

## Easy

1. Let's try to find a pattern. Notice that the first row has one white square and zero black squares. The second row has two white squares and one black square. The third row has three white squares and two black squares. There is the pattern. In the  $n$ th row, there will be  $n$  white squares and  $n - 1$  black squares. We could prove the pattern, but it is clear that the pattern holds for all rows. Therefore, following this pattern, we find that in the 37th row, there will be  $37 - 1 = \boxed{\text{(C) } 36}$  black squares.

2. We can simplify the expression. The first term becomes  $1 + 3 - 4 = 0$ . The second term is  $2 + 5 - 6 = 1$ . Therefore, the answer is just the sum, which is  $0 + 1 = \boxed{\text{(D) } 1}$ .

3. The graph shows that there are twice as many females as males. Let's call the number of males  $m$ . Then since the total population is 480, we make an equation:  $2m + m = 480$ , so  $3m = 480$ , and  $m = 160$ . Therefore, the answer is  $\boxed{\text{(B) } 160}$ .

4. We can first find the minimum possible value of this sum is when  $A = B = 1$ , which is

$$9876 + 132 + 11 = 10019$$

This number has 5 digits. We can easily figure that no matter what digits we use for  $A$  and  $B$ , we cannot exceed 5 digits. Therefore, the answer is  $\boxed{\text{(B) } 5}$ .

5. To achieve the smallest number, we should pick the three smallest numbers in the set: 3, 5, 7. Now, we can check all three ways of doing the next operations.

$$(3 + 5)7 = 8 \cdot 7 = 56$$

$$(3 + 7)5 = 10 \cdot 5 = 50$$

$$(7 + 5)3 = 12 \cdot 3 = 36$$

Our answer is the smallest number, which is  $\boxed{\text{(C) } 36}$ .

6. Call this month "Month 0". Make a table of the fish that Brent and Gretel have each month.

Month / Brent / Gretel

0 / 4 / 128

1 / 16 / 256

2 / 64 / 512

3 / 256 / 1024

4 / 1024 / 2048

5 / 4096 / 4096

We see that in 5 months they will have the same number of fish, giving an answer of  $\boxed{B}$

Solution 2: A more advanced way to solve this would be by using algebra. We let  $x$  represent the number of months it takes till they get the same number of fish. After  $x$  months, the number of fish that Brent has is  $4 \cdot 4^x$ , because each month his fish quadruples. Similarly, Gretel's fish after  $x$  months is  $128 \cdot 2^x$ . Now, we set these two expressions equal and solve for  $x$ .

Our equation is  $4 \cdot 4^x = 128 \cdot 2^x$ . All of these are powers of 2.

After using some laws of exponents, we simplify our equation to  $2^{(2x+2)} = 2^{(7+x)}$ . Since the bases are the same, we know that the exponents must also be the same to preserve equality.

Thus  $2x + 2 = 7 + x$ . Isolating the equation for  $x$ , we see that  $x = 5$ . Our final answer is (B) 5.

7. We can just find each person's final number.

Jose will get  $10 - 1 = 9$ , then  $9 \cdot 2 = 18$ , then  $18 + 2 = 20$ .

Thuy will get  $10 \cdot 2 = 20$ , then  $20 - 1 = 19$ , and then  $19 + 2 = 21$ .

Kareem  $10 - 1 = 9$ , then  $9 + 2 = 11$ , and then  $11 \cdot 2 = 22$ .

Thus, Kareem has the highest number, and the answer is (C) Kareem.

8. We know that 1 is in the top left corner, 8 is in the top right corner, and 64 is in the bottom right corner. We just need to find the bottom left corner. The number in that corner can be found by subtracting 7 from 64 which is 57. Adding the results gives  $1 + 8 + 57 + 64 = \text{span style="border: 1px solid black; padding: 2px;">(A) 130.$

### Medium

1. We can just visualize the cube.  $Z$  goes opposite  $V$  and  $U$  goes opposite  $W$ . Therefore, we know that  $X$  goes opposite (E) Y, since it is the only face left.

2. If the participation increases by 50%, then it is the same as the participation being multiplied by a factor of  $100\% + 50\% = 1 + 0.5 = 1.5$ . Now we can just find the population each year.

In 1997, participation will be  $800 \cdot 1.5 = 1200$ .

In 1998, participation will be  $1200 \cdot 1.5 = 1800$ .

In 1999, participation will be  $1800 \cdot 1.5 = 2700$ , giving an answer of (E) 2700.

3. If we are trying to minimize a difference, let's say  $a - b$ , we need to minimize  $a$  and maximize  $b$ . Since in this problem,  $a$  is a three-digit number and  $b$  is a two-digit number, we set  $a = 245$  and  $b = 96$ . Their difference is therefore the smallest possible, which is  $249 - 96 = \text{span style="border: 1px solid black; padding: 2px;">(C) 149.$

4. To achieve the shortest possible rectangle that has sides 36 and 60, we should make the side opposite the wall 60, because we use it only once.

