Assignment 3;
due Wednesday November 11

Part 1

Exercise 1
Solution

According to [1], Manocha and Zhu (1994) proposed a generalized closed form solution which can be derived for 6 DOF (or less) kinematic chain.

(a) False. It is possible that the target for EE is unreachable.

(b) False. It is possible that the target for EE is unreachable.

(c) False. It is possible that the target for EE is unreachable.

(d) True.

Exercise 2
Solution

(a) False. 3 DoF manipulator with rotation joints can have only 2 dimension work-space

(b) False. Dextrous can be empty.

(c) True. E.g. 1 DoF manipulator with 2 dimension work-space and rotation joint with different length of links.

(d) False.

Exercise 3
Solution

(a) True.

(b) False. Usually we go from trigonometric to transcendental equations.

(c) True. We user FK during solving IK.

(d) True. IK problem needs a very fast computational engine in order to make solution in real-time.

Exercise 4
Solution

(a) False. We have considered the case of revolute joints.

(b) False. Links are not important in IK it can only affect the work-space.

(c) False. It is solution for manipulators with 6DOF's when three consecutive axis intersect.

(d) False.

Part 2

Exercise 1

Solution

DH:

	a_{i-1}	α_{i-1}	d_i	θ_i
1	0	0	0	ϕ
2	0	$-\frac{\pi}{2}$	0	θ
3	0	$\frac{\pi}{2}$	-$(L+d+R_2)$	0

Transformation matrices

$$
{}^0_1T = \begin{bmatrix} cos\phi & -sin\phi & 0 & 0 \\ sin\phi & cos\phi & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

$$
{}^1_2T = \begin{bmatrix} cos\theta & -sin\theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -sin\phi & -cos\phi & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

$$
{}^2_3T = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & L+d+R_2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

$$
{}^0_3T = \begin{bmatrix} cos\phi \cdot sin\theta & -sin\phi & cos\phi \cdot sin\theta & -\left(d+l+R_2\right) cos\phi \cdot sin\theta \\ sin\phi \cdot cos\theta & cos\phi & sin\phi \cdot sin\theta & -\left(d+l+R_2\right) sin\phi \cdot sin\theta \\ -sin\theta & 0 & cos\theta & -\left(d+l+R_2\right) cos\theta \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

$$
\begin{cases} x = -\left(d+l+R_2\right) cos\phi \cdot sin\theta \\ y = -\left(d+l+R_2\right) sin\phi \cdot sin\theta \\ z = -\left(d+l+R_2\right) cos\theta \end{cases}
$$

$$
\begin{cases} y^2 = -\left(d+l+R_2\right)^2 sin^2\phi \cdot sin^2\theta \\ x^2+y^2 = \left(d+l+R_2\right)^2 cos^2\phi sin^2\theta + \left(d+l+R_2\right)^2 sin^2\phi sin^2\theta = \left(d+l+R_2\right)^2 sin^2\theta \\ z^2 = \left(d+l+R_2\right)^2 cos^2\theta \end{cases}
$$

$$
\begin{cases} y^2 = -\left(d+l+R_2\right)^2 sin^2\phi \cdot sin^2\theta \\ \dfrac{z^2}{\left(d+l+R_2\right)^2} = cos^2\theta \\ x^2+y^2 = \left(d+l+R_2\right)^2 \left(1 - \dfrac{z^2}{\left(d+l+R_2\right)^2}\right) \end{cases}
$$

$$\begin{cases} d_1 = \sqrt{x^2 + y^2 + z^2} - l - R_2 \\ d_2 = -\sqrt{x^2 + y^2 + z^2} - l - R_2 \\ \dfrac{z^2}{\left(d + l + R_2\right)^2} = cos^2\theta \\ y^2 = -\left(d + l + R_2\right)^2 sin^2\phi \cdot sin^2\theta \end{cases}$$

