Science Olympiad
Astronomy C
BirdSO Mini Invitational

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Section C Solutions

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Feedback? **Test Code**: *2022BirdSOMini-AstronomyC-Oxygen*

Section C: Calculations

Use the images in Image Set C to answer the following questions. Points are shown for each sub-question, for a total of 70 points.

1. **Big Beak Energy**

Spitzer Space Telescope provides an infrared view of our Galaxy that is full of surprise, and with some imagination, you can spot a bird in the direction of Sagittarius, with a big beak (depicted in image C1). Our estimated distance to this region is 8 kpc.

(a) [2 pts] Assume the beak size is 15 arcmin by 2 arcmin, what is the approximate area of the big beak in steradian?

(b) [2 pts] The red bright features in the eye and above the beak are collected in the 24 micron filter, a MIR wavelength. What is the frequency of 24-micron infrared in Hz?

(c) [2 pts] Calculate the MIR flux reaching us from the eye region, in $\mathrm{W\,m^{-2}}$, if the spectral flux density of this region is 1080 mJy in the MIR.

(d) [3 pts] If the zero point magnitude of 24-micron MIR is 6 Jy, what is the magnitude of the eye region?

(e) [3 pts] What is the region's power in equivalent solar luminosity?

(f) [3 pts] Strong 3-24 μm MIR emission is a hallmark of star formation. What are the emission sources at these wavelengths, and how do they get their energy?

Solution:

(a) 1 Sr = 4.25×10^{10} arcsec^2; So 15×2 arcmin^2 is 2.54×10^{-6} Sr. Or convert arcmin to radian then multiply.

(b) Frequency $f = c/\lambda = 1.237 \times 10^{13}$ Hz or 12.37 THz.

(c) Flux density $= f \times S_v = 1.34 \times 10^{-13}\,\mathrm{W\,m^{-2}}$, recognizing the unit of Jansky is $1 \times 10^{-26}\,\mathrm{W\,m^{-2}\,Hz^{-1}}$ (see also question B8(a) in DSO section).

(d) Zero point magnitude is just the normalization factor that is frequency and filter dependent.

$$M = -2.5 \log_{10}(F_v/F_0) = -2.5 \log_{10}(1.08/6) = 1.86$$

(e) $L/L_{\odot} = 10^{0.4(4.83-M)} = 15.7$.

(f) These are from cold, dense molecular or dust clouds. They absorb the UV radiation from star-formation gas and re-emit in mid-to-far Infrared. Answers referring to the start formation, protostar and its formation mechanism get partial points.

2. **RR Lyrae**

KIC 5559631 is an RR Lyrae star with a pulsing period of 0.621 days. Its light curve is plotted in image C2 folded with the main period to show its variation over the full phase of a cycle. Answer the following questions using this graph.

(a) [3 pts] RR Lyrae itself – the star in Lyra, not the variable star category – has a maximum magnitude (minimum brightness) of 7.13, and a parallax of 0.0044 arcsec. If the KIC 5559631 is just like RR Lyrae, what is the distance to KIC 5559631 in parsec?

(b) [2 pts] Suppose that at maximum magnitude of 15.0 the effective surface temperature is 7400 K, what is the maximum radius of the KIC 5559631 in terms of solar radius?

(c) [2 pts] The pulsation period Φ is governed by the principle of hydrostatic equilibrium, and assume the period is related to the average density (ρ) ratio to solar density ($\rho_\odot$) as below:

$$\Phi = 0.04\sqrt{\frac{\rho_\odot}{\rho}}\ \text{days}$$

What is the mass of KIC 5559631 in terms of solar mass?

(d) [4 pts] Stellar pulsations like that of RR Lyrae occur in a specific regime on the HR diagram, referred to as the instability strip. What physical mechanism determines the blue (hot) and red (cold) boundaries of the strip in the HR diagram?

(e) [2 pts] What appears as noise or scatter in the data actually represents phase and amplitude modulation that occurs over multiple days on a slower time scale. What is the name for this effect?

Solution:

(a) One can use the distance and magnitude of the RR Lyrae star to calculate distance to this DSO. Distance of RR Lyrae is 1/0.0044 parsec, and distance to KIC 5559631 is

$$10^{(m_1-m_2)/5)}/0.0044 = 10^{(15.0-7.13)/5)}/0.0044 = 8522\,\text{pc}$$

(b) The absolute magnitude of RR Lyrae is

$$7.13 + 5\log(10 \times 0.0044) = 0.347$$

Since KIC5559631 has the same maximum magnitude as RR Lyrae, the luminosity is then

$$L = L_\odot \times 10^{(4.83-0.347)/2.5} = 62.1\,\mathrm{L}_\odot$$

Based on Stefan-Boltzmann law, the luminosity is proportional to T^4 and R^2. Radius is

$$R = \sqrt{\frac{L}{L_\odot}\frac{T_\odot^4}{T^4}} = \sqrt{62.1} \times (\frac{5778}{7400})^2 \approx 4.8\,\mathrm{R}_\odot$$

(c) First get density using the period data.

$$\frac{\rho_\odot}{\rho} = \left(\frac{0.621}{0.04}\right)^2, \rho = 0.00415\rho_\odot$$

Then use radius from (b) to get $0.46\,\mathrm{M}_{\odot}$.

