

S&P 500 Stock Price Index: Modeling and Forecasting

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1. Introduction

Standard & Poor's 500 is abbreviated as S&P 500, is an American stock market index based on the market capitalizations of 500 large companies having common stock listed on the NYSE or NASDAQ. It is widely considered as the best representative of American stock market.

Prediction of S&P 500 helps individual investors or fund managers to foresee the future stock trend and provides them hints for their investment plans. The forecast can also help American government to conduct domestic economy policy as well as providing information for foreign government to adjust their future trades with USA.

This project will first conduct a unit root test to see whether the series is random walk. Then the best ARIMA model and the best Holt's Trend Corrected Exponential Model will be selected respectively based on AIC and Durbin-Watson statistic. Then ARIMA model will be compared with smoothing model which performs better based on the in-sample-forecast test. Finally, the Holt's Trend Corrected Exponential Model will be used to forecast S&P value from 2018 November to 2019 October.

2. Data

2.1 Collecting

The data is retrieved from investing.com which is a global financial portal owned by a reliable international company called Fusion Media Limited. It consists of 288 observations of S&P 500 price in US dollar and it is monthly and not-seasonal-adjusted. The timeframe starts from 1994, November to 2018, October, covering 24 years. For actual analysis of model, data until and including 2017, October will be used and the rest of the data from 2017, November to 2018, October is used in the in-sample forecast.

2.2 First glance

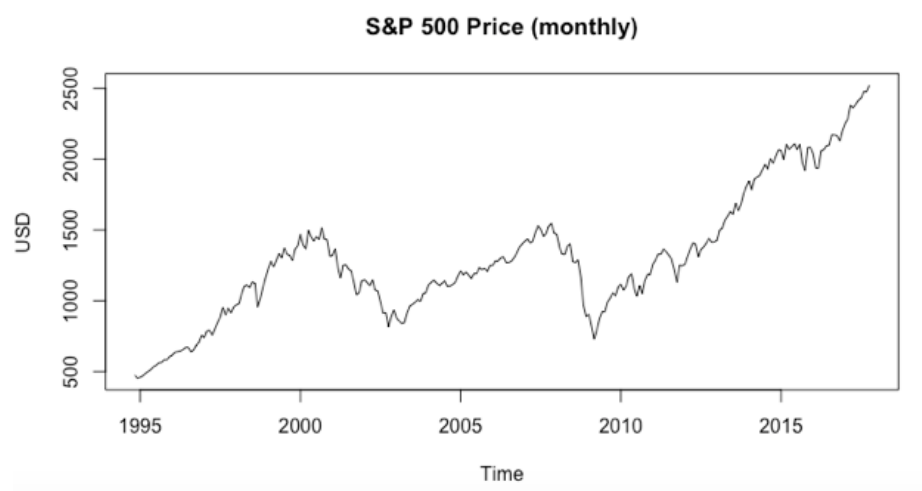
The summary statistics of the data set are presented in Table 1 below (excluding observations from 2017, November to 2018, October). The sample size with 276 data

points would not be too small to generate a statistically representative model. Besides, it would not be too large to exclude the outdated information. Ranging from 200 to 2500, the data has an appropriate order of magnitude. Therefore, there is no need to adjust data's order of magnitude.

Table 2 Summary Statistics of S&P 500 (monthly price in USD)						
variable	n	mean	sd	min	median	max
S&P 500	276	1298.4	451.0323	453.6	1248.9	2521.2

The graph of data series until/including October, 2018 can be seen in Figure 1 below. As the graph shows, the data exhibits a strong increasing trend without obvious seasonal component. Along the trend, there are continuous slight fluctuation which has larger variance after around 1998 year. This may indicate that the log form of data is needed. There are two shocks at 2001-2002 and 2008, which are respectively due to the burst of Internet bubble in 2000s and 2008 subprime crisis. It is notable that the trend climbed up from the new, permanent lower starts instead of following to previous trend. This is a suggestion of stochastic trend (or random walk with drift). Therefore, the assumption of stationarity should be tested.

Figure 1



3. Model

3.1 Decompose of time series

To proceed the analysis, the compose of the data is needed in order to observe the trend, seasonal component and residual more clearly. The decomposition of data is shown in Figure 2 below.

The residual before 1998 seems slightly smaller than the residual from 2003 to 2006 and the residual from 2003 to 2006 seems smaller than those after 2010, which suggests a need for logarithmic formation of the data. However, the growing variance is not conspicuous and the variance seems constant with time in general. To insure the preciseness of the analysis and improve the forecast, it is better to decompose the log form of data (abbreviated as $\ln y$ throughout the paper) to detect the variance of its residuals. The decomposition of $\ln y$ is shown in Figure 3 below. The variance of the $\ln y$ residual seems remain constant with time. Therefore, it is more precise and better to generate models basing on both the original data and $\ln y$ then compare them.

The increasing trend in both Figure 2 and Figure 3 seems to begin anew twice from series' new locations with completely permanent effect. Thus, a random walk with drift is indicated. In both Figure, the range of seasonal component is much smaller than the range of residual. Therefore, the tiny effects from seasonal factors on the series can be ignore. In other words, there is no need to modeling seasonal analysis in the following procedures.

