Advanced Robotics

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1 Part 1

1.1 Problem 1

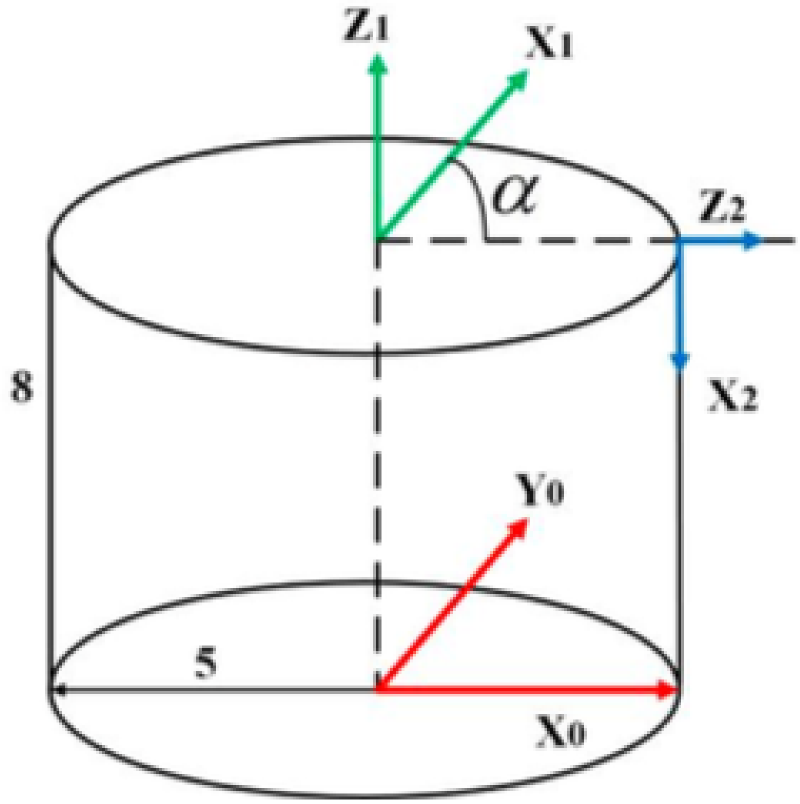

Figure 1: Problem 1 description

1. $^{0}_{1}R = \begin{pmatrix} \cos(\alpha) & \sin(\alpha) & 0 \\ \text{-}\sin(\alpha) & \cos(\alpha) & 0 \\ 0 & 0 & 1 \end{pmatrix}$

   $^{0}P = \begin{pmatrix} 0 \\ 0 \\ 8 \end{pmatrix}$

Thus ${}^0_1T = \begin{pmatrix} \cos(\alpha) & \sin(\alpha) & 0 & 0 \\ \text{-}\sin(\alpha) & \cos(\alpha) & 0 & 0 \\ 0 & 0 & 1 & 8 \\ 0 & 0 & 0 & 1 \end{pmatrix}$

2. ${}^2_1R = \begin{pmatrix} \cos(\alpha) & \sin(\alpha) & 0 \\ \text{-}\sin(\alpha) & \cos(\alpha) & 0 \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ \text{-}1 & 0 & 0 \end{pmatrix} = \begin{pmatrix} 0 & \sin(\alpha) & \cos(\alpha) \\ 0 & \cos(\alpha & \text{-}\sin(\alpha) \\ \text{-}1 & 0 & 0 \end{pmatrix}$

   We know, that ${}^A_BR = ({}^B_AR)^T$, thus ${}^1_2R = \begin{pmatrix} 0 & \sin(\alpha) & \cos(\alpha) \\ 0 & \cos(\alpha & \text{-}\sin(\alpha) \\ \text{-}1 & 0 & 0 \end{pmatrix}^T = \begin{pmatrix} 0 & 0 & \text{-}1 \\ \sin(\alpha) & \cos(\alpha) & 0 \\ \cos(\alpha) & \text{-}\sin(\alpha) & 0 \end{pmatrix}$

   ${}^1P = \begin{pmatrix} 5 \cdot cos(\alpha) \\ \text{-}5 \cdot sin(\alpha) \\ 0 \end{pmatrix}$

   Thus ${}^1_2T = \begin{pmatrix} 0 & 0 & \text{-}1 & 5 \cdot cos(\alpha) \\ \sin(\alpha) & \cos(\alpha) & 0 & \text{-}5 \cdot sin(\alpha) \\ \cos(\alpha) & \text{-}\sin(\alpha) & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$

