Figures 7.15a and 7.15b show the influence of fan pressure ratio and bypass ratio on engine performance. An optimum fan pressure ratio still exists for the turbofan with losses, and the value of the optimum fan pressure ratio is much lower than that for the ideal turbofan.

Figures 7.16a and 7.16b show the variation in specific thrust and thrust specific fuel consumption with bypass ratio and fan pressure ratio. An optimum bypass ratio still exists for the turbofan with losses, and the value of the optimum bypass ratio is much less than that for the ideal turbofan.

7.4.4 Optimum Bypass Ratio α^*

As was true for the turbofan with no losses, we may obtain an expression that allows us to determine the bypass ratio α^* that leads to minimum thrust specific fuel consumption. For a given set of such prescribed variables (τ_r, π_c, π_f, τ_λ, V_0), we may locate the minimum S by taking the partial derivative of S with respect to the bypass ratio α. We consider the case where the exhaust pressures of both the fan stream and the core stream equal the ambient pressure $P_0 = P_9 = P_{19}$. Because the fuel/air ratio is not a function of bypass ratio, we have

$$S = \frac{f}{(1+\alpha)(F/\dot{m}_0)}$$

$$\frac{\partial S}{\partial \alpha} = \frac{\partial}{\partial \alpha}\left[\frac{f}{(1+\alpha)(F/\dot{m}_0)}\right] = 0$$

$$\frac{\partial S}{\partial \alpha} = \frac{-f}{[(1+\alpha)(F/\dot{m}_0)]^2}\frac{\partial}{\partial \alpha}\left[(1+\alpha)\left(\frac{F}{\dot{m}_0}\right)\right] = 0$$

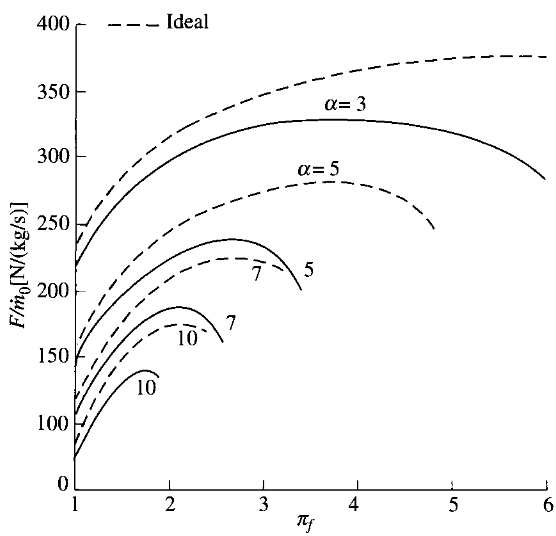

Fig. 7.15a Turbofan engine with losses vs fan pressure ratio: specific thrust.

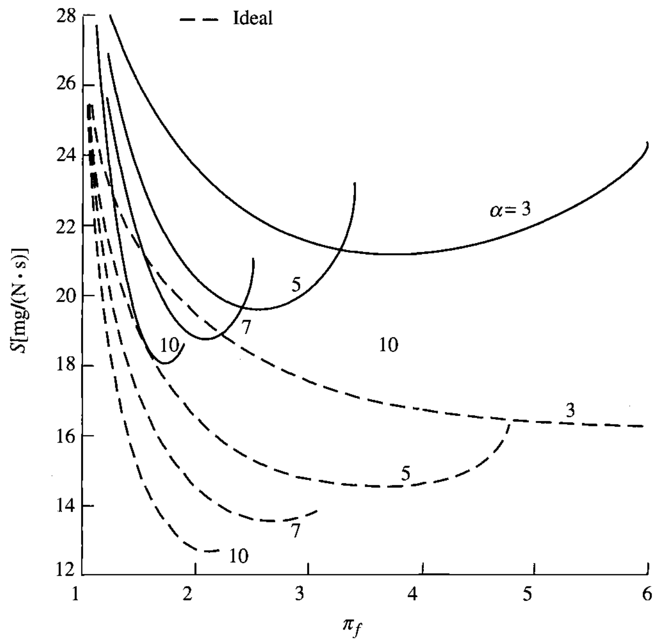

Fig. 7.15b Turbofan engine with losses vs fan pressure ratio: thrust-specific fuel consumption.