Figure 2

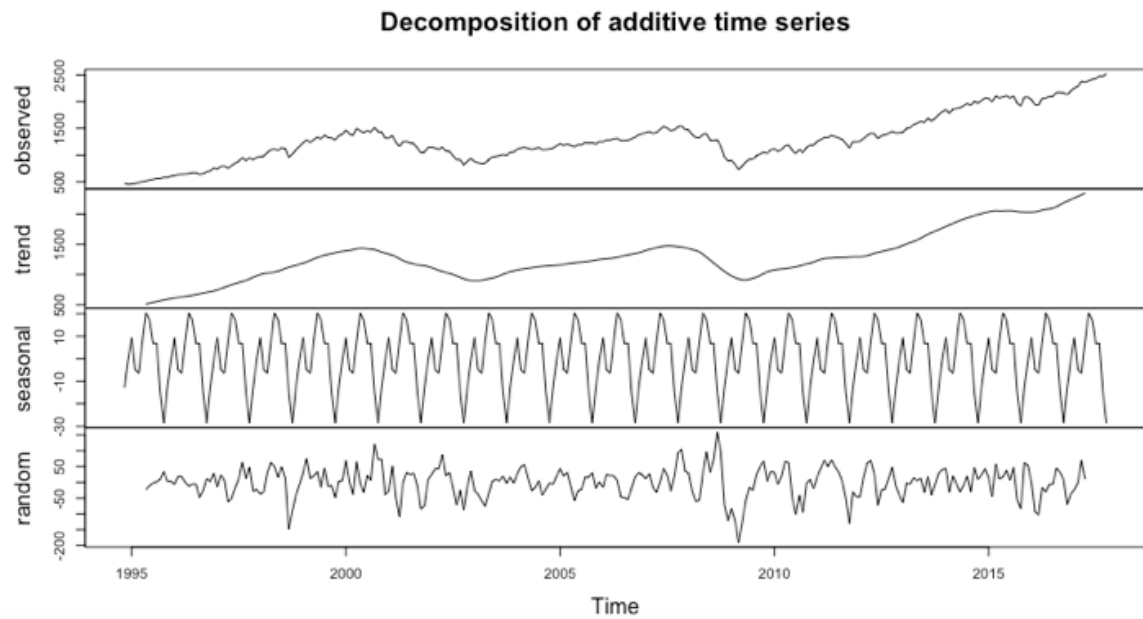
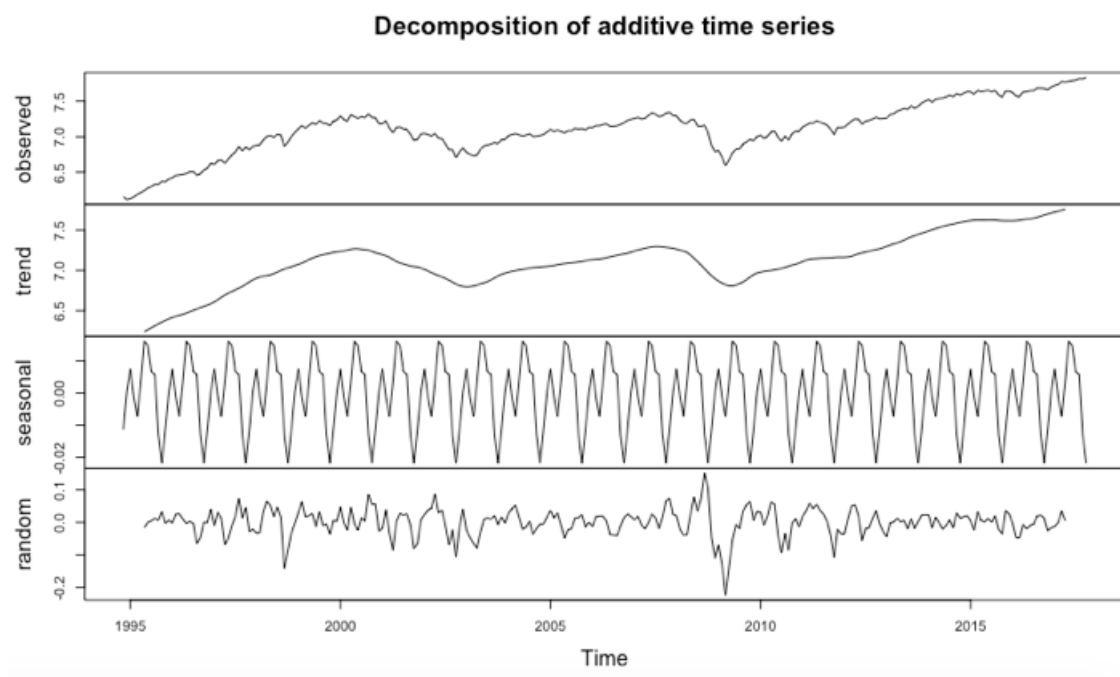


Figure 3



3.2 Stationarity Analysis

To proceed the analysis, unit root should be tested. It is necessary because the recessions in early 2000s and 2008 can either be the evidences of cycle which is autoregressive or the shocks with permanent impact in stochastic trend. The test result

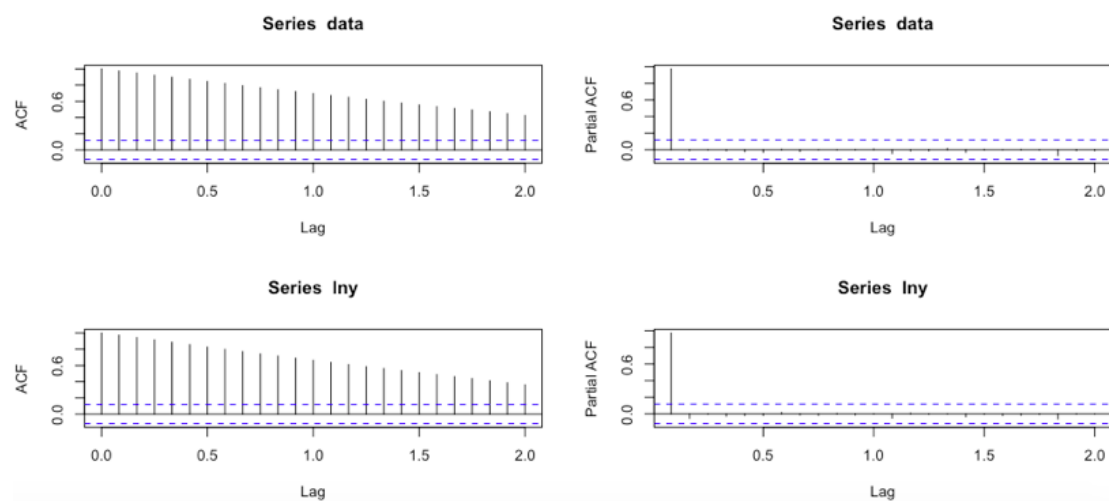
can show whether the data is stationary with deterministic trend which is forecastable and whether it is a random walk with linearly increasing forecast variances. The test results (no unit root or has a unit root) will directly determine the following used modeling methods (deterministic model or stochastic model).

