6
Component Performance

6.1 Introduction

In Chapter 5, we idealized the engine components and assumed that the working fluid behaved as a perfect gas with constant specific heats. These idealizations and assumptions permitted the basic cycle analysis of several types of engines and the analysis of engine performance trends. In this chapter, we will develop the analytical tools that allow us to use realistic assumptions as to component losses and to include the variation of specific heats.

6.2 Variation in Gas Properties

The enthalpy h and specific heat at constant pressure c_p for air (modeled as a perfect gas) are functions of temperature. Also, the enthalpy h and specific heat at constant pressure c_p for a typical hydrocarbon fuel JP-8 and air combustion products (modeled as a perfect gas) are functions of temperature and the fuel/air ratio f. The variations of properties h and c_p for fuel/air combustion products vs temperature are presented in Figs. 6.1a and 6.1b, respectively. The ratio of specific heats γ for fuel/air combustion products is also a function of temperature and of fuel/air ratio. A plot of γ is shown in Fig. 6.2. These figures are based on Eq. (2.64) and the coefficients of Table 2.2. Note that both h and c_p increase and γ decreases with temperature and the fuel/air ratio. Our models of gas properties in the engines need to include changes in both c_p and γ across components where the changes are significant.

In Chapter 7, we will include the variation in c_p and γ through the engine. To simplify the algebra, we will consider c_p and γ to have constant representative values through all engine components except the burner (combustor). The values of c_p and γ will be allowed to change across the burner. Thus we will approximate c_p as c_{pc} (a constant for the engine upstream of the burner) and c_p as c_{pt} (a constant average value for the gases downstream of the burner). Likewise, γ will be γ_c upstream of the burner and γ_t downstream of the burner. The release of thermal energy in the combustion process affects the values of

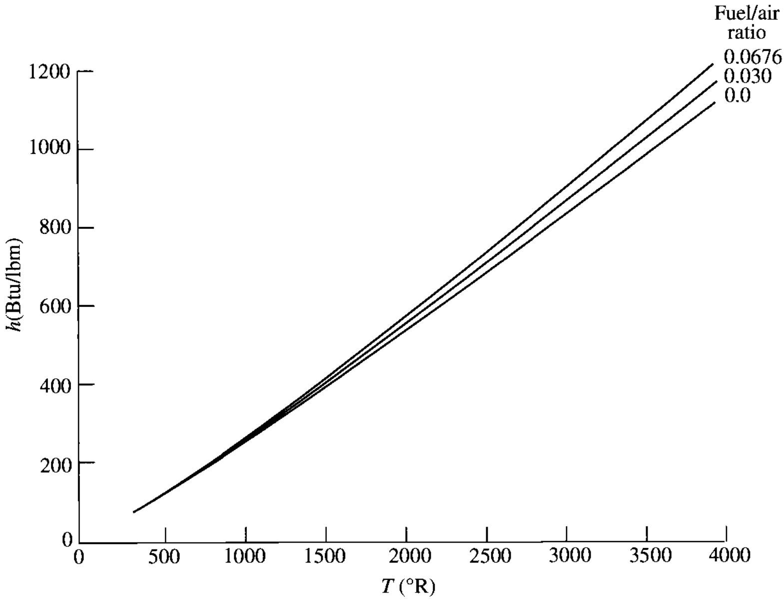

Fig. 6.1a Enthalpy vs temperature for JP-8 and air combustion products.

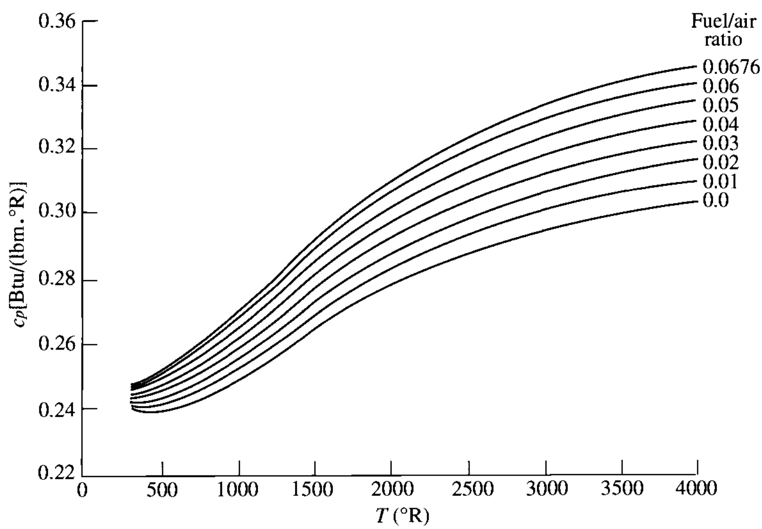

Fig. 6.1b Specific heat c_p vs temperature for JP-8 and air combustion products.

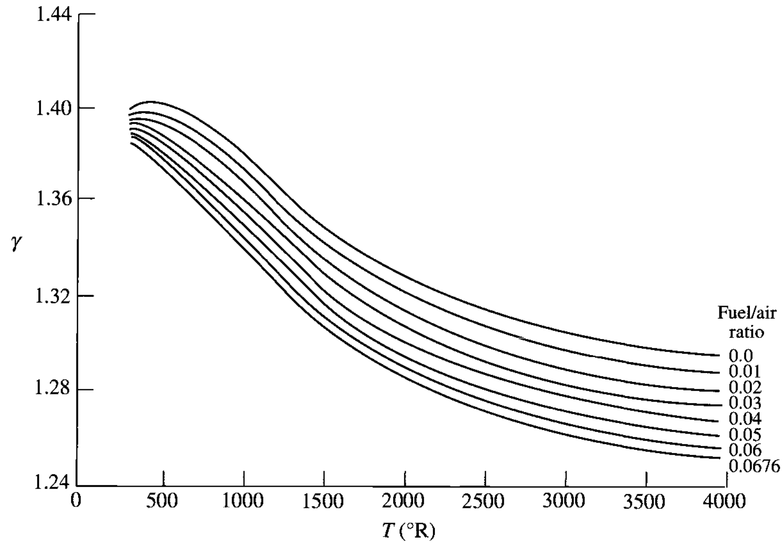

Fig. 6.2 Ratio of specific heats γ vs temperature for JP-8 and air combustion products.

c_{pt} and γ_t, but these two are related by

$$c_{pt} = \frac{\gamma_t}{\gamma_t - 1} R_t = \frac{\gamma_t}{\gamma_t - 1} \frac{\mathcal{R}_u}{\mathcal{M}} \tag{6.1}$$

where

$\mathcal{R}_u$ = universal gas constant
$\mathcal{M}$ = molecular weight

Thus if the chemical reaction causes the vibrational modes to be excited but does not cause appreciable dissociation, then the molecular weight $\mathcal{M}$ will be approximately constant. In this case, a reduction in γ is directly related to an increase in c_p by the formula

$$\frac{c_{pt}}{c_{pc}} = \frac{\gamma_t}{\gamma_t - 1} \frac{\gamma_c - 1}{\gamma_c} \tag{6.2}$$

6.3 Component Performance

In this chapter, each of the engine components will be characterized by *figures of merit* that model the component's performance and facilitate cycle analysis of real airbreathing engines. The total temperature ratio τ, the total pressure ratio π, and the interrelationship between τ and π will be used as much as possible in a component's figure of merit.

6.4 Inlet and Diffuser Pressure Recovery

Inlet losses arise because of the presence of wall friction and shock waves (in a supersonic inlet). Both wall friction and shock losses result in a reduction in total pressure so that $\pi_d < 1$. Inlets are adiabatic to a very high degree of approximation, and so we have $\tau_d = 1$. The inlet's figure of merit is defined simply as π_d.

