Easy

- 1. Let's consider the possible choices for the first digit. As long as the first digit is not greater than 7, we can find the appropriate second digit that makes the sum 7. For example, if our first digit were 4, then our second digit would be 7-4=3. Since our first digit cannot be 0, our first digit must be in the range of 1 to 7 so our answer is just (B) 7.
- 2. Since Brianna's age is half of Aunt Anna's age, we figure that Brianna is 21 years old. Now we can just find Caitlin's age, which is 21 5 = 16. Therefore, our answer is (B) 16.
- 3. We can just work through the answer choices. The reciprocal of -2 is $\frac{1}{-2} = \frac{-1}{2}$. Since $-2 < \frac{-1}{2}$, we know that the answer is (A) -2. We could check through the other choices to confirm our answer, but it is unnecessary.
- 4. We can find that the smallest whole number in the interval is 2 because 1 < 5/3 < 2. The largest whole number in the interval is 6 because $6 < 2\pi < 7$. There are five whole numbers in the interval: 2, 3, 4, 5, and 6. Therefore, our answer is (D) 5.
- 5. We know that the smallest possible prime factor is 2, so if any of our choices are divisible by 2, then that must be the answer. Looking through the choices, we find that 58 is the only multiple of 2, so our answer is (C) 58.
- 6. Since we want to achieve the smallest possible average, we want to pick the 4 smallest even numbers, which are 2, 4, 6, and 8. Thus, the answer is their average, which is $\frac{2+4+6+8}{4} = \boxed{\text{(C) 5}}$.
- 7. We can easily figure out that the next palindrome after 2002 is 2112. The answer is the product of the digits of 2112, which is (B) 4.
- 8. We can just test all possible factors of 24. We soon find that 3 and 8 sum to 11. Therefore, the answer is the larger number: (D) 8.
- 9. We can form the smallest number by using the smallest number for each digit. First, we use the 1. Next, we use the 2. Then, we use the 3. For the next digit, however, we cannot use the 4, because our number must be even and 4 is the only even number left. Therefore, we use the 9 and finally, we use the 4 for the last digit. Our final number is 12394 and our answer is the tens digit, which is (E) 9.
- 10. We can simplify the given expression: (6?3) + 4 (2-1) = 5

$$(6?3) + 4 - 1 = 5$$

$$(6?3) + 3 = 5$$

$$(6?3) = 2$$

Now, it becomes clear that the ? operation should be \div , A

- 11. We sum each triplet and see that all the choices result in 1, except choice D, which sums to 0, instead of 1. Therefore, our answer is (D).
- 12. We can suppose that the school played 11+4=15 games, and it won 11, but lost 4. Therefore, the percentage of games lost is just $\frac{4}{15} \times 100 = 26.\overline{6}\% \approx \boxed{\text{(B) } 27\%}$.
- 13. We can consider all possible additions. There are currently 5 choices for the first letter, 3 choices for the second letter, and 4 choices for the third letter, making total of $5 \cdot 3 \cdot 4 = 60$ license plates.

Adding 2 letters to the start gives $7 \cdot 3 \cdot 4 = 84$ plates.

Adding 2 letters to the middle gives $5 \cdot 5 \cdot 4 = 100$ plates.

Adding 2 letters to the end gives $5 \cdot 3 \cdot 6 = 90$ plates.

Adding a letter to the start and middle gives $6 \cdot 4 \cdot 4 = 96$ plates.

Adding a letter to the start and end gives $6 \cdot 3 \cdot 5 = 90$ plates.

Adding a letter to the middle and end gives $5 \cdot 4 \cdot 5 = 100$ plates.

The most is 100 license plates total. Therefore, the answer is 100 - 60 = 40, (D) 40

Medium

1. We can do casework based on the units digit.

Case 1: 0 gives no solutions, since no real numbers are divisible by 0

Case 2: 1 gives 4 solutions, since all numbers are divisible by 1.

Case 3: 2 has 4 solutions, since every number ending in 2 is even (ie divisible by 2).

Case 4: 3 has 1 solution: 33.

Case 5: 4 has 2 solutions: 24 and 44.

Case 6: 5 has 4 solutions: every number ending in 5 is divisible by 5.

Case 7: 6 has 1 solution: 36.

Case 8: 7 has no solutions.

Case 9: 8 gives 1 solution: 48

Case 10: 9 has no solutions.

We total the solutions, and get 0+4+4+1+2+4+1+0+1+0=17 solutions, therefore giving the final answer (C) 17.

2. We can try to find a pattern for the units digit in each of the terms. 19^1 ends in 9. To find the units digit of 19^2 , we can just multiply the units digit of 19^1 by 9, so 19^2 ends in 9*9=81, so the units digit is 1. The units digit of 19^3 is 1*9=9. We have arrived back at 9 and therefore, there is pattern. All even powers of 19 have a units digit of 1, and all odd powers of 19 have a units digit of 9. So, 19^{19} has a units digit of 9.

The same relation applies for powers of 99, so 99^{99} also has a units digit of 9. 9+9=18 which has a units digit of (D) 8.

