Advanced Robotics

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1 Part 1

1.1 Problem 1

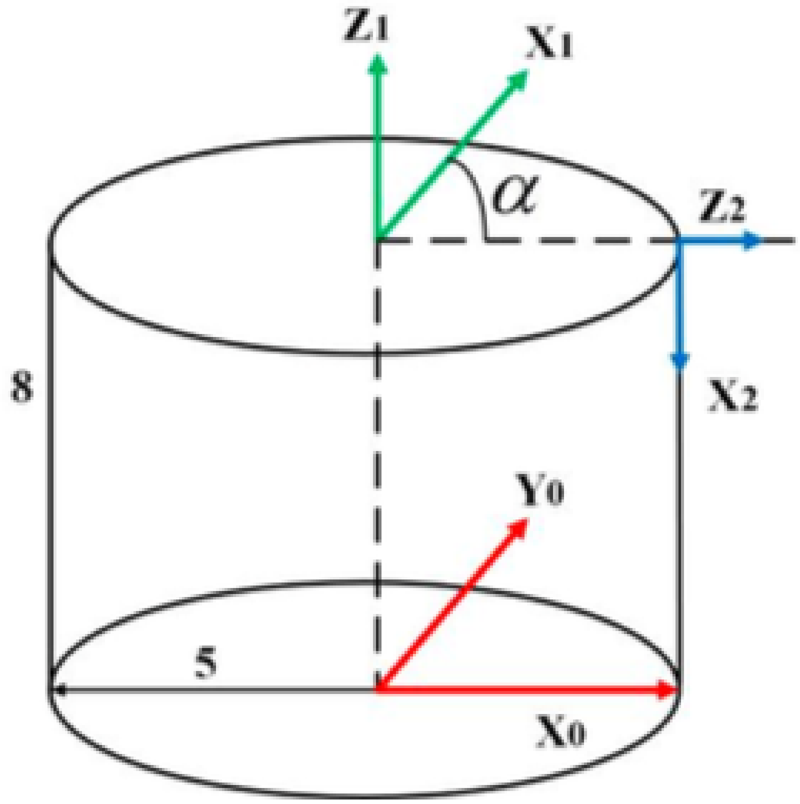

Figure 1: Problem 1 description

1. $^0_1R = \begin{pmatrix} \cos(\alpha) & \sin(\alpha) & 0 \\ \text{-}\sin(\alpha) & \cos(\alpha) & 0 \\ 0 & 0 & 1 \end{pmatrix}$

   $^0P = \begin{pmatrix} 0 \\ 0 \\ 8 \end{pmatrix}$

Thus ${}^0_1T = \begin{pmatrix} \cos(\alpha) & \sin(\alpha) & 0 & 0 \\ \text{-}\sin(\alpha) & \cos(\alpha) & 0 & 0 \\ 0 & 0 & 1 & 8 \\ 0 & 0 & 0 & 1 \end{pmatrix}$

2. ${}^2_1R = \begin{pmatrix} \cos(\alpha) & \sin(\alpha) & 0 \\ \text{-}\sin(\alpha) & \cos(\alpha) & 0 \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ \text{-}1 & 0 & 0 \end{pmatrix} = \begin{pmatrix} 0 & \sin(\alpha) & \cos(\alpha) \\ 0 & \cos(\alpha & \text{-}\sin(\alpha) \\ \text{-}1 & 0 & 0 \end{pmatrix}$

   We know, that ${}^A_BR = ({}^B_AR)^T$, thus ${}^1_2R = \begin{pmatrix} 0 & \sin(\alpha) & \cos(\alpha) \\ 0 & \cos(\alpha & \text{-}\sin(\alpha) \\ \text{-}1 & 0 & 0 \end{pmatrix}^T = \begin{pmatrix} 0 & 0 & \text{-}1 \\ \sin(\alpha) & \cos(\alpha) & 0 \\ \cos(\alpha) & \text{-}\sin(\alpha) & 0 \end{pmatrix}$

   ${}^1P = \begin{pmatrix} 5 \cdot cos(\alpha) \\ \text{-}5 \cdot sin(\alpha) \\ 0 \end{pmatrix}$

   Thus ${}^1_2T = \begin{pmatrix} 0 & 0 & \text{-}1 & 5 \cdot cos(\alpha) \\ \sin(\alpha) & \cos(\alpha) & 0 & \text{-}5 \cdot sin(\alpha) \\ \cos(\alpha) & \text{-}\sin(\alpha) & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$