The *isentropic efficiency* η_d of the diffuser is defined as (refer to Fig. 6.3)

$$\eta_d = \frac{h_{t2s} - h_0}{h_{t0} - h_0} \cong \frac{T_{t2s} - T_0}{T_{t0} - T_0} \tag{6.3}$$

This efficiency can be related to τ_r and π_d to give

$$\eta_d = \frac{\tau_{r(\pi_d)}^{(\gamma-1)/\gamma} - 1}{\tau_r - 1} \tag{6.4}$$

Figure 6.4 gives typical values of π_d for a subsonic inlet. The diffuser efficiency η_d was calculated from π_d by using Eq. (6.4).

In supersonic flight, the flow deceleration in inlets is accompanied by shock waves that can produce a total pressure loss much greater than, and in addition to, the wall friction loss. The inlet's overall pressure ratio is the product of the ram pressure ratio and the diffuser pressure ratio.

Because of shocks, only a portion of the ram total pressure can be recovered. We now define $\pi_{d\max}$ as that portion of π_d that is due to wall friction and define η_r as that portion of π_d due to ram recovery. Thus

$$\pi_d = \pi_{d\max}\eta_r \tag{6.5}$$

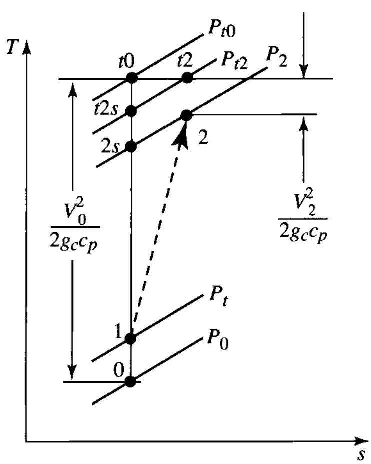

Fig. 6.3 Definition of inlet states.

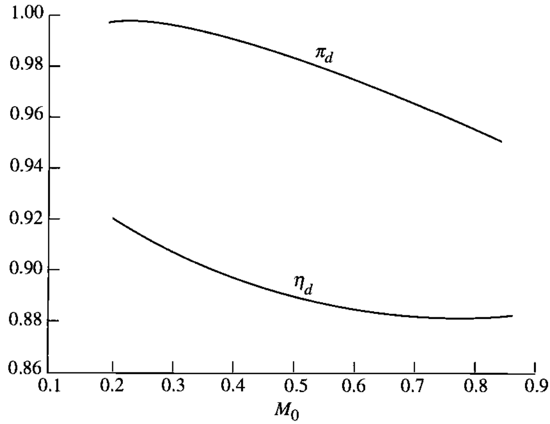

Fig. 6.4 Typical subsonic inlet π_d and η_d.

For subsonic and supersonic flow, a useful reference for the ram recovery η_r is Military Specification 5008B,[36] which is expressed as follows:

$$\eta_r = \begin{cases} 1 & M_0 \leq 1 \\ 1 - 0.075(M_0 - 1)^{1.35} & 1 < M_0 < 5 \\ \dfrac{800}{M_0^4 + 935} & 5 < M_0 \end{cases} \tag{6.6}$$

Because we often do not yet know the details of the inlet in cycle analysis, it is assumed that Military Specification 5008B applies as an ideal goal for ram recovery. The ram recovery of Military Specification 5008B is plotted in Fig. 6.5 vs M_0.

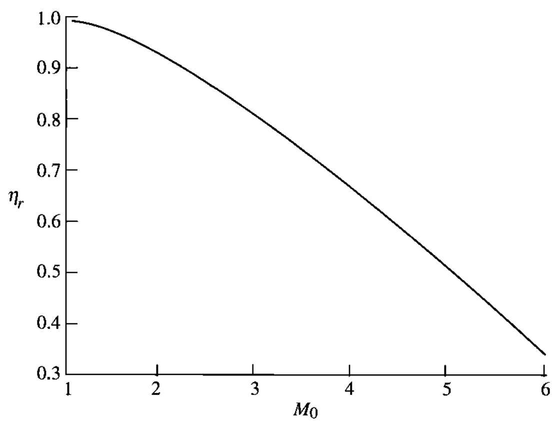

Fig. 6.5 Inlet total pressure recovery η_r of Military Specification 5008B.

6.5 Compressor and Turbine Efficiencies

6.5.1 Compressor Isentropic Efficiency

Compressors are, to a high degree of approximation, adiabatic. The overall efficiency used to measure a compressor's performance is the *isentropic efficiency* η_c, defined by

$$\eta_c = \frac{\text{ideal work of compression for given } \pi_c}{\text{actual work of compression for given } \pi_c} \tag{6.7}$$

Figure 6.6 shows both the ideal and actual compression processes for a given π_c on a T-s diagram. The actual work per unit mass w_c is $h_{t3} - h_{t2}$ [$= c_p (T_{t3} - T_{t2})$], and the ideal work per unit mass w_{ct} is $h_{t3i} - h_{r2}$ [$= c_p(T_{t3i} - T_{t2})$]. Here, h_{t3i} is the ideal (isentropic) compressor leaving total enthalpy. Writing the isentropic efficiency of the compressor η_c in terms of the thermodynamic properties, we have

$$\eta_c = \frac{w_{ci}}{w_c} = \frac{h_{t3i} - h_{t2}}{h_{t3} - h_{t2}}$$

For a calorically perfect gas, we can write

$$\eta_c = \frac{w_{ci}}{w_c} = \frac{c_p(T_{t3\iota} - T_{t2})}{c_p(T_{t3} - T_{t2})} = \frac{\tau_{ci} - 1}{\tau_c - 1}$$

Here τ_{ci} is the ideal compressor temperature ratio that is related to the compressor pressure ratio π_c by the isentropic relationship

$$\tau_{ci} = \pi_{ci}^{(\gamma-1)/\gamma} = \pi_c^{(\gamma-1)/\gamma} \tag{6.8}$$

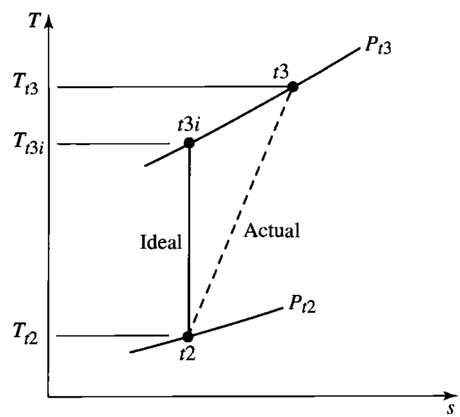

Fig. 6.6 Actual and ideal compressor processes.

Thus we have

$$\eta_c = \frac{\pi_c^{(\gamma-1)/\gamma} - 1}{\tau_c - 1} \tag{6.9}$$

6.5.2 Compressor Stage Efficiency

For a multistage compressor, each stage (set of rotor and stator) will have an isentropic efficiency. Let η_{sj} denote the isentropic efficiency of the jth stage. Likewise, π_{sj} and τ_{sj} represent the pressure ratio and temperature ratio, respectively, for the jth stage. From Eq. (6.9), we can write for the jth stage

$$\eta_{sj} = \frac{\pi_{sj}^{(\gamma-1)/\gamma} - 1}{\tau_{sj} - 1} \tag{6.10}$$

where $\tau_{sj} = T_{tj}/T_{tj-1}$ and $\pi_{sj} = P_{tj}/P_{tj-1}$.

