

To determine if there exists a positive integer (n) such that the cycle decomposition of (f^n) contains a cycle of length 5, we first need to understand the structure of the permutation (f) . The permutation (f) is given by the composition of two disjoint cycles: $f = (1\ 2\ 3\ 4\ 5\ 6) \circ (7\ 8\ 9\ 10)$. This means that (f) acts on the set $\{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$ by permuting the elements in the cycle $(1\ 2\ 3\ 4\ 5\ 6)$ and the elements in the cycle $(7\ 8\ 9\ 10)$ independently. The order of a permutation is the least common multiple (LCM) of the lengths of its disjoint cycles. Here, the lengths of the cycles are 6 and 4. Therefore, the order of (f) is: $\text{order}(f) = \text{lcm}(6, 4) = 12$. This means that $(f^{12}) = \text{id}$, the identity permutation, and no smaller positive power of (f) will be the identity permutation. Now, let's consider the cycle structure of (f^n) for any positive integer (n) . The cycle structure of (f^n) will be determined by the cycle structures of the individual cycles raised to the power (n) . For the cycle $(1\ 2\ 3\ 4\ 5\ 6)$, the length of the cycle is 6. Raising this cycle to the power (n) results in a cycle of length $(\frac{6}{\gcd(6, n)})$. For the cycle $(7\ 8\ 9\ 10)$, the length of the cycle is 4. Raising this cycle to the power (n) results in a cycle of length $(\frac{4}{\gcd(4, n)})$. We need to check if it is possible for either of these lengths to be 5. However, since 5 does not divide 6 or 4, it is impossible for the length of either cycle to be 5. Therefore, there does not exist a positive integer (n) such that the cycle decomposition of (f^n) contains a cycle of length 5. Thus, the answer is: $\boxed{\text{No}}$.

