8
Engine Performance Analysis

8.1 Introduction

This chapter is concerned with predicting the performance of a gas turbine engine and obtaining performance data similar to Figs. 1.14a–1.14e, 1.15a, and 1.15b and the data contained in Appendix B. The analysis required to obtain engine performance is related to, but very different from, the parametric cycle analysis of Chapters 5 and 7. In parametric cycle analysis of a turbojet engine, we independently selected values of the compressor pressure ratio, main burner exit temperature, flight condition, etc. The analysis determined the turbine temperature ratio—it is dependent on the choices of compressor pressure ratio, main burner exit temperature, and flight condition, as shown by Eq. (7.12). In engine performance analysis, we consider the performance of an engine that was built (constructed physically or created mathematically) with a selected compressor pressure ratio and its corresponding turbine temperature ratio. As will be shown in this chapter, the turbine temperature ratio remains essentially constant for a turbojet engine (and many other engine cycles), and its compressor pressure ratio is dependent on the throttle setting (main burner exit temperature T_{t4}) and flight condition (M_0 and T_0). The basic independent and dependent variables of the turbojet engine are listed in Table 8.1 for both parametric cycle analysis and engine performance analysis.

In parametric cycle analysis, we looked at the variation of gas turbine engine cycles where the main burner exit temperature and aircraft flight conditions were specified via the design inputs: T_{t4}, M_0, T_0, and P_0, In addition, the engine cycle was selected along with the compressor pressure ratio, the polytropic efficiency of turbomachinery components, etc. For the combination of design input values, the resulting calculations yielded the specific performance of the engine (specific thrust and thrust specific fuel consumption), required turbine temperature ratio, and the efficiencies of the turbomachinery (fan, compressor, and turbine). The specific combination of design input values is referred to as the engine *design point* or *reference point.* The resulting specific engine thrust and fuel consumption are valid only for the given engine cycle and values of T_{t4}, M_0, T_0, π_c, τ_t, η_c, etc. When we changed any of these values in parametric cycle analysis,

Supporting Material for this chapter is available electronically. See page 869 for instructions to download.

Table 8.1 Comparison of analysis variables

Variable	Parametric cycle	Engine performance
Flight condition (M_0, T_0, and P_0)	Independent	Independent
Compressor pressure ratio π_c	Independent	Dependent
Main burner exit temperature T_{t4}	Independent	Independent
Turbine temperature ratio τ_t	Dependent	Constant

we were studying a "rubber" engine, i.e., one that changes its shape and component design to meet the thermodynamic, fluid dynamic, etc., requirements.

When a gas turbine engine is designed and built, the degree of variability of an engine depends on available technology, the needs of the principal application for the engine, and the desires of the designers. Most gas turbine engines have constant-area flow passages and limited variability (variable T_{t4}; and *sometimes variable* T_{t7} and exhaust nozzle throat area). In a simple constant-flow-area turbojet engine, the performance (pressure ratio and mass flow rate) of its compressor depends on the power from the turbine and the inlet conditions to the compressor. As we will see in this chapter, a simple analytical expression can be used to express the relationship between the compressor performance and the independent variables: throttle setting (T_{t4}) and flight condition (M_0, T_0, P_0).

When a gas turbine engine is installed in an aircraft, its performance varies with flight conditions and throttle setting and is limited by the engine control system. In flight, the pilot controls the operation of the engine directly through the throttle and indirectly by changing flight conditions. The thrust and fuel consumption will thereby change. In this chapter, we will look at how specific engine cycles perform at conditions other than their design (or reference) point.

There are several ways to obtain this engine performance. One way is to look at the interaction and performance of the compressor-burner-turbine combination, known as the *pumping characteristics* of the gas generator. In this case, the performance of the components is known because the gas generator exists. However, in a preliminary design, the gas generator has not been built, and the pumping characteristics are not available. In such a case, the gas generator performance can be estimated by using first principles and estimates of the variations in component efficiencies. In reality, the principal effects of engine performance occur because of the changes in *propulsive efficiency* and *thermal efficiency* (rather than because of changes in component efficiency). Thus a good approximation of an engine's performance can be obtained by simply assuming that the component efficiencies remain constant.

The analysis of engine performance requires a model for the behavior of each engine component over its actual range of operation. The more accurate and complete the model, the more reliable the computed results. Even though the approach (constant efficiency of rotating components and constant total pressure ratio of the other components) used in this textbook gives answers that are perfectly adequate for preliminary design, it is important to know that the usual industrial practice is to use data or correlations having greater accuracy and definition in the form of component "maps." The principal values of the maps are to

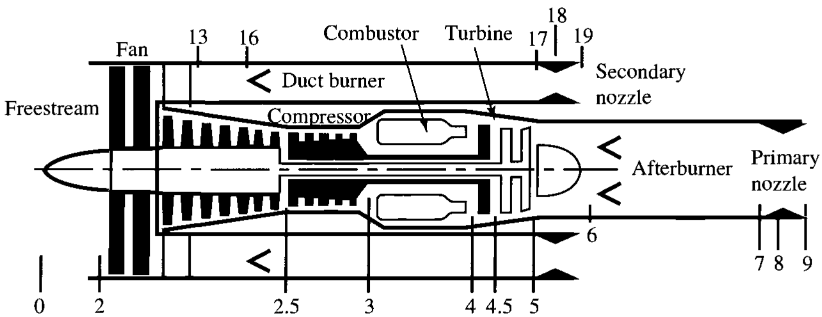

Fig. 8.1 Station numbering for two-spool gas turbine engine.

improve the understanding of component behavior and to slightly increase the accuracy of the results.

8.1.1 Nomenclature

The station numbering used for the performance analysis of the turbojet and turbofan is shown in Fig. 8.1. Note that the turbine is divided into a high-pressure turbine (station 4 to 4.5) and a low-pressure turbine (station 4.5 to 5). The high-pressure turbine drives the high-pressure compressor (station 2.5 to 3), and the low-pressure turbine drives the fan (station 2 to 13) and low-pressure compressor (station 2 to 2.5).

The assembly containing the high-pressure turbine, high-pressure compressor, and connecting shaft is called the *high-pressure spool.* That containing the low-pressure turbine, fan or low-pressure compressor, and connecting shaft is called the *low-pressure spool.* In addition to the τ and π values defined in Table 5.1, the component total temperature ratios and total pressure ratios listed in Table 8.2 are required for analysis of the gas turbine engine with high- and low-pressure spools.

8.1.2 Reference Values and Engine Performance Analysis Assumptions

Functional relationships are used to predict the performance of a gas turbine engine at different flight conditions and throttle settings. These relationships are based on the application of mass, energy, momentum, and entropy considerations to the one-dimensional steady flow of a perfect gas at an engine

Table 8.2 Additional temperature and pressure relationships

$\tau_{cH} = \dfrac{T_{t3}}{T_{t2.5}}$	$\pi_{cH} = \dfrac{P_{t3}}{P_{t2.5}}$	$\tau_{tH} = \dfrac{T_{t4.5}}{T_{t4}}$	$\pi_{tH} = \dfrac{P_{t4.5}}{T_{t4}}$
$\tau_{cL} = \dfrac{T_{t2.5}}{T_{t2}}$	$\pi_{cL} = \dfrac{P_{t2.5}}{P_{t2}}$	$\tau_{tL} = \dfrac{T_{t5}}{T_{t4.5}}$	$\pi_{tL} = \dfrac{P_{t5}}{P_{t4.5}}$
$\tau_c = \tau_{cL}\tau_{cH}$	$\pi_c = \pi_{cL}\pi_{cH}$	$\tau_t = \tau_{tH}\tau_{tL}$	$\pi_t = \pi_{tH}\pi_{tL}$

steady-state operating point. Thus, if

$$f(\tau, \pi) = \text{const}$$

represents a relationship between the two engine variables τ and π at a steady-state operating point, then the constant can be evaluated at a reference condition (subscript R) so that

$$f(\tau, \pi) = f(\tau_R, \pi_R) = \text{const}$$

since $f(\tau, \pi)$ applies to the engine at all operating points. *Sea-level static (SLS) is the normal reference condition (design point) for the value of the gas turbine engine variables.* This technique for replacing constants with reference conditions is frequently used in the analysis to follow.

For conventional turbojet, turbofan, and turboprop engines, we will consider the simple case where the high-pressure turbine entrance nozzle, low-pressure turbine entrance nozzle, and primary exit nozzle (and bypass duct nozzle for the separate-exhaust turbofan) are choked. In addition, we assume that the throat areas where choking occurs in the high-pressure turbine entrance nozzle and the low-pressure turbine entrance nozzle are constant. This type of turbine is known as a *fixed-area turbine* (FAT) engine. These assumptions are true over a wide operating range for modern gas turbine engines. The following performance analyses also include the case(s) of unchoked engine exit nozzle(s).

The following assumptions will be made in the turbojet and turbofan performance analysis:

1) The flow is choked at the high-pressure turbine entrance nozzle, low-pressure turbine entrance nozzle, and the primary exit nozzle. Also the bypass duct nozzle for the turbofan is choked.

2) The total pressure ratios of the main burner, primary exit nozzle, and bypass stream exit nozzle (π_b, π_n, and π_{fn}) do not change from their reference values.

3) The component efficiencies (η_c, η_f, η_b, η_{tH}, η_{tL}, η_{mH}, and η_{mL}) do not change from their reference values.

4) Turbine cooling and leakage effects are neglected.

5) No power is removed from the turbine to drive accessories (or alternately, η_{mH} or η_{mL} includes the power removed but is still constant).

6) Gases will be assumed to be calorically perfect both upstream and downstream of the main burner, and γ_t and c_{pt} do not vary with the power setting (T_{t4}).

7) The term unity plus the fuel/air ratio $(1 + f)$ will be considered as a constant.

Assumptions 4 and 5 are made to simplify the analysis and increase understanding. Reference 12 includes turbine cooling air, compressor bleed air, and power takeoff in the performance analysis. Assumptions 6 and 7 permit easy analysis that results in a set of algebraic expressions for an engine's performance. The performance analysis of an engine with variable gas properties is covered in the Supporting Material Section SM8.3.

8.1.3 Dimensionless and Corrected Component Performance Parameters

Dimensional analysis identifies correlating parameters that allow data taken under one set of conditions to be extended to other conditions. These parameters are useful and necessary because it is always impractical to accumulate experimental data for the bewildering number of possible operating conditions, and because it is often impossible to reach many of the operating conditions in a single, affordable facility.

The quantities of pressure and temperature are normally made dimensionless by dividing each by its respective standard sea-level static values. The dimensionless pressure and temperature are represented by δ and θ, respectively, and given in Eqs. (1.2) and (1.3) When total (stagnation) properties are nondimensionalized, a subscript is used to indicate the station number of that property. The only static properties made dimensionless are freestream, the symbols for which carry no subscripts. Thus

$$\delta_i \equiv \frac{P_{ti}}{P_{\text{ref}}} \tag{8.1a}$$

and

$$\theta_i \equiv \frac{T_{ti}}{T_{\text{ref}}} \tag{8.1b}$$

where $P_{\text{ref}} = 14.696$ psia (101,300 Pa) and $T_{\text{ref}} = 518.69°\text{R}$ (288.2 K).

Dimensionless analysis of engine components yields many useful dimensionless and/or modified component performance parameters. Some examples of these are the compressor pressure ratio, adiabatic efficiency, Mach number at the compressor face, ratio of blade (tip) speed to the speed of sound, and the Reynolds number.

The *corrected mass flow rate* at engine station i used in this analysis is defined as

$$\dot{m}_{ci} \equiv \frac{\dot{m}_i\sqrt{\theta_i}}{\delta_i} \tag{8.2}$$

and is related to the Mach number at station i as shown in the following. From the definition of the mass flow parameter [Eq. (2.75)], we can write the mass flow at station i as

$$\dot{m}_i = \frac{P_{ti}}{\sqrt{T_{ti}}} A_i \times \text{MFP}(M_i)$$

Then

$$\frac{\dot{m}_{ci}}{A_i} = \frac{\dot{m}_{ci}\sqrt{T_{ti}}}{A_i P_{ti}} \frac{P_{\text{ref}}}{\sqrt{T_{\text{ref}}}} = \frac{P_{\text{ref}}}{\sqrt{T_{\text{ref}}}} \text{MFP}(M_i) \tag{8.3}$$

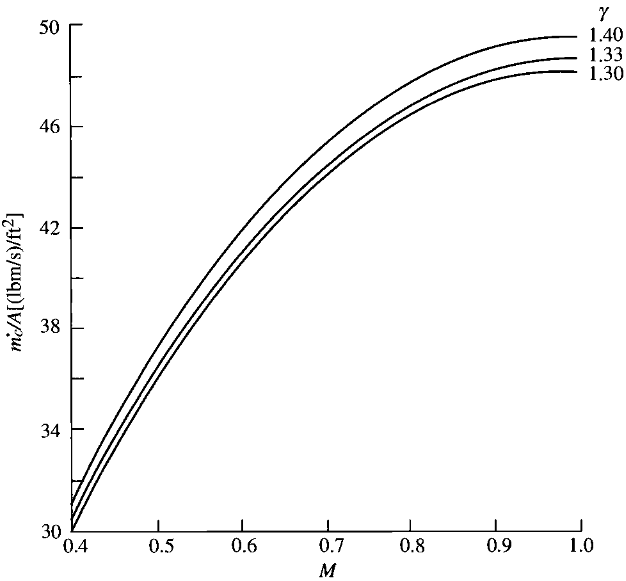

Fig. 8.2 Variation of corrected mass flow per area.

and the corrected mass flow rate per unit area is a function of the Mach number alone for a gas. Equation (8.3) is plotted vs Mach number in Fig. 8.2 for three different γ values. Aircraft gas turbine engines need high thrust or power per unit weight that requires high corrected mass flow rates per unit area.

At the entrance to the fan or compressor (station 2), the design Mach number is about 0.56, which corresponds to a corrected mass flow rate per unit area of about 40 lbm/(s · ft^2). A reduction in engine power will lower the corrected mass flow rate and the corresponding Mach number into the fan or compressor.

The flow is normally choked at the entrance to the turbine (station 4) and the throat of the exhaust nozzle (station 8) for most steady-state operating conditions of interest (the flow is typically unchoked at these stations during engine startup). When the flow is choked at station 4, the corrected mass flow rate per unit area entering the turbine is constant, which helps define the pumping characteristics of the gas generator. As shown later in this chapter, choked flow at both stations 4 and 8 limits the turbine operation. Even if the flow unchokes at a station and the Mach number drops from 1.0 to 0.9, the corrected mass flow rate is reduced less than 1%. Thus the corrected mass flow rate is considered constant when the flow is near or at choking conditions.

Choked flow at station 8 is desired in convergent-only exhaust nozzles to obtain high exit velocity and is required in a convergent-divergent exhaust nozzle to reach supersonic exit velocities. When the afterburner is operated on a turbojet or turbofan engine with choked exhaust nozzle, T_{t8} increases—this requires an increase in the nozzle throat area A_8 to maintain the correct mass flow rate/area ratio corresponding to choked conditions. If the nozzle throat is not increased, the pressure increases and the mass flow rate decreases, which can adversely impact the upstream engine components.

The *corrected engine speed* at engine station i used in this analysis is defined as

$$N_{ci} \equiv \frac{N}{\sqrt{\theta_i}} \tag{8.4}$$

and is related to the *blade* Mach number.

These four parameters represent a first approximation of the complete set necessary to reproduce nature for the turbomachinery. These extremely useful parameters have become a standard in the gas turbine industry and are summarized in Table 8.3.

Three additional corrected quantities have found common acceptance for describing the performance of gas turbine engines: corrected thrust F_c, corrected thrust specific fuel consumption S_c, and corrected fuel mass flow rate $\dot{m}_{fc}$.

The *corrected thrust* is defined as

$$F_c \equiv \frac{F}{\delta_0} \tag{8.5}$$

For many gas turbine engines operating at maximum T_{t4}, the corrected thrust is essentially a function of only the corrected freestream total temperature θ_0.

The *corrected thrust-specific fuel consumption* is defined as

$$S_c \equiv \frac{S}{\sqrt{\theta_0}} \tag{8.6}$$

Table 8.3 Corrected parameters

Parameter	Symbol	Corrected parameter
Total pressure	P_{ti}	$\delta_i = \dfrac{P_{ti}}{P_{\text{ref}}}$
Total temperature	T_{ti}	$\theta_i = \dfrac{T_{ti}}{T_{\text{ref}}}$
Rotational speed	$N = \text{RPM}$	$N_{ci} = \dfrac{N}{\sqrt{\theta_i}}$
Mass flow rate	$\dot{m}_i$	$\dot{m}_{ci} = \dfrac{\dot{m}_t\sqrt{\theta_i}}{\delta_i}$
Thrust	F	$F_c = \dfrac{F}{\delta_0}$
Thrust-specific fuel consumption	S	$S_c = \dfrac{S}{\sqrt{\theta_0}}$
Fuel mass flow rate	$\dot{m}_f$	$\dot{m}_{fc} = \dfrac{\dot{m}_f}{\delta_2\sqrt{\theta_2}}$

and the *corrected fuel mass flow rate is* defined as

$$\dot{m}_{fc} \equiv \frac{\dot{m}_f}{\delta_2\sqrt{\theta_2}} \tag{8.7}$$

Like the corrected thrust, these two corrected quantities collapse the variation in fuel consumption with flight condition and throttle setting.

These three corrected quantities are closely related. By using the equation for thrust-specific fuel consumption

$$S = \frac{\dot{m}_f}{F}$$

$\pi_d = P_{t2}/P_{t0}$, and the fact that $\theta_2 = \theta_0$, the following relationship results between these corrected quantities:

$$S_c = \pi_d \frac{\dot{m}_{fc}}{F_c} \tag{8.8}$$

These extremely useful corrected engine performance parameters have also become a standard in the gas turbine industry and are included in Table 8.3.

8.1.4 Component Performance Maps

8.1.4.1 Compressor and fan performance maps. The performance of a compressor or fan is normally shown by using the total pressure ratio, corrected mass flow rate, corrected engine speed, and component efficiency. Most often this performance is presented in one map showing the interrelationship of all four parameters, like that depicted in Fig. 8.3. Sometimes, for clarity, two maps are used, with one showing the pressure ratio vs corrected mass flow rate/corrected

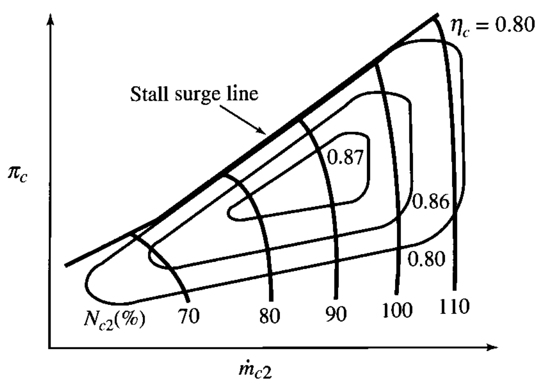

Fig. 8.3 Typical compressor performance map.

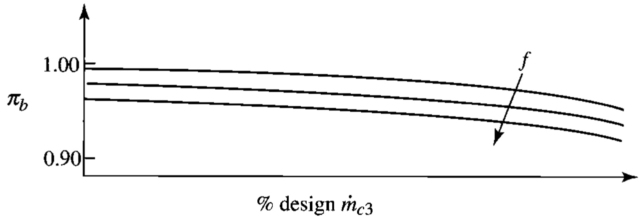

Fig. 8.4a Combustor pressure ratio.

speed and the other showing compressor efficiency vs corrected mass flow rate/corrected speed.

A limitation on fan and compressor performance of special concern is the *stall* or *surge line.* Steady operation above the line is impossible, and entering the region even momentarily is dangerous to the gas turbine engine.

8.1.4.2 Main burner maps. The performance of the main burner is normally presented in terms of its performance parameters that are most important to engine performance: total pressure ratio of the main burner π_b and its combustion efficiency η_b. The total pressure ratio of the main burner is normally plotted vs the corrected mass flow rate through the burner $(\dot{m}_3\sqrt{\theta_3}/\delta_3)$ for different fuel/air ratios f, as shown in Fig. 8.4a. The efficiency of the main burner can be represented as a plot vs the temperature rise in the main burner $T_{t4} - T_{t3}$ or fuel/air ratio f for various values of inlet pressures P_{t3}, as shown in Fig. 8.4b.

8.1.4.3 Turbine maps. The flow through a turbine first passes through stationary airfoils (often called *inlet guide vanes* or *nozzles*) that turn and accelerate the fluid, increasing its tangential momentum. The flow then passes through rotating airfoils (called *rotor blades*) that remove energy from the fluid as they

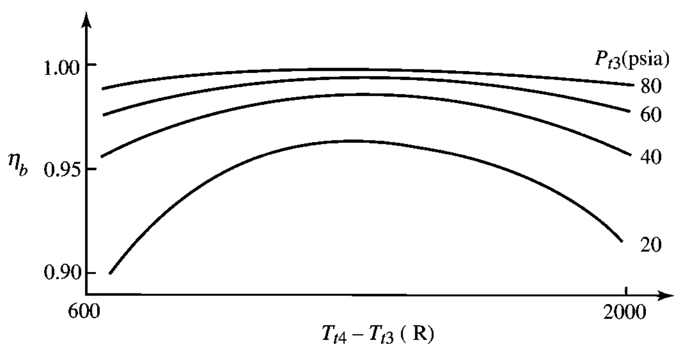

Fig. 8.4b Combustor efficiency.

change its tangential momentum. Successive pairs of stationary airfoils followed by rotating airfoils remove additional energy from the fluid. To obtain a high output power/weight ratio from a turbine, the flow entering the first-stage turbine rotor is normally supersonic, which requires the flow to pass through sonic conditions at the minimum passage area in the inlet guide vanes (nozzles). By using Eq. (8.3), the corrected inlet mass flow rate based on this minimum passage area (throat) will be constant for fixed-inlet-area turbines. This flow characteristic is shown in the typical turbine flow map (Fig. 8.5a) when the expansion ratio across the turbine $[(P_{t4}/P_{t5}) = 1/\pi_t)]$ is greater than about 2 and the flow at the throat is choked.

The performance of a turbine is normally shown by using the total pressure ratio, corrected mass flow rate, corrected turbine speed, and component efficiency. This performance can be presented in two maps or a combined map (similar to that shown for the compressor in Fig. 8.3). When two maps are used, one map shows the interrelationship of the total pressure ratio, corrected mass flow rate, and corrected turbine speed, like that depicted in Fig. 8.5a. The other map shows the interrelationship of turbine efficiency vs corrected mass flow rate/expansion ratio, like that shown in Fig. 8.5b. When a combined map is used, the total pressure ratio of the turbine is plotted vs the product of corrected mass flow rate and the corrected speed, as shown in Fig. 8.5c. This spreads out the lines of constant corrected speed from those shown in Fig. 8.5a, and the turbine efficiency can now be shown. If we tried to add these lines of constant turbine efficiency to Fig. 8.5a, many would coincide with the line for choked flow.

For the majority of aircraft gas turbine engine operation, the turbine efficiency varies very little. In the analysis of this chapter, we consider that the turbine efficiency is constant.

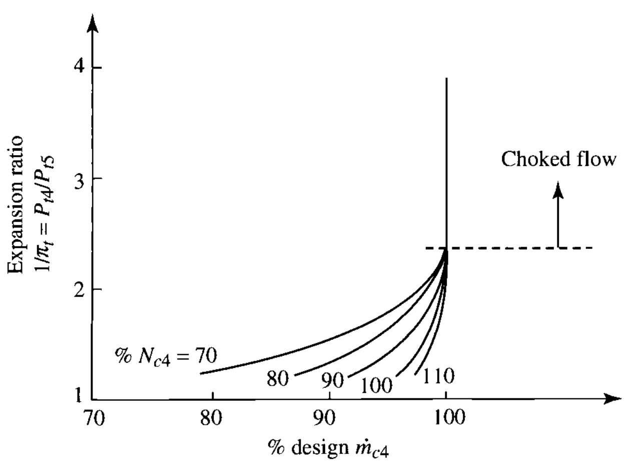

Fig. 8.5a Typical turbine flow map.

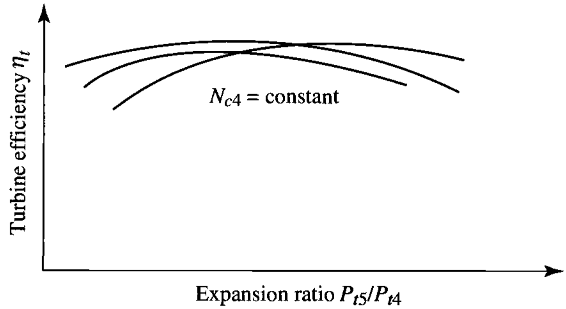

Fig. 8.5b Typical turbine efficiency map.

8.2 Gas Generator

The performance of a gas turbine engine depends on the operation of its gas generator. In this section, algebraic expressions for the pumping characteristics of a simple gas turbine engine are developed.

8.2.1 Conservation of Mass

We consider the flow through a single-spool turbojet engine with constant inlet area to the turbine (A_4 = constant). The mass flow rate into the turbine is equal to the sum of the mass flow rate through the compressor and the fuel flow rate into the main burner. Using the *mass flow parameter* (MFP), we can write

$$\dot{m}_2 + \dot{m}_f = (1 + f)\dot{m}_2 = \dot{m}_4 = \frac{P_{t4}A_4}{\sqrt{T_{t4}}}\mathrm{MFP}(M_4)$$

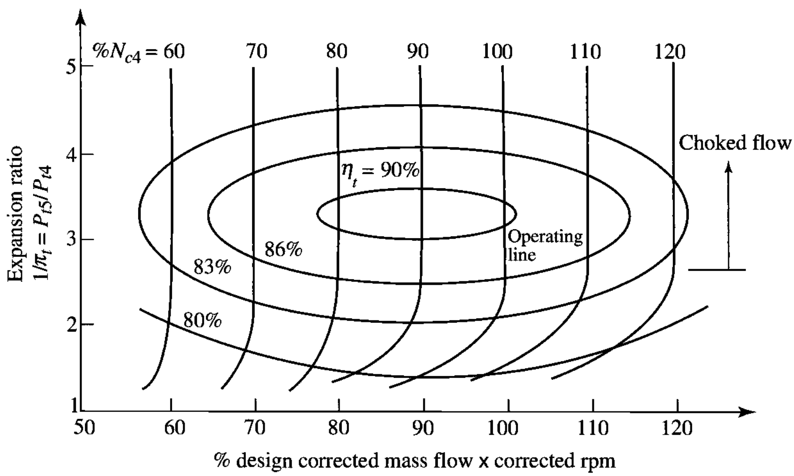

Fig. 8.5c Combined turbine performance map.

With the help of Eq. (8.3), the preceding equation yields the following expression for the compressor corrected mass flow rate:

$$\dot{m}_{c2} = \sqrt{\frac{T_{t2}}{T_{\text{ref}}}}\frac{P_{\text{ref}}}{P_{t2}}\frac{P_{t4}}{\sqrt{T_{t4}}}\frac{A_4}{1+f}\text{MFP}(M_4)$$

Noting that $P_{t4} = \pi_c \pi_b P_{t2}$, we see that

$$\dot{m}_{c2} = \left(\frac{T_{t2}}{T_{t4}}\right)^{1/2} \pi_c \pi_b \frac{P_{\text{ref}}}{\sqrt{T_{\text{ref}}}}\frac{A_4}{1+f}\text{MFP}(M_4) \tag{8.9}$$

Equation (8.9) is a straight line on a compressor map for constant values of $T_{t4}/T_{t2}, A_4, f$, and M_4. Lines of constant T_{t2}/T_{t4} are plotted on a typical compressor map in Figs. 8.6a and 8.6b for constant values of A_4 and f. Note that these lines start at a pressure ratio of 1 and corrected mass flow rate of 0 and are curved for low compressor pressure ratios (see Fig. 8.6b) because station 4 is unchoked. Station 4 chokes at a pressure ratio of about 2. At pressure ratios above 2, these lines are straight and appear to start at the origin (pressure ratio of 0 and mass

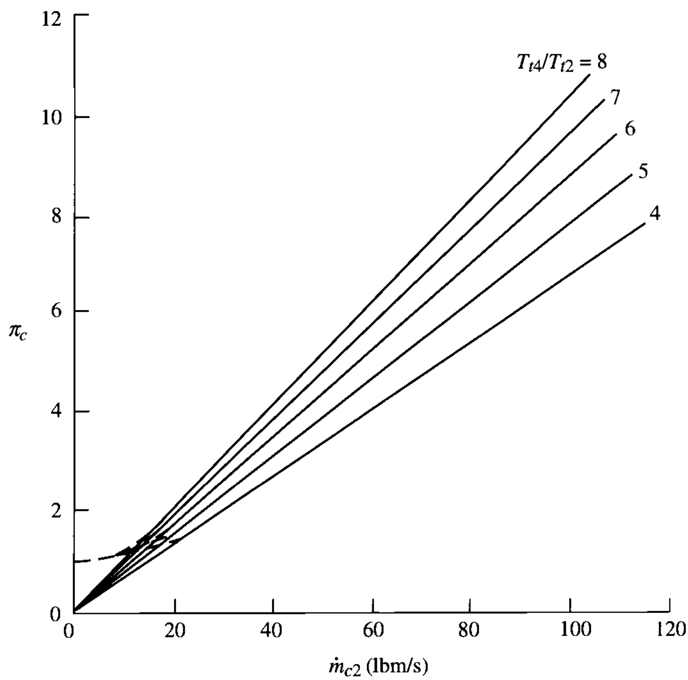

Fig. 8.6a Compressor map with lines of constant T_{t4}/T_{t2}.