Figure 6.7 shows the process for a multistage compressor. Here η_{sj} can be interpreted as the vertical height from A to B divided by the vertical height from A to C. For counting purposes, subscript 0 outside the parentheses is at the inlet, and subscript N is at the outlet of the compressor. Thus $(P_t)_0 = P_{t2}$, $(T_t)_0 = T_{t2}$, $P_{tN} = P_{tN} = P_3$, and $T_{TN} = T_{t3}$. From Eq. (6.9), we have for the overall compressor isentropic efficiency

$$\eta_c = \frac{(P_{t3}/P_{t2})^{(\gamma-1)/\gamma} - 1}{T_{t3}/T_{t2} - 1} = \frac{[P_{tN}/(P_t)_0]^{(\gamma-1)/\gamma} - 1}{T_{tN}/(T_t)_0 - 1} \tag{6.11}$$

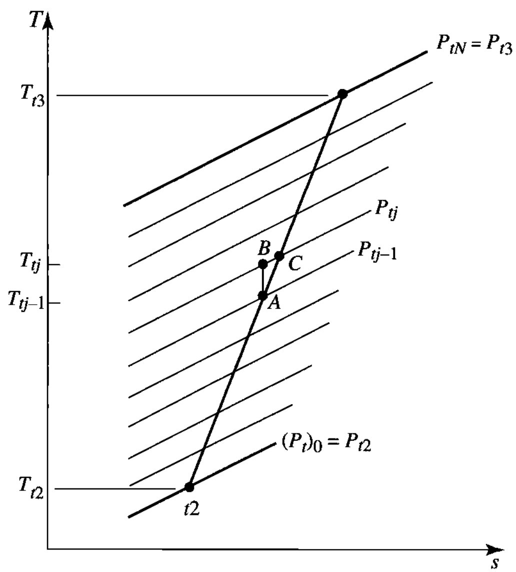

Fig. 6.7 Multistage compressor process and nomenclature.

From Eq. (6.10), we have

$$\frac{T_{tj}}{T_{tj-1}} = 1 + \frac{1}{\eta_{sj}}\left[\left(\frac{P_{tj}}{P_{tj-1}}\right)^{(\gamma-1)/\gamma} - 1\right]$$

and so

$$\frac{T_{tN}}{(T_t)_0} = \prod_{j=1}^{N}\left\{1 + \frac{1}{\eta_{sj}}\left[\left(\frac{P_{tj}}{P_{tj-1}}\right)^{(\gamma-1)/\gamma} - 1\right]\right\}$$

where $\prod$ means product. We note also the requirement that

$$\frac{P_{tN}}{(P_t)_0} = \prod_{j=1}^{N}\frac{P_{tj}}{P_{tj-1}}$$

where

$$\frac{P_{tN}}{(P_t)_0} \equiv \pi_c$$

Thus Eq. (6.11) becomes

$$\eta_c = \frac{[P_{tN}/(P_t)_0]^{(\gamma-1)/\gamma} - 1}{\prod_{j=1}^{N}\{1 + (1/\eta_{sj})[(P_{tj}/P_{tj-1})^{(\gamma-1)/\gamma} - 1]\} - 1} \tag{6.12}$$

We can see from Eq. (6.12) that the isentropic efficiency of a compressor is a function of the compressor pressure ratio, the pressure ratio of each stage, and the isentropic efficiency of each stage. This complex functional form makes the isentropic efficiency of the compressor undesirable for use in cycle analysis. We are looking for a simpler form of the figure of merit that will allow us to vary the compression ratio and still accurately predict the variation of η_c.

Let us consider the special case when each stage pressure ratio and each stage efficiency are the same. In this case

$$\pi_c = \prod_{j=1}^{N}\left(\frac{P_{tj}}{P_{tj-1}}\right) = \pi_s^N$$

Here η_s is the stage efficiency, and π_s is the stage pressure ratio. Also

$$\frac{T_{tj}}{T_{tj-1}} = 1 + \frac{1}{\eta_s}(\pi_s^{(\gamma-1)/\gamma} - 1)$$

and so

$$\frac{T_{tN}}{(T_t)_0} \equiv \tau_c = \left[1 + \frac{1}{\eta_s}(\pi_s^{(\gamma-1)/\gamma} - 1)\right]^N$$

or

$$\tau_c = \left[1 + \frac{1}{\eta_s}\left(\pi_c^{(\gamma-1)/(\gamma N)} - 1\right)\right]^N$$

and

$$\eta_c = \frac{\pi_c^{(\gamma-1)/\gamma} - 1}{[1 + (1/\eta_s)(\pi_c^{(\gamma-1)/(\gamma N)} - 1)]^N - 1}$$

$$= \frac{\pi_s^{N(\gamma-1)/\gamma} - 1}{[1 + (1/\eta_s)(\pi_s^{(\gamma-1)/\gamma} - 1)]^N - 1} \tag{6.13}$$

Note: This is a relationship connecting η_c with η_s for an N-stage compressor with equal stage pressure ratios and equal stage efficiencies.

6.5.3 Compressor Polytropic Efficiency

The *polytropic efficiency* e_c is related to the preceding efficiencies and is defined as

$$e_c = \frac{\text{ideal work of compression for a differential pressure change}}{\text{actual work of compression for a differential pressure change}}$$

Thus

$$e_c = \frac{dw_i}{dw} = \frac{dh_{ti}}{dh_t} = \frac{dT_{ti}}{dT_t}$$

Note that for an ideal compressor, the isentropic relationship gives $T_{ti} = P_{ti}^{(\gamma-1)/\gamma} \times$ constant. Thus

$$\frac{dT_{ti}}{T_t} = \frac{\gamma - 1}{\gamma}\frac{dP_t}{P_t}$$

and

$$e_c = \frac{dT_{ti}}{dT_t} = \frac{dT_{ti}/T_t}{dT_t/T_t} = \frac{\gamma - 1}{\gamma}\frac{dP_t/P_t}{dT_t/T_t}$$

Assuming that the polytropic efficiency e_c is constant, we can obtain a simple relationship between τ_c and π_c as follows:

1) Rewrite the preceding equation as

$$\frac{dT_t}{T_t} = \frac{\gamma - 1}{\gamma e_c}\frac{dP_t}{P_t}$$

2) Integration between states $t2$ and $t3$ gives

$$\ln \frac{T_{t3}}{T_{t2}} = \frac{\gamma - 1}{\gamma e_c} \ln \frac{P_{t3}}{P_{t2}}$$

or

$$\tau_c = \pi_c^{(\gamma-1)/(\gamma e_c)} \tag{6.14}$$

For a state-of-the-art design, the polytropic efficiency is essentially constant. Substitution of Eq. (6.14) into Eq. (6.9) gives

$$\eta_c = \frac{\pi_c^{(\gamma-1)/\gamma} - 1}{\tau_c - 1} = \frac{\pi_c^{(\gamma-1)/\gamma} - 1}{\pi_c^{(\gamma-1)/(\gamma e_c)} - 1} \tag{6.15}$$

Equation (6.15) accurately predicts the relationship between the isentropic efficiency of a compressor and the compressor ratio for a given state-of-the-art polytropic efficiency. This relationship is plotted in Fig. 6.8 for a given value of e_c.

We will use the polytropic efficiency e_c as the figure of merit for the compressor. Equations (6.14) and (6.15) will be used to obtain the total temperature ratio and isentropic efficiency of the compressor, respectively, in the cycle analysis.

