

# Bicycle Model

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## 1 Derivation

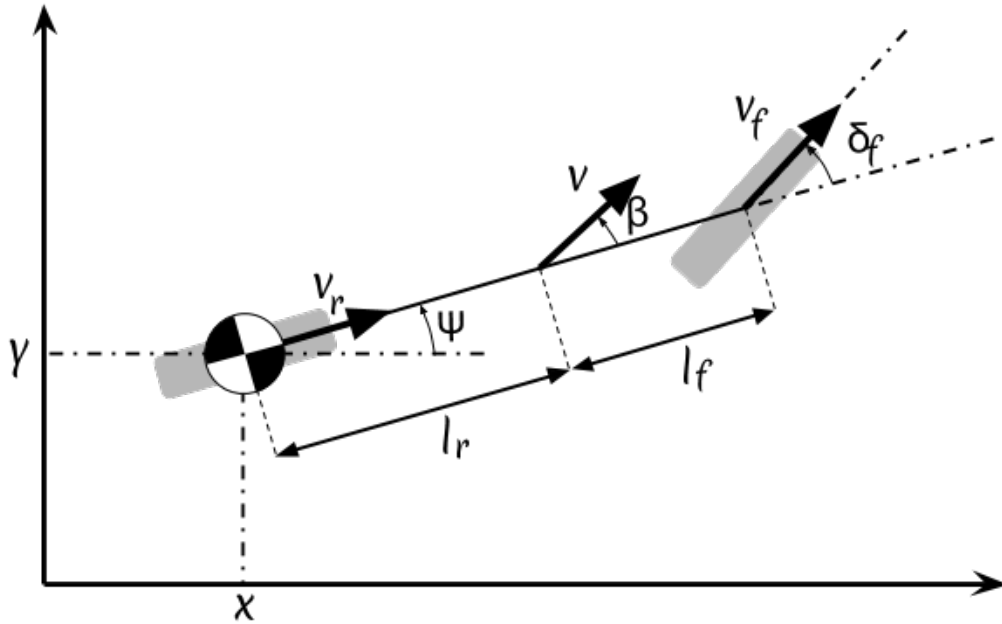


Figure 1: Bicycle model state variable diagram.

Some preliminary relationships:

State variables:

$$z = \begin{bmatrix} x \\ y \\ \psi \\ \delta_f \\ v_f \end{bmatrix}$$

We will use  $v_f$  as the state variable for velocity to avoid discontinuities.

$$v_r = v_f \cos(\delta_f)$$

$$\dot{\psi} = v_f \sin(\delta_f)$$

We will assume a front wheel drive vehicle with wheel torque  $\tau$  and wheel radius  $r$ , generating a force of  $\frac{\tau}{r}$  as an input. Assuming a mass  $m$  and an inertia  $I$  for the vehicle defined as the inertia at the wheel base, not the center of mass, we can derive the following equation for the acceleration of the front wheel:

$$a_f = \frac{\tau}{r} \left( \frac{1}{m \cos(\delta_f)} + \frac{((l_r + l_f) \sin(\delta_f))^2}{I} \right)$$

We will also assume that the steering angle is controlled by some steering rate input  $w$

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\psi} \\ \dot{\delta}_f \\ \dot{v}_f \end{bmatrix} = \begin{bmatrix} v_f \cos(\delta_f) \cos(\psi) \\ v_f \cos(\delta_f) \sin(\psi) \\ \frac{v_f \sin(\delta_f)}{l_r + l_f} \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ w \\ a_f \end{bmatrix}$$