To improve the test result, both formal and informal test methods would be used. When testing, the original data and $\ln y$ without de-seasonalized would be used because they are not impacted by seasonal factors.

3.2.1 Autocorrelation function

If the time series has unit root, the ACF (autocorrelation function) damps extremely slowly or fail to damp at all, besides, the PACF (partial autocorrelation function) is very close to 1 at lag 1 and damp quickly thereafter. The autocorrelation function for original data and $\ln y$ is shown in Figure 4. It suggests a unit root of both data and $\ln y$.

Figure 4

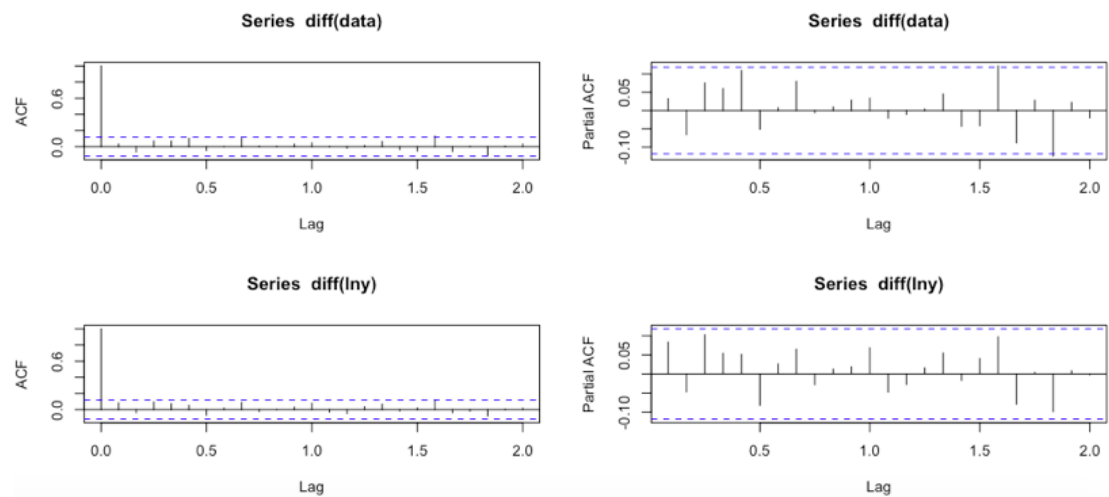


3.2.2 Autocorrelation function of differences

However, for stationary time series which owns high autocorrelation near to 1 may lead to slowly damping ACF and PACF with significance at lag 1 too. Therefore, ACF and PACF of differences should be checked.

For random walk with drift which takes the form: $y_t = \delta + y_{t-1} + \varepsilon_t$, the first difference is $y_t - y_{t-1} = \delta + \varepsilon_t$ which is not autocorrelated at all and should show insignificance on lag 1 in both ACF and PACF. For the stationary data with high autocorrelation less than 1, the first difference Δy_t consists ϕy_{t-1} (ϕ is not 0) and ϕy_{t-1} is correlated to ϕy_{t-2} included in Δy_{t-1} . Therefore, for stationary data, there is significance on lag 1 in ACF and PACF. The autocorrelation function of first difference of original data and $\ln y$ are shown in Figure 5. There is no significance on lag 1 and this is an indication of stochastic trend for both data and $\ln y$.

Figure 5



3.2.3 Augmented Dickey-Fuller test

Though the autocorrelation functions above suggest a unit root, the result is not convincing enough because it is possible to get the same result when the data is stationary. Therefore, the Augmented Dickey-Fuller test, which takes into account the possibility of wrong conclusion, is needed. It improves accuracy by setting a null hypothesis stating the data is non-stationary. In other words, the alternative hypothesis states the data is stationary with deterministic trend. The Augmented Dickey-Fuller test result is shown in Table 3. It shows $|t\text{-statistic}| < |critical\ value|$ at all significance levels. Thus, in both the data and $\ln y$ model, the null hypothesis cannot be rejected, indicating that a stochastic model should be used.

Table 3 Arguemented Dickey-Fuller test				
	t-statistic	1% significance level	5% significance level	10% significance level
data	-0.89	-3.98	-3.42	-3.13
lny	-2.1346	-3.98	-3.42	-3.13

3.2.4 Test drift with OLS

Figure 2 and Figure 3 shows that both data and lny has an increasing trend. Therefore, an OLS regression: $y_t = \delta + y_{t-1} + \varepsilon_t$ is needed to test the drift formally. Table 4 and Table 5 shows the regression result. In both figures it shows that the intercept is significant at all levels. Therefore, the time series is characterized by random walk with drift. The further model analysis will be conducted based on this conclusion.

Table 4 OLS Regression Result for Data			
coefficient	estimate	standard error	p-value and significance
intercept	-8.76E-13	5.52E-14	<2e-16 ***
first lag	1.00E+00	4.02E-17	<2e-16 ***

Table 5 OLS Regression Result for lny			
coefficient	estimate	standard error	p-value and significance
intercept	4.28E-15	2.55E-16	<2e-16 ***
first lag	1.00E+00	3.59E-17	<2e-16 ***

3.3 Stochastic model - ARIMA

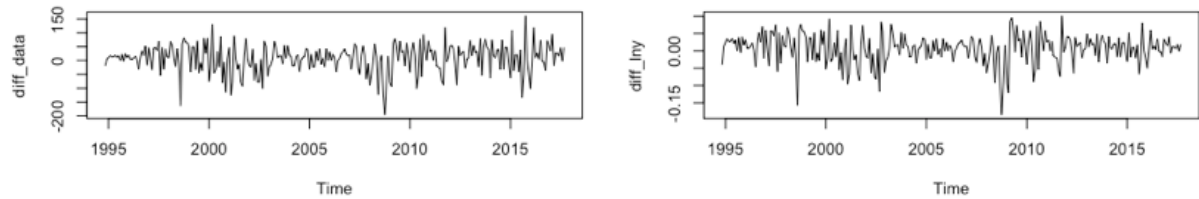
3.3.1 Determine ARIMA and difference

Autoregressive Integrated Moving Average (ARIMA) models include an explicit statistical model for the irregular component of a time series, that allows for non-zero autocorrelations in the irregular component, and it is defined for stationary time series (2016, Coghlan).