6.5.4 Relationship Between Compressor Efficiencies

We have, from Eqs. (6.13) and (6.15), relationships connecting η_c, η_s, and e_c. In this section, we wish to see if η_s formally approaches e_c as we let the number of stages get very large and the pressure ratio per stage get very small. To do this, first we note the relationship

$$1 - x^{1/N} = \frac{y}{N}$$

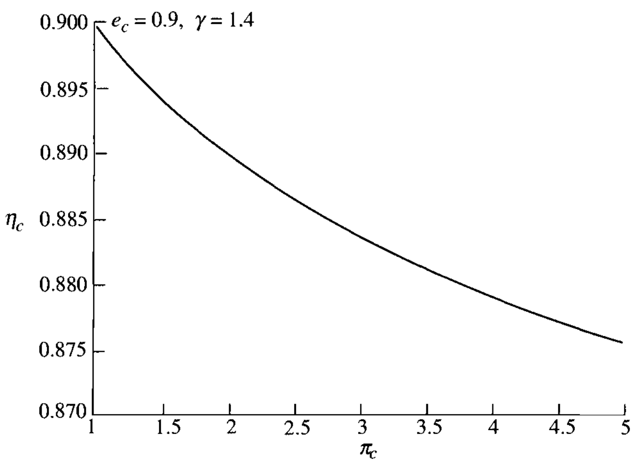

Fig. 6.8 Isentropic efficiency vs compressor pressure ratio for constant polytropic efficiency of 0.9.

then

$$x = \left(1 - \frac{y}{N}\right)^N$$

This may be expanded by the binomial expansion to give

$$x = \left(1 - \frac{y}{N}\right)^N = 1 + N\left(\frac{-y}{N}\right) + \frac{N(N-1)}{1 \cdot 2}\left(\frac{-y}{N}\right)^2 + \cdots \tag{i}$$

If N is very large, this becomes

$$x = 1 - y + \frac{y^2}{2!} - \frac{y^3}{3!} + \cdots = \exp(-y)$$

or

$$y = -\ln x \tag{ii}$$

With these basic relationships established, we now write portions of Eq. (6.13) in the form given in Eq. (i). We consider

$$\left[1 + \frac{1}{\eta_s}(\pi_c^{(\gamma-1)/(\gamma N)} - 1)\right]^N \equiv \left[1 - \frac{1}{\eta_s}(1 - \pi_c^{(\gamma-1)(\gamma N)})\right]^N$$

Then

$$\frac{y}{N} = 1 - \pi_c^{(\gamma-1)/(\gamma N)} = 1 - x^{1/N}$$

where

$$x = \pi_c^{(\gamma-1)/\gamma}$$

Thus for N large, we approach

$$1 - \pi_c^{(\gamma-1)/(\gamma N)} \to \frac{y}{N} = -\frac{1}{N}\ln x = -\frac{1}{N}\ln \pi_c^{(\gamma-1)/\gamma}$$

Then

$$\left[1 - \frac{1}{\eta_s}(1 - \pi_c^{(\gamma-1)/(\gamma N)})\right]^N \to \left(1 + \frac{1}{\eta_s}\frac{1}{N}\ln \pi_c^{(\gamma-1)/\gamma}\right)^N$$
$$= \left(1 + \frac{1}{N}\ln \pi_c^{(\gamma-1)/(\gamma\eta_s)}\right)^N$$

However, as earlier, the expansion for $(1 - z/N)^N$ for large N is just e^{-z}. Here we let

$$z = -\ln \pi_c^{(\gamma-1)/(\gamma N)}$$

and thus

$$e^{-z} = \pi_c^{(\gamma-1)/(\gamma N)}$$

Hence for large N

$$\left[1 - \frac{1}{\eta_s}(1 - \pi_c^{(\gamma-1)/(\gamma N)})\right]^N \to \pi_c^{(\gamma-1)/(\gamma\eta_s)}$$

Thus

$$\eta_c = \frac{\pi_c^{(\gamma-1)/\gamma} - 1}{\pi_c^{(\gamma-1)/(\gamma\eta_s)} - 1} \tag{6.16}$$

for a multistage machine. This expression is identical to Eq. (6.15) with e_c replaced by η_s. Thus for very large N, η_s approaches e_c.

Example 6.1

Say we plan to construct a 16-stage compressor, with each stage pressure ratio the same, given $\pi_c = 25$. Then we have $\pi_s = 25^{1/16} = 1.223$. Say η_s is measured at 0.93. Then, with Eq. (6.14) solved for e_c, we have

$$e_c = \frac{(\gamma - 1)/\gamma \,\ell n\, \pi_s}{\ell n[1 + (1/\eta_s)(\pi_s^{(\gamma-1)/\gamma} - 1)]}$$

$$= 0.9320 \tag{6.17}$$

Then we get two estimates for η_c, one based on constant η_s and another based on constant e_c. From Eq. (6.13), with $\pi_s = 1.223$, we obtain

$$\eta_c = \frac{\pi_c^{(\gamma-1)/\gamma} - 1}{[1 + (1/\eta_s)(\pi_s^{(\gamma-1)/\gamma} - 1)]^N - 1}$$

$$= \frac{25^{1/3.5} - 1}{[1 + (1/0.93)(1.223^{1/3.5} - 1)]^{16} - 1}$$

$$= 0.8965^-$$

From Eq. (6.15) we find, for comparison,

$$\eta_c = \frac{\pi_c^{(\gamma-1)/\gamma} - 1}{\pi_c^{(\gamma-1)/(\gamma e_c)} - 1} = 0.8965^+$$

More simply, if $e_c = \eta_s = 0.93$, then by either Eq. (6.15) or Eq. (6.16), we get $\eta_c = 0.8965$.

The point of all of this is that for a multistage machine, the simplicity and accuracy of using the polytropic efficiency make it a useful concept. Thus from now on we will use Eq. (6.15) to compute the compressor efficiency.

6.5.5 Compressor Stage Pressure Ratio

For a multistage compressor, the energy added is divided somewhat evenly per stage, and each stage increases the total temperature of the flow about the same amount. The total temperature ratio of a stage τ_s that has a total temperature change of ΔT_t can be written as

$$\tau_s = 1 + \frac{\Delta T_t}{T_{ti}}$$

Using the polytropic efficiency e_c to relate the stage pressure ratio π_s to its temperature ratio τ_s, we have

$$\pi_s = (\tau_s)^{(\gamma e_c)/(\gamma-1)} = \left(1 + \frac{\Delta T_t}{T_{ti}}\right)^{(\gamma e_c)/(\gamma-1)}$$

This equation gives the variation of the compressor stage pressure ratio with stage inlet temperature T_{ti} and total temperature change ΔT_t. This equation shows the decrease in stage pressure ratio with increases in stage inlet temperature for stages with the same total temperature change.

Stage pressure ratio results for $\gamma = 1.4$ and $e_c = 0.9$ are plotted in Fig. 6.9 vs $\Delta T_t/T_{ti}$. By using this figure, a 30 K change with 300 K inlet temperature gives a stage pressure ratio of about 1.35. Likewise, a 60°R change with 1000°R inlet temperature gives a stage pressure ratio of about 1.20. Figure 6.9 helps explain the change in stage pressure ratio through a compressor.

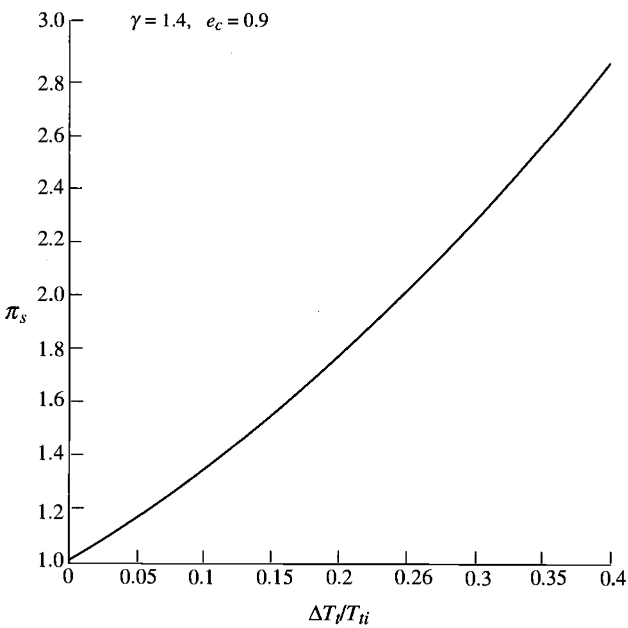

Fig. 6.9 Variation of compressor stage pressure ratio.

