2020 MIT Science Olympiad Invitational Tournament
Sounds of Music: Written KEY

January 25th, 2020

Exploring the World of Science

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Competitor Names: ANSWER KEY

School Name: ______________________________ Team #: ________

DO NOT OPEN THIS EXAM UNTIL INSTRUCTED!

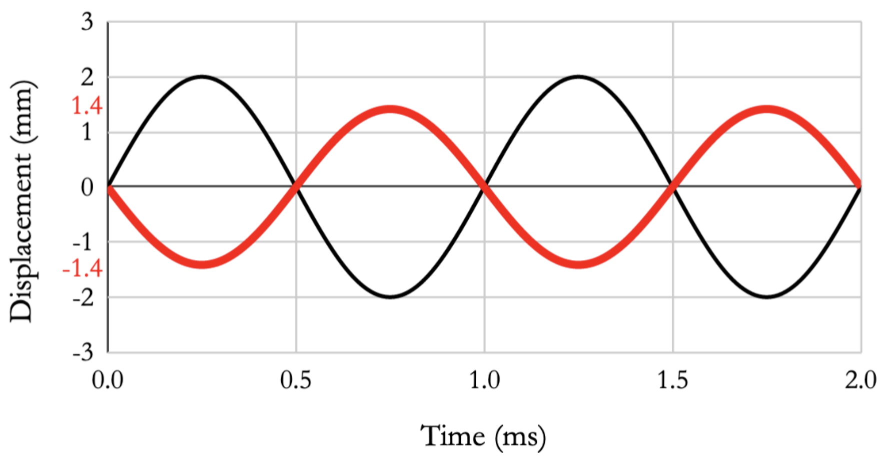
EXAM INFORMATION:

- You will be given **25 minutes** to complete this exam.
- You may use a three-ring binder and two standalone calculators of any type.
- The written exam is divided into two parts: **Section 1: Mathematical Analysis**, 60 points (pgs. 1-6); and **Section 2: Physiology and Behavior**, 40 points (pgs. 7-10). Each page is worth 10 points.
- Once the testing period begins, verify that you have **10 numbered pages** and that your team number is correctly written in the top-left corner on every page. If you are missing a page, let us know immediately.
- For calculations, correct answers alone earn full credit. If your answer is incorrect or incomplete, work will be graded for partial credit.
- Although some questions ask you to answer in a certain number of sentences, you are not required to write in complete sentences. These are merely guidelines for how long your responses should be.
- For all questions, use **343 m/s** for the speed of sound in air.
- Your score on this exam will count for up to 36 points toward your final score and will be scaled using the following formula: (Written Exam score / Highest Written Exam score across all teams) x 36.

WRITTEN EXAM SCORE: 100 **/ 100 points**

SECTION 1: MATHEMATICAL ANALYSIS (60 POINTS)

1) Consider a vibrating guitar string with length L and fixed ends at $x = 0$ and $x = L$. The following graph shows the transverse displacement at ***x* = *L*/2** as a function of time. **(10 points, Tiebreaker #5)**



(a) Is the mode of vibration of the string more likely to be characterized by $n = 3$ or $n = 4$? Justify your answer in 1–2 sentences. [3pts]

It is more likely to be characterized by $n = 3$ because the third harmonic has a vibrating antinode at the center of the string ($x = L/2$), whereas the fourth harmonic has a stationary node there.

(b) Using your value of n from part (a), sketch the transverse displacement at ***x* = *L*/12** as a function of time on the graph above. If the period and/or amplitude is different, clearly label the new value on the graph. [3pts]

(c) Based on your value of n from part (a), what other point(s) on the string would exhibit the same displacement vs. time graph as $x = L/12$? If there are multiple answers, give all of them. [1pt]

$x = L/4$, $x = 3L/4$, and $x = 11L/12$

(d) Using your value of n from part (a), if $L = 0.60$ m and the tension in the string is 72 N, calculate the mass of the string. Include appropriate units and significant figures. **Circle your final answer.** [3pts]

$f = nv/2L \rightarrow v = (2)(1.0 \text{ x } 10^3 \text{ Hz})(0.60 \text{ m})/(3) = 4.0 \text{ x } 10^2 \text{ m/s}$
$v = (\tau/\mu)^{1/2} \rightarrow \mu = (72 \text{ N})/(4.0 \text{ x } 10^2 \text{ m/s})^2 = 4.5 \text{ x } 10^{-4} \text{ kg/m}$
$m = \mu L = (4.5 \text{ x } 10^{-4} \text{ kg/m})(0.60 \text{ m})$ = **$2.7 \text{ x } 10^{-4}$ kg** or **0.27 g**

If $n = 4$ is used instead of $n = 3$, the accepted answer is **$4.8 \text{ x } 10^{-4}$ kg** or **0.48 g**.