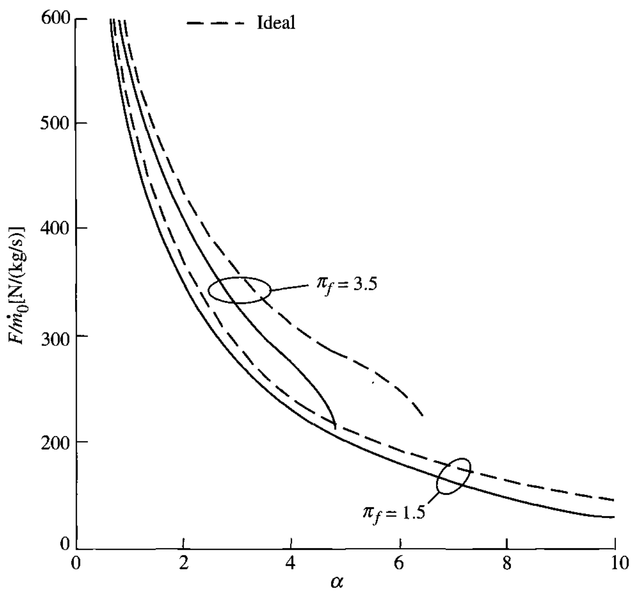

Fig. 7.16a Turbofan engine with losses vs bypass ratio: specific thrust.

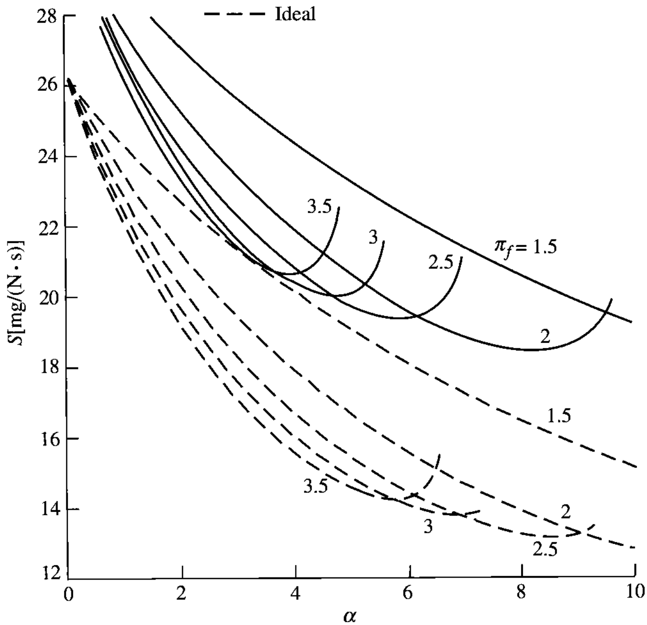

Fig. 7.16b Turbofan engine with losses vs bypass ratio: thrust-specific fuel consumption.

Thus $\partial S/\partial\alpha = 0$ is satisfied by

$$\frac{\partial}{\partial\alpha}\left[\frac{g_c}{V_0}(1+\alpha)\left(\frac{F}{\dot{m}_0}\right)\right] = 0$$

where

$$\frac{g_c}{V_0}(1+\alpha)\left(\frac{F}{\dot{m}_0}\right) = (1+f)\left(\frac{V_9}{V_0} - 1\right) + \alpha\left(\frac{V_{19}}{V_0} - 1\right)$$

Then the optimum bypass ratio is given by the following expression:

$$\begin{aligned}\frac{\partial}{\partial\alpha}&\left[(1+f)\left(\frac{V_9}{V_0} - 1\right) + \alpha\left(\frac{V_{19}}{V_0} - 1\right)\right] \\ &= (1+f)\frac{\partial}{\partial\alpha}\left(\frac{V_9}{V_0}\right) + \frac{V_{19}}{V_0} - 1 = 0\end{aligned} \qquad \text{(i)}$$