6.5.6 Turbine Isentropic Efficiency

Modern turbines are cooled by air taken from the compressors, passed through vanes and rotors, and then remixed with the main flow. From the point of view of the overall flow, the flow is adiabatic; but to be correct, a multiple-stream analysis would have to be applied. Such an analysis is straightforward conceptually, but it is difficult to estimate the various mixing losses, etc., that occur. The concept of isentropic efficiency is still utilized in most such analyses (for the mainstream portion of the flow), and in any case, the isentropic efficiency gives a reasonable approximation to the turbine performance when cooling flow rates are small. Hence, in this text, we consider only the adiabatic case.

In analogy to the compressor isentropic efficiency, we define the *isentropic efficiency* of the turbine by

$$\eta_t = \frac{\text{actual turbine work for a given } \pi_t}{\text{ideal turbine work for a given } \pi_t}$$

The actual and ideal expansion processes for a given π_t are shown in Fig. 6.10 on a T-s diagram. The actual turbine work per unit mass is $h_{t4} - h_{t5}$ $[=c_p(T_{t4} - T_{t5})]$, and the ideal turbine work per unit mass is $h_{t4} - h_{t5i}$ $[=c_p(T_{t4} - T_{t5i})]$. Here, T_{t5i} is the ideal turbine leaving total temperature. Writing the isentropic efficiency of the turbine in terms of the thermodynamic properties, we have

$$\eta_t = \frac{h_{t4} - h_{t5}}{h_{t4} - h_{t5i}} = \frac{T_{t4} - T_{t5}}{T_{t4} - T_{t5i}}$$

or

$$\eta_t = \frac{1 - \tau_t}{1 - \pi_t^{(\gamma-1)/\gamma}} \tag{6.18}$$

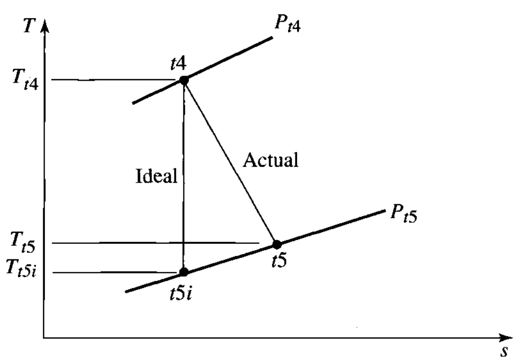

Fig. 6.10 Actual and ideal turbine processes.

6.5.7 Turbine Stage Efficiency

In a completely similar analysis to that for the compressor, the turbine isentropic efficiency can be written in terms of η_{sj} and π_{sj}:

$$\eta_t = \frac{1 - \prod_{j=1}^{N}[1 - \eta_{sj}(1 - \pi_{sj}^{(\gamma-1)/\gamma})]}{1 - \pi_t^{(\gamma-1)/\gamma}} \tag{6.19}$$

When all stages have the same π_s and η_s, the preceding equation reduces to

$$\eta_t = \frac{1 - [1 - \eta_s(1 - \pi_s^{(\gamma-1)/\gamma})]^N}{1 - \pi_t^{(\gamma-1)/\gamma}} \tag{6.20}$$

6.5.8 Turbine Polytropic Efficiency

The *polytropic turbine efficiency* e_t is defined similarly to the turbine isentropic efficiency as

$$e_t = \frac{\text{actual turbine work for a differential pressure change}}{\text{ideal turbine work for a differential pressure change}}$$

Thus

$$e_t = \frac{\mathrm{d}w}{\mathrm{d}w_i} = \frac{\mathrm{d}h_t}{\mathrm{d}h_{ti}} = \frac{\mathrm{d}T_t}{\mathrm{d}T_{ti}}$$

For the isentropic relationship, we have

$$\frac{\mathrm{d}T_{ti}}{T_t} = \frac{\gamma - 1}{\gamma}\frac{\mathrm{d}P_t}{P_t}$$

Thus

$$e_t = \frac{\mathrm{d}T_t}{\mathrm{d}T_{ti}} = \frac{\mathrm{d}T_t/T_t}{\mathrm{d}T_{ti}/T_t} = \frac{\mathrm{d}T_t/T_t}{[(\gamma - 1)/\gamma]\,\mathrm{d}P_t/P_t}$$

Assuming that the polytropic efficiency e_t is constant over the pressure ratio, we integrate the preceding equation to give

$$\pi_t = \tau_t^{\gamma/[(\gamma-1)e_t]} \tag{6.21}$$

and so

$$\eta_t = \frac{1 - \tau_t}{1 - \tau_t^{1/e_t}} \tag{6.22}$$

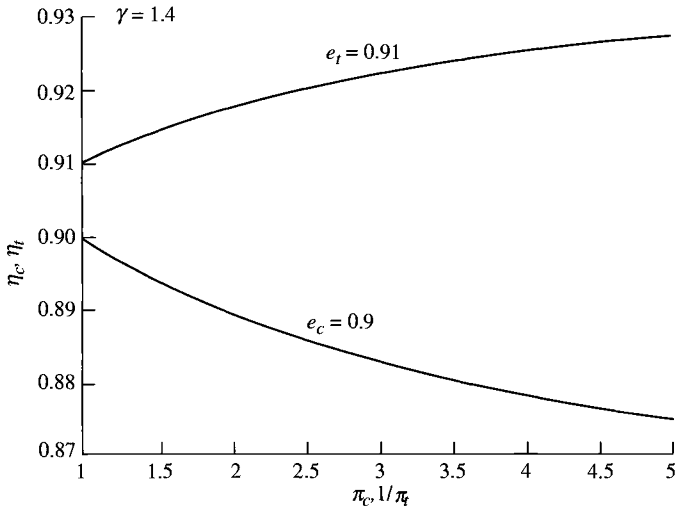

Fig. 6.11 Compressor and turbine efficiencies vs pressure ratio.

or

$$\eta_t = \frac{1 - \pi_t^{(\gamma-1)e_t/\gamma}}{1 - \pi_t^{(\gamma-1)/\gamma}} \tag{6.23}$$

This relationship is plotted in Fig. 6.11 along with Eq. (6.15) for the compressor. Note that the turbine efficiency increases with the turbine expression ratio $1/\pi_t$ for a constant e_t.

In cycle analysis, τ_t is usually first obtained from the work balance. Then π_t can be calculated for a known e_t by using Eq. (6.21), and η_t can be calculated by using either Eq. (6.22) or Eq. (6.23). We will use the polytropic efficiency e_t as the figure of merit for the turbine.

6.6 Burner Efficiency and Pressure Loss

In the burner, we are concerned with two efforts: incomplete combustion of the fuel and total pressure loss. *Combustion efficiency* η_b is defined by

$$\eta_b = \frac{(\dot{m} + \dot{m}_f)h_{t4} - \dot{m}h_{t3}}{\dot{m}_f h_{PR}} = \frac{(\dot{m} + \dot{m}_f)c_{p4}T_{t4} - \dot{m}c_{p3}T_{t3}}{\dot{m}_f h_{PR}}$$

We can approximate c_{p3} as c_{pc} (a constant for the engine upstream of the burner) and c_{p4} as c_{pt} (a constant average value for the gases downstream of the burner). Thus the combustion efficiency is

$$\eta_b = \frac{(\dot{m} + \dot{m}_f)c_{pt}T_{t4} - \dot{m}c_{pc}T_{t3}}{\dot{m}_f h_{PR}} \tag{6.24}$$

The total pressure losses arise from two effects: the viscous losses in the combustion chamber and the total pressure loss due to combustion at finite Mach number. These effects are combined for the purpose of performance analysis in

$$\pi_b = \frac{P_{t4}}{P_{t3}} < 1$$

We will use both η_b and π_b as the figures of merit for the burner. There are similar combustion efficiencies and total pressure ratios for afterburners (augmenters) and duct burners.