2) Consider a hollow pipe with two open ends at $x = 0$ and $x = L$ and **an open hole at $x = 0.40L$.** **(10 points)**

(a) Draw a standing wave diagram showing the displacement amplitude from $x = 0$ to $x = L$ for the lowest harmonic that can be produced when the hole at $x = 0.40L$ is uncovered. Label all displacement nodes and antinodes. [3pts]

(b) Based on your standing wave diagram from part (a), if the air particles at **$x = 0.15L$** are moving rightward at a given point in time, indicate the direction of the net movement of air particles at each of the following positions in the same moment. **Circle one answer per row.** [2pts]

$x = 0.60L$	***moving left***	***moving right***	***no net movement***
$x = 0.75L$	***moving left***	***moving right***	***no net movement***
$x = 0.90L$	***moving left***	***moving right***	***no net movement***

(c) Based on the standing wave in part (a), order the three positions in part (b) ($x = 0.60L$, $x = 0.75L$, $x = 0.90L$) from lowest to highest **pressure amplitude.** [1pt]

$x = 0.60L < x = 0.75L < x = 0.90L$

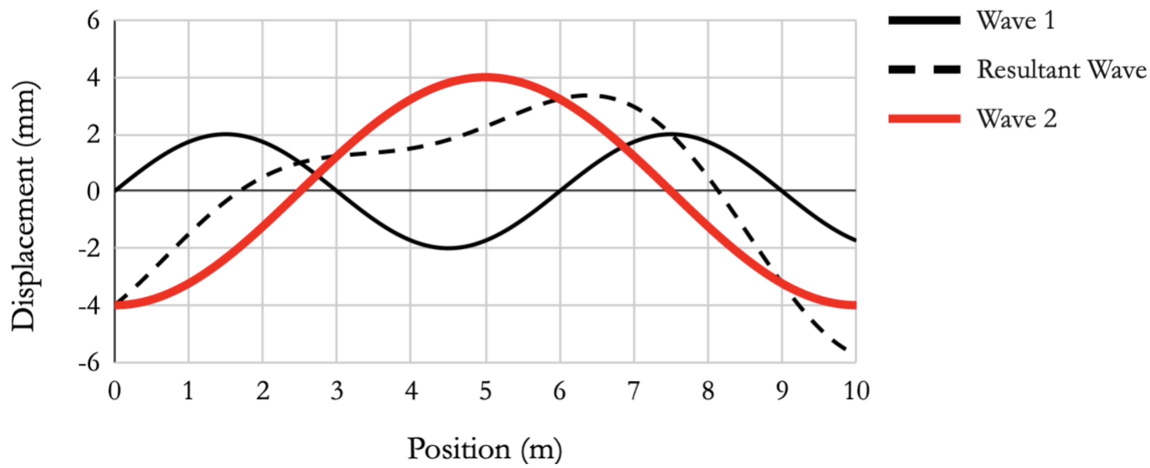
(d) If one end of the pipe is sealed off at $x = L$, calculate the **two** lowest frequencies that can be sounded if the hole at $x = 0.40L$ remains open and the length L of the pipe is 0.80 m. Include appropriate units and significant figures. **Circle both final answers.** [3pts]

Lowest two harmonics: $n = 5$ and $n = 15$ (n must be odd for a closed-end pipe)
5th harmonic: $f = nv/4L = (5)(343\text{ m/s})/(4)(0.80\text{ m})$ = **540 Hz**
15th harmonic: $f = (3)(540\text{ Hz})$ = **1600 Hz**

(e) For the closed-end pipe formed in part (d), at what other value(s) of x could another hole be added without changing the harmonics that can be produced? If there are multiple answers, give all of them. [1pt]

$x = 0.80L$ ($x = 0.64$ m)

3) The following graph shows the displacements of sound waves as a function of position at $t = 0$ s. **(10 points)**



(a) Sinusoidal waves can be modeled by the equation $s = s_{max} \sin(kx - \omega t)$, where k is the angular wavenumber and ω is the angular frequency. If these are sound waves in air at 20.0 ˚C, write an equation for the displacement of **Wave 1 (solid curve)** as a function of position and time. Include appropriate units. **Circle your final answer.** [3pts]

$k = 2\pi/\lambda = 2\pi/(6\text{ m}) = \pi/3\text{ m}^{-1}$ or 1.05 m^{-1}

$\omega = vk = (343\text{ m/s})(\pi/3\text{ m}^{-1}) = 343\pi/3\text{ s}^{-1}$ or 359 s^{-1}

Complete equation: $\boldsymbol{s = (2\text{ mm})\sin(\pi x/3 - 343\pi t/3)}$ or $\boldsymbol{s = (2\text{ mm})\sin(1.05x - 359t)}$

(b) The resultant wave (dashed curve) is formed by the superposition of two sinusoidal waves, Wave 1 (shown) and Wave 2 (not shown). On the graph above, sketch the displacement of **Wave 2** from $x = 0$ m to $x = 10$ m. [2pts]

