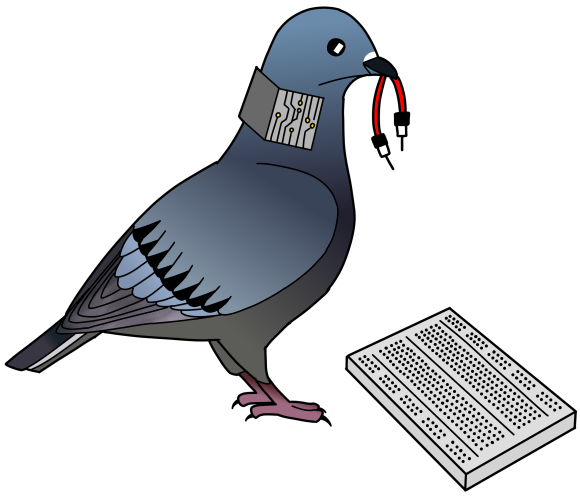
2021 BirdSO
Science Olympiad C-Division
Circuit Lab

March 7th, 2021



Drawn and colored by Ruth, Gwennie, Crystal, and Allen!

(A) (55 questions, 100 points): Multiple choices, T/F and Fill-In-Blanks
(B) (20 questions, 100 points): Short-Answers questions

Total score: _____ /200 Rank: _____ / _____

Instructions
1) Follows standard sig-fig rules and include units when appropriate.
2) One to two sentences should be sufficient for short-answer questions.
3) Test length is 50 minutes
4) Some problems are adapted from Circuit textbooks and Physics Olympiad training materials. Use this for your own educational purpose only

Event Supervisor: Lei Jiang

Page score: Team name/number:

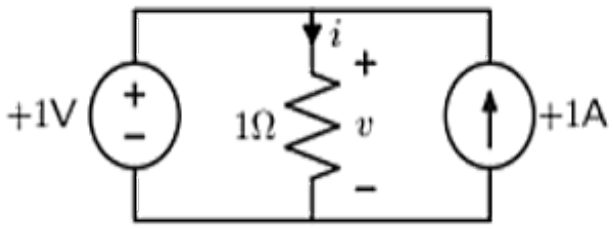
A T/F, Fill-in-Blank, and Multiple Choices

1. [**T/F**] When two positive point charges move away from each other, the magnitude of their electrostatic potential energy increases. False
2. [**T/F**] [Not scored for ambiguity] The potential difference across the terminal of a car battery is same during charging and discharging. False
3. [**T/F**] If a magnet is brought closer to a current-carry loop along its axis, the current always decreases in the loop. False
4. [**T/F**] A truth table illustrates how the input level of a gate responds to all possible output combinations. False
5. [**T/F**] A transistor amplifier device in common emitter configuration has a low input impedance. True
6. [**T/F**] The ratio of electrical field ($\vec{E}$) and magnetic field ($\vec{B}$) has the dimension of velocity. True
7. [**T/F**] If a U-shaped tube filled with diamagnetic solutions has one of its arms placed in between the poles of a strong magnet with the free-surface perpendicular to the field, the level of the solution falls. True
8. [**T/F**] A NOR gate output is LOW If any of its inputs is LOW False
9. [**T/F**] A NAND gate output is LOW only if all the inputs are HIGH True
10. [**T/F**] An OR gate output is HIGH only if all the inputs are HIGH. False

11. The conductivity of semiconductor increases with increase in temperature because [1 point]
    (A) both number density of charge carriers and relaxation time increases.
    (B) number density of charge carriers increases.
    (C) number density of current carriers increases, relaxation time decrease but effect is much less than number density increases.
    (D) relaxation time increases.

12. What is the current going through the resistor below?

    (A) 1 A
    (B) 2 A
    (C) 0 A
    (D) Cannot be determined
    (E) None of the above



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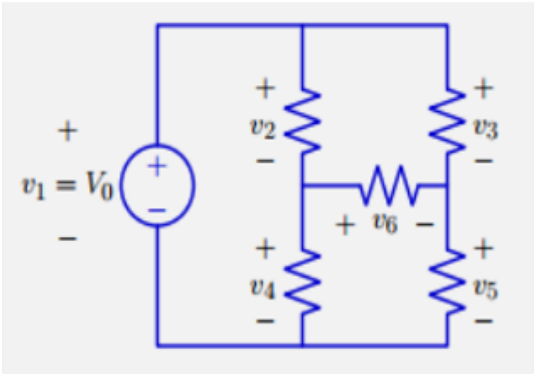
13. A cell of internal resistance r drives current through an external resistance R. The power delivered by the cell to the external resistance will be maximum when [1 point]

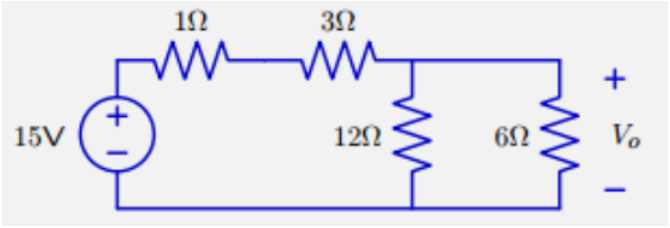
(A) $R = 0.001r$
(B) $R = r$
(C) $R = 2r$
(D) $R = 1000r$

14. [Not scored for ambiguity] How many KVL equations can be written for the circuit to the right [1 point]



(A) 3
(B) 4
(C) 5
(D) 6
(E) 7

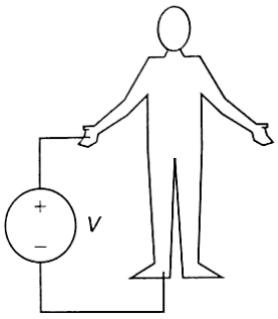
15. Which of the following is true? [1 point]

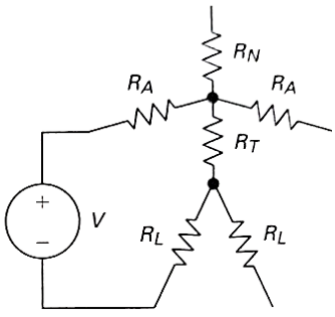


(A) $V_o \leq 3V$
(B) $3V < V_o \leq 6V$
(C) $6V < V_o \leq 9V$
(D) $9V < V_o \leq 12V$
(E) $V_o \leq 3V$

16. The main risk of human electrical injury is to the nerve system. The current level for physiological reactions are (1) Barely perceptible at 3-5mA, (2) Extreme pain at 35-50mA, (3) Muscle paralysis at 50-70mA (4) Heart stoppage at 500 mA+, (5) Skin burns at 1.2A+
Typical human body model below has arm resistance at 400 Ω, leg resistance at 200 Ω, and trunk resistance at 50 Ω. [1 point]

What is the level of risk for a 200V source?





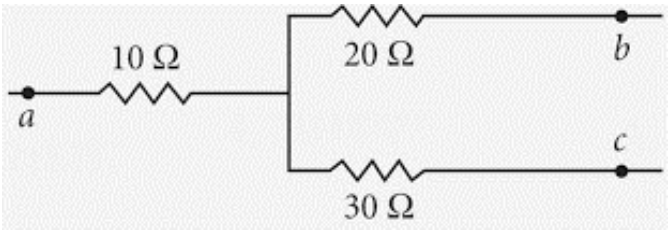
(A) Barely perceptible
(B) Extreme pain
(C) Muscle paralysis
(D) Heart stoppage
(E) Burn

17. For a p-type semiconductor, which of the following is true? [1 point]

(A) Electrons are the majority carriers, and pentavalent atoms are the dopants.
(B) Holes are the majority carriers and trivalent atoms are the dopants.
(C) Electrons are the majority carriers and trivalent atoms are the dopants
(D) Holes are the majority carriers and pentavalent atoms are the dopants.