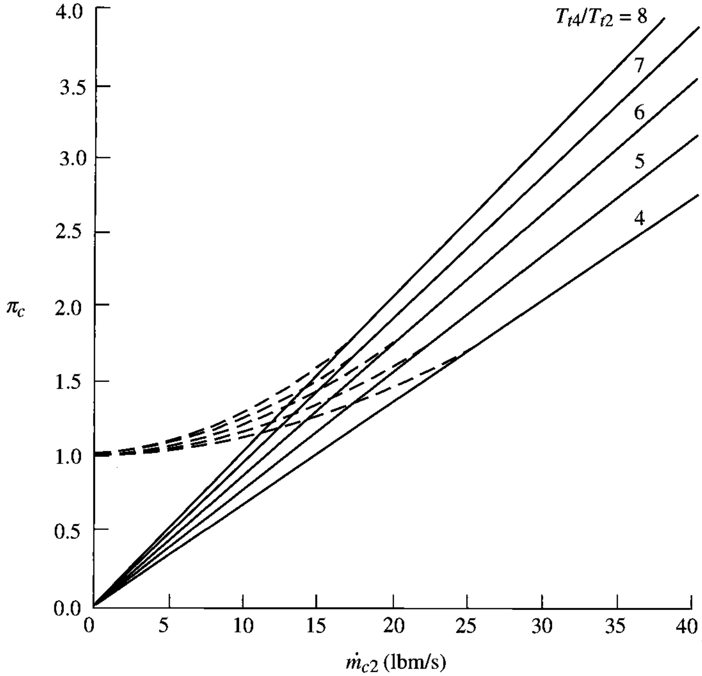

Fig. 8.6b Compressor map origin with lines of constant T_{t4}/T_{t2}.

flow rate of 0). The lines of constant T_{t2}/T_{t4} show the general characteristics required to satisfy conservation of mass and are independent of the turbine. For a given T_{t4}/T_{t2}, any point on that line will satisfy mass conservation for engine stations 2 and 4. The actual operating point of the compressor depends on the turbine and exhaust nozzle.

Equation (8.9) can be written simply for the case when station 4 is choked (the normal situation in gas turbine engines) as

$$\dot{m}_{c2} = C_1 \frac{\pi_c}{\sqrt{T_{t4}/T_{t2}}} \tag{8.10}$$

For an engine or gas generator, the specific relationship between the compressor pressure ratio and corrected mass flow rate is called the *compressor operating line* and depends on the characteristics of the turbine. The equation for the operating line is developed later in this section.

8.2.2 Turbine Characteristics

Before developing the equations that predict the operating characteristics of the turbine, we write the mass flow parameter at any station i in terms of the mass flow rate, total pressure, total temperature, area, and Mach number. Since

$$\frac{\dot{m}_i\sqrt{T_{ti}}}{P_{ti}A_i} = \text{MFP}(M_i) = \sqrt{\frac{\gamma_i g_c}{R_i}} M_i \left(1 + \frac{\gamma_i - 1}{2} M_i^2\right)^{-(\gamma_i+1)/[2(\gamma_i-1)]}$$

Then, for $M_i = 1$,

$$\frac{\dot{m}_i\sqrt{T_{ti}}}{P_{ti}A_i} = \text{MFP}(M_i = 1) = \sqrt{\frac{\gamma_i g_c}{R_i}} \left(\frac{2}{\gamma_i + 1}\right)^{(\gamma_i+1)/[2(\gamma_i-1)]} = \frac{\Gamma_i}{\sqrt{R_i/g_c}} \tag{8.11a}$$

where

$$\Gamma_i \equiv \sqrt{\gamma_i}\left(\frac{2}{\gamma_i + 1}\right)^{(\gamma_i+1)/[2(\gamma_i-1)]} \tag{8.11b}$$

For a turbojet engine, the flow is choked ($M = 1$) in the turbine inlet guide vanes (station 4) and nearly at the throat of the exhaust nozzle (station 8). Thus the corrected mass flow rate per unit area is constant at station 4 and

$$\dot{m}_4 = \frac{P_{t4}A_4}{\sqrt{T_{t4}}} \frac{\Gamma_4}{\sqrt{R_4/g_c}} \qquad \dot{m}_8 = \frac{P_{t8}A_8}{\sqrt{T_{t8}}} \text{MFP}(M_8) \tag{i}$$

For a simple turbojet engine, the mass flow rate through the turbine is equal to that through the exhaust nozzle, or

$$\dot{m}_8 = \dot{m}_4$$

Using Eq. (i) then, we have

$$\frac{\sqrt{T_{t8}/T_{t4}}}{P_{t8}/P_{t4}} = \frac{A_8}{A_4} \frac{\text{MFP}(M_8)}{\Gamma_4/\sqrt{R_4/g_c}}$$

or

$$\frac{\sqrt{\tau_t}}{\pi_t} = \frac{A_8}{A_4} \frac{\text{MFP}(M_8)}{\Gamma_4/\sqrt{R_4/g_c}} \tag{8.12a}$$

where

$$\pi_t = \left(1 - \frac{1 - \tau_t}{\eta_t}\right)^{\gamma_t/(\gamma_t-1)} \tag{8.12b}$$

For constant turbine efficiency η_t, constant values of R and Γ, constant areas at stations 4 and 8, and choked flow at station 8, Eqs. (8.12a) and (8.12b) can be satisfied only by constant values of the turbine temperature ratio τ_t and the turbine pressure ratio π_t. Thus we have

$$\tau_t, \ \pi_t \text{ constant} \qquad \text{for } M_8 = 1 \text{ and constant } A_4 \text{ and } A_8$$

If the exhaust nozzle unchokes and/or its throat area is changed, then both τ_t and π_t will change. Consider a turbine with reference values of $\eta_t = 0.90$ and $\tau_t = 0.80$ when the exhaust nozzle is choked and the gas has $\gamma = 1.33$. From Eqs. (8.12a) and (8.12b), $\pi_t = 0.363174$ and $A_8/A_4 = 2.46281$ at reference conditions. Figure 8.7a shows plots of Eq. (8.12a) for different values of the area ratio A_8/A_4 times the mass flow parameter at station 8 [MFP(M_8)] and Eq. (8.12b). Because of the relative slopes of these equations, the changes of both τ_t and π_t with A_8 and M_8 can be found by using the following functional iteration scheme, starting with an initial value of τ_t: 1) solve for π_t, using Eq. (8.12a); 2) calculate a new τ_t, using Eq. (8.12b); 3) repeat steps 1 and 2 until successive values of τ_t are within a specified range (say, ± 0.0001). The results of this iteration, plotted in Figs. 8.7b and 8.7c, show that when the Mach number M_8 is

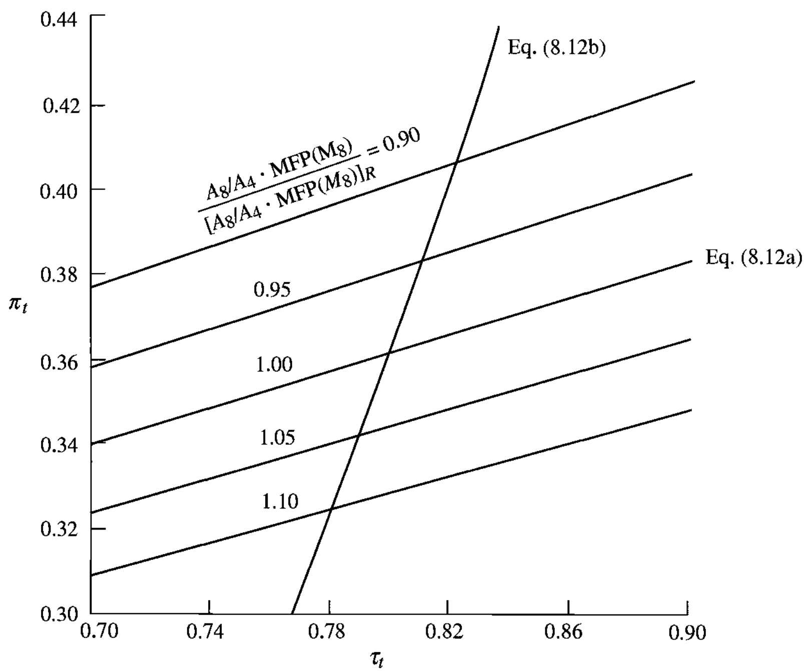

Fig. 8.7a Plot of turbine performance Eqs. (8.12a) and (8.12b).

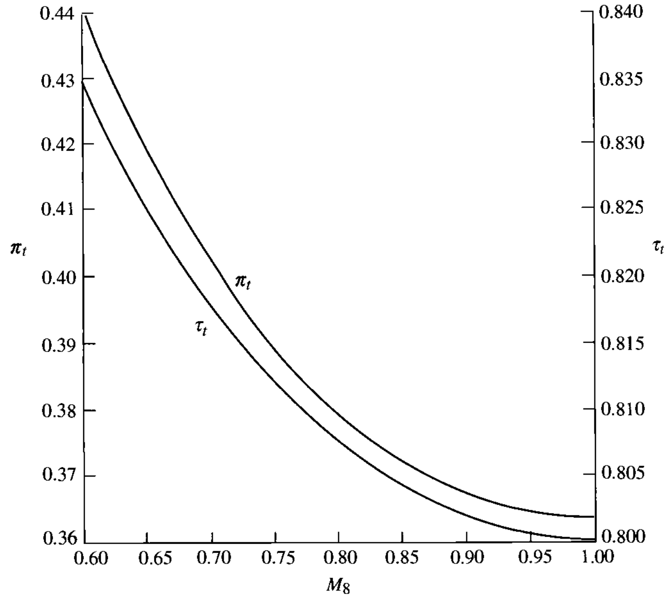

Fig. 8.7b Variation of turbine performance with exhaust nozzle Mach number.

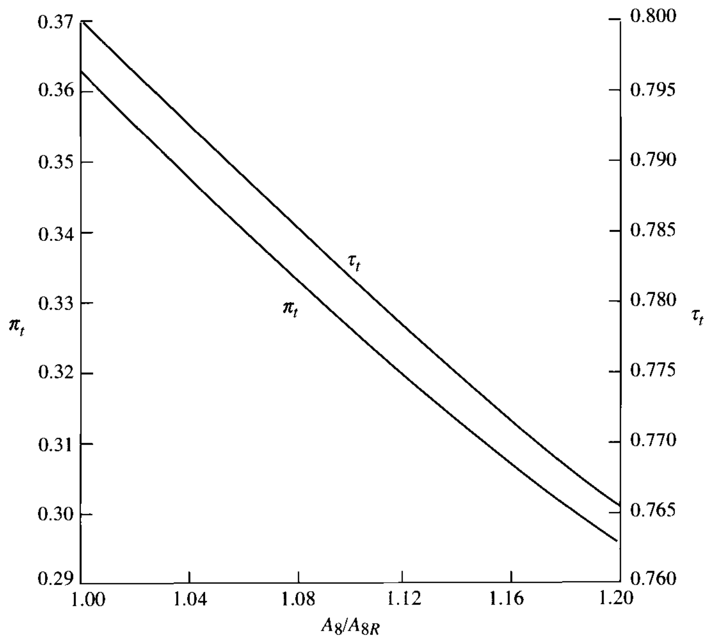

Fig. 8.7c Variation of turbine performance with exhaust nozzle area.

reduced from choked conditions ($M_8 = 1$), both τ_t and π_t increase; and when the exhaust nozzle throat area A_8 is increased from its reference value, both τ_t and π_t decrease. A decrease in τ_t, with its corresponding decrease in π_t, will increase the turbine power per unit mass flow and change the pumping characteristics of the gas generator.

8.2.3 Compressor Operating Line

From a work balance between the compressor and turbine, we write

$$\dot{m}_2 c_{pc}(T_{t3} - T_{t2}) = \eta_m \dot{m}_2 (1 + f) c_{pt}(T_{t4} - T_{t5})$$

or

$$\tau_c = 1 + \frac{T_{t4}}{T_{t2}} \frac{c_{pt}}{c_{pc}} \eta_m (1 + f)(1 - \tau_t) \tag{8.13}$$

where

$$\pi_c = [1 + \eta_c(\tau_c - 1)]^{\gamma_c/(\gamma_c - 1)} \tag{ii}$$

Combining Eqs. (8.13) and (ii) gives

$$\pi_c = \left\{1 + \frac{T_{t4}}{T_{t2}} \left[\frac{c_{pt}}{c_{pc}} \eta_c \eta_m (1 + f)(1 - \tau_t)\right]\right\}^{\gamma_c/(\gamma_c - 1)} \tag{8.14}$$

where the term in square brackets can be considered a constant when τ_t is constant. Solving Eq. (8.14) for the temperature ratio gives

$$\frac{T_{t4}}{T_{t2}} = C_2[(\pi_c)^{(\gamma_c - 1)/\gamma_c} - 1]$$

where C_2 represents the reciprocal of the constant term within the square brackets of Eq. (8.14). Combining this equation with Eq. (8.10) gives an equation for the *compressor operating line* that can be written as

$$\dot{m}_{c2} = \frac{\pi_c}{\sqrt{\pi_c^{(\gamma_c - 1)/\gamma_c} - 1}} \frac{C_1}{\sqrt{C_2}} \qquad \text{for constant } \tau_t \tag{8.15}$$

We can plot the *compressor operating line*, using Eq. (8.15), on the compressor map of Fig. 8.6a, giving the compressor map with operating line shown in Fig. 8.8. This compressor operating line shows that for each value of the temperature ratio T_{t2}/T_{t4} there is one value of compressor pressure ratio and corrected

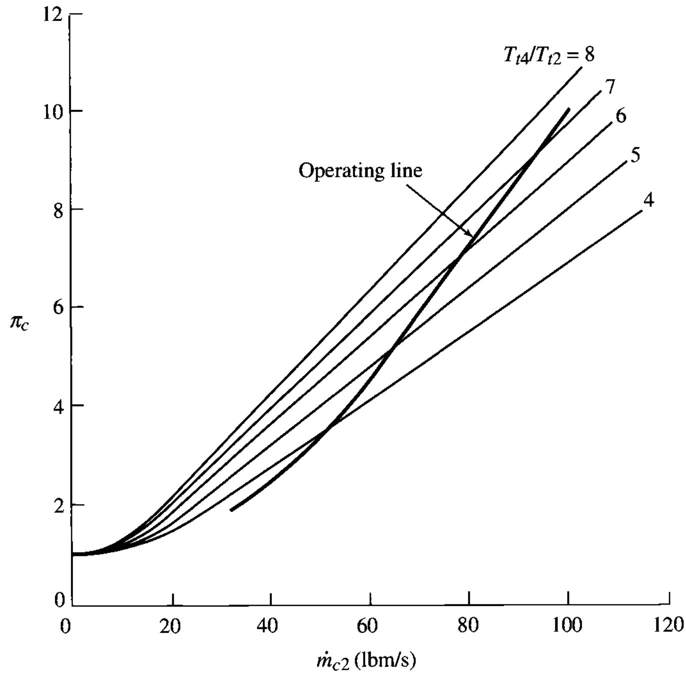

Fig. 8.8 Compressor map with operating line.

mass flow rate. One can also see that for a constant value of T_{t2}, both the compressor pressure ratio and the corrected mass flow rate will increase with increases in throttle setting (increases in T_{t4}). In addition, when at constant T_{t4}, the compressor pressure ratio and corrected mass flow rate will decrease with increases in T_{t2} due to higher speed and/or lower altitude (note: $T_{t2} = T_{t0} = T_0\tau_r$). The curving of the operating line in Fig. 8.8 at pressure ratios below 4 is due to the exhaust nozzle being unchoked ($M_8 < 1$), which increases the value of τ_t (see Fig. 8.7b).

The compressor operating line defines the pumping characteristics of the gas generator. As mentioned earlier, changing the throat area of the exhaust nozzle A_8 will change these characteristics. It achieves this change by shifting the compressor operating line. Increasing A_8 will decrease τ_t (see Fig. 8.7c). This decrease in τ_t will increase the term within the square brackets of Eq. (8.14) that corresponds to the reciprocal of constant C_2 in Eq. (8.15). Thus an increase in A_8 will decrease the constant C_2 in Eq. (8.15). For a constant T_{t4}/T_{t2}, this shift in the operating line will increase both the corrected mass flow rate and the pressure ratio of the compressor, as shown in Fig. 8.9 for a 20% increase in A_8. For some compressors, an increase in the exhaust nozzle throat area A_8 can keep engine operation away from the surge.

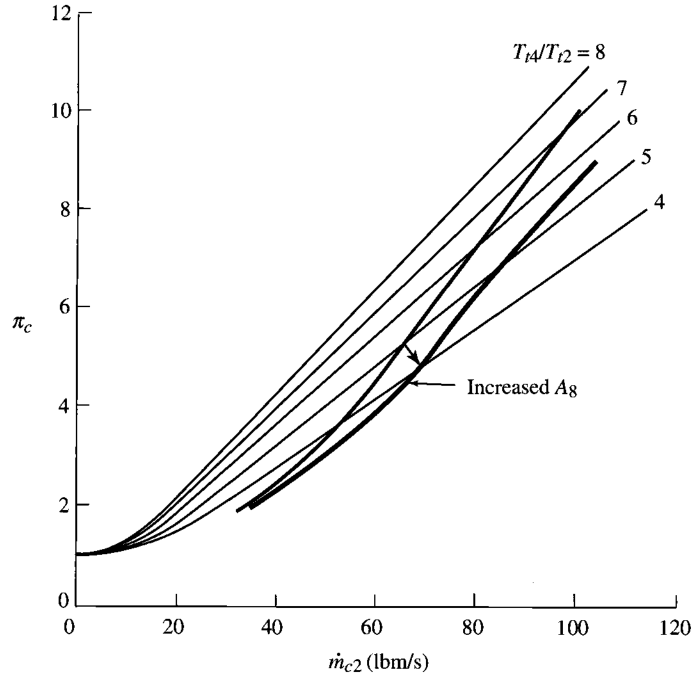

Fig. 8.9 Effect of exhaust nozzle area on compressor operating line.

8.2.4 Engine Controls

The engine control system will control the gas generator operation to keep the main burner exit temperature T_{t4}, the compressor's pressure ratio π_c, rotational speed N, exit total pressure T_{t3}, and exit total pressure P_{t3} from exceeding specific maximum values. Limiting T_{t4} and π_c has the most dominating effect and is included in the following analysis. An understanding of the influence of the engine control system on compressor performance during changing flight conditions and throttle settings can be gained by recasting Eqs. (8.10) and (8.14) in terms of the dimensionless total temperature at station 0 (θ_0). We note that

$$T_{t0} = T_{\text{ref}} \frac{T_{t0}}{T_{\text{ref}}} = T_{\text{ref}}\,\theta_0$$

and

$$\theta_0 = \frac{T_0}{T_{\text{ref}}} \tau_r = \frac{T_0}{T_{\text{ref}}} \left(1 + \frac{\gamma - 1}{2} M_0^2\right) \tag{8.16}$$

Equation (8.16) and Figs. 8.10 and 8.11 show that θ_0 includes the influence of both the altitude (through the ambient temperature T_0) and the flight Mach number. Although Fig. 8.10 shows the direct influence of Mach number and

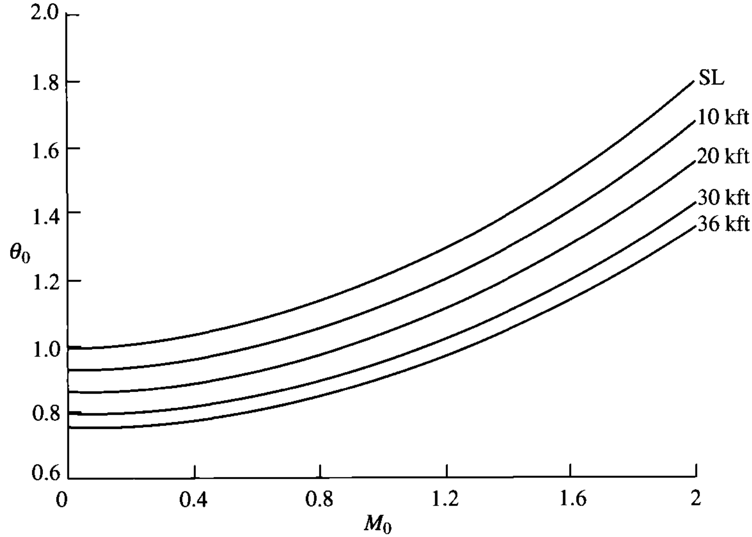

Fig. 8.10 θ_0 **vs Mach number at different altitudes (standard day).**

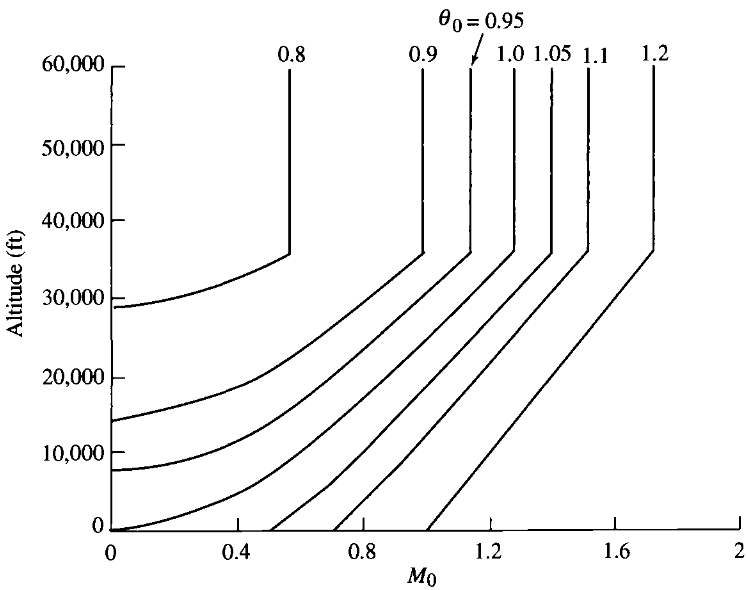

Fig. 8.11 θ_0 **vs Mach number and altitude (standard day).**

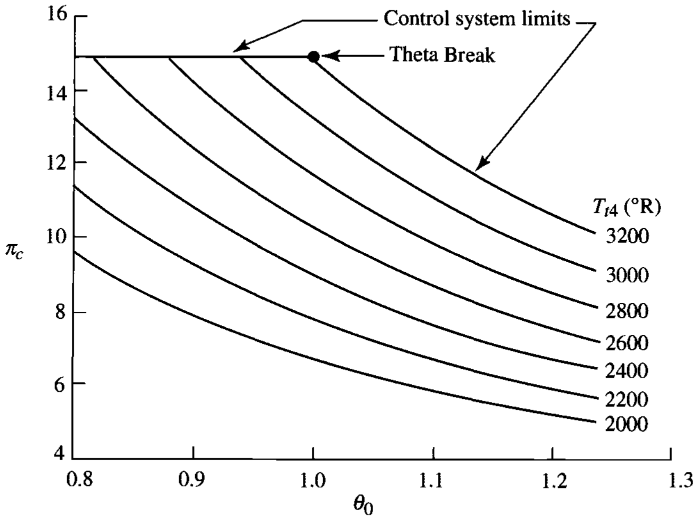

Fig. 8.12 Compressor pressure ratio vs θ_0 and T_{t4}.

altitude on θ_0, Fig. 8.11 is an easier plot to understand in terms of aircraft flight conditions (Mach number and altitude).

Using Eq. (8.16) and the fact that $T_{t2} = T_{t0}$, we can write Eq. (8.14) as

$$\pi_c = \left(1 + \frac{T_{t4}}{\theta_0} K_1\right)^{\gamma_c/(\gamma_c - 1)} \tag{8.17}$$

where K_1 is a constant. Equation (8.17) is plotted in Fig. 8.12 for the turbojet engine of Example 8.1.

By using Eqs. (8.17) and (8.10), the corrected mass flow rate through the compressor can be expressed as

$$\dot{m}_{c2} = \left(\frac{\theta_0}{T_{t4}}\right)^{1/2} \left(1 + \frac{T_{t4}}{\theta_0} K_1\right)^{\gamma_c/(\gamma_c - 1)} K_2 \tag{8.18}$$

where K_2 is a constant. Equation (8.18) is plotted in Fig. 8.13 for the turbojet engine of Example 8.1.

8.2.5 Theta Break

This concept is explained in Appendix D of *Aircraft Engine Design*, by Mattingly et al. (Ref. 12, pp. 525, 526) and is reprinted here with permission of the publisher:

The unique point, so visually striking in [Fig. 8.12], at which the control logic must switch from limiting π_c to limiting T_{t4}, is known as the theta break, or $\theta_{0\ \text{break}}$.

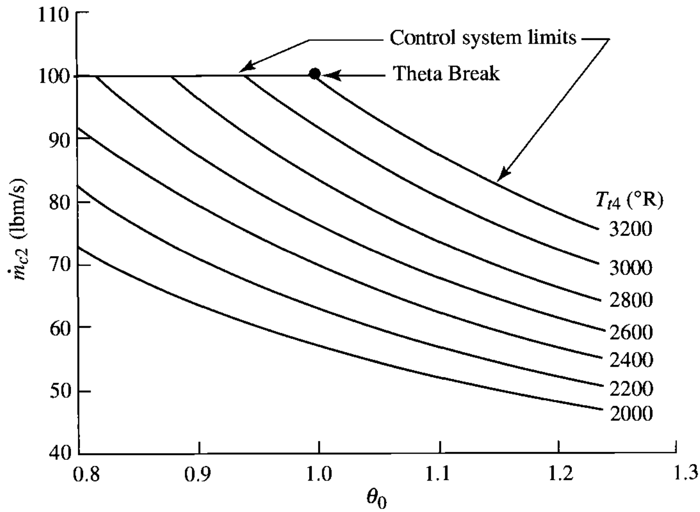

Fig. 8.13 Compressor corrected mass flow rate vs θ_0 and T_{t4}.

Returning briefly to [Fig. 8.12], you will find it very convenient to visualize that at any point in the flight envelope to the left of the theta break $\pi_c = \pi_{c\,\max}$ and $T_{t4} < T_{t4\,\max}$, while at any point in the right of the theta break $\pi_c < \pi_{c\,\max}$ and $T_{t4} = T_{t4\,\max}$. The relationship of the instantaneous value of θ_0 to $\theta_{0\,\text{break}}$ has important consequences to engine cycle performance. On the one hand, when $\theta_0 < \theta_{0\,\text{break}}$ and $T_{t4} < T_{t4\,\max}$, the specific thrust of the engine is less than its inherent material capabilities would make possible. On the other hand, when $\theta_0 > \theta_{0\,\text{break}}$ and $\pi_c < \pi_{c\,\max}$, the specific fuel consumption is more than its inherent thermal efficiency would make possible.

The designer would therefore strongly prefer to have the engine always operate at or very near $\theta_0 = \theta_{0\,\text{break}}$, but this is impossible because every aircraft has a flight envelope with a range of θ_0 [see Fig. 8.11]. The best available compromise is to choose a $\theta_{0\,\text{break}}$ that provides the best balance of engine performance over the expected range of flight conditions.

It is interesting to note that, since early commercial and military aircraft primarily flew at or near $\theta_0 = 1$, they were successfully designed with $\theta_{0\,\text{break}} = 1$. Consequently, several generations of propulsion engineers took it for granted that aircraft engines always operated at $\pi_{c\,\max}$ and $T_{t4\,\max}$ under standard sea-level static conditions. However, the special requirements of more recent aircraft such as [the F-22 Raptor ($\theta_0 > 1.2$ at supercruise)] have forced designers to select theta breaks different from 1.0. These engines may operate either at $\pi_{c\,\max}$ or $T_{t4\,\max}$ at standard sea-level static conditions, but never both.

