GENERAL ASSEMBLY

INTRO TO DATA SCIENCE

LOGISTIC REGRESSION

	continuous	categorical
supervised	regression	classification
unsupervised	dimension reduction	clustering

Q: What is **logistic regression***?*

A: A generalization of the linear regression model to classification problems.

In linear regression, we used a set of input variables to predict the value of a continuous response variable.

In logistic regression, we use a set of input variables to predict probabilities of class membership.

In linear regression, we used a set of input variables to predict the value of a continuous response variable.

In logistic regression, we use a set of input variables to predict probabilities of class membership.

NOTE

Class membership is not always binary, however, that is what we will focus on for this class.

In linear regression, we used a set of input variables to predict the value of a continuous response variable.

In logistic regression, we use a set of input variables to predict probabilities of class membership.

These probabilities can then mapped to class labels, thus predicting the class for each observation.

When performing linear regression, we use the following function:

$$y = \beta_0 + \beta_1 x$$

When performing logistic regression, we use the following form:

$$\pi = \Pr(y = 1 \mid x) = \frac{e^{\beta_0 + \beta_1 x}}{1 + e^{\beta_0 + \beta_1 x}}$$

When performing linear regression, we use the following function:

$$y = \beta_0 + \beta_1 x$$

When performing logistic regression, we use the following form:

$$\pi = \Pr(y = 1 \mid x) = \frac{e^{\beta_0 + \beta_1 x}}{1 + e^{\beta_0 + \beta_1 x}}$$

Probability of y = 1, given x

Quiz: *Create a plot of the logistic function.*

$$\pi = \frac{e^{\beta_0 + \beta_1 x}}{1 + e^{\beta_0 + \beta_1 x}}$$

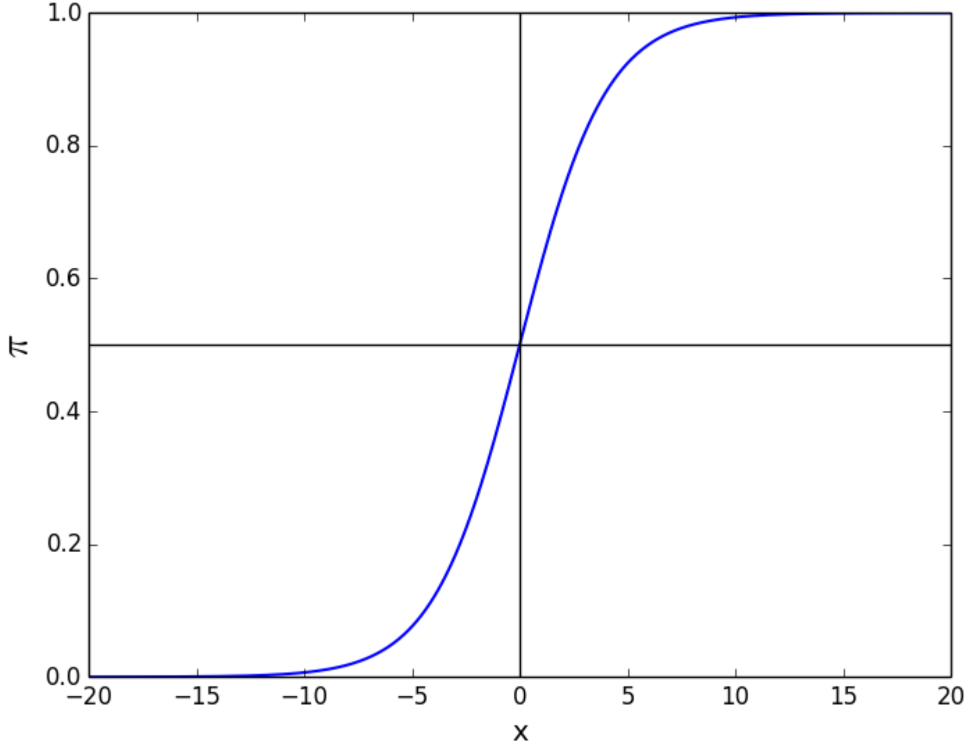
Quiz: *Create a plot of the logistic function.*

$$\pi = \frac{e^{\beta_0 + \beta_1 x}}{1 + e^{\beta_0 + \beta_1 x}}$$

How would you describe the shape of the function?

