TECHNISCHE UNIVERSITÄT MÜNCHEN
Zentrum Mathematik

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Computational Plasma Physics, SS 2022
http://www-m16.ma.tum.de/Allgemeines/CompPlasmaPhys22
Exercise Sheet 5 (06. 06. 2022)

1D constant-coefficient advection

This week we will be considering a 1D advection equation

$$\frac{\partial}{\partial t} f(t,x) + v \frac{\partial}{\partial x} f(t,x) = 0 \tag{1}$$

with some constant $v \in \mathbb{R}$, in some time interval $t \in [0,T]$ on a domain $[a,b]$ with periodic boundary conditions

$$f(t,a) = f(t,b) \qquad \forall\, t \in [a,b] \tag{2}$$

We will approximate the function f by vectors f_i^n at time t^n and grid-point x_i.

1. Implement an Euler upwind scheme:

$$f_i^{n+1} = f_i^n - \frac{\Delta t}{h} \left(v_-(f_{i+1}^n - f_i^n) + v_+(f_i^n - f_{i-1}^n) \right) \tag{3a}$$

$$v_- = \min(0,v) \qquad v_+ = \max(0,v) \tag{3b}$$

2. Implement a Lax-Wendroff scheme:

$$f_i^{n+1} = f_i^n - \frac{v\Delta t}{2h}(f_{i+1}^n - f_{i-1}^n) + \frac{v^2 \Delta t^{\,2}}{2h^2}(f_{i+1}^n - 2f_i^n + f_{i-1}^n) \tag{4}$$

3. Test your code with the parameters $a = 0, b = 1, \Delta t = 0.03, T = 0.9, N = 100$, try both $v = +2$ and $v = -2$, and take the initial condition

$$f(0,x) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{\left(x - \frac{a+b}{2}\right)^2}{2\sigma^2} \right) \tag{5}$$

   for which the analytical solution of (1) reads $f(t,x) = f(0, x - vt)$. Take $\sigma = 0.05$ for your tests. Plot your approximation and the analytical solution for different times. What do you observe? Also plot the maximum of your approximation and the maximum of the analytical solution as functions of time.

4. The discrete mass is defined as

$$m_h = \int f(t,x)\, \mathrm{d}x \tag{6}$$

   Compute the discrete mass and plot it versus time. What do you observe? Does this correspond to your expectation?