Problem set 4
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Problem 1

We can rewrite the initial probel min the following form:

$$\min_{\omega,\xi} \frac{1}{n}\xi e + \frac{1}{2C}\omega^T\omega$$

S.T.

$$\phi(x)\omega^T - y \leq \xi + \varepsilon e$$

$$\phi(x)\omega^T - y \geq -\xi - \varepsilon e$$

$$\xi \geq 0$$

Where e^T is (1, 1 ... $1)^n$

Problem 2

Dual problem has the form:

$$L(\xi,\omega,\lambda,\mu,\nu) = \frac{1}{n}\xi e+\frac{1}{2C}\omega^T\omega+\lambda^T(\phi(x)\omega^T-y-\xi-\varepsilon e)+\mu^T(y-\xi-\varepsilon e-\phi(x)\omega^T)-\nu^T\xi$$

$$g(\lambda,\mu,\nu) = min_{\xi,\omega}L(\xi,\omega,\lambda,\mu,\nu)$$

$$\frac{dL}{\partial\xi} = \frac{1}{n}e - \lambda - \mu - \nu$$

$$\frac{dL}{\partial\omega} = \frac{1}{C}\omega + \phi(x)(\lambda - \mu)$$

The dual problem is:

$$max_{\lambda,\mu,\nu}g(\lambda,\mu,\nu)$$

S.T

$$\lambda,\mu,\nu \geq 0$$

$$\frac{dL}{\partial\xi} = 0$$

We can rewrite this as:

$$\omega = C\phi(x)(\mu - \lambda)$$

$$max_{\lambda,\mu,\nu}\frac{1}{2}C(\phi(x)(\mu-\lambda))^T\phi(x)(\mu-\lambda)+(\lambda^T-\mu^T)\phi(x)C(\phi(x)(\mu-\lambda))^T-\lambda^T(y+\varepsilon e)+\mu^T(y-\varepsilon e)$$

S.T.

$$\lambda, \mu, \nu \geq 0$$

$$\frac{1}{n}e - \lambda - \mu - \nu = 0$$

After simplification we can get:

$$\omega = C\phi(x)(\mu - \lambda)$$

$$max_{\lambda,\mu,\nu} - \frac{1}{2}(\mu - \alpha)K(x,x)(\mu - \alpha) + y^T(\mu - \lambda) - \varepsilon e^T(\mu + \lambda)$$

S.T.

$$\lambda, \mu \geq 0$$

$$\frac{1}{n}e - \lambda - \mu \leq 0$$

Where $K(x,x) = \phi(x)^T\phi(x)$
The dimansionality of the dual problem is 2n.
Problem 3

The prediction is defined by the following formula:

$$\hat{y}(x_{new}) = C(\phi(x)(\mu - \lambda))^T\phi(x_{new}) = C(\mu^T - \lambda^T)\phi(x)^T\phi(x_{new}) = C(\mu^T - \lambda^T)K(x, x_{new})$$

Problem 4

We want the problem to be in the following form:

$$min_x \frac{1}{2}x^TPx + q^Tx$$

S.T.

$$Gx \leq h$$

So we have to define the following:

$$x = \begin{pmatrix} \lambda_1 \\ \vdots \\ \lambda_n \\ \mu_1 \\ \vdots \\ \mu_n \end{pmatrix}$$

$$P = \begin{pmatrix} K & -K \\ -K & K \end{pmatrix}$$

$$q = \begin{pmatrix} y_1 + \varepsilon \\ \vdots \\ y_n + \varepsilon \\ -y_1 + \varepsilon \\ \vdots \\ -y_n + \varepsilon \end{pmatrix}$$

$$G = \begin{pmatrix} -I(n) & 0 \\ 0 & -I(n) \\ I(n) & I(n) \end{pmatrix}$$

$$h = \begin{pmatrix} zeros((2n, 1)) \\ \frac{C}{n} ones((n, 1)) \end{pmatrix}$$

Due to the definition of Kernel function it is positive semidefinite. Thus P is also positive semidefinite.