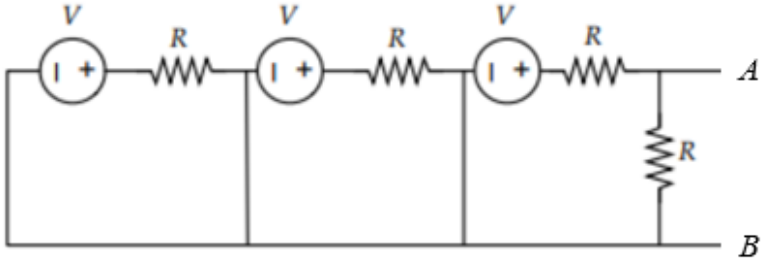
18. A section of a circuit is shown below with voltage at points a, b, c as 30 V, 12 V, and 2 V respectively. The current through the 10 Ω resistor is _____ (A). [1 point]

(A) 10
(B) 5
(C) 2.5
(D) 2
(E) 1



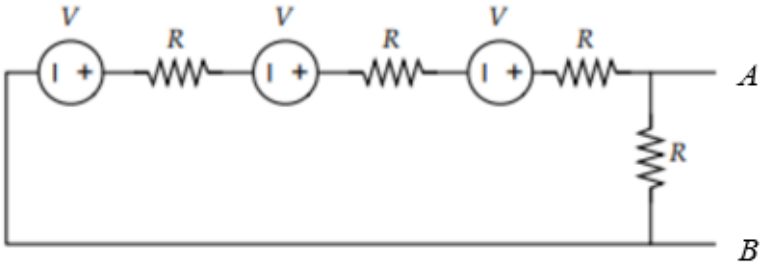
19. What is the voltage delta A-B? [1 point]

(A) 3 V
(B) 1.5 V
(C) 1 V
(D) 0.75 V
(E) 0.5 V



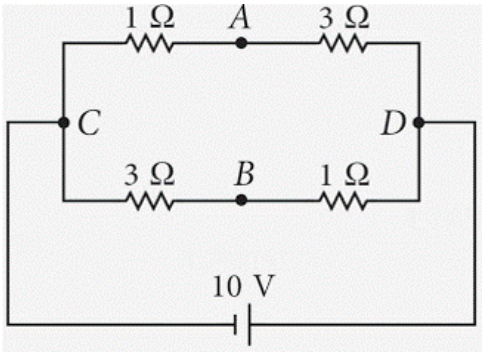
20. What is the voltage delta A-B? [1 point]

(A) 3 V
(B) 1.5 V
(C) 1 V
(D) 0.75 V
(E) 0.5 V



21. What is the voltage delta A-B? [1 point]

(A) 2 V
(B) 5 V
(C) 7.5 V
(D) 9.5 V



22. A wire has a resistance of 16 Ω. It is melted and drawn out into a wire of half its length, with uniform cross section. What is the most reasonable resistance for the new wire? [1 point]

(A) 32 Ω
(B) 16 Ω
(C) 8 Ω
(D) 4 Ω
(E) 2 Ω
(F) 1 Ω

23. What is the capacitance of a 1-meter long cable where the central conductor is 1-mm in diameter and the shield is 5-mm in diameter? [2 points]

(A) 15 pF
(B) 40 pF
(C) 34.5 pF
(D) 36 pF

24. An electric bulb rated as 500W for 100V is used in circuit with a 200V supply. What is the resistance that must be put in series with the bulb so that it delivers 500W safely? [2 points]

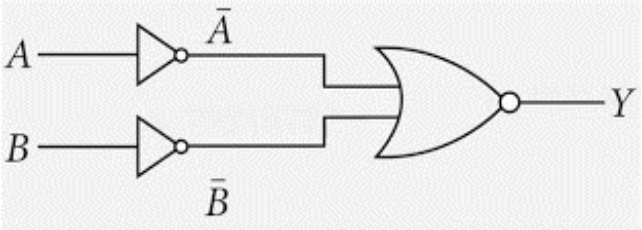
(A) 50 Ω
(B) 15 Ω
(C) 5.2 Ω
(D) 7.5 Ω
(E) 20 Ω
(F) 10 Ω

25. A coil with mean area of 500 cm^2 and 1000 turns is held perpendicular to a uniform field of 0.4 gauss. The coil is turned 180 $=^o$ in one-tenth second. What is the closest estimate to the average induced emf? [2 points]

(A) 0.4 V
(B) 0.04 V
(C) 7.50 V
(D) 2.04 V
(E) 4.00 V
(F) 40.0 V

26. Which logic gate is represented by the following combinations? [2 points]

(A) OR
(B) NAND
(C) AND
(D) NOR
(E) NOT



27. The frequency band of a radio station is 5 MHz to 10 MHz. What is the the corresponding wavelength band? [2 points]

(A) 10-30 m
(B) 300-600 m
(C) 1-2 m
(D) 30-60 m
(E) 10-20 m
(F) 3-6 m

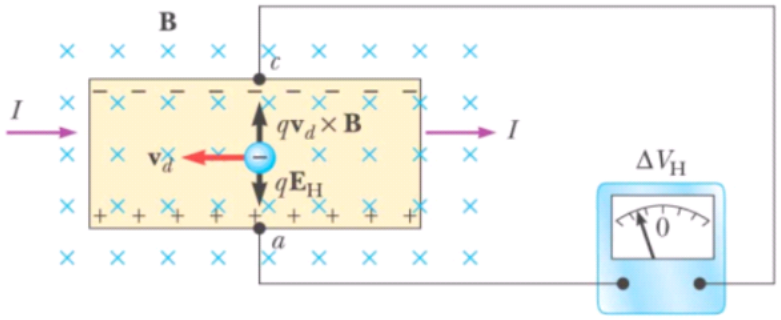
28. A 10V Zener diode along with a series resistance is connected across a 40V supply. Calculate the minimum resistance required if the maximum Zener current is 50mA. [2 points]

(A) 500 Ω
(B) 7‘Ω
(C) 600 Ω
(D) 200 Ω

29. A ferroelectric material has the following attributes (Mark ALL correct answers) [2 points]

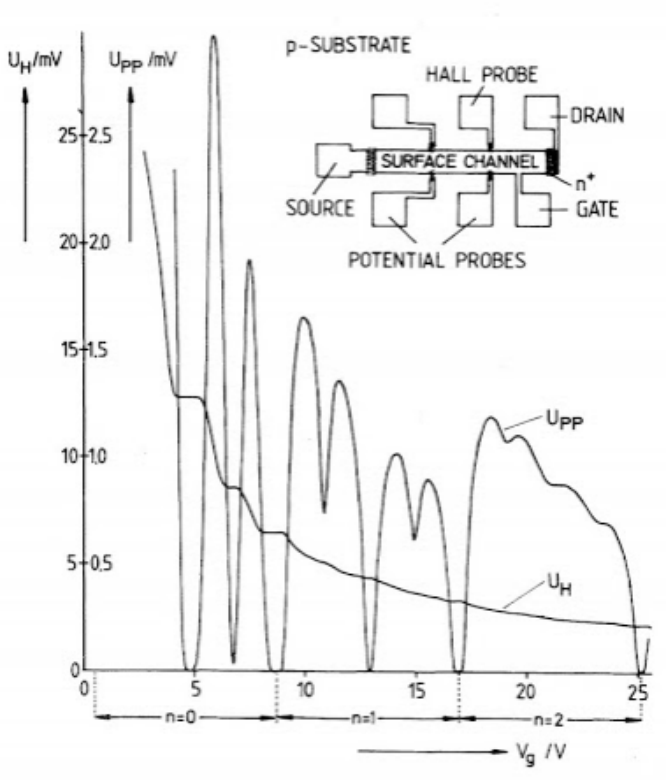
(A) It has an asymmetric crystal structure.
(B) It exhibits a structural transition across the Curie temperature.
(C) It shows a polarization vs. electric field hysteresis loop with the application of electric field.
(D) It is the same as an electret.
(E) It can be used to build capacitor with negative capacitance.
(F) It has stronger quantum spin interactions than paramagnetic materials.

