

Theory of Computation

Exercise 1: (Mathematic preliminary, Language, String)

1. Let $\Sigma = \{a, b\}$ and $L = \{aa, bb\}$. Describe \bar{L} by a set notation.

1. $\bar{L} = \{a, b\}^* - \{aa, bb\}$

2. $\bar{L} = \{w \mid |w| \geq 3\} \cup \{\lambda, a, b, ab, ba\}$

2. Find five strings which are in each of the following languages.

a) $L = \{w \in \{a\}^*: |w| \bmod 3 \neq |w| \bmod 2\}$

Handwritten notes at the top: $1 \bmod 2 = 1$, $7 \bmod 2 = 1$, $9 \bmod 4 = 1$

2. Find five strings which are in each of the following languages.

a) $L = \{w \in \{a\}^* : |w| \bmod 3 \neq |w| \bmod 2\}$

Handwritten analysis for part a):

$L = \{ \cancel{a}, \cancel{aa}, aa, aaa, aaaa, aaaaa, \cancel{aaaaa}, \cancel{aaaaaa}, a^8 \}$

Modulo calculations for each string:

- \cancel{a} : $1 \bmod 3 = 1, 1 \bmod 2 = 1$ (not in L)
- \cancel{aa} : $2 \bmod 3 = 2, 2 \bmod 2 = 0$ (not in L)
- aa : $2 \bmod 3 = 2, 2 \bmod 2 = 0$ (not in L)
- aaa : $3 \bmod 3 = 0, 3 \bmod 2 = 1$ (in L)
- $aaaa$: $4 \bmod 3 = 1, 4 \bmod 2 = 0$ (in L)
- $aaaaa$: $5 \bmod 3 = 2, 5 \bmod 2 = 1$ (in L)
- \cancel{aaaaa} : $6 \bmod 3 = 0, 6 \bmod 2 = 0$ (not in L)
- \cancel{aaaaaa} : $7 \bmod 3 = 1, 7 \bmod 2 = 1$ (not in L)
- a^8 : $8 \bmod 3 = 2, 8 \bmod 2 = 0$ (in L)

b) $L = \{w \in \{a, b\}^*: n_a(w) \geq n_b(w) + 1\}$

Where $n_a(w)$ means the number of a's in string w.

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$$L = \{a, aa, aaa, aab, aba\}$$