Example 8.1

We now consider compressor operation at different T_{t4} and θ_0, specifically a compressor that has a pressure ratio of 15 and corrected mass flow rate of 100 lbm/s for T_{t2} of 518.7°R (sea-level standard) and T_{t4} of 3200°R. At these conditions, θ_0 is 1, and constants K_1 and K_2 in Eqs. (8.17) and (8.18) are 3.649×10^{-4} and 377-1, respectively. In addition, we assume that an engine control system limits π_c to 15 and T_{t4} to 3200°R. By using Eqs. (8.17) and (8.18), the compressor pressure ratio and corrected mass flow rate are calculated for various values of T_{t4} and θ_0. Figures 8.12 and 8.13 show the resulting variation of compressor pressure ratio and corrected mass flow rate, respectively, with flight condition θ_0 and throttle setting T_{t4}. Note that at θ_0 above 1.0, the compressor pressure ratio and corrected mass flow rate are limited by the maximum combustor exit temperature T_{t4} of 3200°R. The compressor pressure ratio limits performance at θ_0 below 1.0. Thus $\theta_{0\,\text{break}} = 1.0$.

8.2.6 Variation in Engine Speed

As will be shown in Chapter 9, the change in total enthalpy across a fan or compressor is proportional to the rotational speed N squared. For a calorically perfect gas, we can write

$$T_{t3} - T_{t2} = K_1 N^2$$

or

$$\tau_c - 1 = \frac{K_1}{T_{\text{ref}}} N_{c2}^2 \tag{i}$$

where N_{c2} is the compressor corrected speed. The compressor temperature ratio is related to the compressor pressure ratio through the efficiency, or

$$\tau_c - 1 = (\pi_c^{(\gamma_c-1)/\gamma_c} - 1)/\eta_c$$

Combining this equation with Eq. (i), rewriting the resulting equation in terms of pressure ratio and corrected speed, rearranging into variable and constant terms, and equating the constant to reference values give for constant compressor efficiency

$$\frac{\pi_c^{(\gamma_c-1)/\gamma_c} - 1}{N_{c2}^2} = \frac{\eta_c K_1}{T_{\text{ref}}} = \frac{\pi_{cR}^{(\gamma_c-1)/\gamma_c} - 1}{N_{c2R}^2} \tag{ii}$$

Solving Eq. (ii) for the corrected speed ratio N_{c2}/N_{c2R}, we have

$$\frac{N_{c2}}{N_{c2R}} = \sqrt{\frac{\pi_c^{(\gamma_c-1)/\gamma_c} - 1}{\pi_{cR}^{(\gamma_c-1)/\gamma_c} - 1}} \tag{8.19a}$$

This equation can also be used to estimate the variation in engine speed N with flight condition. Equation (8.19a) is plotted in Fig. 8.14 for a reference compressor pressure ratio of 16. Note that a reduction in compressor pressure ratio from 16 to 11 requires only a 10% reduction in corrected speed N_c. Equation (8.19a)

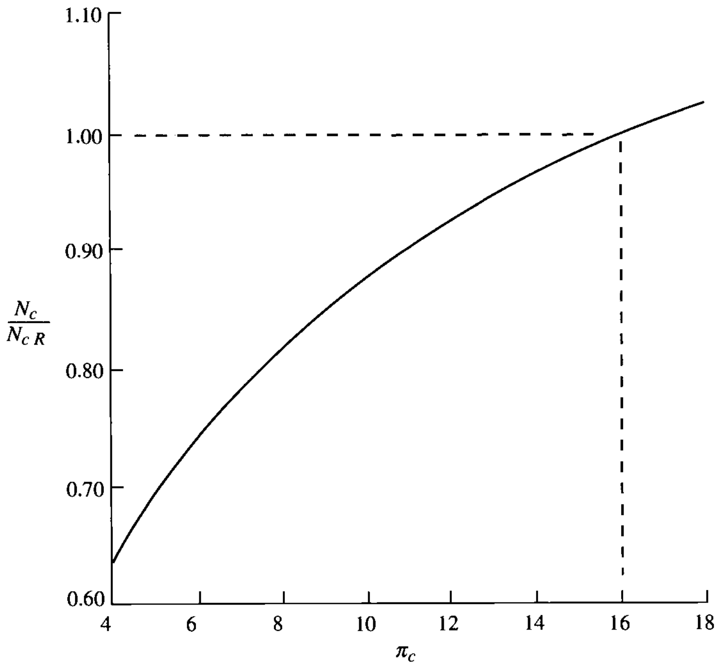

Fig. 8.14 Variation in corrected speed with compressor pressure ratio.

can be written in terms of T_{t4}/θ_0 by using Eq. (8.17), yielding

$$\frac{N_{c2}}{N_{c2R}} = \sqrt{\frac{T_{t4}/\theta_0}{(T_{t4}/\theta_0)_R}} \tag{8.19b}$$

Because the compressor and turbine are connected to the same shaft, they have the same rotational speed N, and we can write the following relationship between their corrected speeds:

$$N_{c2} = \frac{1}{\sqrt{T_{\text{ref}}}}\sqrt{\frac{T_{t4}}{\theta_0}}N_{c4} \tag{8.20}$$

Comparison of Eqs. (8.19b) and (8.20) gives the result that the corrected turbine speed is constant, or

$$N_{c4} = \text{const} \tag{8.21}$$

This result may surprise one at first. However, given that the turbine's temperature ratio τ_t, pressure ratio π_t, and efficiency η_t are considered constant in this analysis, the turbine's corrected speed must be constant (see Fig. 8.5c).

8.2.7 Gas Generator Equations

The pumping characteristics of a simple gas generator can be represented by the variation of the gas generator's parameter ratios with corrected compressor speed. The equations for the gas generator's pressure and temperature ratios, corrected air mass flow and fuel flow rates, compressor pressure ratio, and corrected compressor speed can be written in terms of T_{t4}/T_{t2}, reference values (subscript R), and other variables. The gas generator's pressure and temperature ratios are given simply by

$$\frac{P_{t6}}{P_{t2}} = \pi_c \pi_b \pi_t \tag{8.22}$$

$$\frac{T_{t6}}{T_{t2}} = \frac{T_{t4}}{T_{t2}} \tau_t \tag{8.23}$$

From Eq. (8.10) and referencing, the corrected mass flow rate can be written as

$$\frac{\dot{m}_{c2}}{\dot{m}_{c2R}} = \frac{\pi_c}{\pi_{cR}} \sqrt{\frac{(T_{t4}/T_{t2})_R}{T_{t4}/T_{t2}}} \tag{8.24}$$

where the compressor pressure ratio is given by Eq. (8.17), rewritten in terms of T_{t4}/T_{t2}, or

$$\pi_c = \left[1 + \frac{T_{t4}/T_{t2}}{(T_{t4}/T_{t2})_R}(\pi_{cR}^{(\gamma_c-1)/\gamma_c} - 1)\right]^{\gamma_c/(\gamma_c-1)} \tag{8.25}$$

Equation (8.19b) for the corrected speed can be rewritten in terms of T_{t4}/T_{t2} as

$$\frac{N_{c2}}{N_{c2R}} = \sqrt{\frac{T_{t4}/T_{t2}}{(T_{t4}/T_{t2})_R}} \tag{8.26}$$

An expression for the corrected fuel flow rate results from Eqs. (7.9), (8.2), and (8.7) as follows. Solving Eq. (7.9) for the fuel flow rate gives

$$\dot{m}_f = \dot{m}_0 \frac{c_{pt}T_{t4} - c_{pc}T_{t3}}{\eta_b h_{PR} - c_{pt}T_{t4}}$$

From Eqs. (8.2) and (8.7), this equation becomes

$$\dot{m}_{fc} = \frac{\dot{m}_{c2}}{\theta_2} \frac{c_{pt}T_{t4} - c_{pc}T_{t3}}{\eta_b h_{PR} - c_{pt}T_{t4}}$$

or

$$\dot{m}_{fc} = \dot{m}_{c2} \frac{T_{t4}/T_{t2} - \tau_c(c_{pc}/c_{pt})}{\eta_b h_{PR}/(c_{pt}T_{\text{ref}}) - T_{t4}/T_{\text{ref}}} \tag{8.27}$$

where by using Eq. (8.13) and referencing, τ_c is given by

$$\tau_c = 1 + (\tau_{cR} - 1)\frac{T_{t4}/T_{t2}}{(T_{t4}/T_{t2})_R} \tag{8.28}$$

Equations (8.22–8.28) constitute a set of equations for the pumping characteristics of a simple gas generator in terms of T_{t4}/T_{t2} and reference values. Only Eq. (8.27) for the corrected fuel flow rate has the term T_{t4}/T_{ref} that is not strictly a function of T_{t4}/T_{t2}. The first term in the denominator of Eq. (8.27) has a magnitude of about 130, and T_{t4}/T_{ref} has a value of about 6 or smaller. Thus the denominator of Eq. (8.28) does not vary appreciably, and the corrected fuel flow rate is a function of T_{t4}/T_{t2} and reference values. In summary, the pumping characteristics of the gas generator are a function of only the temperature ratio T_{t4}/T_{t2}.

Example 8.2

We want to determine the characteristics of a gas generator with a maximum compressor pressure ratio of 15, a compressor corrected mass flow rate of 100 lbm/s at T_{t2} of 518.7°R (sea-level standard), and a maximum T_{t4} of 3200°R. This is the same compressor we considered in Example 8.1 (see Figs. 8.12 and 8.13). We assume the compressor has an efficiency η_c of 0.8572 ($e_c = 0.9$), and the burner has an efficiency η_b of 0.995 and a pressure ratio π_b of 0.96. In addition, we assume the following gas constants: $\gamma_c = 1.4$, $c_{pc} = 0.24$ Btu/(lbm · °R), $\gamma_t = 1.33$, and $c_{pt} = 0.276$ Btu/(lbm · °R).

By using Eq. (7.10), the reference fuel/air ratio f_R is 0.03381 for $h_{PR} = 18{,}400$ Btu/lbm, and the corrected fuel flow rate is 12,170 lb/h. From Eq. (7.12), the turbine temperature ratio τ_t is 0.8124. Assuming $e_t = 0.9$, Eqs. (7.13) and (7.14) give the turbine pressure ratio π_t as 0.3943 and the turbine efficiency η_t, as 0.910. The reference compressor temperature ratio τ_{cR} is 2.3624.

Calculations were done over a range of T_{t4} with $T_{t2} = 518.7$°R and using Eqs. (8.22–8.28). The resulting gas generator pumping characteristics are plotted in Fig. 8.15. We can see that the compressor pressure ratio and corrected fuel flow rate decrease more rapidly with decreasing corrected speed than corrected airflow rate. As discussed previously, the gas generator's pumping characteristics are a function of only T_{t4}/T_{t2}, and Fig. 8.15 shows this most important relationship in graphical form.

Because the maximum T_{t4} is 3200°R and the maximum pressure ratio is 15, the operation of the gas generator at different inlet conditions (T_{t2}, P_{t2}) and/or different throttle setting (T_{t4}) can be obtained from Fig. 8.15. For example, consider a 100°F day (T_{t2}) at sea level with maximum power. Here $T_{t2} = 560$°R, $P_{t2} = 14.7$ psia, and $T_{t4} = 3200$°R; thus $T_{t4}/T_{t2} = 5.71$, and Fig. 8.15 gives the following data: $N_c/N_{cR} = 0.96$, $\dot{m}_c/\dot{m}_{cR} = 0.88$, $\pi_c/\pi_{cR} = 0.84$, $\dot{m}_{fc}/\dot{m}_{fcR} = 0.78$, $T_{t6}/T_{t2} = 4.6$, and $P_{t6}/P_{t2} = 4.8$. With these data, the pressures,

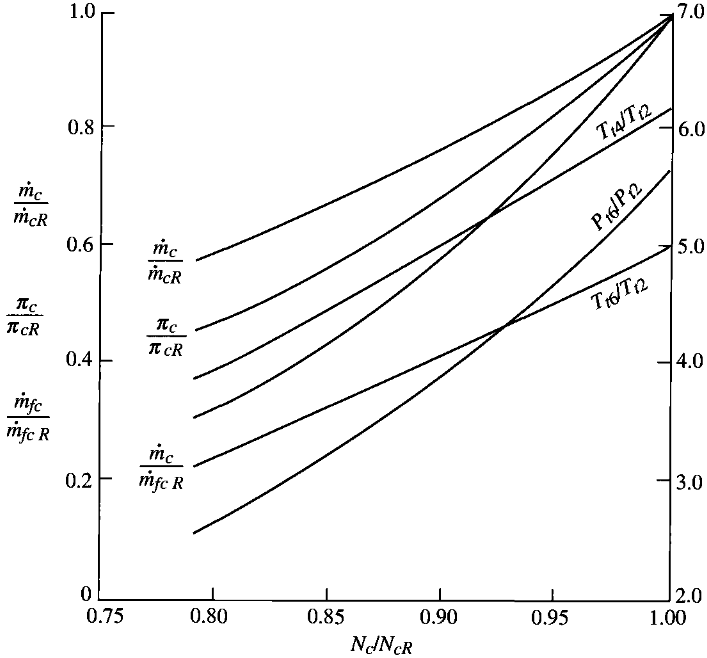

Fig. 8.15 Gas generator pumping characteristics.

temperatures, and flow rates can be calculated as follows:

$$\dot m = \left(\frac{P_{t2}}{P_{t2R}}\sqrt{\frac{T_{t2R}}{T_{t2}}}\right)\left(\frac{\dot m_c}{\dot m_{cR}}\right)\dot m_{cR} = 1 \times \sqrt{\frac{518.7}{560}}\,(0.88)(100) = 84.7\ \text{lbm/s}$$

$$\dot m_f = \frac{P_{t2}}{P_{t2R}}\sqrt{\frac{T_{t2}}{T_{t2R}}}\frac{\dot m_{fc}}{\dot m_{fcR}}\dot m_{fcR} = 1 \times \sqrt{\frac{560}{518.6}}\,(0.78)(12{,}170) = 9860\ \text{lbm/h}$$

$$T_{t6} = \frac{T_{t6}}{T_{t2}}T_{t2} = 4.6(560) = 2576^\circ\text{R}$$

$$P_{t6} = \frac{P_{t6}}{P_{t2}}P_{t2} = 4.8(14.7) = 70.6\ \text{psia}$$

$$\pi_c = \frac{\pi_c}{\pi_{cR}}\pi_{cR} = 0.84(15) = 12.6$$

As another example, consider flight at Mach 0.6 and 40 kft ($\theta = 0.7519$, $\delta = 0.1858$) with maximum throttle. Since T_{t2} ($=418.1^\circ$R) is less than T_{t2R}, the maximum value for T_{t4} is 2579.4°R (= 3200 × 418.1/518.7), and the compressor has a pressure ratio of 15 and corrected mass flow rate of 100 lbm/s. The air mass flow rate is reduced to 20.7 lbm/s and the mass fuel flow rate is reduced to 2030 lbm/h.

8.3 Turbojet Engine

In this section, the performance equations of the single-spool turbojet engine, shown in Fig. 8.16, are developed and the results are studied. We assume choked flow at stations 4 and 8. In addition, the throttle (T_{t4}), flight conditions (M_0, T_0, and P_0), and the ambient pressure/exhaust pressure ratio P_0/P_9 can be independently varied for this engine. The performance equations for this turbojet can be obtained easily by adding inlet and exhaust nozzle losses to the single-spool gas generator studied in the previous section.

This engine has five independent variables (T_{t4}, M_0, T_0, P_0, and P_0/P_9). The performance analysis develops analytical expressions for component performance in terms of these independent variables. We have six dependent variables for the single-spool turbojet engine: engine mass flow rate, compressor pressure ratio, compressor temperature ratio, burner fuel/air ratio, exit temperature ratio T_9/T_0, and exit Mach number. A summary of the independent variables, dependent variables, and constants or knowns for this engine is given in Table 8.4.

The thrust for this engine is given by

$$\frac{F}{\dot{m}_0} = \frac{a_0}{g_c}\left[(1+f)\frac{V_9}{a_0} - M_0 + (1+f)\frac{R_t}{R_c}\frac{T_9/T_0}{V_9/a_0}\frac{1 - P_0/P_9}{\gamma_c}\right] \tag{i}$$

where

$$\frac{T_9}{T_0} = \frac{T_{t4}\tau_t}{(P_{t9}/P_9)^{(\gamma_t - 1)/\gamma_t}} \tag{ii}$$

$$\frac{P_{t9}}{P_9} = \frac{P_0}{P_9}\pi_r \pi_d \pi_c \pi_b \pi_t \pi_n \tag{iii}$$

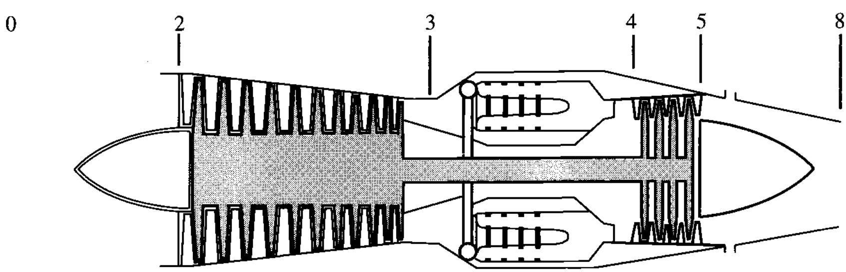

Fig. 8.16 Single-spool turbojet engine. (Courtesy of Pratt & Whitney.)

Table 8.4 Performance analysis variables for single-spool turbojet engine

	Variables		
Component	Independent	Constant or known	Dependent
Engine	M_0, T_0, P_0		$\dot{m}_0$
Diffuser		$\pi_d = f(M_0)$	
Compressor		η_c	π_c, τ_c
Burner	T_{t4}	π_b, η_b	f
Turbine		π_t, τ_t	
Nozzle	P_9/P_0	π_n	M_9, T_9/T_0
Total number	5		6

$$M_9 = \sqrt{\frac{2}{\gamma_t - 1}\left[\left(\frac{P_{t9}}{P_9}\right)^{(\gamma_t-1)/\gamma_t} - 1\right]} \tag{iv}$$

and

$$\frac{V_9}{a_0} = M_9\sqrt{\frac{\gamma_t R_t T_9}{\gamma_c R_c T_0}} \tag{v}$$

The thrust-specific fuel consumption for this engine is given by

$$S = \frac{f}{F/\dot{m}_0} \tag{vi}$$

where

$$f = \frac{\tau_\lambda - \tau_r \tau_c}{h_{PR}\eta_b/(c_{pc}T_0) - \tau_\lambda} \tag{vii}$$

Equations (i–vii) can be solved for given T_{t4}, M_0, T_0, P_0, P_0/P_9, gas properties with expressions for τ_λ, π_r, τ_r, π_d, π_c, τ_c, and engine mass flow rate in terms of the five independent variables and other dependent variables. In the previous section, we developed Eq. (8.28), repeated here, for the compressor's temperature ratios in terms of T_{t4}/T_{t2} and reference values:

$$\tau_c = 1 + (\tau_{cR} - 1)\frac{T_{t4}/T_{t2}}{(T_{t4}/T_{t2})_R}$$

The compressor pressure ratio is related to its temperature ratio by its efficiency.

An equation for the engine mass flow rate follows from the mass flow parameter (MFP) written for station 4 with choked flow and the definitions of component π values. We write

$$\dot{m}_0 = \frac{P_{t4}}{\sqrt{T_{t4}}}\frac{A_4}{1+f}\text{MFP}(1) = \frac{P_0\pi_r\pi_d\pi_c}{\sqrt{T_{t4}}}\left[\frac{\pi_b A_4}{1+f}\text{MFP}(1)\right]$$

Because the terms within the square brackets are considered constant, we move the variable terms to the left side of the equation, and, using referencing, equate the constant to reference values:

$$\frac{\dot{m}_0\sqrt{T_{t4}}}{P_0\pi_r\pi_d\pi_c} = \frac{\pi_b A_4}{1+f}\text{MFP}(1) = \left(\frac{\dot{m}_0\sqrt{T_{t4}}}{P_0\pi_r\pi_d\pi_c}\right)_R$$

Solving for the engine mass flow rate, we get

$$\dot{m}_0 = \dot{m}_{0R}\frac{P_0\pi_r\pi_d\pi_c}{(P_0\pi_r\pi_d\pi_c)_R}\sqrt{\frac{T_{t4R}}{T_{t4}}} \tag{8.29}$$

Relationships for τ_λ, π_r, τ_r, and π_d follow from their equations in Chapter 7.

The throat area of the exhaust nozzle is assumed to be constant. With P_0/P_9 an independent variable, the exit area of the exhaust nozzle A_9 must correspond to the nozzle pressure ratio P_{t9}/P_9. An expression for the exhaust nozzle exit area follows from the mass flow parameter and other compressible flow properties. The subscript t is used in the following equations for the gas properties (γ, R, and Γ) at stations 8 and 9. Using Eq. (8.11) for choked flow at station 8 gives

$$\dot{m}_8 = \frac{P_{t9}A_8}{\sqrt{T_{t8}}}\frac{\Gamma_t}{\sqrt{R_t/g_c}} \tag{i}$$

From the equation for the mass flow parameter [Eq. (2.76)], the mass flow rate at station 9 is

$$\dot{m}_9 = \frac{P_{t9}A_9}{\sqrt{T_{t9}}}\frac{\sqrt{\gamma_t}}{\sqrt{R_t/g_c}}M_9\left(1+\frac{\gamma_t-1}{2}M_9^2\right)^{-(\gamma_t+1)/2(\gamma_t-1)]} \tag{ii}$$

Using the nozzle relationships $T_{t8} = T_{t9}$ and $\pi_n = P_{t9}/P_{t8}$ and equating the mass flow rate at station 8 [Eq. (i)] to that at station 9 [Eq. (ii)] give

$$\frac{A_9}{A_8} = \frac{\Gamma_t}{\sqrt{\gamma_t}}\frac{1}{\pi_n}\frac{1}{M_9}\left(1+\frac{\gamma_t-1}{2}M_9^2\right)^{(\gamma_t+1)/[2(\gamma_t-1)]}$$

Replacing the Mach number at station 9 by using

$$M_9 = \sqrt{\frac{2}{\gamma_t - 1}[(P_{t9}/P_9)^{(\gamma_t-1)/\gamma_t} - 1]}$$

gives

$$\frac{A_9}{A_8} = \Gamma_t \sqrt{\frac{\gamma_t - 1}{2\gamma_t}} \frac{1}{\pi_n} \frac{(P_{t9}/P_9)^{(\gamma_t+1)/(2\gamma_t)}}{\sqrt{(P_{t9}/P_9)^{(\gamma_t-1)/\gamma_t} - 1}} \tag{8.30}$$

Because the throat area A_8 is constant, Eq. (8.30) can be used to obtain the ratio of the exit area A_9 to a reference exit area A_{9R} that can be written as

$$\frac{A_9}{A_{9R}} = \left[\frac{P_{t9}/P_9}{(P_{t9}/P_9)_R}\right]^{(\gamma_t+1)/(2\gamma_t)} \sqrt{\frac{(P_{t9}/P_9)_R^{(\gamma_t-1)/\gamma_t} - 1}{(P_{t9}/P_9)^{(\gamma_t-1)/\gamma_t} - 1}} \tag{8.31}$$

8.3.1 *Summary of Performance Equations—Single-Spool Turbojet Without Afterburner*

INPUTS:

Choices	
Flight parameters:	M_0, T_0 (K, °R), P_0 (kPa, psia)
Throttle setting:	T_{t4} (K, °R)
Exhaust nozzle setting:	P_0/P_9
Design constants	
π:	$\pi_{d\max}$, π_b, π_t, π_n
τ:	τ_t
η:	η_c, η_b, η_m
Gas properties:	γ_c, γ_t, c_{pc}, c_{pt} [kJ/(Kg · K), Btu/(lbm · °R)]
Fuel:	h_{PR}, (kJ/kg, Btu/lbm)
Reference conditions	
Flight parameters:	M_{0R}, T_{0R} (K, °R), P_{0R} (kPa, psia), τ_{rR}, π_{rR}
Throttle setting:	T_{t4R} (K, °R)
Component behavior:	T_{dR}, π_{cR}, τ_{cR}

OUTPUTS:

Overall performance:	F(N, lbf), $\dot{m}_0$ (kg/s, lbm/s), f,
	$S\left(\frac{\text{mg/s}}{\text{N}}, \frac{\text{lbm/h}}{\text{lbf}}\right)$, η_P, η_T, η_O
Component behavior:	π_d, π_c, τ_c, f, M_9, N/N_R

EQUATIONS:

$$R_c = \frac{\gamma_c - 1}{\gamma_c} c_{pc} \tag{8.32a}$$

$$R_t = \frac{\gamma_t - 1}{\gamma_t} c_{pt} \tag{8.32b}$$

$$a_0 = \sqrt{\gamma_c R_c g_c T_0} \tag{8.32c}$$

$$V_0 = a_0 M_0 \tag{8.32d}$$

$$\tau_r = 1 + \frac{\gamma_c - 1}{2} M_0^2 \tag{8.32e}$$

$$\pi_r = \tau^{\gamma_c/(\gamma_c - 1)} \tag{8.32f}$$

$$\eta_r = 1 \qquad \text{for } M_0 \leq 1 \tag{8.32g}$$

$$\eta_r = 1 - 0.075(M_0 - 1)^{1.35} \qquad \text{for } M_0 > 1 \tag{8.32h}$$

$$\pi_d = \pi_{d\max} \eta_r \tag{8.32i}$$

$$T_{t2} = T_0 \tau_r \tag{8.32j}$$

$$\tau_c = 1 + (\tau_{cR} - 1) \frac{T_{t4}/T_{t2}}{(T_{t4}/T_{t2})_R} \tag{8.32k}$$

$$\pi_c = [1 + \eta_c(\tau_c - 1)]^{\gamma_c/(\gamma_c - 1)} \tag{8.32l}$$

$$\tau_\lambda = \frac{c_{pt} T_{t4}}{c_{pc} T_0} \tag{8.32m}$$

$$f = \frac{\tau_\lambda - \tau_r \tau_c}{h_{PR} \eta_b / (c_p T_0) - \tau_\lambda} \tag{8.32n}$$

$$\dot{m}_0 = \dot{m}_{0R} \frac{P_0 \pi_r \pi_d \pi_c}{(P_0 \pi_r \pi_d \pi_c)_R} \sqrt{\frac{T_{t4R}}{T_{t4}}} \tag{8.32o}$$

$$\frac{P_{t9}}{P_9} = \frac{P_0}{P_9} \pi_r \pi_d \pi_c \pi_b \pi_t \pi_n \tag{8.32p}$$

$$M_9 = \sqrt{\frac{2}{\gamma_t - 1} \left[\left(\frac{P_{t9}}{P_9} \right)^{(\gamma_t - 1)/\gamma_t} - 1 \right]} \tag{8.32q}$$

$$\frac{T_9}{T_0} = \frac{T_{t4} \tau_t / T_0}{(P_{t9}/P_0)^{(\gamma_t - 1)/\gamma_t}} \tag{8.32r}$$

$$\frac{V_9}{a_0} = M_9 \sqrt{\frac{\gamma_t R_t}{\gamma_c R_c} \frac{T_9}{T_0}} \tag{8.32s}$$

$$\frac{F}{\dot{m}_0} = \frac{a_0}{g_c}\left[(1+f)\frac{V_9}{a_0} - M_0 + (1+f)\frac{R_t}{R_c}\frac{T_9/T_0}{V_9/a_0}\frac{1-P_0/P_9}{\gamma_c}\right] \tag{8.32t}$$

$$F = \dot{m}_0\left(\frac{F}{\dot{m}_0}\right) \tag{8.32u}$$

$$S = \frac{f}{F/\dot{m}_0} \tag{8.32v}$$

$$\eta_T = \frac{a_0^2[(1+f)(V_9/a_0)^2 - M_0^2]}{2g_c f h_{PR}} \tag{8.32w}$$

$$\eta_P = \frac{2g_c V_0(F/\dot{m}_0)}{a_0^2[(1+f)(V_0/a_0)^2 - M_0^2]} \tag{8.32x}$$

$$\eta_o = \eta_P \eta_T \tag{8.32y}$$

$$\frac{N}{N_R} = \sqrt{\frac{T_0\tau_r}{T_{0R}\tau_{rR}}\frac{\pi_c^{(\gamma_c-1)/\gamma_c} - 1}{\pi_{cR}^{(\gamma_c-1)/\gamma_c} - 1}} \tag{8.32z}$$

$$\frac{A_9}{A_{9R}} = \left[\frac{P_{t9}/P_9}{(P_{t9}/P_9)_R}\right]^{(\gamma_t+1)/(2\gamma_t)}\sqrt{\frac{(P_{t9}/P_9)_R^{(\gamma_t-1)/\gamma_t} - 1}{(P_{t9}/P_9)^{(\gamma_t-1)/\gamma_t} - 1}} \tag{8.32aa}$$

Example 8.3

We consider the performance of the turbojet engine of Example 7.1 sized for a mass flow rate of 50 kg/s at the reference condition and altitude of 12 km. We are to determine this engine's performance at an altitude of 9 km, Mach number of 1.5, reduced throttle setting ($T_{t4} = 1670°\text{R}$), and exit to ambient pressure ratio (P_0/P_9) of 0.955.