(d) The instability strip is defined by location of the Helium-II partial ionization zone at different stage of evolution.
The blue side limit is due to high surface temperature at which partial ionization occurs very close to the surface, limiting the pulsation. The red-side limit corresponds to strong convection effects at lower temperature that prevent pressure buildup, or countering the positive feedback from opacity mechanism.

(e) Blazhko Effect

3. **Magnetic Cataclysmic Variables**

A rare CV binary system was found with strong eclipsing of the secondary star accompanied by the flaring activities. A series of light curves are shown in image C3A around the phase of inferior conjunction at different epochs, three showing strong flaring, and the two during weak flaring. The period of eclipsing is found to be 0.3 days. A model light curve for the binary with a white dwarf and a secondary star is shown as an overlay to represent the ideal eclipsing binary behavior. Here the mass ratio is assumed as 0.5, and the inclination angle indicates an almost edge-on orbit ($i = 81°$).

(a) [3 pts] The variation in magnitude of the model curve is quite significant. What is the equivalent ratio of radius for the eclipsing WD versus that of the secondary star, based on this variation? Assuming WD has much lower surface brightness.

(b) [5 pts] Does your answer match the characteristics of a WD size and typical CV companion star? Explain briefly the eclipsing magnitude. (Note the strong contrast in eclipsing magnitude of the strong vs. weak flaring case.)

(c) [3 pts] Assuming during the eclipse, the obtained spectrum (Depicted as the red spectra in image C3B) is representative of the secondary star. What category of star is this?

(d) [2 pts] Extracted velocity dispersion based on Hα emission varies on order of minutes (Shown in the insert at lower right of image C3B). During the peak flaring, the range of velocity is $\pm 1000\,\mathrm{km\,s^{-1}}$. What would be the original width of the Hα emission line at the peak flaring stage in nm?

(e) [3 pts] Radial velocity measured from absorption lines is $240\,\mathrm{km\,s^{-1}}$ for the secondary star. Based on the inclination ($i = 81°$) and period (0.3 days), what is the distance in km between the secondary star and the center of mass for the binary system? Assume circular motion.

(f) [3 pts] What would be the total orbital radius in AU, assuming a mass ratio of 2:1 between WD and the secondary star?

(g) [3 pts] The mass function of a binary is defined as

$$\frac{m_1^3 \sin^3 i}{(m_1 + m_2)^2}$$

where m_1 represents the mass of the WD, and m_2 represents the mass of the companion. $i = 81°$ is the inclination of the orbit. Calculate the binary mass function in solar mass.

(h) [2 pts] Based on the mass ratio assumption of 2:1, what is the white dwarf mass in solar mass?

Solution:

(a) The change in magnitude on the graph is 1.5. The flux change is then

$$\Delta F = 10^{(m-M)/2.5} = 1/4$$

Using the approximation that the flux ratio is proportional to projected area of the blocking WD,

$$\Delta F \approx (R_1^2 - R_2^2)/R_1^2$$

therefore

$$R_2/R_1 = \sqrt{1 - \frac{1}{4}} = 0.866$$

(b) WD size should be much smaller than the companion star which are typically red giants or at least much larger than earth radius. Mostly the accreting material around the WD is causing the large magnitude change.
The gas transferred from the secondary star is filling the orbits around the WD, creating a shrouded region (Roche lobe) that is much bigger than the WD itself. The flare region, with denser gas (stronger influxes of gas ionized by the strong magnetic field) is potentially the main area blocking the secondary star.

(c) M-type based on the continuum part of the spectrum (or Red dwarf).

(d) Hα wavelength is 656 nm, so $\Delta\lambda = 2 \times 1000/300,000\lambda = 4.37$ nm. Some students interpreted the question as asking the wavelength itself, which get only partial point (if including the width).

(e) Based on kinematics, velocity $\times$ period = circumference, so $R = 1.0 \times 10^6$ km. Inclination angle is only a small correction (almost edge-on).

(f) $2\pi R = P(V_1 + V_2)/\sin i$, and $V_1/V_2 = m_2/m_1 = 2$, so $R = 1.5 \times 10^6$ km, i.e. 0.01 AU.

