7
Parametric Cycle Analysis of Real Engines

7.1 Introduction

In Chapter 5, we idealized the engine components and assumed that the working fluid behaved as a perfect gas with constant specific heats. These idealizations and assumptions permitted the basic parametric analysis of several types of engine cycles and the analysis of engine performance trends. In Chapter 6, we looked at the variation of specific heat with temperature and fuel/air ratio and developed component models and figures of merit. This allows us to use realistic assumptions as to component losses and to include the variation of specific heats in engine cycle analysis. In this chapter, we develop the cycle analysis equations for many engine cycles, analyze their performance, and determine the effects of real components by comparison with the ideal engines of Chapter 5. We begin our analysis with the turbojet engine cycle and treat the simpler ramjet engine cycle as a special case of the turbojet ($\pi_c = 1,\ \tau_c = 1,\ \pi_t = 1,\ \tau_t = 1$).

7.2 Turbojet

We will now compute the behavior of the turbojet engine including component losses, the mass flow rate of fuel through the components, and the variation of specific heats. Our analysis still assumes one-dimensional flow at the entrance and exit of each component. The variation of specific heats will be approximated by assuming a perfect gas with constant specific heat c_{pc} upstream of the main burner (combustor) and a perfect gas with different constant specific heat c_{pt} downstream of the main burner.

The turbojet engine with station numbering is shown in Fig. 7.1, and the T-s diagram for this cycle with losses is plotted in Fig. 7.2. Figure 7.2 shows the total states for all engine stations along with the static states for stations 0 and 9.

7.2.1 Cycle Analysis

In this section we develop a system of equations to analyze the turbojet engine cycle. The steps of cycle analysis are applied to the turbojet engine and presented next in the order listed in Section 5.4.

Supporting Material for this chapter is available electronically. See page 869 for instructions to download.

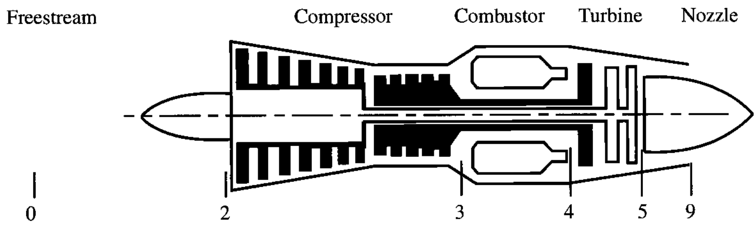

Fig. 7.1 Station numbering of turbojet engine.

Step 1: For uninstalled thrust,

$$F = \frac{1}{g_c}(\dot{m}_9 V_9 - \dot{m}_0 V_0) + A_9(P_9 - P_0)$$

$$\frac{F}{\dot{m}_0} = \frac{a_0}{g_c}\left(\frac{\dot{m}_9}{\dot{m}_0}\frac{V_9}{a_0} - M_0\right) + \frac{A_9 P_9}{\dot{m}_0}\left(1 - \frac{P_0}{P_9}\right) \tag{7.1}$$

We note that

$$\begin{aligned}
\frac{A_9 P_9}{\dot{m}_0}\left(1 - \frac{P_0}{P_9}\right) &= \frac{\dot{m}_9}{\dot{m}_0}\frac{A_9 P_9}{\rho_9 A_9 V_9}\left(1 - \frac{P_0}{P_9}\right) \\
&= \frac{\dot{m}_9}{\dot{m}_0}\frac{P_9}{[P_9/(R_9 T_9)]V_9}\left(1 - \frac{P_0}{P_9}\right) \\
&= \frac{\dot{m}_9}{\dot{m}_0}\frac{R_9 T_9}{V_9}\left(1 - \frac{P_0}{P_9}\right) \\
&= \frac{\dot{m}_9}{\dot{m}_0}\frac{R_9 T_9}{V_9}\frac{\gamma_0 R_0 g_c T_0}{\gamma_0 R_0 g_c T_0}\left(1 - \frac{P_0}{P_9}\right) \\
&= \frac{\dot{m}_9}{\dot{m}_0}\frac{R_9 T_9}{R_0 T_0}\frac{a_0^2}{\gamma_0 g_c V_9}\left(1 - \frac{P_0}{P_9}\right)
\end{aligned}$$

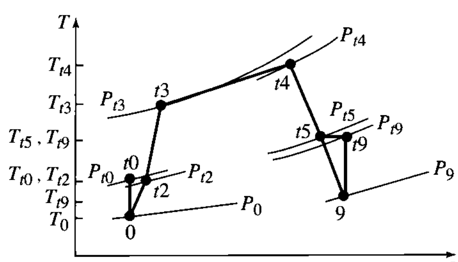

Fig. 7.2 The *T-s* diagram for turbojet engine.

$$\frac{A_9P_9}{\dot{m}_0}\left(1-\frac{P_0}{P_9}\right)=\frac{a_0}{g_c}\left(\frac{\dot{m}_9}{\dot{m}_0}\frac{R_9}{R_0}\frac{T_9/T_0}{V_9/a_0}\frac{1-P_0/P_9}{\gamma_0}\right) \tag{7.2}$$

For the case of the turbojet cycle, the mass ratio can be written in terms of the fuel/air ratio f:

$$\frac{\dot{m}_9}{\dot{m}_0}=1+f$$

Using Eq. (7.2) with gas property subscripts c and t for engine stations 0 to 9, respectively, we can write Eq. (7.1) as

$$\frac{F}{\dot{m}_0}=\frac{a_0}{g_c}\left[(1+f)\frac{V_9}{a_0}-M_0+(1+f)\frac{R_t}{R_c}\frac{T_9/T_0}{V_9/a_0}\frac{1-P_0/P_9}{\gamma_c}\right] \tag{7.3}$$

Step 2:

$$\left(\frac{V_9}{a_0}\right)^2=\frac{a_9^2M_9^2}{a_0^2}=\frac{\gamma_9R_9g_cT_9}{\gamma_0R_0g_cT_0}M_9^2$$

For the turbojet cycle, this equation becomes

$$\left(\frac{V_9}{a_0}\right)^2=\frac{\gamma_tR_tT_9}{\gamma_cR_cT_0}M_9^2 \tag{7.4}$$

Note: From the definition of τ_λ given in Eq. (5.7), we have

$$\tau_\lambda=\frac{c_{pt}T_{t4}}{c_{pc}T_0} \tag{7.5}$$

Step 3: We have

$$M_9^2=\frac{2}{\gamma_t-1}\left[\left(\frac{P_{t9}}{P_9}\right)^{(\gamma_t-1)/\gamma_t}-1\right] \tag{7.6}$$

where

$$\frac{P_{t9}}{P_9}=\frac{P_0}{P_9}\pi_r\pi_d\pi_c\pi_b\pi_t\pi_n \tag{7.7}$$

Step 4: We have

$$\frac{T_9}{T_0}=\frac{T_{t9}/T_0}{(P_{t9}/P_9)^{(\gamma_t-1)/\gamma_t}}$$

where

$$\frac{T_{t9}}{T_0} = \tau_r \tau_d \tau_c \tau_b \tau_t \tau_n \tag{7.8}$$

Step 5: Application of the first law of thermodynamics to the burner gives

$$\dot{m}_0 c_{pc} T_{t3} + \eta_b \dot{m}_f h_{PR} = \dot{m}_4 c_{pt} T_{t4} \tag{7.9}$$

Dividing the preceding equation by $\dot{m}_0 c_{pc} T_0$ and using the definitions of temperature ratios give

$$\tau_r \tau_c + f \frac{\eta_b h_{PR}}{c_{pc} T_0} = (1 + f)\tau_\lambda$$

Solving for the fuel/air ratio gives

$$f = \frac{\tau_\lambda - \tau_r \tau_c}{\eta_b h_{PR}/(c_{pc} T_0) - \tau_\lambda} \tag{7.10}$$

Step 6: The power balance between the turbine and compressor, with a mechanical efficiency η_m of the turbine compressor coupling, gives

$$\begin{gathered} \text{Power into compressor} = \text{net power from turbine} \\ \dot{m}_0 c_{pc}(T_{t3} - T_{t2}) = \eta_m \dot{m}_4 c_{pt}(T_{t4} - T_{t5}) \end{gathered} \tag{7.11}$$

Dividing the preceding equation by $\dot{m}_0 c_{pc} T_{t2}$ gives

$$\tau_c - 1 = \eta_m (1 + f) \frac{\tau_\lambda}{\tau_r} (1 - \tau_t)$$

Solving for the turbine temperature ratio gives

$$\tau_t = 1 - \frac{1}{\eta_m (1 + f)} \frac{\tau_r}{\tau_\lambda} (\tau_c - 1) \tag{7.12}$$

This expression enables us to solve for τ_t, from which we then obtain

$$\pi_t = \tau_t^{\gamma_t/[(\gamma_t - 1)e_t]} \tag{7.13}$$

We note that η_t will be given in terms of e_t by [Eq. (6.22)]

$$\eta_t = \frac{1 - \tau_t}{1 - \tau_t^{1/e_t}} \tag{7.14}$$

We also require the calculation of τ_c to allow determination of τ_t. Thus we note from Eq. (6.14)

$$\tau_c = \pi_c^{(\gamma_c - 1)/(\gamma_c e_t)} \tag{7.15}$$

Then also, from Eq. (6.15),

$$\eta_c = \frac{\pi_c^{(\gamma_c - 1)/\gamma_c} - 1}{\tau_c - 1} \tag{7.16}$$

Step 7: The equation for specific thrust cannot be simplified for this analysis.

Step 8: The equation for the thrust specific fuel consumption is

$$S = \frac{f}{F/\dot{m}_0} \tag{7.17}$$

Step 9: From the definitions of propulsive and thermal efficiency, one can easily show that for the turbojet engine

$$\eta_P = \frac{2g_c V_0 (F/\dot{m}_0)}{a_0^2[(1+f)(V_9/a_0)^2 - M_0^2]} \tag{7.18}$$

and

$$\eta_T = \frac{a_0^2[(1+f)(V_9/a_0)^2 - M_0^2]}{2g_c f h_{PR}} \tag{7.19}$$

Now we have all of the equations needed for analysis of the turbojet cycle. For convenience, this system of equations is listed (in the order of calculation) in the following section for easier calculation.

7.2.2 Summary of Equations—Turbojet Engine

INPUTS:

$$M_0,\ T_0(\text{K, °R}),\ \gamma_c, c_{pc}\left(\frac{\text{kJ}}{\text{kg}\cdot\text{K}}, \frac{\text{Btu}}{\text{lbm}\cdot\text{°R}}\right),\ \gamma_t,\ c_{pt}\left(\frac{\text{kJ}}{\text{kg}\cdot\text{K}}, \frac{\text{Btu}}{\text{lbm}\cdot\text{°R}}\right)$$

$$h_{PR}\left(\frac{\text{kJ}}{\text{kg}}, \frac{\text{Btu}}{\text{lbm}}\right),\quad \pi_{d\max},\ \pi_b,\ \pi_n,\ e_c,\ e_t,\ \eta_b,\ \eta_m,\ P_0/P_9,\ T_{t4}(\text{K, °R,})\ \pi_c$$

OUTPUTS:

$$\frac{F}{\dot{m}_0}\left(\frac{\text{N}}{\text{kg/s}}, \frac{\text{lbf}}{\text{lbm/s}}\right), f,\ S\left(\frac{\text{mg/s}}{\text{N}}, \frac{\text{lbm/h}}{\text{lbf}}\right), \eta_T,\ \eta_P,\ \eta_O,\ \eta_c,\ \eta_t,\ \text{etc.}$$

EQUATIONS:

$$R_c = \frac{\gamma_c - 1}{\gamma_c} c_{pc} \tag{7.20a}$$

$$R_t = \frac{\gamma_t - 1}{\gamma_t} c_{pt} \tag{7.20b}$$

$$a_0 = \sqrt{\gamma_c R_c g_c T_0} \tag{7.20c}$$

$$V_0 = a_0 M_0 \tag{7.20d}$$

$$\tau_r = 1 + \frac{\gamma_c - 1}{2} M_0^2 \tag{7.20e}$$

$$\pi_r = \tau_r^{\gamma_t/(\gamma_t - 1)} \tag{7.20f}$$

$$\eta_r = 1 \quad \text{for } M_0 \le 1 \tag{7.20g}$$

$$\eta_r = 1 - 0.075(M_0 - 1)^{1.35} \quad \text{for } M_0 > 1 \tag{7.20h}$$

$$\pi_d = \pi_{d\max} \eta_r \tag{7.20i}$$

$$\tau_\lambda = \frac{c_{pt} T_{t4}}{c_{pc} T_0} \tag{7.20j}$$

$$\tau_c = \pi_c^{(\gamma_c - 1)/(\gamma_c e_c)} \tag{7.20k}$$

$$\eta_c = \frac{\pi_c^{(\gamma_c - 1)/\gamma_c} - 1}{\tau_c - 1} \tag{7.20l}$$

$$f = \frac{\tau_\lambda - \tau_r \tau_c}{h_{PR} \eta_b / (c_{pc} T_0) - \tau_\lambda} \tag{7.20m}$$

$$\tau_t = 1 - \frac{1}{\eta_m (1 + f)} \frac{\tau_r}{\tau_\lambda} (\tau_c - 1) \tag{7.20n}$$

$$\pi_t = \tau_t^{\gamma_t [(\gamma_t - 1) e_t]} \tag{7.20o}$$

$$\eta_t = \frac{1 - \tau_t}{1 - \tau_t^{1/e_t}} \tag{7.20p}$$

$$\frac{P_{t9}}{P_9} = \frac{P_0}{P_9} \pi_r \pi_d \pi_c \pi_b \pi_t \pi_n \tag{7.20q}$$

$$M_9 = \sqrt{\frac{2}{\gamma_t - 1} \left[\left(\frac{P_{t9}}{P_9} \right)^{(\gamma_t - 1)/\gamma_t} - 1 \right]} \tag{7.20r}$$

$$\frac{T_9}{T_0} = \frac{\tau_\lambda \tau_t}{(P_{t9}/P_9)^{(\gamma_t - 1)/\gamma_t}} \frac{c_{pc}}{c_{pt}} \tag{7.20s}$$

$$\frac{V_9}{a_0} = M_9\sqrt{\frac{\gamma_t R_t T_9}{\gamma_c R_c T_0}} \tag{7.20t}$$

$$\frac{F}{\dot{m}_0} = \frac{a_0}{g_c}\left[(1+f)\frac{V_9}{a_0} - M_0 + (1+f)\frac{R_t T_9/T_0}{R_c V_9/a_0}\frac{(1-P_0/P_9)}{\gamma_c}\right] \tag{7.20u}$$

$$S = \frac{f}{F/\dot{m}_0} \tag{7.20v}$$

$$\eta_T = \frac{a_0^2[(1+f)(V_9/a_0)^2 - M_0^2]}{2g_c f h_{PR}} \tag{7.20w}$$

$$\eta_P = \frac{2g_c V_0(F/\dot{m}_0)}{a_0^2[(1+f)(V_9/a_0)^2 - M_0^2]} \tag{7.20x}$$

$$\eta_O = \eta_P \eta_T \tag{7.20y}$$

7.2.3 Examples—Turbojet Engine

We begin this section with a single example calculation for a turbojet engine. The other examples involve multiple calculations to investigate trends in engine performance.

Example 7.1

Consider a turbojet engine operating at high speed with the following input data.

INPUTS:

$$M_0 = 2,\ T_0 = 216.7\ \text{K},\ \gamma_c = 1.4,\ c_{pc} = 1.004\ \text{kJ/(kg}\cdot\text{K)},\ \gamma_t = 1.3$$
$$c_{pt} = 1.239\ \text{kJ/(kg}\cdot\text{K)},\ h_{PR} = 42{,}800\ \text{kJ/kg},\ \pi_{d\max} = 0.95,\ \pi_b = 0.94$$
$$\pi_n = 0.96,\ e_c = 0.9,\ e_t = 0.9,\ \eta_b = 0.98,\ \eta_m = 0.99,\ P_0/P_9 = 0.5$$
$$T_{t4} = 1800\ \text{K},\ \pi_c = 10$$

EQUATIONS:

$$R_c = \frac{\gamma_c - 1}{\gamma_c}c_{pc} = \frac{0.4}{1.4}(1.004) = 0.2869\ \text{kJ/(kg}\cdot\text{K)}$$

$$R_t = \frac{\gamma_t - 1}{\gamma_t}c_{pt} = \frac{0.3}{1.3}(1.239) = 0.2859\ \text{kJ/(kg}\cdot\text{K)}$$

$$a_0 = \sqrt{\gamma_c R_c g_c T_0} = \sqrt{1.4 \times 286.9 \times 1 \times 216.7} = 295.0\ \text{m/s}$$

$$V_0 = a_0 M_0 = 295.0 \times 2 = 590\ \text{m/s}$$

$$V_0 = a_0 M_0 = 295.0 \times 2 = 590\ \text{m/s}$$

$$\tau_r = 1 + \frac{\gamma_c - 1}{2} M_0^2 = 1 + 0.2 \times 2^2 = 1.8$$

$$\pi_r = \tau_r^{\gamma_c/(\gamma_c - 1)} = 1.8^{3.5} = 7.82445$$

$$\eta_r = 1 - 0.075(M_0 - 1)^{1.35} = 1 - 0.075(1^{1.35}) = 0.925$$

$$\pi_d = \pi_{d\max} \eta_r = 0.95 \times 0.925 = 0.87875$$

$$\tau_\lambda = \frac{c_{pt} T_{t4}}{c_{pc} T_0} = \frac{1.239 \times 1800}{1.004 \times 216.7} = 10.2506$$

$$\tau_c = \pi_c^{(\gamma_c - 1)/(\gamma_c e_c)} = 10^{1/(3.5 \times 0.9)} = 2.0771$$

$$\eta_c = \frac{\pi_c^{(\gamma_c - 1)/\gamma_c} - 1}{\tau_c - 1} = \frac{10^{1/3.5} - 1}{2.0771 - 1} = 0.8641$$

$$f = \frac{\tau_\lambda - \tau_r \tau_c}{h_{PR} \eta_b / (c_{pc} T_0) - \tau_\lambda}$$

$$= \frac{10.2506 - 1.8 \times 2.0771}{42{,}800 \times 0.98/(1.004 \times 216.7) - 10.2506} = 0.03567$$

$$\tau_t = 1 - \frac{1}{\eta_m (1 + f)} \frac{\tau_r}{\tau_\lambda} (\tau_c - 1)$$

$$= 1 - \frac{1}{0.99 \times 1.03567} \frac{1.8}{10.2506} (2.0771 - 1) = 0.8155$$

$$\pi_t = \tau_t^{\gamma_t/[(\gamma_t - 1)e_t]} = 0.8155^{1.3/(0.3 \times 0.9)} = 0.3746$$

$$\eta_t = \frac{1 - \tau_t}{1 - \tau_t^{1/e_t}} = \frac{1 - 0.8155}{1 - 0.8155^{1/0.9}} = 0.9099$$

