

Section 6, Exercise 1

Assume $H = 0$, $J = 1$, and state 1 is the microstate where all spins are $s_i = +1$. In microstate 2, one spin has flipped. (Assume this flipped spin is not at an edge.) What is the difference in the energies of the two microstates, $E_1 - E_2$?

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With $H = 0$ and $J = -1$, we have

$$E = - \sum_{\text{pairs}} s_i s_j \quad (1)$$

Assume we have N_{pairs} pairs. In Microstate 1, all spins are up, so

$$E_1 = - \sum_{i=0}^{N_{\text{pairs}}} (1)(1) = -N_{\text{pairs}} \quad (2)$$

In Microstate 2, all spins are up except for one. Of N_{pairs} pairs, only 4 pairs are affected by this change, so

$$\begin{aligned} E_2 &= - \sum_{i=0}^{N_{\text{pairs}}-4} (1)(1) + - \sum_{i=0}^4 (-1)(1) \\ &= -(N_{\text{pairs}} - 4) - 4(-1) \\ &= -N_{\text{pairs}} + 4 + 4 \\ &= 8 - N_{\text{pairs}} \end{aligned} \quad (3)$$

Therefore

$$E_1 - E_2 = -N_{\text{pairs}} - (8 - N_{\text{pairs}}) = -8 \quad (4)$$

Section 6, Exercise 2

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Magnetization is the sum of the spins on the lattice. In the given microstate, we know that at least 1 of N spins must be up, so we have

$$M = 1 + \sum_{i=0}^{N-1} s_i \equiv 1 + M_{\text{other}} \quad (5)$$

where we've defined M_{other} as the sum of spins other than the one we know is up. Then that one spin flips, so we have

$$M_{\text{new}} = -1 + \sum_{i=1}^{N-1} s_i = -1 + M_{\text{other}} = -1 + (M - 1) = M - 2 \quad (6)$$

Section 6, Exercise 3

1. On an $n \times n$ lattice with periodic boundary conditions, what is the maximum distance between two spins in the same row?
2. On an $n \times n$ lattice with periodic boundary conditions, how many other spins will a given spin be paired with to calculate the correlation function? Assume we are only considering pairs of spins in the same row or column.

Section 6, Exercise 3

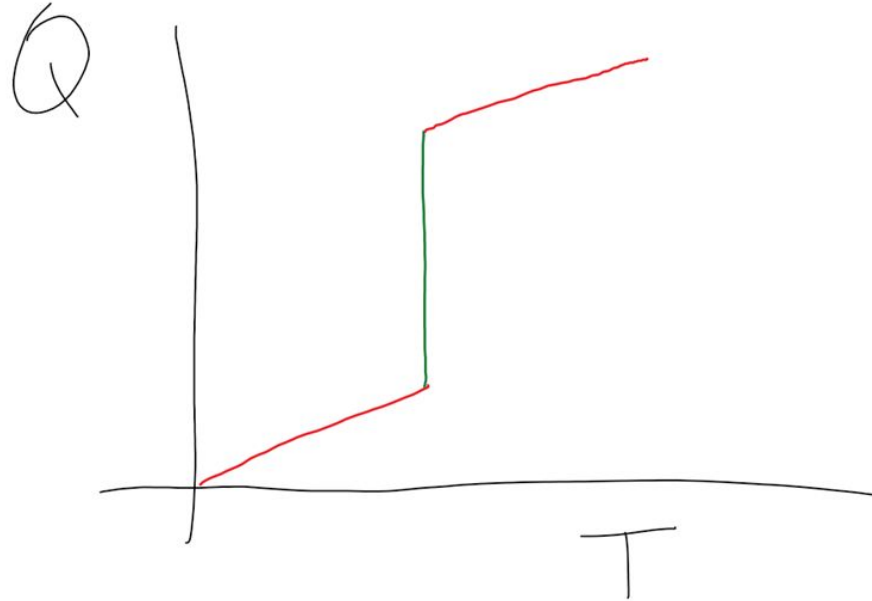
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1. $n/2$ if n is even, or $(n-1)/2$ if n is odd
2. $2(n-1)$

(See code)

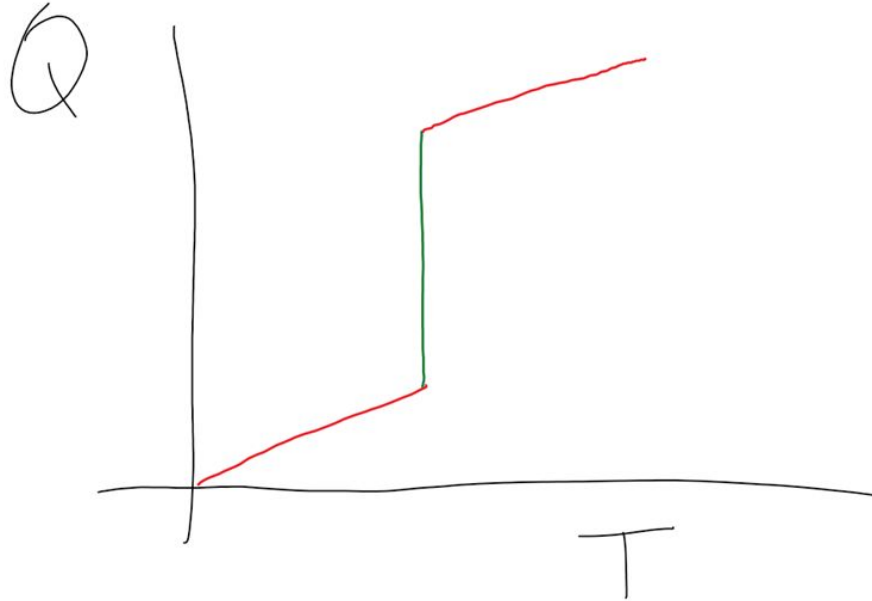
Section 6, Exercise 4

The sketch shows the heat added (Q) vs the temperature of some substance at constant pressure. Explain what process is happening in the green portion of the curve.



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A phase change (i.e solid to liquid, liquid to gas). This is an example of a first-order phase transition.