REFERENCE:

$$T_0 = 216.7\ \text{K},\quad \gamma_c = 1.4,\quad c_{pc} = 1.004\ \text{kJ/(kg}\cdot\text{K)},\quad \gamma_t = 1.3$$
$$c_{pt} = 1.239\ \text{kJ/(kg}\cdot\text{K)},\quad T_{t4} = 1800\ \text{K},\quad M_0 = 2,\quad \pi_c = 10$$
$$\tau_c = 2.0771,\quad \eta_c = 0.8641,\quad \pi_t = 0.8155,\quad \pi_t = 0.3746$$
$$\pi_{d\max} = 0.95,\quad \pi_d = 0.8788,\quad \pi_b = 0.94,\quad \pi_n = 0.96$$
$$\eta_b = 0.98,\quad \eta_m = 0.99, P_0/P_9 = 0.5,\quad h_{PR} = 42{,}800\ \text{kJ/kg}$$
$$f = 0.03567,\quad P_{t9}/P_9 = 11.62,\quad F/\dot{m}_0 = 806.9\ \text{N/(kg/s)}$$
$$S = 44.21\ \text{(mg/s)/N},\quad P_0 = 19.40\ \text{kPa (12 km)},\quad \dot{m}_0 = 50\ \text{kg/s}$$
$$F = \dot{m}_0 \times (F/\dot{m}_0) = 50 \times 806.9 = 40{,}345\ \text{N}$$

OFF-DESIGN CONDITION:

$$T_0 = 229.8\ \text{K},\quad P_0 = 30.8\ \text{kPa (9 km)},\quad M_0 = 1.5$$
$$P_0/P_9 = 0.955,\quad T_{t4} = 1670\ \text{K}$$

EQUATIONS:

$$R_c = \frac{\gamma_c - 1}{\gamma_c} c_{pc} = \frac{0.4}{1.4}(1.004) = 0.2869\,\text{kJ/(kg} \cdot \text{K)}$$

$$R_t = \frac{\gamma_t - 1}{\gamma_t} c_{pt} = \frac{0.3}{1.3}(1.239) = 0.2859\,\text{kJ/(kg} \cdot \text{K)}$$

$$a_0 = \sqrt{\gamma_c R_c g_c T_0} = \sqrt{1.4 \times 286.9 \times 1 \times 229.8} = 303.8 \text{ m/s}$$

$$V_0 = a_0 M_0 = 303.8 \times 1.5 = 455.7\,\text{m/s}$$

$$\tau_r = 1 + \frac{\gamma_c - 1}{2} M_0^2 = 1 + 0.2 \times 1.5^2 = 1.45$$

$$\pi_r = \tau_r^{\gamma_c/(\gamma_c - 1)} = 1.45^{3.5} = 3.671$$

$$\eta_r = 1 - 0.075(M_0 - 1)^{1.35} = 1 - 0.075(0.5)^{1.35} = 0.9706$$

$$\pi_d = \pi_{d\max}\eta_r = 0.95 \times 0.9706 = 0.9220$$

$$\tau_\lambda = \frac{c_{pt}T_{t4}}{c_{pc}T_0} = \frac{1.2329 \times 1670}{1.004 \times 229.8} = 8.9682$$

$$T_{t2} = T_0\tau_r = 229.8 \times 1.45 = 333.2\,\text{K}$$

$$\tau_{rR} = 1 + \frac{\gamma_c - 1}{2} M_{0R}^2 = 1 + 0.2 \times 2^2 = 1.80$$

$$T_{t2R} = T_{0R}\tau_{rR} = 216.7 \times 1.8 = 390.1\,\text{K}$$

$$\pi_{rR} = \tau_{rR}^{\gamma_c/(\gamma_c - 1)} = 1.8^{3.5} = 7.824$$

$$\tau_c = 1 + (\tau_{cR} - 1)\frac{T_{t4}/T_{t2}}{(T_{t4}/T_{t2})_R}$$

$$= 1 + (2.0771 - 1)\frac{1670/333.2}{1800/390.1} = 2.170$$

$$\pi_c = [1 + \eta_c(\tau_c - 1)]^{\gamma_c/(\gamma_c - 1)} = [1 + 0.8641(2.170 - 1)]^{3.5} = 11.53$$

$$f = \frac{\tau_\lambda - \tau_r\tau_c}{h_{PR}\eta_b/(c_{pc}T_0) - \tau_\lambda}$$

$$= \frac{8.9682 - 1.45 \times 2.170}{42{,}800 \times 0.98/(1.004 \times 229.8) - 8.9682} = 0.03368$$

$$\frac{P_{t9}}{P_9} = \frac{P_0}{P_9}\pi_r\pi_d\pi_c\pi_b\pi_t\pi_n$$

$$= 0.955 \times 3.671 \times 0.9220 \times 11.53 \times 0.94 \times 0.3746 \times 0.96 = 12.60$$

$$M_9 = \sqrt{\frac{2}{\gamma_t - 1}[(P_{t9}/P_9)^{(\gamma_t - 1)/\gamma_t} - 1]} = \sqrt{\frac{2}{0.3}(12.60^{0.3/1.3} - 1)} = 2.301$$

$$\frac{T_9}{T_0} = \frac{\tau_\lambda \tau_t}{(P_{t9}/P_9)^{(\gamma_t-1)/\gamma_t}} \frac{c_{pc}}{c_{pt}} = \frac{8.9682 \times 0.8155}{12.60^{0.3/1.3}} \frac{1.004}{1.239} = 3.303$$

$$\frac{V_9}{a_0} = M_9 \sqrt{\frac{\gamma_t R_t T_9}{\gamma_c R_c T_0}} = 2.301 \sqrt{\frac{1.3 \times 285.9}{1.4 \times 286.9}(3.303)} = 4.023$$

$$\frac{F}{\dot{m}_0} = \frac{a_0}{g_c}\left[(1+f)\frac{V_9}{a_0} - M_0 + (1+f)\frac{R_t}{R_c}\frac{T_9/T_0}{V_9/a_0}\frac{1 - P_0/P_9}{\gamma_c}\right]$$

$$= 303.8\left(1.03368 \times 4.023 - 1.5 + 1.03368 \frac{285.9}{286.9}\frac{3.303}{4.023}\frac{0.045}{1.4}\right)$$

$$= 303.8(2.6585 + 0.0272) = 815.9\,\mathrm{N/(kg/s)}$$

$$S = \frac{f}{F/\dot{m}_0} = \frac{0.03368 \times 10^6}{815.9} = 41.28\,\mathrm{(mg/s)/N}$$

$$\dot{m}_0 = \dot{m}_{0R} \frac{P_0 \pi_r \pi_d \pi_c}{(P_0 \pi_r \pi_d \pi_c)_R} \sqrt{\frac{T_{t4R}}{T_{t4}}}$$

$$\dot{m}_0 = 50 \frac{30.8 \times 3.671 \times 0.9220 \times 11.53}{19.4 \times 7.824 \times 0.8788 \times 10} \sqrt{\frac{1800}{1670}} = 46.78\,\mathrm{kg/s}$$

$$F = \dot{m}_0 \frac{F}{\dot{m}_0} = 46.78 \times 815.9 = 38{,}170\,\mathrm{N}$$

$$\eta_T = \frac{a_0^2[(1+f)(V_9/a_0)^2 - M_0^2]}{2 g_c f h_{PR}}$$

$$= \frac{303.8^2[(1.03368)(4.023^2) - 1.5^2]}{2 \times 1 \times 0.03368 \times 42{,}800 \times 1{,}000} = 46.36\%$$

$$\eta_P = \frac{2 g_c V_0 (F/\dot{m}_0)}{a_0^2[(1+f)(V_9/a_0)^2 - M_0^2]}$$

$$= \frac{2 \times 1 \times 455.7 \times 815.9}{303.8^2[(1.03368)(4.023^2) - 1.5^2]} = 55.64\%$$

$$\eta_O = \eta_P \eta_T = 0.4635 \times 0.5564 = 25.79\%$$

$$\frac{N}{N_R} = \sqrt{\frac{T_0 \tau_r}{T_{0R} \tau_{rR}} \frac{\pi_c^{(\gamma_c-1)/\gamma_c} - 1}{\pi_{cR}^{(\gamma_c-1)/\gamma_c} - 1}}$$

$$= \sqrt{\frac{229.8 \times 1.45}{216.7 \times 1.8} \frac{11.53^{0.4/1.4} - 1}{10^{0.4/1.4} - 1}} = 0.9278$$

$$\frac{\dot{m}_{c2}}{\dot{m}_{c2R}} = \frac{\pi_c}{\pi_{cR}} \sqrt{\frac{(T_{t4}/T_{t2})_R}{T_{t4}/T_{t2}}} = \frac{11.53}{10} \sqrt{\frac{1800/390.1}{1670/333.2}} = 1.106$$

$$\frac{A_9}{A_{9R}} = \left[\frac{P_{t9}/P_9}{(P_{t9}/P_9)_R}\right]^{(\gamma_t+1)/(2\gamma_t)} \sqrt{\frac{(P_{t9}/P_9)_R^{(\gamma_t-1)/\gamma_t} - 1}{(P_{t9}/P_9)^{(\gamma_t-1)/\gamma_t} - 1}}$$

$$\frac{A_9}{A_{9R}} = \left(\frac{12.60}{11.62}\right)^{2.3/1.3} \sqrt{\frac{11.62^{0.3/1.3} - 1}{12.60^{0.3/1.3} - 1}} = 1.052$$

Example 8.4

Consider a turbojet engine composed of the gas generator of Example 8.2, an inlet with $\pi_{d\max} = 0.99$, and an exhaust nozzle with $\pi_n = 0.99$ and $P_0/P_9 = 1$. The reference engine has the following values.

REFERENCE:

$$T_0 = 518.7°R, \quad \gamma_c = 1.4, \quad c_{pc} = 0.24 \text{ Btu/(lbm} \cdot \text{°R)}, \quad \gamma_t = 1.33$$

$$c_{pt} = 0.276 \text{ Btu/(lbm} \cdot \text{°R)}, \quad T_{t4} = 3200\text{°R}, \quad M_0 = 0$$

$$\pi_c = 15, \quad \eta_c = 0.8572, \quad \tau_t = 0.8124, \quad \pi_t = 0.3943$$

$$\pi_{d\max} = 0.99, \quad \pi_b = 0.96, \quad \pi_n = 0.99, \quad \eta_b = 0.995$$

$$\eta_m = 0.99, \quad P_0/P_9 = 1, \quad P_0 = 14.696 \text{ psia (sea level)}$$

$$P_{t9}/P_9 = 5.5653, \quad \dot{m}_0 = 100 \text{ lbm/s}, \quad F/\dot{m}_0 = 113.42 \text{ lbf/(lbm/s)}$$

$$F = \dot{m}_0 \times (F/\dot{m}_0) = 100 \times 113.42 = 11{,}342 \text{ lbf}$$

This engine has a control system that limits the compressor pressure ratio π_c to 15 and the combustor exit total temperature T_{t4} to 3200°R. Calculation of engine performance using Eqs. (8.32a–8.32aa) with full throttle at altitudes of sea level, 20 kft, and 40 kft over a range of flight Mach numbers gives the results shown in Figs. 8.17–8.22. Note the breaks in the plots of thrust, engine mass flow rate, compressor pressure ratio, and station 2 corrected mass flow rate at a Mach/altitude combination of about 0.9/20 kft and 1.3/40 kft. To the left of these breaks, the combustor exit temperature T_{t4} is below its maximum of 3200°R, and the compressor pressure ratio π_c is at its maximum of 15. To the right of these breaks, the combustor exit temperature T_{t4} is at its maximum of 3200°R, and the compressor pressure ratio π_c is below its maximum of 15. At the break, both the compressor pressure ratio and combustor exit temperature are at their maximum values. This break corresponds to the engine's theta break of 1.0.

The designer of a gas generator's turbomachinery needs to know the maximum power requirements of the compressor and turbine. Because the turbine drives the compressor, the maximum requirements of both occur at the same conditions. Consider the following power balance between the compressor and turbine:

$$\dot{W}_c = \eta_m \dot{W}_t$$

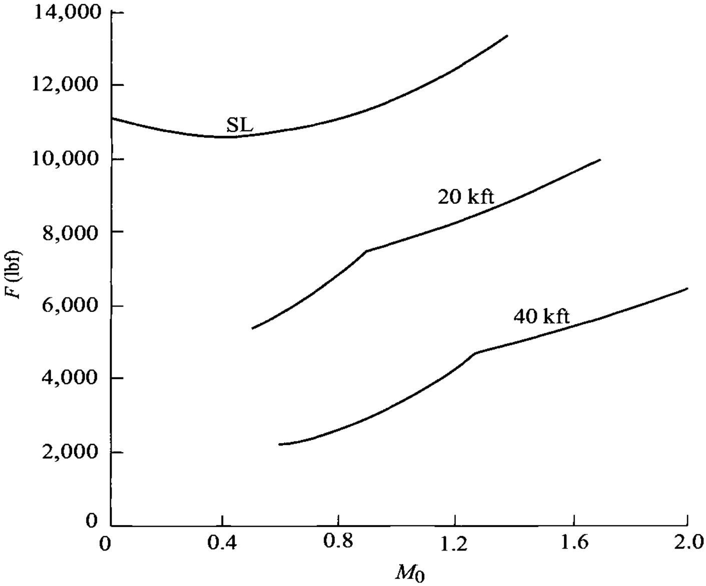

Fig. 8.17 Maximum thrust F of a turbojet vs M_0.

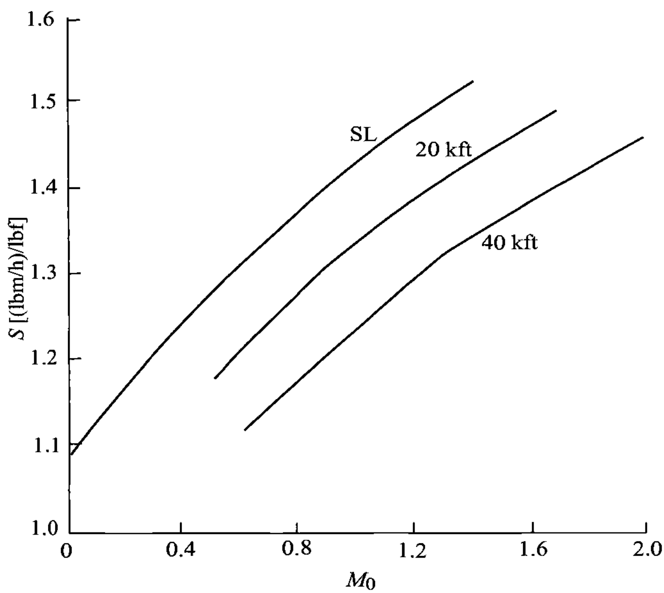

Fig. 8.18 Thrust-specific fuel consumption S of a turbojet vs M_0.

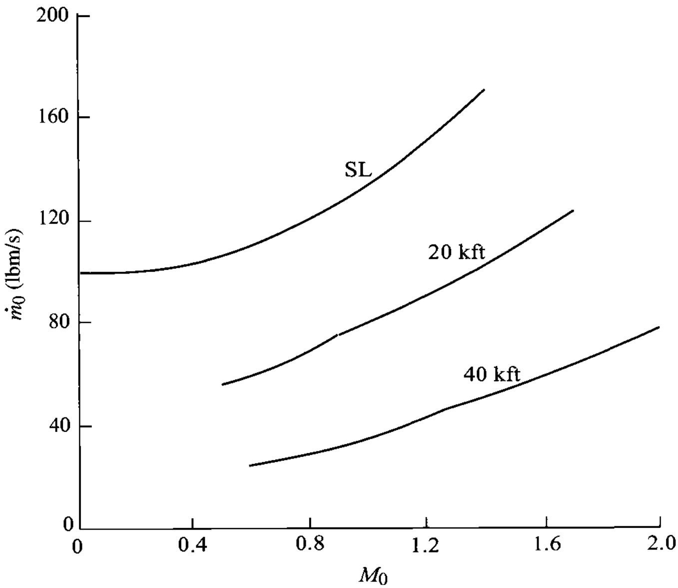

Fig. 8.19 Engine mass flow rate of a turbojet vs M_0.

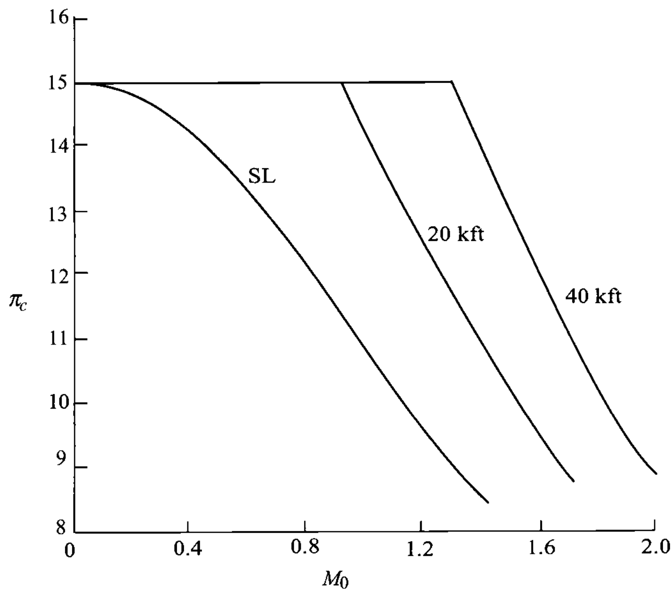

Fig. 8.20 Compressor pressure ratio of a turbojet vs M_0.

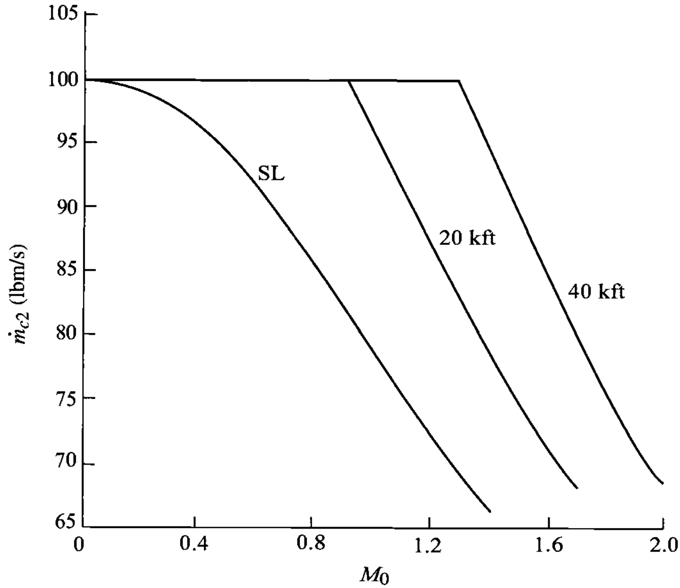

Fig. 8.21 Compressor corrected mass flow rate of a turbojet vs M_0.

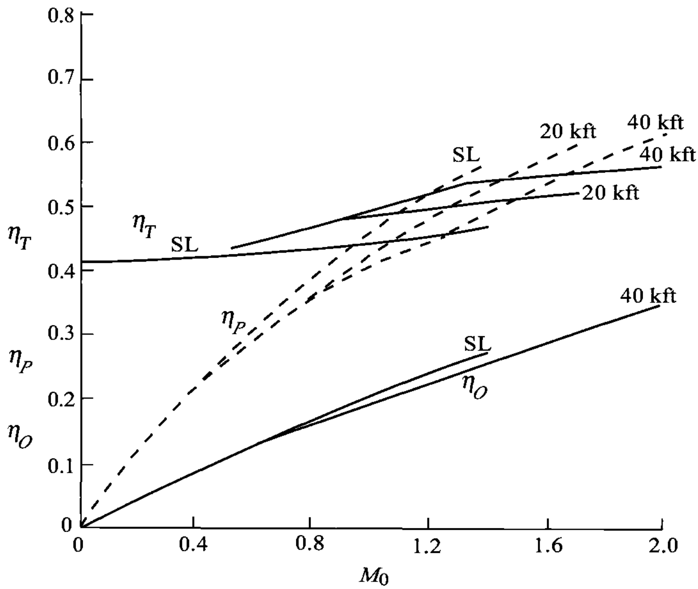

Fig. 8.22 Turbojet efficiencies vs M_0.

Rewriting turbine power in terms of its mass flow rate, total temperatures, etc., gives

$$\dot{W}_c = \eta_m \dot{m}_0 (1+f) c_{pt} (T_{t4} - T_{t5})$$

or

$$\dot{W}_c = \dot{m}_0 T_{t4} [\eta_m (1+f) c_{pt} (1 - \tau_t)]$$

Because the terms within the square braces of the preceding equation are considered constant, the maximum compressor power will be at the flight condition having maximum engine mass flow rate at maximum T_{t4}. From Fig. 8.19, the maximum compressor or turbine power corresponds to the maximum engine mass flow rate at sea level and Mach 1.4.

At an altitude of 20 kft and a Mach number of 0.8, engine performance calculations at reduced throttle (T_{t4}) using Eqs. (8.32a–8.32aa) were performed, and some of these results are given in Fig. 8.23. The typical variation in thrust specific fuel consumption S with thrust F is shown in this figure. As the throttle is reduced, the thrust specific fuel consumption first reduces before increasing. This plot of thrust specific fuel consumption S vs thrust F is commonly called the *throttle hook* because of its shape.

We stated at the beginning of this chapter that the principal efficiencies that affect engine performance are the thermal efficiency and the propulsive

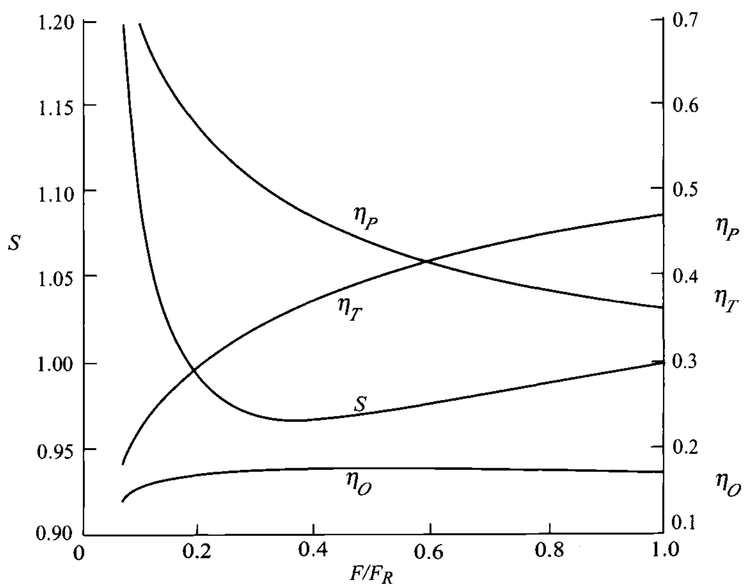

Fig. 8.23 Turbojet performance at partial throttle.

efficiency. Figure 8.23 shows the very large changes in both propulsive and thermal efficiency with engine thrust. Note that as thrust is reduced from its maximum, the increase in propulsive efficiency more than offsets the decrease in thermal efficiency such that the overall efficiency increases and the thrust specific fuel consumption decreases until about 40% of maximum thrust. Below 40% thrust, the decrease in thermal efficiency dominates the increase in propulsive efficiency and the overall efficiency decreases, and the thrust specific fuel consumption increases with reduced thrust.

8.3.2 Corrected Engine Performance

The changes in maximum thrust of a simple turbojet engine can be presented in a corrected format that essentially collapses the thrust data. Consider the thrust equation for the turbojet engine as given by

$$F = \frac{\dot{m}_0}{g_c}[(1+f)V_9 - V_0]$$

where

$$V_9 = \sqrt{2g_c c_{pt} T_{t4}\tau_t[1 - (\pi_r \pi_d \pi_c \pi_b \pi_t \pi_n)^{(-\gamma_t - 1)/\gamma_t}]}$$

and

$$V_0 = M_0 a_0 = M_0\sqrt{\gamma_c R_c T_0}$$

Note that the engine mass flow rate is related to the compressor corrected mass flow rate by

$$\dot{m}_0 = \dot{m}_{c0} = \frac{\delta_0}{\sqrt{\theta_0}} = \dot{m}_{c2}\frac{\delta_2}{\sqrt{\theta_2}} = \dot{m}_{c2}\frac{\pi_d \delta_0}{\sqrt{\theta_0}}$$

The engine thrust can now be written as

$$F = \frac{\dot{m}_{c2}}{g_c}\frac{\pi_d \delta_0}{\sqrt{\theta_0}}[(1+f)V_9 - V_0]$$

Dividing the thrust by the dimensionless total pressure at station 0 gives

$$\frac{F}{\delta_0} = \frac{\dot{m}_{c2}\pi_d}{g_c}\left[(1+f)\frac{V_9}{\sqrt{\theta_0}} - \frac{V_0}{\sqrt{\theta_0}}\right] \tag{8.33a}$$

where

$$\frac{V_9}{\sqrt{\theta_0}} = \sqrt{\frac{T_{t4}}{T_{t2}}}\sqrt{2g_c c_{pt} T_{\text{ref}}\tau_t[1 - \pi_r \pi_d \pi_c \pi_b \pi_t \pi_n)^{-(\gamma_t - 1)/\gamma_t}]} \tag{8.33b}$$

and

$$\frac{V_0}{\sqrt{\theta_0}} = \frac{M_0}{\sqrt{\tau_r}} a_{SL} \tag{8.33c}$$

The maximum thrust for the turbojet engine of Example 8.4 can be determined by using the preceding equations. Figures 8.17, 8.20, and 8.21 show the variation of the maximum thrust F, compressor pressure ratio, and corrected mass flow rate from this turbojet engine at full throttle vs flight Mach number M_0. The corrected thrust F/δ_0 of this engine is plotted vs flight condition θ_0 in Fig. 8.24. The variation of T_{t4}/T_{t2}, compressor pressure ratio, corrected mass flow rate, and corrected fuel flow rate are plotted vs θ_0 in Fig. 8.25. The representation of the engine thrust, as corrected thrust vs θ_0, essentially collapses the thrust data into one line for θ_0 greater than 1.0. The discussion that follows helps one see why the plot in Fig. 8.24 behaves as shown. When θ_0 is less than 1.0, we observe the following:

1) The compressor pressure ratio is constant at its maximum value of 15 (see Fig. 8.25).

2) The compressor corrected mass flow rate is constant at its maximum value of 100 lbm/s (see Fig. 8.25).

3) The value of T_{t4}/T_{t2} is constant at its maximum value of 6.17 (see Fig. 8.25).

4) The corrected exit velocity given by Eq. (8.33b) is essentially constant.

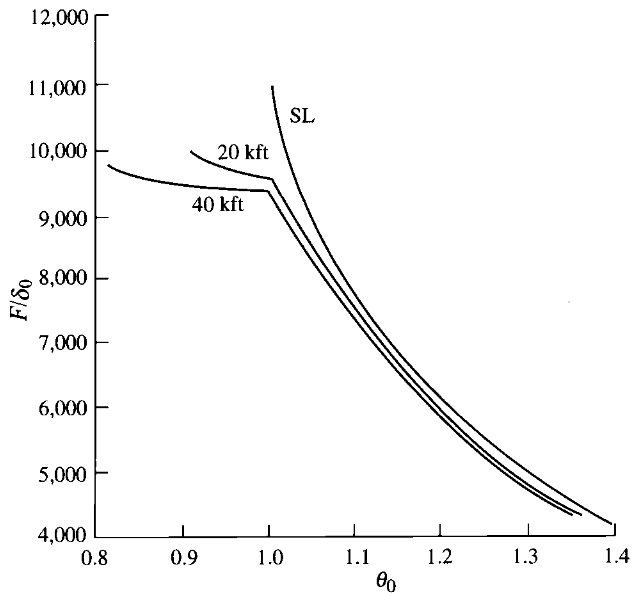

Fig. 8.24 Maximum corrected thrust (F/δ_0) of a turbojet vs θ_0.

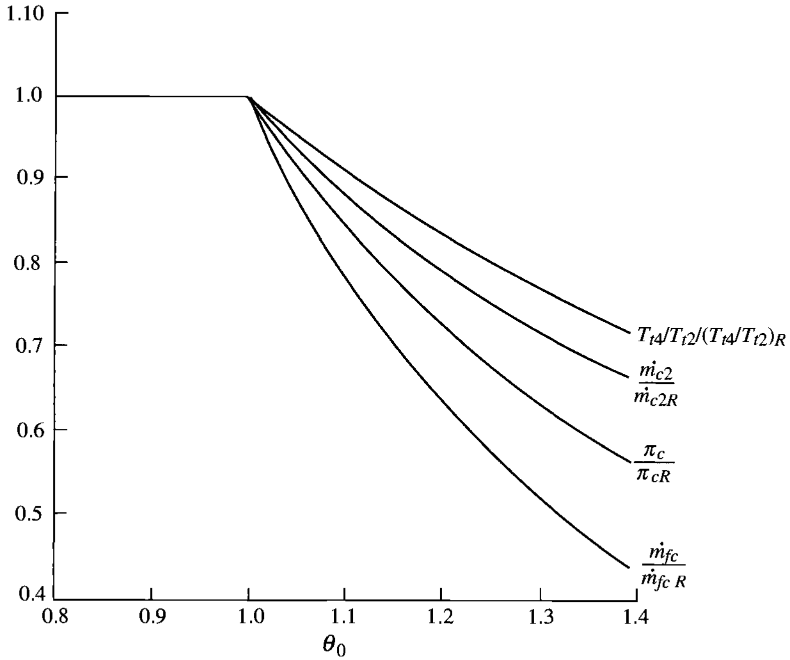

Fig. 8.25 Maximum throttle characteristics of a turbojet vs θ_0.