30. Diagram below illustrates the _____ effect. [2 points]

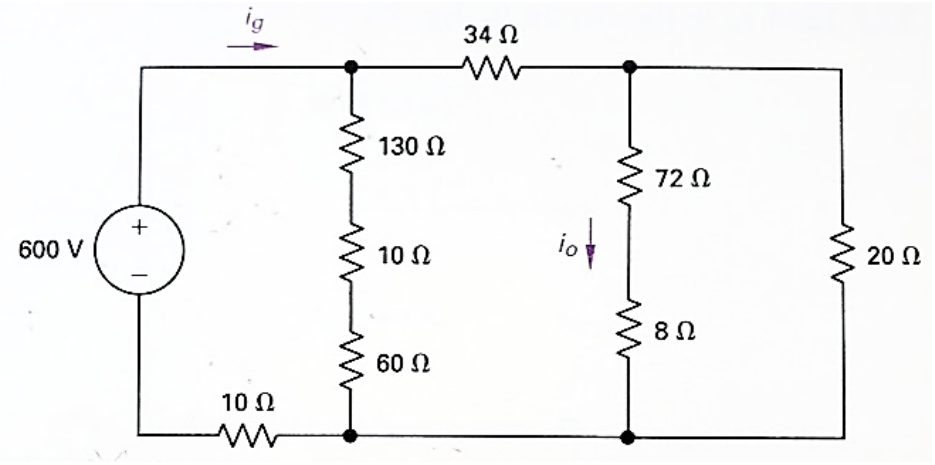


Answer: Hall effect

31. The data below is first demonstration of _____ effect using a narrow layer of conducting semiconductor. [2 points]
Answer: Quantum Hall effect

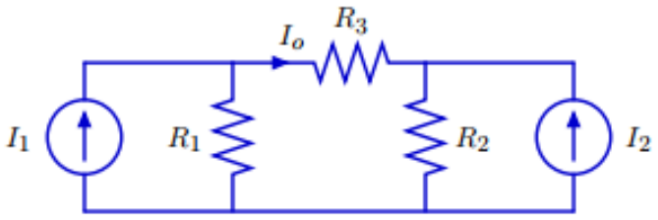


32. The currents are i_g = _____ (A) and i_o = _____ (A) in this circuit. [2 points]

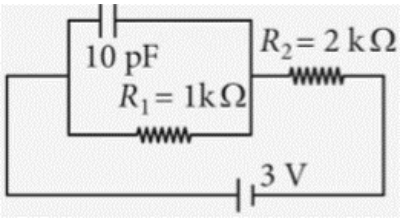


Answer: 12A, 1.92A

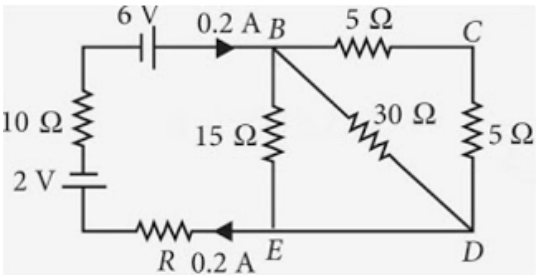
33. The current I_0=_____ (A) if $I_1 = 1.0$A, $I_2 = 0.5$A, $R_1 = 200\Omega$, $R_2 = 100\Omega$, $R_3 = 300\Omega$. [2 points]
Answer: 0.25A



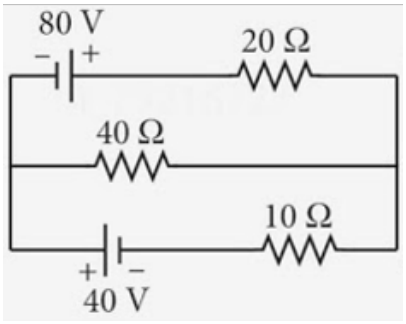
34. The voltage and current passing through resistor R_2 in the circuit is _____V and _____ mA, respectively.
[2 points]
Answer: 2 V, 1 mA



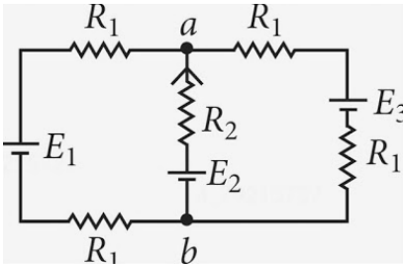
35. In the following circuit, the value of R is _____ Ω, and the voltage between point B and E is _____ V.
[3 points]
Answer: 5Ohm, 1 V



36. The current in the 40 Ω resistor is _____ (A), and the current through the 20 Ω resistor is _____ (A).
[3 points]
Answer: 0 A, 1A

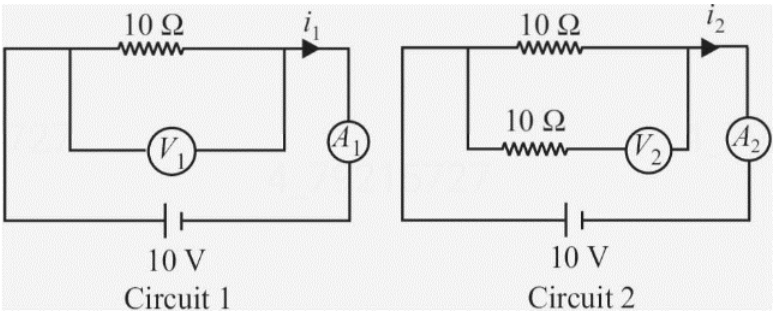


37. For the circuit below, with $R_1 = 1.0\Omega$, $R_2 = 2.0\Omega$, $E_1 = 2$ V and $E_2 = E_3 = 4$ V. The potential difference between the points a and b = _____ V.
[3 points]
Answer: 3.3 V



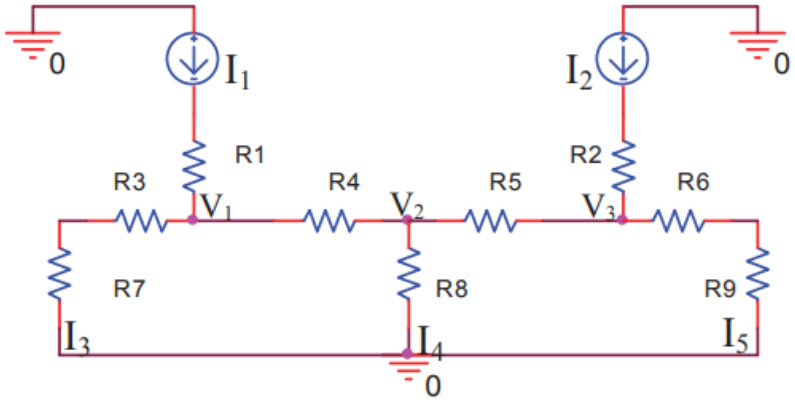
38. In the two circuits below, the reading of voltmeter and ammeter should be
[2 points]

