Easy

- 1. First, we count the small triangles. There are three small triangles. Next, there is one medium triangle consisting of the two right small triangles. Finally, there exists the large triangle, which is the entire figure. Therefore, the answer is $2 + 1 + 1 = (E) \ 5$.
- 2. To find the area of the shaded region, we need to find the area of the circle and subtract it from the area of the entire rectangle.

The diameter of the circle is 1, so we find that the radius is 1/2 which makes the area

$$(1/2)^2\pi = \pi/4.$$

The rectangle has area $2 \times 3 = 6$, so the area of the shaded region is $6 - \pi/4$. We can then easily figure that this is closest to (E) 5.

- 3. Since opposite sides of a parallelogram are congruent, we know that AD = BC = 4. If we let the base be AD, we know that the height is the difference of the y-coordinates of A and B, which is 2. The area of a parellogram is the product of the base and the height. Therefore, the answer is $4 \cdot 2 = (B) \ 8$.
- 4. We could approach the problem by counting the area of each square and triangle inside the polygon, but if we stop to inspect the figure, we can notice that the triangle on the top row matches the hole in the bottom row. Therefore, we can perfectly fit the top triangle into the bottom, like a puzzle piece, which conveniently creates a 2×3 rectangle, which has area $2 \times 3 = \boxed{(B) \ 6}$.
- 5. Each of the small shaded triangles has side length of $\frac{1}{4} \times 8 = 2$, so the area is $\frac{1}{2} \times 2 \times 2 = 2$. We can count to find that there are 10 of these triangles is the shaded area. Therefore the shaded area is $2 \times 10 = 6$ (B) 20.
- 6. There is one shaded triangle, which is congruent to an unshaded triangle. Next, there is one medium shaded trapezoid, which is congruent to an unshaded trapezoid. Finally, there is one large shaded trapezoid, which is congruent to an unshaded trapezoid. Therefore, the area of the shaded region is equal to the area of the unshaded region. Thus, we find that the shaded region makes up $\frac{1}{2}$ of the entire square, so the answer is (E).
- 7. Since the hour hand is at 7 and the minute hand is at 12, we know that the angle makes up 5/12 of the circle which is the clock. Since a circle is 360° , so the measure of the smaller angle is

$$\frac{5}{12} \cdot 360^{\circ} = 150^{\circ} \rightarrow \boxed{D}$$

8. There are two cases for the numbers: 11, 12, 13, 14, 15, 16 and 10, 11, 12, 13, 14, 15.

In the second case, we can find the common sum, which would be (10 + 11 + 12 + 13 + 14 + 15)/3 = 25, because there are three 3 common sums in the whole cube. In this case, 11 must be paired with 14, but it is not.

Thus, the only possibility is the first case: 11, 12, 13, 14, 15, 16 and the sum of the six numbers is (E) 81

- 9. The Triangle Inequality states that the sum of any two sides in a triangle is greater than the third side. By Triangle Inequality, we know that 6.5 + s > 10 and therefore we find s > 3.5. The smallest whole number that satisfies this is (B) 4.
- 10. The sum of the angles in a triangle is 180°. Therefore, we can find that $\angle ABE = 180 60 40 = 80$ °. Next, $\angle CBD = 180 80 = 100$ °, because $\angle ABC = 180$.

Now we can find $\angle BDC$ because we know the other two angles in the triangle.

$$\angle BDC = 180 - 100 - 30 = (B) 50^{\circ}$$

Medium

1. There are many ways to approach this problem.

A simple solution is to divide the square into 16 congruent smaller squares. The shaded square is formed from 4 half-squares, so its area is 2. Therefore, the ratio 2 to 16 is 1/8, so the answer is $\boxed{\mathbf{C}}$.

2. We can call x the sidelength of one square and y the sidelength of the other square, where x > y. The perimeter of the first square is 4x and the perimeter of the second square is 4y. Since the perimeter of one is 3 times the other's, then $4x = 3(4y) \rightarrow x = 3y$. Now we can find the ratio of area of the larger square over the area of the smaller square, which is

$$\frac{x^2}{y^2} = \frac{(3y)^2}{y^2} = \frac{9y^2}{y^2} = \boxed{\text{(E) 9}}$$

- 3. We know that the area of the shaded region is the area of the entire parallelogram ABCD minus the area of the triangle ABE. The area of parallelogram ABCD is the base times the height, which is $10 \cdot 8 = 80$. The area of the triangle ABE is slightly more tricky, because the base AE is unknown. However, since AD = BC, we know that AE + ED = 10, so $AE + 6 = 10 \rightarrow AE = 4$. Now we can find the area of triangle ABE, which is $\frac{1}{2} \cdot 4 \cdot 8 = 16$. Now, the answer is just the difference of the two areas that we calculated, which is $80 16 = \boxed{\text{(D) } 64}$.
- 4. There are 26 5 = 21 pairs of consecutive exits. To find the maximum number of miles of one of these, we must set the other 20 equal to the minimum number, which is 5. These 20 exits contribute a total of (5)(20) = 100 miles. The longest distance must therefore be whatever's left, which is $118 100 = \boxed{(C) \ 18}$.