5) The corrected flight velocity [Eq. (8.33c)] increases in a nearly linear manner with M_0.

6) The corrected thrust [Eq. (8.33a)] decreases slightly with increasing θ_0.

When θ_0 is greater than 1.0, we observe the following:

1) The compressor pressure ratio decreases with increasing θ_0.

2) The compressor corrected mass flow rate decreases with increasing θ_0.

3) The value of T_{t4} is constant at its maximum value of 3200°R.

4) The corrected exit velocity given by Eq. (8.33b) decreases with increasing θ_0.

5) The corrected flight velocity [Eq. (8.33c)] increases in a nearly linear manner with M_0.

6) The corrected thrust [Eq. (8.33a)] decreases substantially with increasing θ_0.

As shown in Fig. 8.24, the trend in maximum corrected thrust F/δ_0 of this turbojet dramatically changes at the $\theta_{0\,\mathrm{break}}$ value of 1.0. Both the compressor pressure ratio π_c and combustor exit temperature T_{t4} are at their maximum values when θ_0 is 1.0. The engine control system varies the fuel flow to the combustor to keep π_c and T_{t4} under control. The control system maintains π_c at its maximum for θ_0 values less than 1.0, and T_{t4} at its maximum for θ_0 values greater than 1.0. These same kinds of trends are observed for many other gas turbine aircraft engines.

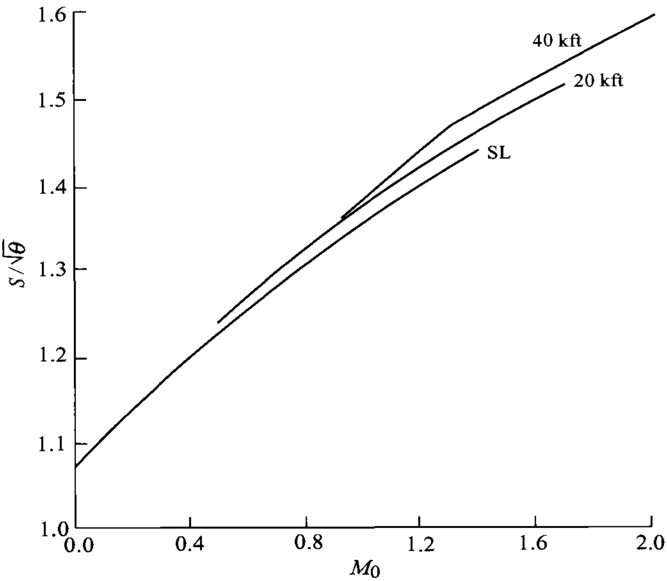

Fig. 8.26 $S/\sqrt{\theta}$ **of a turbojet vs** M_0**.**

The thrust-specific fuel consumption S of this turbojet at maximum thrust is plotted vs Mach number in Fig. 8.18. If the values of S are divided by the square root of the corrected ambient temperature, then the curves for higher altitudes are shifted up and we get Fig. 8.26. Note that these curves could be estimated by a straight line. Equations (1.36a–1.36f) are based on this nearly linear relationship with flight Mach number M_0. When the corrected thrust-specific fuel consumption [S_c, see Eq. (8.8)] is plotted vs θ_0, the spread in fuel consumption data is substantially reduced, as shown in Fig. 8.27. One could estimate that the corrected thrust specific fuel consumption has a value of about 1.24 for most flight conditions.

8.3.3 Throttle Ratio

The *throttle ratio* (TR) is defined as the ratio of the maximum value of T_{t4} to the value of T_{t4} at sea-level static (SLS) conditions. In equation form, the throttle ratio is

$$\mathrm{TR} \equiv \frac{(T_{t4})_{\max}}{(T_{t4})_{\mathrm{SLS}}} \tag{8.34a}$$

The throttle ratio for the simple turbojet engine and compressor of Figs. 8.17–8.27 has a value of 1.0. Both the compressor performance and engine performance curves change shape at a θ_0 value of 1.0. This change in shape of the performance curves occurs at the simultaneous maximum of π_c and T_{t4}. The fact that both the throttle ratio and dimensionless total temperature

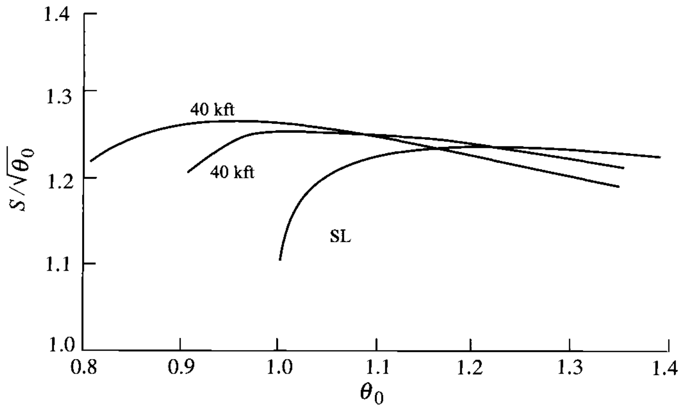

Fig. 8.27 $S/\sqrt{\theta_0}$ **of a turbojet vs** θ_0**.**

θ_0 have a value of 1.0 at the simultaneous maximum is not a coincidence but is a direct result of compressor-turbine power balance given by Eq. (8.17). At the simultaneous maximum of π_c and T_{t4}, the throttle ratio equals θ_0:

$$\text{TR} = \theta_0 \qquad \text{at max } \pi_c \text{ and max } T_{t4}$$

or

$$\text{TR} = \theta_{0\,\text{break}} \tag{8.34b}$$

High-performance fighters want gas turbine engines whose thrust does not drop off as fast with increasing θ_0 as that of Fig. 8.24. The value of θ_0, where the corrected maximum thrust F/δ_0 curves change slope, can be increased by increasing the maximum T_{t4} of the preceding example turbojet engine.

Example 8.5

Again, we consider the example turbojet engine with a compressor that has a compressor pressure ratio of 15 and corrected mass flow rate of 100 lbm/s for T_{t2} of 518.7°R and T_{t4} of 3200°R. The maximum π_c is maintained at 15, and the maximum T_{t4} is increased from 3200 to 3360°R (TR = 1.05). The variation in thrust, thrust-specific fuel consumption, compressor pressure ratio, and corrected mass flow rate of this turbojet engine at full throttle are plotted vs flight Mach number M_0 in Figs. 8.28, 8.29, 8.30, and 8.31, respectively. Figure 8.32 shows the corrected thrust F/δ_0 plotted vs θ_0. Comparing Figs. 8.17 and 8.28, we note that the thrust of both engines are the same at sea-level static, and the engine with a throttle ratio of 1.05 has higher thrust at high Mach numbers. Figures 8.24 and 8.32 show that changing the throttle ratio from 1.0 to 1.05 changes the θ_0 value at which the curves change shape $\theta_{0\,\text{break}}$ and increases the corrected thrust at θ_0 values greater than 1.0.

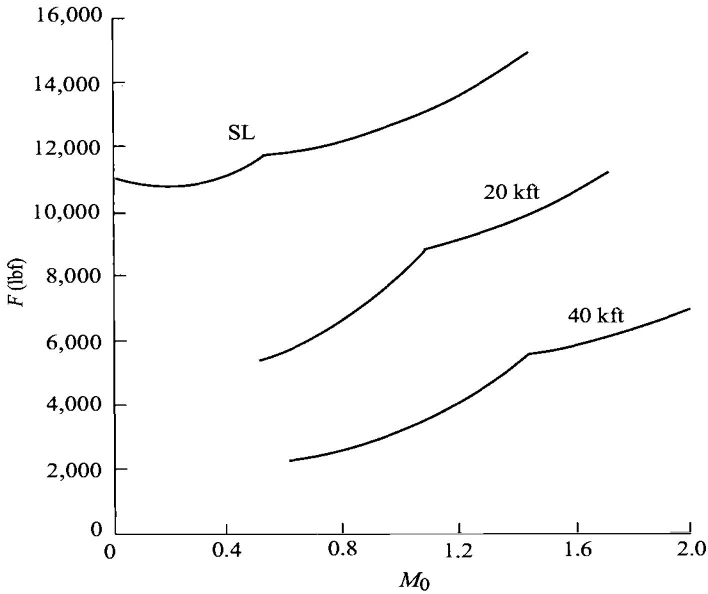

Fig. 8.28 Maximum thrust F of improved turbojet vs M_0.

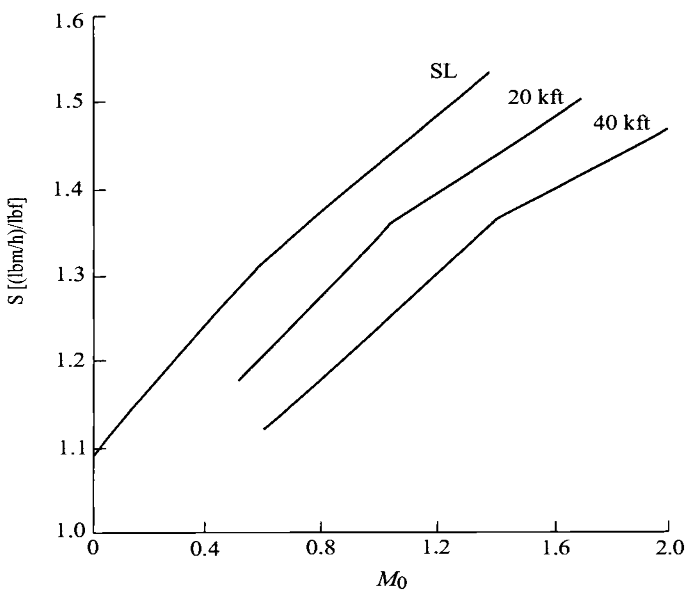

Fig. 8.29 Thrust-specific fuel consumption S of improved turbojet vs M_0.

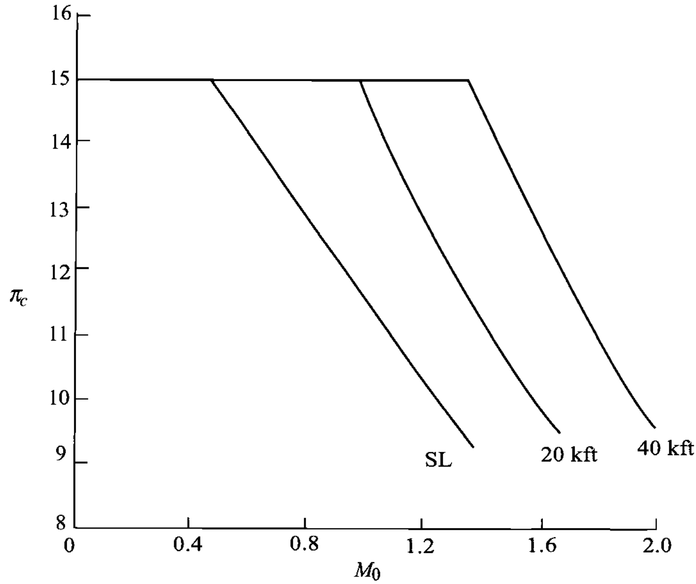

Fig. 8.30 Compressor pressure ratio of improved turbojet vs M_0.

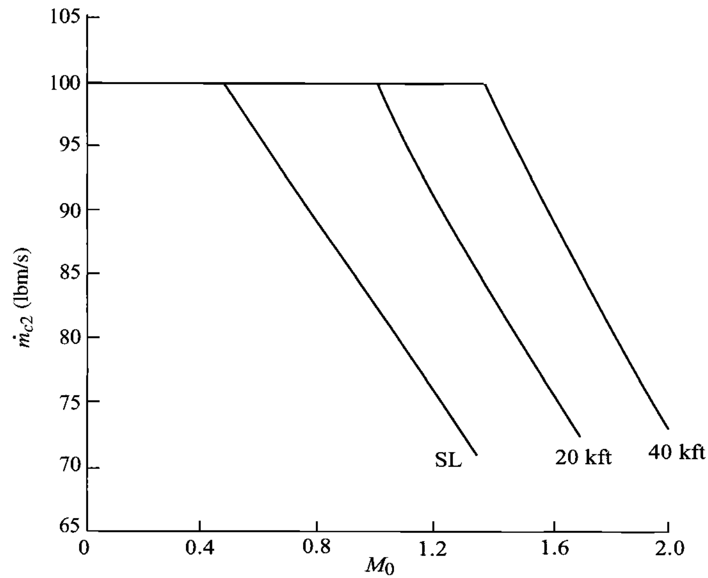

Fig. 8.31 Compressor corrected mass flow rate of improved turbojet vs M_0.

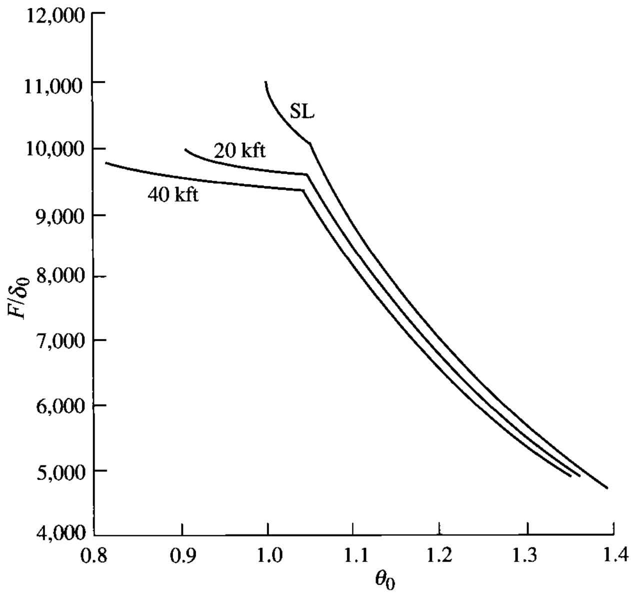

Fig. 8.32 Maximum corrected thrust F/δ_0 of improved turbojet vs θ_0.

Because the compressor and turbine are connected to the same shaft, they have the same rotational speed N, and we can write the following relationship between their corrected speeds:

$$N_{c2} = 1/\sqrt{T_{\text{ref}}}\sqrt{T_{t4}/\theta_0}\,N_{c4} \tag{8.35}$$

Recall that for constant turbine efficiency and choked flow at stations 4 and 8, the correct turbine speed N_{c4} was found to be constant. For maximum thrust engine conditions where θ_0 is less than the throttle ratio, the corrected rotational speed of the compressor N_{c2} and the ratio T_{t4}/θ_0 are constant. Equation (8.35) shows that the corrected speed of the turbine N_{c4} must also be constant at these engine conditions. At $\theta_0 = \text{TR}$, T_{t4} is maximum, the corrected rotational speed of the compressor N_{c2} is constant, and the shaft rotational speed N increases by the square root of θ_0. Thus an engine with a throttle ratio of 1.05 can have a shaft rotational speed at $\theta_0 = \text{TR}$ that is 1.0247 times the maximum speed at sea-level static conditions. This is commonly referred to as an *overspeed* of 2.47%.

8.3.4 Turbine Performance Relationships—Dual-Spool Engines

Two-spool engines, like the turbojet engine shown in Fig. 8.33 and the turbofan engine of Fig. 8.1, are designed with choked flow at engine

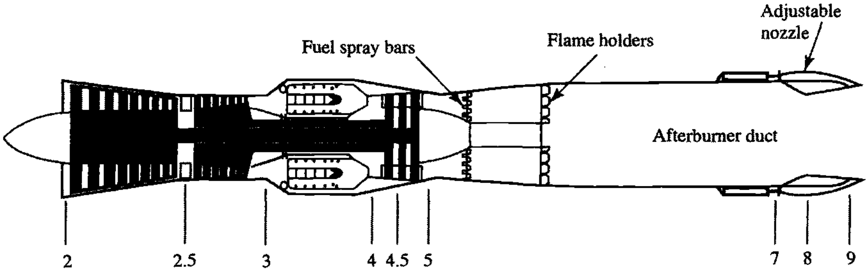

Fig. 8.33 Dual-spool afterburning turbojet engine. (Courtesy of Pratt & Whitney.)

stations 4, 4.5, and 8. Under some operating conditions, the flow may unchoke at station 8. The resulting high-pressure turbine and low-pressure turbine performance relationships are developed in this section for later use.

8.3.4.1 High-pressure turbine. Because the mass flow rate at the entrance to the high-pressure turbine equals that entering the low-pressure turbine,

$$\dot{m}_4 = \frac{P_{t4}}{\sqrt{T_{t4}}} A_4 \text{MFP}(M_4) = \dot{m}_{4.5} = \frac{P_{t4.5}}{\sqrt{T_{t4.5}}} A_{4.5} \text{MFP}(M_{4.5})$$

We assume that the areas are constant and the flow is choked at stations 4 and 4.5. Then

$$\frac{P_{t4}/P_{t4.5}}{\sqrt{T_{t4}/T_{t4.5}}} = \frac{\sqrt{\tau_{tH}}}{\pi_{tH}} = \text{const}$$

Thus for constant η_{tH}, we have

$$\text{constant values of } \pi_{tH}, \tau_{tH}, \dot{m}_{c4}, \text{ and } \dot{m}_{c4.5} \tag{8.36}$$

8.3.4.2 Low-pressure turbine. Because the mass flow rate at the entrance to the low-pressure turbine equals that at the exit nozzle throat,

$$\dot{m}_{4.5} = \frac{P_{t4.5}}{\sqrt{T_{t4.5}}} A_{4.5} \text{MFP}(M_{4.5}) = \dot{m}_8 = \frac{P_{t8}}{\sqrt{T_{t8}}} A_8 \text{MFP}(M_8)$$

We assume that the areas are constant at stations 4.5 and 8 and the flow is choked at station 4.5, and so

$$\frac{P_{t4.5}/P_{t5}}{\sqrt{T_{t4.5}/T_{t8}}} \frac{1}{\text{MFP}(M_8)} = \frac{\sqrt{\tau_{tL}}/\pi_{tL}}{\text{MFP}(M_8)} = \frac{A_8 \pi_{AB}}{A_{4.5} \text{MFP}(M_{4.5})}$$

Using the reference condition to evaluate the constant on the right-hand side of the preceding equation gives

$$\pi_{tL} = \pi_{tLR}\sqrt{\frac{\tau_{tL}}{\tau_{tL\ R}}}\frac{\text{MFP}(M_{8R})}{\text{MFP}(M_8)} \tag{8.37}$$

where

$$\tau_{tL} = 1 - \eta_{tL}(1 - \pi_{tL}^{(\gamma_t-1)/\gamma_t}) \tag{8.38}$$

If station 8 is choked at the reference condition and at the current operating point, then π_{tL} and τ_{tL} are constant and equal their reference values.

8.4 Turbojet Engine with Afterburning

The dual-spool afterburning turbojet engine (Fig. 8.33) is normally designed with choked flow at stations 4, 4.5, and 8. For the afterburning turbojet engine, the variable-area exhaust nozzle is controlled by the engine control system so that the upstream turbomachinery is unaffected by the afterburner operation. In other words, the exhaust nozzle throat area A_8 is controlled during afterburner operation such that the turbine exit conditions (P_{t5}, T_{t5}, and M_5) remain constant. Because the exhaust nozzle has choked flow at its throat at all operating conditions of interest, Eqs. (8.37) and (8.38) for constant efficiency of the low-pressure turbine require that π_{tL} and τ_{tL} be constant:

$$\text{constant values of } \pi_{tL} \text{ and } \tau_{tL} \tag{8.39}$$

This engine has six independent variables (T_{t4}, T_{t7}, M_0, T_0, P_0, and P_0/P_9) and nine dependent variables. These performance analysis variables are summarized in Table 8.5.

Table 8.5 Performance analysis variables for dual-spool afterburning turbojet engine

	Variables		
Component	Independent	Constant or known	Dependent
Engine	M_0, T_0, P_0	$\dot{m}_0$	
Diffuser		$\pi_d = f(M_0)$	
Fan		η_{cL}	π_{cL}, τ_{cL}
High-pressure compressor		η_{cH}	π_{cH}, τ_{cH}
Burner	T_{t4}	π_b, η_b	f
High-pressure turbine		π_{tH}, τ_{tH}	
Low-pressure turbine		π_{tL}, τ_{tL}	
Afterburner	T_{t7}	π_{AB}, η_{AB}	f_{AB}
Nozzle	$\dfrac{P_9}{P_0}$	π_n	M_9, $\dfrac{T_9}{T_0}$
Total number	6		9

8.4.1 Analysis of Compressors

8.4.1.1 High-pressure compressor (τ_{cH}, π_{cH}). The power balance between the high-pressure turbine and the high-pressure compressor (high-pressure spool) gives

$$\eta_{mH}\dot{m}_4 c_{pt}(T_{t4} - T_{t4.5}) = \dot{m}_2 c_{pc}(T_{t3} - T_{t2.5})$$

Rewriting this equation in terms of temperature ratios, rearranging into variable and constant terms, and equating the constant to reference values give

$$\frac{\tau_r \tau_{cL}(\tau_{cH} - 1)}{T_{t4}/T_0} = \eta_{mH}(1+f)(1-\tau_{tH}) = \left[\frac{\tau_r \tau_{cL}(\tau_{cH} - 1)}{T_{t4}/T_0}\right]_R$$

Solving for τ_{cH} gives

$$\tau_{cH} = 1 + \frac{T_{t4}/T_0}{(T_{t4}/T_0)_R}\frac{(\tau_r\tau_{cL})_R}{\tau_r\tau_{cL}}(\tau_{cH} - 1)_R \tag{8.40}$$

From the definition of compressor efficiency, π_{cH} is given by

$$\pi_{cH} = [1 + \eta_{cH}(\tau_{cH} - 1)]^{\gamma_c/(\gamma_c - 1)} \tag{8.41}$$

8.4.1.2 Low-pressure compressor (τ_{cL}, π_{cL}). From a power balance between the low-pressure compressor and low-pressure turbine, we get

$$\eta_{mL}\dot{m}_{4.5} c_{pt}(T_{t4.5} - T_{t5}) = \dot{m}_2 c_{pc}(T_{t2.5} - T_{t2})$$

Rewriting this equation in terms of temperature ratios, rearranging into variable and constant terms, and equating the constant to reference values give

$$\frac{\tau_r(\tau_{cL} - 1)}{T_{t4}/T_0} = \eta_{mL}(1+f)\tau_{tH}(1-\tau_{tL}) = \left[\frac{\tau_r(\tau_{cL} - 1)}{T_{t4}/T_0}\right]_R$$

Solving for τ_{cL} gives

$$\tau_{cL} = 1 + \frac{T_{t4}/T_0}{(T_{t4}/T_0)_R}\frac{(\tau_r)_R}{\tau_r}(\tau_{cL} - 1)_R \tag{8.42}$$

where

$$\pi_{cL} = [1 + \eta_{cL}(\tau_{cL} - 1)]^{\gamma_c/(\gamma_c - 1)} \tag{8.43}$$

8.4.2 Mass Flow Rate

Since

$$\dot{m}_4 = \dot{m}_0 + \dot{m}_f = \dot{m}_0(1+f)$$

and

$$\dot{m}_4 = \frac{P_{t4}}{\sqrt{T_{t4}}}A_4\,\mathrm{MFP}(M_4)$$

thus

$$\dot{m}_0 = \frac{P_{t4}}{\sqrt{T_{t4}}} \frac{A_4 \, \mathrm{MFP}(M_4)}{1+f} = \frac{P_0 \pi_r \pi_d \pi_{cL} \pi_{cH} \pi_b}{\sqrt{T_{t4}}} \frac{A_4 \, \mathrm{MFP}(M_4)}{1+f}$$

For A_4, M_4, $1+f$, and π_b essentially constant, the preceding expression can be rewritten as

$$\frac{\dot{m}_0 \sqrt{T_{t4}}}{P_0 \pi_r \pi_d \pi_{cL} \pi_{cH}} = \frac{\pi_b A_4 \, \mathrm{MFP}(M_4)}{1+f} = \left(\frac{\dot{m}_0 \sqrt{T_{t4}}}{P_0 \pi_r \pi_d \pi_{cL} \pi_{cH}} \right)_R$$

or

$$\frac{\dot{m}_0}{\dot{m}_{0R}} = \frac{P_0 \pi_r \pi_d \pi_{cL} \pi_{cH}}{(P_0 \pi_r \pi_d \pi_{cl} \pi_{cH})_R} \sqrt{\frac{T_{t4R}}{T_{t4}}} \tag{8.44}$$

8.4.3 Summary of Performance Equations—Turbojet With and Without Afterburner

INPUTS:

Choices	
Flight parameters:	M_0, T_0(K, °R), P_0(kPa, psia)
Throttle setting:	T_{t4} (K, °R), T_{t7} (K, °R)
Exhaust nozzle setting:	P_0/P_9
Design constants	
π:	$\pi_{d\max}$, π_b, π_{tH}, π_{tL}, π_{AB}, π_n
τ:	τ_{tH}, τ_{tL}
η:	η_{cL}, η_{cH}, η_b, η_{AB}, η_{mH}, η_{mL}
Gas properties:	γ_c, γ_t, γ_{AB}, c_{pc}, c_{pt}, c_{pAB} [kJ/(kg · K), Btu/(lbm · °R)]
Fuel:	h_{PR} (kJ/kg, Btu/lbm)
Reference conditions	
Flight parameters:	M_{0R}, T_{0R} (K, °R), P_{0R} (kPa, psia) τ_{rR}, π_{rR}
Throttle setting:	T_{t4R} (K, °R)
Component behavior:	π_{dR}, π_{cLR}, π_{cHR}, τ_{cLR}, τ_{cHR}

OUTPUTS:

Overall performance:	F(N, lbf), $\dot{m}_0$ (kg/s, lbm/s), f_O, $S\left(\frac{\mathrm{mg/s}}{N}, \frac{\mathrm{lbm/h}}{\mathrm{lbf}}\right)$, η_P, η_T, η_O
Component behavior:	π_{cL}, π_{cH}, τ_{cL}, τ_{cH}, f, f_{AB}, M_9, $(N/N_R)_{\mathrm{LPspool}}$, $(N/N_R)_{\mathrm{HPspool}}$

EQUATIONS:

$$R_c = \frac{\gamma_c - 1}{\gamma_c} c_{pc} \tag{8.45a}$$

$$R_t = \frac{\gamma_t - 1}{\gamma_t} c_{pt} \tag{8.45b}$$

$$a_0 = \sqrt{\gamma_c R_c g_c T_0} \tag{8.45c}$$

$$V_0 = a_0 M_0 \tag{8.45d}$$

$$\tau_r = 1 + \frac{\gamma_c - 1}{2} M_0^2 \tag{8.45e}$$

$$\pi_r = \tau_r^{\gamma_c/(\gamma_c - 1)} \tag{8.45f}$$

$$\eta_r = 1 \qquad \text{for } M_0 \le 1 \tag{8.45g}$$

$$\eta_r = 1 - 0.075(M_0 - 1)^{1.35} \qquad \text{for } M_0 > 1 \tag{8.45h}$$

$$\pi_d = \pi_{d\max} \eta_r \tag{8.45i}$$

$$\tau_{cL} = 1 + \frac{T_{t4}/T_0}{(T_{t4}/T_0)_R} \frac{(\tau_r)_R}{\tau_r} (\tau_{cL} - 1)_R \tag{8.45j}$$

$$\pi_{cL} = [1 + \eta_{cL}(\tau_{cL} - 1)]^{\gamma_c/(\gamma_c - 1)} \tag{8.45k}$$

$$\tau_{cH} = 1 + \frac{T_{t4}/T_0}{(T_{t4}/t_0)_R} \frac{(\tau_r \tau_{cL})_R}{\tau_r \tau_{cL}} (\tau_{cH} - 1)_R \tag{8.45l}$$

$$\pi_{cH} = [1 + \eta_{cH}(\tau_{cH} - 1)]^{\gamma_c/(\gamma_c - 1)} \tag{8.45m}$$

$$\tau_\lambda = \frac{c_{pt} T_{t4}}{c_{pc} T_0} \tag{8.45n}$$

$$f = \frac{\tau_\lambda - \tau_r \tau_{cL} \tau_{cH}}{h_{PR} \eta_b / (c_{pc} T_0) - \tau_\lambda} \tag{8.45o}$$

$$\dot{m}_0 = \dot{m}_{0R} \frac{P_0 \pi_r \pi_d \pi_{cL} \pi_{cH}}{(P_0 \pi_r \pi_d \pi_{cL} \pi_{cH})_R} \sqrt{\frac{T_{t4R}}{T_{t4}}} \tag{8.45p}$$

