A)
            A = A.reshape((4, 4))

            ### calculate X ###
            U, S, VH = np.linalg.svd(A)
            X = VH[-1]
            X = X / X[-1]
            X = X[:-1].reshape((3, 1))

            ### Count in-front-of-camera point ###
            C = (-1) * R.T @ t
            view_direction = (R[2, :].T).reshape(3,1)
            if ((X-C).T @ view_direction) > 0:
                # in front!
                print('inner product: ', (X-C).T @ view_direction)
                cnt_of_front_points[i] += 1
        print('cnt: ', cnt_of_front_points)

    max_p2_idx = np.argmax(cnt_of_front_points)
    return P2_chooses[max_p2_idx]
```



6. Do the triangulation to get 3D points.

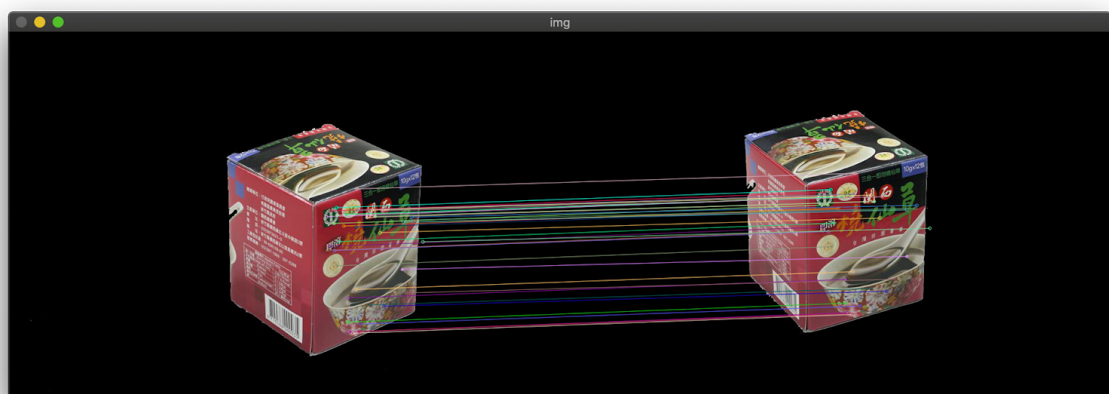
```
def Triangulation_wesuck(x1,x2,P1,P2):
    X = []

