Skolkovo Institute of Science and Technology

Homework 3. Matrix Calculus

Bayesian Methods Advanced Machine Learning, Spring 2016, Term 3

Start Date: Wednesday, February 10.
Submission Deadline: **Thursday, February 18, 23:59.**

Questions regarding this assignment should be sent to *bayesml@gmail.com*. Please use the following prefix for the subject: [BMML Skoltech 2016]

1. Prove Woodbury's matrix identity:

$$(A + UCV)^{-1} = A^{-1} - A^{-1}U(C^{-1} + VA^{-1}U)^{-1}VA^{-1}.$$

   Dimensions: $A \in \mathbb{R}^{n\times n}$, $C \in \mathbb{R}^{m\times m}$, $U \in \mathbb{R}^{n\times m}$, $V \in \mathbb{R}^{m\times n}$.

2. Find $\frac{\partial}{\partial X} \operatorname{tr}(AX^{-1}B)$.

3. Let $p(\boldsymbol{x}) = \mathcal{N}(\boldsymbol{x}|\boldsymbol{\mu}, \Sigma)$ and $\boldsymbol{x} = [\boldsymbol{x}_a, \boldsymbol{x}_b]$. Find $p(\boldsymbol{x}_a)$.

4. Let $p(\boldsymbol{x}) = \mathcal{N}(\boldsymbol{x}|\boldsymbol{\mu}, \Sigma)$. Find $\mathbb{E}(\boldsymbol{x} - \boldsymbol{a})^T B(\boldsymbol{x} - \boldsymbol{a})$. Hint: $\Sigma = \mathbb{E}(\boldsymbol{x} - \boldsymbol{\mu})(\boldsymbol{x} - \boldsymbol{\mu})^T$.

5. Let $p(\boldsymbol{x}) = \mathcal{N}(\boldsymbol{x}|\boldsymbol{\mu}, \Sigma)$, $p(\boldsymbol{y}|\boldsymbol{x}) = \mathcal{N}(\boldsymbol{y}|A\boldsymbol{x}, \Gamma)$. Prove that $p(\boldsymbol{y}) = \mathcal{N}(\boldsymbol{y}|A\boldsymbol{\mu}, \Gamma + A\Sigma A^T)$. Hint: use "completing the square" technique. See, for example, Bishop "Pattern Recognition and Machine Learning", chapter 2.3.1.