(g) Following the formula below using solar units:

$$\frac{m_1^3 \sin^3 i}{(m_1 + m_2)^2} = \frac{V_{rad}^3 P_{orb}}{2\pi G} = 0.43$$

So the mass function is about $0.43\,\mathrm{M}_\odot$.

(h) Based on binary mass function in answer (g) and the $m_1/(m_1 + m_2) = 2/3$,

$$m_1 = (3/2)^2 \times 0.43 = 0.97\,\mathrm{M}_\odot$$

The problem is modified based on paper by Garnivich et al. 2021 *Confirmation of a Second Propeller: A High-Inclination Twin of AE Aquarii* (https://arxiv.org/pdf/2102.08377.pdf). Any manipulation and mis-interpretation of the data/math is the problem writer's fault.

4. **Pulsating Stars**

For variable stars and pulsars, O-C diagrams (observed-minus-calculated) are extremely powerful to detect subtle changes in the time series and its periods. Images C4A through C4C shows the O-C diagram for the three following objects. A is the amplitude of the light curve in the V band and p is the period in days. The general fit for the O-C curve is the function $f(E) = a + bE + cE^2$ where E is the epoch (number of cycles).

Object	A	p	a	b	c
A	0.06	3.969	-0.146	1.125×10^{-4}	2.787×10^{-7}
B	0.6	10.152	-0.01657	-2.592×10^{-2}	-4.987×10^{-7}
C	~ 2	75.41	-7.639	6.023×10^{-2}	-9.658×10^{-5}

(a) [4 pts] Object A has a $T_{\text{eff}} \sim 7000\,\text{K}$. What is the period change in (second/year)? Based on this, describe the most likely evolution path object A is going through in the H-R diagram and the reason.

(b) [4 pts] For object B, what is the change of the period and its magnitude in (second/year)? Describe the type of the object. What is the most likely evolution path object B is going through on the H-R diagram?

(c) [5 pts] Image C4C shows a clear wave with a period of 9323.3 days. After filtering out the sinusoidal component, a parabolic fit of the longer-term variation is listed in the table for Object C. What is the type of the star and what direction it is evolving to in the H-R diagram?

(d) [5 pts] Rather than removing the sinusoidal wave, it can also be assumed that the variation from one cycle to the next is random with magnitude e, and that the variable $u(x)$ represents the variation in O-C value accumulated over x period, compared to the ephemeris. The average of cycle-to-cycle variation $< u(x) >$ is proportional to the number of cycles x if the period fluctuation e is significant. Image C4D shows such post-processing of variability in the O-C diagram.

What is the period fluctuation e in (days) based on Image C4D? How does it compare with the long-term period changes based on O-C parabola fit?

Solution:

The O-C diagram is an interesting technique that is perhaps a bit lost in the modern advanced statistical analysis world of Astronomy. The book by John Percy's *Understanding Variable Stars* the AAVSO website and this link (http://jjherm.es/research/omc.html) has good explanations. Basic formulus:

$$O - C = \Delta t_0 + \Delta P \times E + \frac{1}{2} P \times (\frac{dP}{dt}) E^2$$

where ΔP is the offset between observed period vs. estimate, while dP/dt is the change in period over time, and E is the epoch.

So the period change over time can be extracted based on the parabolic shape, i.e. using the term c in the table above. If the O-C curve appears concave up, then the period is increasing; and if the curve appears concave-down, then the period is decreasing. Amplitude and period (A and p) give hints on what type of variables.

(a) Based on equation above,

$$c \text{ (days)} = 2.787 \times 10^{-7} = \frac{1}{2} P \cdot \dot{P} = \frac{1}{2} \cdot 3.969 \cdot \dot{P} \text{ (days)}$$

Converting to second/year,

$$\dot{P} = 1.4 \times 10^{-7} \times 3.156 \times 10^{7} \frac{\text{sec}}{\text{year}} = 4.43 \text{ sec/yr}$$

The period change is 4.43 sec/yr. Increasing period indicates larger radius, so it is going through the instability strip from left to right (high T_{eff} suggests first time. *The plot is for the Polaris*)

(b) This is a Cepheid (based on magnitude and period), and period change is -3.10 sec/yr (period getting shorter). It is going through the instability strip from right to left — likely the second or fourth crossing of the instability strip.

(c) This is a RV Tauri star (larger amplitude, much longer period, less regular). The decreasing period is consistent with its post-red giant phase, evolving to the blue in the HR diagram.
Answers include Asymptotic Giant branch are given partial points.

(d) Based on John Percy's paper *The Nature of the Period Changes in RV Tauri Stars* (1997),

$$< u(x) >^2 = e^2 x + \text{const}$$

so the slope is basically e^2 so fluctuation in period is 0.7 days. (accept 0.6 - 0.9 days). This is much larger than the period change of 81 sec/year for the overall reduction of periods.
Answer recognizing by dimension analysis of the chart axis gets partial points.