$$\begin{cases} d_1 = \sqrt{x^2 + y^2 + z^2} - l - R_2 \\ d_2 = -\sqrt{x^2 + y^2 + z^2} - l - R_2 \\ \theta = arccos\left(\pm\sqrt{\dfrac{z}{(d + l + R_2)}}\right) \\ y^2 = -\left(d + l + R_2\right)^2 sin^2\phi \cdot sin^2\theta \end{cases}$$

$$\begin{cases} d_1 = \sqrt{x^2 + y^2 + z^2} - (l + R_2) \\ d_2 = -\sqrt{x^2 + y^2 + z^2} - (l + R_2) \\ \theta = arccos\left(\pm\dfrac{z}{d + l + R_2}\right) \\ \phi = arcsin\left(\pm\dfrac{y \cdot sin\theta}{d + l + R_2}\right) \end{cases}$$

Exercise 2

Solution

DH:

	a_{i-1}	α_{i-1}	d_i	θ_i
1	0	0	$-L_1$	0
2	0	$-\frac{\pi}{2}$	0	θ
3	$-L2$	$\frac{\pi}{2}$	p	ϕ
4	n	0	0	0

Transformation matrices

$$
{}^0_1T = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & -L_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

$$
{}^1_2T = \begin{bmatrix} cos\theta & -sin\theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -sin\theta & -cos\theta & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

$$
{}^2_3T = \begin{bmatrix} cos\phi & -sin\phi & 0 & -L_2 \\ 0 & 0 & -1 & p \\ -sin\phi & -cos\phi & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

$$
{}^3_4T = \begin{bmatrix} 1 & 0 & 0 & n \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

$$
{}^0_4T = \begin{bmatrix} WE & WE & WE & ncos\theta \cdot cos\phi - L_2cos\theta - psin\theta \\ WE & WE & WE & nsin\phi \\ WE & WE & WE & -nsin\theta \cdot cos\phi + L_2sin\theta - pcos\theta - L_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

$$
\begin{cases} \phi = arcsin\frac{y}{n} \\ x = nsin\theta \cdot cos\phi - L_2cos\theta - p \cdot sin\theta \\ z = -nsin\theta \cdot cos\phi - L_2sit\theta - p \cdot cos\theta - L_1 \end{cases}
$$

$$
\begin{cases} \phi = arcsin\frac{y}{n} \\ x = nsin\theta \cdot cos\phi - L_2cos\theta - p \cdot sin\theta \\ z = -nsin\theta \cdot cos\phi - L_2sit\theta - p \cdot cos\theta - L_1 \\ u = tg\frac{\theta}{2} \end{cases}
$$

$$\begin{cases} \phi = arcsin\dfrac{y}{n} \\ x = \dfrac{(1-u^2)(n \cdot cos\phi - L_2) - p \cdot 2u}{1+u^2} \\ z = -nsin\theta \cdot cos\phi - L_2 sit\theta - p \cdot cos\theta - L_1 \\ u = tg\dfrac{\theta}{2} \end{cases}$$

$$\begin{cases} \phi = arcsin\dfrac{y}{n} \\ u_{1,2} = p \pm \sqrt{p^2 - x^2 + (ncos\phi - L_2)^2} \\ \theta = 2arcth(p \pm \sqrt{p^2 - x^2 + (ncos\phi - L_2)^2}) \\ L_1 = L_2 \cdot sin\theta - pcos\theta + nsin\theta cos\phi - z \end{cases}$$

Exercise 3

Solution

DH:

	a_{i-1}	α_{i-1}	d_i	θ_i
1	0	0	$-p$	0
2	0	$-\frac{\pi}{2}$	$-\omega$	0
3	0	0	0	θ
4	r	0	0	0

