Solutions match the problem in level of difficulty. You don't need much prior knowledge to understand solutions to easy problems; more scientific background is assumed for solutions to harder problems.

**1**. E – the lowest frequency of a drumhead is given by $f \propto \frac{\sqrt{T}}{r\sqrt{\sigma}}$, where $\sigma$ is the mass per area, and $T$ is the tension per length.

**2**. C – the equation for the speed of sound in an ideal gas is $\sqrt{\frac{\gamma RT}{m}}$. The molar mass $m$ doesn't change, nor does the specific heat ratio $\gamma$, because these depend only on the gas, not the average energy of its particles. $R$ is a universal constant, so only $T$ changes. By $K = \frac{3}{2}k_B T$, the absolute temperature of the gas in Kelvin increases by 10%, so the new speed of sound is $\sqrt{1.1} = 1.05$ times greater, leading to choice (C).

**3**. A – pipe $C$ is obviously longer and therefore is lower in frequency than the others. Due to a larger radius, pipe $B$ is a tiny amount lower than pipe $A$ in frequency. This is known as end correction, where the effective length of a pipe increases as radius increases.

**4**. C – because the sound sources are in phase (that is, no phase shift or offset occurs), the sound pressures from each source are the same for points that are equally distant from either source. Therefore, point equidistant from both sources must have complete *con*structive interference, so these may be eliminated. These include choices (A), (B), (D), and (E), so choice (C) is all that remains. This is because the path length difference between the sources at point $C$ is contingent upon $f$, and we aren't told what $f$ is. There must exist some $f$ where the path length difference to be exactly half of a wavelength, resulting in complete destructive interference, so a silent region is a possibility at point $C$, but not at $A$ nor $B$.

**5**. B – by Bernoulli's equation, $p+\rho gh+\frac{1}{2}\rho v^2 = \text{constant}$. The density $\rho$ of the fluid is a bulk property so it is constant; $g$ is also constant on Earth's surface, and even for very large increases in $h$ by human standards, the gravitational acceleration would barely budge. $v$ is constant by the continuity equation as the problem states that the pipe has constant radius. So, the only two things that change in the equation are $h$ and $p$, so as $h$ goes up, $p$ must go down.

**6**. E – the angular wavenumber $k$ of a wave is $\frac{2\pi}{\lambda}$. Because $k$ is proportional to $1/\lambda$, it is also proportional to $f$ given no change in $v$. The speed of sound in air does not change by changing frequency, so the new angular wavenumber must be three times its original value.

**7**. D – from $A$ to $E$, the frequencies of the strings are respectively 400 Hz, 460 Hz, 230 Hz, 115 Hz, and 920 Hz. These are calculated by $f_1 = \frac{v}{2L}$.

**8**. C – only these strings have harmonics equal to 460 Hz. These are therefore the only strings for which the tuning fork drives resonance.

**9**. D – the Doppler Effect is unrelated to the distance of the observer and source, and only to their relative velocities and the emitted frequency.

**10**. D – Solution 1: the cafeteria is the largest of these four. Because reverberations are sound waves that reflect and return to a source, time delay happens between when a sound is released and when it hits the walls and reflects. This time delay is greater for larger rooms than smaller rooms, since the waves must travel further.

Solution 2: by Sabine's Law, the reverberation time is proportional to $\frac{V}{v_{\text{sound}}S}$, where $V$ is the volume of a room. Therefore, greater $V$ means greater reverberation.

**11**. C – the derivation of the equation $f_n = \frac{n}{2L}\sqrt{\frac{T}{\mu}}$ assumes gravity is negligible relative to the tension in the string. There is, of course, gravity in space, but the gravitational force on the string in space is just as negligible as it was before on Earth, so the string must have the same fundamental frequency as it would have on Earth. The fact that local observers are doing the measurements in the question is to tell you to ignore effects of general relativity.

**12**. D – a sine wave, triangle wave, and square wave all exhibit reflectional symmetry around an amplitude. The wave depicted does not show symmetry around areas of high particle density, so it is none of these.

**13**. B – we are told the values of $f_n$, $v$, and $L$. With $f_n = \frac{nv}{2L}$, we can calculate

$$n = \frac{2Lf_n}{v} = \frac{2(720\text{ Hz})(0.75\text{ m})}{360\text{ m/s}}$$

and so $n = 3$. Therefore, half a wavelength takes up one third of the string, and so a full wavelength takes up two-thirds of the string, or 0.50 meters.