However,

$$\frac{1}{2V_9/V_0}\frac{\partial}{\partial\alpha}\left[\left(\frac{V_9}{V_0}\right)^2\right]=\frac{\partial}{\partial\alpha}\left(\frac{V_9}{V_0}\right)$$

Thus Eq. (i) becomes

$$\left(\frac{V_9}{V_0}\right)_{\alpha^*}=-\frac{1+f}{2}\frac{\partial/\partial\alpha[(V_9/V_0)^2]}{V_{19}/V_0-1} \tag{ii}$$

Note that

$$\begin{aligned}\left(\frac{V_9}{V_0}\right)^2&=\frac{1}{M_0^2}\left(\frac{V_9}{a_0}\right)^2=\frac{1}{[2/(\gamma_c-1)](\tau_r-1)}\left(\frac{V_9}{a_0}\right)^2\\&=\frac{1}{[2/(\gamma_c-1)](\tau_r-1)}M_9^2\frac{\gamma_t R_t T_9}{\gamma_c R_c T_0}\end{aligned}$$

Using Eqs. (7.41) and (7.42), we have

$$\left(\frac{V_9}{V_0}\right)^2=\frac{\tau_\lambda\tau_t}{\tau_r-1}\left[1-\left(\frac{P_{t9}}{P_9}\right)^{-(\gamma_t-1)/\gamma_t}\right] \tag{iii}$$

where

$$\frac{P_{t9}}{P_9}=\pi_r\pi_d\pi_c\pi_b\pi_t\pi_n \tag{iv}$$

Combining Eqs. (iii) and (iv), we obtain

$$\left(\frac{V_9}{V_0}\right)^2=\frac{\tau_\lambda\tau_t}{\tau_r-1}\left[1-\frac{1}{\Pi(\pi_t)^{(\gamma_t-1)/\gamma_t}}\right] \tag{v}$$

where

$$\Pi=(\pi_r\pi_d\pi_c\pi_b\pi_n)^{(\gamma_t-1)/\gamma_t} \tag{7.56}$$

Noting that

$$\pi_t^{(\gamma_t-1)/\gamma_t}=\tau_t^{1/e_t}$$

we see that then Eq. (v) becomes

$$\left(\frac{V_9}{V_0}\right)^2 = \frac{\tau_\lambda}{\tau_r - 1}\left(\tau_t - \frac{1}{\Pi}\tau_t^{-(1-e_t)/e_t}\right) \tag{vi}$$

To evaluate the partial derivative of Eq. (ii), we apply the chain rule to Eq. (vi) as follows:

$$\frac{\partial}{\partial\alpha}\left[\left(\frac{V_9}{V_0}\right)^2\right] = \frac{\partial\tau_t}{\partial\alpha}\frac{\partial}{\partial\tau_t}\left[\left(\frac{V_9}{V_0}\right)^2\right]$$

$$= \frac{\partial\tau_t}{\partial\alpha}\frac{\tau_\lambda}{\tau_r - 1}\left(1 + \frac{1-e_t}{e_t}\frac{\tau_t^{-1/e_t}}{\Pi}\right) \tag{vii}$$

Since

$$\tau_t = 1 - \frac{1}{\eta_m(1+f)}\frac{\tau_r}{\tau_\lambda}[\tau_c - 1 + \alpha(\tau_f - 1)]$$

then

$$\frac{\partial\tau_t}{\partial\alpha} = -\frac{\tau_r(\tau_f - 1)}{\eta_m\tau_\lambda(1+f)} \tag{viii}$$

Combining Eqs. (ii), (vii), and (viii) yields

$$\left(\frac{V_9}{V_0}\right)_{\alpha^*} = \frac{1}{2\eta_m(\tau_r - 1)}\frac{\tau_r(\tau_f - 1)}{V_{19}/V_0 - 1}\left(1 + \frac{1-e_t}{e_t}\frac{\tau_t^{-1/e_t}}{\Pi}\right)$$