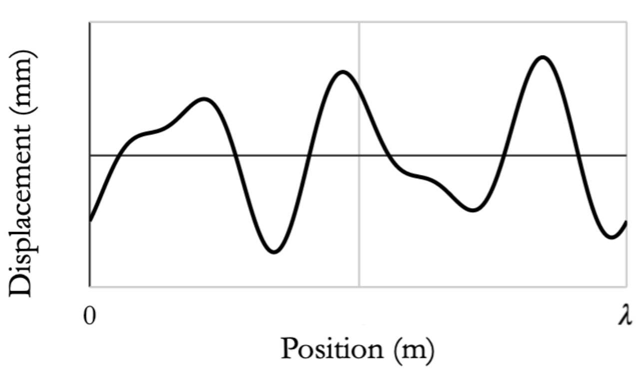
(c) What is the wavelength of the resultant wave? Include appropriate units. [1pt]

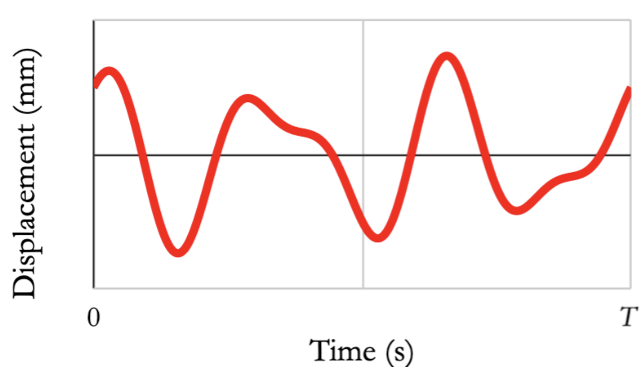
30 m

(d) At which positions does constructive interference occur between Waves 1 and 2? **Circle all that apply.** [2pts]

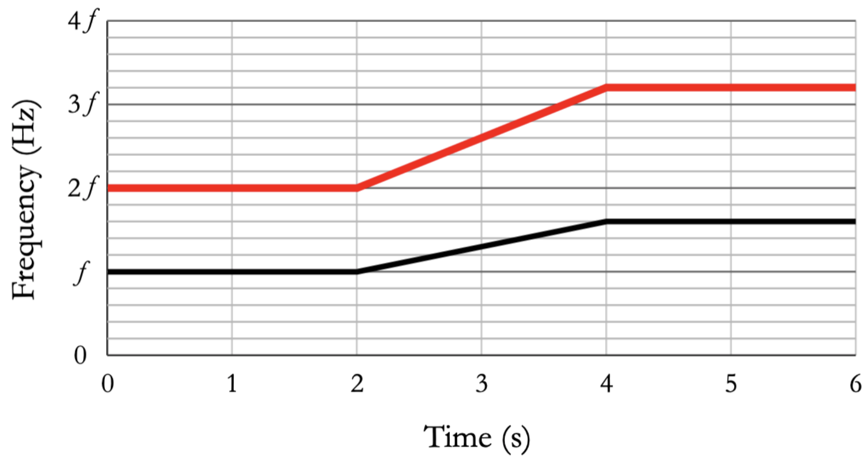
x = 4.0 m **x = 7.0 m** **x = 8.0 m** **x = 13.0 m** **x = –2.0 m**

(e) The graph on the left shows one complete waveform of the resultant wave at $t = 0$ s. On the right set of axes, sketch the displacement at **$x = \lambda/2$** for one period of the resultant wave as a function of time. [2pts]





4) The following graph shows the **fundamental frequency** of the notes played by an instrument as a function of time. The scale of the *y*-axis is based on multiples of the initial frequency *f*. **(10 points)**



(a) Which of the following instruments could have produced this sound? **Circle all possible answers.** [2pts]

bassoon ***euphonium*** ***harpsichord*** ***marimba*** ***trombone*** ***voice***

(b) To the nearest whole number, determine the number of semitones between the initial and final pitches. [2pts]

$f'/f = 2^{n/12} \rightarrow n = 12 \log(1.6)/\log(2) = \mathbf{8}$

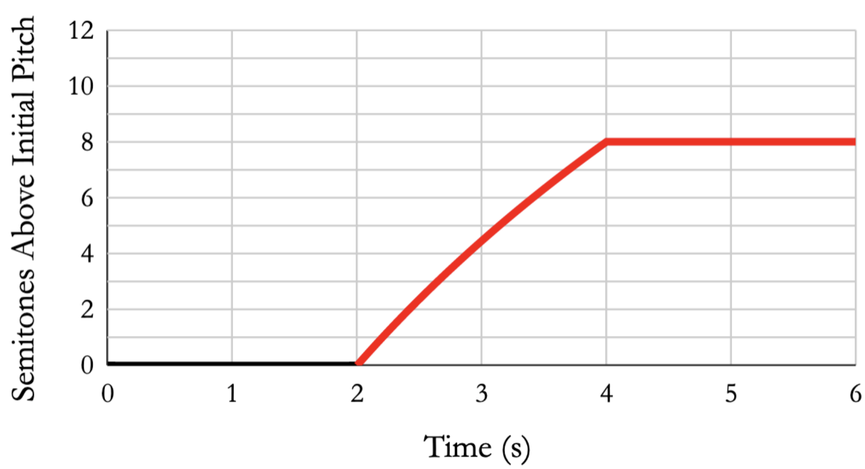
(c) Based on part (b), give **two** enharmonic names for the interval between the initial and final pitches. [2pts]