According to the stationarity analysis above, the data appears to be a random walk with drift which takes the form $y_t = \delta + y_{t-1} + \varepsilon_t$; $\varepsilon_t \sim WN(0, \sigma^2)$. In this form, y_t is not covariance stationary but the first difference in y_t and y_{t-1} are stationary component. Therefore, if the differences of the data and lny appear to be stationary, then ARIMA model can be used.

The first difference of data and lny can be seen in Figure 6. In Figure 6, it looks like white noise and its mean and variance seem to remain constant overtime, indicating its stationarity. Thus, the ARIMA model can be used when series is differenced 1 time.

Figure 6



3.3.2 Select Best ARIMA Model

The best ARIMA model would be selected according to the AIC (Akaike Information Criterion) which is effectively an estimate of forecast error variance. AIC takes into account both forecast risk and the number of estimated parameters. The smaller the AIC is, the better the model is. The reported AIC for data and lny can be found in Table 6 and Table 7. For data, the best model is ARIMA(0,1,0) with AIC=2950.128. For lny, the best model is ARIMA(0,1,1) with AIC=-948.933. Both model trends (Figure 7) fit well with the series despite of slight lag.

Table 6 AIC for Data					Table 7 AIC for lny				
	< MA(q)/AR(p) >					< MA(q)/AR(p) >			
	P=0	p=1	p=2	p=3		P=0	p=1	p=2	p=3
q=0	2950.128	2951.839	2952.648	2953.088	q=0	-948.778	-948.7101	-947.3094	-948.2126
q=1	2951.793	2951.897	2953.437	2952.518	q=1	-948.933	-948.7819	-946.8056	-947.1335
q=2	2952.558	2953.464	2955.427	2954.481	q=2	-947.7231	-946.8022	-946.02	-944.9731
q=3	2952.646	2952.45	2954.358	2956.273	q=3	-948.2837	-947.2739	-945.5159	-944.6548

Both residuals (Figure 8, Figure 9 and Table 8) of these two models are more or less normally distributed with mean around 0 and the Durbin-Watson statistic are near to 2, suggesting they are white noise. It seems two model are equally fit. However, based on AIC, stochastic model of lny is showed to bring less forecast error variance than stochastic model of original data.

Figure 7

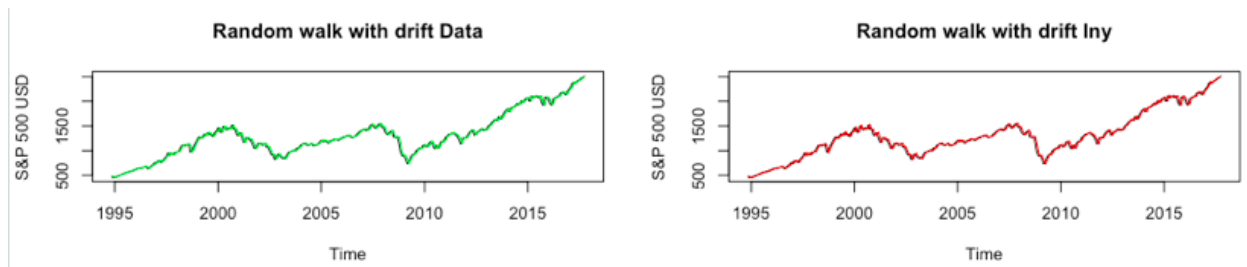


Figure 8

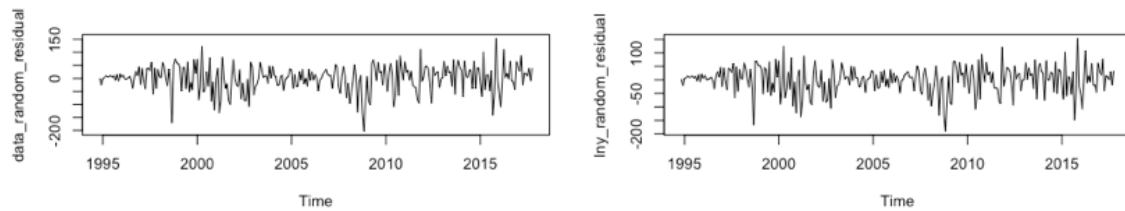


Figure 9

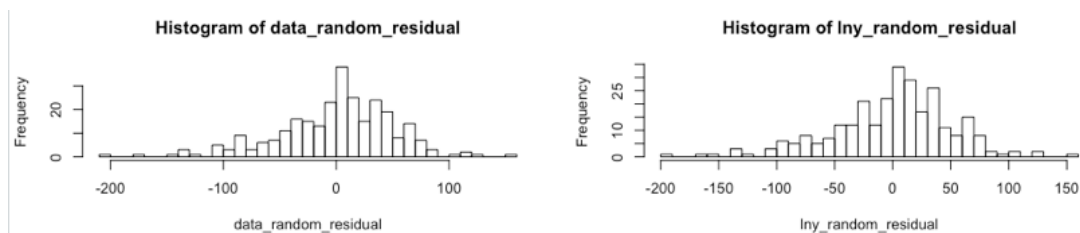
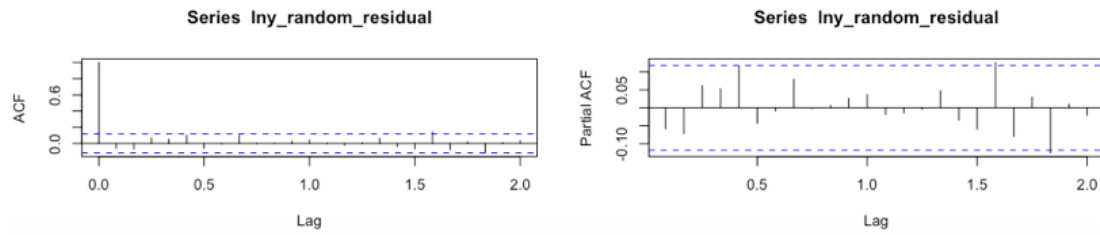