$$\frac{P_{t9}}{P_9} = \frac{P_0}{P_9} \pi_r \pi_d \pi_c \pi_b \pi_t \pi_n$$

$$= 0.5 \times 7.824 \times 0.8788 \times 10 \times 0.94 \times 0.3746 \times 0.96 = 11.621$$

$$M_9 = \sqrt{\frac{2}{\gamma_t - 1} \left[\left(\frac{P_{t9}}{P_9} \right)^{(\gamma_t - 1)/\gamma_t} - 1 \right]}$$

$$= \sqrt{\frac{2}{0.3} (11.621^{0.3/1.3} - 1)} = 2.253$$

$$\frac{T_9}{T_0} = \frac{\tau_\lambda \tau_t}{(P_{t9}/P_9)^{(\gamma_t - 1)/\gamma_t}} \frac{c_{pc}}{c_{pt}} = \frac{10.2506 \times 0.8155}{11.621^{0.3/1.3}} \frac{1.004}{1.239} = 3.846$$

$$\frac{V_9}{a_0} = M_9\sqrt{\frac{\gamma_t R_t T_9}{\gamma_c R_c T_c}} = 2.253\sqrt{\frac{1.3 \times 0.2859}{1.4 \times 0.2869}(3.846)} = 4.250$$

$$\frac{F}{\dot{m}_0} = \frac{a_0}{g_c}\left[(1+f)\frac{V_9}{a_0} - M_0 + (1+f)\frac{R_t T_9/T_0}{R_c V_9/a_0}\frac{1 - P_0/P_9}{\gamma_c}\right]$$

$$= \frac{295}{1}\left(1.03567 \times 4.250 - 2 + 1.03567\frac{0.2859}{0.2869}\frac{3.846}{4.250}\frac{0.5}{1.4}\right)$$

$$= 295(2.4016 + 0.3336) = 806.9 \text{ N/(kg/s)}$$

$$S = \frac{f}{F/\dot{m}_0} = \frac{0.03567}{806.9} \times 10^6 = 44.21\text{(mg/s)/N}$$

$$\eta_T = \frac{a_0^2[(1+f)(V_9/a_0)^2 - M_0^2]}{2g_c f h_{PR}}$$

$$= \frac{295.0^2[(1.03835)4.250^2) - 2^2]}{2 \times 1 \times 0.03567 \times 42{,}800 \times 1000} = 41.92\%$$

$$\eta_P = \frac{2g_c V_0(F/\dot{m}_0)}{a_0^2[(1+f)(V_9/a_0)^2 - M_0^2]}$$

$$= \frac{2 \times 1 \times 590 \times 806.9}{295^2[1.03567(4.250^2) - 2^2]} = 74.39\%$$

$$\eta_O = \eta_P \eta_T = 0.4192 \times 0.7439 = 31.18\%$$

Example 7.2

Now we consider the turbojet cycle with losses over the same range of Mach numbers and compressor pressure ratios as analyzed for the ideal turbojet cycle and plotted in Figs. 5.8, 5.9, and 5.10.

INPUTS:

$$M_0 = 0 \to 3,\ T_0 = 390°\text{R},\ \gamma_c = 1.4,\ c_{pc} = 0.24 \text{ Btu/(lbm} \cdot °\text{R)}$$
$$\gamma_t = 1.35,\ c_{pt} = 0.262 \text{ Btu/(lbm} \cdot °\text{R)},\ h_{PR} = 18{,}400 \text{ Btu/lbm}$$
$$\pi_{d\max} = 0.98,\quad \pi_b = 0.98,\quad \pi_n = 0.98,\quad e_c = 0.92,\ e_t = 0.91$$
$$\eta_b = 0.99,\quad \eta_m = 0.98,\ P_0/P_9 = 1,\quad T_{t4} = 3000°\text{R},\quad \pi_c = 1 \to 30$$

The results of the analysis are plotted vs compressor pressure ratio in Figs. 7.3a–7.3d and vs flight Mach number in Figs. 7.4a–7.4d. When compared to the corresponding figures for the ideal turbojet cycle, the following can be concluded for the turbojet cycle with losses:

a) Specific thrust $F/\dot{m}_0$. Comparing Fig. 7.3a to Fig. 5.8a and Fig. 7.4a to Fig. 5.9a, one can see that the variation of specific thrust with compressor pressure ratio or Mach number is not appreciably changed and the magnitudes are nearly equal. At high Mach numbers, the effect of the losses causes the

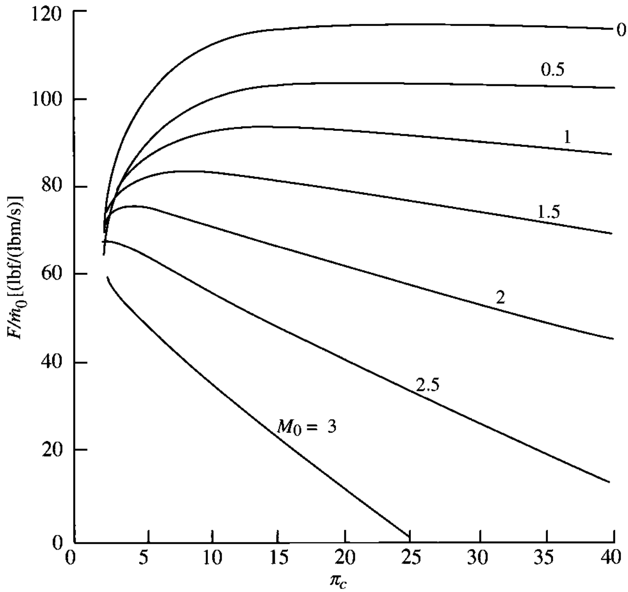

Fig. 7.3a Turbojet performance vs π_c: specific thrust.

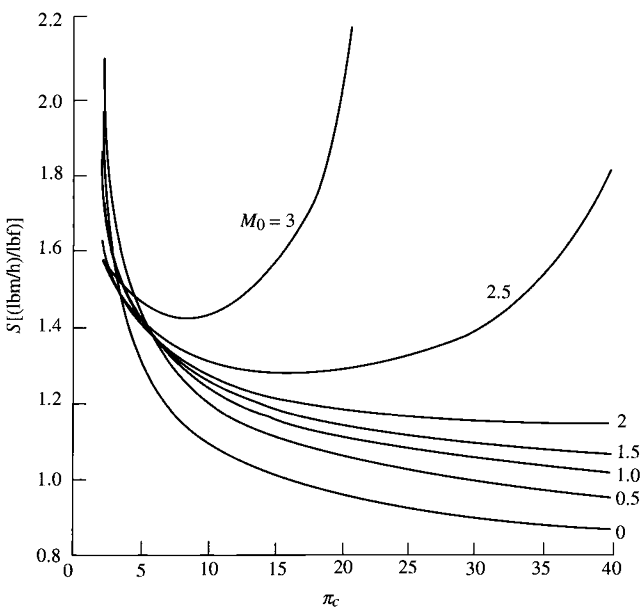

Fig. 7.3b Turbojet performance vs π_c: thrust-specific fuel consumption.

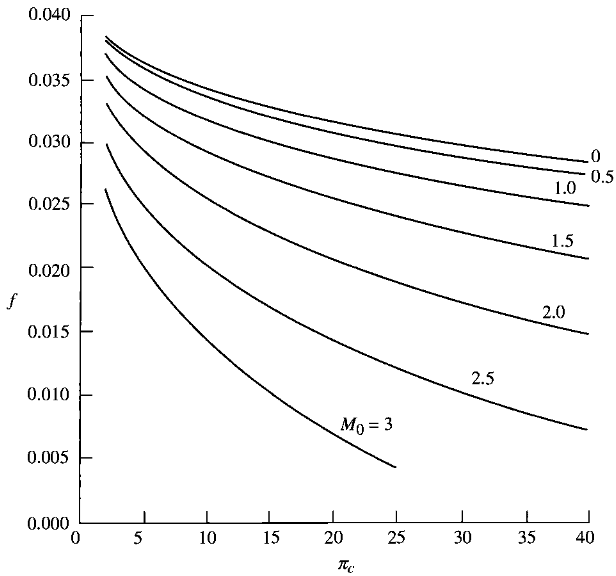

Fig. 7.3c Turbojet performance vs π_c: fuel/air ratio.

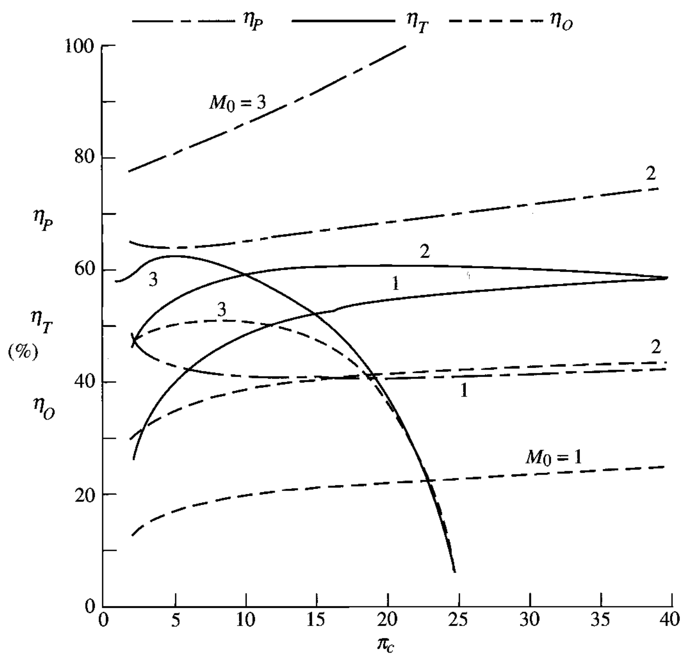

Fig. 7.3d Turbojet performance vs π_c: efficiencies.

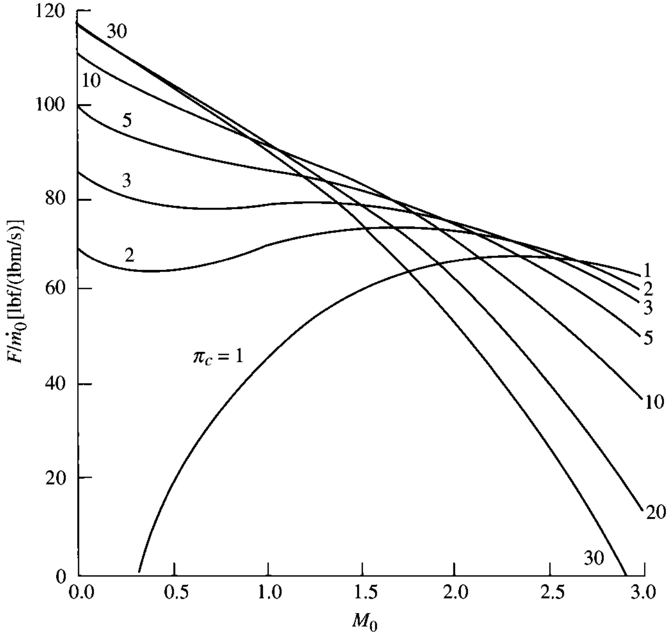

Fig. 7.4a Turbojet performance vs M_0: specific thrust.

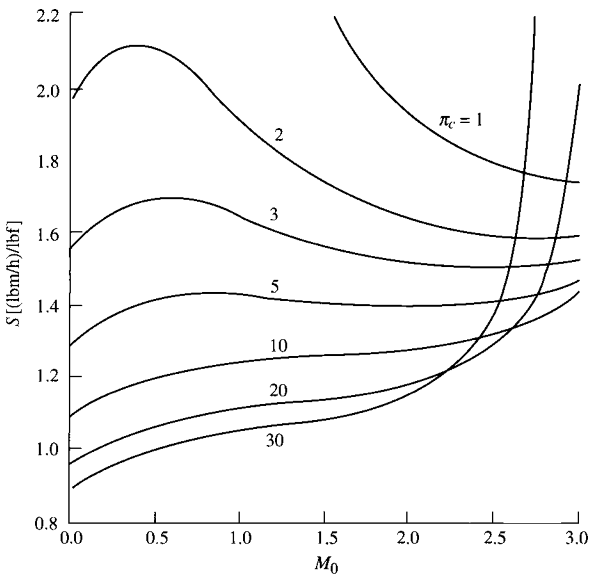

Fig. 7.4b Turbojet performance vs M_0: thrust-specific fuel consumption.

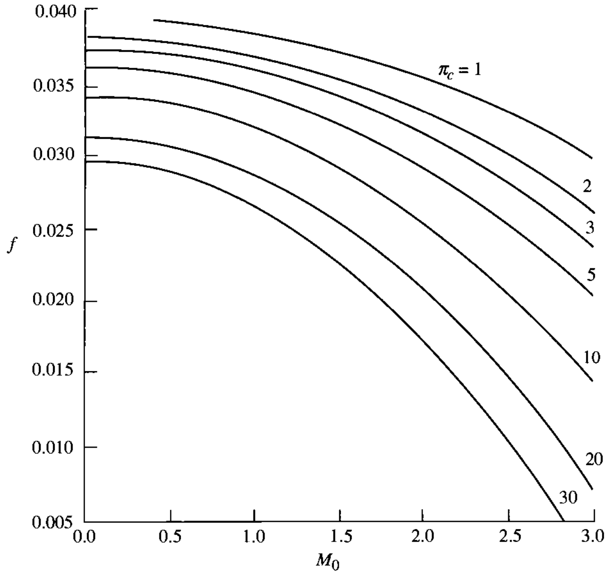

Fig. 7.4c Turbojet performance vs M_0: fuel/air ratio.

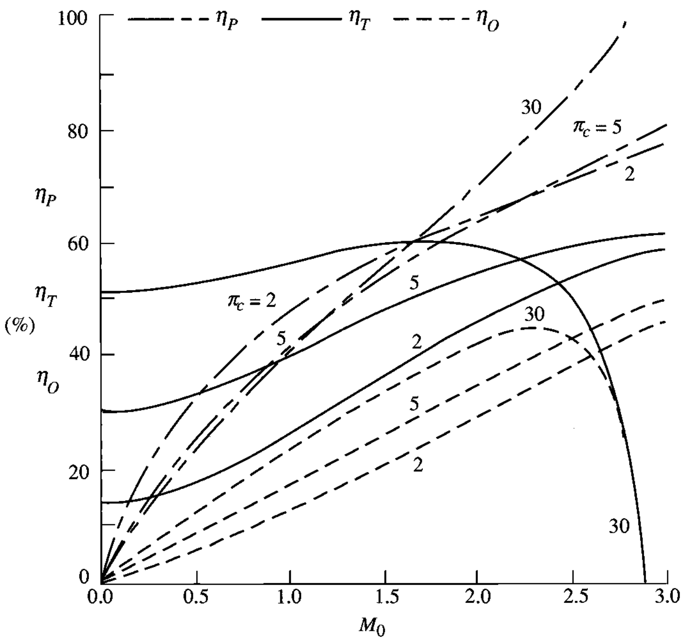

Fig. 7.4d Turbojet performance vs M_0: efficiencies.

thrust to go to zero at a lower compressor pressure ratio. For a Mach number, the compressor pressure ratio that gives maximum specific thrust is lower than that of the ideal turbojet. Also, the ramjet cycle ($\pi_c = 1$) does not have thrust for Mach numbers less than 0.3.

b) Thrust specific fuel consumption S. Comparison of Fig. 7.3b to Fig. 5.8b and Fig. 7.4b to Fig. 5.9b shows that the values of fuel consumption are larger for the engine with losses. The thrust specific fuel consumption no longer continues to decrease with increasing compressor pressure ratio, and there is now a compressor pressure ratio giving minimum fuel consumption for a given Mach number.

c) Fuel/air ratio f. Comparing Fig. 7.3c to Fig. 5.8c and Fig. 7.4c to Fig. 5.9c, we see that the values of the fuel/air ratio are larger for the turbojet with losses. The main reasons for this increase in fuel/air ratio are the increase in specific heat across the main burner, the inefficiency of the combustion process, and the larger mass flow rate exiting the main burner.

d) Propulsive efficiency η_P. Comparison of Fig. 7.3d to Fig. 5.8d and Fig. 7.4d to Fig. 5.9d shows that the propulsive efficiencies are a little larger for the turbojet with losses. This is due mainly to the decrease in exhaust velocity for the engine with losses.

e) Thermal efficiency η_T. Comparing Fig. 7.3d to Fig. 5.8d and Fig. 7.4d to Fig. 5.9d, we can see that the engines with losses have lower thermal efficiency. Also, the thermal efficiency of high-compressor-pressure-ratio engines at high Mach go toward zero because the thrust goes to zero before the fuel flow rate.

f) Overall efficiency η_O. One can see that the overall efficiencies are lower for the turbojet engines with losses by comparison of Fig. 7.3d to Fig. 5.8d and Fig. 7.4d to Fig. 5.9d. This is mainly due to the decrease in thermal efficiency of the engines with losses.

Example 7.3

The effect of compressor efficiency on the performance of a turbojet engine cycle at Mach 2.0 is investigated in the following. The compressor pressure ratio was varied over the range of 1 to 40 for two different compressor polytropic efficiencies to give the results indicated in Fig. 7.5. Also included on this plot are the results of ideal cycle analysis. The input data for this analysis are listed here.

INPUTS:

$$M_0 = 2,\ T_0 = 390^\circ\text{R},\ \gamma_c = 1.4,\ c_{pc} = 0.24\ \text{Btu/(lbm}\cdot{}^\circ\text{R)}$$
$$\gamma_t = 1.33,\ c_{pt} = 0.276\ \text{Btu/(lbm}\cdot{}^\circ\text{R)},\ h_{PR} = 18{,}400\ \text{Btu/lbm}$$
$$\pi_{d\max} = 0.98,\quad \pi_b = 0.98,\quad \pi_n = 0.98,\quad e_c = 0.92 \text{ and } 0.89,\quad e_t = 0.91$$
$$\eta_b = 0.99,\quad \eta_m = 0.98,\ P_0/P_9 = 1,\quad T_{t4} = 3000^\circ\text{R},\quad \pi_c = 1 \to 40$$

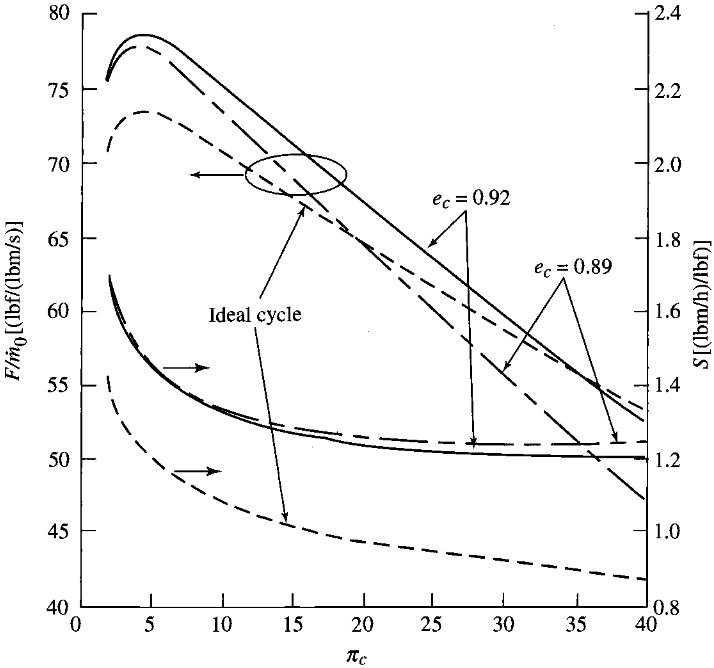

Fig. 7.5 Effect of compressor polytropic efficiency of turbojet cycle.

