Free final state problem

given $s_i(T), i = I$

we have boundary conditions for other state:

$$\lambda_j(T) = \frac{\partial h(s^*(T))}{\partial s_j}, \text{ for } i \neq j$$

Solution:

Suppost terminal function $h(s(T)) = 0$

then we have

① from boundary fixed condition $\frac{T^5}{120}\alpha + \frac{T^4}{24}\beta + \frac{T^3}{6}\gamma = \Delta p = p_f - p_0 - v_0 T - \frac{1}{2}a_0$

② $\lambda_2 = \frac{\partial h(s^*(T))}{\partial s_2} = 2\alpha T + 2\beta = 0 \Rightarrow \beta = -\alpha T$

③ $\lambda_3 = \text{''} = -\alpha T^2 \cancel{2}\beta t - 2\gamma = 0 \Rightarrow \gamma = \frac{\alpha}{2}T^2$

put ② and ③ into ①, get $\frac{\alpha}{20}T^5 = \Delta p$

$$\therefore \Rightarrow \begin{cases} \alpha = \frac{20}{T^5}\Delta p \\ \beta = -\frac{20}{T^4}\Delta p \\ \gamma = \frac{10}{T^3}\Delta p \end{cases} \quad ④$$

put ④ into function of J, where $J = \gamma^2 + \beta\gamma T + \frac{1}{3}\beta^2 T^2 + \frac{1}{3}\alpha\gamma T^2 + \frac{1}{4}\alpha\beta T^3 + \frac{1}{20}\alpha^2 T^4$

$$= \frac{\Delta p^2}{T6}(-80 + 200 - 100)$$

$$= \frac{20}{T6}(\Delta p^2)$$

$$= \frac{20}{T6}\left(p_f - p_0 - v_0 T - \frac{1}{2}a_0 T^2\right)^2$$

let $\frac{dJ}{dT} = 0$ we get 4 solutions, judge by plot of the function

the optimal $T^* = -\frac{2v_0 - \sqrt{2}\sqrt{2v_0^2 - 3a_0 p_0 + 3a_0 p_f}}{a_0}$