Homework 4

Phys 4350

Turn in your code as well as images of your plots. Remember to use axis labels with units. If a numerical answer is required, it should be printed out when the program is run.

1. In the cannonball example from lecture, we predicted the trajectory of the cannonball including wind along the direction of the launch. Now I want you to consider the case where a wind is blowing at an angle with respect to the launch direction, blowing the cannonball off path. You will also use a different function than what we used in class to model the air density as a function of height.

   Use Euler's method to predict the trajectory of a cannonball launched at an angle of 37° with an initial speed of 550 m/s. Assume the projectile is launched from sea level ($y = 0$) towards the east. Assume a horizontal wind is blowing with a speed of 11 m/s (around 25 mph), oriented at an angle of 22° north of east. Incorporate the effects of the air resistance, assuming the air density varies as a function of height as $\rho = \rho_0 \left(1 - \frac{ay}{T_0}\right)^b$, where $a = 6.5 \times 10^{-3}$ K/m, $b = 2.5$, and $T_0 = 300$ K. You may assume that the ratio of the drag factor (D in $F_{\text{air}} = Dv^2$) to the mass has a value $D/m = 3.2 \times 10^{-5}$ at sea level ($y = 0$). Plot the three-dimensional trajectory of the cannonball (y vs x, z). What is the range of the cannonball? At the position it lands, how far displaced is it from due east?

   Notes: Let y be the vertical direction as we have in previous examples. Let the z axis point towards the east (the direction of launch), and let the x axis point towards the north. In other words, the xz plane is the surface of the Earth, and the y axis represents height above the Earth. Note that the wind has both x and z components. I strongly recommend drawing some pictures and writing out all the equations before trying to code this up. In our first lecture, I showed an example of one way to make a 3D plot; pay close attention to the axes, since the y axis should be vertical. Note that you can click and drag the plot to adjust the viewing angle.

2. The Van der Pol oscillator is a non-conservative, oscillating system with non-linear damping. (For a non-conservative system, the total work done depends on the path taken. Another way of stating it is that for a non-conservative system, the total work done for a closed path with the same starting and ending points is not zero.) The Van der Pol oscillator was first studied in electrical circuits, but has been applied to other

problems. The differential equation, including a driving force term, is given by

$$\frac{d^2x}{dt^2} - \mu(1 - x^2)\frac{dx}{dt} + x - A\sin(\omega t) = 0 \tag{1}$$

where x is position, μ is a constant which represents the non-linearity and strength of the damping, A is the amplitude of the driving force function, and ω is the angular frequency of the driving force.

Use the fourth-order Runge-Kutta method to plot $x(t)$ vs t from $t = 0$ to $t = 150$ s with initial conditions $x = 1$ and $\frac{dx}{dt} = 0$ at $t = 0$. Then make a phase space plot of $\frac{dx}{dt}$ vs x. Make both plots for each of the following parameter sets:

(a) $\mu = 0.1,\ A = 0$

(b) $\mu = 1,\ A = 0$

(c) $\mu = 10,\ A = 0$

(d) $\mu = 8,\ A = 1.2,\ \omega = 2\pi/10$

What is the effect of increasing the parameter μ when the driving force is zero ($A = 0$)? Which of the above parameter sets results in chaotic behavior?