**14**. C – it's irrelevant whether or not the velocity of the source relative to the observer is greater than the speed of sound, thereby eliminating (E). This is because sound is always emitted relative to the ground, so the sound will overtake and reach a subsonic observer no matter what. Using sign conventions for the Doppler effect, (C) is the correct choice.

Sneaky alternative method: if you hate the sign conventions for the Doppler Effect, we know that the observed frequency must be smaller than $f$ because both the observer and source are getting farther away from each other. That means both the numerator and denominator must make the observed frequency lower, so the numerator must become smaller and the denominator must become bigger. This means there has to be a minus sign in the numerator and a plus sign in the denominator. This leads to choice (C), which is the correct answer.

**15**. A – the Doppler Effect may be written as $\frac{f_{\text{obs}}}{f_{\text{src}}} = \frac{v + v_{\text{obs}}}{v + v_{\text{src}}}$. Notice that this is a ratio. Because the size of an interval is the same if the ratio between two notes is the same, the Doppler Effect simply multiplies the frequencies of each note by some number, so the ratio between notes in the scale remain the same. As a consequence, so do the intervals between the notes, and therefore the major scale remains a major scale regardless of Doppler shift.

**16**. E – I is true by definition. II is true of all waves. (Fun fact: this includes light waves!) III is true because all musical compositions have a definitive ending point, but noise does not necessarily end at any particular time.

**17**. E – II is true by definition of dispersion. III is a logical consequence of II because any non-sinusoidal waveform may be considered the sum of multiple sine waves of different frequencies. Because each frequency travels at a different speed, more rapidly propagating frequencies will leave behind the slower ones, breaking apart from the initial group of waves the composed the original non-sinusoidal wave packet. Statement I is false—the phase velocity is the velocity of any particular frequency; the group velocity is the velocity of the group of waves collectively, or the velocity of the packet of all the waves together.[Note 2] When dispersion occurs, some waves move faster than others, so the group velocity can't equal the phase velocity for each wave in the medium.

**18**. E – the fundamental frequency of a tuning fork is the frequency with the most energy, and all other frequencies are significantly higher and significantly more quiet. So, people will hear mainly, if not only, the

[2] Actually, this isn't quite true—I've oversimplified. But it's a nice mental shortcut to think about the group velocity as the velocity of a group. The true definition of the group velocity is $d\omega/dk$, where $k$ is the angular wavenumber of the sound. You'll notice it's the same $k$ as in question 6. You won't (or at least shouldn't) need to memorize the actual definition of group velocity for sounds of music, but it's helpful to grasp the intuitive meaning.

fundamental frequency, which is by definition a pure tone.

**19**. B – I know accordions have keyboards, but a keyboard doesn't really tell you what class of the Hornbostel Sachs system an instrument belongs to. The vibrations from an accordion always comes from air, which is pushed across reeds when the accordionist's hands stretch and compress the bellows back and forth.

**20**. A – it is physical improbable to use a pedal to do anything but (A) so you didn't have to know anything about harps to answer this question. The displacement of the string depends on how much the player plucks the string away from equilibrium position, not on a pedal; Young's Modulus (aka the stiffness) for the string depends on what the string is made out of and cannot be changed by pressing a pedal; and while it is *possible* to make a string play only odd harmonics by releasing one end from tension, it seems quite unlikely and bizarre that a manufacturer would ever make an instrument that does so with the press of a pedal.

**21**. B – there are 88 keys on a piano and there are 12 half steps in an octave, so there are seven octaves plus 4 keys we still must account for. Four keys, each of which is one half step from the next, is essentially an *increase* of three half steps, so the interval size of three half steps is a minor third. (In more concrete terms, an A and a C are only three half steps apart, but they span four keys: A, B♭, B, and C.)

**22**. A – by definition

**23**. A – F is a half step above E, and valve 2 is responsible for lowering pitch by a half step, so it must be removed to do the reverse and increase pitch by a half step. A good binder should list the effects of each of the three main valves on the trumpet.

**24**. C – organology is the study of the classification of musical instruments. Psychophysics is the study of perception of physical quantities; acoustic physics is well, physics (aka most of this test); phonetics is the study of the pronunciation, including the physics of the human vocal tract and physics of the ear; and ethnomusicology is the study of music in relation to culture.

**25**. E – both phons and sones are units of loudness, which is the human perception of intensity. Mels, on the other hand, are units of pitch, and do not apply.

**26**. E – the vertical axis refers to the amplitude of a sound, so units of pressure are allowed (I and III). Because equal loudness contours do not assume people are getting more or less distant as they are perceiving a sound stimulus, intensity and pressure amplitude are interchangeable, so units of intensity are also allowed.