An expression for τ_t is obtained by squaring the preceding equation, substituting for $(V_9/V_0)^2$ by using Eq. (vi), and then solving for the first τ_t within parentheses on the right side of Eq. (vi). The resulting expression for the turbine temperature ratio τ_t^* corresponding to the optimum bypass ratio α^* is

$$\tau_t^* = \frac{\tau_t^{-(1-e_t)/e_t}}{\Pi} + \frac{1}{\tau_\lambda(\tau_r - 1)}\left[\frac{1}{2\eta_m}\frac{\tau_r(\tau_f - 1)}{V_{19}/V_0 - 1}\left(1 + \frac{1-e_t}{e_t}\frac{\tau_t^{-1/e_t}}{\Pi}\right)\right]^2 \tag{7.57}$$

Because Eq. (7.57) is an equation for τ_t^* in terms of itself, in addition to other known values, an iterative solution is required. A starting value of τ_t^*, denoted by τ_{ti}^*, is obtained by solving Eq. (7.57) for the case when $e_t = 1$, which gives

$$\tau_{ti}^* = \frac{1}{\Pi} + \frac{1}{\tau_\lambda(\tau_r - 1)}\left[\frac{1}{2\eta_m}\frac{\tau_r(\tau_f - 1)}{V_{19}/V_0 - 1}\right]^2 \tag{7.58}$$

This starting value can be substituted into the right-hand side of Eq. (7.57), yielding a new value of τ_t^*. This new value of τ_t^* is then substituted into Eq. (7.57), and another new value of τ_t^* is calculated. This process continues until the change in successive calculations of τ_t^* is less than some small number (say, 0.0001). Once the solution for τ_t^* is found, the optimum bypass ratio α^* is calculated by using Eq. (7.45), solved for α:

$$\alpha^* = \frac{\eta_m(1+f)\tau_\lambda(1-\tau_t^*) - \tau_r(\tau_c - 1)}{\tau_r(\tau_f - 1)} \tag{7.59}$$

When the optimum bypass ratio α^* is desired in calculating the parametric engine cycle performance, Eqs. (7.56), (7.57), (7.58), and (7.59) replace the equation for τ_t contained in the summary of equations and α^* is an output.

Example 7.8

Because the optimum-bypass-ratio turbofan cycle has two design variables, its performance with losses can be understood by performing a parametric analysis, plotting the results vs values of the design variables, and comparing results to the performance of the optimum-bypass-ratio ideal turbofan. Figures 7.17–7.19 are plots for optimum-bypass-ratio turbofan engines with the following input values (the same input used for the parametric analysis of the turbofan engine with losses in Example 7.7). The results for the ideal optimum-bypass-ratio turbofan engine cycle are shown in dashed lines. Unless shown otherwise, the Mach number, compressor pressure ratio, and fan pressure ratio are the values listed under *Baseline*:

$T_0 = 216.7$ K	$\pi_{d\max} = 0.98$	$e_c = 0.90$	*Baseline*
$\gamma_c = 1.4$	$\pi_b = 0.98$	$e_t = 0.91$	$M_0 = 0.9$
$c_{pc} = 1.004$ kJ/(kg · K)	$\pi_n = \pi_{fn} = 0.98$	$e_f = 0.88$	$\pi_c = 24$
$\gamma_t = 1.35$	$\eta_b = 0.99$	$h_{PR} = 42{,}800$ kJ/kg	$\pi_f = 2$
$c_{pt} = 1.096$ kJ/(kg · K)	$\eta_m = 0.98$	$T_{t4} = 1670$ K	
$\dfrac{P_0}{P_9} = 1$	$\dfrac{P_0}{P_{19}} = 1$		

Figures 7.17a and 7.17b show the following characteristics of the optimum bypass-ratio turbofan engine:

1) The compressor pressure ratio has very little effect on the specific thrust.
2) Increasing the fan pressure ratio increases the specific thrust.
3) The optimum bypass ratio increases with π_c and decreases with π_f.
4) Specific fuel consumption decreases with increasing π_c.
5) Specific fuel consumption increases with increasing π_f.