

①

$$(0) \quad \frac{\partial \rho}{\partial t} + \nabla \cdot \mathbf{J} = 0; \quad \text{continuity equation}$$

$$(1) \quad \frac{\partial \mathbf{E}}{\partial t} - \nabla \times \mathbf{B} = -\mathbf{J};$$

$$\nabla \cdot (\nabla \times \mathbf{A}) = 0$$

$$(2) \quad \frac{\partial \mathbf{B}}{\partial t} + \nabla \times \mathbf{E} = 0;$$

$$(3) \quad \nabla \cdot \mathbf{E} = \rho;$$

$$(4) \quad \nabla \cdot \mathbf{B} = 0;$$

$$\frac{\partial (\nabla \cdot \mathbf{E})}{\partial t} = -\nabla \cdot \mathbf{J} = \frac{\partial \rho}{\partial t}; \quad [\text{by } \nabla \cdot (1)]$$

$$\frac{\partial (\nabla \cdot \mathbf{B})}{\partial t} = \frac{\partial}{\partial t} 0 = 0; \quad [\text{by } \nabla \cdot (2)],$$

Hence, if (3) and (4) are true at  $t = 0$ , they are true for  $t \geq 0$  as well.

$$(1^*) \quad -\frac{\partial \mathbf{E}}{\partial t} + \nabla \times \mathbf{B} = 0$$

$$(2^*) \quad \frac{\partial \mathbf{B}}{\partial t} + \nabla \times \mathbf{E} = 0;$$

$$\begin{pmatrix} \text{rot } F_x \\ \text{rot } F_y \\ \text{rot } F_z \end{pmatrix} = \begin{pmatrix} \frac{\partial F_z}{\partial y} - \frac{\partial F_y}{\partial z} \\ \frac{\partial F_x}{\partial z} - \frac{\partial F_z}{\partial x} \\ \frac{\partial F_y}{\partial x} - \frac{\partial F_x}{\partial y} \end{pmatrix}$$

$$- \frac{E_x^{n+1,i,j,k} - E_x^{n,i,j,k}}{\Delta t} + \frac{B_z^{n,i,j,k} - B_z^{n,i,j-1,k}}{h_y} = 0$$

$$- \frac{B_y^{n,i,j,k} - B_y^{n,i,j,k-1}}{h_z} = 0$$

$$- \frac{E_y^{n+1,i,j,k} - E_y^{n,i,j,k}}{\Delta t} + \frac{B_x^{n,i,j,k} - B_x^{n,i,j,k-1}}{h_z} = 0$$

$$- \frac{B_z^{n,i,j,k} - B_z^{n,i-1,j,k}}{h_x} = 0$$

$$- \frac{E_z^{n+1,i,j,k} - E_z^{n,i,j,k}}{\Delta t} + \frac{B_y^{n,i,j,k} - B_y^{n,i-1,j,k}}{h_x} = 0$$

$$= \frac{B_x^{n,i,j,k} - B_x^{n,i,j-1,k}}{h_y} = 0$$

$$\frac{B_x^{n+1,i,j,k} - B_x^{n,i,j,k}}{\Delta t} + \frac{E_z^{n,i,j,k} - E_z^{n,i,j-1,k}}{h_y} - \frac{E_y^{n,i,j,k} - E_y^{n,i,j,k-1}}{h_z} = 0$$

$$\frac{B_y^{n+1,i,j,k} - B_y^{n,i,j,k}}{\Delta t} + \frac{E_x^{n,i,j,k} - E_x^{n,i,j,k-1}}{h_z} - \frac{E_z^{n,i,j,k} - E_z^{n,i-1,j,k}}{h_x} = 0$$

$$\frac{B_z^{n+1,i,j,k} - B_z^{n,i,j,k}}{\Delta t} + \frac{E_y^{n,i,j,k} - E_y^{n,i-1,j,k}}{h_x} - \frac{E_x^{n,i,j,k} - E_x^{n,i,j-1,k}}{h_y} = 0$$



$$E_x^{n+1,i,j,k} = E_x^{n,i,j,k} + \Delta t \left[ \frac{B_z - B_z}{h_y} + \frac{B_y - B_y}{h_z} \right]$$

$$E_y^{n+1,i,j,k} = E_y^{n,i,j,k} + \Delta t \left[ \frac{B_x - B_x}{h_z} + \frac{B_z - B_z}{h_x} \right]$$

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$$B_x^{n+1,i,j,k} = B_x^{n,i,j,k} - \Delta t \left[ \frac{E_z - E_z}{h_y} + \frac{E_y - E_y}{h_z} \right]$$

$$B_y^{n+1,i,j,k} = B_y^{n,i,j,k} - \Delta t \left[ \frac{E_x - E_x}{h_z} + \frac{E_z - E_z}{h_x} \right]$$

$$B_z^{n+1,i,j,k} = B_z^{n,i,j,k} - \Delta t \left[ \frac{E_y - E_y}{h_x} + \frac{E_x - E_x}{h_y} \right]$$