

**Assignment 1;
due Friday November 7**

Part 1

**Exercise 1
Solution**

The system is the closed kinematic chain.

- (a) False
- (b) False
- (c) False
- (d) True.

**Exercise 2
Solution**

- (a) FALSE. SCARA robots can consist only of 4 rotational axes
- (b) TRUE. 3 rotational axes can give us 3 DoF. The forth one cannot add one more.
- (c) FALSE. The Chebyshev linkage has 4 rotational joints and only one DoF.
- (d) FALSE. The explanation is like in previous example.

**Exercise 3
Solution**

- (a) TRUE. By definition.
- (b) FALSE. By definition.
- (c) FALSE. Because of b)
- (d) FALSE. Because of a)

**Exercise 4
Solution**

- (a) FALSE. We can choose coordinates frames in the end effector.
- (b) TRUE. We can choose coordinates frames in the end effector, so it could be many matrices. If there is only one base frame there is only one DH matrix.
- (c) FALSE. Because we can change direction of X and Z axes. However, multiplication of all DH matrices will give us translation from base frame to the frame in the end effector and it will be unique.
- (d) FALSE. The same explanation as in c)

Part 2

Exercise 1

Solution

$$\text{DH : } \begin{array}{ccccc} & a_{i-1} & \alpha_{i-1} & d_i & \theta_i \\ 1 & 0 & 0 & 0 & \theta_1 \\ 2 & 15 & -\frac{\pi}{2} & 0 & \theta_2 \\ 3 & 20 & 0 & 0 & 0 \end{array}$$

$${}^0_1T = \begin{pmatrix} \cos(\theta_1) & -\sin(\theta_1) & 0 & 0 \\ \sin(\theta_1) & \cos(\theta_1) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$${}^1_2T = \begin{pmatrix} \cos(\theta_2) & -\sin(\theta_2) & 0 & 15 \\ 0 & 0 & 1 & 0 \\ -\sin(\theta_2) & -\cos(\theta_2) & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$${}^2_3T = \begin{pmatrix} 1 & 0 & 0 & 20 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$${}^0_3T = {}^0_1T {}^1_2T {}^2_3T = \begin{pmatrix} \cos(\theta_1)\cos(\theta_2) & -\cos(\theta_1)\sin(\theta_2) & -\sin(\theta_1) & 20\cos(\theta_1)\cos(\theta_2) + 15\sin(\theta_1) \\ \sin(\theta_1)\cos(\theta_2) & -\sin(\theta_1)\sin(\theta_2) & \cos(\theta_1) & 20\sin(\theta_1)\cos(\theta_2) + 15\sin(\theta_1) \\ -\sin(\theta_2) & -\cos(\theta_2) & 0 & -20\sin(\theta_2) \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Exercise 2

Solution

$$\text{(a) DH : } \begin{array}{ccccc} & a_{i-1} & \alpha_{i-1} & d_i & \theta_i \\ 1 & 0 & 0 & 0 & \theta_1 \\ 2 & 0.3 & \frac{\pi}{2} & 0 & \theta_2 \\ 3 & 1 & 0 & 0 & \theta_3 \\ 4 & 0.2 & -\frac{\pi}{2} & 0 & \theta_4 \\ 5 & 1.5 & 0 & 0 & \theta_5 \\ 6 & 0 & \frac{\pi}{2} & 0 & \theta_6 \end{array}$$

$${}^0_1T = \begin{pmatrix} \cos(\theta_1) & -\sin(\theta_1) & 0 & 0 \\ \sin(\theta_1) & \cos(\theta_1) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$${}^1_2T = \begin{pmatrix} \cos(\theta_2) & -\sin(\theta_2) & 0 & 0.3 \\ 0 & 0 & -1 & 0 \\ \sin(\theta_2) & \cos(\theta_2) & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$${}^2_3T = \begin{pmatrix} \cos(\theta_3) & -\sin(\theta_3) & 0 & 1 \\ \sin(\theta_3) & \cos(\theta_3) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$${}^3_4T = \begin{pmatrix} \cos(\theta_4) & -\sin(\theta_4) & 0 & 0.2 \\ 0 & 0 & 1 & 0 \\ -\sin(\theta_4) & -\cos(\theta_4) & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$${}^4_5T = \begin{pmatrix} \cos(\theta_5) & -\sin(\theta_5) & 0 & 1.5 \\ \sin(\theta_5) & \cos(\theta_5) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$${}^5_6T = \begin{pmatrix} \cos(\theta_6) & -\sin(\theta_6) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ \sin(\theta_6) & \cos(\theta_6) & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$${}^0_6T = {}^0_1T {}^1_2T {}^2_3T {}^3_4T {}^4_5T {}^5_6T =$$