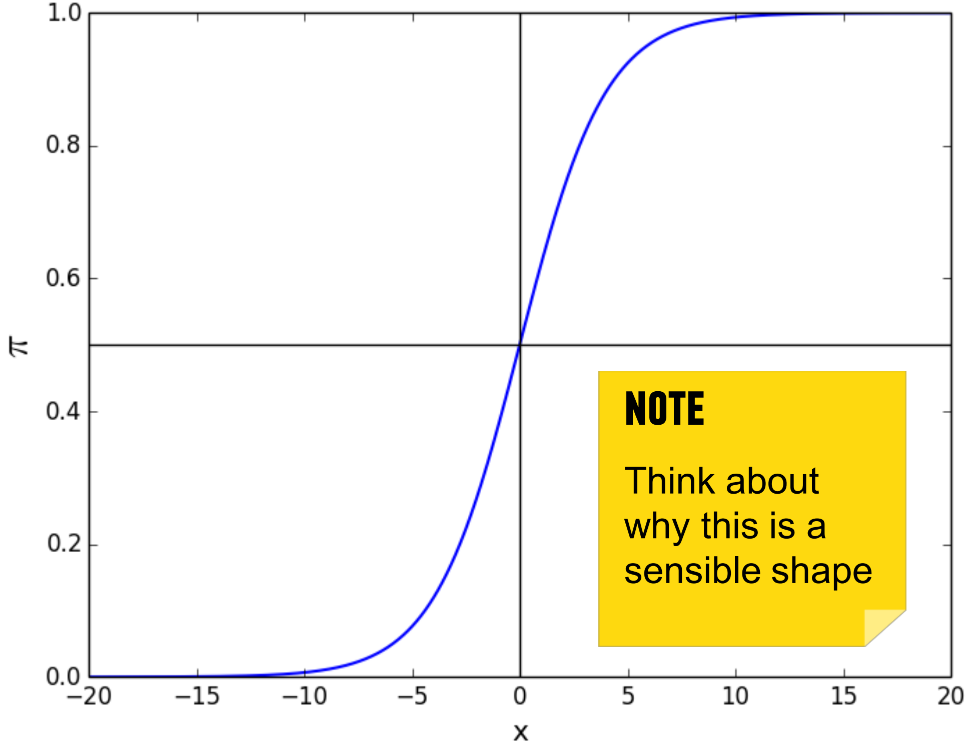
The logistic function takes on an "S" shape, where y is bounded by [0, 1]

$$\pi = \frac{e^{\beta_0 + \beta_1 x}}{1 + e^{\beta_0 + \beta_1 x}}$$



The logistic function takes on an "S" shape, where y is bounded by [0, 1]