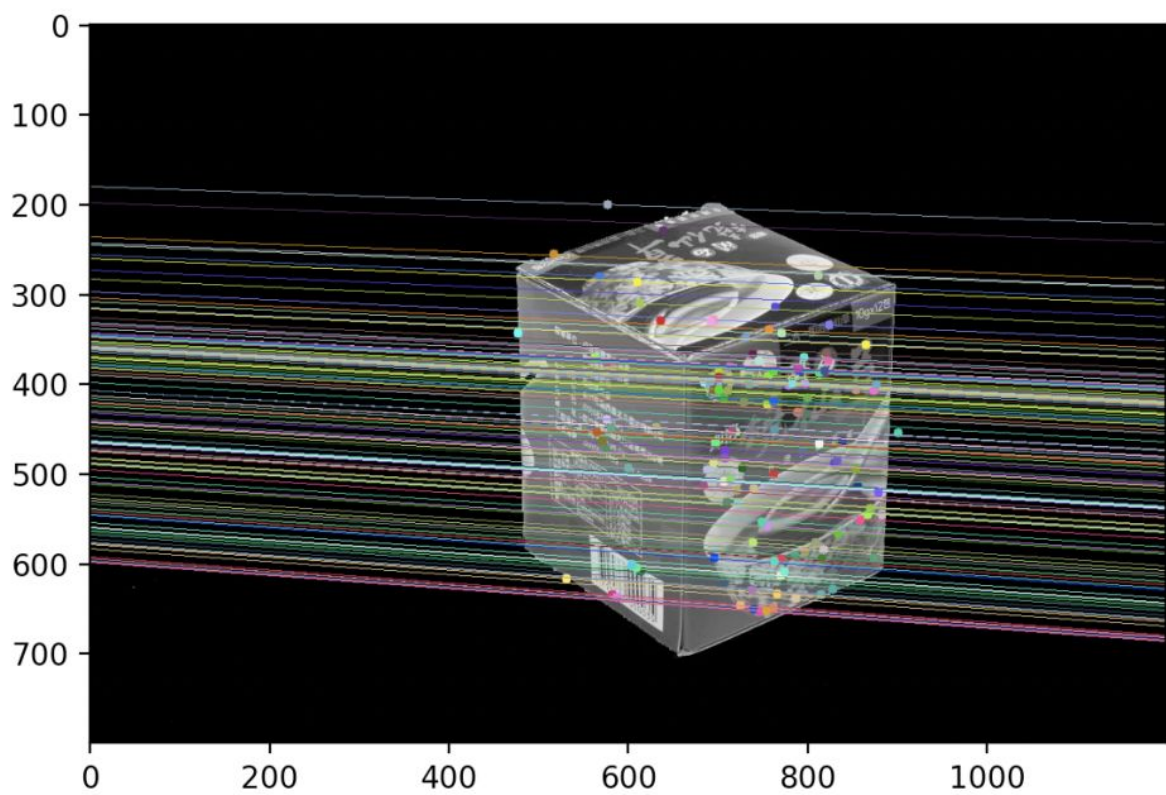
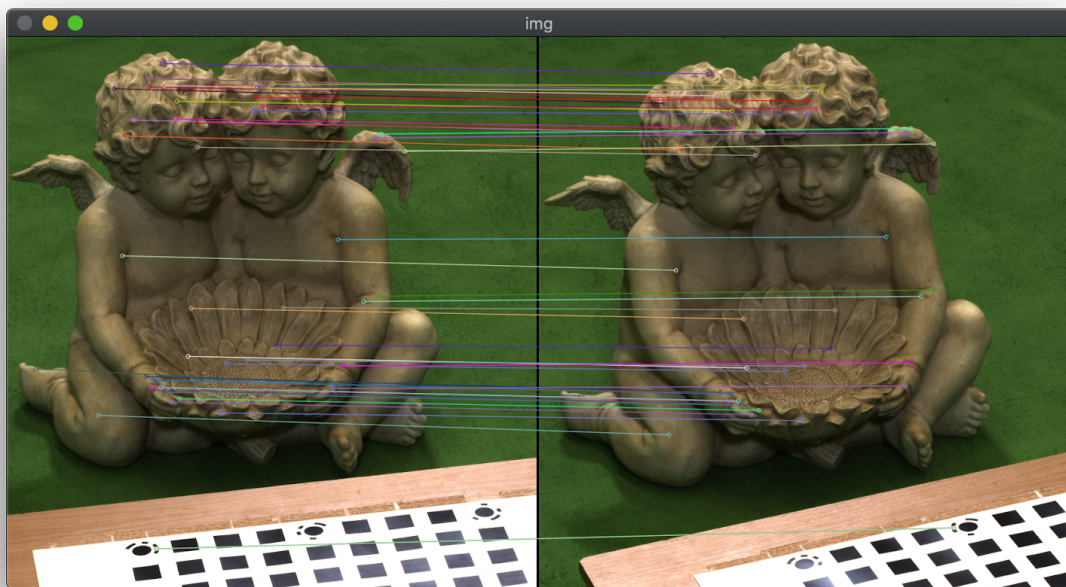
    # print(u_1.shape)
    P2_1 = P2[0].reshape((P2[0].shape[0], 1))
    P2_2 = P2[1].reshape((P2[1].shape[0], 1))
    P2_3 = P2[2].reshape((P2[2].shape[0], 1))
    P1_1 = P1[0].reshape((P1[0].shape[0], 1))
    P1_2 = P1[1].reshape((P1[1].shape[0], 1))
    P1_3 = P1[2].reshape((P1[2].shape[0], 1))
    # print(P_1.shape)
    # print(u_1 @ P_0.T)
    for i in range(len(x1)):
        u_1, v_1 = x1[i, 0], x1[i, 1]
        u_2, v_2 = x2[i, 0], x2[i, 1]
        A = [u_1 * P1_3 - P1_1,
             v_1 * P1_3 - P1_2,
             u_2 * P2_3 - P2_1,
             v_2 * P2_3 - P2_2]
        A = np.array(A)
        A = A.reshape((4, 4))
        U, S, VH = np.linalg.svd(A)
        X1 = VH[-1]
        X1 = X1 / X1[-1]
        X1 = X1[:-1].reshape(3, 1)
        X.append(X1)

    return np.array(X)
```

## C. Experimental Result

### 1. Epipolar and matching points





## 2. Triangulation and matlab



## D. Discussion

### 1. Appropriate amount of interest points?

Originally, we set the ratio of good distance ratio to be 0.3, which made the number of good matching points to be too little. So we tried a lot and find out that the result seemed to be good when the ratio was 0.8 and select the top 25% points. After that, we can get appropriate pairs of matching points.

### 2. Appropriate geometric distance when doing RANSAC?

During the RANSAC part, we calculate the geometric distance  $|x'Fx|$  and use a threshold  $d$  to points which are closer. After a lot of trials, we finally pick  $d$  to be 0.01.

### 3. The randomness when picking points for RANSAC to compute fundamental matrix?

Due to the randomness of points for calculating fundamental matrix, it is possible that the picked points are not good enough so that the result of fundamental matrix and epipolar lines will be quite weird. Therefore, we need to try a few times to make sure the picked points are appropriate.

### 4. The epipolar lines seemed to be correct, but the 3D-reconstructed structure becomes wrong?

The result of fundamental matrix and epipolar lines seemed to be correct. But the final result of 3D model was distorted. However, we had no idea which part was wrong.

## E. Conclusion

1. Tuning of parameters (e.g. `good_matching_ratio`, `gemotric_distance` threshold, etc.) is required to reach a good enough result.
2. The randomness when picking points for RANSAC to calculate fundamental matrix will somehow affect the result.
3. We failed at reconstruct 3D model.

## F. Work Assignment Plan

1. 黃聖祺-Coding
2. 林恭白-Coding, Report
3. 楊上萱-Coding