6.7 Exhaust Nozzle Loss

The primary loss due to the nozzle has to do with the over- or underexpansion of the nozzle. In addition, there will be a loss in total pressure from turbine to exit. Thus we may have

$$P_9 \neq P_0 \qquad \text{and} \qquad \pi_n = \frac{P_{t9}}{P_{t8}} < 1$$

We still have $\tau_n = 1$, because the nozzle is very nearly adiabatic. We will use π_n as the figure of merit for the nozzle.

6.8 Mechanical Efficiency of Power Shaft

We define the mechanical efficiency of a shaft to account for the loss or extraction of power on that shaft. The mechanical efficiency η_m is defined as the ratio of the power leaving the shaft that enters the compressor, $\dot{W}_c$, to the power entering the shaft from the turbine, $\dot{W}_t$. This can be written in equation form as

$$\eta_m = \frac{\text{power leaving shaft to compressor}}{\text{power entering shaft from turbine}} = \frac{\dot{W}_c}{\dot{W}_t}$$

The mechanical efficiency η_m is less than one due to losses in power that occur from shaft bearings and power extraction for driving engine acessories like oil and fuel pumps. For multishaft engines, each shaft will have a mechanical efficiency associated with the power transfer on the shaft.

6.9 Summary of Component Figures of Merit (Constant c_p Values)

Table 6.1 summarizes the ideal and actual behaviors of gas turbine engine components with calorically perfect gases. Note that for a compressor with constant polytropic efficiency, the isentropic efficiency follows from Eq. (6.15). We will use these figures of merit in the following chapter.

At a particular level of technological development, the polytropic efficiency e_c for the compressor can be considered to be a constant, and thus the compressor pressure ratio π_c determines the compressor efficiency η_c [see Eq. (6.16)]. Similarly, the polytropic efficiency e_t for the turbine can be considered to be a

Table 6.1 Summary of component figures of merit (constant c_p values)

Component	Ideal behavior	Actual behavior	Figure of merit
Inlet	Adiabatic and reversible (isentropic)	Adiabatic, not reversible	
	$\tau_d = 1,\ \pi_d = 1$	$\tau_d = 1,\ \pi_d < 1$	π_d
Compressor	Adiabatic and reversible (isentropic)	Adiabatic, not reversible	
	$w_c = c_p T_{t2}(\tau_c - 1)$	$w_c = c_{pc} T_{t2}(\tau_c - 1)$	
	$\pi_c = \tau_c^{\gamma(\gamma-1)}$	$\pi_c = [1 + \eta_c(\tau_c - 1)]^{\gamma_c/(\gamma_c-1)}$	η_c
		$\tau_c = 1 + \frac{1}{\eta_c}(\pi_c^{(\gamma_c-1)/\gamma_c} - 1)$	
		$\tau_c = (\pi_c)^{(\gamma_c-1)/(\gamma_c e_c)}$	e_c
Burner	No total pressure loss, 100% combustion	Total pressure loss, combustion <100%	
	$\pi_b = 1$	$\pi_b < 1$	π_b
	$\dot{m} c_p T_{t4} - \dot{m} c_p T_{t3} = \dot{m}_f h_{PR}$	$(\dot{m} + \dot{m}_f) c_{pt} T_{t4} - \dot{m} c_{pc} T_{t3} = \eta_b \dot{m}_f h_{PR}$	η_b
Turbine	Adiabatic and reversible (isentropic)	Adiabatic, not reversible	
	$w_t = c_p T_{t4}(1 - \tau_t)$	$w_t = c_{pt} T_{t4}(1 - \tau_t)$	
	$\pi_t = \tau_t^{\gamma/(\gamma-1)}$	$\pi_t = [1 - \frac{1}{\eta_t}(1 - \tau_t)]^{\gamma_t/(\gamma_t-1)}$	η_t
		$\tau_t = 1 - \eta_t(1 - \pi_t^{(\gamma_t-1)/\gamma_t})$	
		$\pi_t = \tau_t^{\gamma_t/[(\gamma_t-1)e_t]}$	e_t
Nozzle	Adiabatic and reversible	Adiabatic, not reversible	
	$\tau_n = 1,\ \pi_n = 1$	$\tau_n = 1,\ \pi_n < 1$	π_n

constant, and thus the turbine temperature ratio τ_t determines the turbine efficiency η_t [see Eq. (6.22)]. For the analysis in the following chapter, we will use the polytropic efficiencies as input data and will calculate the resulting component efficiencies.

The values of these figures of merit have changed as technology has improved over the years. In addition, the values of the figures of merit for the diffuser and nozzle depend on the application. For example, a commercial airliner with engine nacelles and convergent, fixed-area exhaust nozzles will typically have much higher values of π_d and π_n than a supersonic fighter with its engines in the airframe and convergent-divergent, variable-area exhaust nozzles. Table 6.2 lists typical values for the figures of merit that correspond to different periods in the evolution of engine technology (called *levels of technology*) and the application.

Table 6.2 Component efficiencies, total pressure ratios, and temperature limits

Component	Figure of merit	Type[a]	Level of technology[b]			
			1	2	3	4
Diffuser	$\pi_{d\,\max}$	A	0.90	0.95	0.98	0.995
		B	0.88	0.93	0.96	0.98
		C	0.85	0.90	0.94	0.96
Compressor	e_c		0.80	0.84	0.88	0.90
Fan	e_f		0.78	0.82	0.86	0.89
Burner	π_b		0.90	0.92	0.94	0.95
	η_b		0.88	0.94	0.99	0.999
Turbine	e_t	Uncooled	0.80	0.85	0.89	0.90
		Cooled		0.83	0.87	0.89
Afterburner	π_{AB}		0.90	0.92	0.94	0.95
	η_{AB}		0.85	0.91	0.96	0.99
Nozzle		D	0.95	0.97	0.98	0.995
	π_n	E	0.93	0.96	0.97	0.98
		F	0.90	0.93	0.95	0.97
Mechanical shaft	η_m	Shaft only	0.95	0.97	0.99	0.995
		With power takeoff	0.90	0.92	0.95	0.97
Maximum T_{t4}		(K)	1110	1390	1780	2000
		(R)	2000	2500	3200	3600
Maximum T_{t7}		(K)	1390	1670	2000	2220
		(R)	2500	3000	3600	4000

[a]A = subsonic aircraft with engines in nacelles
B = subsonic aircraft with engine(s) in airframe
C = supersonic aircraft with engine(s) in airframe
D = fixed-area convergent nozzle
E = variable-area convergent nozzle
F = variable-area convergent-divergent nozzle

[b]Notes: Stealth may reduce $\pi_{d\max}$, π_{AB}, and π_n. The levels of technology can be thought of as representing the technical capability for 20-yr increments in time beginning in 1945. Thus level 3 of technology presents typical component design values for the time period 1985–2005.

6.10 Component Performance with Variable c_p

In Section 2.6.6 of Chapter 2, we developed the relationships [Eqs. (2.53), (2.54), and (2.57)] for the thermodynamic properties h, ϕ, and s of a perfect gas with variable specific heat. In addition, the reduced pressure P_r and reduced volume v_r were defined by Eqs. (2.55) and (2.56), respectively. We will use these properties to describe the performance of engine components when the variation of specific heat is to be included. We will use Appendix D or the computer program AFPROP to obtain the thermodynamic properties.