Table 8 Summary of Random Walk Residual				
	median	mean	standard deviation	Durbin-Watson statistic
data	5.89431	0.00168	51.29835	1.932985
lny	5.1317	-0.4792	51.42205	2.11401

Therefore, according to the comparison above, the ARIMA (0,1,1) of lny performs better for it has smaller AIC. The function of selected model ARIMA (0,1,1) is: $y_t = \delta + y_{t-1} + \theta \varepsilon_{t-1} + \varepsilon_t$. Besides, the ACF and PACF (Figure 10) shows no significance at first lag, indicating no more serial correlation left again. This shows that the data cannot be explain more and the ARIMA (0,1,1) is appropriate for forecast.

Figure 10



3.4 Exponential Smoothing

Because the series is random walk, its component may be changing over time. Thus, the exponential smoothing method, which weights the observed value unequally, may be effective for forecasting. Since both data and lny display an increasing trend according to Figure 2 and Figure 3, the Holt's Trend Corrected Exponential Model will be used. In this model, the point forecast made in period T for y_{T+h} is $\hat{y}_{T+h} = I_T + hb_T$. The estimate of growth rate is $b_T = \beta [I_T - I_{T-1}] + (1-\beta) b_{T-1}$ and the estimate of the level is $I_T = \alpha y_T + (1-\alpha) [I_{T-1} + b_{T-1}]$.

The smoothing model summary for data and lny are shown in Table 9. Alpha is close to 1, indicating that much weight is placed on the most recent observations when making forecast. Beta is close to 0, suggesting that the recent slope of trend component has little impact in forecast. Both Figure 11 and Figure 12 shows that the model fit the data well despite of slight lag.

Table 9 Smoothing Model Summary

	alpha	beta
data	1	0.04678917
lny	1	0.09867163

Figure 11

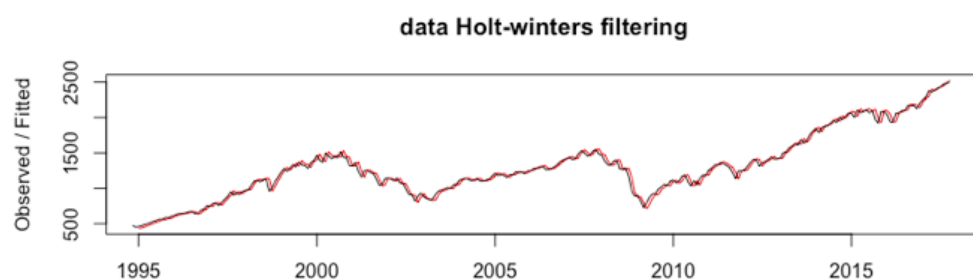
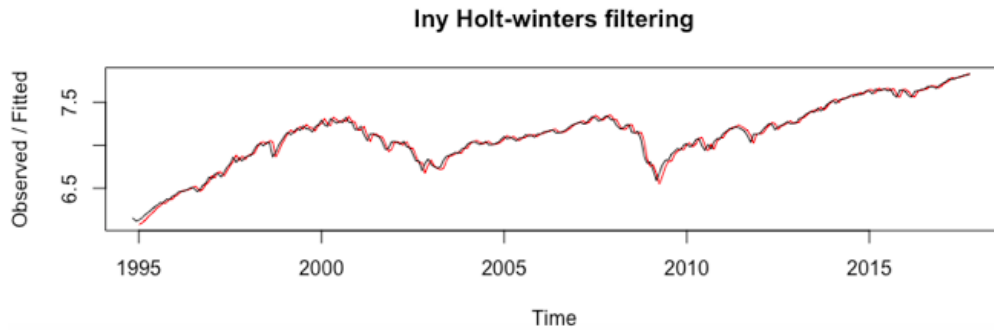


