Q2: Build an ego-graph of the linear modelled robot. Select the best trajectory with the model and cost function enclosed.

(1) Integration

$$p_{iT} = p_{i0} + v_{i0} T + \frac{1}{2} a_i T^2$$

$$v_{iT} = v_{i0} + a_i T$$

$$i = x, y, z$$

(2) Optimal solution

The solution process is shown in the enclosed ‘chap4_2.mlx’.
First, get the cost function with single variable T, and known parameters P0, Pt, V0.
Then get the first order differential of dJ(T)/dT = 0.
The result is a 4th order polynomial equation.
I select to solve it by computing the eigenvalue of the companion matrix. The algorithm is shown here:

4 Companion matrices

Finding **roots of polynomials is *equivalent* to finding eigenvalues**. Not only can you find eigenvalues by solving for the roots of the characteristic polynomial, but you can conversely find roots of *any* polynomial by turning into a matrix and finding the eigenvalues.

Given the degree-n polynomial:

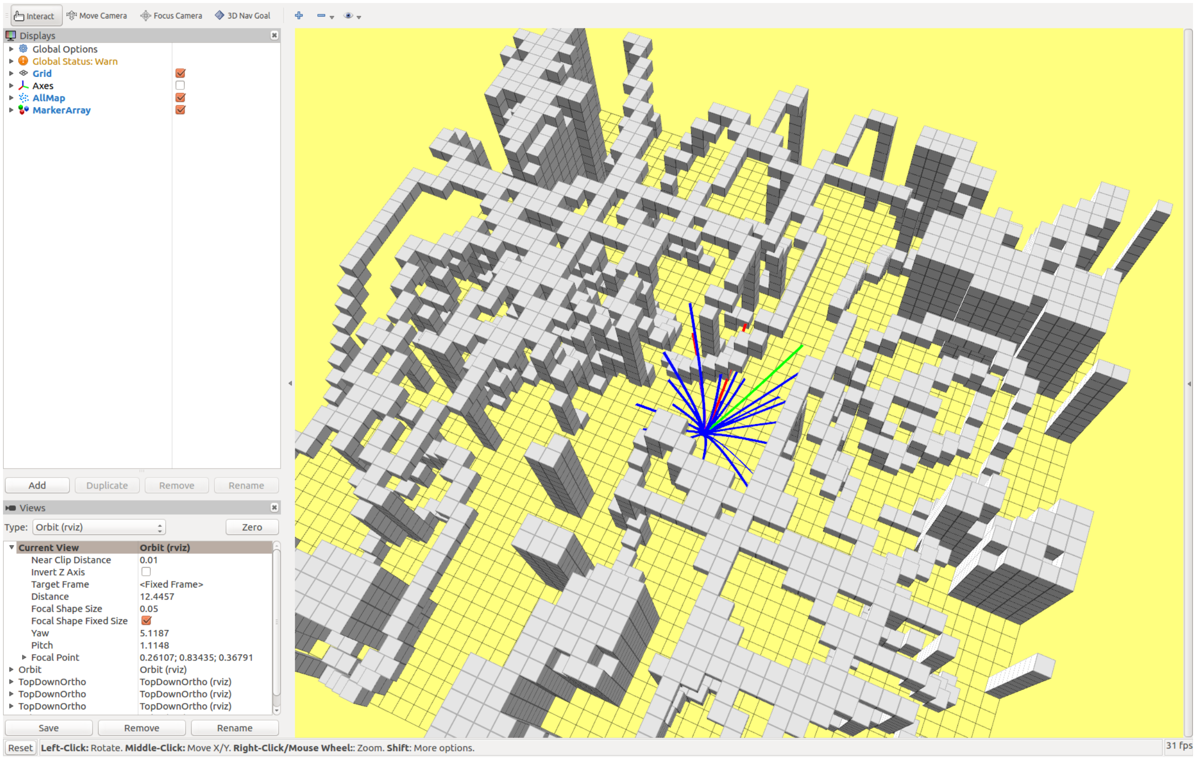
$$p(z) = c_0 + c_1 z + \cdots + c_{n-1} z^{n-1} + z^n \ ,$$

(notice that the z^n coefficient is 1), we define the $n \times n$ **companion matrix**

$$C = \begin{pmatrix} 0 & 1 & 0 & \dots & 0 \\ 0 & 0 & 1 & \dots & 0 \\ 0 & \ddots & \ddots & \ddots & \vdots \\ \vdots & \vdots & \ddots & 0 & 1 \\ -c_0 & -c_1 & \dots & -c_{n-2} & -c_{n-1} \end{pmatrix} .$$

The amazing fact is that the *characteristic polynomial* $\det(C - \lambda I) = p(\lambda)$, and so the **eigenvalues of C are the roots of p**.

and the excurate result is here:


Interact
Move Camera
Focus Camera
3D Nav Goal
Displays
Global Options
Global Status: Warn
Grid
Axes
AllMap
MarkerArray
Add
Duplicate
Remove
Rename
Views
Type: Orbit (rviz)
Zero
Current View Orbit (rviz)
Near Clip Distance 0.01
Invert Z Axis
Target Frame <Fixed Frame>
Distance 12.4457
Focal Shape Size 0.05
Focal Shape Fixed Size
Yaw 5.1187
Pitch 1.1148
Focal Point 0.26107; 0.83435; 0.36791
Orbit Orbit (rviz)
TopDownOrtho TopDownOrtho (rviz)
TopDownOrtho TopDownOrtho (rviz)
TopDownOrtho TopDownOrtho (rviz)
Save
Remove
Rename
Reset
Left-Click: Rotate. Middle-Click: Move X/Y. Right-Click/Mouse Wheel:: Zoom. Shift: More options.
31 fps