It can be seen from Fig. 7.5 that the ideal turbojet analysis gives the basic trends for the lower values of the compressor pressure ratio. As the compressor pressure ratio increases, the effect of engine losses increases. At low compressor pressure ratio, the ideal analysis predicts a lower value of specific thrust than the engine with losses because the momentum of the fuel is neglected in the ideal case.

It can also be seen from Fig. 7.5 that a prospective designer would be immediately confronted with a design choice, because the compressor pressure ratio leading to maximum specific thrust is far from that leading to minimum fuel consumption. Clearly, a short-range interceptor would better suit a low compressor pressure ratio with the resultant high specific thrust and lightweight (small compressor) engine. Conversely, the designer of a long-range transport would favor an engine with high compressor pressure ratio and low specific fuel consumption. Thus we see what should be obvious—before an engine can be correctly designed, the mission (use) for which it is being designed must be precisely understood.

Another aspect of the designer's dilemma becomes apparent when the curves obtained for the two different compressor polytropic efficiencies in Fig. 7.5 are compared. For example, if a designer chooses a compressor pressure ratio of 35 for use in a supersonic transport because the compressor design group has promised a compressor with $e_c = 0.92$, and then the group delivers a compressor with $e_c = 0.89$, clearly the choice $\pi_c = 35$ would be quite inappropriate. That is, such a compressor would have a higher pressure ratio than that leading to minimum fuel consumption. Thus the designer would have a compressor that

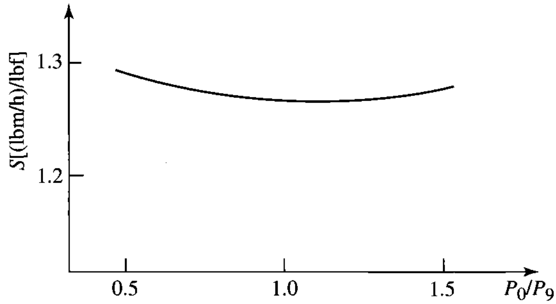

Fig. 7.6 Effect of nozzle off-design conditions on thrust-specific fuel consumption.

was heavier (and more expensive) than that leading to a minimum specific fuel consumption, he or she would also have lower specific thrust, and finally the designer would have an acute need to change employers.

The effect of nozzle off-design conditions can be invesigated by considering the engine to have the same parameters as those indicated previously, but with various values of P_0/P_9. As an example, we consider an engine with $\pi_c = 16$ and $e_c = 0.92$ to obtain the thrust specific fuel consumption information plotted in Fig. 7.6.

It is apparent that for small exit nozzle off-design conditions ($0.8 < P_0/P_9 < 1.2$), the thrust and thrust specific fuel consumption vary only slightly with the exit pressure mismatch. This result indicates that the best nozzle design should be determined by considering the external flow behavior (boattail drag, etc.). Note also that for a long-range transport or passenger aircraft, a 1% change in specific fuel consumption is very significant.

Example 7.4

Let's consider another example in which we let the Mach number vary and see how the engine performance changes. The flight conditions, design limits, component performance figures of merit, and design choices are listed in the following. We will compare the turbojet engine with losses with the corresponding ideal engine. The trends that were obtained for the ideal engine are generally true for the engine with losses; exceptions are noted in the following discussion.

INPUTS:

$$M_0 = 1 \to 4,\ T_0 = 216.7\text{ K},\ \gamma_c = 1.4,\ c_{pc} = 1.004\text{ kJ/(kg}\cdot\text{ K)},\ \gamma_t = 1.35$$
$$c_{pt} = 1.096\text{ kJ/(kg}\cdot\text{ K)},\ h_{PR} = 42{,}800\text{ kJ/kg},\ \pi_{d\max} = 0.98,\ \pi_b = 0.98$$
$$\pi_n = 0.96,\ e_c = 0.89,\ e_t = 0.91,\ \eta_b = 0.99,\ \eta_m = 0.98$$
$$P_0/P_9 = 1,\ T_{t4} = 1670\text{ K},\ \pi_c = 8\text{ and }24$$

The specific thrust vs Mach number is plotted in Fig. 7.7a for both compressor pressure ratios. The specific thrust is approximately the same as that for the ideal

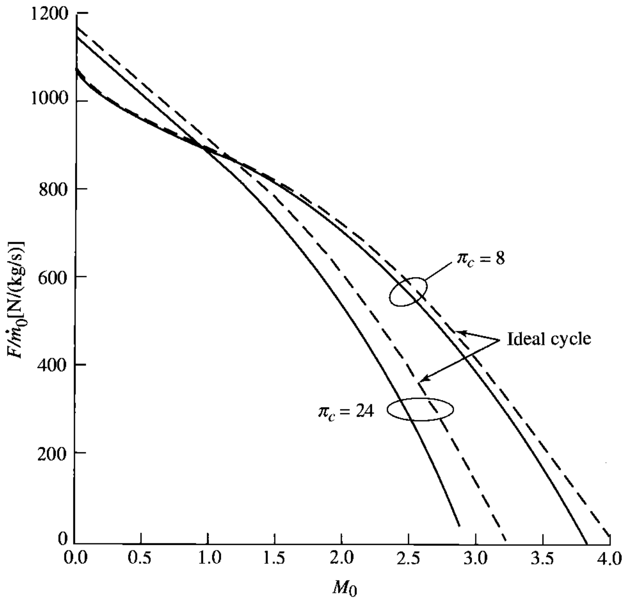

Fig. 7.7a Specific thrust for two compressor pressure ratios.

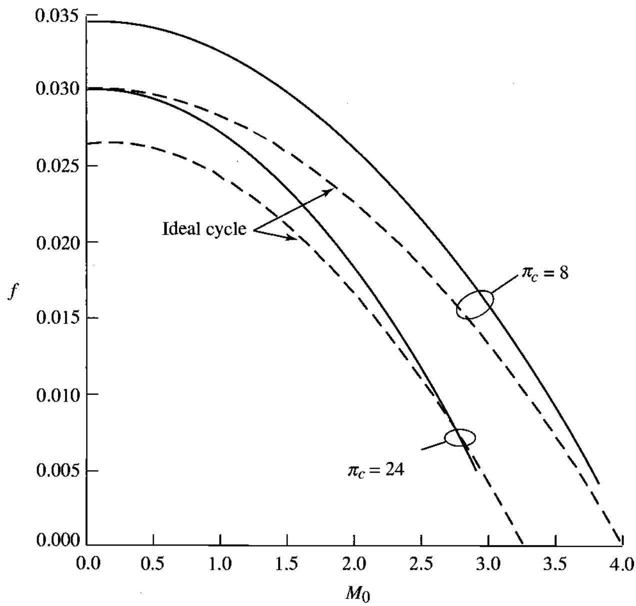

Fig. 7.7b Fuel/air ratio for two compressor pressure ratios.

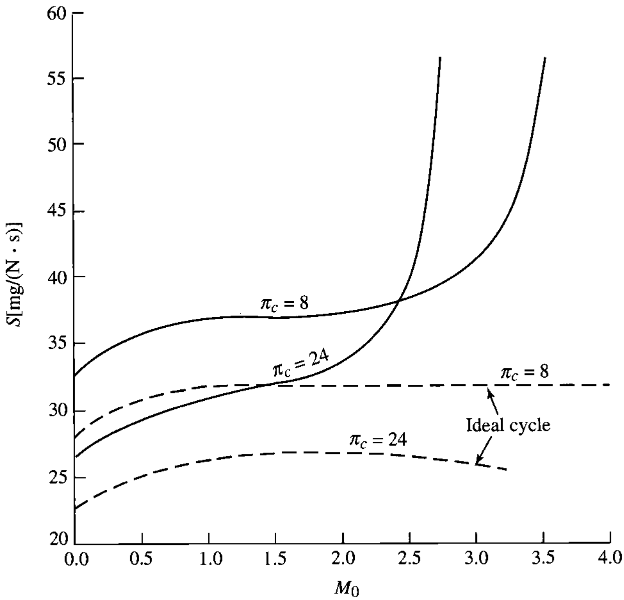

Fig. 7.7c Specific fuel consumption for two compressor pressure ratios.

engine. While the exit velocity decreases slightly for the engine with losses, this is compensated by the extra mass flow leaving the engine due to fuel addition.

The fuel/air ratio f vs Mach number is plotted in Fig. 7.7b for both compressor pressure ratios. The fuel/air ratio is considerably higher for the engine with losses. There are three reasons for this:

1) The extra mass due to fuel addition, neglected in the ideal case, must be heated to the temperature of the products of combustion leaving the burner. This requires extra fuel.

2) The combustion process is not 100% efficient, and so extra fuel is required.

3) Most important, the change in gas properties, i.e., the increase in the specific heat at constant pressure c_P, means that more energy is needed to increase the temperature of the products of combustion than if the gas remained as air with the low-temperature properties. This is true since $h_t = c_p T_t$ and the fuel burned goes to increasing h_t directly and T_t only indirectly.

The thrust-specific fuel consumption S vs Mach number is plotted in Fig. 7.7c for both compressor pressure ratios. Because the thrust is approximately the same for the ideal engine and the corresponding engine with losses, and because the required fuel/air ratio is higher for the engine with losses, the thrust-specific fuel consumption is considerably higher for the engine with losses. For the example given here, the value for the engine with losses is generally higher by 30–40%. However, the two values really diverge for high flight Mach numbers. The thrust specific fuel consumption starts increasing toward infinity at $M_0 > 2.0$ for a compressor pressure ratio of 8 and at $M_0 > 1.5$ for a compressor pressure ratio of 24. This follows from the definition of the thrust specific fuel consumption—the fuel flow rate divided by the thrust. For the engine with losses, the thrust goes to zero before the fuel flow rate does. This indicates that the thermal efficiency for the turbojet at high speeds goes to zero; i.e., there is a heat input from the fuel, but no net power output because of the component inefficiencies. This is not the case for the ideal engines where the thermal efficiency always increases with flight Mach number.

7.3 Turbojet with Afterburner

The turbojet engine with afterburner is shown in Fig. 7.8, and the temperature-entropy plot of this engine with losses is shown in Fig. 7.9. The numbering system indicated in these figures is the industry standard.[30]

For the analysis of this engine, we remind the reader of the following definitions for the afterburner:

$$
\begin{aligned}
\pi_{AB} &= \frac{P_{t7}}{P_{t6}} \qquad f_{AB} = \frac{\dot{m}_{fAB}}{\dot{m}_0} \\
\tau_{AB} &= \frac{T_{t7}}{T_{t6}} \qquad \tau_{\lambda AB} = \frac{c_{pAB}}{c_{pc}}\frac{T_{t7}}{T_0} \\
\eta_{AB} &= \frac{(\dot{m}_0 + \dot{m}_f + \dot{m}_{fAB})c_{pAB}T_{t7} - (\dot{m}_0 + \dot{m}_f)c_{pt}T_{t6}}{\dot{m}_{fAB}h_{PR}}
\end{aligned}
\tag{7.21}
$$

Note that stations 6 and 7 are used for these afterburner parameters. We assume isentropic flow from station 5 to station 6 in the following analysis. Thus Fig. 7.9 shows the afterburning process going from station 5 to 7.

We also note that

$$
\tau_{\lambda AB} = \frac{c_{pAB}}{c_{pc}}\frac{T_{t4}}{T_0}\frac{T_{t5}}{T_{t4}}\frac{T_{t8}}{T_{t5}} = \frac{c_{pAB}}{c_{pc}}\tau_\lambda \tau_t \tau_{AB} \tag{7.22}
$$

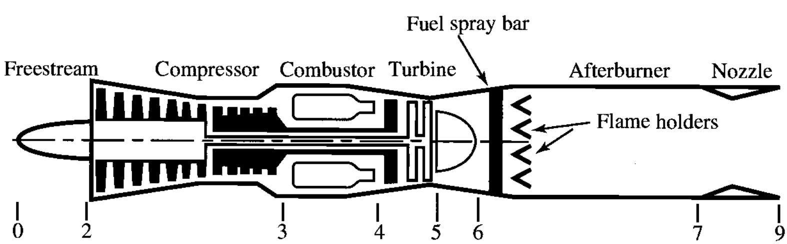

Fig. 7.8 Ideal afterburning turbojet engine with station numbering.

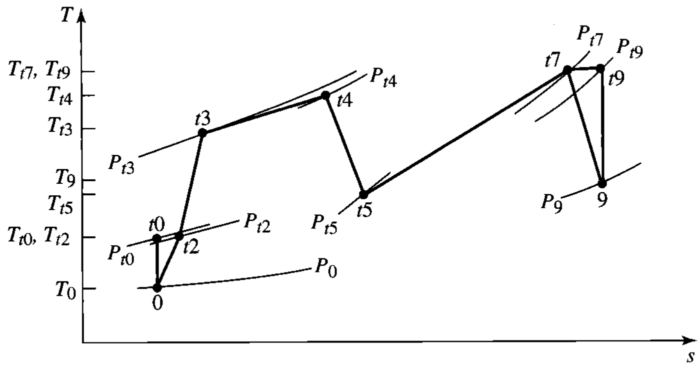

Fig. 7.9 The T-s diagram for afterburning turbojet engine.

and

$$\frac{\dot{m}_9}{\dot{m}_0} = \frac{\dot{m}_0 + \dot{m}_f + \dot{m}_{f\mathrm{AB}}}{\dot{m}_0} = 1 + f + f_{\mathrm{AB}} \tag{7.23}$$

7.3.1 Cycle Analysis

The expression for the thrust will be the same as that already obtained for the turbojet without afterburning except that the effects of fuel addition in the afterburner must be included. Application of the steps of cycle analysis (see Section 5.4) is listed next.

Step 1: The specific thrust equation becomes

$$\frac{F}{\dot{m}_0} = \frac{a_0}{g_c}\left[(1 + f + f_{\mathrm{AB}})\frac{V_9}{a_0} - M_0 + (1 + f + f_{\mathrm{AB}}) \times \frac{R_{\mathrm{AB}}}{R_c}\frac{T_9/T_0}{V_9/a_0}\frac{1 - P_0/P_9}{\gamma_c}\right] \tag{7.24}$$

Step 2: As before,

$$\left(\frac{V_9}{a_0}\right)^2 = \frac{\gamma_{\mathrm{AB}} R_{\mathrm{AB}} T_9}{\gamma_c R_c T_0} M_9^2 \tag{7.25}$$

Step 3: We have

$$M_9^2 = \frac{2}{\gamma_{\mathrm{AB}} - 1}\left[\left(\frac{P_{t9}}{P_9}\right)^{(\gamma_{\mathrm{AB}}-1)/\gamma_{\mathrm{AB}}} - 1\right] \tag{7.26}$$

where

$$\frac{P_{t9}}{P_9} = \frac{P_0}{P_0}\pi_r \pi_d \pi_c \pi_b \pi_t \pi_{AB} \pi_n \tag{7.27}$$

Step 4: We have

$$\frac{T_9}{T_0} = \frac{T_{t9}/T_0}{(P_{t9}/P_9)^{(\gamma_{AB}-1)/\gamma_{AB}}}$$

where

$$\frac{T_{t9}}{T_0} = \tau_{\lambda AB}\frac{c_{pc}}{c_{pAB}} \tag{7.28}$$

Step 5: Application of the steady flow energy equation to the main combustor gives, [Eq. (7.10)]

$$f = \frac{\tau_\lambda - \tau_r \tau_c}{\eta_b h_{PR}/(c_{pc}T_0) - \tau_\lambda}$$

Application of the steady flow energy equation to the afterburner gives

$$(\dot{m}_0 + \dot{m}_f)c_{pt}T_{t6} + \eta_{AB}\dot{m}_{fAB}h_{PR} = (\dot{m}_0 + \dot{m}_f + \dot{m}_{fAB})c_{pAB}T_{t7}$$

This equation can be solved for the afterburner fuel/air ratio f_{AB}, giving

$$f_{AB} = (1+f)\frac{\tau_{\lambda AB} - \tau_\lambda \tau_t}{\eta_{AB}h_{PR}/(c_{pc}T_0) - \tau_{\lambda AB}} \tag{7.29}$$

Step 6: The power balance between the turbine and compressor is unaffected by the addition of the afterburner. Thus Eqs. (7.12)–(7.16) apply to this engine cycle.

Step 7: Not used in this analysis.

Step 8: The thrust specific fuel consumption S is expressed in terms of both the main burner and afterburner fuel/air ratios as

$$S = \frac{f + f_{AB}}{F/\dot{m}_0} \tag{7.30}$$

Step 9: From the definitions of propulsive and thermal efficiency, one can easily show that for the afterburning turbojet engine

$$\eta_P = \frac{2g_c V_0(F/\dot{m}_0)}{a_0^2[(1+f+f_{AB})(V_9/a_0)^2 - M_0^2]} \tag{7.31}$$

$$\eta_T = \frac{a_0^2[(1+f+f_{AB})(V_9/a_0)^2 - M_0^2]}{2g_c(f+f_{AB})h_{PR}} \tag{7.32}$$

Now we have all of the equations needed for analysis of the afterburning turbojet cycle. For convenience, this system of equations is listed (in the order of calculation) in the following section for easier calculation.