Hard

1. After we finish both folds, the square becomes a triangle that has an area of $\frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}$ of the original square.

Since 9 square inches is the area for $\frac{1}{4}$ of the square, we can find that $9 \times 4 = 36$ square inches is the area of the entire square PQRS.

Since the area of a square is 36, we find that the side length is $\sqrt{36} = 6$ inches, because the area of a square is just the side length squared.

If each side is 6 inches, the total perimeter must be $6 \times 4 = \boxed{\text{(D) } 24}$

2. We can consider the contributions of the tops of the three layers and the sides of the layers separately.

Layer 1 has 4 side unit squares. Similarly, layer 2 has $4 \times 2 = 8$ side unit squares, and layer 3 has $4 \times 3 = 12$ side unit squares. Therefore, the sides of the pyramid contribute 4 + 8 + 12 = 24 for the surface area.

Instead of directly calculating the area of the tops of the layers, we realize that when combined, they form the same arrangement of unit cubes as the bottom of the pyramid, which is a 3×3 square. Therefore, this contributes 9 for the surface area.

Thus, the artist paints a total of $24 + 9 = 33 \rightarrow \boxed{\text{C}}$ square meters.

3. The original 12 edges still remain. Therefore, in addition to the original 12 edges, each original vertex contributes 3 new edges.

There are 8 original vertices, so there are $12 + 3 \times 8 = \boxed{\text{(C) 36}}$ edges in the new figure.

4. We deduce that we must use some cubes of edge length 1 and some cubes of edge length 2, because if none of the cubes have edge length 2, then all of the cubes have edge length 1, meaning they all are the same size, which is a contradiction.

2

Since it is impossible to split a cube of edge 3 into two or more non-overlapping cubes of edge 2 with extra unit cubes, we know that there must be one $2 \times 2 \times 2$ cube and $3^3 - 2^3 = 19$ unit cubes.

The total number of cubes, N, is 1 + 19 = (E) 20.

- 5. We should notice that with each change, 3/4 of the black space from the previous change remains. Since there are 5 changes, the fractional part of the triangle that remains black after $(3/4)^5 = 243/1024$, so the answer is \boxed{C} .
- 6. We find that the side of the square is made up of two radii of length 3, so the area of the square is $(3+3)^2 = 36$. The unshaded region of the square is composed of four quarter circles, so it has the area of one whole circle, which is $3^2\pi = 9\pi$. The area of the shaded region is the area of the entire square minus the area of the unshaded region $36 9\pi$. We can compute that this is closest to (A) 7.7.
- 7. Model the amount left in the container as follows:

After the first pour $\frac{1}{2}$ remains, after the second $\frac{1}{2} \times \frac{2}{3} = \frac{1}{3}$ remains, after the third $\frac{1}{3} \times \frac{3}{4} = \frac{1}{4}$ remains. So there is a pattern. After each pour, the water left =

 $\frac{1}{\text{(the number of times that we pour+1)}}.$

So to get one tenth, the number of times that we pour is 10 - 1 = 9, so the answer is (D) 9.

- 8. The radius of each of the four semicircles is 2, because it is just half of the sidelength of the small square. If we draw the line straight down the middle of square ABCD, we notice this is equal to the side length of the big square. It is also equal to the sum of two radii and the length of the smaller square. Therefore, the side length of the big square is 4 + 2 + 2 = 8. The answer is the area of ABCD, which is $8^2 = \boxed{\text{(E) } 64}$.
- 9. We can imagine removing the cubes that do not fit the description of the problem.

In the top face, there are $2 \times 2 = 4$ cubes in the center of the top face that do not fit the description, so we will remove these.

Now, we move to one layer down and follow the same process to remove 4 more cubes in the second layer.

Finally, we repeat this in the third layer, and remove 4 more cubes..

We do not consider the bottom layer, because these cubes touch the bottom of the box.

The total number of cubes that we removed is 4+4+4=12 out of the $4\times4\times4=64$ cubes.

Therfore, our answer is the number of cubes which remain, which is $64 - 12 = \boxed{\text{(B) } 52}$

10. All small triangles are congruent in each iteration of the diagram. The number of shaded triangles follows this pattern:

 $0, 1, 3, 6, \dots$

Each time, the number 1, 2, 3, 4, 5... is added to the previous term. Thus, the first eight terms are:

0, 1, 3, 6, 10, 15, 21, 28

In the eighth diagram, we see that there will be 28 shaded triangles.

Now, the total number of small triangles follows the pattern:

 $1, 4, 9, 16, \dots$

which is the pattern of "square numbers". Thus, the eighth triangle will have $8^2 = 64$ small triangles in total.

The ratio of shaded to total triangles will be the fraction of the whole figure that's shaded, since all triangles are congruent. Therefore, the answer is $\frac{28}{64} = \frac{7}{16}$, which is \boxed{C} .