**27**. A – if you look up any of these things, the Fletcher–Munson effect is the only thing relevant to the graph in the question. The Nyquist–Fourier effect is also made up.

**28**. E – combination tones are auditory illusions where we hear a note equal in frequency to either the sum or difference of multiples of the frequency of two other notes; in this question, the type of combination tone is a *difference tone*, although a sum tone of 1800 Hz is possible as well. Combination tones are not the same as beats because beats are a frequency of oscillation of the *intensity* of two sounds. That is, 802 Hz and 800 Hz would form beats of 2 Hz, resulting in a note that increases then decreases in *loudness* twice a second, but you wouldn't hear a *note* of 2 Hz as its frequency is too low.

**29**. A – a tritone is an interval of exactly six half steps. In 12-TET, this means that the ratio between the higher note and the lower is $\sqrt[12]{2}^6 = \sqrt{2}$.

**30**. B – All half steps in Pythagorean tuning are of ratio 256/243 (= $2^8/3^4$), so (A) is true. In (C), (D), and (E), by "pure," we mean that the ratio between notes is equal to a ratio in the harmonic series; that is, an octave is a ratio of 2, a perfect fifth is a ratio of 3/2, a perfect fourth has a ratio of 4/3, and so forth. Although not all intervals in Pythagorean temperament are pure, it is well-known for being extraordinarily pure, so all of these answers you could easily guess to be true, and correctly so. (This is in contrast to 12-tone equal temperament, for which no intervals are pure because $\sqrt[12]{2}$ is irrational, and therefore cannot form ratios of integers.) (E) is true in pretty much every temperament; the octave is defined to be a ratio of 2, so most temperaments try to preserve this because notes an octave apart sound "the same" to listeners. (B) should be obviously untrue: as long as an instrument can be built to play like this, the instrument is considered to be in Pythagorean tuning.

**31**. B

**32**. E – suppose that we were to play the notes of a C major scale, except starting on D, and ending on the D an octave above. This would give us D–E–F–G–A–B–C–D, or in terms of whole steps (W) and half steps (h), W–h–W–W–W–h–W, which is the sequence of the Dorian mode.

**33**. C – a melodic minor scale has the same first five notes as the natural minor scale (which we usually just call a minor scale), but the sixth and seventh notes are raised by a half step.

**34**. D – from left to right, these time signatures may be classified as irrational (a type of complex or odd time signature), additive, fractional, compound triple, and mixed.

**35**. A – tambourines are the only unpitched instrument of these four, and this staff gives merely rhythms but omits pitch.

**36**. This is known as *rubato*. It is worth noting that *rubato* in current usage may refer to either the definition given in this question, or to any time where the player may temporarily disregard the strictly defined tempo.

**37**. Flexure is a noun which means 'bending.'

**38**. A compression is a region of greater particle density.

**39**. Examples of acceptable answers:

- Hot air is less dense than cold air, yet sound travels faster in hot air.
- Ice is less dense than water, but the speed of sound is higher in solids than liquids. Compare the speed of sound in ice with that in water.
- The density of seawater increases at greater depths, but the speed of sound in the ocean does not consistently increase or decrease with depth.
- Speed of sound is 343 m/s in air, but it is significantly higher in less dense gaseous media of $H_2$ (about 1200 m/s, depending on temperature) and helium (about 1000 m/s).
- The density of the Earth only increases monotonically with depth, but the speed of *P* waves does not. (*P* waves are essentially sound waves that travel through the Earth as seismic waves. The wave speed formulas for sound and *P* waves are identical.)

**40**. $\frac{3\sqrt{2}}{2}$

**41**. *Tremolo* has two definitions, either of which would be accepted for one point: fluctuations in the loudness of a note, or rapid alternations between a series of distinct notes. Another point is earned for comparing the definition to that of *vibrato*, which is fluctuations in the pitch of a single note.

**42**. *Parallel* refers to keys that begin on the same note. So, B minor and B major are mutually parallel. The notes of B major are B, C♯, D♯, E, F♯, G♯, A♯ (and B). I gave full credit regardless of whether B was repeated.

**43**. *Relative* refers to keys that have the same notes. The relative major of any minor has a tonic at an interval three half steps above the starting note of the minor. The note three half steps above B is D, so the relative major of B minor is D. The notes are D, E, F♯, G, A, B, C♯ (and D). I gave full credit regardless of whether or not D was repeated.