The notation π_a [see Eq. (5.3)] represents a component's total pressure ratio. However, we will use a modified definition for τ_a to represent the ratio of total enthalpies [see original definition given by Eq. (5.4)]. Thus

$$\tau_a = \frac{\text{total enthalpy leaving component } a}{\text{total enthalpy entering component } a} \tag{6.25}$$

6.10.1 Freestream Properties

From the definition of the total enthalpy and total pressure, we can write

$$\tau_r = \frac{h_{t0}}{h_0} = \frac{h_0 + V_0^2/(2g_c)}{h_0} \qquad \text{and} \qquad \pi_r = \frac{P_{r0}}{P_0} = \frac{P_{rt0}}{P_{r0}} \tag{6.26}$$

6.10.2 Inlet

Because the inlet is assumed to be adiabatic, then

$$\tau_d = \frac{h_{t2}}{h_{t0}} = 1 \tag{6.27}$$

By using Eq. (2.57), the inlet total pressure ratio can be expressed in terms of the entropy change as follows:

$$\pi_d = \frac{P_{t2}}{P_{r0}} = \exp\left(-\frac{s_2 - s_0}{R}\right) \tag{6.28}$$

6.10.3 Compressor

The variable τ_c represents the total enthalpy ratio of the compressor, and π_c represents its total pressure ratio, or

$$\tau_c = \frac{h_{t3}}{h_{t2}} \qquad \text{and} \qquad \pi_c = \frac{P_{t3}}{P_{t2}} \tag{6.29}$$

The polytropic efficiency of the compressor e_c can be written as

$$e_c = \frac{\mathrm{d}h_{ti}}{\mathrm{d}h_t} = \frac{\mathrm{d}h_{ti}/T_t}{\mathrm{d}h_t/T_t}$$

By using the Gibbs equation [Eq. (2.31)], the numerator in the preceding equation can be expressed as

$$\frac{\mathrm{d}h_{ti}}{T_t} = R\frac{\mathrm{d}P_t}{P_t}$$

Thus

$$e_c = R\frac{\mathrm{d}P_t/P_t}{\mathrm{d}h_t/T_t} \tag{6.30}$$

For a constant polytropic efficiency, integration of the preceding equation between states $t2$ and $t3$ gives

$$\phi_{t3} - \phi_{t2} = \frac{R}{e_c} \ln \frac{P_{t3}}{P_{t2}}$$

Thus the compressor pressure ratio π_c can be written as

$$\pi_c = \frac{P_{t3}}{P_{t2}} = \exp\left(e_c \frac{\phi_{t3} - \phi_{t2}}{R}\right)$$

or

$$\pi_c = \left(\frac{P_{rt3}}{P_{rt2}}\right)^{e_c} \tag{6.31}$$

The reduced pressure at state $t2$ can be obtained from Appendix D or the computer program AFPROP, given the temperature at state $t2$. If the values of π_c and e_c are also known, one can get the reduced pressure at state $t3$ by using

$$P_{rt3} = P_{rt2}\pi_c^{1/e_c} \tag{6.32}$$

Given the reduced pressure at state $t3$, the total temperature and total enthalpy can be obtained from Appendix D or the computer program AFPROP.

The isentropic efficiency of a compressor can be expressed as

$$\eta_c = \frac{h_{t3i} - h_{t2}}{h_{t3} - h_{t2}} \tag{6.33}$$

This equation requires that the total enthalpy be known at states $t2$, $t3$, and $t3i$.

Example 6.2

Air at 1 atm and 540°R enters a compressor whose polytropic efficiency is 0.9. If the compressor pressure ratio is 15, determine the leaving total properties and compressor isentropic efficiency. From Appendix D for $f = 0$, we have

$$h_{t2} = 129.02\,\text{Btu/lbm} \qquad \text{and} \qquad P_{rt2} = 1.384$$

The exit total pressure is 15 atm. The reduced pressures at stations $t3$ and $t3i$ are

$$P_{rt3} = P_{rt2}\pi_c^{l/e_c} = 1.384 \times 15^{1/0.9} = 28.048$$
$$P_{rt3\iota} = P_{rt2}\pi_c = 1.384 \times 15 = 20.76$$

Linear interpolation of Appendix D, with $f = 0$ and using the preceding reduced pressures, gives the following values of total enthalpy and total temperature:

$$h_{t3} = 304.48\,\text{Btu/lbm} \qquad \text{and} \qquad T_{t3} = 1251.92°\text{R}$$
$$h_{t3\iota} = 297.67\,\text{Btu/lbm} \qquad \text{and} \qquad T_{t3i} = 1154.58°\text{R}$$

The compressor isentropic efficiency is

$$\eta_c = \frac{279.67 - 129.02}{304.48 - 129.02} = 0.8586$$

6.10.4 Burner

The τ_b represents the total enthalpy ratio of the burner, and π_b represents its total pressure ratio, or

$$\tau_b = \frac{h_{t4}}{h_{t3}} \qquad \text{and} \qquad \pi_b = \frac{P_{t4}}{P_{t3}} \tag{6.34}$$

From the definition of burner efficiency, we have

$$\eta_b = \frac{(\dot{m} + \dot{m}_f)h_{t4} - \dot{m}h_{t3}}{\dot{m}_f h_{PR}}$$

or

$$\eta_b = \frac{(1+f)h_{t4} - h_{t3}}{fh_{PR}} \tag{6.35}$$

where f is the ratio of the fuel flow rate to the airflow rate entering the burner, or $f = \dot{m}_f/\dot{m}$.

In the analysis of gas turbine engines, we determine the fuel/air ratio f and normally specify η_b, h_{PR}, and T_{t4}. The enthalpy at station t_3 (h_{t3}) will be known from analysis of the compressor. Equation (6.35) can be solved for the fuel/air ratio f, giving

$$f = \frac{h_{t4} - h_{t3}}{\eta_b h_{PR} - h_{t4}} \tag{6.36}$$

Note: The value of h_{t4} is a function of the fuel/air ratio f, and thus the solution of Eq. (6.36) is iterative.

6.10.5 Turbine

The τ_t represents the total enthalpy ratio of the turbine, and π_t represents its total pressure ratio or

$$\tau_t = \frac{h_{t5}}{h_{t4}} \qquad \text{and} \qquad \pi_t = \frac{P_{t5}}{P_{t4}} \tag{6.37}$$

The *polytropic efficiency of a turbine* is defined as

$$e_t = \frac{\mathrm{d}h_t}{\mathrm{d}h_{ti}}$$

which can be written as

$$e_t = \frac{1}{R}\frac{\mathrm{d}h_t/T_t}{\mathrm{d}P_t/P_t} \tag{6.38}$$

For constant polytropic efficiency, the following relationships are obtained by integration from state $t4$ to $t5$:

$$\pi_t = \frac{P_{t5}}{P_{t4}} = \exp\frac{\phi_{t5} - \phi_{t4}}{Re_t} \tag{6.39}$$

and

$$\pi_t = \left(\frac{P_{rt5}}{P_{rt4}}\right)^{1/e_t} \tag{6.40}$$

The *isentropic efficiency of the turbine* is defined as

$$\eta_t = \frac{h_{t4} - h_{t5}}{h_{t4} - h_{r5i}} \tag{6.41}$$

where each total enthalpy is a function of the total temperature T_t and fuel/air ratio f.