minor sixth, augmented fifth

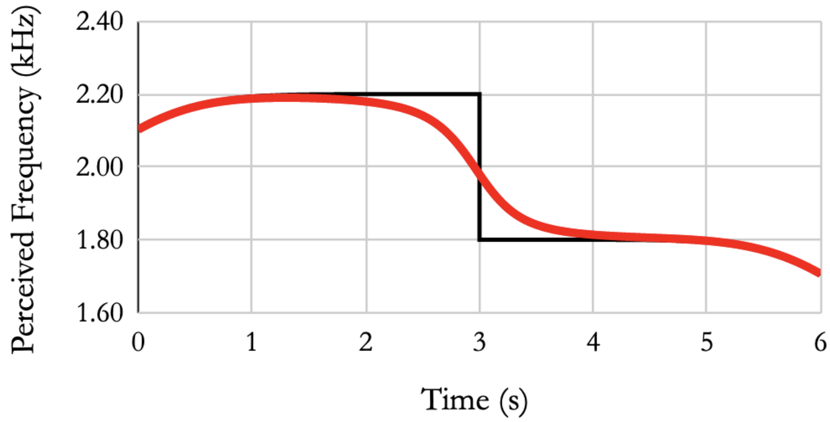
(d) On the graph **above**, quantitatively sketch the frequency of the **second harmonic** as a function of time. [1pt]

(e) Circle the phrase that completes this sentence: The interval between the initial and final fundamental pitches is (***smaller than*** / ***the same as*** / ***larger than***) the interval between the initial and final second harmonic pitches. [1pt]

(f) On the axes below, sketch a curve for the **pitch** of the fundamental as a function of time. The region of the curve from 2 s < *t* < 4 s can be drawn qualitatively, without any additional calculations. [2pts]



5) A fire truck blares a siren of **constant frequency** and drives past you while you stand directly on the edge of the road. The following graph shows the frequency you perceive as a function of time. **(10 points, Tiebreaker #4)**



(a) Circle the phrases that complete this sentence: Relative to the ground, the speed of the sound waves in front of the truck is (***less than*** / ***equal to*** / ***greater than***) the speed of the waves behind the truck, and the wavelength of the waves in front of the truck is (***shorter than*** / ***equal to*** / ***longer than***) the wavelength behind the truck. [1pt]

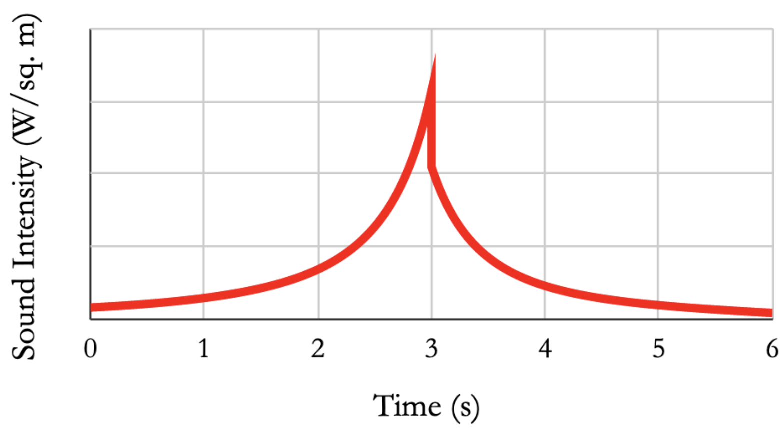
(b) Circle the phrases that complete this sentence: The truck is (***speeding up*** / ***slowing down***) from 0 s < t < 1 s and (***speeding up*** / ***slowing down***) from 5 s < t < 6 s. [1pt]

(c) Calculate the speed of the truck from 2 s < t < 4 s to at least two significant figures **and** the actual frequency of the siren to at least three significant figures. Include appropriate units. **Circle both final answers.** [3pts]

2.20 kHz = f(343 m/s)/(343 m/s − v_{truck})
1.80 kHz = f(343 m/s)/(343 m/s + v_{truck})
Divide equations and solve: v_{truck} = **34 m/s**, f = **1.98 kHz**

(d) On the graph above, qualitatively sketch the frequency you would perceive as a function of time if you were standing a fixed distance away from the road, rather than directly on the edge. [2pts]