**44**. By Nyquist's Criterion, it is limited to frequencies below 22050 Hz (1 point), exclusive (1 point).

**45**. 2 points for writing "1" for all three items. The frequency is equal in each region.

**46**. Due to refraction, sound moves along the wall (1 point), but exits in the same direction it entered (1 point), but the observer does not know this.

Alternative explanation: because sound moves along the wall more quickly than in air, all wave fronts enter and exit approximately vertically

**47**. This question wasn't worth any real points

Note: the first free-response question was weighted twice as much as the second.

**48**. **8 points total**

(a) 2 points

For having $\dfrac{v_{\text{air}}}{2L}$ or $f = \dfrac{v_{\text{air}}}{2L}$

Subtract one point if $v_{\text{s}}$ is used in place of $v_{\text{air}}$, if the harmonic $n$ is included in the final expression, or if both errors are made

Zero points if any other error is made

(b) 2 points

For checking "Softer" — 1 point

For justifying that resonance cannot occur when the airstream goes directly through one end of the pipe and exits through the other end — 1 point

Example: standing waves cannot occur because the air moves directly into and out of the pipe. Without standing waves, the loudness decreases.

Example: the air cannot reflect back into the pipe when the air goes directly from the mouth to the pipe, so no resonance occurs. Therefore, no tone is made and the sound is more quiet.

Remarks: since the student moves the pipe closer to his mouth, doesn't that mean the pipe gets more air, so the loudness increases? No. The student has essentially extended his mouth to the end of the pipe, as though the pipe is an oversized drinking straw. All air blown must travel through the end of the pipe and out, rather than reflecting at the end to create standing waves. The air merely moves from his real mouth to the PVC mouth (i.e. the end of the pipe) before venturing out into the world, instead of bumping back and forth as it would in a standing wave—this is reasonable, since the air does *not* return back into his mouth. In contrast, when the student creates the open pipe frequency—the one you found in part (a)—sound radiates from both sides of the pipe, rather than flowing out from the side opposite to the student.

If this still does not make sense, imagine that the student has a large supply of water in his mouth, except that this water is special because it, uh, acts exactly like air (lol just imagine it's colored air that you can see, I guess). He expels all the water through the PVC pipe. The water will simply flow out at the end; it will not create a wave inside the pipe since it will not reflect back into the pipe.

(c) 2 points

For checking "no change" 1 point

For a correct justification 1 point

See part (b) for examples of correct justification; the reasoning is the same for this part because the velocity more or less doesn't matter as long as it can't produce resonance, so part (b) and (c) won't differ

(d) 2 points

For checking "louder" 1 point

For a reasonable justification (anything that makes sense scientifically should be awarded credit) 1 point

Remarks: actually, it's irrelevant that the mouthpiece comes from a saxophone. The mouthpiece of any concert reed instrument would have sufficed. Why? A reed mouthpiece is a specialized device designed with the particular goal of creating resonance within a pipe, more efficiently than with the mouth alone. Humans have been refining the design of musical instruments for centuries, so inevitably any reed instrument that has survived the evolution into modern instruments is sufficiently loud that people are willing to use it. At any rate, for any type of reed mouthpiece you could answer that the reed more efficiently converts energy from the student's mouth to sound energy (i.e. less energy is lost to the environment due to entropy), but I believe this is so *not obvious* that any answer that is scientifically reasonable deserves credit.

Note that I would also accept "it cannot be determined" if the justification stated the resonance generated by a blower who blew at *just the right speed* could be just as loud as the resonance generated by a mouthpiece. Such a Goldilocks speed could be achieved in theory, even if it would be cumbersome to achieve in practice.

I would also accept "softer" if the justification stated that the pipe becomes a closed pipe when the PVC is added, and given that the pipe is probably short, the new tone would be less loud on an equal loudness contour. (While intensity is a physical quantity with units of energy per surface area, loudness is not a physical quantity. Instead, loudness is the subjective perception of relative intensity, so a listener won't necessarily hear two tones with the same intensity as having the same loudness.) However, because I don't think many people will come up with these justifications, the primary answer I have is "louder." Selecting "it cannot be determined" or "softer" is an all-or-nothing two points; the response either gets two points or none depending on whether a good justification is present.

Unrelated: Why use $\phi$ for the symbol of loudness? It's because I couldn't find a good commonly used symbol other than $L$, which is length, and so I picked *ph*i by analogy with *ph*on.