(A) $V_2 > V_1$ and $i_1 > i_2$
(B) $V_2 > V_1$ and $i_1 = i_2$
(C) $V_2 = V_1$ and $i_1 > i_2$
(D) $V_2 = V_1$ and $i_1 = i_2$
(E) $V_2 < V_1$ and $i_1 < i_2$
(F) $V_2 > V_1$ and $i_1 = i_2$



39. In the circuit below, $I_1 = I_2 = 1\text{A}$. Assume $R_1 = R_2 = R_3 = R_4 = R_5 = R_6 = 1\Omega$, $R_7 = R_8 = R_9 = 0.333\Omega$. What would be the ratio of I_3/I_4? [2 points]

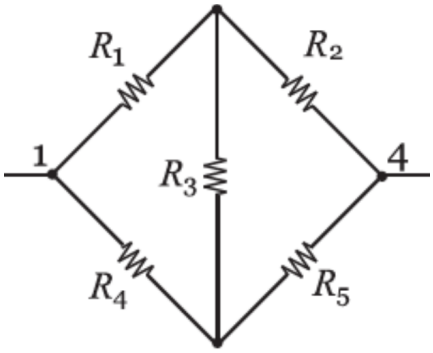
(A) 0.6 (B) 0.833 (C) 1 (D) 1.5 (E) 1.667



40. In the same circuit with $I_1 = I_2 = 1\text{A}$, what would be the ratio of I_3/I_4 if $R_1 = R_2 = R_3 = R_6 = 1\Omega$, $R_4 = R_5 = R_7 = R_8 = R_9 = 2\Omega$. [2 points]

(A) 0.6 (B) 0.833 (C) 1 (D) 1.5 (E) 1.667

41. Select the closest value (unit: Ω) for the equivalent resistance between node 1 and node 4 in the following circuit: ($R_1 = R_5 = 26\Omega$, $R_2 = R_4 = 10\Omega$, $R_3 = 6\Omega$) [2 points]

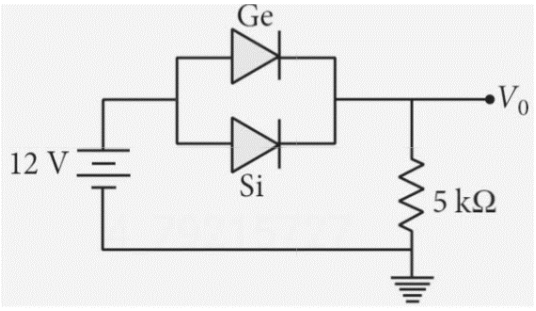


(A) 85/6
(B) 46/3
(C) 15.0
(D) 14.0
(E) 6.44

42. A parallel plate capacitor of 20 μF is charged by a voltage source at a rate of 2.5 V/s. The conduction current through the connecting wires and the displacement current through the plates of the capacitor would be respectively: [2 points]

(A) 0, 0 μA (B) 0, 50 μA (C) 50, 50 μA (D) 50, 0 μA

43. Threshold voltage for Ge- and Si- diodes are 0.3 V and 0.7 V, respectively. In this diagram, if Ge diode connection is reversed, the value of V_0 changes by [2 points]

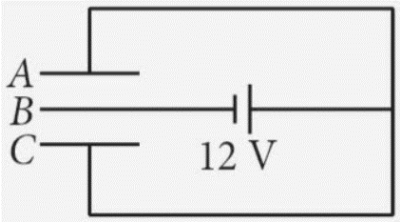


(A) 0.2 V
(B) 0.4 V
(C) 0.6 V
(D) 0.8 V

44. A parallel plate capacitor has 1 μF capacitance. One of the plate is given $+2\mu$C charge and the other plate, $+4\mu$C charge. The potential difference developed across the capacitor is [2 points]

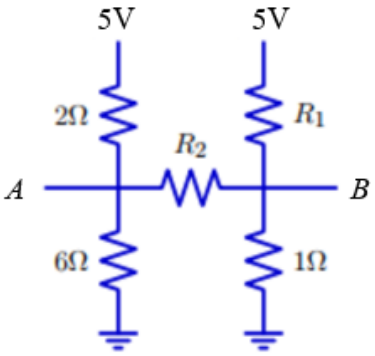
(A) 3 V (B) 1 V (C) 2 V (D) 5 V (E) Not enough info

45. Three plates A, B, and C have separation of 3 μm between A and B, and 6 μm between B and C. If the area of the plate is 50 cm^2 each, what is the energy stored when they are charged fully by the 12 V battery? [2 points]



(A) 2μJ (B) 1.6μJ (C) 5μJ (D) 3.2μJ

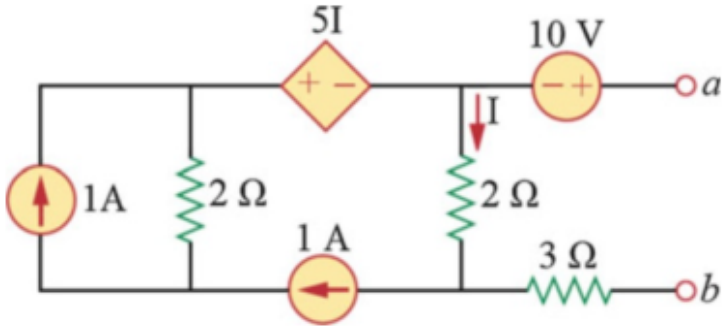
46. What are the values of R_1 and R_2 so that $V_A = 3$V and $V_B = 2$V? [3 points]



(A) $R_1 = 3\Omega$, $R_2 = 3\Omega$
(B) $R_1 = 3\Omega$, $R_2 = 2\Omega$
(C) $R_1 = 2\Omega$, $R_2 = 2\Omega$
(D) $R_1 = 1\Omega$, $R_2 = 3\Omega$
(E) $R_1 = 3\Omega$, $R_2 = 1\Omega$

47. For the following open circuit (a-b), what is the current I? [3 points]

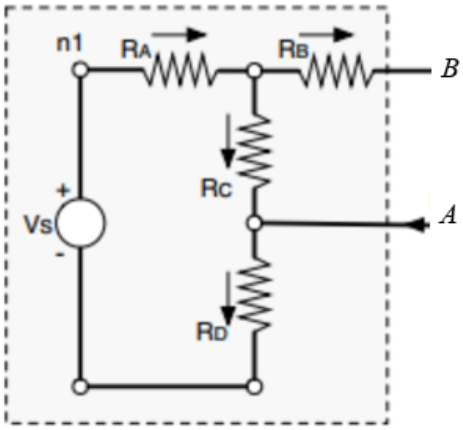
(A) 2 A (B) 1 A (C) 3 A (D) 1.5 A (E) 0.5 A (F) 0 A



48. For the same circuit in the question above, what is the Thevenin voltage and Thevenin resistance seen from terminal a-b? [2 points]

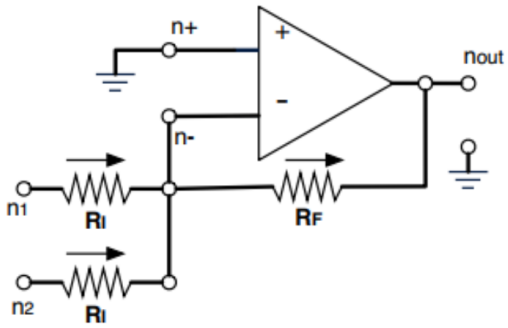
(A) 15V, 2Ω (B) 15V, 4Ω (C) 12V, 5Ω (D) 15V, 5Ω (E) None of these are correct