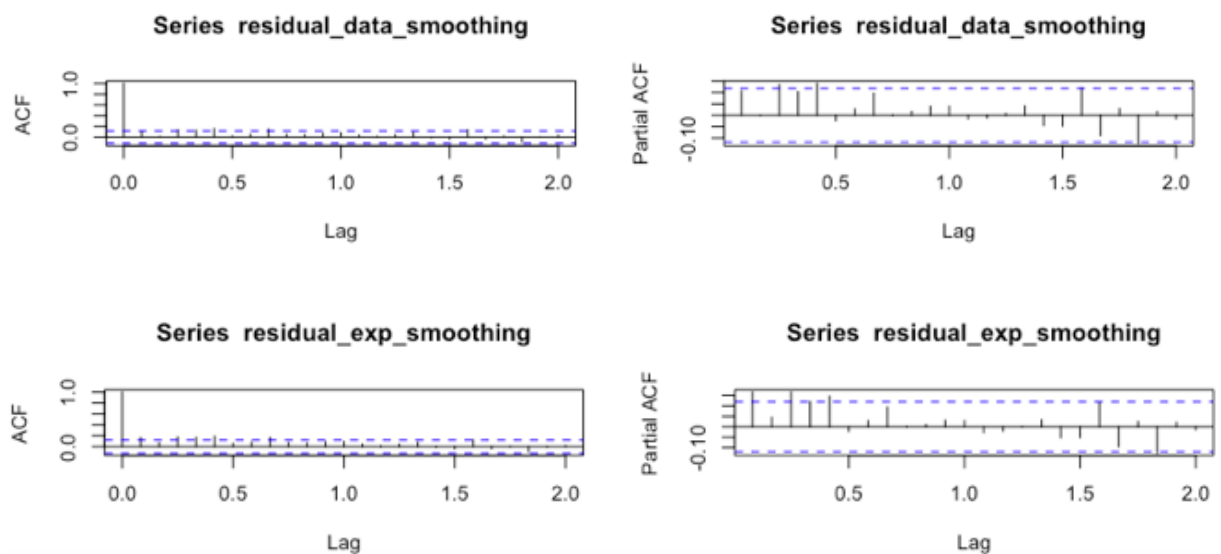
Figure 12



To select the better smoothing model, the Durbin-Watson statistic and residual summary should be checked. Table 10 shows that the smoothing model of data has a closer Durbin-Watson statistic than lny model and smaller standard deviation. Besides, in Figure 13 the PACF of the residual in data model shows no significance in first lag while in lny model the significance on first lag is shown. This indicates that data smoothing model has less autocorrelation in residual and explain the series better than the lny smoothing model. Therefore, data smoothing model should be more appropriate to forecast than lny smoothing model.

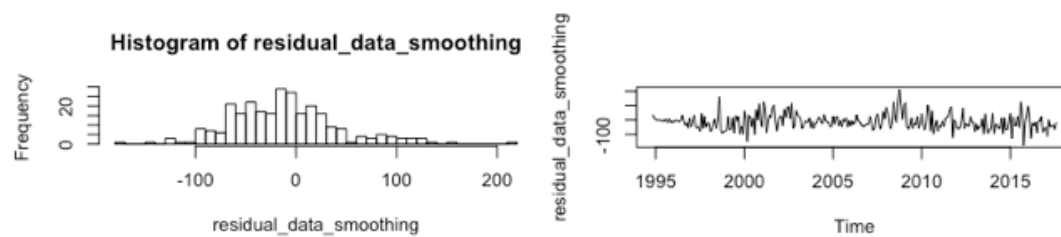
Table 10 Summary of Smoothing Model Residual				
	median	mean	standard deviation	Durbin-Watson statistic
data	-14.81	-11.87	54.86257	1.703803
lny	-18.21	-15	58.53137	1.564402

Figure 13



In Figure 14, the histogram of the residuals of data smoothing model are shown. It is more or less normally distributed with mean close to 0. In Figure 15, the plot of residuals looks like white noise with constant mean and variance over time. This suggests that under the data smoothing model the series cannot be explained furthermore and the model is fit for the series.

Figure 14 Figure 15



3.5 Comparing Smoothing Model and ARIMA

Since two models (random walk with drift model of lny or smoothing model of data) are both appropriate for forecast, to determine which model, in-sample forecast should be conducted. The model which has less in-sample forecast error should be chosen.

Because random walk model has linearly increasing variance and smoothing model can only produce short term forecast, the in-sample forecast period should be short. It is from 2017, November to 2018, October with 12 observation and the confident level is 95 percent. In Figure 15, Figure 16 and Table 12, the in-sample forecasts both under estimate the original data. However, the data move inside the boundaries, indicating both predictions are reliable and appropriate. Table 11 shows that the stochastic model has higher residual standard deviation at 2.7 than smoothing model. Therefore, data smoothing model is more appropriate in forecasting than stochastic model.

Figure 15

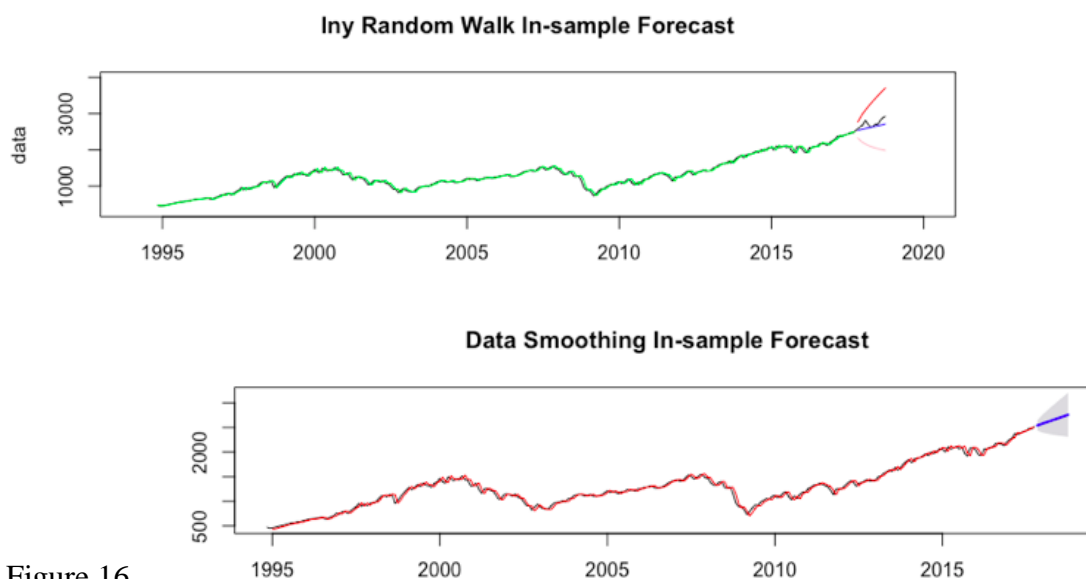


Figure 16

Table 11 Summary of Residuals of In-sample Forecast

	median	mean	standard deviation
stochastic	-101.344	-105.655	76.055
smoothing	-90.6	-84.25	73.25909
difference value			2.79591

Table 12 Smoothing Model In-sample Forecast (Difference=Forecast-Data)

data/month	11	12	1	2	3	4
difference	-43.42	-89.82	-112.87	-229.91	-112.9	-15.26
	5	6	7	8	9	10
difference	-9.48	-68.48	-38.56	-138.52	-197.95	-210.82

In conclusion, based on the argument above, the Holt's Trend Corrected Exponential Smoothing Model is the best model to forecast the series.

