TECHNISCHE UNIVERSITÄT MÜNCHEN
**Zentrum Mathematik**

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**Computational Plasma Physics, SS 2022**
http://www-m16.ma.tum.de/Allgemeines/CompPlasmaPhys22
**Exercise Sheet 6 (27. 06. 2022)**

# 1D constant-coefficient advection with Fourier spectral method

This week we will again consider a 1D advection equation

$$\frac{\partial}{\partial t} f(t,x) + v \frac{\partial}{\partial x} f(t,x) = 0 \tag{1}$$

with some constant $v \in \mathbb{R}$, in some time interval $t \in [0,T]$ on a domain $[a,b]$ with periodic boundary conditions

$$f(t,a) = f(t,b) \qquad \forall\, t \in [a,b] \tag{2}$$

We will approximate the function $f$ by vectors $f_i^n$ at time $t^n$ and grid-point $x_i$.

Last week we saw that the Euler upwind scheme and the Lax-Wendroff scheme preserved the discrete mass but not the shape of the function $f(t,x)$.

This week we will look at the Fourier spectral collocation method using the FFT-module of `numpy`[1]:

1. Create a vector $f_0$ of the initial function values
2. Perform the FFT and followed by a shift of the modes to yield $\hat{f}_0$.
3. Derive a time-step update rule for the Fourier modes $\hat{f}_k$ from the Fourier ansatz

$$f(t,x) = \sum_k \hat{f}_k(t) \exp\left(\frac{2\pi i k x}{L}\right) \tag{3}$$

   for the advection equation (1).

4. Implement a time-loop where $\hat{f}$ gets updated and test your code with the parameters $a = 0, b = 2\pi, \Delta t = 0.01, T = 1, N = 400, v = 2$. Use the initial condition and corresponding analytical solution with $\sigma = 0.2$ from last weeks sheet.
5. Compute the error in the $L^1$-, $L^2$, $L^\infty$-norm in each time-step and plot it versus time.
6. *Optional*: Examine the error in the $L^1$-, $L^2$, $L^\infty$-norm: what happens for larger $N$?

[1] Hint: Use the `numpy` module `fft`, it contains functions such as `fft`, `ifft`, `fftshift`, and `ifftshift`.