7.3.2 Summary of Equations—Afterburning Turbojet Engine

INPUTS:

$$M_0,\ T_0(\mathrm{K}, {}^\circ\mathrm{R}),\ \gamma_c,\ c_{pc}\left(\frac{\mathrm{kJ}}{\mathrm{kg\cdot K}}, \frac{\mathrm{Btu}}{\mathrm{lbm\cdot {}^\circ R}}\right),\ \gamma_t,\ c_{pt}\left(\frac{\mathrm{kJ}}{\mathrm{kg\cdot K}}, \frac{\mathrm{Btu}}{\mathrm{lbm\cdot {}^\circ R}}\right)$$

$$h_{PR}\left(\frac{\mathrm{kJ}}{\mathrm{kg}}, \frac{\mathrm{Btu}}{\mathrm{lbm}}\right),\ \gamma_{AB},\ c_{pAB}\left(\frac{\mathrm{kJ}}{\mathrm{kg\cdot K}}, \frac{\mathrm{Btu}}{\mathrm{lbm\cdot {}^\circ R}}\right),\ \pi_{d\max},\ \pi_b,\ \pi_{AB},\ \pi_n,\ e_c,\ e_t$$

$$\eta_b,\ \eta_{AB}, \eta_m,\ P_0/P_9,\ T_{t4}(\mathrm{K}, {}^\circ\mathrm{R}),\ T_{t7}(\mathrm{K}, {}^\circ\mathrm{R}),\ \pi_c$$

OUTPUTS:

$$\frac{F}{\dot{m}_0}\left(\frac{\mathrm{N}}{\mathrm{kg/s}}, \frac{\mathrm{lbf}}{\mathrm{lbm/s}}\right), f, f_{AB},\ S\left(\frac{\mathrm{mg/s}}{N}, \frac{\mathrm{lbm/h}}{\mathrm{lbf}}\right),\ \eta_T,\ \eta_P,\ \eta_O,\ \eta_c, \eta_t,\ \text{etc.}$$

EQUATIONS:

Equations (7.20a–7.20p) and the following:

$$R_{AB} = \frac{\gamma_{AB} - 1}{\gamma_{AB}} c_{pAB} \tag{7.33a}$$

$$\tau_{\lambda AB} = \frac{c_{pAB}}{c_{pc}} \frac{T_{t7}}{T_0} \tag{7.33b}$$

$$f_{AB} = (1+f)\frac{\tau_{\lambda AB} - \tau_\lambda \tau_t}{\eta_{AB} h_{PR}/(c_{pc}T_0) - \tau_{\lambda AB}} \tag{7.33c}$$

$$\frac{P_{t9}}{P_9} = \frac{P_0}{P_9}\pi_r \pi_d \pi_c \pi_b \pi_t \pi_{AB} \pi_n \tag{7.33d}$$

$$\frac{T_9}{T_0} = \frac{T_{t7}/T_0}{(P_{t9}/P_9)^{(\gamma_{AB}-1)/\gamma_{AB}}} \tag{7.33e}$$

$$M_9^2 = \frac{2}{\gamma_{AB}-1}\left[\left(\frac{P_{t9}}{P_9}\right)^{(\gamma_{AB}-1)/\gamma_{AB}} - 1\right] \tag{7.33f}$$

$$\frac{V_9}{a_0} = M_9\sqrt{\frac{\gamma_{AB}R_{AB}T_9}{\gamma_c R_c T_0}} \tag{7.33g}$$

$$\frac{F}{\dot{m}_0} = \frac{a_0}{g_c}\left[(1+f+f_{AB})\frac{V_9}{a_0} - M_0 + (1+f+f_{AB}) \times \frac{R_{AB}}{R_c}\frac{T_9/T_0}{V_9/a_0}\frac{1-P_0/P_9}{\gamma_c}\right] \tag{7.33h}$$

$$S = \frac{f+f_{AB}}{F/\dot{m}_0} \tag{7.33i}$$

$$\eta_P = \frac{2g_c V_0(F/\dot{m}_0)}{a_0^2[(1+f+f_{AB})(V_9/a_0)^2 - M_0^2]} \tag{7.33j}$$

$$\eta_T = \frac{a_0^2[(1+f+f_{AB})(V_9/a_0)^2 - M_0^2]}{2g_c(f+f_{AB})h_{PR}} \tag{7.33k}$$

$$\eta_O = \eta_P \eta_T \tag{7.33l}$$

Example 7.5

We consider an example similar to that considered for the "dry" turbojet of Example 7.3 so that we can directly compare the effects of afterburning. Thus we have the following input data.

INPUTS:

$$M_0 = 2,\ T_0 = 390°\text{R},\ \gamma_c = 1.4,\ c_{pc} = 0.24\ \text{Btu/(lbm} \cdot °\text{R)}$$
$$\gamma_t = 1.33,\ c_{pt} = 0.276\ \text{Btu/(lbm} \cdot °\text{R)},\ h_{PR} = 18{,}400\ \text{Btu/lbm}$$
$$\gamma_{AB} = 1.30,\ c_{pAB} = 0.295\ \text{Btu/(lbm} \cdot °\text{R)},\ \pi_{d\max} = 0.98,\ \pi_b = 0.98$$
$$\pi_{AB} = 0.98,\ \pi_n = 0.98,\ e_c = 0.89,\ e_t = 0.91,\ \eta_b = 0.99$$
$$\eta_{AB} = 0.96,\ \eta_m = 0.98,\ P_0/P_9 = 1,\ T_{t4} = 3000°\text{R}$$
$$T_{t7} = 3500°\text{R},\ \pi_c = 2 \rightarrow 14$$

In this example, we limit the maximum compressor pressure ratio to 14 because the total temperature leaving the compressor T_{t3} will exceed 1200°F

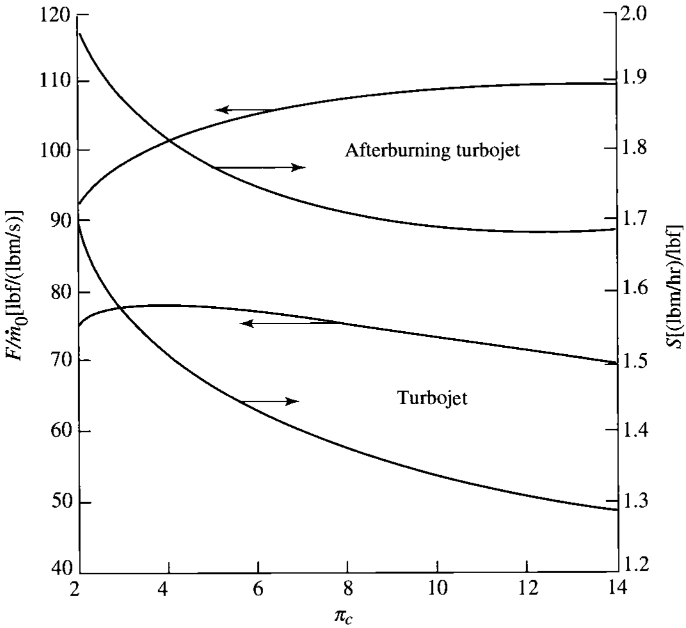

Fig. 7.10 Performance of turbojet engine with and without afterburner.

(temperature limit of current compressor materials) at higher pressure ratios. For the afterburning turbojet, we take

$$\pi_n \pi_{AB} = 0.98^2 \quad \text{afterburner on}$$
$$\pi_n \pi_{AB} = 0.98 \quad \text{afterburner off}$$

The results are indicated in Fig. 7.10. Note that operation of the afterburner will increase both the specific thrust and the fuel consumption. The magnitude of the increases depends on the compressor pressure ratio. A compressor pressure ratio of 12 gives good specific thrust and fuel consumption for afterburner operation and reasonable performance without the afterburner.

The design compressor pressure ratio will depend on the aircraft and its mission (use), which requires an in-depth analysis.

7.4 Turbofan—Separate Exhaust Streams

A turbofan engine with station numbering is shown in Fig. 7.11. A temperature vs entropy plot for the flow through the fan and the engine core is shown in Fig. 7.12. The effect of engine losses can be seen by comparing Fig. 7.12 with Figs. 5.21 and 5.22. The exit velocity of both the fan stream and the engine core stream is reduced by engine losses.

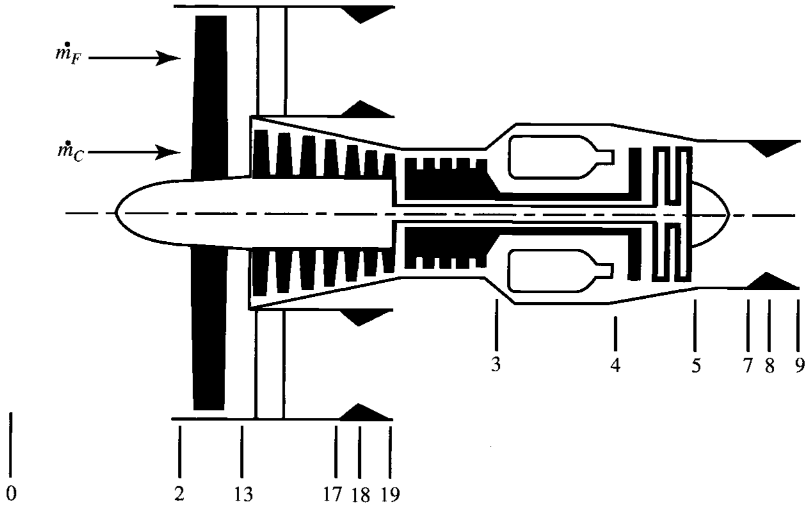

Fig. 7.11 Station numbering of turbofan engine.

7.4.1 Cycle Analysis

The assumptions for the analysis of the turbofan engine cycle with losses are as follows:

1) Perfect gas upstream of main burner with constant properties γ_c, R_c, c_{pc}, etc.
2) Perfect gas downstream of main burner with constant properties γ_t, R_t, c_{pt}, etc.
3) All components are adiabatic (no turbine cooling).

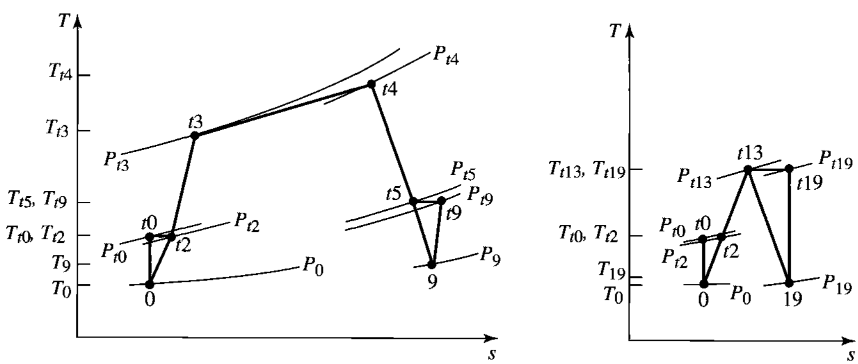

Fig. 7.12 The *T*-*s* diagram of turbofan engine with losses (not to scale).

4) The efficiencies of the compressor, fan, and turbine are described through the use of (constant) polytropic efficiencies e_c, e_f, and e_t, respectively.

The steps of cycle analysis are applied to the turbofan engine and presented next in the order listed in Section 5.4. We will apply the steps of cycle analysis to both the fan stream and the engine core stream.

7.4.1.1 Fan stream. Steps 1–4 are as follows.

Step 1: Uninstalled thrust of fan stream F_F:

$$F_F = \frac{\dot{m}_F}{g_c}(V_{19} - V_0) + A_{19}(P_{19} - P_0)$$

Using Eq. (7.2) for the fan stream gives

$$\frac{F_F}{\dot{m}_F} = \frac{a_0}{g_c}\left(\frac{V_{19}}{a_0} - M_0 + \frac{T_{19}/T_0}{V_{19}/a_0}\frac{1 - P_0/P_{19}}{\gamma_c}\right) \tag{7.34}$$

Step 2:

$$\left(\frac{V_{19}}{a_0}\right)^2 = \frac{T_{19}}{T_0}M_{19}^2 \tag{7.35}$$

Step 3: We have

$$M_{19}^2 = \frac{2}{\gamma_c - 1}\left[\left(\frac{P_{t19}}{P_{19}}\right)^{(\gamma_c-1)/\gamma_c} - 1\right] \tag{7.36a}$$

where

$$\frac{P_{t19}}{P_{19}} = \frac{P_0}{P_{19}}\pi_r\pi_d\pi_f\pi_{fn} \tag{7.36b}$$

Step 4: We have

$$\frac{T_{19}}{T_0} = \frac{T_{t19}/T_0}{(P_{t19}/P_{19})^{(\gamma_c-1)/\gamma_c}} \tag{7.37a}$$

where

$$\frac{T_{t19}}{T_0} = \tau_r\tau_f \tag{7.37b}$$

7.4.1.2 Engine core stream. Steps 1–5 are the same as for the turbojet engine cycle with losses.

Step 1: Uninstalled thrust:

$$F_C = \frac{1}{g_c}(\dot{m}_9V_9 - \dot{m}_CV_0) + A_9(P_9 - P_0)$$

or

$$\frac{F_C}{\dot{m}_C} = \frac{a_0}{g_c}\left[(1+f)\frac{V_9}{a_0} - M_0 + (1+f)\frac{R_t}{R_c}\frac{T_9/T_0}{V_9/a_0}\frac{1-P_0/P_9}{\gamma_c}\right] \tag{7.38}$$

where the fuel/air ratio for the main burner is defined as

$$f \equiv \frac{\dot{m}_f}{\dot{m}_C} \tag{7.39}$$

Step 2:

$$\left(\frac{V_9}{a_0}\right)^2 = \frac{\gamma_t R_t T_9}{\gamma_c R_c T_0} M_9^2 \tag{7.40}$$

Step 3: We have

$$M_9^2 = \frac{2}{\gamma_t - 1}\left[\left(\frac{P_{t9}}{P_9}\right)^{(\gamma_t-1)/\gamma_t} - 1\right] \tag{7.41a}$$

where

$$\frac{P_{t9}}{P_9} = \frac{P_0}{P_9}\pi_r \pi_d \pi_c \pi_b \pi_t \pi_n \tag{7.41b}$$

Step 4: We have

$$\frac{T_9}{T_0} = \frac{T_{t9}/T_0}{(P_{t9}/P_9)^{(\gamma_t-1)/\gamma_t}} \tag{7.42a}$$

where

$$\frac{T_{t9}}{T_0} = \tau_r \tau_d \tau_c \tau_b \tau_t \tau_n = \frac{c_{pc}}{c_{pt}}\tau_\lambda \tau_t \tag{7.42b}$$

Step 5: Application of the first law of thermodynamics to the burner gives

$$\dot{m}_C c_{pc} T_{t3} + \eta_b \dot{m}_f h_{PR} = \dot{m}_4 c_{pt} T_{t4}$$

By using the definitions of the temperature ratios and fuel/air ratio, the preceding equation becomes

$$\tau_r \tau_c + f\frac{\eta_b h_{PR}}{c_{pc} T_0} = (1+f)\tau_\lambda$$

Solving for f, we get

$$f = \frac{\tau_\lambda - \tau_r \tau_c}{\eta_b h_{PR}/(c_{pc} T_0) - \tau_\lambda} \tag{7.43}$$

Step 6: The power balance between the turbine, compressor, and fan, with a mechanical efficiency η_m of the coupling between the turbine and compressor and fan, gives

$$\underbrace{\dot{m}_C c_{pc}(T_{t3} - T_{t2})}_{\text{power into compressor}} + \underbrace{\dot{m}_F c_{pc}(T_{t13} - T_{t2})}_{\text{power into fan}} = \underbrace{\eta_m \dot{m}_4 c_{pt}(T_{t4} - T_{t5})}_{\text{net power from turbine}} \tag{7.44}$$

Dividing the preceding equation by $\dot{m}_C c_{pc} T_{t2}$ and using the definitions of temperature ratios, fuel/air ratio, and the bypass ratio [α, Eq. (5.46)], we obtain

$$\tau_c - 1 + \alpha(\tau_f - 1) = \eta_m(1 + f)\frac{\tau_\lambda}{\tau_r}(1 - \tau_t)$$

Solving for the turbine temperature ratio gives

$$\tau_t = 1 - \frac{1}{\eta_m(1 + f)}\frac{\tau_r}{\tau_\lambda}[\tau_c - 1 + \alpha(\tau_f - 1)] \tag{7.45}$$

Equations (7.13–7.16) are used to obtain the unknown pressure or temperature ratio and efficiencies of the turbine and compressor. For the fan, the following equations apply:

$$\tau_f = \pi_f^{(\gamma_c - 1)/(\gamma_c e_f)} \tag{7.46}$$

$$\eta_f = \frac{\pi_f^{(\gamma_c - 1)/\gamma_c} - 1}{\tau_f - 1} \tag{7.47}$$

Step 7: Combining the thrust equations for the fan stream and the engine core stream, we obtain

$$\begin{aligned}\frac{F}{\dot{m}_0} = {} & \frac{1}{1 + \alpha}\frac{a_0}{g_c}\left[(1 + f)\frac{V_9}{a_0} - M_0 + (1 + f)\frac{R_t T_9/T_0}{R_c V_9/a_0}\frac{1 - P_0/P_9}{\gamma_c}\right] \\ & + \frac{\alpha}{1 + \alpha}\frac{a_0}{g_c}\left(\frac{V_{19}}{a_0} - M_0 + \frac{T_{19}/T_0}{V_{19}/a_0}\frac{1 - P_0/P_{19}}{\gamma_c}\right)\end{aligned} \tag{7.48}$$

Step 8: The thrust specific fuel consumption S is

$$S = \frac{\dot{m}_f}{F} = \frac{\dot{m}_f/\dot{m}_C}{(\dot{m}_0/\dot{m}_C)F/\dot{m}_0}$$

or

$$S = \frac{f}{(1 + \alpha)F/\dot{m}_0} \tag{7.49}$$

Step 9: Expressions for the propulsive efficiency η_p and thermal efficiency η_T are listed next for the case of $P_9 = P_{19} = P_0$. Development of these equations is left as an exercise for the reader.

$$\eta_P = \frac{2M_0[(1+f)(V_9/a_0) + \alpha(V_{19}/a_0) - (1+\alpha)M_0]}{(1+f)(V_9/a_0)^2 + \alpha(V_{19}/a_0)^2 - (1+\alpha)M_0^2} \tag{7.50}$$

$$\eta_T = \frac{a_0^2[(1+f)(V_9/a_0)^2 + \alpha(V_{19}/a_0)^2 - (1+\alpha)M_0^2]}{2g_c f h_{PR}} \tag{7.51}$$

7.4.2 Summary of Equations—Separate-Exhaust-Stream Turbofan Engine

INPUTS:

$$M_0,\ T_0(\text{K},\ ^\circ\text{R}),\ \gamma_c,\ c_{pc}\left(\frac{\text{kJ}}{\text{kg}\cdot\text{K}}, \frac{\text{Btu}}{\text{lbm}\cdot{}^\circ\text{R}}\right),\ \gamma_t,\ c_{pt}\left(\frac{\text{kJ}}{\text{kg}\cdot K}, \frac{\text{Btu}}{\text{lbm}\cdot{}^\circ\text{R}}\right)$$

$$h_{PR}\left(\frac{\text{kJ}}{\text{kg}}, \frac{\text{Btu}}{\text{lbm}}\right),\ \pi_{d\max},\ \pi_b,\ \pi_n,\ \pi_{fn},\ e_c,\ e_f,\ e_t,\ \eta_b$$

$$\eta_m,\ P_0/P_9,\ P_0/P_{19},\ T_{t4}(\text{K},\ ^\circ\text{R}),\ \pi_c,\ \pi_f,\ \alpha$$

OUTPUTS:

$$\frac{F}{\dot{m}_0}\left(\frac{\text{N}}{\text{kg/s}}, \frac{\text{lbf}}{\text{lbm/s}}\right), f,\ S\left(\frac{\text{mg/s}}{\text{N}}, \frac{\text{lbm/h}}{\text{lbf}}\right),\ \eta_T,\ \eta_P,\ \eta_O,\ \eta_c,\ \eta_t,\ \text{etc.}$$