49. What is the equivalent resistance R_{TH} and Thevenin voltage V_{TH} of the following circuit at port A-B? $V_S = 10\text{V}$, $R_A = 1000\Omega$, $R_B = R_C = R_D = 1500\Omega$. [3 points]



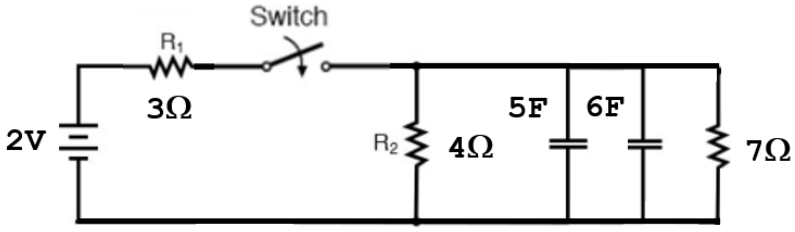
(A) $R_{TH} = 1500\Omega$, $V_{TH} = 5.00\text{V}$
(B) $R_{TH} = 1500\Omega$, $V_{TH} = -5.00\text{V}$
(C) $R_{TH} = 1500\Omega$, $V_{TH} = 3.75\text{V}$
(D) $R_{TH} = 1750\Omega$, $V_{TH} = -3.75\text{V}$
(E) $R_{TH} = 3000\Omega$, $V_{TH} = 5\text{V}$
(F) $R_{TH} = 3000\Omega$, $V_{TH} = -5\text{V}$

50. What is the function of the following circuit? [3 points]



(A) $V_{out} = (V_1 + V_2)R_F/R_I$
(B) $V_{out} = (V_1 - V_2)R_F/R_I$
(C) $V_{out} = -(V_1 + V_2)R_F/R_I$
(D) $V_{out} = -(V_1 - V_2)R_F/R_I$

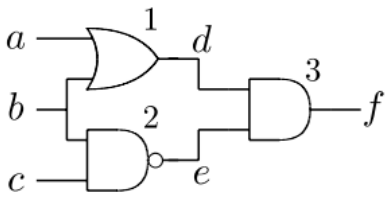
51. Which of the following value is the closest to the time constant for the circuit with switched closed at $t = 0$ sec? [3 points]



(A) 30.10 sec
(B) 15.15 sec
(C) 7.60 sec
(D) 15.12 sec
(E) 10.02 sec
(F) 7.50 sec

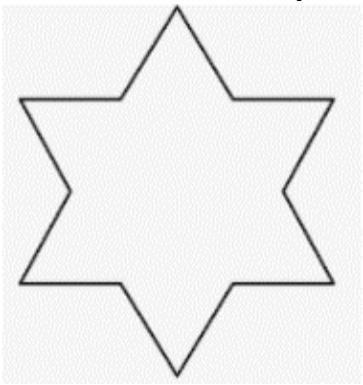
52. Which of the following is **NOT** a valid possible outcome for d, e, f?
(Mark ALL the answers) [3 points]

(A) 0, 1, 0
(B) 0, 1, 1
(C) 1, 1, 1
(D) 1, 0, 0
(E) 0, 0, 0
(F) 0, 0, 1



53. A symmetric star shaped conducting wire loop is carrying a steady-state current I, with top to bottom tip distance being $4a$. The magnitude of magnetic field at center of the loop is [3 points]

(A) $\frac{\mu_0 I}{4\pi a}\ 3[2-\sqrt{3}]$
(B) $\frac{\mu_0 I}{4\pi a}\ 3[\sqrt{3}-1]$
(C) $\frac{\mu_0 I}{4\pi a}\ 6[\sqrt{3}-1]$
(D) $\frac{\mu_0 I}{4\pi a}\ 6[\sqrt{3}+1]$



54. A potential difference of 220V is applied to a coil at temperature of 15^oC and the initial current is 10A before the coil heats up. Finally the current falls to 5A. Find the mean temperature of the coil, given thermal coefficient of resistance is $\frac{1}{234}{}^o\text{C}^{-1}$ at 0^oC. [2 points]

(A) 350 ^{o}C
(B) 150 ^{o}C
(C) 225 ^{o}C
(D) 125 ^{o}C
(E) 264 ^{o}C
(F) 500 ^{o}C

55. The frequency of an alternating current is 50 Hz. What is the minimum time taken by the current to reach its peak value from its RMS value? [3 points]

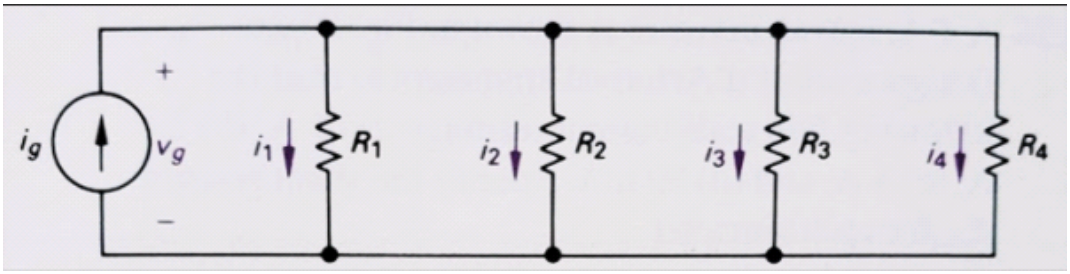
(A) 0.02 sec
(B) 0.005 sec
(C) 0.01 sec
(D) 0.0025 sec
(E) 0.015 sec

B Short Answers

(One to two sentences explaining how you solved and what law is applied could earn partial points even if answer is wrong. Don't need to write essays. Last 10 questions may be more challenging, so a bit more explaination may be needed in your solution approach)

56. [**4 points**] What are the resistor values (R_1 to R_4) that meet the following current ratio, assume $i_g = 10$ mA, and $v_g = 4$ V?

$$i_1 = 4i_2;\ i_2 = 8i_3;\ i_3 = 5i_4.$$



Answer: R1 = 515 Ohm, R2=2.06 kOhm, R3=16.48 kOhm, R4=82.4 kOhm.

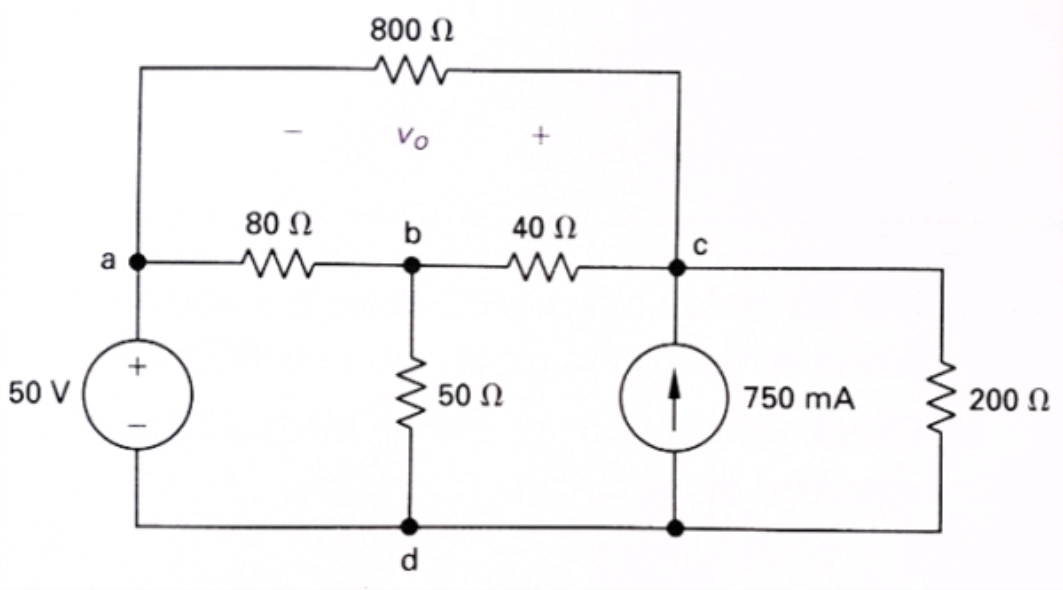
57. [**5 points**] Resistors 1Ω, 2Ω, ... 100Ω are connected in a single loop in the order of increasing resistance. One terminal of an ideal battery with emf of 10V is connected between the 1Ω resistor and the 100Ω resistor.

