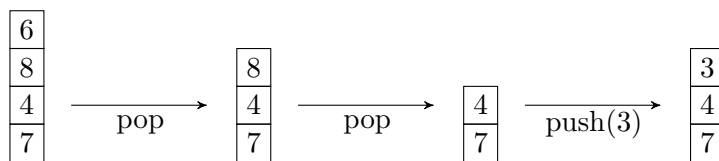


Chapter 7

Stacks and Queues

7.1. Stack

The stack is a basic data structure in which the insertion of new elements takes place at the top and deletion of elements also takes place from the top. The idea of the stack can be illustrated by plates stacked on top of one another. Each new plate is placed on top of the stack of plates, and plates can only be taken off the top of the stack.



The stack can be represented by an array in which we should also remember the size of the stack and we must be sure to declare sufficient space for the array (in the following implementation we can store N elements).

7.1: Push / pop operations — $O(1)$.

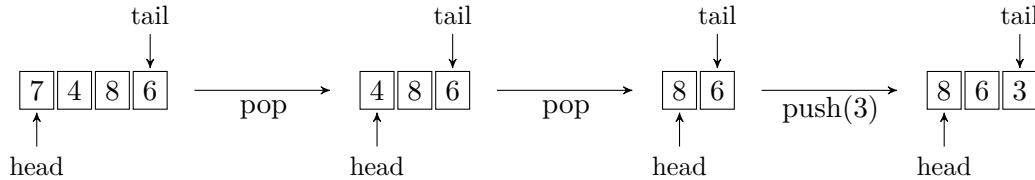
```

1 stack = [0] * N
2 size = 0
3 def push(x):
4     global size
5     stack[size] = x
6     size += 1
7 def pop():
8     global size
9     size -= 1
10    return stack[size]
  
```

The push function adds an element to the stack. The pop function returns and removes the most recently pushed element from the stack. We shouldn't perform a pop operation on an empty stack.

7.2. Queue

The queue is a basic data structure in which new elements are inserted at the back but old elements are removed from the front. The idea of the queue can be illustrated by a line of customers in a grocery store. New people join the back of the queue and the next person to be served is the first one in the line.



The queue can be represented by an array in which we should also remember the front (head) and back (tail) of the queue. we must be sure to declare sufficient space for the array (in the following implementation we can store $N - 1$ elements).

7.2: Push / pop / size / empty operations — $O(1)$.

```
1 queue = [0] * N
2 head, tail = 0, 0
3 def push(x):
4     global tail
5     tail = (tail + 1) % N
6     queue[tail] = x
7 def pop():
8     global head
9     head = (head + 1) % N
10    return queue[head]
11 def size():
12    return (tail - head + N) % N
13 def empty():
14    return head == tail
```

Notice that in the above implementation we used cyclic buffer (you can read about it more at http://en.wikipedia.org/wiki/Circular_buffer).

The push function adds an element to the queue. The pop function returns and removes an element from the front of the queue. We shouldn't perform a pop operation on an empty queue (the empty function or the size function can be used to check whether the queue is empty).

7.3. Exercises

Problem: You are given a zero-indexed array A consisting of n integers: a_0, a_1, \dots, a_{n-1} . Array A represents a scenario in a grocery store, and contains only 0s and/or 1s:

- 0 represents the action of a new person joining the line in the grocery store,
- 1 represents the action of the person at the front of the queue being served and leaving the line.

The goal is to count the minimum number of people who should have been in the line before the above scenario, so that the scenario is possible (it is not possible to serve a person if the line is empty).

Solution: We should remember the size of the queue and carry out a simulation of people arriving at and leaving the grocery store. If the size of the queue becomes a negative number then that sets the lower limit for the number of people who had to stand in the line previously. We should find the smallest negative number to determine the size of the queue during the whole simulation.

7.3: Golden solution — $O(n)$.

```
1 def groceryStore(A):
2     n = len(A)
3     size, result = 0, 0
4     for i in xrange(n):
5         if (A[i] == 0):
6             size += 1
7         else:
8             size -= 1
9             result = max(result, -size);
10    return result
```

The total time complexity of the above algorithm is $O(n)$. The space complexity is $O(1)$ because we don't store people in the array, but only remember the size of the queue.