1.2 Problem 2

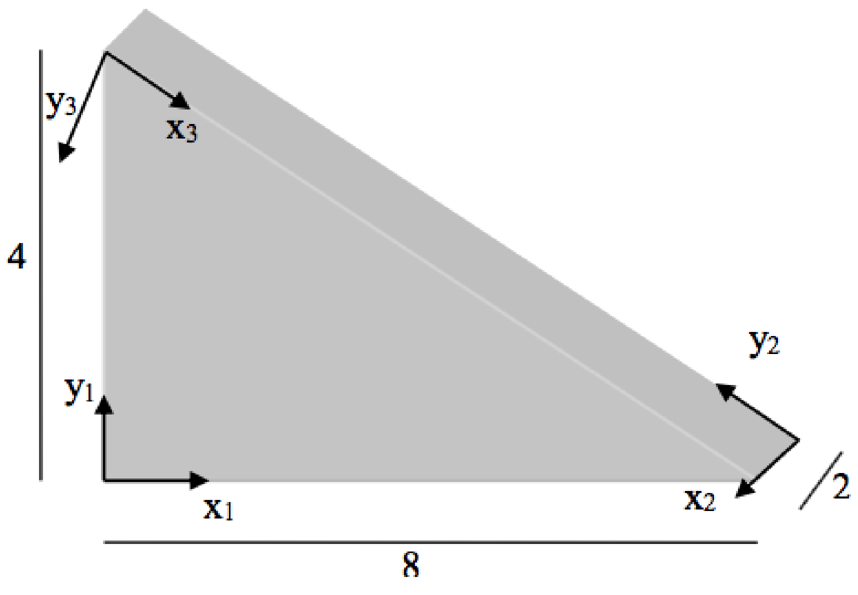

Figure 2: Problem 2 description

1. Let us define $\alpha = \text{arctg}(0.5)$

   ${}^2_1R = \begin{pmatrix} \cos(\alpha) & \text{-}\sin(\alpha) & 0 \\ \sin(\alpha) & \cos(\alpha) & 0 \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 0 & 0 & \text{-}1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix} = \begin{pmatrix} 0 & \text{-}\sin(\alpha) & \text{-}\cos(\alpha) \\ 0 & \cos(\alpha) & \text{-}\sin(\alpha) \\ 1 & 0 & 0 \end{pmatrix}$

   ${}^2P = \begin{pmatrix} 2 \\ 8 \cdot cos(\alpha) \\ 8 \cdot sin(\alpha) \end{pmatrix}$

Thus $^1_2T = \begin{pmatrix} 0 & \text{-sin}(\alpha) & \text{-cos}(\alpha) & 2 \\ 0 & \cos(\alpha) & \text{-sin}(\alpha) & 8{\cdot}cos(\alpha) \\ 1 & 0 & 0 & 8{\cdot}sin(\alpha) \\ 0 & 0 & 0 & 1 \end{pmatrix}$

2. $^3_2R = \begin{pmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 & -1 \\ -1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$

   $^3P = \begin{pmatrix} 2 \\ 0 \\ 4\sqrt{5} \end{pmatrix}$

   Thus $^3_2T = \begin{pmatrix} 0 & 0 & -1 & 2 \\ -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 4\sqrt{5} \\ 0 & 0 & 0 & 1 \end{pmatrix}$

3. $^3_1R = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix} \cdot \begin{pmatrix} \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} & 0 \\ \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} & 0 \\ -\frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} & 0 \\ 0 & 0 & -1 \end{pmatrix}$

   $^3P = \begin{pmatrix} 4{\cdot}sin(\alpha) \\ 8{\cdot}sin(\alpha) \\ 0 \end{pmatrix}$

   Thus $^3_1T = \begin{pmatrix} \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} & 0 & 4{\cdot}sin(\alpha) \\ -\frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} & 0 & 8{\cdot}sin(\alpha) \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$

4. We know, that $^A_BT = \left(\begin{array}{ccc|c} & ^A_BR & & ^AP \\ 0 & 0 & 0 & 1 \end{array}\right)$

   Furthermore, $^B_AT = \left(\begin{array}{ccc|c} & ^B_AR & & -^B_AR \cdot ^A P \\ 0 & 0 & 0 & 1 \end{array}\right)$

   Hence, $^1_3T = \begin{pmatrix} \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} & 0 & -2\sqrt{2} \cdot sin(\alpha) \\ -\frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} & 0 & -4\sqrt{2} \cdot sin(\alpha) \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$

1.3 Problem 3

Time domain transfer function:

$$u(t) = K_p e(t) + K_i \int_0^t e(t)dt + K_d \frac{de(t)}{dt}$$

S-domain transfer function:

$$\frac{U(s)}{E(s)} = K_p + \frac{K_i}{s} + K_d s$$

PI controllers eliminate steady-state error (oscillations around the set point), but increases overshooting. PD controllers are almost never used; they reduce overshoot and settling time. PID controllers dynamics are similar to PI controllers, but, due to the derivative term, overshooting and settling time decrease, and one can increase K_i.