(e) On the axes below, qualitatively sketch the **sound intensity** at your location as a function of time when you stand directly on the edge of the road (original scenario). [3pts]



6) In a Cartesian plane, a row of microphones is placed along the line **x = 5.00 m.** **(10 points, Tiebreaker #2)**

(a) A constant-power 30.0-W speaker located at (0.00 m, 1.00 m) emits a constant 400.0-Hz tone. Calculate the highest sound intensity level (in dB) recorded along the line x = 5.00 m **to at least one decimal place.** Assume spherical sound propagation in three dimensions with no friction or reflection. **Circle your final answer.** [2pts]

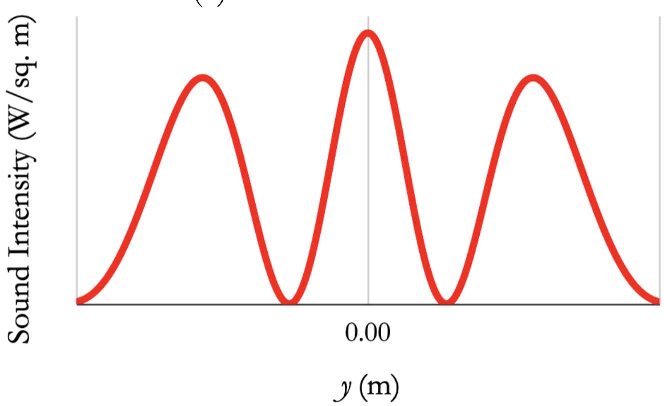
$I = P/(4\pi r^2) = (30.0\text{ W})/(4\pi(5.00\text{ m})^2) = 0.0955\text{ W/m}^2$

$\beta = 10\log(I/I_0) = 10\log((0.0955\text{ W/m}^2)/(1.0 \times 10^{-12}\text{ W/m}^2)) = \mathbf{109.8\ dB}$

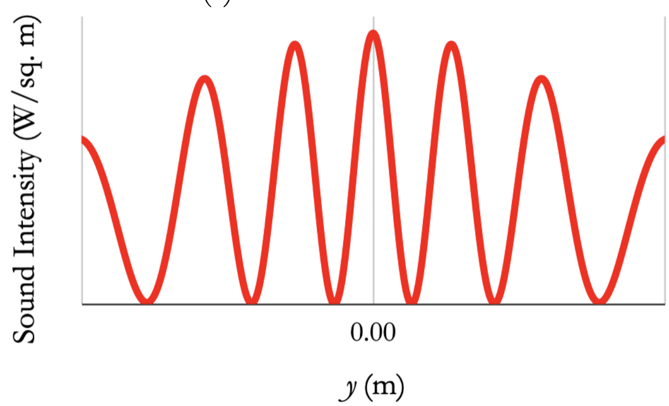
(b) In addition to the speaker at (0.00 m, 1.00 m), a second identical 30.0-W speaker at (0.00 m, −1.00 m) also emits a 400.0-Hz tone. Both speakers emit sound completely in phase with each other. On the left set of axes, qualitatively sketch the sound intensity as a function of y-position along the row of microphones at x = 5.00 m. [2pts]

(c) The two 30.0-W speakers (still in phase) now emit constant 800.0-Hz tones instead of 400.0-Hz tones. On the right set of axes, qualitatively sketch the sound intensity as a function of y-position along the row of microphones at x = 5.00 m. Any differences between this graph and part (b) should be clearly visible. [2pts]