**49**. **8 points**

(a) 1 point

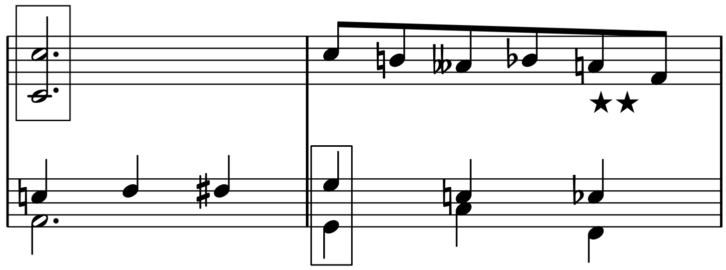
For stating that the quarter note gets the beat

(b) 1 point

For stating that the piece is six steps lower

(c) i. 1 point

For drawing a box around the stacked dotted half note in the treble staff in measure 5, or around the first stacked quarter note in the bass staff in measure 6



For those who are curious, for a hinged-hinged bar or a hinged-free bar, overtones occur according to $f_n \propto n^2 f_1$. Therefore, the first overtone is four times greater in frequency than the fundamental—more than an octave.

ii. 1 point

For a correct definition

A hinged end is a point in a bar where an object (a hinge) causes particles to not displace, but particles in the surrounding regions of the bar can displace freely.

(d) 1 point

For stating the interval is a "major third"

I didn't award credit for only stating "third" because the question asks for the interval quality ("major") too

(e) 1 point

For writing "A"

(f) 2 points

For checking "I" — 1 point

For a valid explanation — 1 point

Examples of acceptable responses:

- An I chord is played in beat 2 of measure 1, so a I chord is logical to play in beat 1 as well.
- Both ii° and IV$^+$ introduce dissonant minor seconds with C and E♭, while a I chord does not.
- The piece is in a major key, and both ii° and IV$^+$ are highly unusual in major keys.

Possible points on the test: 75 = multiple choice (35) + short answer (16) + FRQ 1 (2×8) + FRQ 2 (8)

## How was your instrument scored?

The pitch score test was scored according to the rules manual. I used the same tuning app as the app used at the 2019 Science Olympiad National Tournament. The tuner is available at `www.pascioly.org/sounds`. I ran Audacity simultaneously as a backup app.

The procedure for the song score test was adapted from `www.soinc.org/sites/default/files/uploaded_files/2020_Event_Logistics_Manual_100819.pdf`. Here, a "point" refers to a point of your overall score, such that the written test is worth exactly 45 points and the pitch test score is exactly 36 points. I am not using "point" to refer to a unit of score on the written test.

| **Criterion** | 3 points | 2 points | 0 points |
|---|---|---|---|
| Time | For playing "Twinkle Twinkle Little Star" in 15 seconds or less | N/A | For duration longer than 15 seconds |
| Rhythm | For playing "Twinkle Twinkle Little Star" with a rhythm mostly matching the arrangement in the rules manual.<br><br>Most teams received 3 points for rhythm. | For rhythm that didn't match that of the rules manual, but was recognizable as "Twinkle Twinkle Little Star" | For not playing "Twinkle Twinkle Little Star" |
| Pitch | For having a pitch within $F_2$ and $F_3$ inclusive<br><br>For pitch that produces a recognizable version of "Twinkle Twinkle Little Star" | For a melody that was a believable but unconvincing rendition of "Twinkle Twinkle Little Star" and student did not correct mistakes | Wait... what song did you just play?<br>*OR*<br>For not playing within the skipped range, as specified by the rules manual |

If your team had a complete instrument, you probably got 8 or 9 points. There was pretty much no fair way for me to give you anything less. Anything that wasn't completely wrong deserved at least some credit, but it would be hard to declare a fair cutoff between 2 points and 1 point. So, I didn't award only 1 point for any category. As long as your instrument was in the range specified in the rules, most people did not receive zero in any category.

## Share your thoughts!

I'd appreciate any remarks on what you thought of the test. If you'd like to discuss questions, don't hesitate to shoot me an email! My email address is on the front cover of this test. You're also welcome to discuss questions with me even if you received this test through a trade or through your team's test exchange.

You can also anonymously give your feedback at the link below.

`http://scioly.web.unc.edu/rate-my-tests-gz839918`

## Errata

The original version of this test and key included several errors. In the solution for question 1, I incorrectly wrote that $\sigma$ was the "density per area," when it is actually the area density. For question 5, I should have swapped the order of the underlined quantities "pressure" and "speed" above the choices. That is, the one on the left should have been on the right, and vice versa. This version of the answer key has corrected these errors, as well as various minor grammatical errors.