Transformation matrices

$$
{}^0_1T = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & -p \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

$$
{}^1_2T = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & -\omega \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

$$
{}^2_3T = \begin{bmatrix} cos\theta & -sin\theta & 0 & 0 \\ sin\theta & cos\theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

$$
{}^3_4T = \begin{bmatrix} 1 & 0 & 0 & r \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

$$
{}^0_4T = \begin{bmatrix} cos\theta & -sin\theta & 0 & r\left(cos\theta+1\right) \\ 0 & 0 & 1 & -w \\ -sin\theta & -cos\theta & 0 & -p-r\cdot sin\theta \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

$$
\begin{cases} x = r\left(cos\theta+1\right) \\ y = -\omega \\ z = -r\cdot sin\theta - p \end{cases}
$$

$$
\begin{cases} \omega = -y \\ \theta = arccos\left(\dfrac{x}{r}-1\right) \\ p = -\left(z + r\sqrt{1-\left(\dfrac{x}{r}-1\right)^2}\right) \end{cases}
$$

Exercise 4

Solution

DH:

	a_{i-1}	α_{i-1}	d_i	θ_i
1	0	0	0	α
2	0	$\frac{\pi}{2}$	a	$\frac{\pi}{2}$
3	0	0	-b	β
4	0	$-\frac{\pi}{2}$	0	γ
5	0	0	-c	0

Transformation matrices

$$^0_1T = \begin{bmatrix} cos\alpha & -sin\alpha & 0 & 0 \\ sin\alpha & cos\alpha & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$^1_2T = \begin{bmatrix} 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & -a \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$^2_3T = \begin{bmatrix} cos\beta & -sin\beta & 0 & 0 \\ sin\beta & cos\beta & 0 & 0 \\ 0 & 0 & 1 & -b \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$^3_4T = \begin{bmatrix} cos\gamma & -sin\gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -sin\gamma & -cos\gamma & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$^4_5T = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & -c \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$^0_5T = \begin{bmatrix} -c_\alpha s_\beta c_\gamma - s_\alpha s_\gamma & -c_\alpha s_\beta c_\gamma - s_\alpha c_\gamma & -c_\alpha c_\beta & -c_\alpha s_\beta c_\gamma - s_\alpha s_\gamma + c \cdot c_\alpha c_\beta + (a-b) \cdot s_\alpha \\ -s_\alpha s_\beta c_\gamma - c_\alpha s_\gamma & s_\alpha s_\beta s_\gamma + c_\alpha c_\gamma & -s_\alpha c_\beta & -s_\alpha s_\beta s_\gamma - c_\alpha s_{gamma} - c \cdot s_\alpha c_\beta + (b-a) \cdot c_\alpha \\ c_\beta c_\gamma & -c_\beta s_\gamma & -s_\beta & c_\beta c_\gamma + c \cdot s_\beta \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Exercise 5

Solution

Here we will try geometric solution instead of algebraic.

$x_1 = x_0 - L_3 sin\phi$

$y_1 = y_0 - L_3 cos\phi$

$x_1^2 + y_1^2 = L_1^2 + L_2^2 - 2L_1L_2cos\theta_2$

$\theta_2 = arccos\left(\frac{x_1^2 + y_1^2 - L_1^2 - L_2^2}{2L_1L_2}\right)$

$\gamma = \frac{\pi}{2} - \theta_1 - arctg(\frac{y_1}{x_1})$

$L_2^2 = L_1^2 + x_1^2 + y_1^2 - 2L_1\sqrt{x_1^2 + y_1^2}cos\gamma$

$\theta_1 = arcsin\left(\frac{L_1^2 - L_2^2 + x_1^2 + y_1^2}{2L_1\sqrt{x_1^2 + y_1^2}}\right) - arctg\left(\frac{y_1}{x_1}\right)$

$\theta_3 = \phi - \theta_1 - \theta_2$

List of references

[1] CLOSED FORM AND GENERALIZED INVERSE KINEMATIC SOLUTIONS FOR ANIMATING THE HUMAN ARTICULATED STRUCTURE. Kwan W. CHIN