Assignment 1;
due Friday November 7

Part 1

Exercise 1
Solution

The system is the closed kinematic chain.

(a) False

(b) False

(c) False

(d) True.

Exercise 2
Solution

(a) FALSE. SCARA robots can consist only of 4 rotational axes

(b) TRUE. 3 rotational axes can give us 3 DoF. The forth one cannot add one more.

(c) FALSE. The Chebyshev linkage has 4 rotational joints and only one DoF.

(d) FALSE. The explanation is like in previous example.

Exercise 3
Solution

(a) TRUE. By definition.

(b) FALSE. By definition.

(c) FALSE. Because of b)

(d) FALSE. Because of a)

Exercise 4
Solution

(a) FALSE. We can choose coordinates frames in the end effector.

(b) TRUE. We can choose coordinates frames in the end effector, so it could be many matrices. If there is only one base frame there is only one DH matrix.

(c) FALSE. Because we can change direction of X and Z axes. However, multiplication of all DH matrices will give us translation from base frame to the frame in the end effector and it will be unique.

(d) FALSE. The same explanation as in c)

Part 2

Exercise 1
Solution

DH :

	a_{i-1}	α_{i-1}	d_i	θ_i
1	0	0	0	θ_1
2	0	$-\frac{\pi}{2}$	0	θ_2
3	20	0	0	0

$$ {}^0_1T = \begin{pmatrix} cos(\theta_1) & -sin(\theta_1) & 0 & 0 \\ sin(\theta_1) & cos(\theta_1) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} $$

$$ {}^1_2T = \begin{pmatrix} cos(\theta_2) & -sin(\theta_2) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -sin(\theta_2) & -cos(\theta_2) & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} $$

$$ {}^2_3T = \begin{pmatrix} 1 & 0 & 0 & 20 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} $$

$$ {}^0_3T = {}^0_1T{}^1_2T{}^2_3T = \begin{pmatrix} cos(\theta_1)cos(\theta_2) & -cos(\theta_1)sin(\theta_2) & -sin(\theta_1) & 20cos(\theta_1)cos(\theta_2) \\ sin(\theta_1)cos(\theta_2) & -sin(\theta_1)sin(\theta_2) & cos(\theta_1) & 20sin(\theta_1)cos(\theta_2) \\ -sin(\theta_2) & -cos(\theta_2) & 0 & -20sin(\theta_2) \\ 0 & 0 & 0 & 1 \end{pmatrix} $$

Exercise 2
Solution

(a) DH :

	a_{i-1}	α_{i-1}	d_i	θ_i
1	0	0	0	θ_1
2	-0.3	$-\frac{\pi}{2}$	0	θ_2
3	1	0	0	θ_3
4	0	$\frac{\pi}{2}$	0.2	θ_4
5	1.5	0	0	θ_5
6	0	$-\frac{\pi}{2}$	0	θ_6

$$ {}^0_1T = \begin{pmatrix} cos(\theta_1) & -sin(\theta_1) & 0 & 0 \\ sin(\theta_1) & cos(\theta_1) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} $$

$$ {}^1_2T = \begin{pmatrix} cos(\theta_2) & -sin(\theta_2) & 0 & -0.3 \\ 0 & 0 & 1 & 0 \\ -sin(\theta_2) & -cos(\theta_2) & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} $$

$$ {}^2_3T = \begin{pmatrix} cos(\theta_3) & -sin(\theta_3) & 0 & 1 \\ sin(\theta_3) & cos(\theta_3) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} $$

$$
{}^3_4T = \begin{pmatrix} cos(\theta_4) & -sin(\theta_4) & 0 & 0 \\ 0 & 0 & -1 & -0.2 \\ sin(\theta_4) & cos(\theta_4) & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}
$$

$$
{}^4_5T = \begin{pmatrix} cos(\theta_5) & -sin(\theta_5) & 0 & 1.5 \\ sin(\theta_5) & cos(\theta_5) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}
$$

$$
{}^5_6T = \begin{pmatrix} cos(\theta_6) & -sin(\theta_6) & 0 & 0 \\ 0 & 0 & -1 & 0 \\ -sin(\theta_6) & -cos(\theta_6) & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}
$$