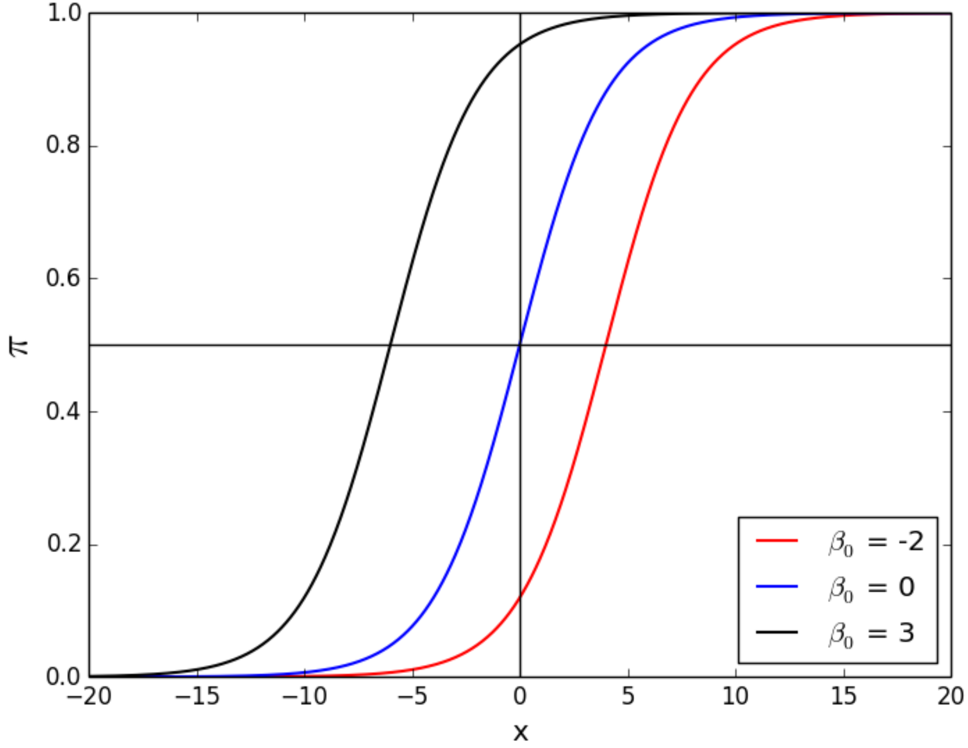
$$\pi = \frac{e^{\beta_0 + \beta_1 x}}{1 + e^{\beta_0 + \beta_1 x}}$$



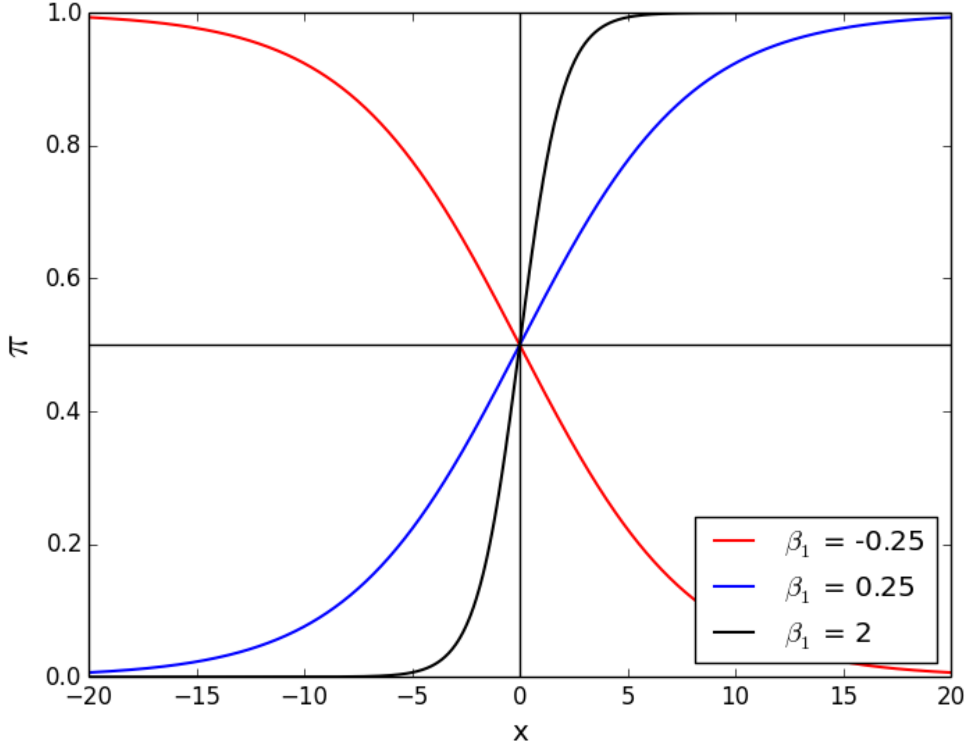
NOTE

Think about why this is a sensible shape

Changing the β_0 value shifts the function horizontally.