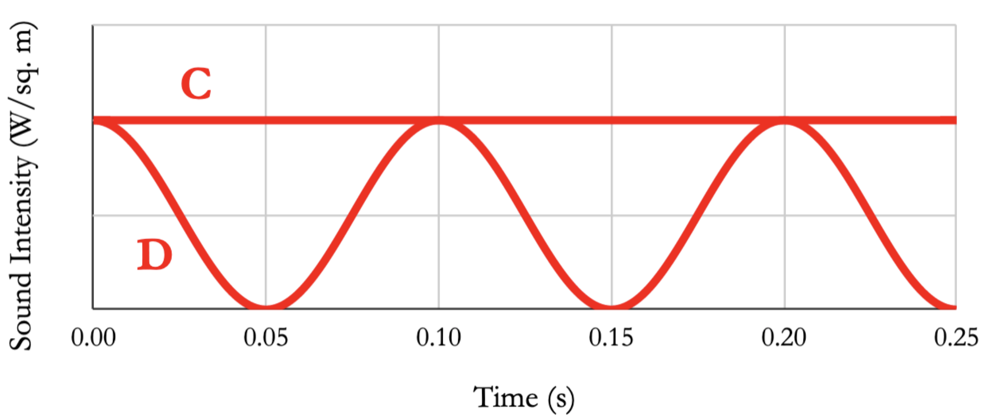
(b) Both tones at 400.0 Hz



(c) Both tones at 800.0 Hz



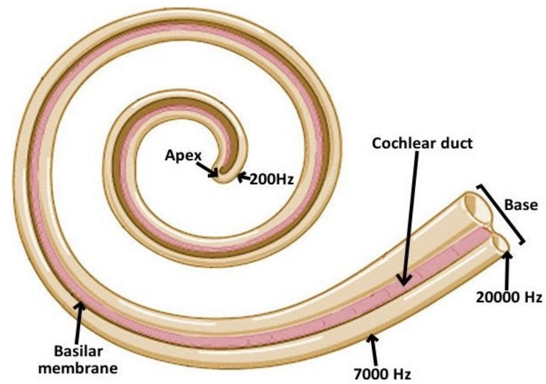
(d) Now, one speaker emits a tone at 800.0 Hz, and the other emits a tone at 810.0 Hz. On the axes below, sketch curves showing **sound intensity vs. time at (5.00 m, 0.00 m)** for the setup in part (c) (two 800.0-Hz tones) **and** for the setup described here (800.0-Hz and 810.0-Hz tones). Label the two curves "C" and "D," respectively. [4pts]



SECTION 2: PHYSIOLOGY AND BEHAVIOR (40 POINTS)

7) Sound waves that enter the inner ear vibrate the basilar membrane (image below), causing sensory neurons to fire and send signals to the brain via the cochlear nerve. There are two predominant theories for auditory transduction: place theory and temporal theory. According to place theory, frequency is encoded by the location at which sound maximally vibrates the basilar membrane. On the other hand, temporal theory proposes that sensory neurons encode frequency by firing at the same rate as the frequency of the sound wave. **(10 points, Tiebreaker #1)**

(a) Place theory suggests that each point along the basilar membrane only vibrates in response to a specific frequency of sound. In one sentence, name and describe the physical phenomenon responsible for this specificity. [2pts]



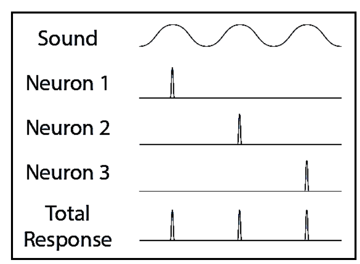
This is an example of resonance, where the frequency of a sound matches the natural oscillation frequency of an object and increases the amplitude of the object's vibrations.

(b) The above diagram indicates the points at which frequencies of 200 Hz, 7000 Hz, and 20000 Hz are transduced according to place theory. Based on this information, circle the phrase that completes the following sentence: You would expect the stiffness of the basilar membrane to be greatest (***at the apex*** / ***at the base*** / ***in the center***). [1pt]

(c) The absolute refractory period (the minimum time between successive firings) of an auditory neuron is 1.60 ms. Based on temporal theory, what range of frequencies can a single neuron encode? Include appropriate units. [2pts]

$T \geq 1.60$ ms → $f \leq (1/0.00160$ s$)$ → **625 Hz and below (0 Hz < f ≤ 625 Hz** or **20 Hz ≤ f ≤ 625 Hz)**

(d) Volley theory extends temporal theory by proposing that multiple neurons can fire in staggered intervals to encode frequencies higher than a single neuron can encode (image right). Based on your answer to part (c), at least how many neurons are needed to encode a frequency of 5000 Hz according to volley theory? [1pt]

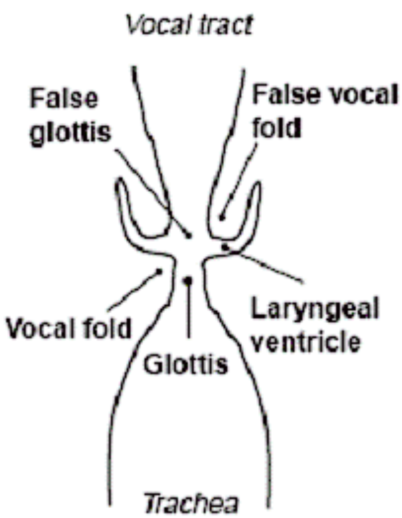


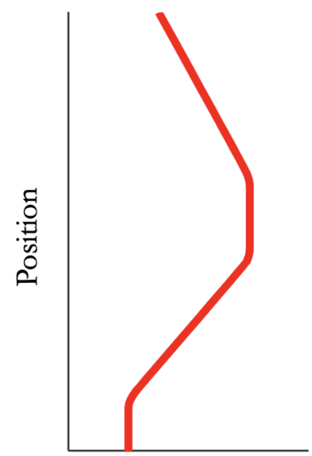
(5000 Hz)/(625 Hz) = **8**

(e) A single earbud in a person's ear plays tones at 400 Hz, 600 Hz, 800 Hz, and 1000 Hz. What frequency will the person primarily perceive? Does this result support place theory or temporal/volley theory? Justify your answer to the second question in 1–2 sentences. [4pts]

