Bicycle Model

Josh Bennett, jjbennet@andrew.cmu.edu

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1 Derivation

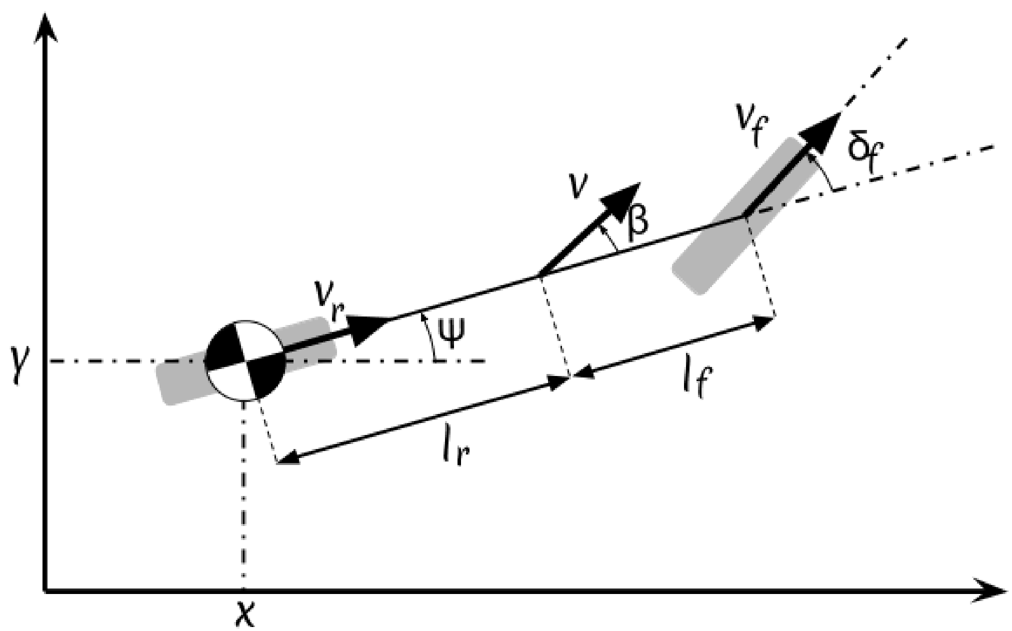

Figure 1: Bicycle model state variable diagram.

Some preliminary relationships:

State variables:

$$z = \begin{bmatrix} x \\ y \\ \psi \\ \delta_f \\ v_f \end{bmatrix}$$

We will use v_f as the state variable for velocity to avoid discontinuities.

$$v_r = v_f \cos(\delta_f)$$

$$\dot{\psi} = v_f \sin(\delta_f)$$

We will assume a front wheel drive vehicle with wheel torque τ and wheel radius r, generating a force of $\frac{\tau}{r}$ as an input. Assuming a mass m and an inertia I for the vehicle defined as the inertia at the wheel base, not the center of mass, we can derive the following equation for the acceleration of the front wheel:

$$a_f = \frac{\tau}{r}\left(\frac{1}{m\cos(\delta_f)} + \frac{((l_r + l_f)\sin(\delta_f))^2}{I}\right)$$

We will also assume that the steering angle is controlled by some steering rate input w

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\psi} \\ \dot{\delta_f} \\ \dot{v_f} \end{bmatrix} = \begin{bmatrix} v_f \cos(\delta_f)\cos(\psi) \\ v_f \cos(\delta_f)\sin(\psi) \\ v_f \sin(\delta_f) \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ w \\ a_f \end{bmatrix}$$