1.2 Problem 2

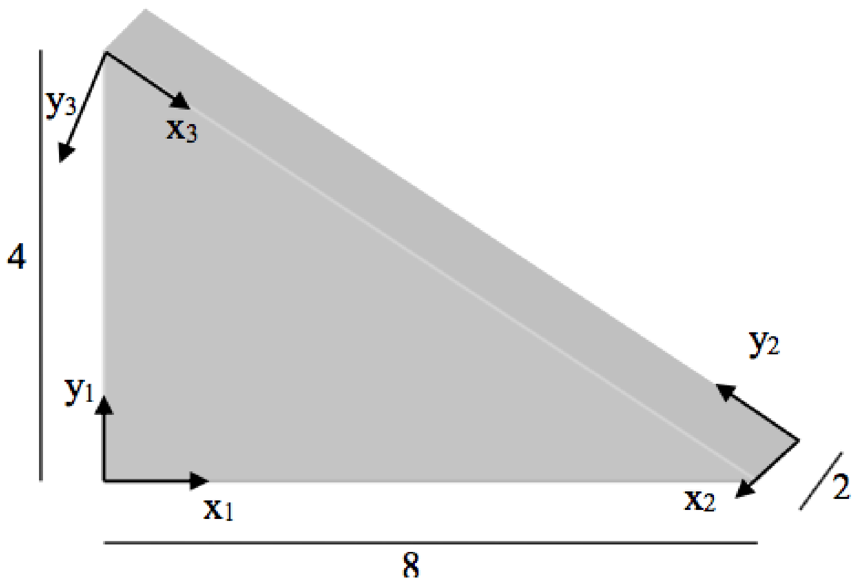

Figure 2: Problem 2 description

1. Let us define $\alpha = \text{arctg}(0.5)$

   ${}^2_1R = \begin{pmatrix} \cos(\alpha) & \text{-}\sin(\alpha) & 0 \\ \sin(\alpha) & \cos(\alpha) & 0 \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 0 & 0 & \text{-}1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix} = \begin{pmatrix} 0 & \text{-}\sin(\alpha) & \text{-}\cos(\alpha) \\ 0 & \cos(\alpha) & \text{-}\sin(\alpha) \\ 1 & 0 & 0 \end{pmatrix}$

   ${}^2P = \begin{pmatrix} 2 \\ 8 \cdot cos(\alpha) \\ 8 \cdot sin(\alpha) \end{pmatrix}$

Thus $^1_2T = \begin{pmatrix} 0 & \text{-sin}(\alpha) & \text{-cos}(\alpha) & 2 \\ 0 & \cos(\alpha) & \text{-sin}(\alpha) & 8{\cdot}cos(\alpha) \\ 1 & 0 & 0 & 8{\cdot}sin(\alpha) \\ 0 & 0 & 0 & 1 \end{pmatrix}$

2. $^3_2R = \begin{pmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 & -1 \\ -1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$

   $^3P = \begin{pmatrix} 2 \\ 0 \\ 4\sqrt{5} \end{pmatrix}$

   Thus $^3_2T = \begin{pmatrix} 0 & 0 & -1 & 2 \\ -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 4\sqrt{5} \\ 0 & 0 & 0 & 1 \end{pmatrix}$

3. $^3_1R = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix} \cdot \begin{pmatrix} \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} & 0 \\ \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} & 0 \\ -\frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} & 0 \\ 0 & 0 & -1 \end{pmatrix}$

   $^3P = \begin{pmatrix} 4{\cdot}sin(\alpha) \\ 8{\cdot}sin(\alpha) \\ 0 \end{pmatrix}$

   Thus $^3_1T = \begin{pmatrix} \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} & 0 & 4{\cdot}sin(\alpha) \\ -\frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} & 0 & 8{\cdot}sin(\alpha) \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$

4. We know, that $^A_BT = \left(\begin{array}{ccc|c} & ^A_BR & & ^AP \\ 0 & 0 & 0 & 1 \end{array}\right)$

   Furthermore, $^B_AT = \left(\begin{array}{ccc|c} & ^B_AR & & -^B_AR \cdot ^A P \\ 0 & 0 & 0 & 1 \end{array}\right)$

   Hence, $^1_3T = \begin{pmatrix} \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} & 0 & -2\sqrt{2} \cdot sin(\alpha) \\ -\frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} & 0 & -4\sqrt{2} \cdot sin(\alpha) \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$

1.3 Problem 3

1.4 Problem 4

Differentiator approximation:

$$T_d s \sim \frac{T_d s}{1 + \gamma T_d s}$$

Closed-loop transfer function:

$$\frac{C(s)}{R(s)} = \frac{G(s)}{1 + G(s)H(s)}$$

$$G(s) = \frac{1}{\gamma} \qquad H(s) = \frac{1}{T_d s}$$

$$\frac{C(s)}{R(s)} = \frac{G(s)}{1 + G(s)\,H(s)} = \frac{\gamma^{-1}}{1 + \frac{1}{\gamma T_d s}} = \frac{\gamma^{-1}}{\frac{\gamma T_d s + 1}{\gamma T_d s}} = \frac{\gamma^{-1} \cdot \gamma T_d s}{\gamma T_d s + 1} = \frac{T_d s}{1 + \gamma T_d s}$$

1.5 Problem 5