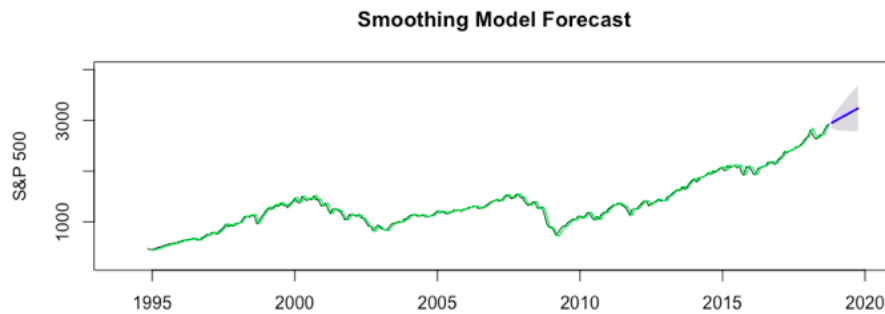
4. Out-of-sample Forecast

Holt's Trend Corrected Exponential Smoothing Model leads to less accuracy when the forecast is made further in the future, thus the forecast period should be short. Here, it is only one year ahead from 2018, November to 2019, October with 12 observations. The confidence intervals are 95%. Table 13 shows the summary of smoothing model for out-of-sample forecast, which is quite similar to the smoothing model made by data from 1994 November to 2018 October. Figure 17 shows the model trend almost fits the series completely in green line, despite of its slight lag. In 2018 Nov. to 2019 Oct., there is a linearly upward trend with growing forecast variance. Table 14 shows the forecast value from 2952 to 3235 during the forecast period.

Table 13 Summary of Smoothing Model for Out-of-sample Forecast

	alpha	beta
data	1	0.04528134

Figure 17

**Table 14 Forecast Value From 2018 Nov. to 2019 Oct.**

month	Nov-18	Dec-18	Jan-19	Feb-19	Mar-19	Apr-19
value	2952.066	2977.842	3003.618	3029.394	3055.17	3080.946
month	May-19	Jun-19	May-08	Aug-19	Sep-19	Oct-19
value	3106.722	3132.498	3158.274	3184.05	3209.826	3235.602

5. Conclusion

To find the best fitted model, at the beginning, unit root was tested to determine whether the data should use stochastic model. After the stationarity analysis, selection basing on AIC shows that ARIMA model with log form of data turned out to perform better than ARIMA model with original data. Then exponential smoothing method is used to estimate the data. Finally, smoothing model with data without log form turns to perform better than ARIMA model with smaller in-sample-forecast residual variance. The model fit the series very well despite of a slight lag and is appropriate for forecast.

However, there are some critics about it. First, the model can only forecast values in short term. Second, the predict variance is growing with less accuracy along the time. Third, this model cannot predict the shock which may have large impact on the series.

Generally speaking, even though the model has shortages, it still can give a hint about the future for investors and government.

Reference

S&P 500 (SPX)(2018). [online]Available at<<https://www.investing.com/indices/us-spx-500-historical-data>> (26/11/2018)

Avril Coghlan, (2016) A Little Book of R For Time Series *Release 0.2* . [online]Available at<https://learn.cbs.dk/pluginfile.php/914399/mod_resource/content/1/a-little-book-of-r-for-time-series.pdf> (26/11/2018)

Appendix

Augmented Dickey-Fuller test

data

```
#####  
# Augmented Dickey-Fuller Test Unit Root Test #  
#####  
  
Test regression trend  
  
Call:  
lm(formula = z.diff ~ z.lag.1 + 1 + tt + z.diff.lag)  
  
Residuals:  
      Min       1Q   Median       3Q      Max   
-201.778  -29.626   4.752   35.716  153.380  
  
Coefficients:  
              Estimate Std. Error t value Pr(>|t|)      
(Intercept)  9.71357    10.95035   0.887   0.376      
z.lag.1      -0.01079     0.01213  -0.890   0.374      
tt           0.08183     0.06694   1.222   0.223      
z.diff.lag   0.03733     0.06243   0.598   0.550      
  
Residual standard error: 52.44 on 261 degrees of freedom  
Multiple R-squared:  0.006756, Adjusted R-squared:  -0.004661  
F-statistic: 0.5918 on 3 and 261 DF,  p-value: 0.6209  
  
Value of test-statistic is: -0.89 2.1176 0.7515  
  
Critical values for test statistics:  
      1pct  5pct 10pct  
tau3 -3.98 -3.42 -3.13  
phi2  6.15  4.71  4.05  
phi3  8.34  6.30  5.36
```

lny

```
#####
# Augmented Dickey-Fuller Test Unit Root Test #
#####

Test regression trend

Call:
lm(formula = z.diff ~ z.lag.1 + 1 + tt + z.diff.lag)

Residuals:
    Min       1Q   Median       3Q      Max
-0.184049 -0.021810  0.003836  0.026932  0.097543

Coefficients:
              Estimate Std. Error t value Pr(>|t|)
(Intercept)  1.929e-01  8.750e-02   2.205   0.0283 *
z.lag.1      -2.791e-02  1.308e-02  -2.135   0.0337 *
tt           8.018e-05  5.437e-05   1.475   0.1415
z.diff.lag   9.055e-02  6.143e-02   1.474   0.1417
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.04315 on 261 degrees of freedom
Multiple R-squared:  0.02357,    Adjusted R-squared:  0.01235
F-statistic: 2.1 on 3 and 261 DF,  p-value: 0.1006

Value of test-statistic is: -2.1346 2.8154 2.3138

Critical values for test statistics:
      1pct  5pct 10pct
tau3 -3.98 -3.42 -3.13
phi2  6.15  4.71  4.05
phi3  8.34  6.30  5.36
```

