Quantum Computing Intro Course

Qiskit Hackathon

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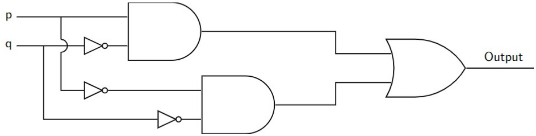
Outline

1. Review: Classical Computing
2. Qubits and Gates
3. Superposition
4. Bloch Sphere
5. Bell States Lab

Classical Computing

p	q	$p \wedge q$	$p \vee q$
T	T	T	T
T	F	F	T
F	T	F	T
F	F	F	F



Qubits

Definition

The **Dirac notation** can be used to represent states. Each state corresponds to a complex vector.

Example

We will represent quantum bits (qubits) zero and one are represented as follows:

$$|0\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \; |1\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

Quantum Gates

Definition

Pauli matrices:

$$X = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad Y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \quad Z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

We have the following identities:

$$X^2 = Y^2 = Z^2 = -iXYZ = I$$

$$XY = iZ \quad YZ = iX \quad ZX = iY \quad YX = -iZ \quad ZY = -iX \quad XZ = -iY$$

Definition

The **Pauli group** $\mathscr{P}_n$ consists of matrices of the form $i^k P_1 \otimes ... \otimes P_n$ where $P_j \in \{I, X, Y, Z\}$

Advanced Quantum Gates

Definition

The Clifford group (up to scalars) is the normalizer of the Pauli group, ie $\mathscr{C}_n = \{C \mid C\mathscr{P}_nC^{-1} \subset \mathscr{P}_n\}$

Example

H, $CNOT = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$, and $S = \begin{pmatrix} 1 & 0 \\ 0 & i \end{pmatrix}$ are Clifford operators.

Remark

If P is Pauli and C is Clifford, then $CPC^{-1} = Q$ is Pauli, ie the Clifford group acts on the Pauli group by conjugation.

Quantum Circuits

Definition

The **tensor product**, denoted by $\otimes$, is an operation of two matrices or vectors which results in a block matrix or vector.

$$A \otimes B = \begin{pmatrix} a_{1,1}B \dots a_{1,n}B \\ \vdots \ddots \vdots \\ a_{m,1}B \dots a_{m,n}B \end{pmatrix}$$

Example

We can use the tensor product to extend states and gates to multiple qubits. Given states $|0\rangle$ and $|1\rangle$ as defined above we can calculate the tensor product of the two states as follows:

$$|0\rangle \otimes |1\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \otimes \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \cdot 0 \\ 1 \cdot 1 \\ 0 \cdot 0 \\ 0 \cdot 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix} = |01\rangle$$

Qiskit Lab 1.1 - your first quantum circuit

Let's try some coding with Qiskit!

Quantum State

Definition

A quantum state $|\phi\rangle$ is a complex linear combinations of basis states of the form

$$|\phi\rangle = \alpha_1|00\rangle + \alpha_2|01\rangle + \alpha_3|10\rangle + \alpha_4|11\rangle$$

where:

$$|\alpha_1|^2 + |\alpha_2|^2 + |\alpha_3|^2 + |\alpha_4|^2 = 1$$

In other words, they are unit vectors. A non-basis state is said to be in a **superposition**.

Probability of Outcome

Definition

We measure the **probability of outcome** of a qubit is defeined as

$$P(\psi_n) = |\alpha_n|^2$$

Example

Let $|\psi\rangle = \frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle$. The probability of measurement outcome of $|0\rangle$:

$$P(|0\rangle) = |\frac{1}{\sqrt{2}}| = \left(\frac{1}{\sqrt{2}}\right)^2 = \frac{1^2}{\sqrt{2}^2} = \frac{1}{2}$$

$$P(|1\rangle) = 1 - P(|0\rangle) = 1 - \frac{1}{2} = \frac{1}{2}$$

Therefore, the probability of measuring $|1\rangle$ or $|0\rangle$ would be 50%.