Changing the β_1 value changes the slope of the curve



I. INTERPRETATION

In order to interpret the outputs of a logistic function we must understand the difference between probability and odds.

The odds of an event are given by the ratio of the probability of the event by its complement:

In order to interpret the outputs of a logistic function we must understand the difference between probability and odds.

The odds of an event are given by the ratio of the probability of the event by its complement:

$$Odds = \frac{\pi}{1-\pi}$$

In order to interpret the outputs of a logistic function we must understand the difference between probability and odds.

The odds of an event are given by the ratio of the probability of the event by its complement:

$$Odds = \frac{\pi}{1-\pi}$$

QUESTION

What is the range of the odds?

Quiz: *You're trying to determine whether a customer will convert or not. The customer conversion rate is 33.33%. what are the odds that a customer will convert?*

Take 2 minutes and work this out.

Quiz: *You're trying to determine whether a customer will convert or not. The customer conversion rate is 33.33%. what are the odds that a customer will convert?*

Take 2 minutes and work this out.

$$Odds = \frac{\pi}{1-\pi}$$

Quiz: *You're trying to determine whether a customer will convert or not. The customer conversion rate is 33.33%. what are the odds that a customer will convert?*

Take 2 minutes and work this out.

$$Odds = \frac{\pi}{1-\pi} = \frac{.3333}{.6666} = \frac{1}{2}$$

Quiz: *You're trying to determine whether a customer will convert or not. The customer conversion rate is 33.33%. what are the odds that a customer will convert?*

Take 2 minutes and work this out.

$$Odds = \frac{\pi}{1-\pi} = \frac{.3333}{.6666} = \frac{1}{2}$$

NOTE

This means that for every customer that converts you will have two customers that do not convert

Notice if we take the logarithm of the odds, we return a linear equation

$$\log\left(\frac{\pi}{1-\pi}\right) = \log\left(e^{\beta_0 + \beta_1 x}\right) = \beta_0 + \beta_1 x$$

Notice if we take the logarithm of the odds, we return a linear equation

$$\log\left(\frac{\pi}{1-\pi}\right) = \log\left(e^{\beta_0 + \beta_1 x}\right) = \beta_0 + \beta_1 x$$

NOTE

What is the range of the logit function?

Notice if we take the logarithm of the odds, we return a linear equation

$$\log\left(\frac{\pi}{1-\pi}\right) = \log(e^{\beta_0 + \beta_1 x}) = \beta_0 + \beta_1 x$$

This simple relationship between the odds ratio and the parameter β is what makes logistic regression such a powerful tool.

In linear regression, the parameter β_1 represents the change in the ***response variable*** *for a unit change in x.*

*In linear regression, the parameter β_1 represents the change in the **response variable** for a unit change in x.*

*In logistic regression, β_1 represents the change in the **log-odds** for a unit change in x.*

*In linear regression, the parameter β_1 represents the change in the **response variable** for a unit change in x.*

*In logistic regression, β_1 represents the change in the **log-odds** for a unit change in x.*

*This means that e^{β_1} gives us the change in the **odds** for a unit change in x.*

Q: How to determine whether a coefficient is significant?

A: This is based off of the p-value, just as with the linear regression

Example: *Suppose we are interested in mobile purchase behavior. Let y be a class label denoting purchase/no purchase, and let x denote whether phone was an iPhone.*

Example: *Suppose we are interested in mobile purchase behavior. Let y be a class label denoting purchase/no purchase, and let x denote whether phone was an iPhone.*

We perform a logistic regression, and we get $\beta_1 = 0.693$.

Example: *Suppose we are interested in mobile purchase behavior. Let y be a class label denoting purchase/no purchase, and let x denote whether phone was an iPhone.*

We perform a logistic regression, and we get $\beta_1 = 0.693$.