1.4 Problem 4

Differentiator approximation:

$$T_d s \sim \frac{T_d s}{1 + \gamma T_d s}$$

Closed-loop transfer function:

$$\frac{C(s)}{R(s)} = \frac{G(s)}{1 + G(s)H(s)}$$

$$G(s) = \frac{1}{\gamma} \qquad H(s) = \frac{1}{T_d s}$$

$$\frac{C(s)}{R(s)} = \frac{G(s)}{1 + G(s)\,H(s)} = \frac{\gamma^{-1}}{1 + \frac{1}{\gamma T_d s}} = \frac{\gamma^{-1}}{\frac{\gamma T_d s + 1}{\gamma T_d s}} = \frac{\gamma^{-1} \cdot \gamma T_d s}{\gamma T_d s + 1} = \frac{T_d s}{1 + \gamma T_d s}$$

1.5 Problem 5

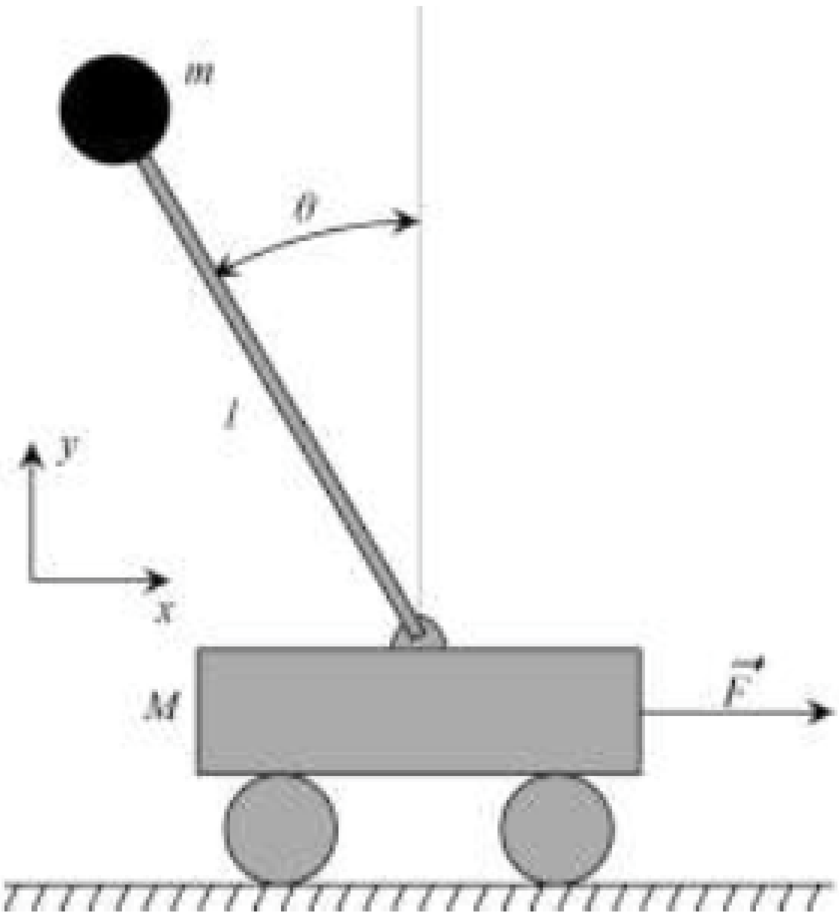

Figure 3: Inverted pendulum on a cart

Inverted pendulum on a cart problem can be described by the following set of equations:

$$\begin{cases} (M+m)\ddot{x} - ml\ddot{Q} \cdot cos(\theta) + ml\dot{Q}^2 \cdot sin(\theta) = F \\ l\ddot{Q} - g \cdot sin(\theta) = \ddot{x} \cdot cos(\theta) \end{cases}$$

When we are near the equilibrium point we can assume, that $sin(\theta) = \theta$, $cos(\theta) = 1$, $\dot{Q}^2 = 0$. Thus, linearized model is :

$$\begin{cases} (M+m)\ddot{x} - ml\ddot{Q} + mlQ\dot{Q}^2 = F \\ l\ddot{Q} - gQ = \ddot{x} \end{cases}$$

Laplace transform will give:

$$\begin{cases} (M+m)X(S)S^2 - mlQ(S)S^2 = F(S) \\ lQ(S) - gQ(S) = X(S)S^2 \end{cases}$$

So, $\dfrac{Q(S)}{F(S)} = \dfrac{1}{MLS^2 - (M+m)g}$ and $\dfrac{X(S)}{F(S)} = \dfrac{lS^2 - g}{MLS^4 - (M+m)gS^2}$