Putting a state in a Superposition

Example

The Hadamard gate

$$H = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$$

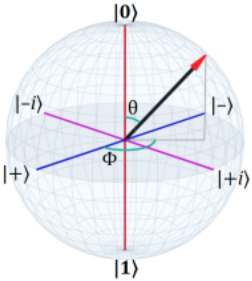
$$H|0\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 1 \end{pmatrix} = \frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle$$

Bloch Sphere

Definition

The equation for a state on the bloch sphere is given as:

$$|\psi\rangle = cos(\frac{\theta}{2})|0\rangle + e^{i\phi}sin(\frac{\theta}{2})|1\rangle$$



Pauli Eigenvectors and the Bloch Sphere

Matrix	Eigenvalue	Eigenvector
$X = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$	$+1$	$\lvert + \rangle = \frac{\lvert 0 \rangle + \lvert 1 \rangle}{\sqrt{2}}$
	-1	$\lvert - \rangle = \frac{\lvert 0 \rangle - \lvert 1 \rangle}{\sqrt{2}}$
$Y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$	$+1$	$\lvert + i \rangle = \frac{\lvert 0 \rangle + i \lvert 1 \rangle}{\sqrt{2}}$
	-1	$\lvert - i \rangle = \frac{\lvert 0 \rangle - i \lvert 1 \rangle}{\sqrt{2}}$
$Z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$	$+1$	$\lvert 0 \rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$
	-1	$\lvert 1 \rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

Rotations on the Bloch Sphere

Definition

The rotation matrices rotate about the x, y, or z axis by θ.

$$R_x = \begin{bmatrix} cos(\frac{\theta}{2}) & -isin(\frac{\theta}{2}) \\ -isin(\frac{\theta}{2}) & cos(\frac{\theta}{2}) \end{bmatrix} \; R_y = \begin{bmatrix} cos(\frac{\theta}{2}) & -sin(\frac{\theta}{2}) \\ sin(\frac{\theta}{2}) & cos(\frac{\theta}{2}) \end{bmatrix} \; R_z = \begin{bmatrix} e^{-i\frac{\psi}{2}} & 0 \\ 0 & e^{i\frac{\psi}{2}} \end{bmatrix}$$

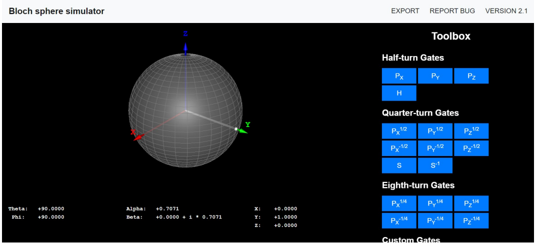
Example

Setting $\theta = \pi$ will have the same result as applying one of the Pauli matrices:

$$R_x(\pi)|0\rangle = \begin{bmatrix} 0 & -i \\ -i & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ -i \end{bmatrix} = -i|1\rangle = |1\rangle$$

(global phase can be ignored).

Lab 1.2: Rotations on a Bloch Sphere



Bell States

Definition

Bell states aka EPR are maximally entangled states:

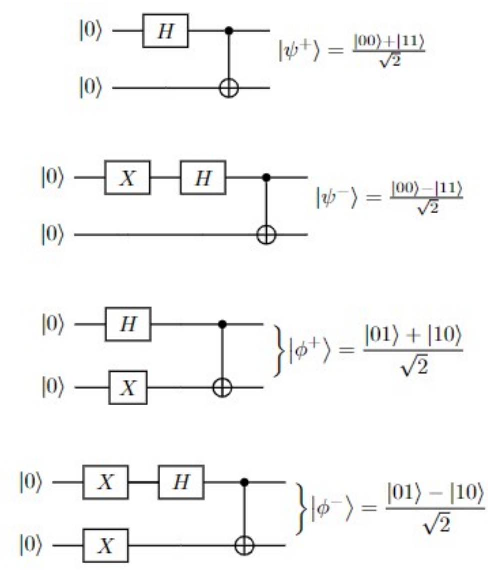
$$|\phi^+\rangle = \frac{|01\rangle + |10\rangle}{\sqrt{2}}$$

$$|\phi^-\rangle = \frac{|01\rangle - |10\rangle}{\sqrt{2}}$$

$$|\psi^+\rangle = \frac{|00\rangle + |11\rangle}{\sqrt{2}}$$

$$|\psi^-\rangle = \frac{|00\rangle - |11\rangle}{\sqrt{2}}$$

Lab 1.3: Circuits for Bell States in Qiskit



End of Course 1

15 minute break!