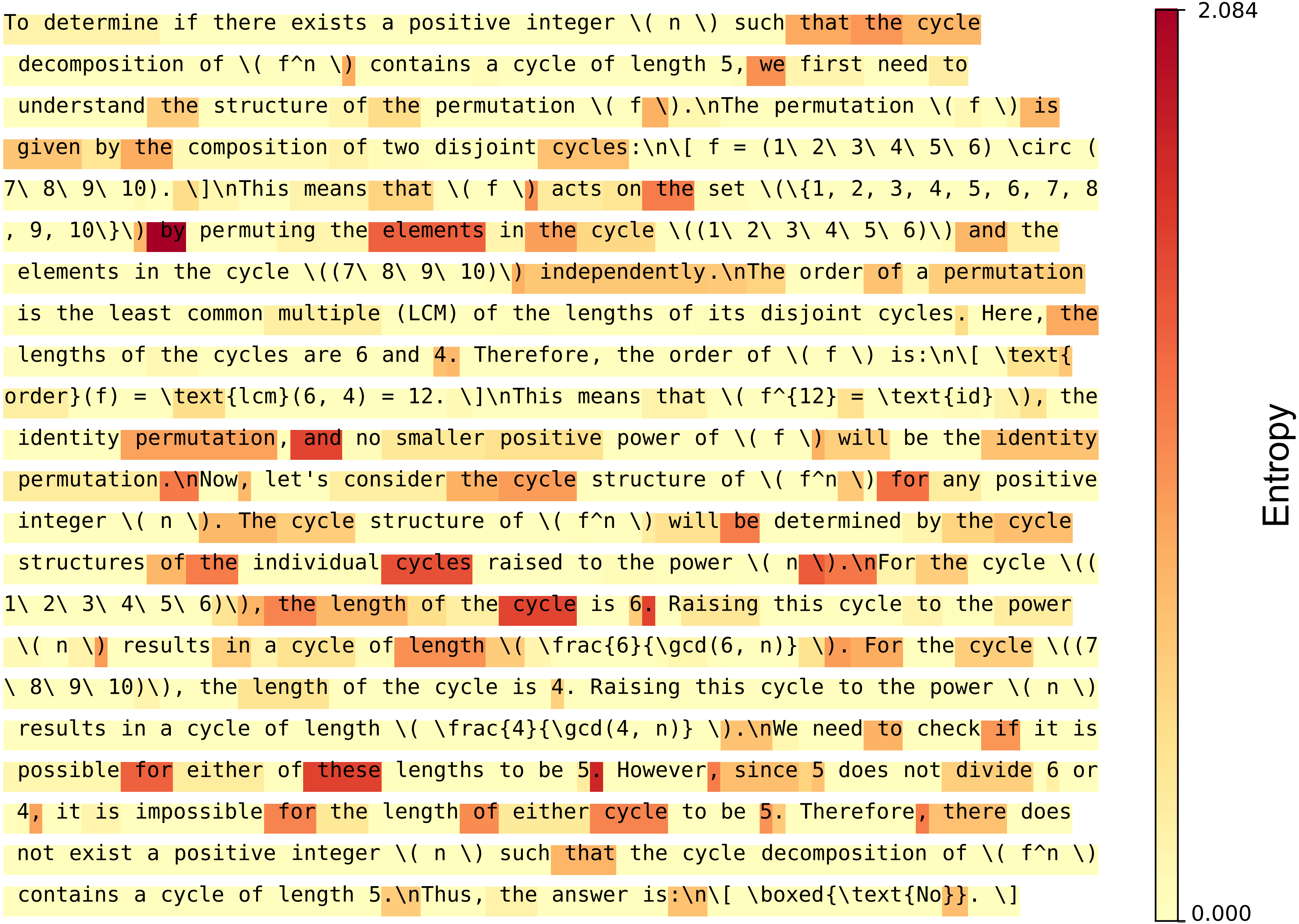
To determine if there exists a positive integer \(n \) such that the cycle
decomposition of \(f^n \) contains a cycle of length 5, we first need to
understand the structure of the permutation \(f \).\nThe permutation \(f \) is
given by the composition of two disjoint cycles:\n\[f = (1\ 2\ 3\ 4\ 5\ 6) \circ (
7\ 8\ 9\ 10). \]\nThis means that \(f \) acts on the set \(\{1, 2, 3, 4, 5, 6, 7, 8
, 9, 10\}\) by permuting the elements in the cycle \((1\ 2\ 3\ 4\ 5\ 6)\) and the
elements in the cycle \((7\ 8\ 9\ 10)\) independently.\nThe order of a permutation
is the least common multiple (LCM) of the lengths of its disjoint cycles. Here, the
lengths of the cycles are 6 and 4. Therefore, the order of \(f \) is:\n\[\text{
order}(f) = \text{lcm}(6, 4) = 12. \]\nThis means that \(f^{12} = \text{id} \), the
identity permutation, and no smaller positive power of \(f \) will be the identity
permutation.\nNow, let's consider the cycle structure of \(f^n \) for any positive
integer \(n \). The cycle structure of \(f^n \) will be determined by the cycle
structures of the individual cycles raised to the power \(n \).\nFor the cycle \((
1\ 2\ 3\ 4\ 5\ 6)\), the length of the cycle is 6. Raising this cycle to the power
\(n \) results in a cycle of length \(\frac{6}{\gcd(6, n)} \). For the cycle \((7
\ 8\ 9\ 10)\), the length of the cycle is 4. Raising this cycle to the power \(n \)
results in a cycle of length \(\frac{4}{\gcd(4, n)} \).\nWe need to check if it is
possible for either of these lengths to be 5. However, since 5 does not divide 6 or
4, it is impossible for the length of either cycle to be 5. Therefore, there does
not exist a positive integer \(n \) such that the cycle decomposition of \(f^n \)
contains a cycle of length 5.\nThus, the answer is:\n\[\boxed{\text{No}}. \]
2.084
0.000
Entropy