Example 6.3

Products of combustion ($f = 0.0338$) at 20 atm and 3000°R enter a turbine whose polytropic efficiency is 0.9. If the total enthalpy of the flow through the turbine decreases 100 Btu/lbm, determine the leaving total properties and turbine isentropic efficiency. From Appendix D for $f = 0.0338$, we have

$$h_{t4} = 828.75\ \text{Btu/lbm} \qquad \text{and} \qquad P_{rt4} = 1299.6$$

The exit total enthalpy is 728.75 Btu/lbm. From linear interpolation of Appendix D, the total temperature and reduced pressure at station $t5$ are

$$T_{t5} = 2677.52°\text{R} \qquad \text{and} \qquad P_{rt5} = 777.39$$

From Eq. (6.40), the turbine pressure ratio is

$$\pi_t = \left(\frac{P_{rt5}}{P_{rt4}}\right)^{1/e_t} = \left(\frac{777.39}{1299.6}\right)^{1/0.9} = 0.5650$$

and the reduced pressure at state $t5i$ is

$$P_{rt5i} = P_{rt4}\pi_t = 1299.6 \times 0.5650 = 734.3$$

Linear interpolation of Appendix D, with $f = 0.0338$ and using the preceding reduced pressure, gives the following values of total enthalpy and total temperature for state $t5i$:

$$h_{t5\iota} = 718.34\ \text{Btu/lbm} \qquad \text{and} \qquad T_{t5\iota} = 2643.64°\text{R}$$

The turbine isentropic efficiency is

$$\eta_t = \frac{828.75 - 728.75}{828.75 - 718.34} = 0.9057$$

6.10.6 Nozzle

Because the nozzle is assumed to be adiabatic, then

$$\tau_n = \frac{h_{t9}}{h_{t8}} = 1 \tag{6.42}$$

By using Eq. (2.57), the inlet total pressure ratio can be expressed in terms of the entropy change as follows:

$$\pi_n = \frac{P_{t9}}{P_{t8}} = \exp\left(-\frac{s_9 - s_8}{R}\right) \tag{6.43}$$

The exit velocity is obtained from the difference between the total and static enthalpies as follows:

$$V_9 = \sqrt{2g_c(h_{t9} - h_9)} \tag{6.44}$$

where h_9 is obtained from the reduced pressure at state 9 (P_{r9}) given by

$$P_{r9} = \frac{P_{rt9}}{P_{t9}/P_9} \tag{6.45}$$

Problems

6.1 Calculate the total pressure recovery η_r, using Eq. (6.6), and the total pressure ratio across a normal shock in air for Mach numbers 1.25, 1.50, 1.75, and 2.0. How do they compare?

6.2 The isentropic efficiency for a diffuser with a calorically perfect gas can be written as

$$\eta_d = \frac{(P_{t2}/P_0)^{(\gamma-1)/\gamma} - 1}{T_{t0}/T_0 - 1}$$

Show that this equation can be rewritten in terms τ_r and π_d as

$$\eta_d = \frac{\tau_r \pi_d^{(\gamma-1)/\gamma} - 1}{\tau_r - 1}$$

6.3 Starting from Eqs. (6.14) and (6.21), show that the polytropic efficiency for the compressor and turbine can be expressed as

$$e_c = \frac{\gamma_c - 1}{\gamma_c}\frac{\ell n\, \pi_c}{\ell n\, \tau_c} \qquad e_t = \frac{\gamma_t}{\gamma_t - 1}\frac{\ell n\, \tau_t}{\ell n\, \pi_t}$$

6.4 A J-57B afterburning turbojet engine at maximum static power on a sea-level, standard day ($P_0 = 14.696$ psia, $T_0 = 518.7°$R, and $V_0 = 0$) has the data listed in Appendix B.

(a) Calculate the adiabatic and polytropic efficiencies of the low-pressure and high-pressure compressors. Assume $\gamma = 1.4$ for the low-pressure compressor and $\gamma = 1.39$ for the high-pressure compressor.
(b) Calculate the adiabatic and polytropic efficiencies of the turbine. Assume $\gamma = 1.33$.
(c) Calculate π_b, τ_b, π_{AB}, and τ_{AB}.

6.5 A J-57B afterburning turbojet engine had 167 lbm/s of air at 167 psia and 660°F enter the combustor (station 3) and products of combustion at 158 psia and 1570°F leave the combustor (station 4). If the fuel flow into the combustor was 8520 lbm/h, determine the combustor efficiency η_b, assuming $h_{PR} = 18{,}400$ Btu/lbm, $c_{pc} = 0.25$ Btu/(lbm · °R), and $c_{pt} = 0.26$ Btu/(lbm · °R).

6.6 A J-57B afterburning turbojet engine had 169.4 lbm/s of air at 36 psia and 1013°F enter the afterburner (station 6) and products of combustion at 31.9 psia and 2540°F leave the afterburner (station 7). If the fuel flow into the afterburner was 25,130 lbm/h, determine the afterburner efficiency η_{AB}, assuming $h_{PR} = 18{,}400$ Btu/lbm, $c_{pt} = 0.27$ Btu/(lbm · °R), and $c_{PAB} = 0.29$ Btu/(lbm · °R).

6.7 A JT9D high-bypass-ratio turbofan engine at maximum static power on a sea-level, standard day ($P_0 = 14.696$ psia, $T_0 = 518.7$°R, and $V_0 = 0$) has the data listed in Appendix B.
(a) Calculate the adiabatic and polytropic efficiencies of the fan. Assume $\gamma = 1.4$.
(b) Calculate the adiabatic and polytropic efficiencies of both the low-pressure and high-pressure compressors. Assume $\gamma = 1.4$ for the low-pressure compressor and $\gamma = 1.39$ for the high-pressure compressor.
(c) Calculate the adiabatic polytropic efficiencies of the turbine. Assume $\gamma = 1.35$.
(d) Calculate the power (horsepower and kilowatts) into the fan and compressors. Assume $Rg_c = 1716\ \text{ft}^2/(\text{s}^2 \cdot °\text{R})$ and the γ of parts b and c.
(e) Calculate the power (horsepower and kilowatts) from the turbine for a mass flow rate of 251 lbm/s. Assume $\gamma = 1.31$ and $Rg_c = 1716\ \text{ft}^2/(\text{s}^2 \cdot °\text{R})$.
(f) How do the results of parts d and e compare?

6.8 The isentropic efficiency for a nozzle η_n is defined as

$$\eta_n = \frac{h_{t7} - h_9}{h_{t7} - h_{9s}}$$

where the states $t7$, 9, and $9s$ are shown in Fig. P6.1. For $P_9 = P_0$, show that the isentropic efficiency for the nozzle in a turbojet engine can be written

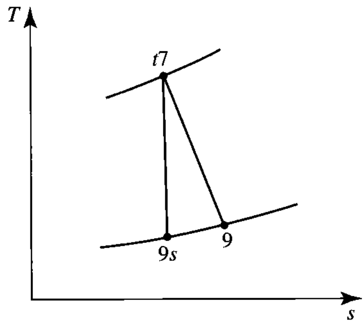

Fig. P6.1

for a calorically perfect gas as

$$\eta_n = \frac{\mathrm{PIT}^{(\gamma-1)/\gamma} - 1}{\mathrm{PIT}^{(\gamma-1)/\gamma} - (\pi_n)^{(\gamma-1)/\gamma}}$$

where PIT $= \pi_r \pi_d \pi_c \pi_b \pi_t \pi_n$.

6.9 Repeat Problem 6.4 with variable gas properties, using Appendix D or program AFPROP. Compare your results to those of Problem 6.4.

6.10 Repeat Problem 6.5 with variable gas properties, using Appendix D or program AFPROP. Compare your results to those of Problem 6.5.

6.11 Repeat Problem 6.6 with variable gas properties, using Appendix D or program AFPROP. Compare your results to those of Problem 6.6.

6.12 A JT9D high-bypass-ratio turbofan engine at maximum static power on a sea-level, standard day ($P_0 = 14.696$ psia, $T_0 = 518.7°$R, and $V_0 = 0$) has the data listed in Appendix B. Assuming 100% efficient combustion and $h_{PR} = 18{,}400$ Btu/lbm, calculate the fuel/air ratio, using Appendix D or program AFPROP and starting with an initial guess of 0.03. Now repeat Problem 6.7 with variable gas properties, using Appendix D or program AFPROP. Compare your results to those of Problem 6.7.