EQUATIONS:

$$R_c = \frac{\gamma_c - 1}{\gamma_c} c_{pc} \tag{7.52a}$$

$$R_t = \frac{\gamma_t - 1}{\gamma_t} c_{pt} \tag{7.52b}$$

$$a_0 = \sqrt{\gamma_c R_c g_c T_0} \tag{7.52c}$$

$$V_0 = a_0 M_0 \tag{7.52d}$$

$$\tau_r = 1 + \frac{\gamma_c - 1}{2} M_0^2 \tag{7.52e}$$

$$\pi_r = \tau_r^{\gamma_c/(\gamma_c-1)} \tag{7.52f}$$

$$\eta_r = 1 \quad \text{for } M_0 \le 1 \tag{7.52g}$$

$$\eta_r = 1 - 0.075(M_0 - 1)^{1.35} \quad \text{for } M_0 > 1 \tag{7.52h}$$

$$\pi_d = \pi_{d\max}\eta_r \tag{7.52i}$$

$$\tau_\lambda = \frac{c_{pt}T_{t4}}{c_{pc}T_0} \tag{7.52j}$$

$$\tau_c = \pi_c^{(\gamma_c-1)/(\gamma_c e_c)} \tag{7.52k}$$

$$\eta_c = \frac{\pi_c^{(\gamma_c-1)/\gamma_c} - 1}{\tau_c - 1} \tag{7.52l}$$

$$\tau_f = \pi_f^{(\gamma_c-1)/(\gamma_c e_f)} \tag{7.52m}$$

$$\eta_f = \frac{\pi_f^{(\gamma_c-1)/\gamma_c} - 1}{\tau_f - 1} \tag{7.52n}$$

$$f = \frac{\tau_\lambda - \tau_r\tau_c}{\eta_b h_{PR}/(c_{pc}T_0) - \tau_\lambda} \tag{7.52o}$$

$$\tau_t = 1 - \frac{1}{\eta_m(1+f)}\frac{\tau_r}{\tau_\lambda}[\tau_c - 1 + \alpha(\tau_f - 1)] \tag{7.52p}$$

$$\pi_t = \tau_t^{\gamma_t/[(\gamma_t-1)e_t]} \tag{7.52q}$$

$$\eta_t = \frac{1 - \tau_t}{1 - \tau_t^{1/e_t}} \tag{7.52r}$$

$$\frac{P_{t9}}{P_9} = \frac{P_0}{P_9}\pi_r\pi_d\pi_c\pi_b\pi_t\pi_n \tag{7.52s}$$

$$M_9 = \sqrt{\frac{2}{\gamma_t - 1}\left[\left(\frac{P_{t9}}{P_9}\right)^{(\gamma_t-1)/\gamma_t} - 1\right]} \tag{7.52t}$$

$$\frac{T_9}{T_0} = \frac{\tau_\lambda\tau_t}{(P_{t9}/P_9)^{(\gamma_t-1)/\gamma_t}}\frac{c_{pc}}{c_{pt}} \tag{7.52u}$$

$$\frac{V_9}{a_0} = M_9\sqrt{\frac{\gamma_t R_t T_9}{\gamma_c R_c T_0}} \tag{7.52v}$$

$$\frac{P_{t19}}{P_{19}} = \frac{P_0}{P_{19}}\pi_r\pi_d\pi_f\pi_{fn} \tag{7.52w}$$

$$M_{19} = \sqrt{\frac{2}{\gamma_c - 1}\left[\left(\frac{P_{t19}}{P_{19}}\right)^{(\gamma_c-1)/\gamma_c} - 1\right]} \tag{7.52x}$$

$$\frac{T_{19}}{T_0} = \frac{\tau_r \tau_f}{(P_{t19}/P_{19})^{(\gamma_c-1)/\gamma_c}} \tag{7.52y}$$

$$\frac{V_{19}}{a_0} = M_{19}\sqrt{\frac{T_{19}}{T_0}} \tag{7.52z}$$

$$\begin{aligned}\frac{F}{\dot{m}_0} = {} & \frac{1}{1+\alpha}\frac{a_0}{g_c}\left[(1+f)\frac{V_9}{a_0} - M_0 + (1+f)\right.\\ & \left.\times \frac{R_t T_9/T_0}{R_c V_9/a_0}\frac{1 - P_0/P_9}{\gamma_c}\right] + \frac{\alpha}{1+1\alpha}\frac{a_0}{g_c}\\ & \times \left(\frac{V_{19}}{a_0} - M_0 + \frac{T_{19}/T_0}{V_{19}/a_0}\frac{1 - P_0/P_{19}}{\gamma_c}\right)\end{aligned} \tag{7.52aa}$$

$$S = \frac{f}{(1+\alpha)F/\dot{m}_0} \tag{7.55ab}$$

$$\text{Thrust ratio (FR)} = \frac{(1+f)\dfrac{V_9}{a_0} - M_0 + (1+f)\dfrac{R_t T_9/T_0}{R_c V_9/a_0}\dfrac{1 - P_0/P_9}{\gamma_c}}{\dfrac{V_{19}}{a_0} - M_0 + \dfrac{T_{19}/T_0}{V_{19}/a_0}\dfrac{1 - P_0/P_{19}}{\gamma_c}} \tag{7.52ac}$$

$$\eta_P = \frac{2M_0[(1+f)V_9/a_0 + \alpha(V_{19}/a_0) - (1+\alpha)M_0]}{(1+f)(V_9/a_0)^2 + \alpha(V_{19}/a_0)^2 - (1+\alpha)M_0^2} \tag{7.52ad}$$

$$\eta_T = \frac{a_0^2[(1+f)(V_9/a_0)^2 + \alpha(V_{19}/a_0)^2 - (1+\alpha)M_0^2]}{2g_c f h_{PR}} \tag{7.52ae}$$

$$\eta_O = \eta_P \eta_T \tag{7.52af}$$

7.4.3 Exit Pressure Conditions

Separate-stream turbofan engines are generally used with subsonic aircraft, and the pressure ratio across both primary and secondary nozzles is not very large. As a result, often convergent-only nozzles are utilized. In this case, if the nozzles are choked, we have

$$\frac{P_{t19}}{P_{19}} = \left(\frac{\gamma_c + 1}{2}\right)^{\gamma_c/(\gamma_c-1)} \quad \text{and} \quad \frac{P_{t9}}{P_9} = \left(\frac{\gamma_t + 1}{2}\right)^{\gamma_t/(\gamma_t-1)} \tag{7.53}$$

Thus

$$\frac{P_0}{P_{19}} = \frac{P_{t19}/P_{19}}{P_{19}/P_0} = \frac{[(\gamma_c + 1)/2]^{\gamma_c/(\gamma_c - 1)}}{\pi_r \pi_d \pi_f \pi_{fn}} \tag{7.54}$$

and

$$\frac{P_0}{P_9} = \frac{P_{t9}/P_9}{P_9/P_0} = \frac{[(\gamma_t + 1)/2]^{\gamma_t/(\gamma_t - 1)}}{\pi_r \pi_d \pi_c \pi_b \pi_t \pi_n} \tag{7.55}$$

Note that these two expressions are valid only when both P_9 and P_{19} are greater than P_0. If these expressions predict P_9 and P_{19} less than P_0, the nozzles will not be choked. In this case, we take $P_{19} = P_0$ and/or $P_9 = P_0$.

Example 7.6

As our first example for the turbofan with losses, we calculate the performance of a turbofan engine cycle with the following input data.

INPUTS:

$$M_0 = 0.8,\ T_0 = 390°\text{R},\ \gamma_c = 1.4,\ c_{pc} = 0.240\ \text{Btu/(lbm} \cdot °\text{R)}$$
$$\gamma_t = 1.33,\ c_{pt} = 0.276\ \text{Btu/(lbm} \cdot °\text{R)},\ h_{PR} = 18{,}400\ \text{Btu/lbm}$$
$$\pi_{d\max} = 0.99,\quad \pi_b = 0.96,\quad \pi_n = 0.99,\quad \pi_{fn} = 0.99,\ e_c = 0.90,\ e_f = 0.89$$
$$\pi_t = 0.89, \eta_b = 0.99, \eta_m = 0.99,\ P_0/P_9 = 0.9, P_0/P_{19} = 0.9$$
$$T_{t4} = 3000°\text{R},\quad \pi_c = 36,\ \pi_f = 1.7,\quad \alpha = 8$$

EQUATIONS:

$$R_c = \frac{\gamma_c - 1}{\gamma_c} c_{pc} = \frac{0.4}{1.4}(0.24 \times 778.16) = 53.36\ \text{ft} \cdot \text{lbf/(lbm} \cdot °\text{R)}$$

$$R_t = \frac{\gamma_t - 1}{\gamma_t} c_{pt} = \frac{0.33}{1.33}(0.276 \times 778.16) = 53.29\ \text{ft} \cdot \text{lbf/(lbm} \cdot °\text{R)}$$

$$a_0 = \sqrt{1.4 \times 53.36 \times 32.174 \times 390} = 968.2\ \text{ft/s}$$

$$V_0 = a_0 M_0 = 968.2 \times 0.8 = 774.6\ \text{ft/s}$$

$$\tau_r = 1 + \frac{\gamma_c - 1}{2} M_0^2 = 1 + 0.2 \times 0.8^2 = 1.128$$

$$\pi_r = \tau_r^{\gamma_c/(\gamma_c - 1)} = 1.128^{3.5} = 1.5243$$

$$\eta_r = 1 \quad \text{since } M_0 < 1$$

$$\pi_d = \pi_{d\max} \eta_r = 0.99$$

$$\tau_\lambda = \frac{c_{pt}T_{t4}}{c_{pc}T_0} = \frac{0.276 \times 3000}{0.240 \times 390} = 8.846$$

$$\tau_c = \pi_c^{(\gamma_c-1)/(\gamma_c e_c)} = 36^{1/(3.5\times 0.9)} = 3.119$$

$$\eta_c = \frac{\pi_c^{(\gamma_c-1)/\gamma_c} - 1}{\tau_c - 1} = \frac{36^{1/3.5} - 1}{3.119 - 1} = \frac{1.784}{2.119} = 84.2\%$$

$$\tau_f = \pi_f^{(\gamma_c-1)/\gamma_c e_f)} = 1.7^{1/(3.5\times 0.89)} = 1.1857$$

$$\eta_f = \frac{\pi_f^{(\gamma_c-1)/\gamma_c} - 1}{\tau_f - 1} = \frac{1.7^{1/35} - 1}{1.1857 - 1} = \frac{0.1637}{0.1857} = 88.2\%$$

$$f = \frac{\tau_\lambda - \tau_r\tau_c}{h_{PR}\eta_b/(c_pT_0) - \tau_\lambda}$$

$$= \frac{8.846 - 1.128 \times 3.119}{18{,}400 \times 0.99/(0.24 \times 390) - 8.846} = 0.02868$$

$$\tau_t = 1 - \frac{1}{\eta_m(1+f)}\frac{\tau_r}{\tau_\lambda}[\tau_c - 1 + \alpha(\tau_f - 1)]$$

$$= 1 - \frac{1}{0.99(1.02868)}\frac{1.128}{8.846}[3.119 - 1 + 8(1.1857 - 1)]$$

$$= 0.54866$$

$$\pi_t = \tau_t^{\gamma_t/[(\gamma_t-1)e_t]} = 0.54866^{1.33/(0.33\times 0.89)} = 0.06599$$

$$\eta_t = \frac{1 - \tau_t}{1 - \tau_t^{1/e_t}} = \frac{1 - 0.54866}{1 - 0.54866^{1/0.89}} = 92.0\%$$

$$\frac{P_{t9}}{P_9} = \frac{P_0}{P_9}\pi_r\pi_d\pi_c\pi_b\pi_t\pi_n$$

$$= 0.9 \times 1.5243 \times 0.99 \times 36 \times 0.96 \times 0.06599 \times 0.99 = 3.066$$

$$M_9 = \sqrt{\frac{2}{\gamma_t - 1}\left[\left(\frac{P_{t9}}{P_9}\right)^{(\gamma_t-1)/\gamma_t} - 1\right]}$$

$$= \sqrt{\frac{2}{0.33}(3.066^{0.33/1.33} - 1)} = 1.394$$

$$\frac{T_9}{T_0} = \frac{8.846 \times 0.54866}{3.066^{0.33/1.33}}\frac{0.240}{0.276} = 3.196$$

$$\frac{V_9}{a_0} = M_9\sqrt{\frac{\gamma_t R_t T_9}{\gamma_c R_c T_0}} = 1.394\sqrt{\frac{1.33 \times 53.29}{1.40 \times 53.36}(3.196)} = 2.427$$

$$M_{19} = \sqrt{\frac{2}{\gamma_c - 1}\left[\left(\frac{P_{t19}}{P_{19}}\right)^{(\gamma_c-1)/\gamma_c} - 1\right]}$$

$$= \sqrt{\frac{2}{0.4}(2.286^{1/3.5} - 1)} = 1.154$$

$$\frac{T_{19}}{T_0} = \frac{\tau_r \tau_f}{(P_{t19}/P_{19})^{(\gamma_c-1)/\gamma_c}} = \frac{1.128 \times 1.1857}{2.286^{1/3.5}} = 1.0561$$

$$\frac{V_{19}}{a_0} = M_{19}\sqrt{\frac{T_{19}}{T_0}} = 1.154\sqrt{1.0561} = 1.186$$

$$\frac{F}{\dot{m}_0} = \frac{1}{1+\alpha}\frac{a_0}{g_c}\left[(1+f)\frac{V_9}{a_0} - M_0 + (1+f)\frac{R_t T_9/T_0}{R_c V_9/A_0}\frac{1-P_0/P_9}{\gamma_c}\right]$$

$$+\frac{\alpha}{1+\alpha}\frac{a_0}{g_c}\left(\frac{V_{19}}{a_0} - M_0 + \frac{T_{19}/T_0}{V_{19}/a_0}\frac{1-P_0/P_{19}}{\gamma_c}\right)$$

$$= \frac{968.2}{9 \times 32.174}\left(1.02868 \times 2.427 - 0.8 + 1.02868\frac{53.29}{53.36}\frac{3.196}{2.427}\frac{0.1}{1.4}\right)$$

$$+\frac{8 \times 968.2}{9 \times 32.174}\left(1.186 - 0.8 + \frac{1.0561}{1.186}\frac{0.1}{1.4}\right)$$

$$= 3.3436(1.79324 + 3.59684) = 18.02 \text{ lbf/(lbm/s)}$$

$$S = \frac{f}{(1+\alpha)F/\dot{m}_0} = \frac{3600 \times 0.02868}{9 \times 18.02} = 0.6366 \text{ (lbm/h)/lbf}$$

$$FR = \frac{1.79324}{3.59684/8} = 3.988$$

$$\eta_P = \frac{2M_0[(1+f)V_9/a_0 + \alpha(V_{19}/a_0) - (1+\alpha)M_0]}{(1+f)(V_9/a_0)^2 + \alpha(V_{19}/a_0)^2 - (1+\alpha)M_0^2}$$

$$= \frac{2 \times 0.8(1.02868 \times 2.427 + 8 \times 1.186 - 9 \times 0.8)}{1.02868 \times 2.427^2 + 8 \times 1.186^2 - 9 \times 0.8^2}$$

$$\eta_T = \frac{a_0^2[(1+f)(V_9/a_0)^2 + \alpha(V_{19}/a_0)^2 - (1+\alpha)M_0^2]}{2g_c f h_{PR}}$$

$$= \frac{968.2^2(1.02868 \times 2.427^2 + 8 \times 1.186^2 - 9 \times 0.8^2)}{2 \times 32.174 \times 0.02868 \times 18{,}400 \times 778.16} = 40.98\%$$

$$\eta_O = \eta_T \eta_P = 0.4098 \times 0.6627 = 27.16\%$$

Example 7.7

Because the turbofan cycle has three design variables, its performance with losses can be understood by performing a parametric analysis, plotting the results vs values of the design variables, and comparing results to the performance of the ideal turbofan. Figures 7.13–7.16 are plots for turbofan engines with $P_9 = P_{19} = P_0$ and the following input values. Unless shown otherwise, the Mach number, compressor pressure ratio, and fan pressure ratio are the values listed under *Baseline*:

$T_0 = 216.7$ K	$\pi_{d\max} = 0.98$	$e_c = 0.90$	*Baseline*
$\gamma_c = 1.4$	$\pi_b = 0.98$	$e_t = 0.91$	$M_0 = 0.9$
$c_{pc} = 1.004$ kJ/(kg · K)	$\pi_n = \pi_{fn} = 0.98$	$e_f = 0.88$	$\pi_c = 24$
$\gamma_t = 1.35$	$\eta_b = 0.99$	$h_{PR} = 42{,}800$ kJ/kg	$\pi_f = 2$
$c_{pt} = 1.096$ kJ/(kg · K)	$\eta_m = 0.98$	$T_{t4} = 1670$ K	

Figures 7.13a, 7.13b, and 7.13c show the influence of compressor pressure ratio and bypass ratio on engine performance. As the bypass ratio increases, the difference in specific thrust between the engine cycle with losses and the "ideal" engine cycle increases. The major difference between the engine cycle's thrust specific fuel consumption for the two models is due to the much higher "fuel/air" ratio for the "real" engine.

Figures 7.14a and 7.14b show the influence of Mach number and bypass ratio on engine performance. The engine's specific thrust is reduced more than that of the ideal engine at high Mach number because of the increasing inlet total pressure loss. The limiting Mach number for economical operation of a turbofan engine with a specific bypass ratio is much lower for the engine with losses than for the ideal engine.

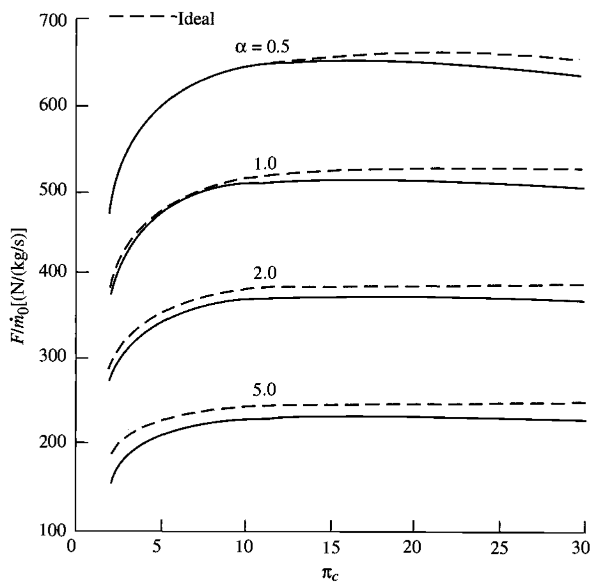

Fig. 7.13a Turbofan engine with losses vs compressor pressure ratio: specific thrust.