Without afterburner:

$$R_{AB} = R_t \quad c_{pAB} = c_{pt} \quad \gamma_{AB} = \gamma_t \quad T_{t7} = T_{t4} \tau_{tH} \tau_{tL} \quad \pi_{AB} = 1 \quad f_{AB} = 0 \tag{8.45q}$$

With afterburner:

$$R_{AB} = \frac{\gamma_{AB} - 1}{\gamma_{AB}} c_{pAB} \tag{8.45r}$$

$$\tau_{\lambda AB} = \frac{c_{pAB} T_{t7}}{c_{pc} T_0} \tag{8.45s}$$

$$f_{AB} = \frac{\tau_{\lambda AB} - \tau_\lambda \tau_{tH} \tau_{tL}}{h_{PR} \eta_{AB}/(c_{pc} T_0) - \tau_{\lambda AB}} \tag{8.45t}$$

Remainder of equations:

$$\frac{P_{t9}}{P_9} = \frac{P_0}{P_9} \pi_r \pi_d \pi_{cL} \pi_{cH} \pi_b \pi_{tH} \pi_{tL} \pi_{AB} \pi_n \tag{8.45u}$$

$$M_9 = \sqrt{\frac{2}{\gamma_{AB-1}}\left[\left(\frac{P_{t9}}{P_9}\right)^{(\gamma_{AB}-1)/\gamma_{AB}} - 1\right]} \tag{8.45v}$$

$$\frac{T_9}{T_0} = \frac{T_{t7}/T_0}{(P_{t9}/P_9)^{(\gamma_{AB}-1)/\gamma_{AB}}} \tag{8.45w}$$

$$\frac{V_9}{a_0} = M_9 \sqrt{\frac{\gamma_{AB} R_{AB}}{\gamma_c R_c} \frac{T_9}{T_0}} \tag{8.45x}$$

$$f_O = f + f_{AB} \tag{8.45y}$$

$$\frac{F}{\dot{m}_0} = \frac{a_0}{g_c}\left[(1+f_0)\frac{V_9}{a_0} - M_0 + (1+f_O)\frac{R_{AB}}{R_c}\frac{T_9/T_0}{V_9/a_0}\frac{1-P_0/P_9}{\gamma_c}\right] \tag{8.45z}$$

$$F = \dot{m}_0\left(\frac{F}{\dot{m}_0}\right) \tag{8.45aa}$$

$$S = \frac{f_O}{F/\dot{m}_0} \tag{8.45ab}$$

$$\eta_T = \frac{a_0^2[(1+f_O)(V_9/a_0)^2 - M_0^2]}{2 g_c f_O h_{PR}} \tag{8.45ac}$$

$$\eta_P = \frac{2 g_c V_0 (F/\dot{m}_0)}{a_0^2[(1+f_O)(V_9/a_0)^2 - M_0^2]} \tag{8.45ad}$$

$$\eta_O = \eta_P \eta_T \tag{8.45ae}$$

$$\left(\frac{N}{N_R}\right)_{\text{LPspool}} = \sqrt{\frac{T_0 \tau_r}{T_{0R} \tau_{rR}} \frac{\pi_{cL}^{(\gamma-1)/\gamma} - 1}{\pi_{cLR}^{(\gamma-1)/\gamma} - 1}} \tag{8.45af}$$

$$\left(\frac{N}{N_R}\right)_{\text{HPspool}} = \sqrt{\frac{T_0 \tau_r \tau_{cL}}{T_{0R} \tau_{rR} \tau_{cLR}} \frac{\pi_{cH}^{(\gamma-1)/\gamma} - 1}{\pi_{cHR}^{(\gamma-1)/\gamma} - 1}} \tag{8.45ag}$$

$$\frac{A_9}{A_{9R}} = \left[\frac{P_{t9}/P_9}{(P_{t9}/P_9)_R}\right]^{(\gamma_t+1)/(2\gamma_t)} \sqrt{\frac{(P_{t9}/P_9)_R^{(\gamma_t-1)/\gamma_t} - 1}{(P_{t9}/P_9)^{(\gamma_t-1)/\gamma_t} - 1}} \tag{8.45ah}$$

Example 8.6

In this example, we consider the variation in engine performance of the dry turbojet in Example 8.3 with Mach number M_0, altitude, ambient temperature T_0, ambient pressure P_0, and throttle setting T_{t4}. The compressor pressure ratio is limited to 12.3, and the combustor exit temperature T_{t4} is limited to 1800 K. Figures 8.34, 8.35, 8.36, 8.37, and 8.38 show the variations of thrust, thrust specific fuel consumption S, engine mass flow, engine corrected mass flow, and compressor pressure ratio with Mach number, respectively.

At θ_0 values below 1.2, the engine is at the maximum compressor pressure ratio of 12.3 and T_{t4} is below its maximum value of 1800 K. At θ_0 values above 1.2, the engine is at its maximum T_{t4} value of 1800 K and the compressor pressure ratio is below its maximum value of 12. This engine has a throttle ratio ($\theta_{0\,\text{break}}$) of 1.2.

The corrected engine mass flow, shown in Fig. 8.37, is used in sizing the engine inlet area. For this engine, the maximum corrected engine mass flow is constant for subsonic Mach numbers and decreases with increasing supersonic Mach numbers.

The effect of the inlet total pressure loss on the engine mass flow rate can be seen by comparison of the variations of corrected engine mass flow and compressor pressure ratio, shown in Figs. 8.37 and 8.38. The compressor corrected mass flow rate is constant when the compressor pressure ratio is constant. The drop-off in corrected engine mass flow rate (Fig. 8.37) at Mach numbers greater than 1 is due to the decrease from unity of the inlet total pressure recovery.

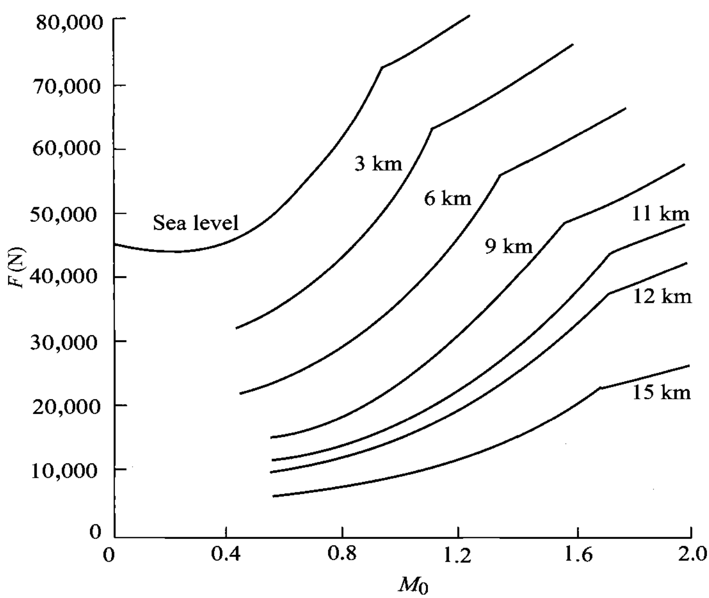

Fig. 8.34 Maximum thrust of dry turbojet.

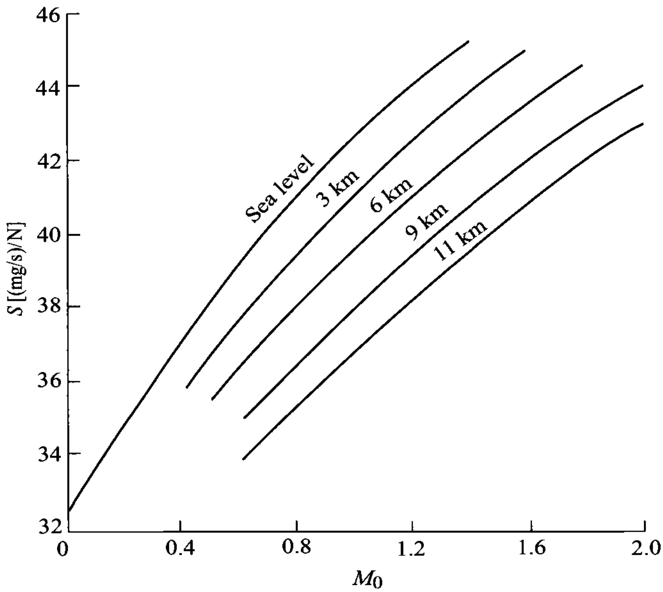

Fig. 8.35 Thrust-specific fuel consumption of dry turbojet at maximum thrust.

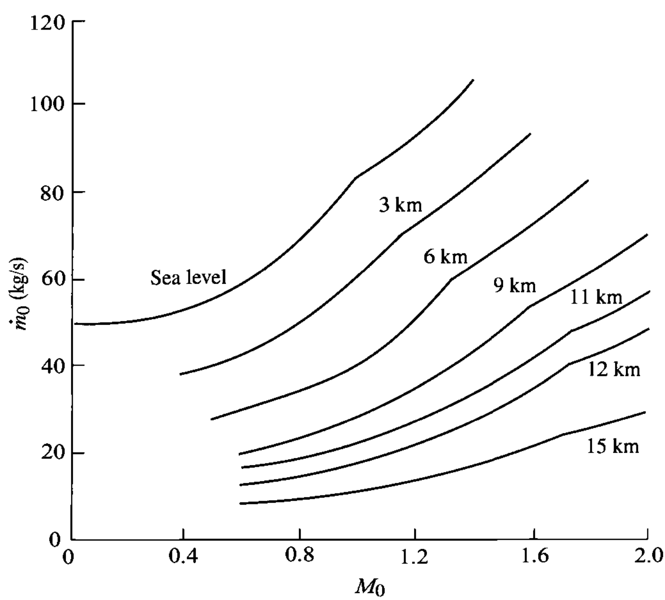

Fig. 8.36 Mass flow rate of dry turbojet at maximum thrust.

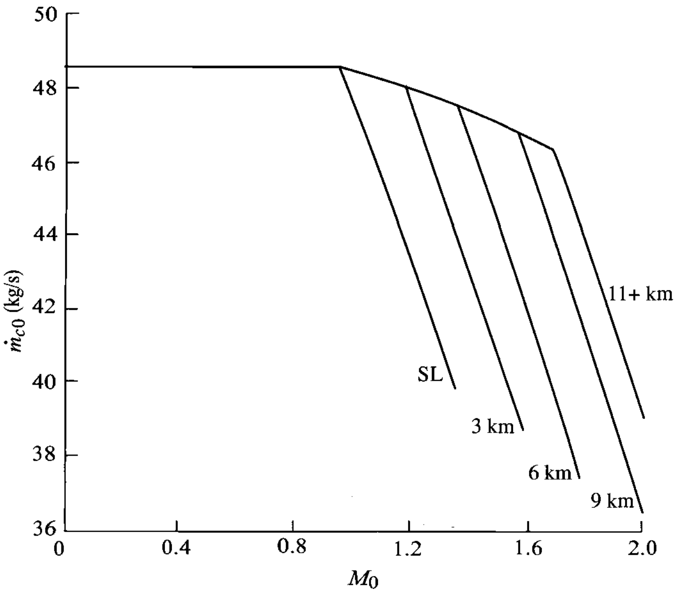

Fig. 8.37 Corrected mass flow rate of dry turbojet at maximum thrust.

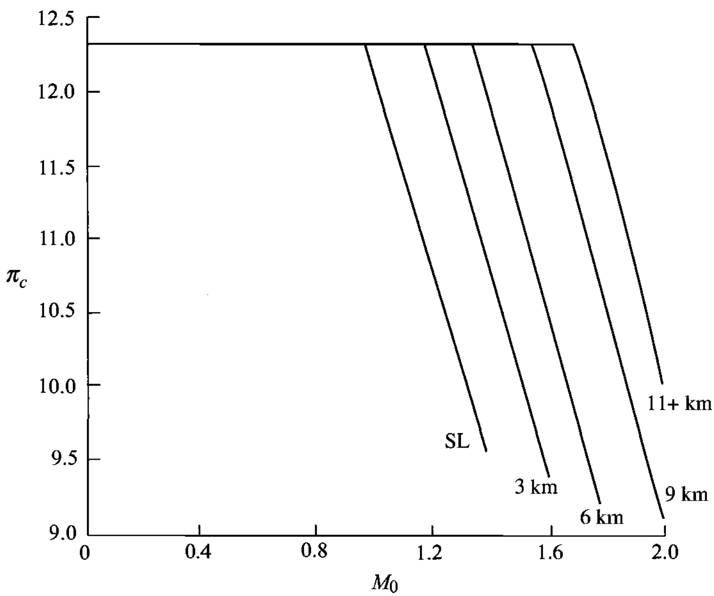

Fig. 8.38 Compressor pressure ratio of dry turbojet at maximum thrust.

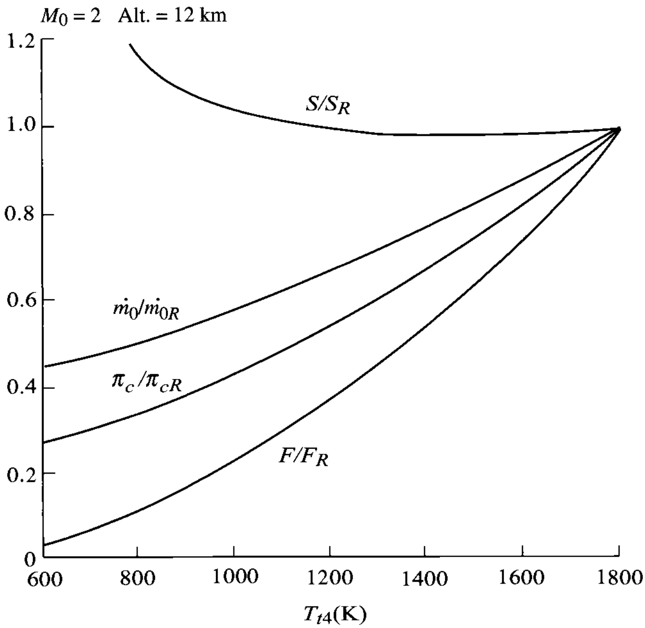

Fig. 8.39 Performance of dry turbojet at partial throttle.

Figure 8.39 shows the variation of engine performance with reduction in engine throttle T_{t4}. Here each engine parameter is compared to its value at the reference condition. As T_{t4} is reduced, the thrust specific fuel consumption initially decreases a little and then increases substantially. The thrust, mass flow rate, and compressor pressure ratio decrease as T_{t4} is reduced.

Figures 8.40 and 8.41 show the variations of engine performance with changes in ambient temperature T_0 and pressure P_0, respectively. Note that decreases in temperature improve thrust F and thrust specific fuel consumption S. The combined effect of ambient temperature and pressure T_0 and P_0 can be seen in the plot vs altitude shown in Fig. 8.42.

Example 8.7

The performance of an afterburning turbojet with a throttle ratio of 1 is considered. The engine performance, when the afterburner is operating at its maximum exit temperature, is commonly referred to as *maximum* or *wet*. This afterburning turbojet engine has a maximum thrust of 25,000 lbf. The terms *military* and *dry* refer to the engine's performance when the afterburner is off (not operating) and the engine core is at maximum operating conditions. The reference conditions and operating limits for the afterburning turbojet engine are as follows.

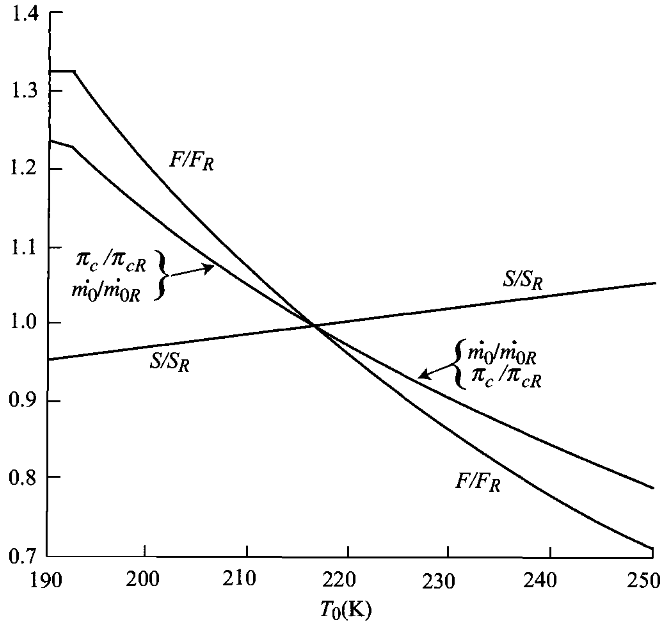

Fig. 8.40 Performance of dry turbojet at maximum thrust vs T_0.

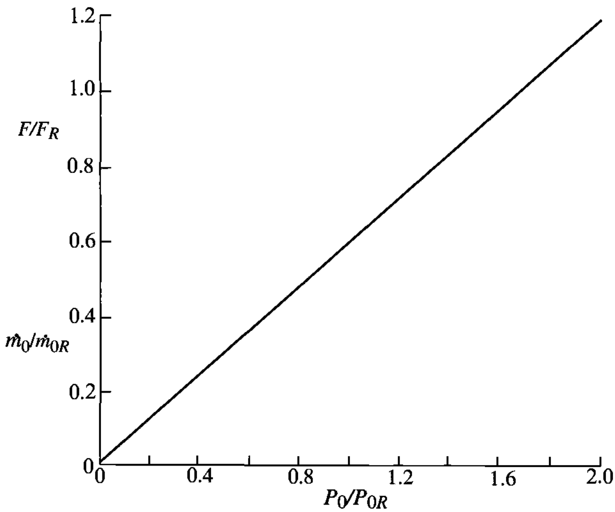

Fig. 8.41 Performance of dry turbojet at maximum thrust vs P_0.

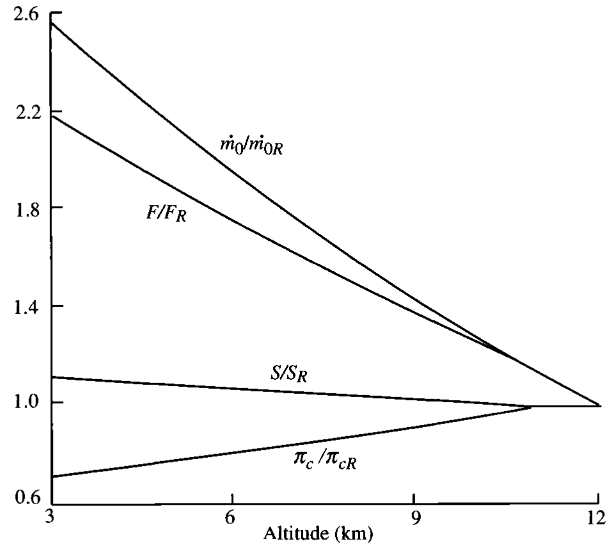

Fig. 8.42 Performance of dry turbojet at maximum thrust vs altitude.

REFERENCE:

$$\text{Sea-level static } (T_0 = 518.7°\text{R},\ P_0 = 14.696 \text{ psia})$$

$$\pi_c = 20, \quad \pi_{cL} = 5, \quad \pi_{cH} = 4, \quad e_{cL} = 0.9$$

$$e_{cH} = 0.9, \quad e_{tH} = 0.9, e_{tL} = 0.9, \quad \pi_{d\max} = 0.98$$

$$\pi_b = 0.96, \quad \pi_n = 0.98, \quad T_{t4} = 3200°\text{R}$$

$$c_{pc} = 0.24 \text{ Btu/(lbm}\cdot°\text{R)}, \quad \gamma_c = 1.4, \quad c_{pt} = 0.295 \text{ Btu/(lbm}\cdot°\text{R)}$$

$$\gamma_t = 1.3, \quad \eta_b = 0.995, \quad \eta_{mL} = 0.995, \quad \eta_{mH} = 0.995$$

$$h_{PR} = 18{,}400 \text{ Btu/lbm}, \quad T_{t7} = 3600°\text{R}, \quad c_{p\text{AB}} = 0.295 \text{ Btu/(lbm}\cdot°\text{R)}$$

$$\gamma_{\text{AB}} = 1.3, \quad \pi_{\text{AB}} = 0.94, \quad \eta_{\text{AB}} = 0.95, \quad \eta_{cL} = 0.8755$$

$$\eta_{cH} = 0.8791, \quad \eta_{tH} = 0.9062, \quad \eta_{tL} = 0.9050, \quad \pi_{tH} = 0.5466$$

$$\tau_{tH} = 0.8821, \quad \pi_{tL} = 0.6127, \quad \tau_{tL} = 0.9033, \quad M_8 = 1$$

$$M_9 = 1.85, \quad f = 0.0358, \quad f_{\text{AB}} = 0.0195, \quad f_O = 0.0554$$

$$F = 25{,}000 \text{ lbf}, \quad S = 1.4473 \text{ (lbm/h)/lbf}, \quad \dot{m}_0 = 181.57 \text{ lbm/s}$$

OPERATION:

Maximum $T_{t4} = 3200°R$	Maximum $T_{t7} = 3600°R$
Mach number: 0 to 2	Altitudes (kft): 0, 20, and 40

The wet and dry performances of this afterburning turbojet are compared in Figs. 8.43 and 8.44. Note that the wet thrust is about 20% greater than the dry thrust, and the thrust at 40-kft altitude is about 25% of its sea-level value. The thrust specific fuel consumption at 40-kft altitude is much higher than expected for Mach numbers below about 1.3. This high S is due to the reduction in T_{t4} below maximum for $\theta_0 < TR$, which lowers the temperature of the gas entering the afterburner and increases the temperature rise across the afterburner.

The partial-throttle performance of the afterburning turbjoet is shown in Fig. 8.45 at flight conditions of sea-level static and Mach 1.5 at 40 kft. These curves are commonly called *throttle hooks* because of their shape. At sea-level static conditions, the minimum thrust specific fuel consumption of about 1.02 (lbm/h)/lbf occurs at a thrust of about 4300 lbf (about 20% of dry thrust). At partial-power levels this low, the change in component efficiency can cause the fuel consumption of a real engine to be very different from that predicted here. Because the engine models used to generate these curves are based on constant component efficiencies, the results at significantly reduced throttle settings can be misleading. Comparison of this figure with the

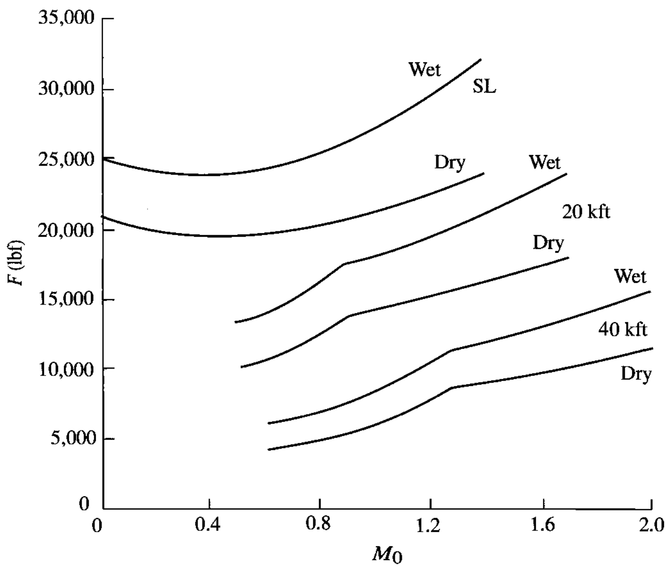

Fig. 8.43 Maximum wet and dry thrust of afterburning turbojet.

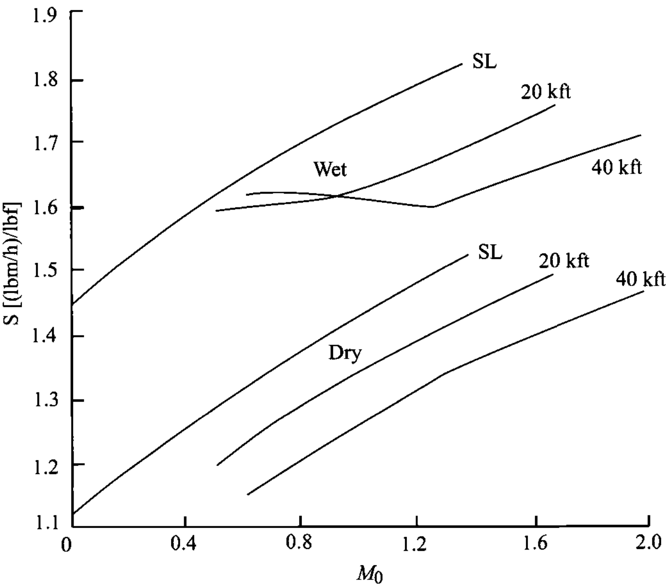

Fig. 8.44 Maximum wet and dry thrust-specific fuel consumption of afterburning turbojet.

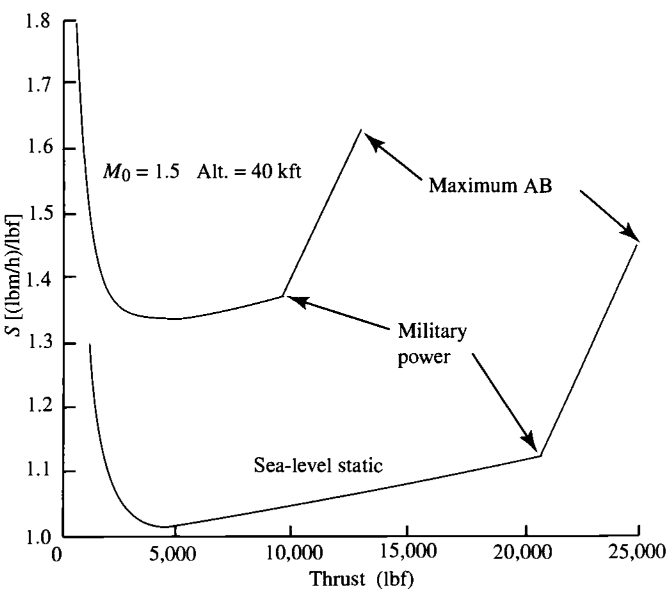

Fig. 8.45 Partial-throttle performance of afterburning turbojet.

partial-power performance of the advanced fighter engine of Fig. 1.14e shows that the trends are correct. The advanced turbofan engine of Fig. 1.14e has lower thrust specific fuel consumption mainly because it is a low-bypass-ratio turbofan engine.

8.5 Turbofan Engine—Separate Exhausts and Convergent Nozzles

The turbofan engines used on commercial subsonic aircraft typically have two spools and separate exhaust nozzles of the convergent type, as shown in Fig. 8.46. For ease of analysis, we will consider a turbofan engine whose fan exit state (13) is the same as the low-pressure compressor exit state (2.5). Thus

$$\tau_f = \tau_{cL} \qquad \text{and} \qquad \pi_f = \pi_{cL}$$

The exhaust nozzles of these turbofan engines have fixed throat areas that will be choked when the exhaust total pressure/ambient static pressure ratio is equal to or larger than $[(\gamma + 1)/2]^{\gamma/(\gamma-1)}$. When an exhaust nozzle is unchoked, the nozzle exit pressure equals the ambient pressure and the exit Mach number is subsonic.

Choked flow at stations 4 and 4.5 of the high-pressure spool during engine operation requires [Eq. (8.36)]

$$\text{constant values of } \pi_{tH},\ \tau_{tH},\ \dot{m}_{c4},\ \text{and } \dot{m}_{c4.5}$$

Because the exhaust nozzles have fixed areas, this gas turbine engine has 4 independent variables (T_{t4}, M_0, T_0, and P_0). We will consider the case when both exhaust nozzles may be unchoked, resulting in 11 dependent variables. The performance analysis variables and constants are summarized in Table 8.6.

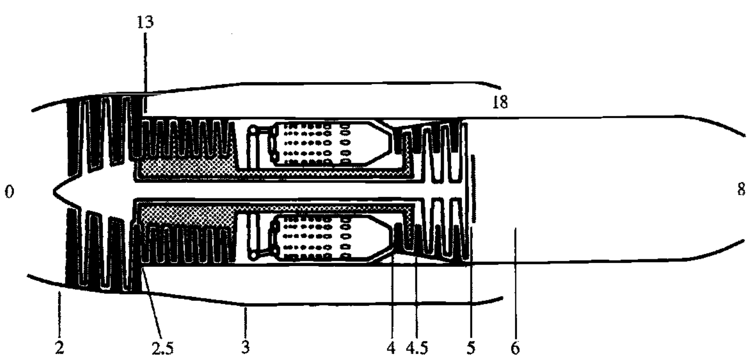

Fig. 8.46 Turbofan engine with separate exhausts. (Courtesy of Pratt & Whitney.)