Between which two resistors should the other terminal be connected so that the battery current is the smallest possible? (3 pts)

What is that current? (2 pts)

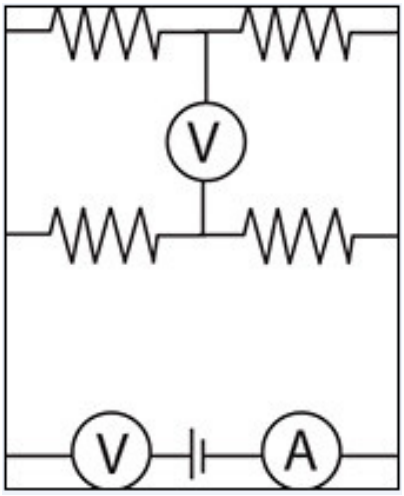
Answer: Treat first part of the chain as equivalent $R_1 = n(n+1)/2\Omega$, and $R_2 = 100\times101/2 - R_1\Omega$. So the maximum resistance is when R_1 and R_2 are balanced. So the ideal solution is between resistor 71 and resistor 72. Then using the equivalent $R_1||R_2$, one get 8mA of current.

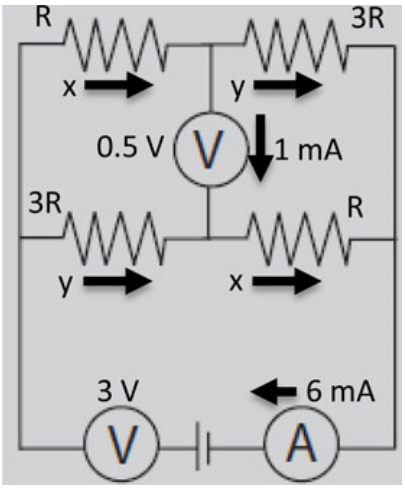
58. **[5 points]** What is the voltage level v_o (across the 800 Ω resistor) in the circuit below?



Answer: 3.2V; Nodal analysis or Superposition works.

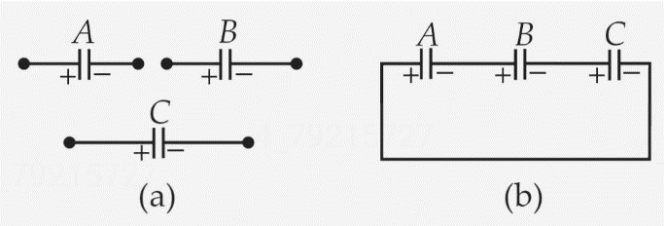
59. **[5 points]** The circuit shown below contains four resistors: two of them have resistance R, and the other two have 3R. The voltmeters are identical. The readings of the measuring devices are 6 mA, 0.5V and 3.0V. Find R.





Answer: Since voltage is nonzero, the R and 3R must be alternated at top and bottom rows. The current through top voltmeter is 1mA (1/6 of bottom current). Using nodal balance, current is 3.5mA and 2.5 mA at top two resistors (R and 3R). From there one can get R = 125 Ω.

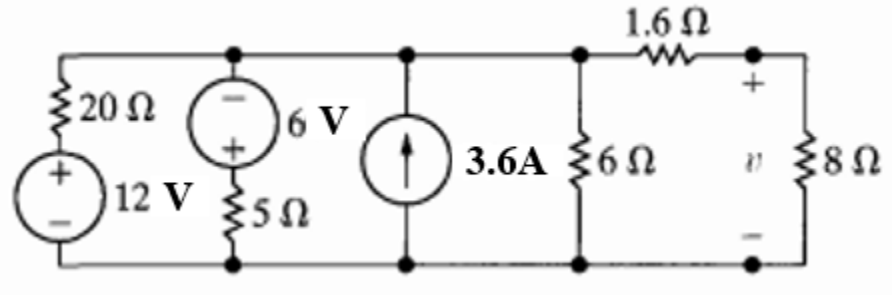
60. **[6 points]** Three capacitors, $C_A = 1\mu$F, C_B and $C_C = 2\mu$F, were initially charged to a potential difference of $V_A = 10$V, $V_B = 40$V, and $V_C = 60$V, respectively, and kept separate as in (a). Now if we connect them in the configuration (b) with + and -ve signs representing their initial polarities. What is the approximate total amount of heat produced (in μJ) by the time steady state is reached?



Answer is 3025. Apply conservation of charge, find $Q_{A,B,C}$ in final stage (3 algebraic equations). $U = 1/2Q^2/\text{C}$, and get change $U_{start} - U_{end} = 3025\mu$J

61. **[5 points]** A parallel plate capacitor with $C = 12\text{pF}$ is charged by a battery to a potential difference of V=12V between its plates. The batter is now disconnected and material with $\kappa = 6.0$ is inserted between the plate.
(a) What is the potential energy (pJ) before the material is inserted?
(b) What is the potential energy (pJ) after the material has been inserted?
Answer: $U = 1/2CV^2 = 864$ pJ. After insertion, $U = (q^2/2\kappa)C$ - where C is increased, charge q remains the same. So energy = 144 pJ

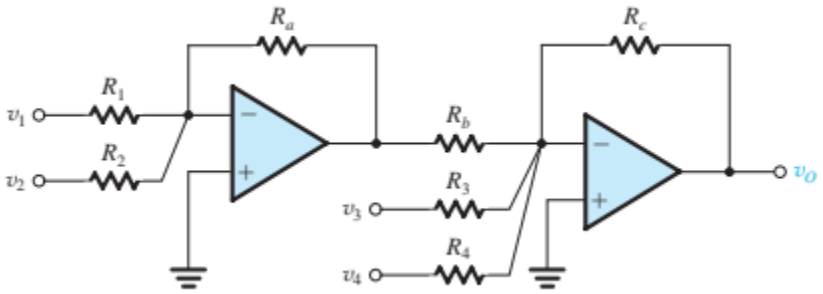
62. **[6 points]** What is the voltage across the 8Ω resistor? (Hint: Using source transformation may be quicker)
How much power does the 12V source deliver to the entire circuit?



Answer: 4.8V, 3.74W

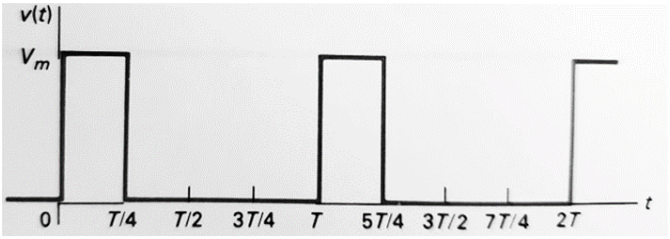
63. **[6 points]** Recommend proper resistances for R_1, R_2, R_3 and R_a, R_b, R_c to achieve the following output. Consider input v_4 as floating.