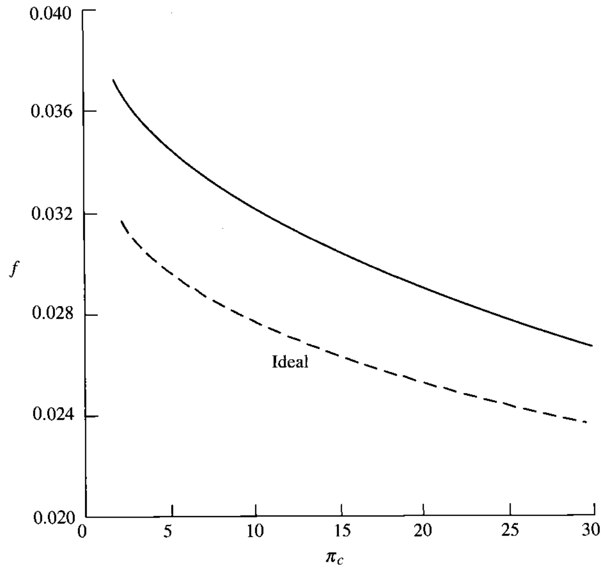

Fig. 7.13b Turbofan engine with losses vs compressor pressure ratio: fuel/air ratio.

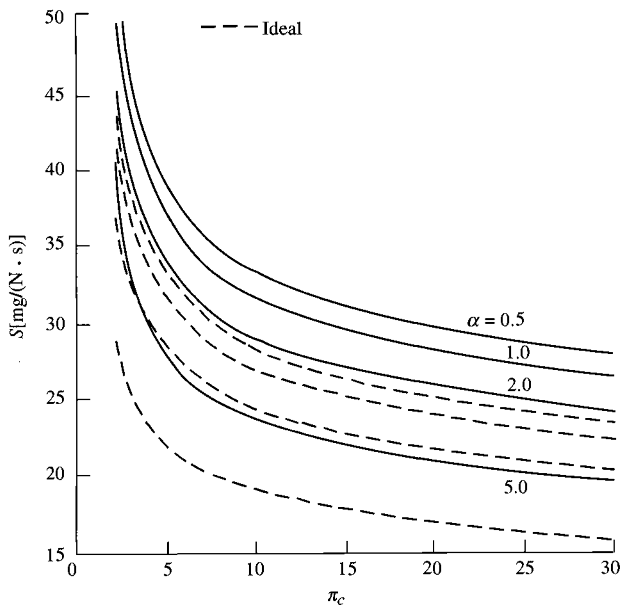

Fig. 7.13c Turbofan engine with losses vs compressor pressure ratio: thrust-specific fuel consumption.

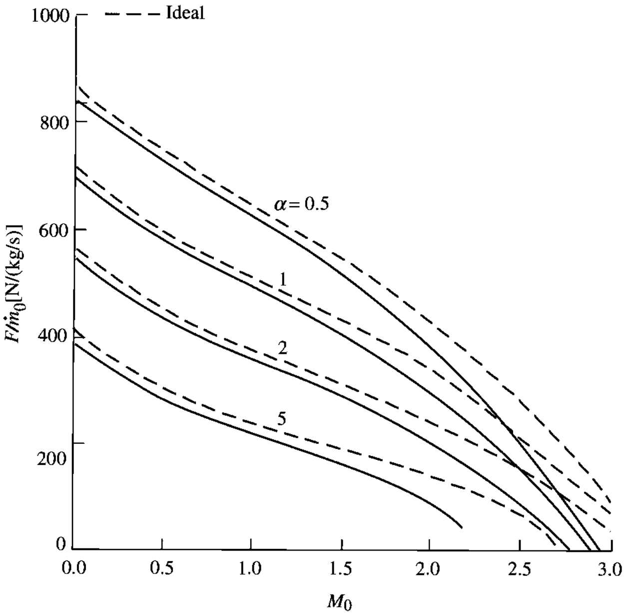

Fig. 7.14a Turbofan engine with losses vs flight Mach number: specific thrust.

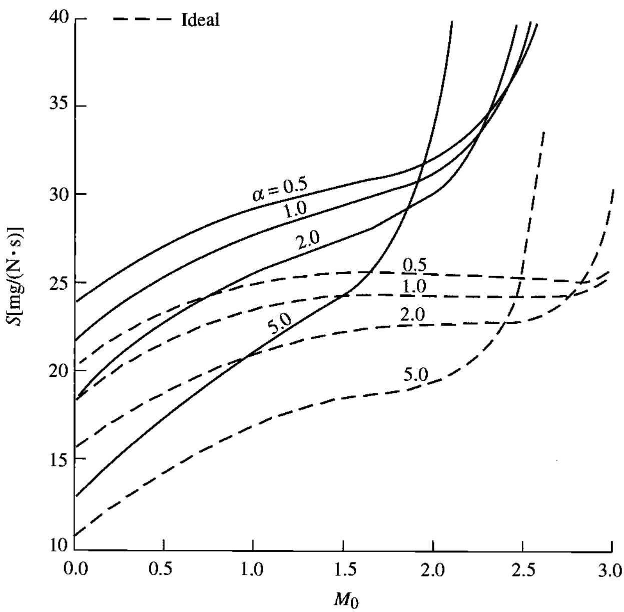

Fig. 7.14b Turbofan engine with losses vs flight Mach number: thrust-specific fuel consumption.

Figures 7.15a and 7.15b show the influence of fan pressure ratio and bypass ratio on engine performance. An optimum fan pressure ratio still exists for the turbofan with losses, and the value of the optimum fan pressure ratio is much lower than that for the ideal turbofan.

Figures 7.16a and 7.16b show the variation in specific thrust and thrust specific fuel consumption with bypass ratio and fan pressure ratio. An optimum bypass ratio still exists for the turbofan with losses, and the value of the optimum bypass ratio is much less than that for the ideal turbofan.

7.4.4 Optimum Bypass Ratio α^*

As was true for the turbofan with no losses, we may obtain an expression that allows us to determine the bypass ratio α^* that leads to minimum thrust specific fuel consumption. For a given set of such prescribed variables (τ_r, π_c, π_f, τ_λ, V_0), we may locate the minimum S by taking the partial derivative of S with respect to the bypass ratio α. We consider the case where the exhaust pressures of both the fan stream and the core stream equal the ambient pressure $P_0 = P_9 = P_{19}$. Because the fuel/air ratio is not a function of bypass ratio, we have

$$S = \frac{f}{(1+\alpha)(F/\dot{m}_0)}$$

$$\frac{\partial S}{\partial \alpha} = \frac{\partial}{\partial \alpha}\left[\frac{f}{(1+\alpha)(F/\dot{m}_0)}\right] = 0$$

$$\frac{\partial S}{\partial \alpha} = \frac{-f}{[(1+\alpha)(F/\dot{m}_0)]^2}\frac{\partial}{\partial \alpha}\left[(1+\alpha)\left(\frac{F}{\dot{m}_0}\right)\right] = 0$$

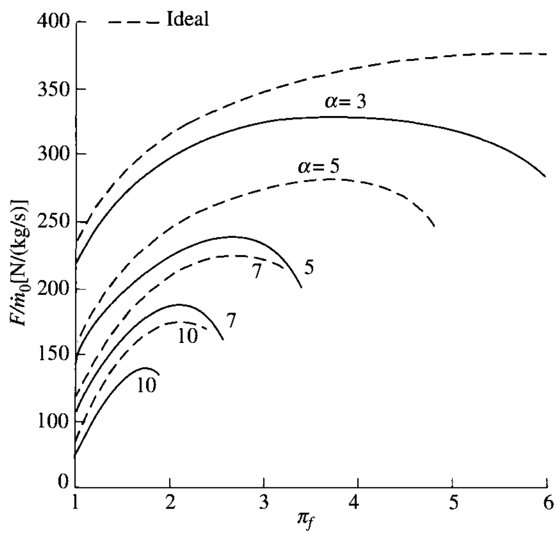

Fig. 7.15a Turbofan engine with losses vs fan pressure ratio: specific thrust.

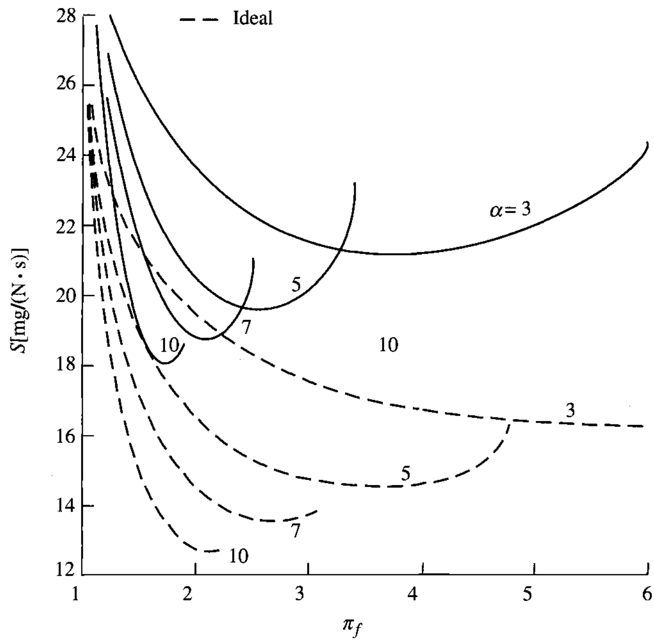

Fig. 7.15b Turbofan engine with losses vs fan pressure ratio: thrust-specific fuel consumption.

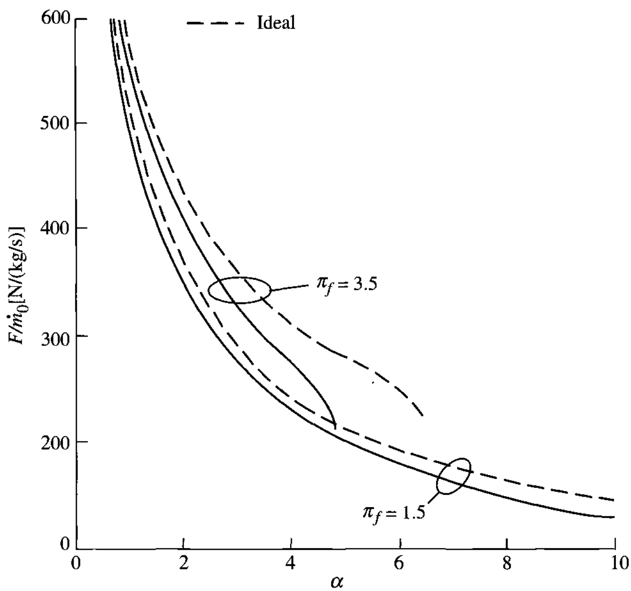

Fig. 7.16a Turbofan engine with losses vs bypass ratio: specific thrust.

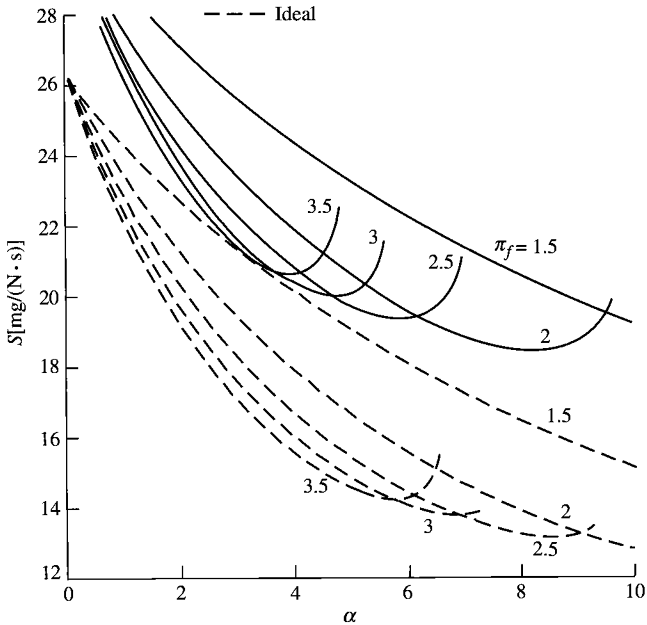

Fig. 7.16b Turbofan engine with losses vs bypass ratio: thrust-specific fuel consumption.

Thus $\partial S/\partial\alpha = 0$ is satisfied by

$$\frac{\partial}{\partial\alpha}\left[\frac{g_c}{V_0}(1+\alpha)\left(\frac{F}{\dot{m}_0}\right)\right] = 0$$

where

$$\frac{g_c}{V_0}(1+\alpha)\left(\frac{F}{\dot{m}_0}\right) = (1+f)\left(\frac{V_9}{V_0} - 1\right) + \alpha\left(\frac{V_{19}}{V_0} - 1\right)$$

Then the optimum bypass ratio is given by the following expression:

$$\frac{\partial}{\partial\alpha}\left[(1+f)\left(\frac{V_9}{V_0} - 1\right) + \alpha\left(\frac{V_{19}}{V_0} - 1\right)\right] = (1+f)\frac{\partial}{\partial\alpha}\left(\frac{V_9}{V_0}\right) + \frac{V_{19}}{V_0} - 1 = 0 \qquad \text{(i)}$$

However,

$$\frac{1}{2V_9/V_0}\frac{\partial}{\partial\alpha}\left[\left(\frac{V_9}{V_0}\right)^2\right]=\frac{\partial}{\partial\alpha}\left(\frac{V_9}{V_0}\right)$$

Thus Eq. (i) becomes

$$\left(\frac{V_9}{V_0}\right)_{\alpha^*}=-\frac{1+f}{2}\frac{\partial/\partial\alpha[(V_9/V_0)^2]}{V_{19}/V_0-1} \tag{ii}$$

Note that

$$\left(\frac{V_9}{V_0}\right)^2=\frac{1}{M_0^2}\left(\frac{V_9}{a_0}\right)^2=\frac{1}{[2/(\gamma_c-1)](\tau_r-1)}\left(\frac{V_9}{a_0}\right)^2$$
$$=\frac{1}{[2/(\gamma_c-1)](\tau_r-1)}M_9^2\frac{\gamma_t R_t T_9}{\gamma_c R_c T_0}$$

Using Eqs. (7.41) and (7.42), we have

$$\left(\frac{V_9}{V_0}\right)^2=\frac{\tau_\lambda\tau_t}{\tau_r-1}\left[1-\left(\frac{P_{t9}}{P_9}\right)^{-(\gamma_t-1)/\gamma_t}\right] \tag{iii}$$

where

$$\frac{P_{t9}}{P_9}=\pi_r\pi_d\pi_c\pi_b\pi_t\pi_n \tag{iv}$$

Combining Eqs. (iii) and (iv), we obtain

$$\left(\frac{V_9}{V_0}\right)^2=\frac{\tau_\lambda\tau_t}{\tau_r-1}\left[1-\frac{1}{\Pi(\pi_t)^{(\gamma_t-1)/\gamma_t}}\right] \tag{v}$$

where

$$\Pi=(\pi_r\pi_d\pi_c\pi_b\pi_n)^{(\gamma_t-1)/\gamma_t} \tag{7.56}$$

Noting that

$$\pi_t^{(\gamma_t-1)/\gamma_t}=\tau_t^{1/e_t}$$

we see that then Eq. (v) becomes

$$\left(\frac{V_9}{V_0}\right)^2 = \frac{\tau_\lambda}{\tau_r - 1}\left(\tau_t - \frac{1}{\Pi}\tau_t^{-(1-e_t)/e_t}\right) \tag{vi}$$

To evaluate the partial derivative of Eq. (ii), we apply the chain rule to Eq. (vi) as follows:

$$\frac{\partial}{\partial\alpha}\left[\left(\frac{V_9}{V_0}\right)^2\right] = \frac{\partial\tau_t}{\partial\alpha}\frac{\partial}{\partial\tau_t}\left[\left(\frac{V_9}{V_0}\right)^2\right]$$

$$= \frac{\partial\tau_t}{\partial\alpha}\frac{\tau_\lambda}{\tau_r - 1}\left(1 + \frac{1 - e_t}{e_t}\frac{\tau_t^{-1/e_t}}{\Pi}\right) \tag{vii}$$

Since

$$\tau_t = 1 - \frac{1}{\eta_m(1+f)}\frac{\tau_r}{\tau_\lambda}[\tau_c - 1 + \alpha(\tau_f - 1)]$$

then

$$\frac{\partial\tau_t}{\partial\alpha} = -\frac{\tau_r(\tau_f - 1)}{\eta_m\tau_\lambda(1+f)} \tag{viii}$$

Combining Eqs. (ii), (vii), and (viii) yields

$$\left(\frac{V_9}{V_0}\right)_{\alpha^*} = \frac{1}{2\eta_m(\tau_r - 1)}\frac{\tau_r(\tau_f - 1)}{V_{19}/V_0 - 1}\left(1 + \frac{1 - e_t}{e_t}\frac{\tau_t^{-1/e_t}}{\Pi}\right)$$

An expression for τ_t is obtained by squaring the preceding equation, substituting for $(V_9/V_0)^2$ by using Eq. (vi), and then solving for the first τ_t within parentheses on the right side of Eq. (vi). The resulting expression for the turbine temperature ratio τ_t^* corresponding to the optimum bypass ratio α^* is

$$\tau_t^* = \frac{\tau_t^{-(1-e_t)/e_t}}{\Pi} + \frac{1}{\tau_\lambda(\tau_r - 1)}\left[\frac{1}{2\eta_m}\frac{\tau_r(\tau_f - 1)}{V_{19}/V_0 - 1}\left(1 + \frac{1 - e_t}{e_t}\frac{\tau_t^{-1/e_t}}{\Pi}\right)\right]^2 \tag{7.57}$$

Because Eq. (7.57) is an equation for τ_t^* in terms of itself, in addition to other known values, an iterative solution is required. A starting value of τ_t^*, denoted by τ_{ti}^*, is obtained by solving Eq. (7.57) for the case when $e_t = 1$, which gives

$$\tau_{ti}^* = \frac{1}{\Pi} + \frac{1}{\tau_\lambda(\tau_r - 1)}\left[\frac{1}{2\eta_m}\frac{\tau_r(\tau_f - 1)}{V_{19}/V_0 - 1}\right]^2 \tag{7.58}$$

This starting value can be substituted into the right-hand side of Eq. (7.57), yielding a new value of τ_t^*. This new value of τ_t^* is then substituted into Eq. (7.57), and another new value of τ_t^* is calculated. This process continues until the change in successive calculations of τ_t^* is less than some small number (say, 0.0001). Once the solution for τ_t^* is found, the optimum bypass ratio α^* is calculated by using Eq. (7.45), solved for α:

$$\alpha^* = \frac{\eta_m(1+f)\tau_\lambda(1-\tau_t^*) - \tau_r(\tau_c - 1)}{\tau_r(\tau_f - 1)} \tag{7.59}$$

When the optimum bypass ratio α^* is desired in calculating the parametric engine cycle performance, Eqs. (7.56), (7.57), (7.58), and (7.59) replace the equation for τ_t contained in the summary of equations and α^* is an output.