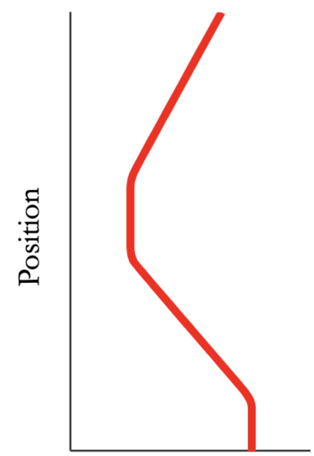
The person will primarily perceive a frequency of 200 Hz (the missing fundamental). This result supports temporal/volley theory because this combination of frequencies has an overall frequency of 200 Hz and would maximally stimulate neurons at a rate of 200 Hz. In place theory, regions encoding different frequencies are spatially separated along the membrane, so there would be no reason for stimulation of the regions encoding 400 Hz, 600 Hz, etc. to activate the 200-Hz region in an entirely different location.

8) In the human respiratory system, air flows upward from the trachea through the glottis (the open space between the vocal folds) and into the vocal tract. During phonation, air passes through the larynx and vibrates the vocal folds, which rapidly open and close to create sound waves. **(10 points)**





(a) Flow Velocity



(b) Air Pressure

Note: For parts (a) and (b), exact curve shape and concavity do not matter; only the general trend is important.

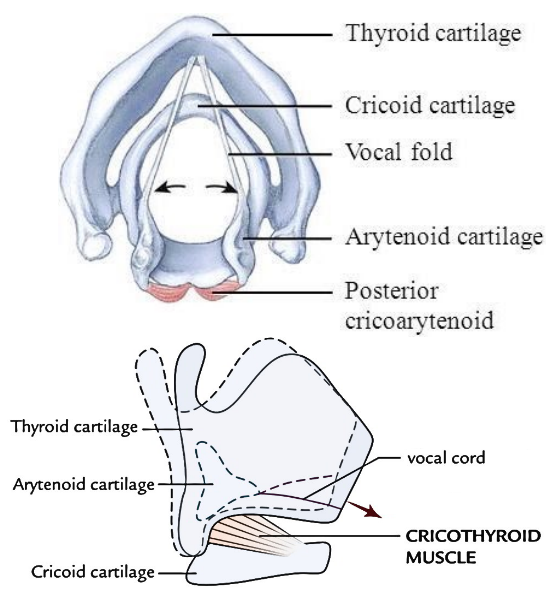
(a) On the left set of axes, qualitatively sketch the **linear airflow velocity** through the larynx as a function of position (velocity increases to the right). The positions on the vertical axis align with the diagram to the left. [2pts]

(b) On the right set of axes, qualitatively sketch the **air pressure** in the larynx as a function of position. The positions on the vertical axis align with the diagram to the left. **Justify your answer below in one sentence.** [4pts]

According to Bernoulli's principle, an increase in flow velocity corresponds with a decrease in fluid pressure.

(c) When producing sound, the vocal folds can be modeled as vibrating strings. They are attached anteriorly to the thyroid cartilage and posteriorly to the arytenoid cartilages (image right). When the cricothyroid muscle contracts and pulls the thyroid cartilage forward and downward (image bottom-right), what does this physically do to the vocal folds? How does this affect the sound produced? [3pts]

This stretches and increases the tension in the vocal folds, thereby increasing the frequency (pitch) of the sound.



(d) Circle the phrases that complete this sentence: The vocal folds of a soprano are (***shorter*** / ***longer***) and have (***less mass*** / ***more mass***) than the vocal folds of an alto. [1pt]

9) Bats use echolocation to detect obstacles and prey in their surroundings. **(10 points, Tiebreaker #3)**

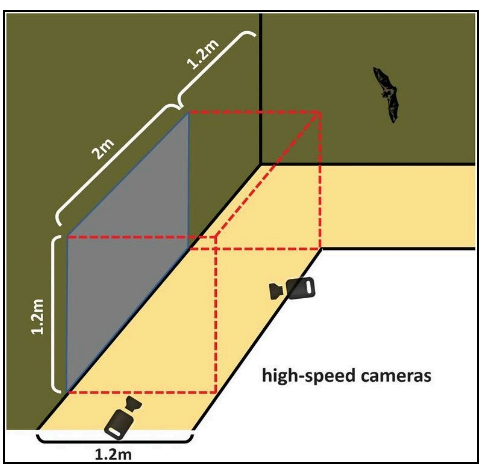
(a) Bats perceive the properties of a reflected sound wave and compare them to the original sound wave emitted. For each of the following properties of the returning signal, what information does the bat learn about nearby objects in its surroundings? (Be specific—simply writing "location" is insufficient.) [3pts]

Property	**Information Provided to the Bat**
Intensity	distance to objects
Frequency	motion/velocity of objects
Interaural time difference	angle/direction in which objects are located