$v_o = 2v_1 + v_2 - 4v_3$.



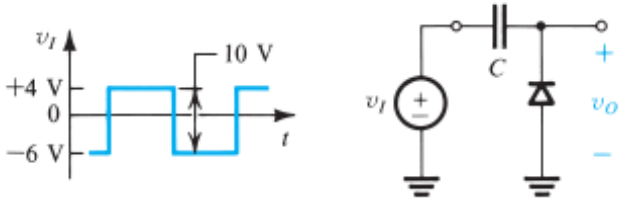
Answer: Any answer with the following ratio are correct. R1:R2 =1:2, Ra = Rb = Rc = R2; Rb:R3 = 4:1. Typical values in kΩ range is best

64. **[5 points]** The following square wave profile is applied to an 100Ω resistor. The value of V_m is 20V, and the time period T is 5 ms.
a) What is the average DC power? (2 pts)
b) What is the total average power delivered to the 100Ω resistor? (3 pts)



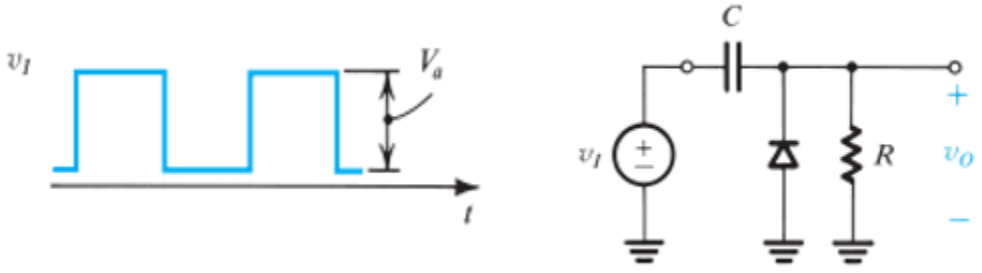
Answer: DC power is $V_{avg}^2/R = 5^2/100 = 0.25$ W. Total average power is related to RMS voltage, $V_{rms} = \sqrt{(20^2 \times (T/4)/T} = 10$ V; So total power $= 10^2/100 = 1$W.

65. **[5 points]** For a circuit with a capacitor C and diode powered by voltage source V_I, the input waveform for V_I is shown below.
a) What would be the output voltage maximum and minimum, assuming ideal diode?
b) What is the maximum voltage value across the capacitor during the cycle where it is charged to have the polarity shown below?
b) Describe the role of the diode in this case?



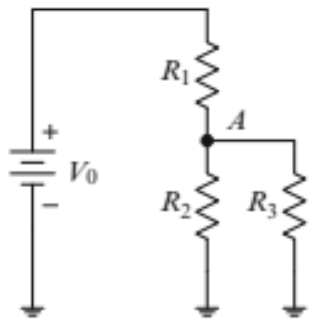
Answer:Output voltage will be between 0 and 10V. The capacitor can be charged to the maximum negative value, 6V The diode clamp the output voltage at zero V, so shift the waveform to 0-10V.

66. **[5 points]** If we add a load resistor to the output as shown below, and change the input (V_I) waveform, describe the change in output profile, and the role of diode during the cycle?



Answer:There will be an RC decay during the where V_0 dips below V_a, and output waveform could dip below zero level when V_I drops by V_a. But diode will help restore the change in such case so the lower bound will be close to threshold voltage of the diode.

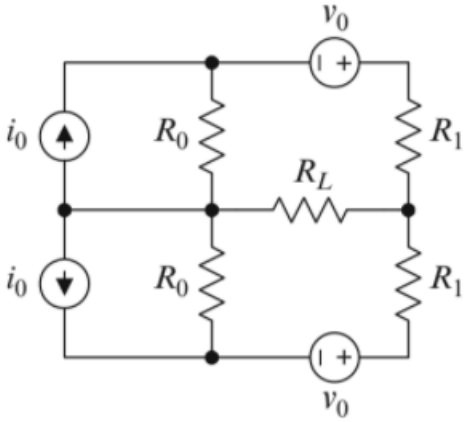
67. **[4 points]** If a sinusoidal waveform is applied in the following circuit with V_0 =50V. The power delivered by the source is 90.2 mW, the voltage V_A equals 32 V, and $R_3 = 91k\Omega$.
Assume $V(t) = V_0\cos(2\pi ft)$, find R_1 and R_2.



Answer: $R_{eq} = 50^2/0.0902/2 = 13.86k\Omega$. $R_2||R_3 = 32/50 \times 13.86 = 8.87k\Omega$, so $R_2 = 9.83k\Omega$. $R_1 = 18/32 \times 8.87 = 4.99k\Omega$.

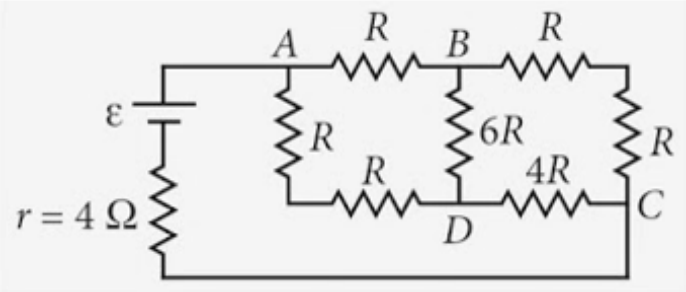
68. **[4 points]** What is the expression of average current for a current profile $i(t) = [\cos(\theta_1 + \omega_1 t) + \cos(\theta_2 + \omega_2 t)]^2$?
Here $\omega_2 > \omega_1 > 0$ and θ_1, θ_2 are arbitrary angles.
Answer: 1

69. **[6 points]** In the following circuit, the voltage and current sources follow these profiles: $v_0 = V_0\cos(2\omega t)$, and $i_0 = I_0\cos(\omega t)$. $I_0 = 2$ mA, $V_0 = 25$ V.
Assume the resistance $R_0 = 10k\Omega$, $R_1 = 100\Omega$, and the load resistor $R_L = 5k\Omega$. What is the power dissipated on the load resistor?



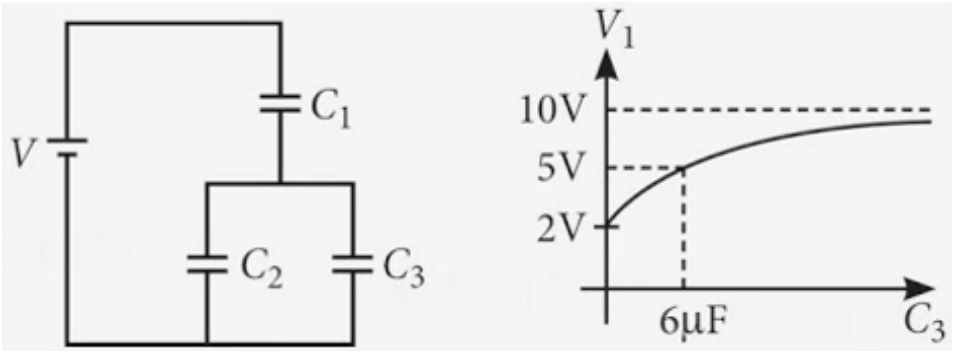
Answer: $P = \frac{2R_L(R_0^2 I_0^2 + V_0^2)}{(R_0 + R_1 + 2R_L)^2} = 25$ mW. Can also use symmetry to analyze half of the circuit.