Example 7.8

Because the optimum-bypass-ratio turbofan cycle has two design variables, its performance with losses can be understood by performing a parametric analysis, plotting the results vs values of the design variables, and comparing results to the performance of the optimum-bypass-ratio ideal turbofan. Figures 7.17–7.19 are plots for optimum-bypass-ratio turbofan engines with the following input values (the same input used for the parametric analysis of the turbofan engine with losses in Example 7.7). The results for the ideal optimum-bypass-ratio turbofan engine cycle are shown in dashed lines. Unless shown otherwise, the Mach number, compressor pressure ratio, and fan pressure ratio are the values listed under *Baseline*:

$T_0 = 216.7$ K	$\pi_{d\max} = 0.98$	$e_c = 0.90$	*Baseline*
$\gamma_c = 1.4$	$\pi_b = 0.98$	$e_t = 0.91$	$M_0 = 0.9$
$c_{pc} = 1.004$ kJ/(kg · K)	$\pi_n = \pi_{fn} = 0.98$	$e_f = 0.88$	$\pi_c = 24$
$\gamma_t = 1.35$	$\eta_b = 0.99$	$h_{PR} = 42{,}800$ kJ/kg	$\pi_f = 2$
$c_{pt} = 1.096$ kJ/(kg · K)	$\eta_m = 0.98$	$T_{t4} = 1670$ K	
$\dfrac{P_0}{P_9} = 1$	$\dfrac{P_0}{P_{19}} = 1$		

Figures 7.17a and 7.17b show the following characteristics of the optimum bypass-ratio turbofan engine:

1) The compressor pressure ratio has very little effect on the specific thrust.
2) Increasing the fan pressure ratio increases the specific thrust.
3) The optimum bypass ratio increases with π_c and decreases with π_f.
4) Specific fuel consumption decreases with increasing π_c.
5) Specific fuel consumption increases with increasing π_f.

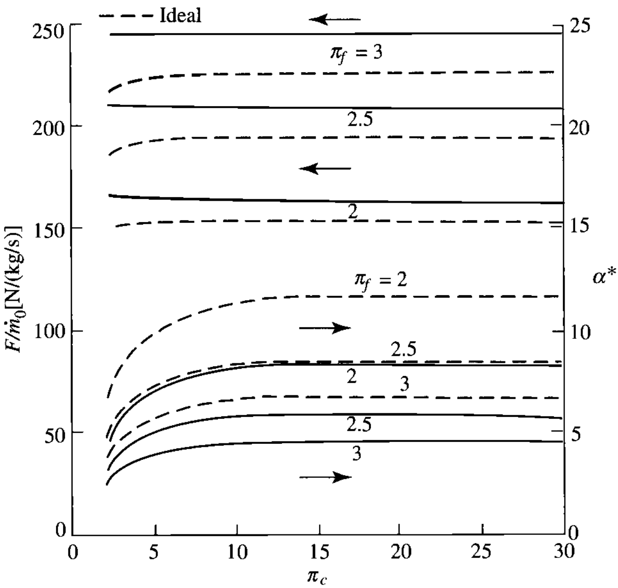

Fig. 7.17a Optimum-bypass-ratio turbofan engine vs π_c: specific thrust and optimum bypass ratio.

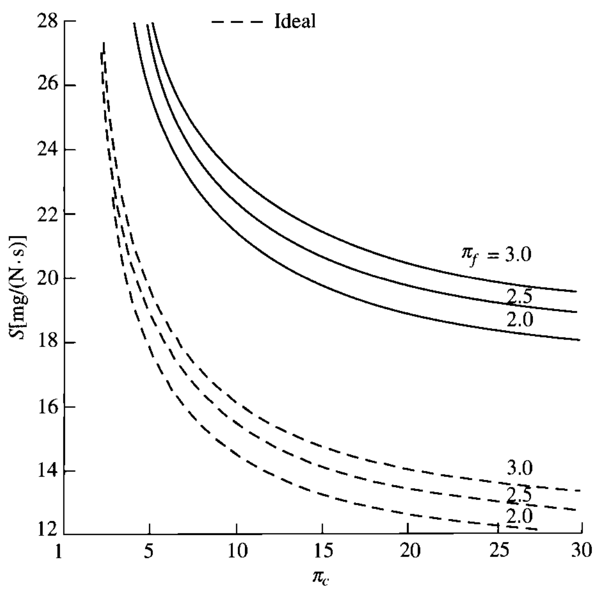

Fig. 7.17b Optimum-bypass-ratio turbofan engine vs π_c: thrust-specific fuel consumption.

The effect of flight Mach number on the performance of the optimum-bypass-ratio turbofan engine as shown in Figs. 7.18a and 7.18b has the following characteristics:

1) The specific thrust decreases with Mach number up to a Mach number of about 1.5.
2) Increasing the fan pressure ratio increases the specific thrust.
3) The optimum bypass ratio decreases with increasing M_0 and π_f.
4) The optimum turbofan is a turbojet engine at a Mach number of about 2.5.
5) Specific fuel consumption increases with increasing M_0 and π_f.

Figures 7.19a and 7.19b show the following characteristics of the optimum-bypass-ratio turbofan engine with respect to fan pressure ratio and flight Mach number:

1) Increasing the fan pressure ratio increases the specific thrust.
2) Increasing the flight Mach number decreases the specific thrust.
3) The optimum bypass ratio decreases with π_f and increases with M_0.
4) Specific fuel consumption increases with increasing π_f.
5) Specific fuel consumption increases with increasing M_0.

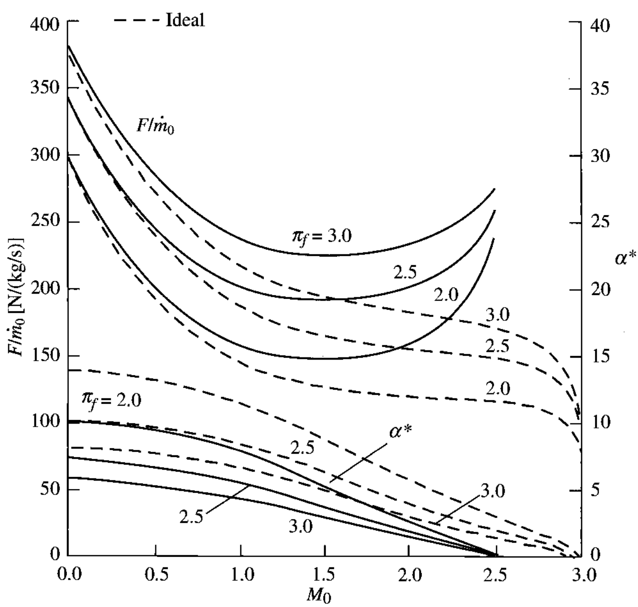

Fig. 7.18a Optimum-bypass-ratio turbofan engine vs Mach number: specific thrust and optimum bypass ratio.

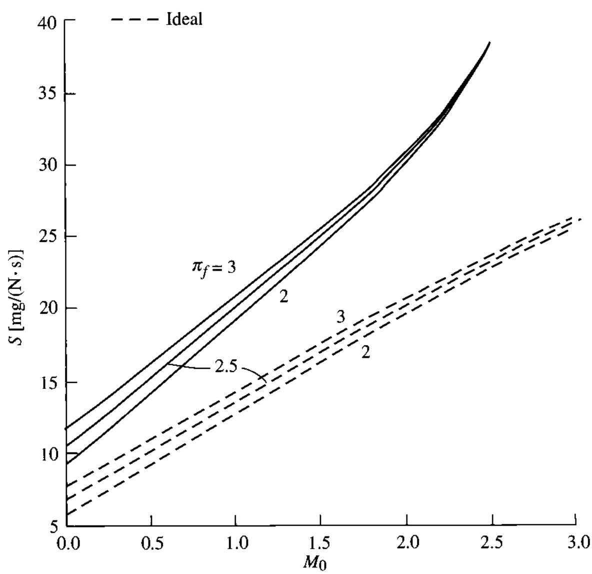

Fig. 7.18b Optimum-bypass-ratio turbofan engine vs Mach number: thrust-specific fuel consumption.

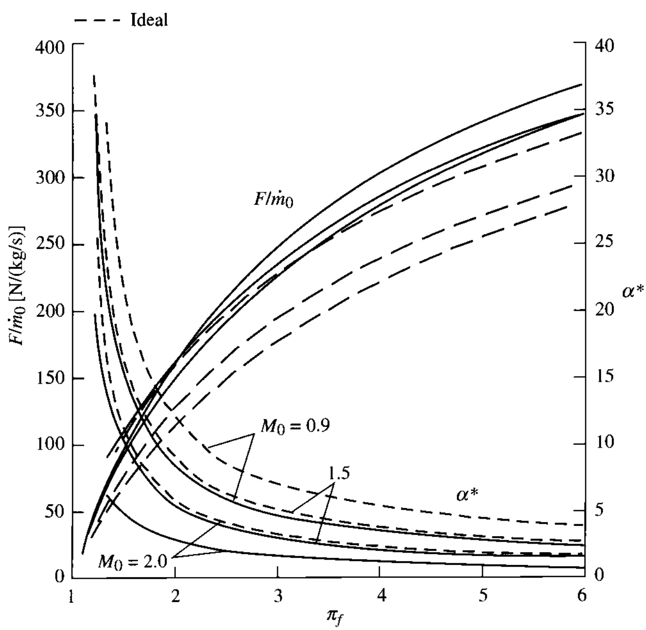

Fig. 7.19a Optimum-bypass-ratio turbofan engine vs τ_f: specific thrust and optimum bypass ratio.

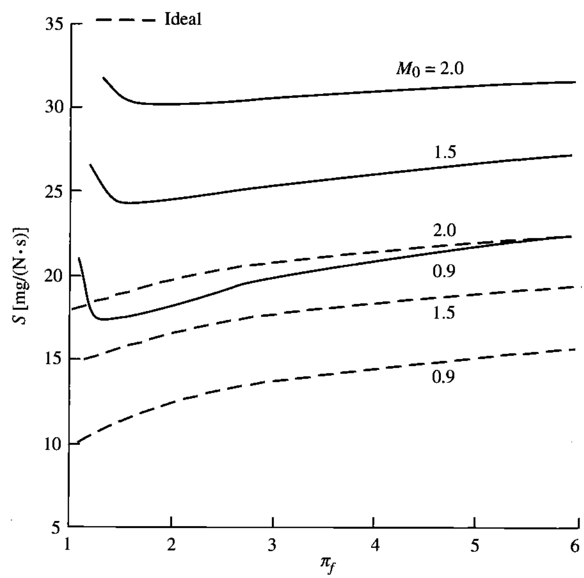

Fig. 7.19b Optimum-bypass-ratio turbofan engine vs π_f: thrust specific fuel consumption.

Problems

7.1 Develop a set of equations for parametric analysis of a ramjet engine with losses. Calculate the performance of a ramjet with losses over a Mach number range of 1 to 3 for the following input data:

$$\begin{array}{llll} \pi_{d\max} = 0.95 & T_0 = 217 \text{ K} & \gamma_c = 1.4 & c_{pc} = 1.004 \text{ kJ/(kg} \cdot \text{K)} \\ \pi_b = 0.94 & \eta_b = 0.96 & \gamma_t = 1.3 & c_{pt} = 1.235 \text{ kJ/(kg} \cdot \text{K)} \\ \pi_n = 0.95 & \dfrac{P_0}{P_9} = 1 & T_{14} = 1800 \text{ K} & h_{PR} = 42{,}800 \text{ kJ/kg} \end{array}$$

Compare your results to those obtained from the PARA computer program.

7.2 Why are the polytropic efficiencies used for the fans, compressors, and turbines in parametric engine cycle analysis rather than the isentropic efficiencies?

7.3 Calculate and compare the performance of turbojet engines with the basic data of Example 7.1 for components with technology level 2 values in Table 6.2 (assume cooled turbine and the same diffuser and nozzle values as in Example 7.1). Comment on the changes in engine performance.

7.4 Using the PARA computer program, compare the performance of turbojet engines with the basic data of Example 7.1 for the polytropic efficiencies of component technology levels 1, 2, 3, and 4 in Table 6.2 (assume uncooled turbine). Comment on the improvements in engine performance.

7.5 Using the PARA computer program, compare the performance of turbojet engines with the basic data of Example 7.3 for the polytropic efficiencies of component technology levels 1, 2, 3, and 4 in Table 6.2 (assume uncooled turbine). Comment on the changes in optimum compressor pressure ratio and improvements in engine performance.

7.6 Using the PARA computer program, find the range of compressor pressure ratios that give turbojet engines with specific thrust greater than 88 lbf/(lbm/s) and thrust specific fuel consumption below 1.5 (lbm/h)/lbf at $M_0 = 1.5$, $T_0 = 390°\text{R}$, and component performance of technology level 3 in Table 6.2 (assume type C diffuser, cooled turbine, and type F nozzle). Determine the compressor pressure ratio giving maximum specific thrust. Assume $\eta_m = 0.99$, $\gamma_c = 1.4$, $c_{pc} = 0.24$ Btu/(lbm · °R), $\gamma_t = 1.3$, $c_{pt} = 0.296$ Btu/(lbm · °R), $h_{PR} = 18{,}400$ Btu/lbm, and $P_0/P_9 = 1$.

7.7 Using the PARA computer program, find the range of compressor pressure ratios that give turbojet engines with specific thrust greater than 950 N/(kg/s) and thrust specific fuel consumption below 40 (mg/s)/N at $M_0 = 0.9$, $T_0 = 216.7$ K, and component performance of technology level 3 in Table 6.2 (assume type C diffuser, cooled turbine, and type F nozzle). Determine the compressor pressure ratio giving maximum specific thrust. Assume $\eta_m = 0.99$, $\gamma_c = 1.4$, $c_{pc} = 1.004$ kJ/(kg · K), $\gamma_t = 1.3$, $c_{pt} = 1.239$ kJ/(kg · K), $h_{PR} = 42{,}800$ kJ/kg, and $P_0/P_9 = 1$.

7.8 For a single-spool turbojet engine with losses, determine the compressor exit T_t and P_t, the turbine exit T_t and P_t, and the nozzle exit Mach number M_9 for the following input data:

$M_0 = 0.8$	$\pi_c = 9$	$T_{t4} = 1780$ K	$h_{PR} = 42{,}800$ kJ/kg
$P_0 = 29.92$ kPa	$T_0 = 229$ K	$\gamma_c = 1.4$	$c_{pc} = 1.004$ kJ/(kg · K)
$\pi_{d\max} = 0.95$	$\pi_b = 0.904$	$\gamma_t = 1.3$	$c_{pt} = 1.239$ kJ/(kg · K)
$e_c = 0.85$	$e_t = 0.88$	$\eta_b = 0.99$	$\eta_m = 0.98$
$\pi_n = 0.98$	$\dfrac{P_0}{P_9} = 0.8$		

Compare your results with those obtained from the PARA computer program.

7.9 Products of combustion enter the afterburner (station 6) at a rate of 230 lbm/s with the following properties: $T_{t6} = 1830°\text{R}$, $P_{t6} = 38$ psia, $M_6 = 0.4$, $\gamma = 1.33$, $c_P = 0.276$ Btu/(lbm · °R), and $R = 53.34$ ft · lbf/

(lbm · °R). Assume a calorically perfect gas and $\eta_{AB} = 0.95$.
(a) Determine the flow area at station 6 in square feet.
(b) With the afterburner off, determine the area (ft^2) of the exhaust nozzle's choked throat (station 8) for $P_{t8}/P_{t6} = 0.97$.
(c) With the afterburner on, determine the afterburner fuel flow rate (lbm/s) and the area (ft^2) of the exhaust nozzle's choked throat (station 8) for $P_{t8}/P_{t6} = 0.94$ and $T_{t8} = 3660°R$. *Assume* that the gas leaving the operating afterburner is a calorically perfect gas with $\gamma = 1.3$, $c_p = 0.297$ Btu/(lbm · °R), and the same gas constant. Also assume the properties at station 6 do not change and $h_{PR} = 18{,}400$ Btu/lbm.

7.10 Calculate and compare the performance of afterburning turbojet engines with the basic data of Example 7.5 but with combustion temperatures of level 4 in Table 6.2 for compressor pressure ratios of 4, 8, and 12. Comment on the improvements in engine performance.

7.11 Using the PARA computer program, find the range of compressor pressure ratios that give afterburning turbojet engines with specific thrust greater than 118 lbf/(lbm/s) and thrust specific fuel consumption below 1.7 (lbm/h)/lbf at $M_0 = 1.5$, $T_0 = 390°R$, and component performance of technology level 3 in Table 6.2 (assume type C diffuser, cooled turbine, and type F nozzle). Determine the compressor pressure ratio giving maximum specific thrust. Assume $\eta_m = 0.99$, $\gamma_c = 1.4$, $c_{pc} = 0.24$ Btu/(lbm · °R), $\gamma_t = 1.3$, $c_{pc} = 0.296$ Btu/(lbm · °R), $\gamma_{AB} = 1.3$, $c_{pAB} = 0.296$ Btu/(lbm · °R), $h_{PR} = 18{,}400$ Btu/lbm, and $P_0/P_9 = 1$.

7.12 Using the PARA computer program, find the range of compressor pressure ratios that give afterburning turbojet engines with specific thrust greater than 1250 N/(kg/s) and thrust specific fuel consumption below 45 (mg/s)/N at $M_0 = 0.9$, $T_0 = 216.7$ K, and component performance of technology level 3 in Table 6.2 (assume type C diffuser, cooled turbine, and type F nozzle). Determine the compressor pressure ratio giving maximum specific thrust. Assume $\eta_m = 0.99$, $\gamma_c = 1.4$, $c_{pt} = 1.004$ kJ/(kg · K), $\gamma_t = 1.3$, $c_{pt} = 1.239$ kJ/(kg · K), $\gamma_{AB} = 1.3$, $c_{pAB} = 0.239$ kJ/(kg · K), $h_{PR} = 42{,}800$ kJ/kg, and $P_0/P_9 = 1$.

7.13 Using the PARA computer program, calculate and compare the performance of afterburning turbojet engines with the basic data of Example 7.5 for the different combustion temperatures and component technologies of levels 2, 3, and 4 in Table 6.2 (assume cooled turbine, type B diffuser, and type F nozzle). Comment on the improvements in engine performance.

7.14 Show that the propulsive efficiency and thermal efficiency of a turbofan engine with separate exhausts are given by Eqs. (7.50) and (7.51), respectively.

7.15 Calculate the performance of a turbofan engine with the basic data of Example 7.6 but with a fan pressure ratio of 1.65 and a bypass ratio of 10. Comment on the improvement in engine performance. Compare your results to those of the PARA computer program.

7.16 Using the PARA computer program, compare the performance of turbofan engines with the basic data of Example 7.6 for the polytropic efficiencies of component technology levels 2, 3, and 4 in Table 6.2 (assume cooled turbine, type A diffuser, and type D nozzle). Comment on the improvement in engine performance.