Q: What does this mean?

Example: *Suppose we are interested in mobile purchase behavior. Let y be a class label denoting purchase/no purchase, and let x denote whether phone was an iPhone.*

We perform a logistic regression, and we get $\beta_1 = 0.693$.

In this case the odds ratio is exp(0.693) = 2, meaning the likelihood of purchase is twice as high if the phone is an iPhone.

Once we understand the basic form for logistic regression, we can easily extend the definition to include multiple input values.

Logit function

$$\log\left(\frac{\pi}{1-\pi}\right) = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \ldots + \beta_p x_p$$

Once we understand the basic form for logistic regression, we can easily extend the definition to include multiple input values.

$$\log\left(\frac{\pi}{1-\pi}\right) = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \ldots + \beta_p x_p$$

Logistic function

$$\pi = \frac{e^{\beta_0 + \beta_1 x_1 + \beta_2 x_2 + \ldots + \beta_p x_p}}{1 + e^{\beta_0 + \beta_1 x_1 + \beta_2 x_2 + \ldots + \beta_p x_p}}$$

II. EXERCISE: PREDICTING DEFAULT

This data set contains 10,000 records associated with credit card accounts with the following four fields:

Default	Binary variable indicating whether the credit card holder defaulted on their credit card obligations
Student	Binary variable indicating whether the credit card holder is a student
Balance	Continuous variable recording the credit card holders current outstanding balance
Income	Continuous variable representing the total annual income for the credit card holder

Part I: Exploration

1) *Read in Default.csv and convert all data to numeric*
2) *Split the data into train and test sets*
3) *Create a histogram of all variables*
4) *Create a scatter plot of the income vs. balance*
5) *Mark defaults with a different color (and symbol)*
6) *What can you infer from this plot?*

Part II: Logistic Regression

1) *Run a logistic regression on the balance variable*
   - *Use the training set*
   - *Use the* `statsmodels.formula.api` *module and* `smf.logit()` *function*
2) *Is the β value associated with balance significant?*
3) *Predict the probability of default for someone with a balance of $1.2k and $1.5k*
4) *Plot the fitted logistic function overtop of the data points*
5) *Create predictions using the test set*
6) *Compute the overall accuracy, the sensitivity and specificity*

III. Q&A

Q: What is a Generalized Linear Model (GLM)?

A: Briefly, GLMs generalize the distribution of the ***error term****, and allow the conditional mean of the response variable to be related to the linear model by a* **link function***.*

Q: What is the error distribution and link function for the logistic regression?

A: The error term follows a [Bernoulli distribution], and the logit is the link function that connects us to the linear predictor.

Q: Is the logit the only link function used for the Bernoulli distribution?

A: No, other link functions include the [probit] the [tobit] model. However, the logit simplifies things nicely and is probably the most commonly used.

Q: What is the difference between $\frac{e^{\beta_0+\beta_1 x}}{1+e^{\beta_0+\beta_1 x}}$ *and* $\frac{1}{1+e^{-\beta_0-\beta_1 x}}$ *?*

A: Nothing, these are equivalent expressions.

If you want to prove this to yourself (a) plot both equations, or (b) multiply both numerator and denominator by $\frac{1}{e^{\beta_0+\beta_1 x}}$ *.*

Q: Why not use a linear regression to predict probabilities of class membership?

A: The linear regression will make predictions that don't make sense (e.g., probability outside of [0,1])

A: Transforming the linear regression into a step function will produce heteroskedastic errors

Q: How do we derive coefficients using maximum likelihood?
A: We find the coefficients that are the most likely, given the observed data. Formally, we estimate the coefficients that maximize the likelihood function. This is done using an iterative procedure.

Notation for the product of a series

$$L(\beta_0, \beta) = \prod_{i=1}^{n} p(x_i)^{y_i}(1 - p(x_i))^{1-y_i}$$

Check out this [link](), for details on the estimation of the coefficients.