Each of the 2 sides of length 36 contribute  $\frac{36}{12} + 1 = 4$  fence posts and finally, the only side of length 60 contributes  $\frac{60}{12} + 1 = 6$  fence posts. However, we already counted the corners where the 36 foot fence meets the 60 foot fence, so we subtract  $6 - 2 = 4$ . In total, we calculate that there are  $4 + 4 + 4 = \text{span style="border: 1px solid black; padding: 2px;">(B) 12 fence posts.$

5. The first six gallons are irrelevant. This is because the trip did not start yet. Then, we start with the odometer at 56,200 miles, and a full gas tank. The total gas consumed by the car during the trip is equal to the total gas the driver had to buy to make the tank full again (because the tank was originally full), which is,  $12 + 20 = 32$  gallons. The distance covered is  $57,060 - 56,200 = 860$  miles. Hence the average MPG ratio is  $860/32 \approx \text{span style="border: 1px solid black; padding: 2px;">(D) 26.9.$

6. We first need to find the number of people who received the same score on both tests. We see that 2 people got an  $A$  on both tests, 4 people got a  $B$  on both tests, 5 people got a  $C$  on both tests, 1 person got a  $D$  on both tests, and no student received an  $F$  on both tests. We then divide the sum by 30 (the total students) and find the percentage.

So, we have

$$\frac{2 + 4 + 5 + 1}{30}$$

which simplifies to

$$\frac{12}{30} = \frac{4}{10} = \frac{40}{100} = 40\%$$

D.

7. Since we want to find is the number of hamburgers sold in the winter, let's call it  $w$ . From the graph, we know that in Spring, 4.5 million hamburgers were sold, in the Summer was 5 million and in the Fall was 4 million. We know that the number of hamburgers sold in Fall is exactly  $\frac{1}{4}$  of the total number of hamburgers sold, so we can say that

$$4 \times \text{Fall} = \text{Spring} + \text{Winter} + \text{Fall} + \text{Summer}$$

$$4 \times 4 = 4.5 + 5 + 4 + x$$

$$16 = x + 13.5$$

$$2.5 = x$$

Therefore, the answer is (A) 2.5.

### Hard

1. The maximum amount of days any given month can have is 31, and the smallest two digit primes are 11, 13, and 17. There are a few different sums that can be deduced from the following numbers, which are 24, 30, and 28, all of which represent the three days. Therefore, since Brittany says that the other two people's uniform numbers is earlier, so that means Caitlin and Ashley's numbers must add up to 24. Similarly, Caitlin says that the other two people's uniform numbers is later, so the sum must add up to 30. This leaves 28 as today's date. From this, Caitlin was referring to the uniform wearers 13 and 17, telling us that her number is 11, giving our solution as (A) 11.

2. The four squares we already have assemble nicely into four sides of the cube. Let the central one be the bottom, and fold the other three upwards to get the front, right, and back side. Currently, our box is missing its left side and its top side. We can try all possibilities that would fold to one of these two places.

$A$  would be the top side - OK  $B$  would be the left side - OK  $C$  would cause the figure to not be foldable at all  $D$  would be the left side - OK  $E$  would be the top side - OK  $F$  is the same case as  $B$  - OK  $G$  is the same case as  $C$   $H$  is the same case as  $A$  - OK In total, there are (E) 6 good possibilities.

3. Since each student has 5 classes, and there are 1200 students, we know that there are a total of  $5 \times 1200 = 6000$  classes among themselves.

Each class has 30 students, so there must be  $\frac{6000}{30} = 200$  total classes in the school. Each class has 1 teachers, so the teachers teach a total of 200 classes among themselves.

Each teacher teaches 4 classes, so the number of teachers is just  $\frac{200}{4} = \text{(E) } 50$ .

4. We can compute the ratio of the sales of the group with more sales and the sales of the group with less sales in each month. If we do that, then we find the largest is in (B) Feb because the ratio of drums to bugles is  $5/3 \approx 1.67$ .

5. We know that we should not try performing the operation 100 times so we look for a pattern.

After 1 press, the calculator displays  $\frac{1}{1-5} = -\frac{1}{4}$

After 2 presses, the calculator displays  $\frac{1}{1--\frac{1}{4}} = \frac{1}{\frac{5}{4}} = \frac{4}{5}$

After 3 presses, the calculator displays  $\frac{1}{1-\frac{4}{5}} = \frac{1}{\frac{1}{5}} = 5$

Aha, this is same as the original number. Therefore, after every three presses, the display will be 5. On press  $3 \cdot 33 = 99$ , the display will be 5. Therefore, 100 presses results in the same number as 1 press, so our answer is (A)  $-0.25$ .

6. For Mary to show Jane wrong, it does not help if she finds a card with an even number. Instead, she must find a card with an odd number on one side, and a vowel on the other side, because this contradicts Jane's statement. The only card that could possibly have this property is the card with the odd number 3, which is answer choice (A) 3.

7. In the one-digit numbers, there is only 1 two.

The number of two-digit numbers with a two in the tens place is 10 and the number with a two in the ones place is 9. Therefore the digit two is used  $10 + 9 = 19$  times for the two digit numbers.

Now, Pat has only  $22 - 1 - 19 = 2$  remaining twos. The last two numbers with a two that he can write are 102 and 112. He can continue numbering the last couple pages without a two until he encounters 120, with the last number he writes being (D) 119.

8. We can find the number of students who prefer tennis in each school and then find the total percentage. In the first school, we can find that  $2000 \cdot 22\% = 2000 \cdot 0.22 = 440$  students prefer tennis.

In the second school,  $2500 \cdot 40\% = 2500 \cdot 0.40 = 1000$  students prefer tennis.

In total,  $440 + 1000 = 1440$  students prefer tennis out of a total of  $2000 + 2500 = 4500$  students

Now, we find the percentage  $\frac{1440}{4500} \cdot 100\% = \frac{32}{100} \cdot 100\% = 32\%$  of the students in both schools prefer tennis, giving answer C.

9. In June, Ana gets  $2000 \cdot (1.2) = 2400$

In July, Ana's pay is  $2400 \cdot (0.8) = 1920$ , giving our answer of (A) 1920 dollars