OLS regression test

lny

```
Call:
lm(formula = lny ~ lag(lny))

Residuals:
    Min       1Q   Median       3Q      Max
-3.140e-16 -2.380e-17 -1.380e-17  6.100e-18  3.472e-15

Coefficients:
              Estimate Std. Error  t value Pr(>|t|)
(Intercept)  4.277e-15  2.553e-16  1.675e+01  <2e-16 ***
lag(lny)     1.000e+00  3.587e-17  2.788e+16  <2e-16 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 2.143e-16 on 274 degrees of freedom
Multiple R-squared:  1,    Adjusted R-squared:  1
F-statistic: 7.771e+32 on 1 and 274 DF,  p-value: < 2.2e-16
```

data

```
Call:
lm(formula = lny ~ lag(lny))

Residuals:
    Min       1Q   Median       3Q      Max
-3.140e-16 -2.380e-17 -1.380e-17  6.100e-18  3.472e-15

Coefficients:
            Estimate Std. Error  t value Pr(>|t|)
(Intercept) 4.277e-15  2.553e-16  1.675e+01  <2e-16 ***
lag(lny)     1.000e+00  3.587e-17  2.788e+16  <2e-16 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 2.143e-16 on 274 degrees of freedom
Multiple R-squared:  1,    Adjusted R-squared:  1
F-statistic: 7.771e+32 on 1 and 274 DF,  p-value: < 2.2e-16
```

ARIMA model

data

```
[1] 0.000 0.000 2950.128
[1] 0.000 1.000 2951.793
[1] 0.000 2.000 2952.558
[1] 0.000 3.000 2952.646
[1] 1.000 0.000 2951.839
[1] 1.000 1.000 2951.897
[1] 1.000 2.000 2953.464
[1] 1.00 3.00 2952.45
[1] 2.000 0.000 2952.648
[1] 2.000 1.000 2953.437
[1] 2.000 2.000 2955.427
[1] 2.000 3.000 2954.358
[1] 3.000 0.000 2953.088
[1] 3.000 1.000 2952.518
[1] 3.000 2.000 2954.481
[1] 3.000 3.000 2956.273
> best.order1
[1] 0 1 0
```

```
Call:
arima(x = data, order = best.order1, xreg = 1:T, optim.control = list(maxit = 1000))

Coefficients:
      1:T
      7.4507
s.e.  3.0934

sigma^2 estimated as 2632:  log likelihood = -1473.06,  aic = 2950.13
```

lny

```

[1] 0.000 0.000 -948.778
[1] 0.0000 1.0000 -948.9333
[1] 0.0000 2.0000 -947.7231
[1] 0.0000 3.0000 -948.2837
[1] 1.0000 0.0000 -948.7101
[1] 1.0000 1.0000 -948.7819
[1] 1.0000 2.0000 -946.8022
[1] 1.0000 3.0000 -947.2739
[1] 2.0000 0.0000 -947.3094
[1] 2.0000 1.0000 -946.8056
[1] 2.00 2.00 -946.02
[1] 2.0000 3.0000 -945.5159
[1] 3.0000 0.0000 -948.2126
[1] 3.0000 1.0000 -947.1335
[1] 3.0000 2.0000 -944.9731
[1] 3.0000 3.0000 -944.6548
> best.order5
[1] 0 1 1

```

```

Call:
arima(x = lny, order = best.order5, xreg = 1:T, optim.control = list(maxit = 1000))

```

```

Coefficients:
      ma1      1:T
    0.0942  0.0061
s.e.  0.0643  0.0028

```

```

sigma^2 estimated as 0.001817: log likelihood = 477.47, aic = -948.93

```

```

Call:
arima(x = lny, order = best.order5, xreg = 1:T, optim.control = list(maxit = 1000))

```

```

Coefficients:
      ma1      1:T
    0.0942  0.0061
s.e.  0.0643  0.0028

```

```

sigma^2 estimated as 0.001817: log likelihood = 477.47, aic = -948.93

```

```

Training set error measures:
              ME      RMSE      MAE      MPE      MAPE      MASE      ACF1
Training set 3.640026e-05 0.04255431 0.03176913 0.0003157817 0.4513406 0.9570339 -0.005898346

```

Exponential smoothing

data

Holt-Winters exponential smoothing with trend and without seasonal component.

```
Call:
HoltWinters(x = data, gamma = FALSE)
```

```
Smoothing parameters:
alpha: 1
beta : 0.04678917
gamma: FALSE
```

```
Coefficients:
      [,1]
a 2521.20000
b   19.53117
> summary(data_smoothing_model)
      Length Class  Mode
fitted      822   mts   numeric
x           276   ts    numeric
alpha        1  -none- numeric
beta         1  -none- numeric
gamma         1  -none- logical
coefficients 2  -none- numeric
seasonal      1  -none- character
SSE           1  -none- numeric
call         3  -none- call
```

lny

Holt-Winters exponential smoothing with trend and without seasonal component.

```
Call:
HoltWinters(x = lny, gamma = FALSE)
```

```
Smoothing parameters:
alpha: 1
beta : 0.09867163
gamma: FALSE
```

```
Coefficients:
      [,1]
a  7.83249026
b  0.01045315
> summary(lny_smoothing_model)
      Length Class  Mode
fitted      822   mts   numeric
x           276   ts    numeric
alpha        1  -none- numeric
beta         1  -none- numeric
gamma         1  -none- logical
coefficients 2  -none- numeric
seasonal      1  -none- character
SSE           1  -none- numeric
call         3  -none- call
```

Out-of-sample Forecast

Holt-Winters exponential smoothing with trend and without seasonal component.

Call:

```
HoltWinters(x = a, gamma = FALSE)
```

Smoothing parameters:

alpha: 1

beta : 0.04528134

gamma: FALSE

Coefficients:

[,1]

a 2926.29000

b 25.77598

```
> summary(original_model)
```

	Length	Class	Mode
fitted	858	mts	numeric
x	288	ts	numeric
alpha	1	-none-	numeric
beta	1	-none-	numeric
gamma	1	-none-	logical
coefficients	2	-none-	numeric
seasonal	1	-none-	character
SSE	1	-none-	numeric
call	3	-none-	call