70. [**4 points**] In the circuit below, the battery emf is 8V, with internal resistance of 4 Ω shown on the left. What is the resistance R such that the power delivered to the network is maximum? What is the maximum power?



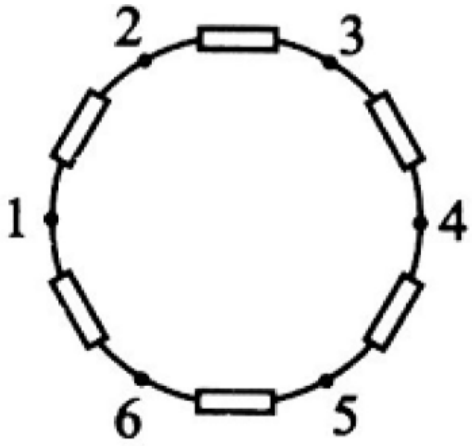
Answer: MaxPower is when resistance is equal to internal resistance; The 6-ohm resistor carries zero current, so the effective resistance is 2R. So R=2 Ohm. Power = 4W.

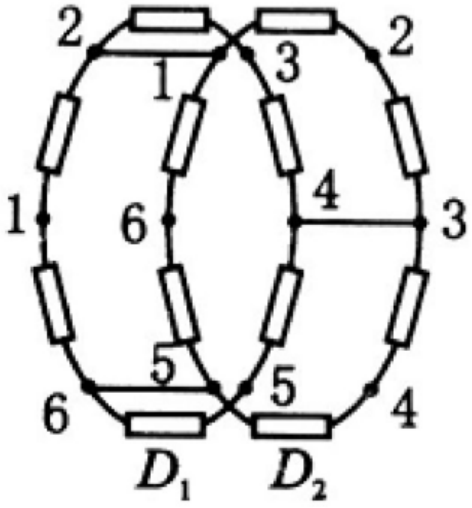
71. [**5 points**] If capacitor C_3 has variable capacitance, we observed the voltage drop V_1 across C_1 versus C_3 as shown in the right curve.
What are the value of C_1 and C_2?



Answer: As C_3 goes to infinity, $V_1 = V = 10\text{V}$; $(C_1V_1 = C_{eq} \times V)$; As $C_3 \to 0$, $V_1 = 2\text{V}$, so $C_1 = 4C_2$; At $C_3 = 6\mu\text{F}$, $5\text{V} = 10\text{V} \times (C_2 + 6)/(C_1 + C_2 + 6)$, so $C_1 = 8\mu\text{F}$, $C_2 = 2\mu\text{F}$.

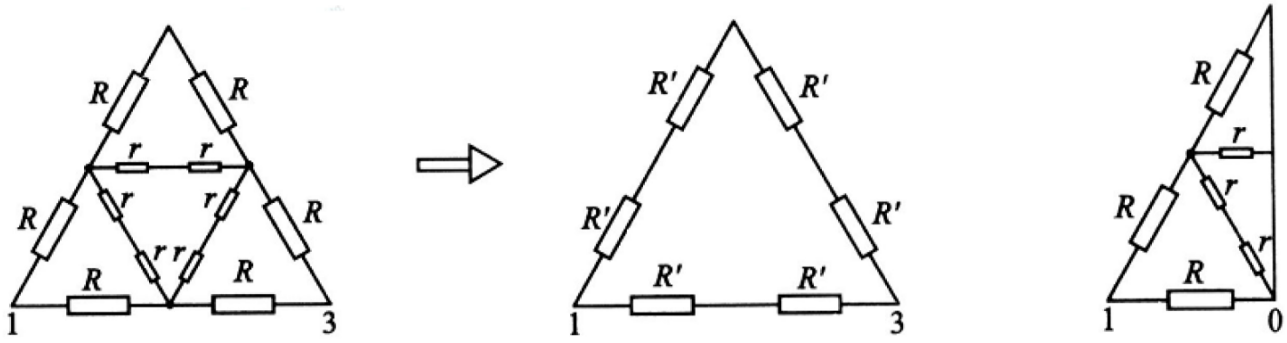
72. **[6 points]** Six resistors with resistance R forms a ring, with node 1 through 6 (left)
1) If we have two identical rings, and ring 1 (D_1) nodes 2, 4, 6 are connected to ring 2 (D_2) nodes 1, 2, 5 (right). What is the equivalent resistance between node 1 and 3 in ring 1?
2) If we add a third ring (D_3), with nodes 1, 3, 5, connected to D_2's nodes 2, 4, 6, what becomes the effective resistance between node 1 and 3 in ring 1?





Answer:

For the first problem, one can simplify each ring into resistor triangles, so the circuit can be converted to the embedded triangles as shown below in the left-most graph. The natural simplification is shown in the middle with node 1-3 resistance equal to R'.
If we represent the individual resistor as R, and use r as the individual resistor in the embedded ring, we can apply symmetry and reduce the problem to finding the resistance between node 1 and node 0 in the lower right graph.
If the general equivalent circuit is solved, we can find that the Node 1-3 equivalent-$R' = 3/4R + 1/4(R||r)$ where r is the individual R of first ring.
For the first question, $R = r$, so with D_2 added, the overall $R_{1,3}$ drops to 7/8R.
One can iterate to get the R_{eq} when D_3 is added, so 13/15R is the answer to 2nd question.



73. **[6 points] Magnetic Resonance Imaging** uses the influence of external magnetic field to shift the spin equilibrium of protons, and when the field is removed, measures the release of energy from the realignment of proton spin. Fundamentally, each spin-up and spin-down orientation of a proton behaves as a magnetic dipole in an external magnetic field.
a) (4 pts) What is the energy difference (in eV) between spin-up and spin-down orientation of a proton in a typical MRI is done with a 1.5 T magnetic field? Assume each proton has a magnetic moment of $\mu_p = 8.8 \times 10^{-8}$ eV/T.
b) (2 pts) There is an excess of protons in the spin-up state, which is useful for MRI function. Describe briefly why this is the case.

Answer:
a) $E = \mu\dot{B}$, so $\delta E = 2\mu \times B = 2 \times 8.8 \times 10^{-8} \times 1.5 = 2.6 \times 10^{-7}$ eV.
b) Based on Maxwell-Boltzmann statistics, proton with higher energy has higher number density. Actual excess is about 10^{-5}x at room temperature.
Note similar principles are not being used to design spintronics, devices that rely on spin orientation rather than electron charge for digital operations.

74. **[5 points] Order-of-Magnitude 1** Due to weak conductivity of salt water (3 S/m), researchers can measure the tidal velocity based on satellite magnetic field. In one or two sentences describes the mechanism and principle (laws) behind this effect. Assume tidal current of maximum velocity of 30 cm/sec, estimate the magnitude of magnetic field change from tidal effect in unit of Tesla.
Answer: Current is induced by flow of seawater through geomagnetic field (Faraday's law, or Lentz law). The time varying electric current produce time-varying magnetic fields (3 pts). Magnitude estimate: Accept any answer within range of 1- 10nT (half point for 10X off).

75. **[3 points] Order-of-Magnitude 2** Studies on migratory birds like European Robins suggested that the main navigation mechanism is through visual sensitivity to magnetic field, a magnetic compass that is chemical in nature. What is the magnetic field strength bird can recognize?

Answer: The surface of earth has a field strength of 0.5G, or 10^{-5} T; Accept any answer between 1-10 uT; Half point for 10X off on lower side.