

## Section 2, Exercise 1

Write a program to calculate the area of the trapezoid bounded by  $x_1 = 1.5$ ,  $x_2 = 2$ , the x-axis, and a straight line drawn between  $f(x_1)$  and  $f(x_2)$ , where  $f(x) = x^2$ .

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```
1 def f(x):  
2     return x**2  
3  
4 x1=1.5  
5 x2=2.0  
6 dx=x2-x1  
7 area = 0.5*(f(x1)+f(x2))*dx  
8 print(area)  
9  
10
```

1.5625

## Section 2, Exercise 2

Suppose you have two data points  $(x_1, y_1)$  and  $(x_2, y_2)$ , and you want to estimate what the value of  $y$  would be for some value of  $x$  between  $x_1$  and  $x_2$  using *linear interpolation*. What does that mean? How does this relate to a numerical integral using the trapezoidal rule?

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An interpolation is an educated guess about the  $y$  value between two known points. A linear interpolation assumes that the function is a smooth line between the two known points.

With a linear interpolation, the extracted  $y$  value for an  $x$  halfway in between the two known points is the average of the  $y$  value at the two known points. The trapezoidal rule, by doing an average, is making a linear interpolation between the two give points.

## Section 2, Exercise 3

Can you think of other ways of representing the vector field with a plot, other than the arrow plot? You have to depict both the magnitude and direction, which you could do separately or on the same plot.

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A density plot, which represents density by shading or color, could be used to represent the magnitude of the vector.

A similar color-based plot could be used to show the direction, provided that the scale only represents numbers from 0 to  $2\pi$ .

Keep the arrows to represent direction, but make all the arrows the same size. Use the color of the area to represent the magnitude.

## **Section 2, Exercise 4**

Suppose you have a list of numbers, and you want to determine which number is closest to some value. Make up a list of numbers, and write a program that finds the one closest to 5 and prints it out.

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```
#here's my list:
a = [5.001, 5.000002, 4.739, 4.999999999999999, 5.28, 4.5, 6.2]
print(a)

#here we search through the list
diff=10
closest=10
for i in a:
    if abs(i-5)<diff:
        diff=abs(i-5)
        closest=i
print("closest value to 5 in this list is: ",closest)
```

```
>>>
= RESTART: C:\Users\lisaw\OneDrive\Documents\Teaching\4350\2023Fall\Lecture Notes\Section2-NumericalCalculus\exercises\Exercise4.py
[5.001, 5.000002, 4.739, 4.999999999999999, 5.28, 4.5, 6.2]
closest value to 5 in this list is:  4.999999999999999
>>> |
```



## Section 2, Exercise 5

Write a program which uses the forward difference to approximate the derivative of  $f(x) = x^5$ , evaluated at  $x = 0.5$ , using  $h = 1e - 5$ . Also find the true value of the integral and compare the values.

## Section 2, Exercise 5

Write a program which uses the forward difference to approximate the derivative of  $f(x) = x^5$ , evaluated at  $x = 0.5$ , using  $h = 1e - 5$ . Also find the true value of the integral and compare the values.

```
>>>  
= RESTART: C:\Users\lisaw\OneDrive\Documents\Teaching\4350\2023Fall\Lecture Notes\Section2-NumericalCalculus\exercises\Exercise5.py  
Numerical approximation = 0.3125125  
True value = 0.3125  
Difference = 1.25e-05  
Note that the approximation error is  $0.5 \cdot h \cdot f'(x) = 1.25e-05$   
The rounding error is much smaller,  $\sim 6.25e-13$ 
```