Table 8.6 Performance analysis variables for separate-exhaust turbofan engine

	Variables		
Component	Independent	Constant or known	Dependent
Engine	M_0, T_0, P_0		$\dot{m}_0$, α
Diffuser		$\pi_d = f(M_0)$	
Fan			π_f, τ_f
High-pressure compressor			π_{cH}, τ_{cH}
Burner	T_{t4}	π_b, η_b	f
High-pressure turbine		π_{tH}, τ_{tH}	
Low-pressure turbine			π_{tL}, τ_{tL}
Core exhaust nozzle		π_n	M_9
Fan exhaust nozzle		π_{fn}	M_{19}
Total number	4		11

8.5.1 Engine Analysis

8.5.1.1 Low-pressure turbine (τ_{tL}, π_{tL}). Equations (8.37) and (8.38) apply for the low-pressure turbine temperature and pressure ratios of this turbofan engine:

$$\pi_{tL} = \pi_{tLR}\sqrt{\frac{\tau_{tL}}{\tau_{tLR}}}\frac{\text{MFP}(M_{9R})}{\text{MFP}(M_9)}$$

where

$$\tau_{tL} = 1 - \eta_{tL}(1 - \pi_{tL}^{(\gamma_t-1)/\gamma_t})$$

If station 9 is choked at the reference condition and at off-design conditions, then π_{tL} and τ_{tL} are constant.

8.5.1.2 Bypass ratio α. An expression for the engine bypass ratio at any operating condition is obtained by first relating the mass flow rates of the core and fan streams to their reference values. For the engine core, we have

$$\dot{m}_C = \frac{\dot{m}_4}{1+f} = \frac{P_{t4}A_4}{\sqrt{T_{t4}}}\frac{\text{MFP}(M_4)}{1+f}$$

Thus

$$\frac{\dot{m}_C}{\dot{m}_{CR}} = \frac{P_{t4}}{P_{t4R}}\sqrt{\frac{T_{t4R}}{T_{t4}}} \tag{i}$$

For the fan stream, we have

$$\dot{m}_F = \frac{P_{t19}A_{19}}{\sqrt{T_{t19}}}\mathrm{MFP}(M_{19})$$

Thus

$$\frac{\dot{m}_F}{\dot{m}_{FR}} = \frac{P_{t19}}{P_{t19R}}\sqrt{\frac{T_{t19R}}{T_{t19}}}\frac{\mathrm{MFP}(M_{19})}{\mathrm{MFP}(M_{19R})} \tag{ii}$$

Combining Eqs. (i) and (ii) to obtain the equation for the bypass ratio α yields

$$\alpha = \alpha_R \frac{\pi_{cHR}}{\pi_{cH}}\sqrt{\frac{\tau_\lambda/(\tau_r\tau_f)}{[\tau_\lambda/(\tau_r\tau_f)]_R}}\frac{\mathrm{MFP}(M_{19})}{\mathrm{MFP}(M_{19R})} \tag{8.46}$$

8.5.1.3 Engine mass flow $\dot{m}_0$. The engine mass flow rate can be written simply as

$$\dot{m}_0 = (1+\alpha)\dot{m}_C$$

Then from Eq. (i), we have

$$\dot{m}_0 = \dot{m}_{0R}\frac{1+\alpha}{1+\alpha_R}\frac{P_0\pi_r\pi_d\pi_f\pi_{cH}}{(P_0\pi_r\pi_d\pi_f\pi_{cH})_R}\sqrt{\frac{T_{t4R}}{T_{t4}}} \tag{8.47}$$

8.5.1.4 High-pressure compressor (τ_{cH}, π_{cH}). The power balance between the high-pressure turbine and the high-pressure compressor [high-pressure (HP) spool] gives

$$\eta_{mH}\dot{m}_4 c_{pt}(T_{t4} - T_{t4.5}) = \dot{m}_{2.5}c_{pc}(T_{t3} - T_{t2.5})$$

Rewriting this equation in terms of temperature ratios, rearranging into variable and constant terms, and equating the constant to reference values give

$$\frac{\tau_r\tau_f(\tau_{cH}-1)}{T_{t4}/T_0} = \eta_{mH}(1+f)(1-\tau_{tH}) = \left[\frac{\tau_r\tau_f(\tau_{cH}-1)}{T_{t4}/T_0}\right]_R$$

Solving for τ_{cH} gives

$$\tau_{cH} = 1 + \frac{T_{t4}/T_0}{(T_{t4}/T_0)_R}\frac{(\tau_r\tau_f)_R}{\tau_r\tau_f}(\tau_{cH}-1)_R \tag{8.48}$$

From the definition of compressor efficiency, π_{cH} is given by

$$\pi_{cH} = [1 + \eta_{cH}(\tau_{cH} - 1)]^{\gamma_c/(\gamma_c - 1)} \tag{8.49}$$

8.5.1.5 Fan (τ_f, π_f). From a power balance between the fan and low-pressure turbine, we get

$$\eta_{mL}\dot{m}_{4.5}c_{pt}(T_{t4.5} - T_{t5}) = (\dot{m}_C + \dot{m}_F)c_{pc}(T_{t13} - T_{t2})$$

Rewriting this equation in terms of temperature ratios, rearranging variable and constant terms, and equating the constant to reference values give

$$(1 + \alpha)\frac{\tau_r(\tau_f - 1)}{T_{t4}/T_0} = \eta_{mL}(1 + f)\tau_{tH}(1 - \tau_{tL}) = \left[(1 + \alpha)\frac{\tau_r(\tau_f - 1)}{T_{t4}/T_0}\right]_R$$

Solving for τ_f gives

$$\tau_f = 1 + \frac{1 - \tau_{tL}}{(1 - \tau_{tL})_R}\frac{\tau_\lambda/\tau_r}{(\tau_\lambda/\tau_r)_R}\frac{1 + \alpha_R}{1 + \alpha}(\tau_{fR} - 1) \tag{8.50}$$

where

$$\pi_f = [1 + (\tau_f - 1)\eta_f]^{\gamma_c/(\gamma_c - 1)} \tag{8.51}$$

8.5.2 Solution Scheme

The principal dependent variables for the turbofan engine are π_{tL}, τ_{tL}, α, τ_{cH}, π_{cH}, τ_f, π_f, M_9, and M_{19}. These variables are dependent on each other plus the engine's independent variables—throttle setting and flight condition. The functional interrelationship of the dependent variables can be written as

$$\begin{aligned}
\tau_{cH} &= f_1(\tau_f) & M_9 &= f_6(\pi_f, \pi_{cH}, \pi_{tL})\\
\pi_{cH} &= f_2(\tau_{cH}) & \pi_{tL} &= f_7(\tau_{tL}, M_9)\\
\tau_f &= f_3(\tau_{tL}, \alpha) & \tau_{tL} &= f_8(\pi_{tL})\\
\pi_f &= f_4(\tau_f) & \alpha &= f_9(\tau_f, \pi_{cH}, M_{19})\\
M_{19} &= f_5(\pi_f)
\end{aligned}$$

This system of nine equations is solved by functional iteration, starting with reference quantities as initial values for π_{tL}, τ_{tL}, and τ_f. The following equations are calculated for the nine dependent variables in the order listed until successive values of τ_{tL} do not change more than a specified amount (say, 0.0001):

$$\tau_{cH} = 1 + \frac{T_{t4}/T_0}{(T_{t4}/T_0)_R}\frac{(\tau_r \tau_f)_R}{\tau_r \tau_f}(\tau_{cH} - 1)_R \tag{i}$$

$$\pi_{cH} = \left[1 + \eta_{cH}(\tau_{cH} - 1)\right]^{\gamma_c/(\gamma_c - 1)} \tag{ii}$$

$$\pi_f = \left[1 + (\tau_f - 1)\eta_f\right]^{\gamma_c/(\gamma_c - 1)} \tag{iii}$$

$$P_{t19}/P_0 = \pi_r \pi_d \pi_f \pi_{\text{fn}} \tag{iv}$$

$$\text{If} \quad \frac{P_{t19}}{P_0} < \left(\frac{\gamma_c + 1}{2}\right)^{\gamma_c/(\gamma_c - 1)} \quad \text{then} \quad \frac{P_{t19}}{P_{19}} = \frac{P_{t19}}{P_0}$$

$$\text{else} \quad \frac{P_{t19}}{P_{19}} = \left(\frac{\gamma_c + 1}{2}\right)^{\gamma_c/(\gamma_c - 1)} \tag{v}$$

$$M_{19} = \sqrt{\frac{2}{\gamma_c - 1}\left[\left(\frac{P_{t19}}{P_{19}}\right)^{(\gamma_c - 1)/\gamma_c} - 1\right]} \tag{vi}$$

$$\frac{P_{t9}}{P_0} = \pi_r \pi_d \pi_f \pi_{cH} \pi_b \pi_{tH} \pi_{tL} \pi_n \tag{vii}$$

$$\text{If} \quad \frac{P_{t9}}{P_0} < \left(\frac{\gamma_t + 1}{2}\right)^{\gamma_t/(\gamma_t - 1)} \quad \text{then} \quad \frac{P_{t9}}{P_9} = \frac{P_{t9}}{P_0}$$

$$\text{else} \quad \frac{P_{t9}}{P_9} = \left(\frac{\gamma_t + 1}{2}\right)^{\gamma_t/(\gamma_t - 1)} \tag{viii}$$

$$M_9 = \sqrt{\frac{2}{\gamma_t - 1}\left[\left(\frac{P_{t9}}{P_9}\right)^{(\gamma_t - 1)/\gamma_t} - 1\right]} \tag{ix}$$

$$\alpha = \alpha_R \frac{\pi_{cHR}}{\pi_{cH}} \sqrt{\frac{\tau_\lambda/(\tau_r \tau_f)}{[\tau_\lambda/(\tau_r \tau_f)]_R}} \frac{\text{MFP}(M_{19})}{\text{MFP}(M_{19R})} \tag{x}$$

$$\tau_f = 1 + \frac{1 - \tau_{tL}}{(1 - \tau_{tL})_R}\frac{\tau_\lambda/\tau_r}{(\tau_\lambda/\tau_r)_R}\frac{1 + \alpha_R}{1 + \alpha}(\tau_{fR} - 1) \tag{xi}$$

$$\tau_{tL} = 1 - \eta_{tL}\left(1 - \pi_{tL}^{(\gamma_t - 1)/\gamma_t}\right) \tag{xii}$$

$$\pi_{tL} = \pi_{tLR}\sqrt{\frac{\tau_{tL}}{\tau_{tLR}}}\frac{\text{MFP}(M_{9R})}{\text{MFP}(M_9)} \tag{xiii}$$

8.5.3 Summary of Performance Equations—Turbofan Engine with Separate Exhausts and Convergent Nozzles

INPUTS:

Choices

Flight parameters:	M_0, T_0 (K, °R), P_0 (kPa, psia)
Throttle setting:	T_{t4} (K, °R)

Design constants

π:	$\pi_{d\max}$, π_b, π_{tH}, π_n, π_{fn}
τ:	τ_{tH}
η:	η_f, η_{cH}, η_b, η_{mH}, η_{mL}
Gas properties:	γ_c, γ_t, c_{pc}, c_{pt} [kJ/(kg · K), Btu/(lbm · °R)]
Fuel:	h_{PR} (kJ/kg, Btu/lbm)

Reference conditions

Flight parameters:	M_{0R}, T_{0R} (K, °R), P_{0R} (kPa, psia), τ_{rR}, π_{rR}
Throttle setting:	T_{t4R} (K, °R)
Component behavior:	π_{dR}, π_{fR}, π_{cHR}, π_{tL}, τ_{fR}, τ_{cHR}, τ_{rLR}, α_R, M_{9R}, M_{19R}

OUTPUTS:

Overall performance:	F(N, lbf), $\dot{m}_0$ (kg/s, lbm/s), f, $S\left(\dfrac{\text{mg/s}}{\text{N}}, \dfrac{\text{lbm/h}}{\text{lbf}}\right)$, η_P, η_T, η_O
Component behavior:	α, π_f, π_{cH}, π_{tL}, τ_f, τ_{cH}, τ_{tL}, f, M_9, M_{19}, N_{fan}, N_{HPspool}
Exhaust nozzle pressure:	P_0/P_9, P_0/P_{19}

EQUATIONS:

$$R_c = \frac{\gamma_c - 1}{\gamma_c} c_{pc} \tag{8.52a}$$

$$R_t = \frac{\gamma_t - 1}{\gamma_t} c_{pt} \tag{8.52b}$$

$$a_0 = \sqrt{\gamma_c R_c g_c T_0} \tag{8.52c}$$

$$V_0 = a_0 M_0 \tag{8.52d}$$

$$\tau_r = 1 + \frac{\gamma_c - 1}{2} M_0^2 \tag{8.52e}$$

$$\pi_r = \tau_r^{\gamma_c/(\gamma_c - 1)} \tag{8.52f}$$

$$\eta_r = 1 \qquad \text{for } M_0 \le 1 \tag{8.52g}$$

$$\eta_r = 1 - 0.075(M_0 - 1)^{1.35} \qquad \text{for } M_0 > 1 \tag{8.52h}$$

$$\pi_d = \pi_{d\max}\eta_r \tag{8.52i}$$

$$\tau_\lambda = \frac{c_{pt}T_{t4}}{c_{pc}T_0} \tag{8.52j}$$

Initial values:

$$\tau_{tL} = \tau_{tLR} \qquad \tau_f = \tau_{fR} \qquad \tau_{tL} = \tau_{tLR}$$

$$\tau_{cH} = 1 + \frac{\tau_\lambda/\tau_r}{(\tau_\lambda/\tau_r)_R}\frac{\tau_{fR}}{\tau_f}(\tau_{cHR} - 1) \tag{8.52k}$$

$$\pi_{cH} = [1 + (\tau_{cH} - 1)\eta_{cH}]^{\gamma_c/(\gamma_c-1)} \tag{8.52l}$$

$$\pi_f = [1 + (\tau_f - 1)\eta_f]^{\gamma_c/(\gamma_c-1)} \tag{8.52m}$$

Exhaust nozzles:

$$\frac{P_{t19}}{P_0} = \pi_r\pi_d\pi_f\pi_{\text{fn}} \tag{8.52n}$$

$$\text{If} \quad \frac{P_{t19}}{P_0} < \left(\frac{\gamma_c+1}{2}\right)^{\gamma_c/(\gamma_c-1)} \quad \text{then} \quad \frac{P_{t19}}{P_{19}} = \frac{P_{t19}}{P_0}$$

$$\text{else} \quad \frac{P_{t19}}{P_{19}} = \left(\frac{\gamma_c+1}{2}\right)^{\gamma_c/(\gamma_c-1)} \tag{8.52o}$$

$$M_{19} = \sqrt{\frac{2}{\gamma_c - 1}\left[\left(\frac{P_{t19}}{P_{19}}\right)^{(\gamma_c-1)/\gamma_c} - 1\right]} \tag{8.52p}$$

$$\frac{P_{t9}}{P_0} = \pi_r\pi_d\pi_f\pi_{cH}\pi_b\pi_{tH}\pi_{tL}\pi_n \tag{8.52q}$$

$$\text{If} \quad \frac{P_{t9}}{P_0} < \left(\frac{\gamma_t+1}{2}\right)^{\gamma_t/(\gamma_t-1)} \quad \text{then} \quad \frac{P_{t9}}{P_9} = \frac{P_{t9}}{P_0}$$

$$\text{else} \quad \frac{P_{t9}}{P_9} = \left(\frac{\gamma_t+1}{2}\right)^{\gamma_t/(\gamma_t-1)} \tag{8.52r}$$

$$M_9 = \sqrt{\frac{2}{\gamma_t - 1}\left[\left(\frac{P_{t9}}{P_9}\right)^{(\gamma_t-1)/\gamma_t} - 1\right]} \tag{8.52s}$$

$$\alpha = \alpha_R\frac{\pi_{cHR}}{\pi_{cH}}\sqrt{\frac{\tau_\lambda/(\tau_r\tau_f)}{[\tau_\lambda/(\tau_r\tau_f)]_R}}\frac{\text{MFP}(M_{19})}{\text{MFP}(M_{19R})} \tag{8.52t}$$

$$\tau_f = 1 + \frac{1-\tau_{tL}}{(1-\tau_{tL})_R}\frac{\tau_\lambda/\tau_r}{(\tau_\lambda/\tau_r)_R}\frac{1+\alpha_R}{1+\alpha}(\tau_{fR}-1) \tag{8.52u}$$

$$\tau_{tL} = 1 - \eta_{tL}(1-\pi_{tL}^{(\gamma_t-1)/\gamma_t}) \tag{8.52v}$$

$$\pi_{tL} = \pi_{tLR}\sqrt{\frac{\tau_{tL}}{\tau_{tLR}}}\frac{\text{MFP}(M_{9R})}{\text{MFP}(M_9)} \tag{8.52w}$$

If τ_{tL} is not within 0.0001 of its previous value, return to Eq. (8.52k) and perform another iteration.

Remainder of calculations:

$$\dot{m}_0 = \dot{m}_{0R}\frac{1+\alpha}{1+\alpha_R}\frac{P_0\pi_r\pi_d\pi_f\pi_{cH}}{(P_0\pi_r\pi_d\pi_f\pi_{cH})_R}\sqrt{\frac{T_{t4R}}{T_{t4}}} \tag{8.52x}$$

$$f = \frac{\tau_\lambda - \tau_r\tau_f\tau_{cH}}{h_{PR}\eta_b/(c_pT_0) - \tau_\lambda} \tag{8.52y}$$

$$\frac{T_9}{T_0} = \frac{\tau_\lambda\tau_{tH}\tau_{tL}}{(P_{t9}/P_9)^{(\gamma_t-1)/\gamma_t}}\frac{c_{pc}}{c_{pt}} \tag{8.52z}$$

$$\frac{V_9}{a_0} = M_9\sqrt{\frac{\gamma_tR_tT_9}{\gamma_cR_cT_0}} \tag{8.52aa}$$

$$\frac{T_{19}}{T_0} = \frac{\tau_r\tau_f}{(P_{t19}/P_{19})^{(\gamma_c-1)/\gamma_c}} \tag{8.52ab}$$

$$\frac{V_{19}}{a_0} = M_{19}\sqrt{\frac{T_{19}}{T_0}} \tag{8.52ac}$$

$$\begin{aligned}\frac{F}{\dot{m}_0} = {} & \frac{1}{1+\alpha}\frac{a_0}{g_c}\left[(1+f)\frac{V_9}{a_0} - M_0 + (1+f)\frac{R_t}{R_c}\frac{T_9/T_0}{V_9/a_0}\frac{1-P_0/P_9}{\gamma_c}\right] \\ & + \frac{\alpha}{1+\alpha}\frac{a_0}{g_c}\left[\frac{V_{19}}{a_0} - M_0 + \frac{T_{19}/T_0}{V_{19}/a_0}\frac{1-P_0/P_{19}}{\gamma_c}\right]\end{aligned} \tag{8.52ad}$$

$$S = \frac{f}{(1+\alpha)(F/\dot{m}_0)} \tag{8.52ae}$$

$$F = \dot{m}_0\left(\frac{F}{\dot{m}_0}\right) \tag{8.52af}$$

$$\left(\frac{N}{N_R}\right)_{\text{fan}} = \sqrt{\frac{T_0\tau_r}{T_{0R}\tau_{rR}}\frac{\pi_f^{(\gamma_c-1)/\gamma_c}-1}{\pi_{fR}^{(\gamma_c-1)/\gamma_c}-1}} \tag{8.52ag}$$

$$\left(\frac{N}{N_R}\right)_{\text{HPspool}} = \sqrt{\frac{T_0\tau_r\tau_f}{(T_0\tau_r\tau_f)_R}\frac{\pi_{cH}^{(\gamma_c-1)/\gamma_c}-1}{\pi_{cHR}^{(\gamma_c-1)/\gamma_c}-1}} \tag{8.52ah}$$

$$\eta_T = \frac{a_0^2[(1+f)(V_9/a_0)^2 + \alpha(V_{19}/a_0)^2 - (1+\alpha)M_0^2]}{2g_c f h_{PR}} \tag{8.52ai}$$

$$\eta_P = \frac{2g_c V_0(1+\alpha)(F/\dot{m}_0)}{a_0^2[(1+f)(V_9/a_0)^2 + \alpha(V_{19}/a_0)^2 - (1+\alpha)M_0^2]} \tag{8.52aj}$$

$$\eta_O = \eta_P \eta_T \tag{8.52ak}$$

Example 8.8

Given the reference engine (see data) sized for a mass flow rate of 600 lbm/s at 40 kft and Mach 0.8, determine the performance at sea-level static conditions with $T_{t4} = 3000°\text{R}$.

REFERENCE:

$$T_0 = 390°\text{R}, \quad \gamma_c = 1.4, \quad c_{pc} = 0.24\ \text{Btu/(lbm}\cdot°\text{R)}, \quad \gamma_t = 1.33$$

$$c_{pt} = 0.276\ \text{Btu/(lbm}\cdot°\text{R)}, \quad T_{t4} = 3000°\text{R}, \quad M_0 = 0.8$$

$$\pi_c = 36, \quad \pi_f = 1.7, \quad \alpha = 8, \quad \eta_f = 0.8815$$

$$\eta_{cH} = 0.8512, \quad \tau_{tH} = 0.7580, \quad \pi_{tH} = 0.2851, \quad \tau_{tL} = 0.7262$$

$$\pi_{tL} = 0.2349, \quad \eta_{tL} = 0.9068, \quad \eta_b = 0.99, \quad \pi_{d\max} = 0.99$$

$$\pi_b = 0.96, \quad \pi_n = 0.99, \quad \pi_{\text{fn}} = 0.99, \quad \eta_{mH} = 0.9915$$

$$\eta_{mL} = 0.997, \quad P_0 = 2.730\ \text{psia (40 kft)}, \quad \dot{m}_0 = 600\ \text{lbm/s}$$

$$F/\dot{m}_0 = 17.92\ \text{lbf/(lbm/s)}, \quad F = 10{,}750\ \text{lbf}$$

PERFORMANCE CONDITIONS:

$$T_0 = 518.7°\text{R} \qquad P_0 = 14.696\ \text{psia (sea level)} \qquad T_{t4} = 3000°\text{R} \qquad M_0 = 0$$

EQUATIONS:

$$R_c = \frac{\gamma_c - 1}{\gamma_c} c_{pc} = \frac{0.4}{1.4}(0.24 \times 778.16) = 53.36\ \text{ft}\cdot\text{lbf/(lbm}\cdot°\text{R)}$$

$$R_t = \frac{\gamma_t - 1}{\gamma_t} c_{pt} = \frac{0.33}{1.33}(0.276 \times 778.16) = 53.29\ \text{ft}\cdot\text{lbf/(lbm}\cdot°\text{R)}$$

$$a_0 = \sqrt{\gamma_c R_c g_c T_0} = \sqrt{1.4 \times 53.36 \times 32.174 \times 518.7} = 1116.6\ \text{ft/s}$$

$$V_0 = a_0 M_0 = 1116.6 \times 0 = 0\ \text{ft/s}$$

$$\tau_r = 1 \qquad \text{and} \qquad \pi_r = 1$$

$$\pi_d = \pi_{d\max}\eta_r = 0.99 \times 1 = 0.99$$

$$\tau_\lambda = \frac{c_{pt}T_{t4}}{c_{pc}T_0} = \frac{0.276 \times 3000}{0.240 \times 518.7} = 6.651$$

Initial values:

$$\tau_{tL} = \tau_{tLR} = 0.7262 \qquad \tau_f = \tau_{fR} = 1.1857 \qquad \pi_{tL} = \pi_{tLR} = 0.2349$$

Eq. (A)

$$\tau_{cH} = 1 + \frac{\tau_\lambda/\tau_r}{(\tau_\lambda/\tau_r)_R}\frac{\tau_{fR}}{\tau_f}(\tau_{cHR} - 1)$$

$$= 1 + \frac{6.651/1.0}{8.846/1.128}\frac{1.1857}{1.1857}(2.636 - 1) = 2.3875$$

$$\pi_{cH} = [1 + (\tau_{cH} - 1)\eta_{cH}]^{\gamma_c/(\gamma_c-1)}$$

$$= [1 + (2.3875 - 1)(0.8512)]^{3.5} = 15.322$$

$$\pi_f = [1 + (\tau_f - 1)\eta_f]^{\gamma_c/(\gamma_c-1)}$$

$$= [1 + (1.1857 - 1)(0.8815)]^{3.5} = 1.70$$

Exhaust nozzles:

$$\frac{P_{t19}}{P_0} = \pi_r\pi_d\pi_f\pi_{\text{fn}} = 1 \times 0.99 \times 1.70 \times 0.99 = 1.6662$$

Since $\dfrac{P_{t19}}{P_0} < 1.893$ then $P_{19} = P_0$

$$M_{19} = \sqrt{\frac{2}{\gamma_c - 1}\left[\left(\frac{P_{t19}}{P_{19}}\right)^{(\gamma_c-1)/\gamma_c} - 1\right]} = \sqrt{\frac{2}{0.4}(1.6662^{1/3.5} - 1)} = 0.8861$$

$$\frac{P_{t9}}{P_0} = \pi_r\pi_d\pi_f\pi_{cH}\pi_b\pi_{tH}\pi_{tL}\pi_n$$

$$= 1 \times 0.99 \times 1.70 \times 15.322 \times 0.96 \times 0.2851 \times 0.2349 \times 0.99 = 1.6413$$

Since $\dfrac{P_{t9}}{P_0} < 1.851$ then $P_9 = P_0$

$$M_9 = \sqrt{\frac{2}{\gamma_t - 1}\left[\left(\frac{P_{t9}}{P_9}\right)^{(\gamma_t-1)/\gamma_t} - 1\right]}$$

$$= \sqrt{\frac{2}{0.33}(1.6413^{0.33/1.33} - 1)} = 0.8904$$

$$\alpha = \alpha_R \frac{\pi_{cHR}}{\pi_{cH}} \sqrt{\frac{\tau_\lambda/(\tau_r \tau_f)}{\tau_\lambda/(\tau_r \tau_f)]_R}} \frac{\text{MFP}(M_{19})}{\text{MFP}(M_{19R})}$$

$$= 8 \frac{21.177}{15.322} \sqrt{\frac{6.651/1.1857}{8.846/(1.128 \times 1.1857)}} \left(\frac{0.5257}{0.5318}\right) = 10.066$$

$$\tau_f = 1 + \frac{1-\tau_{tL}}{(1-\tau_{tL})_R} \frac{\tau_\lambda/\tau_r}{(\tau_\lambda/\tau_r)_R} \frac{1+\alpha_R}{1+\alpha}(\tau_{fR}-1)$$

$$= 1 + \frac{1-0.7262}{1-0.7262} \frac{6.651/1}{8.846/1.128} \frac{1+8}{1+10.066}(1.1857-1) = 1.1281$$

$$\tau_{tL} = 1 - \eta_{tL}(1-\pi_{tL}^{(\gamma_t-1)/\gamma_t}) = 1 - 0.9068(1-0.2349^{0.33/1.33}) = 0.7262$$

$$\pi_{tL} = \pi_{tLR} \sqrt{\frac{\tau_{tL}}{\tau_{tLR}}} \frac{\text{MFP}(M_{9R})}{\text{MFP}(M_9)} = 0.2349 \sqrt{\frac{0.7262}{0.7262}} \frac{0.5224}{0.5167} = 0.2375$$

Since τ_{tL} is not within 0.0001 of its previous value, return to **Eq. (A)** and do another iteration. These data required 10 iterations, which are summarized in Table 8.7.