$$
{}^0_6T = {}^0_1T{}^1_2T{}^2_3T{}^3_4T{}^4_5T{}^5_6T =
$$

$$
\begin{pmatrix} -C_6S_1S_{4,5} + C_1(C_{2,3}C_{4,5}C_6 - S_{2,3}S_6) & S_1S_{4,5}S_6 - C_1(C_6S_{2,3} + C_{2,3}C_{4,5}S_6) & C_4(C_5S_1 + C_1C_{2,3}S_5) + S_4(C_1C_{2,3}C_5 - S_1S_5) & \frac{1}{20}(C_1(-6 + 20C_2 + 15C_{\theta_2+\theta_3-\theta_4} + 15C_{2,3,4} + 4S_{2,3}) - 30S_1S_4) \\ C_6(C_5(C_2C_3C_4S_1 - C_4S_1S_2S_3 + C_1S_4) + (C_1C_4 - C_{2,3}S_1S_4)S_5) - S_1S_{2,3}S_6 & -C_6S_1S_{2,3} - (C_5(C_2C_3C_4S_1 - C_4S_1S_2S_3 + C_1S_4) + (C_1C_4 - C_{2,3}S_1S_4)S_5)S_6 & -C_1C_{4,5} + C_{2,3}S_1S_{4,5} & \frac{1}{20}(S_1(-6 + 20C_2 + 15C_{\theta_2+\theta_3-\theta_4} + 15C_{2,3,4} + 4S_{2,3}) + 30C_1S_4) \\ C_{4,5}C_6S_{2,3} - C_{2,3}S_6 & -C_{2,3}C_6 + C_{4,5}S_{2,3}S_6 & -S_{2,3}S_{4,5} & \frac{1}{5}C_{2,3} + \frac{1}{2}(-(2 + 3C_3C_4)S_2 - 3C_2C_4S_3) \\ 0 & 0 & 0 & 1 \end{pmatrix}
$$

(b) $${}^0_6T_1 = \begin{pmatrix} 0.1908 & -0.0065 & -0.9816 & 0.7873 \\ -0.9816 & 0 & -0.1908 & -1.4724 \\ -0.0012 & -1.0000 & 0.0064 & -0.4021 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$
{}^0_6T_2 = \begin{pmatrix} -0.0678 & 0.8783 & 0.4733 & -0.6718 \\ 0.2145 & 0.4761 & -0.8528 & 0.3326 \\ 0.9744 & -0.0437 & 0.2206 & 1.8191 \\ 0 & 0 & 0 & 1 \end{pmatrix}
$$

$$
{}^0_6T_3 = \begin{pmatrix} -0.7272 & -0.1891 & 0.6599 & -0.6273 \\ -0.6850 & 0.1387 & -0.7152 & 0.5648 \\ -0.0437 & 0.9721 & 0.2304 & -0.2442 \\ 0 & 0 & 0 & 1 \end{pmatrix}
$$

$$
{}^0_6T_4 = \begin{pmatrix} 0.7857 & -0.2540 & 0.5641 & -1.2800 \\ 0.6163 & 0.2430 & -0.7491 & 0.7469 \\ -0.0532 & -0.9362 & -0.3475 & -0.4925 \\ 0 & 0 & 0 & 1 \end{pmatrix}
$$

$$
{}^0_6T_5 = \begin{pmatrix} 0.4057 & 0.3911 & 0.8261 & -0.7269 \\ -0.5013 & -0.6606 & 0.5589 & 1.2890 \\ -0.7643 & 0.6408 & 0.0719 & -0.7336 \\ 0 & 0 & 0 & 1 \end{pmatrix}
$$

Exercise 3

Solution

Exercise 4

Solution

DH :

	a_{i-1}	α_{i-1}	d_i	θ_i
1	0	0	d_1	0
2	0	$-\frac{\pi}{2}$	d_2	0
3	0	0	$-d_2$	θ_3

$$^0_1T = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_1 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$^1_2T = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & d_2 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$^2_3T = \begin{pmatrix} cos\theta_3 & -sin\theta_3 & 0 & 0 \\ sin\theta_3 & cos\theta_3 & 0 & 0 \\ 0 & 0 & 1 & -d_2 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$^0_3T = \begin{pmatrix} cos\theta_3 & -sin\theta_3 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -sin\theta_3 & -cos\theta_3 & 0 & d_1 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Exercise 5

Solution

DH :

	a_{i-1}	α_{i-1}	d_i	θ_i
1	0	0	0	θ_1
2	0	$-\frac{\pi}{2}$	$-L_1$	θ_2
3	0	0	0	0
4	0	0	d_2	0

$$^0_1T = \begin{pmatrix} cos\theta_1 & -sin\theta_1 & 0 & 0 \\ sin\theta_1 & cos\theta_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$^1_2T = \begin{pmatrix} cos\theta_2 & -sin\theta_2 & 0 & 0 \\ 0 & 0 & 1 & -L_1 \\ sin\theta_2 & cos\theta_2 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$^2_3T = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$^3_4T = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_2 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$
{}^0_4T = \begin{pmatrix} cos(\theta_1)cos(\theta_2) & -cos(\theta_1)sin(\theta_2) & -sin(\theta_1) & -sin(\theta_1)(L_1 + d_2) \\ cos(\theta_2)sin(\theta_1) & -sin(\theta_1)sin(\theta_2) & cos(\theta_1) & cos(\theta_1)(L_1 + d_2) \\ sin(\theta_2) & cos(\theta_2) & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}
$$