$$\begin{pmatrix} -C_6S_1S_{4,5} + C_1(C_{2,3}C_{4,5}C_6 - S_{2,3}S_6) & S_1S_{4,5}S_6 - C_1(C_6S_{2,3} + C_{2,3}C_{4,5}S_6) & -C_4(C_5S_1 + C_1C_{2,3}S_5) + S_4(-C_1C_{2,3}C_5 + S_1S_6) & \frac{1}{10}(C_1(3 + 2C_{2,3} + 5C_2(2 + 3C_3C_4) - 15C_4S_2S_6) - 15S_1S_4) \\ C_6(C_5(C_2C_3C_4S_1 - C_4S_1S_2S_3 + C_1S_4) + (C_1C_4 - C_{2,3}S_1S_4)S_5) - S_1S_{2,3}S_6 & -C_6S_1S_{2,3} - (C_5(C_2C_3C_4S_1 - C_4S_1S_2S_3 + C_1S_4) + (C_1C_4 - C_{2,3}S_1S_4)S_5)S_6 & C_1C_{4,5} - C_{2,3}S_1S_{4,5} & \frac{1}{10}(S_1(3 + 2C_{2,3} + 5C_2(2 + 3C_3C_4) - 15C_4S_2S_6) + 15C_1S_4) \\ C_{4,5}C_6S_{2,3} + C_{2,3}S_6 & C_{2,3}C_6 - C_{4,5}S_{2,3}S_6 & -S_{2,3}S_{4,5} & S_2 + \frac{1}{10}(2 + 15C_1)S_{2,3} \end{pmatrix}$$

$$(b) {}^0_6T_1 = \begin{pmatrix} 0.1908 & -0.0065 & 0.9816 & 1.586 \\ -0.9816 & 0 & 0.1908 & -1.4724 \\ 0.0012 & 1 & 0.0064 & 0.6034 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$${}^0_6T_2 = \begin{pmatrix} -0.0678 & 0.8783 & -0.4733 & -0.2295 \\ 0.2145 & 0.4761 & 0.8528 & 0.7092 \\ -0.9744 & 0.0437 & 0.2206 & -1.5369 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$${}^0_6T_3 = \begin{pmatrix} -0.7272 & -0.1891 & -0.6599 & -0.3338 \\ -0.6850 & 0.1387 & 0.7152 & 0.8347 \\ 0.0437 & -0.9721 & 0.2304 & 0.4429 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$${}^0_6T_4 = \begin{pmatrix} 0.7857 & -0.2540 & -0.5641 & -1.6624 \\ 0.6163 & 0.2430 & 0.7491 & 0.2349 \\ 0.0532 & 0.9362 & -0.3475 & 0.2124 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$${}^0_6T_5 = \begin{pmatrix} 0.4057 & 0.3911 & -0.8261 & -0.1587 \\ -0.5013 & -0.6606 & -0.5589 & 1.8721 \\ 0.7643 & -0.6408 & 0.0719 & 0.9183 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Exercise 3

Solution

Exercise 4

Solution

$$\begin{array}{rcccl}
 & a_{i-1} & \alpha_{i-1} & d_i & 0 \\
 \text{DH : } & 1 & 0 & 0 & d_1 & 0 \\
 & 2 & 0 & -\frac{\pi}{2} & d_2 & 0 \\
 & 3 & 0 & 0 & -d_2 & \theta_3
 \end{array}$$

Exercise 5

Solution

$$\begin{array}{rcccl}
 & a_{i-1} & \alpha_{i-1} & d_i & 0 \\
 & 1 & 0 & 0 & 0 & \theta_1 \\
 \text{DH : } & 2 & 0 & -\frac{\pi}{2} & L_1 & \theta_2 \\
 & 3 & 0 & -\frac{\pi}{2} & d_1 & 0
 \end{array}$$