7.17 Using the PARA computer program, find the range of compressor pressure ratios and fan pressure ratios that give optimum-bypass-ratio, separate-exhaust turbofan engines with specific thrust greater than 13 lbf/(lbm/s) and thrust specific fuel consumption below 1.0 (lbm/h)/lbf at $M_0 = 0.9$, $T_0 = 390°\text{R}$, and component performance of technology level 2 in Table 6.2 (assume type A diffuser, uncooled turbine, and type D nozzle). Assume $\eta_m = 0.99$, $\gamma_c = 1.4$, $c_{pc} = 0.24$ Btu/(lbm · °R), $\gamma_t = 1.3$, $c_{pt} = 0.296$ Btu/(lbm · °R), $h_{PR} = 18{,}400$ Btu/lbm, and $P_0/P_9 = 1$.

7.18 Using the PARA computer program, find the range of compressor pressure ratios and fan pressure ratios that give optimum-bypass-ratio, separate-exhaust turbofan engines with specific thrust greater than 130 N/(kg/s) and thrust specific fuel consumption below 28 (mg/s)/N at $M_0 = 0.8$, $T_0 = 216.7$ K, and component performance of technology level 2 in Table 6.2 (assume type A diffuser, uncooled turbine, and type D nozzle). Assume $\eta_m = 0.99$, $\gamma_c = 1.4$, $c_{pc} = 1.004$ kJ/(kg · K), $\gamma_t = 1.3$, $c_{pt} = 1.239$ kJ/(kg · K), $h_{PR} = 42{,}800$ kJ/kg, and $P_0/P_9 = 1$.

7.19 Calculate the performance of an optimum-bypass-ratio turbofan engine with the basic data of Example 7.8 but with a compressor pressure ratio of 30 and fan pressure ratio of 1.7. Compare your results to those of the PARA computer program.

7.20 Using the PARA computer program, compare the performance of optimum-bypass-ratio turbofan engines with the basic data of Example 7.8 for the polytropic efficiencies of component technology levels 2, 3, and 4 in Table 6.2 (assume cooled turbine, type A diffuser, and type D nozzle). Comment on the improvement in engine performance.

7.21 A stationary gas turbine engine with regeneration is shown in Fig. P7.1. The effectiveness of a regenerator η_{rg} is defined by

$$\eta_{rg} = \frac{T_{t3.5} - T_{t3}}{T_{t5} - T_{t3}}$$

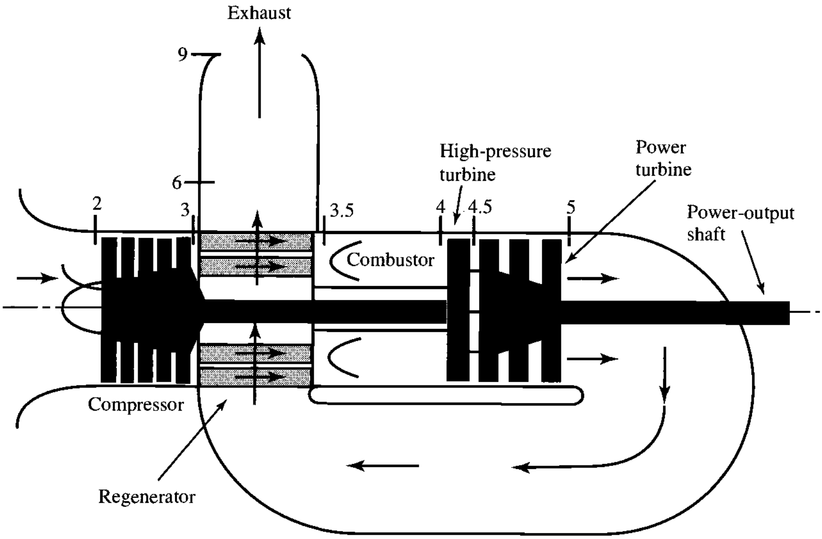

Fig. P7.1

The total pressure ratios across the cold and hot gas paths of the regenerator are defined by

$$\pi_{\text{rg cold}} = \frac{P_{t3.5}}{P_{t3}} \qquad \pi_{\text{rg hot}} = \frac{P_{t6}}{P_{t5}}$$

Using these definitions and others, develop a set of equations for parametric analysis of this turboshaft engine with regeneration and losses.

Problems for Supporting Material

SM7.1 Calculate the performance of an afterburning mixed-flow turbofan engine with the basic data of Example SM7.2 at $M_0 = 0.9$ for a compressor pressure ratio of 30 and a fan pressure ratio of 4. Compare your results to those of the PARA computer program.

SM7.2 Using the PARA computer program, find the range of compressor pressure ratios and corresponding fan pressure ratios that give mixed-flow turbofan engines of 0.5 bypass ratio a specific thrust greater than 55 lbf/(lbm/s) and thrust specific fuel consumption below 1.3 (lbm/h)/lbf at $M_0 = 1.8$, $T_0 = 390°R$, and component performance of technology level 3 in Table 6.2 (assume type C diffuser, cooled turbine, and type F nozzle). Assume $\eta_m = 0.99$, $\gamma_c = 1.4$, $c_{pc} = 0.24$ Btu/(lbm · °R), $\gamma_t = 1.3$, $c_{pt} = 0.296$ Btu/(lbm · °R), $M_6 = 0.5$, $\pi_{M\,\max} = 0.95$, $h_{PR} = 18{,}400$ Btu/lbm, and $P_0/P_9 = 1$.

SM7.3 Using the PARA computer program, find the range of compressor pressure ratios and corresponding fan pressure ratios that give mixed-flow turbofan engines of 0.4 bypass ratio a specific thrust greater than 550 N/(kg/s) and thrust specific fuel consumption below 39 (mg/s)/N at $M_0 = 2.0$, $T_0 = 216.7$ K, and component performance of technology level 3 in Table 6.2 (assume type C diffuser, cooled turbine, and type F nozzle). Assume $\eta_m = 0.99$, $\gamma_c = 1.4$, $c_{pc} = 1.004$ kJ/(kg · K), $\gamma_t = 1.3$, $c_{pt} = 1.239$ kJ/(kg · K), $M_6 = 0.5$, $\pi_{M\max} = 0.95$, $h_{PR} = 42{,}800$ kJ/kg, and $P_0/P_9 = 1$.

SM7.4 Using the PARA computer program, find the range of compressor pressure ratios and corresponding fan pressure ratios that give afterburning mixed-flow turbofan engines of 0.5 bypass ratio a specific thrust greater than 105 lbf/(lbm/s) and thrust specific fuel consumption below 1.845 (lbm/h)/lbf at $M_0 = 1.8$, $T_0 = 390°$R, and component performance of technology level 3 in Table 6.2 (assume type C diffuser, cooled turbine, and type F nozzle). Assume $\eta_m = 0.99$, $\gamma_c = 1.4$, $c_{pc} = 0.24$ Btu/(lbm · °R), $\gamma_t = 1.3$, $c_{pt} = 0.296$ Btu/(lbm · °R), $\gamma_{AB} = 1.3$, $c_{pAB} = 0.296$ Btu/(lbm · °R), $M_6 = 0.5$, $\pi_{M\max} = 0.95$, $h_{PR} = 18{,}400$ Btu/lbm, and $P_0/P_9 = 1$.

SM7.5 Using the PARA computer program, find the range of compressor pressure ratios and corresponding fan pressure ratios that give afterburning mixed-flow turbofan engines of 0.4 bypass ratio a specific thrust greater than 1000 N/(kg/s) and thrust specific fuel consumption below 52.25 (mg/s)/N at $M_0 = 2.0$, $T_0 = 216.7$ K, and component performance of technology level 3 in Table 6.2 (assume type C diffuser, cooled turbine, and type F nozzle). Assume $\eta_m = 0.99$, $\gamma_c = 1.4$, $c_{pc} = 1.004$ kJ/(kg · K), $\gamma = 1.3$, $c_{pt} = 1.239$ kJ/(kg · K), $\gamma_{AB} = 1.3$, $c_{pAB} = 1.239$ kJ/(kg · K), $M_6 = 0.5$, $\pi_{M\max} = 0.95$, $h_{PR} = 42{,}800$ kJ/kg, $M_6 = 0.5$, and $P_0/P_9 = 1$.

SM7.6 Using the PARA computer program, compare the performance of afterburning mixed-flow turbofan engines with the basic data of Example SM7.2 at $M_0 = 0.9$, $\pi_c = 24$, and $\pi_f = 3.5$ for the different combustion temperatures and component technologies of levels 2, 3, and 4 in Table 6.2 (assume cooled turbine, type C diffuser, and type F nozzle). Also assume the same γ, c_p, η_m, and $\pi_{M\max}$. Comment on the improvement in engine performance.

SM7.7 For the mixed-flow turbofan engine with the bypass ratio specified, show that the following functional iteration equation for the fan temperature ratio with matched total pressures entering the mixer can be obtained from Eqs. (SM7.31) and (SM7.32):

$$(\tau_f)_{i+1} = \frac{\tau_r[\alpha - (\tau_c - 1)] + \eta_m(1+f)\tau_\lambda}{\tau_r\alpha + [\eta_m(1+f)\tau_\lambda/(\pi_c\pi_b)^{(\gamma_t-1)e_t/\gamma_t}]^{(\gamma_t-1)/(\gamma_c-1)(\gamma_c/\gamma_t)e_te_f-1}}$$

with the first value of the fan temperature ratio given by

$$(\tau_f)_1 = \frac{\tau_r[\alpha - (\tau_c - 1)] + \eta_m(1+f)\tau_\lambda}{\tau_r\alpha + \eta_m(1+f)\tau_\lambda/(\pi_c\pi_b)^{(\gamma_t-1)e_t/\gamma_t}}$$

SM7.8 Calculate the performance of a turboprop engine with the basic data of Example SM7.4 at a compressor pressure ratio of 20 and turbine temperature ratio of 0.5. Compare your results to those of Example SM7.4 and the PARA computer program.

SM7.9 Using the PARA computer program, find the range of compressor pressure ratios that give turboprop engines with optimum turbine temperature ratio τ_t^* a specific thrust greater than 120 lbf/(lbm/s) and thrust specific fuel consumption below 0.8 (lbm/h)/lbf at $M_0 = 0.7$, $T_0 = 447°\text{R}$, and component performance of technology level 2 in Table 6.2 (assume type A diffuser, uncooled turbine, and type D nozzle). Assume $\eta_{\text{prop}} = 0.83$, $\eta_g = 0.99$, $\eta_{mH} = 0.99$, $\gamma_{mL} = 0.99$, $\gamma_c = 1.4$, $c_{pc} = 0.24\ \text{Btu/(lbm} \cdot °\text{R)}$, $\gamma_t = 1.35$, $c_{pt} = 0.265\ \text{Btu/(lbm} \cdot °\text{R)}$, and $h_{PR} = 18{,}400\ \text{Btu/lbm}$.

SM7.10 Using the PARA computer program, find the range of compressor pressure ratios that give turboprop engines with optimum turbine temperature ratio τ_t^* a specific thrust greater than 1300 N/(kg/s) and thrust specific fuel consumption below 18 (mg/s)/N at $M_0 = 0.6$, $T_0 = 250$ K, and component performance of technology level 2 in Table 6.2 (assume type A diffuser, uncooled turbine, and type D nozzle). Assume $\eta_{\text{prop}} = 0.83$, $\eta_g = 0.99$, $\eta_{mH} = 0.995$, $\gamma_{mL} = 0.995$, $\gamma_e = 1.4$, $c_{pc} = 1.004\ \text{kJ/(kg} \cdot \text{K)}$, $\gamma_t = 1.35$, $c_{pt} = 1.108\ \text{kJ/(kg} \cdot \text{K)}$, and $h_{PR} = 42{,}800\ \text{kJ/kg}$.

SM7.11 Using the PARA computer program, compare the performance of turboprop engines with the basic data of Example SM7.4 with component technologies of levels 1, 2, 3, and 4 in Table 6.2. Comment on the improvement in engine performance.

Gas Turbine Design Problems

7.D1 You are to determine the range of compressor pressure ratios and bypass ratios for turbofan engines with losses that best meet the design requirements for the hypothetical passenger aircraft HP-1.

Hand-Calculate Performance with Losses (HP-1 Aircraft). Using the parametric cycle analysis equations for a turbofan engine with losses and component technology level 4 in Table 6.2 (assume cooled turbine, type A diffuser, and type D nozzle) with $T_{t4} = 1560$ K, hand-calculate the specific thrust and thrust specific fuel consumption for a turbofan engine with a compressor pressure ratio of 36, fan pressure ratio of

1.8, and bypass ratio of 10 at the 0.83 Mach and 11-km altitude cruise condition. Assume $\gamma_c = 1.4$, $c_{pc} = 1.004$ kJ/(kg · K), $\gamma_t = 1.3$, $c_{pt} = 1.235$ kJ/(kg · K), $h_{PR} = 42{,}800$ kJ/kg, and $\eta_m = 0.99$. Compare your answers to results from the parametric cycle analysis program PARA and Design Problem 5.D1.

Computer-Calculated Performance with Losses (HP-1 Aircraft). For the 0.83 Mach and 11-km altitude cruise condition, determine the performance available from turbofan engines with losses. This part of the analysis is accomplished by using the PARA computer program with component technology level 4 in Table 6.2 (assume cooled turbine, type A diffuser, and type D nozzle) and $T_{t4} = 1560$ K. Specifically, you are to vary the compressor pressure ratio from 20 to 40 in increments of 2. Fix the fan pressure ratio at your assigned value of______. Evaluate bypass ratios of 4, 6, 8, 10, 12, and the optimum value. Assume $\gamma_c = 1.4$, $c_{pc} = 1.004$ kg/(kg · K), $\gamma_t = 1.3$, $c_{pt} = 1.235$ kJ/(kg · K), $h_{PR} = 42{,}800$ kJ/kg, and $\eta_m = 0.99$.

Calculate Minimum Specific Thrust at Cruise (HP-1 Aircraft). You can calculate the minimum uninstalled specific thrust at cruise based on the following information:

1) The thrust of the two engines must be able to offset drag at 0.83 Mach and 11-km altitude and have enough excess thrust for P_s of 1.5 m/s. Determine the required installed thrust to attain the cruise condition, using Eq. (1.28). Assuming $\phi_{\text{inlet}} + \phi_{\text{noz}} = 0.02$, determine the required uninstalled thrust.

2) Determine the maximum mass flow into the 2.2-m-diam inlet for the 0.83 Mach and 11-km altitude flight condition, using the equation given in the background section for this design problem in Chapter 1.

3) Using the results of steps 1 and 2, calculate the minimum uninstalled specific thrust at cruise.

4) Perform steps 2 and 3 for inlet diameters of 2.5, 2.75, 3.0, 3.25, and 3.5 m.

Select Promising Engine Cycles (HP-1 Aircraft). Plot thrust specific fuel consumption vs specific thrust (thrust per unit mass flow) for the engines analyzed in the preceding. Plot a curve for each bypass ratio and cross-plot the values of the compressor pressure ratio (see Fig. P5.D1). The result is a carpet plot (a multivariable plot) for the cruise condition. Now draw a dashed horizontal line on the carpet plot corresponding to the maximum allowable uninstalled thrust specific consumption ($S_{\max}$) for the cruise condition (determined in the Chapter 1 portion of this design problem). Draw a dashed vertical line for each minimum uninstalled specific thrust determined in the preceding. Your carpet plots will look similar to the example shown in Fig. P5.D1. What ranges of bypass ratio and compressor pressure ratio look most promising? Compare to the results of Design Problem 5.D1.

7.D2 You are to determine the ranges of compressor pressure ratio and bypass ratio for mixed-flow turbofan engines with losses that best meet the design requirements for the hypothetical fighter aircraft HF-1.

Hand-Calculate Performance with Losses (HF-1 Aircraft). Using the parametric cycle analysis equations for a mixed-flow turbofan engine with losses and component technology level 4 in Table 6.2 (assume cooled turbine, type C diffuser, and type F nozzle) with $T_{t4} = 3250°R$, hand-calculate the specific thrust and thrust specific fuel consumption for an ideal turbofan engine with a compressor pressure ratio of 25 and bypass ratio of 0.5 at the 1.6-Mach and 40-kft altitude supercruise condition. Because the bypass ratio is given, you will need to use the system of equations given in Problem 7.27 to calculate the temperature ratio of the fan. Assume $\gamma_c = 1.4$, $c_{pc} = 0.240$ Btu/(lbm · °R), $\gamma_t = 1.3$, $c_{pt} = 0.296$ Btu/(lbm · °R), $h_{PR} = 18{,}400$ Btu/lbm, $M_6 = 0.4$, $\pi_{M\max} = 0.96$, and $\eta_m = 0.99$. Compare your answers to results from the parametric cycle analysis program PARA and Design Problem 5.D2.

Computer-Calculated Performance with Losses (HF-1 Aircraft). For the 1.6-Mach and 40-kft altitude supercruise condition, determine the performance available from mixed-flow turbofan engines with losses. This part of the analysis is accomplished by using the PARA computer program with component technology level 4 in Table 6.2 (assume cooled turbine, type C diffuser, and type F nozzle) and $T_{t4} = 3250°R$. Specifically, you are to vary the bypass ratio from 0.1 to 1.0 in increments of 0.05. Evaluate compressor pressure ratios of 16, 18, 20, 22, 24, and 28. Assume $\gamma_c = 1.4$, $c_{pc} = 0.240$ Btu/(lbm · °R), $\gamma_t = 1.3$, c_{pt}, $= 0.296$ Btu/(lbm · °R), $h_{PR} = 18{,}400$ Btu/lbm, $M_6 = 0.4$, $\pi_{M\max}$ $=0.96$, and $\eta_m = 0.99$.

Calculate Minimum Specific Thrust at Cruise (HF-1 Aircraft). You can calculate the minimum uninstalled specific thrust at supercruise based on the following information:

1) The thrust of the two engines must be able to offset drag at 1.6-Mach number and 40-kft altitude and 92% of takeoff weight. Assuming $\phi_{\text{inlet}} + \phi_{\text{noz}} = 0.05$, determine the required uninstalled thrust for each engine.

2) The maximum mass flow into a 5-ft^2 inlet for the 1.6-Mach number and 40-kft altitude flight condition is $\dot{m} = \rho AV = \sigma\rho_{\text{ref}}AMa = (0.2471 \times 0.07647)(5)(1.6 \times 0.8671) \times 1116) = 146.3$ lbm/s.

3) Using the results of steps 1 and 2, calculate the minimum uninstalled specific thrust at supercruise.

Select Promising Engine Cycles (HF-1 Aircraft). Plot thrust specific fuel consumption vs specific thrust (thrust per unit mass flow) for the engines analyzed in the preceding. Plot a curve for each bypass ratio, and cross-plot the values of compressor pressure ratio (see Fig. P5.D2). The result is a carpet plot (a multivariable plot) for the supercruise condition. Now draw a dashed horizontal line on the carpet plot corresponding to the maximum allowable uninstalled thrust specific fuel consumption (S_{max}) for the cruise condition (determined in the Chapter 1 portion of this design problem). Draw a dashed vertical line for the minimum uninstalled specific thrust determined in the preceding. Your carpet plots will look similar to the example shown in Fig. P5.D2. What ranges of bypass ratio and compressor pressure ratio look most promising? Compare to the results of Design Problem 5.D2.