3. We can just work through the expression, while doing the innermost parentheses first.

$$[(1 \otimes 2) \otimes 3] - [1 \otimes (2 \otimes 3)]$$

$$\left[\frac{1^2}{2} \otimes 3\right] - \left[1 \otimes \frac{2^2}{3}\right]$$

$$\left[\frac{1}{2} \otimes 3\right] - \left[1 \otimes \frac{4}{3}\right]$$

$$\big[\tfrac{\left(\frac{1}{2}\right)^2}{3}\big] - \big[\tfrac{1^2}{\left(\frac{4}{3}\right)}\big]$$

$$\left[\tfrac{1}{4} \cdot \tfrac{1}{3}\right] - \left[\tfrac{3}{4}\right]$$

$$\frac{1}{12} - \frac{3}{4}$$

$$\frac{1}{12} - \frac{9}{12}$$

 $\frac{-8}{12}$

- $-\frac{2}{3}$. Therefore, the answer is A
- 4. Since you have one coin of each type, 1+5+10+25=41 cents are already determined, leaving you with a total of 102-41=61 cents remaining for 5 coins.

You must have 1 more penny. If you had more than 1 penny, you must have at least 6 pennies to leave a multiple of 5 for the nickels, dimes, and quarters. But you only have 5 more coins to assign.

Now you have 61 - 1 = 60 cents remaining for 4 coins, which may be nickels, quarters, or dimes. You must take maximum 2 more quarters. Let's consider the case where we take 2 quarters. We would then have 10 cents left with two coins left. We cannot take a dime because we must use two coins. We can, however, use two 2 nickels, and complete the amount. Now we find that there is only 1 dime in that combo, so the answer is (A) 1.

5. We can quickly work through all the combinations, eliminating some along the way. Soon, we realize that $\frac{3}{1} - \frac{4}{2} = 1$ so this is right ordering. Therefore, the answer is the sum of W and Y, which is $3 + 4 = \boxed{(E) 7}$.

Hard

- 1. The smallest number divisible by 3, 4, 5, and 6, or their least common multiple, can be found to be 60, because we need to include one factor of 3, two factors of 2, and one factor of 5. When 2 is added to a multiple of number, its remainder when divided by that number is 2. Using this logic, we can find that the desired number is 62, which lies between (B) 60 and 79.
- 2. We could start by looking for a pattern.

$$98, 49, 44, 22, 11, 6, 54, 27, 22, 11, 6, \ldots$$

From here, we see that we have a pattern of $22, 11, 6, 54, 27, \ldots$ after 98, 49, 44

Our problem can be reworded to

Find the 95^{th} term of the sequence that goes $22, 11, 6, 54, 27, 22, 11, 6, 54, 27, 22, \dots$, because the first three terms don't matter.

There are 5 terms in each repetition of the pattern, and 95 leaves no remainder when divided by 5, so the answer is the fifth term in the sequence 22, 11, 6, 54, 27, which is (D) 27.

3. To know whether the remainder when a number is divided by 5, we just need to know its units digit. So we need to find the units digit of 1999^{2000} . We simplify our problem further by realizing that the units digit of 1999^{2000} is the same as the units digit of 9^{2000} . We start by looking for a pattern. 9^1 ends in 9. To find the units digit of 9^2 , we can just multiply the units digit of 9^1 by 9, so 9^2 ends in 9*9=81, so the units digit is 1. The units digit of 19^3 is 1*9=9. We have arrived back at 9 and therefore, there is pattern. All even powers of 9 have a units digit of 1, and all odd powers of 9 have a units digit of 9.

Therefore, 9^{2000} ends in 1, so the remainder when 19^{2000} is divided by 5 is (D) 1

4. The highest possible score is if you get every answer right, to get 5(20) = 100. Since all the answer choices are close to the highest possible score, we can sort of work backwords. The second highest possible score is when you get 19 questions right and leave the remaining one blank, to get a 5(19) + 1(1) = 96. Therefore, no score between 96 and 100 can be achieved, so our answer is (E) 97.

- 5. Since the greatest number possible (7) divided by the smallest number (2) is slightly greater than 3, divide all of the choices by 2, then 3 and see if the resulting answer contains 2,4,5 and 7. While Doing so, you find that 7425/3 = 2475. Therefore, the answer is (D) 7425.
- 6. Since we know how much money Toy had in the beginning, we can try to work through the problem by following the steps. Toy has 36 dollars at the beginning, and then after Amy doubles his money, he has $36 \times 2 = 72$ dollars after this first step. Jan also doubles his money during this step.

In the next step, Jan allows Toy to double his current amount, so Toy now has $72 \times 2 = 144$ dollars after the second step.

Finally, Toy doubles whatever Amy and Jan have. Since Toy ended up with 36, we know that he must have spent 144 - 36 = 108 to double their money. Therefore, just before this third step, Amy and Jan must have together had 108 dollars.

Now we can find the total money in the group. Just before the third step, Toy had 144 dollars. We also figured that before the third step, Amy and Jan must have together had 108 dollars. Since the total money in the group is the same at all times, we know that altogether, the three had 144 + 108 = 252 dollars, and thus, the correct answer is (D) 252 dollars.