(b) At any given moment, a bat emitting multiple signals in succession can only interpret reflections of the last signal it produced. If a stationary bat continually emits one 45-kHz signal every 70. ms, what is the maximum distance at which it can detect objects? Include appropriate units. **Circle your final answer.** [2pts]

Max. distance = ½ (distance traveled by sound) → $d = \frac{1}{2} vt = \frac{1}{2}(343 \text{ m/s})(0.070 \text{ s}) = \mathbf{12 \text{ m}}$

(c) An experiment uses two hallway-like chambers, in which bats are released and directed to fly around a corner (image below). The highlighted part of the wall is made of smooth metal in Chamber A, while the same part of the wall is made of jagged rock in Chamber B. In which chamber, A or B, are the bats more likely to crash into the wall? Justify your answer in 1–2 sentences. [3pts]



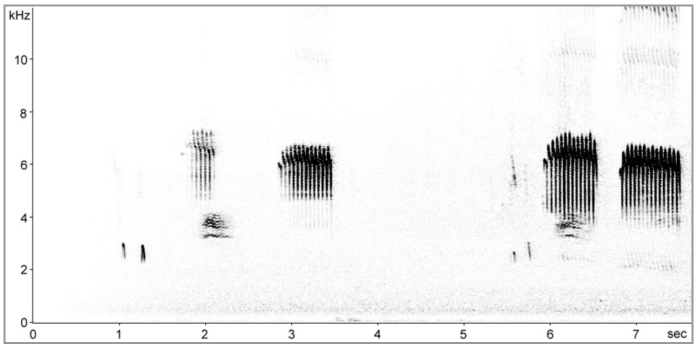
The bats are more likely to crash in Chamber A because all emitted sound waves will reflect off the smooth metal surface at the same angle (specular reflection) and travel away from the bat. In Chamber B, the wall's uneven texture scatters sound waves in many different directions (diffuse reflection), so some signals will return to the bat.

(d) Like bats, dolphins also use echolocation, but underwater instead of in air. Consider a flying bat and a swimming dolphin that emit calls of equal frequency while traveling at the same speed directly toward stationary objects equal distances away. Let t be the time it takes to receive the reflected signal and f be the frequency of the reflected signal. Complete the following comparisons by circling either <, =, or >. [2pts]

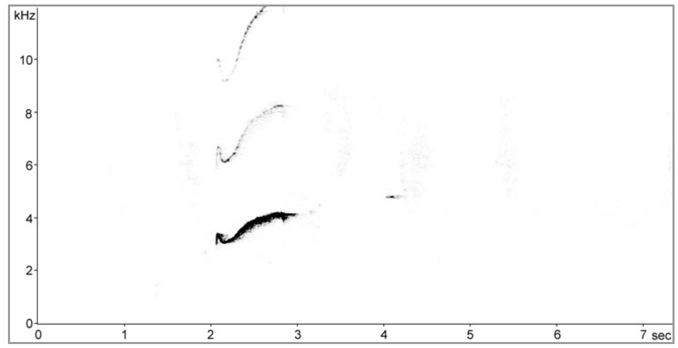
t_{bat} (< = **>**) $t_{dolphin}$ f_{bat} (< = **>**) $f_{dolphin}$

10) Given below are the sonograms for two different North American songbirds (order Passeriformes). **(10 points)**

Bird A (*Bombycilla garrulus*):



Bird B (*Contopus virens*):



(a) Circle the phrases that complete this sentence: Bandwidth is measured in (***seconds*** / ***hertz*** / ***meters***), and Bird A's call has a (***smaller*** / ***larger***) bandwidth than Bird B's call. [1pt]

(b) Based on the sonograms above, answer the following questions by circling either **A** (*B. garrulus*) or **B** (*C. virens*). For each question, justify your selection in 1–2 sentences. [6pts]

Which bird produces sound waves with the waveform on the right? **A** **B**

- Justification: The given waveform is formed by a combination of multiple harmonics, and B's call has clear overtones, whereas A's call only has very faint overtones. The waveform also has a regular, well-defined period, and B's call is a pure tone with a specific frequency, whereas A's call has a wider range of frequencies and thus an ill-defined overall wave period.

Which bird is more likely to live in a densely wooded forest? **A** **B**

- Justification: B's call has a lower frequency than A's, and low frequencies travel further than high frequencies because they diffract more easily around obstacles, are scattered less, and are absorbed less by surfaces. Additionally, the many different frequencies in A's call would be easily distorted and desynchronized by reverberations, whereas a pure tone like B's call is preserved.

(c) Ornithologists often describe the call of *Bombycilla garrulus* as a trill; however, what ornithologists consider a trill is different from the definition of a trill in music. From a musical standpoint, explain in 1–2 sentences why the word "trill" is not a good descriptor of Bird A's call. What other musical term would be more appropriate instead? [3pts]

A musical trill involves a rapid alternation between two adjacent notes, whereas Bird A's call is characterized by the rapid restriking of a single note. It would be better described as a tremolo.