(1)-(2) posses a unique solution.

Proof: Let $\exists$ $\varphi(x)$ and $\psi(x)$, which are both sol-ns of the system:

$$-\varphi''(x) = \rho(x); \quad x \in [a,b] \subset \mathbb{R}; \qquad (1)$$

$$\varphi(a) = \alpha; \quad \varphi'(b) = \gamma \qquad (2);$$

Let $\eta(x) := \varphi(x) - \psi(x); \quad x \in [a;b];$

$\varphi(x)$ and $\psi(x)$ are sol-ns of (1)-(2), $\Rightarrow$

$$\Rightarrow -\eta''(x) = 0; \quad \eta(a) = 0; \quad \eta'(b) = 0; \qquad (3)$$

By solving (3) we get: $\eta(x) = 0; \quad x \in [a,b] \Rightarrow$

$$\Rightarrow \varphi(x) = \psi(x), \quad x \in [a;b];$$

- Let's find an approximation of $\varphi'(x_N)$ of the 2nd order:

$$\varphi'(x_N) = \frac{1}{h}\left(c_1 \varphi_N + c_2 \varphi_{N-1} + c_3 \varphi_{N-2}\right);$$

$$\varphi(x_{N-1}) = \varphi(x_N - h) = \varphi(x_N) - h \cdot \varphi'(x_N) + \frac{h^2}{2} \varphi''(x_N) + $$

$$+ \frac{h^3}{6} \varphi'''(x_N) + O(h^4)$$

$$\varphi(x_{N-2}) = \varphi(x_N - 2h) = \varphi(x_N) - 2h \varphi'(x_N) + 2h^2 \varphi''(x_N) - \frac{4h^3}{3} \varphi'''(x_N) +$$

$$+ O(h^4);$$

$$\varphi(x_{N-2}) - \varphi(x_{N-1}) \cdot 4 + 3\varphi(x_N) = \varphi'(x_N) \cdot 2h + O(h^3)$$

$$\varphi'(x_N) = \frac{\varphi(x_{N-2}) - 4\varphi(x_{N-1}) + 3\varphi(x_N)}{2h} + O(h^2).$$

- Let's consider (1) with $\varphi(x+L) = \varphi(x), \forall x \in \mathbb{R}$ (4);

Necessary condition) $\varphi'(L) - \varphi'(0) = -\int_0^L g(x)\,dx = 0$;

$$\int_0^{2\pi} 4\sin(2x)\,dx = 0;$$

Claim: The solution of (1)+(4) is unique up to a constant α;

Proof: As previously, let's consider $\eta(x) = \varphi(x) - \psi(x)$:

$$-\eta'' = 0, \quad \eta(0) = \eta(L); \quad \eta'(0) = \eta'(L);$$

General solution: $\eta = C_1 x + C_2$;

$\eta(0) = \eta(L) \Rightarrow C_1 = 0$; $\Rightarrow \eta = C_2$

$\Rightarrow \varphi(x) = \psi(x) + C$;

If we add the condition $\varphi(0) = \alpha$, then the solution is unique.

b) A is non-invertible ($\det A = 0$), as $(1, 1 \ldots 1) \in \operatorname{Ker} A$;

$\Rightarrow$ we ~~adot~~ change the last equation (row) by

$$\varphi(0) = \varphi_0 = \alpha; \quad \lambda_i(A') > C^* > 0;$$