Remainder of calculations:

$$\dot{m}_0 = \dot{m}_{0R} \frac{1+\alpha}{1+\alpha_R} \frac{P_0 \pi_r \pi_d \pi_f \pi_{cH}}{(P_0 \pi_r \pi_d \pi_f \pi_{cH})_R} \sqrt{\frac{T_{t4R}}{T_{t4}}}$$

$$= 600 \frac{1+9.103}{1+8} \frac{14.696 \times 1.0 \times 0.99 \times 1.4973 \times 16.555}{2.730 \times 1.524 \times 0.99 \times 1.7 \times 21.176} \sqrt{\frac{3000}{3000}}$$

$$= 1638 \text{ lbm}$$

$$f = \frac{\tau_\lambda - \tau_r \tau_f \tau_{cH}}{h_{PR}\eta_b/(c_p T_0) - \tau_\lambda}$$

$$= \frac{6.651 - 1 \times 1.1387 \times 2.448}{18{,}400 \times 0.99/(0.24 \times 518.7) - 6.651} = 0.02769$$

$$\frac{T_9}{T_0} = \frac{\tau_\lambda \tau_{tH} \tau_{tL}}{(P_{t9}/P_9)^{(\gamma_t-1)/\gamma_t}} \frac{c_{pc}}{c_{pt}}$$

$$= \frac{6.651 \times 0.7580 \times 0.7293}{1.5932^{0.33/1.33}} \frac{0.240}{0.276} = 2.848$$

$$\frac{V_9}{a_0} = M_9 \sqrt{\frac{\gamma_t R_t T_9}{\gamma_c R_c T_0}} = 0.8617 \sqrt{\frac{1.33 \times 53.29}{1.40 \times 53.36}(2.848)} = 1.4165$$

$$\frac{T_{19}}{T_0} = \frac{\tau_r \tau_f}{(P_{t19}/P_{19})^{(\gamma_c-1)/\gamma_c}} = \frac{1.0 \times 1.1387}{1.4675^{1/3.5}} = 1.0205$$

Table 8.7 Summary of internal iterations

i	τ_{cH}	π_{cH}	π_f	M_{19}	M_9	α	τ_f	τ_{tL}	π_{tL}
1	2.3875	15.322	1.7000	0.8861	0.8904	10.07	1.1281	0.7262	0.2375
2	2.4584	16.857	1.4542	0.7299	0.8419	8.833	1.1441	0.7279	0.2408
3	2.4379	16.403	1.5199	0.7766	0.8719	9.233	1.1376	0.7301	0.2391
4	2.4461	16.585	1.4930	0.7580	0.8586	9.077	1.1387	0.7290	0.2398
5	2.4448	16.556	1.4972	0.7609	0.8623	9.102	1.1389	0.7294	0.2396
...	...	...	...	...	...	...	...	...	...
10	2.4448	16.555	1.4973	0.7610	0.8617	9.103	1.1387	0.7293	0.2396

$$\frac{V_{19}}{a_0} = M_{t9}\sqrt{\frac{T_{19}}{T_0}} = 0.7610\sqrt{1.0205} = 0.7688$$

$$\frac{F}{\dot{m}_0} = \frac{a_0}{(1+\alpha)g_c}\left[(1+f)\frac{V_9}{a_0} - M_0 + \alpha\left(\frac{V_{19}}{a_0} - M_0\right)\right]$$

$$= \frac{1116.6/32.174}{1+9.1039}[1.02769(1.4165) + 9.103(0.7688)]$$

$$= 29.04\,\text{lbf/(lbm/s)}$$

$$S = \frac{3600 \times 0.02769}{10.103 \times 29.04}\frac{32.174}{32.174} = 0.3398\ \text{(lbm/h)/lbf}$$

$$F = 1639 \times 29.04 = 47{,}570\,\text{lbf}$$

$$\left(\frac{N}{N_R}\right)_{\text{fan}} = \sqrt{\frac{T_0\tau_r}{T_{0R}\tau_{rR}}\frac{\pi_f^{(\gamma-1)/\gamma} - 1}{\pi_{fR}^{(\gamma-1)/\gamma} - 1}}$$

$$= \sqrt{\frac{518.7 \times 1.0}{390 \times 1.128}\frac{1.4973^{0.4/1.4} - 1}{1.7^{0.4/1.4} - 1}} = 0.938$$

$$\left(\frac{N}{N_R}\right)_{\text{HPspool}} = \sqrt{\frac{T_0\tau_r\tau_f}{(T_{0R}\tau_r\tau_f)_R}\frac{\pi_{cH}^{(\gamma-1)/\gamma} - 1}{\pi_{cHR}^{(\gamma-1)/\gamma} - 1}}$$

$$= \sqrt{\frac{518.7 \times 1.0 \times 1.1387}{390 \times 1.128 \times 1.1857}\frac{16.555^{0.4/1.4} - 1}{21.176^{0.4/1.4} - 1}} = 1.00$$

Example 8.9

In this example, we consider the variation in engine performance of a 270,000-N thrust, high-bypass-ratio turbofan engine with Mach number M_0, altitude, ambient temperature T_0, and throttle setting T_{t4}. The engine reference flight condition is sea-level static with the following values:

$$\alpha = 8 \qquad \pi_f = 1.77 \qquad \pi_{cH} = 20.34 \qquad T_{t4} = 1890\,\text{K} \qquad \dot{m}_0 = 760\,\text{kg/s}$$

For the performance curves drawn in solid lines, the compressor pressure ratio was limited to 36, the combustor exit temperature T_{t4} was limited to 1890 K, and the compressor exit temperature T_{t3} was limited to 920 K. This engine has a throttle ratio TR of 1. At θ_0 values below 1.0, the engine is at the maximum compressor pressure ratio of 36 and T_{t4} is below its maximum value of 1890 K. For these conditions, the flow in the bypass stream is unchoked at sea level and does not choke at altitude until a Mach number of 0.34.

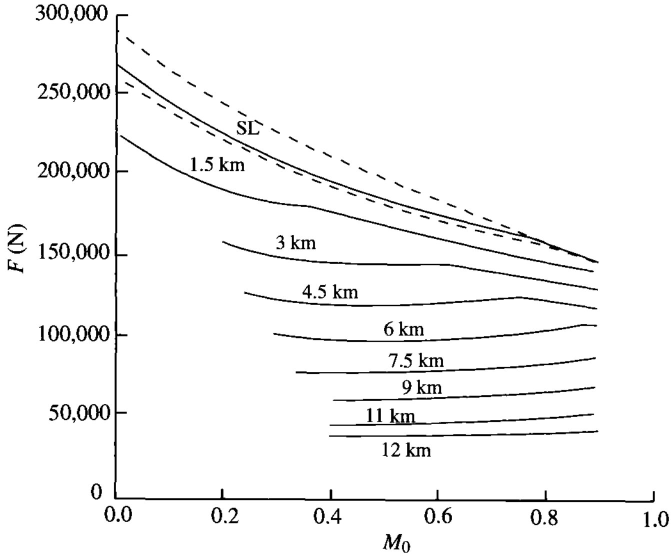

Fig. 8.47 Maximum thrust of high-bypass-ratio turbofan.

Figures 8.47–8.53 show the variations of thrust, thrust specific fuel consumption S, engine mass flow, corrected engine mass flow, bypass ratio, fan pressure ratio π_f, and high-pressure (HP) compressor pressure ratio π_{cH} with Mach number and altitude, respectively. The dashed lines in these figures show the engine performance with the combustor exit temperature T_{t4} limited to 1940 K.

The corrected engine mass flow rate of Fig. 8.50 has the same trend with Mach number and altitude as the fan pressure ratio of Fig. 8.52 and the HP compressor pressure ratio of Fig. 8.53. Both the corrected mass flow rate and the HP compressor pressure ratio reach their maximum values when the bypass stream chokes (flight Mach of 0.34). Figure 8.51 shows that the engine bypass ratio at maximum thrust has a constant minimum value of about 8 when the bypass stream is choked.

The effects of ambient temperature T_0 and altitude on engine performance at maximum thrust are shown in Figs. 8.54 and 8.55, respectively. For ambient temperatures below the reference value of 288.2 K ($\theta_0 < 1.0$), Fig. 8.54 shows that the limit of 36 for the compressor pressure ratio ($\pi_c = \pi_f \pi_{cH}$) holds the engine thrust, bypass ratio, and fan pressure ratio constant. Engine thrust drops off rapidly with T_0 for $\theta_0 > 1.0$. The decreases in engine thrust, fuel consumption, and air mass flow rate with altitude are shown in Fig. 8.55 for a flight Mach number of 0.5. The decrease in engine thrust with altitude for the high-bypass-ratio turbofan engine is much greater than that of the dry turbojet (see Fig. 8.42). If both a high-bypass-ratio turbofan engine and a dry turbojet engine were sized to produce the same thrust at 9 km and 0.8 Mach, the high-bypass-ratio turbofan engine would have much greater thrust at sea-level static conditions. This helps explain the decrease in takeoff length between the

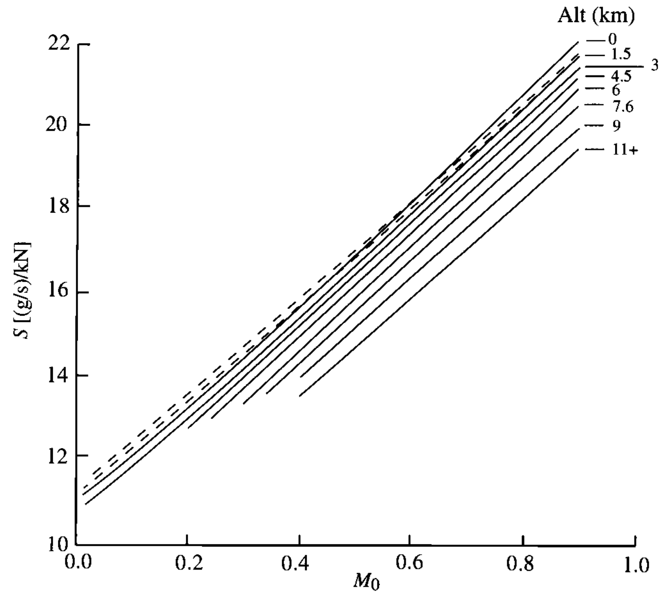

Fig. 8.48 Thrust-specific fuel consumption of high-bypass-ratio turbofan at maximum thrust.

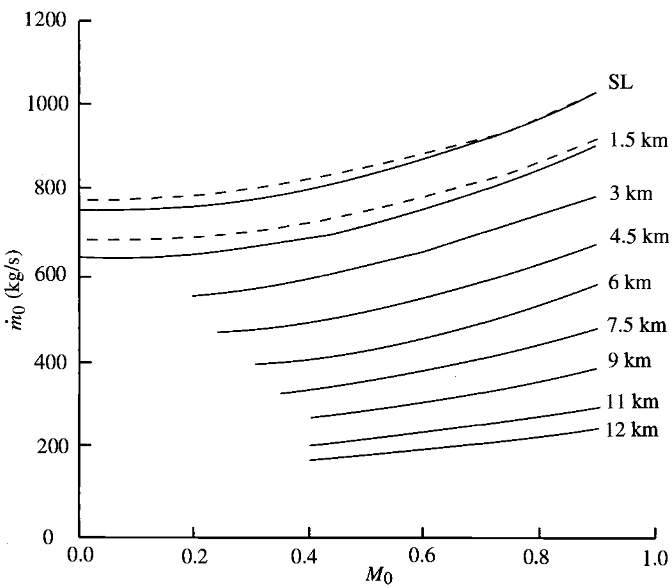

Fig. 8.49 Mass flow rate of high-bypass-ratio turbofan at maximum thrust.

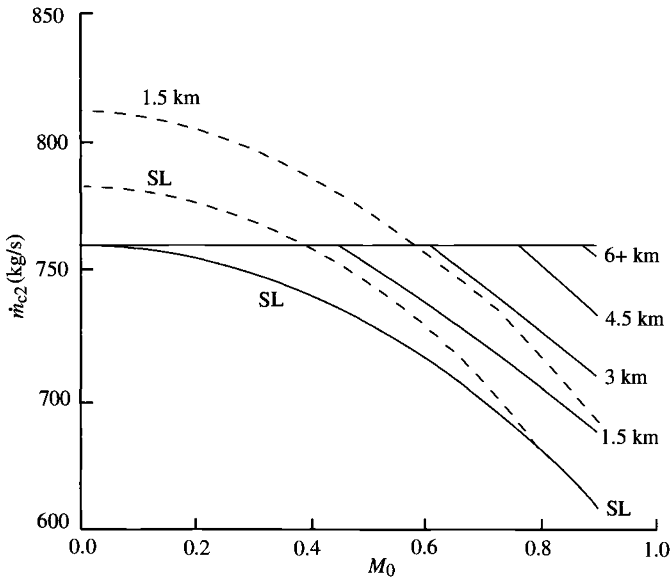

Fig. 8.50 Corrected mass flow rate of high-bypass-ratio turbofan at maximum thrust.

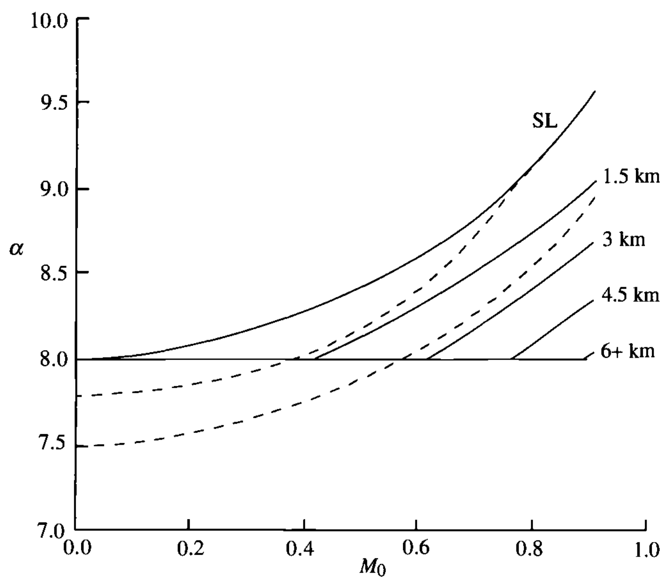

Fig. 8.51 Bypass ratio of high-bypass-ratio turbofan at maximum thrust.

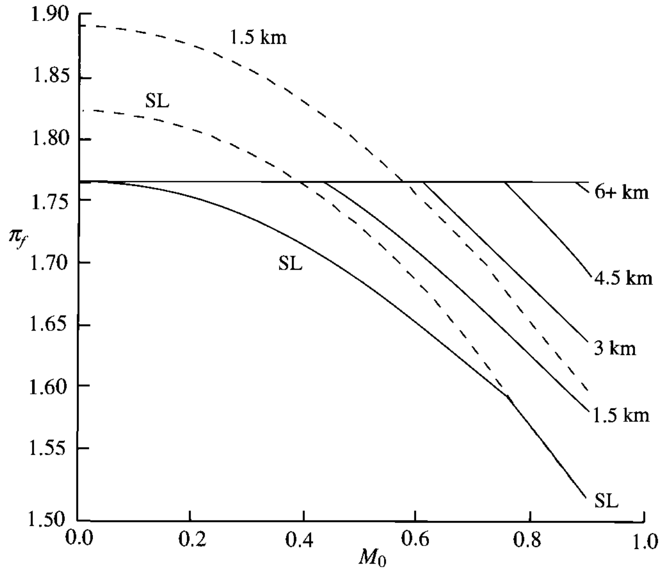

Fig. 8.52 Fan pressure ratio of high-bypass-ratio turbofan at maximum thrust.

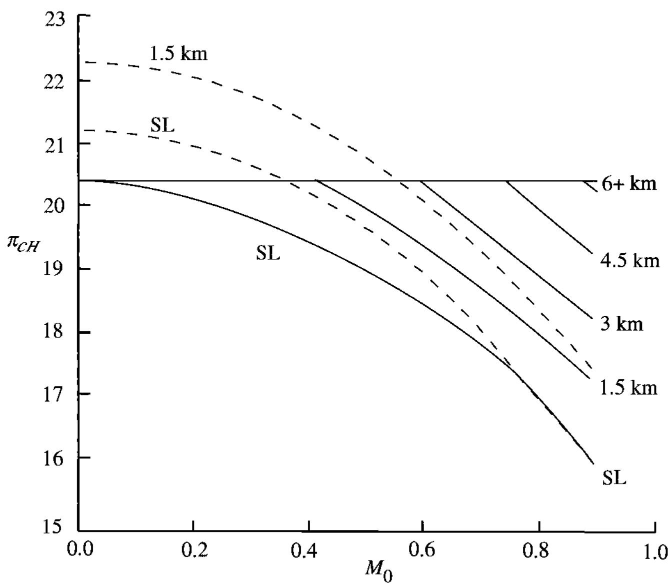

Fig. 8.53 High-pressure compressor pressure ratio of high-bypass-ratio turbofan at maximum thrust.

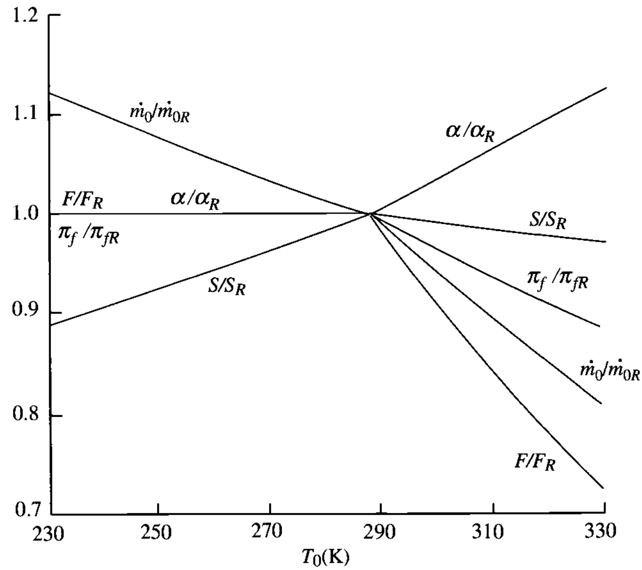

Fig. 8.54 Performance of high-bypass-ratio turbofan vs T_0 at sea-level static, maximum thrust.

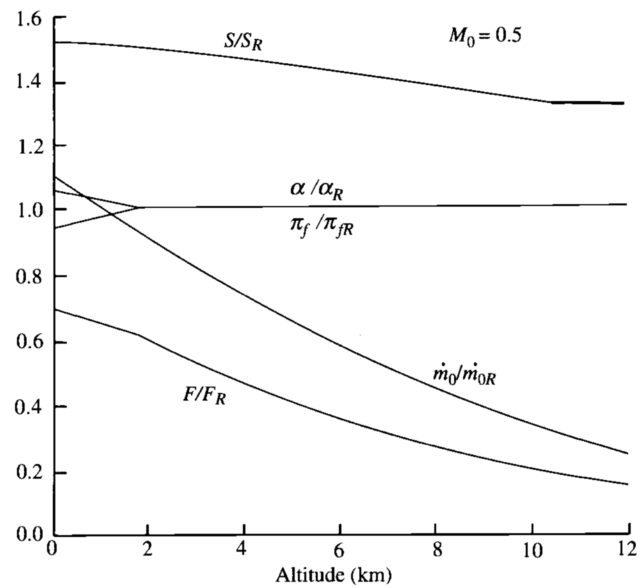

Fig. 8.55 Performance of high-bypass-ratio turbofan at maximum thrust vs altitude.

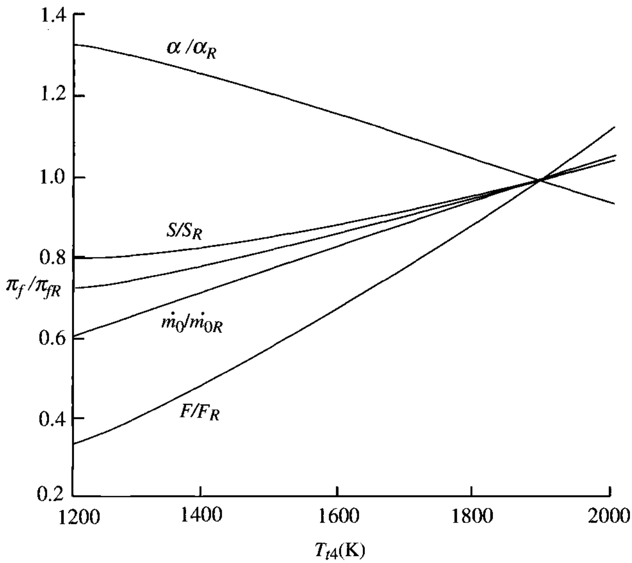

Fig. 8.56 Performance of high-bypass-ratio turbofan vs T_{t4} at sea-level static conditions.

early turbojet-powered passenger aircraft and the modern high-bypass-ratio turbofan-powered passenger aircraft of today.

Figures 8.56 and 8.57 show the effects that changes in combustor exit temperature T_{t4} have on engine performance. As shown in Fig. 8.56, all engine performance parameters except the bypass ratio decrease with reduction in engine throttle T_{t4}. If the throttle were reduced further, the thrust specific fuel consumption S would start to increase. The thrust specific fuel consumption S vs thrust F at partial throttle (partial power) is shown in Fig. 8.57 for two different values of altitude and Mach number. These curves have the classical hook shape that gives them their name of *throttle hook*. Minimum S occurs at about 50% of maximum thrust. At lower throttle settings, the thrust specific fuel consumption rapidly increases.

The characteristics of the low-pressure spool and high-pressure spool for this high-bypass-ratio turbofan engine are shown in Figs. 8.58 and 8.59, respectively. The core flow and/or bypass flow may be choked ($M_9 = 1$ and/or $M_{19} = 1$) at its respective exhaust nozzles, which influences the low-pressure spool. Figure 8.58 shows the characteristics of the low-pressure spool at the flight condition of 9 km and $M_0 = 0.8$ with solid lines, and at the sea-level static flight condition with dashed lines. At sea-level static conditions, the bypass stream is unchoked for all operating conditions of the low-pressure spool, and the core exhaust nozzle unchokes at about 95% of N_{cL}. However, at 9 km and $M_0 = 0.8$, the core exhaust nozzle unchokes at about 78% of N_{cL}, and the bypass stream unchokes at about 61% of N_{cL}. The variation of fan pressure ratio and corrected fuel flow with the corrected speed of the low-pressure spool is unaffected by the

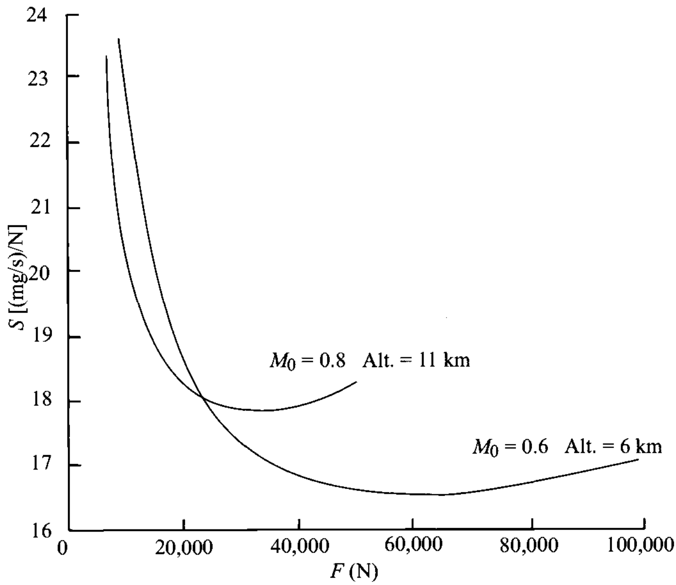

Fig. 8.57 Partial-throttle performance of high-bypass-ratio turbofan.

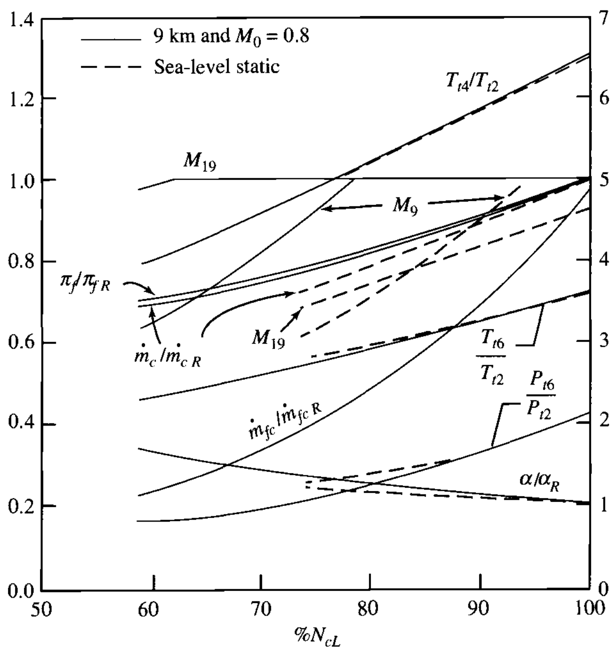

Fig. 8.58 Partial-throttle characteristics of low-pressure spool.

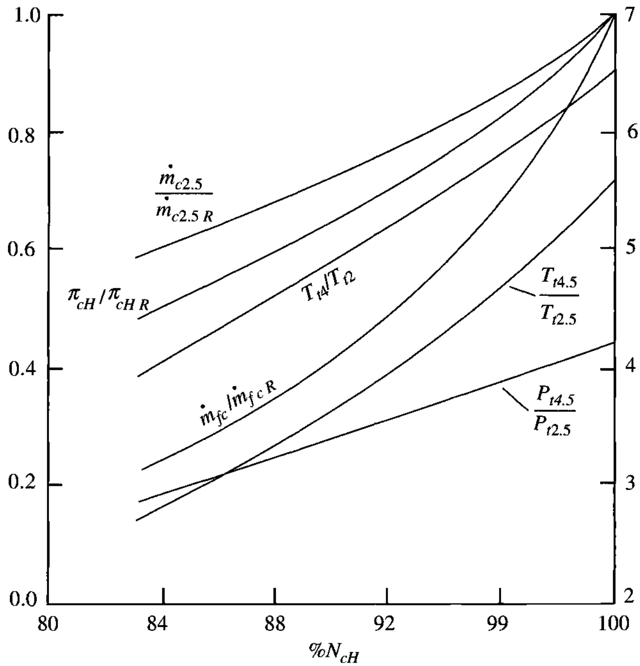

Fig. 8.59 Partial-throttle characteristics of high-pressure spool.

flight condition. The variations in T_{t6}/T_{t2}, P_{t6}/P_{t2}, and α/α_R with corrected speed are small above 80% of N_{cL}. As in the single-spool engine, there is a one-to-one correspondence of the temperature ratio T_{t4}/T_{t2} with the corrected speed of the spool.

The pumping characteristics of the high-pressure spool are shown in Fig. 8.59. These are the same characteristic curves that we found for the gas generator of the single-spool turbojet (Fig. 8.15).

8.5.4 Compressor Stages on Low-Pressure Spool

Modern high-bypass-ratio turbofan engines and other turbofan engines are constructed with compressor stages on the low-pressure spool as shown in Fig. 8.60. This addition of compressor stages to the spool that powers the fan gives a better balance between the high- and low-pressure turbines. This change in engine layout also adds two dependent variables to the nine we had for the performance analysis of the turbofan engine in the previous section. These two new variables are the low-pressure compressor's total temperature ratio τ_{cL} and total pressure ratio π_{cL}.

Because the low-pressure compressor and the fan are on the same shaft, the enthalpy rise across the low-pressure compressor will be proportional to the

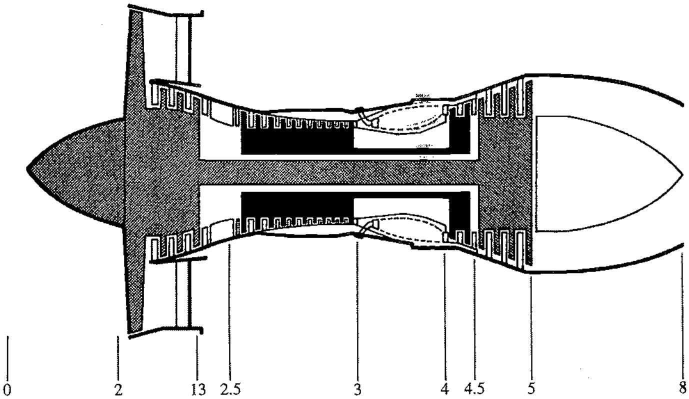

Fig. 8.60 Turbofan engine with compressor stages on low-pressure spool. (Courtesy of Pratt & Whitney.)

enthalpy rise across the fan during normal operation. For a calorically perfect gas, we can write

$$T_{t2.5} - T_{t2} = K(T_{t13} - T_{t2})$$

or

$$\tau_{cL} - 1 = K(\tau_f - 1)$$

Using reference conditions to replace the constant K, we can solve the preceding equation for τ_{cL} as

$$\tau_{cL} = 1 + (\tau_f - 1)\frac{\tau_{cLR} - 1}{\tau_{fR} - 1} \tag{8.53}$$

The pressure ratio for the low-pressure compressor is given by Eq. (8.43):

$$\pi_{cL} = [1 + \eta_{cL}(\tau_{cL} - 1)]^{\gamma_c/(\gamma_c - 1)}$$

In a manner like that used to obtain Eq. (8.46), the following equation for the bypass ratio results:

$$\alpha = \alpha_R \frac{\pi_{cLR}\pi_{cHR}/\pi_{fR}}{\pi_{cL}\pi_{cH}/\pi_f}\sqrt{\frac{\tau_\lambda/(\tau_r\tau_f)}{[\tau_\lambda/(\tau_r\tau_f)]_R}}\frac{\text{MFP}(M_{19})}{\text{MFP}(M_{19R})} \tag{8.54}$$

By rewriting Eq. (8.47) for this engine configuration, the engine mass flow rate is given by

$$\dot{m}_0 = \dot{m}_{0R}\frac{1 + \alpha}{1 + \alpha_R}\frac{P_0\pi_r\pi_d\pi_{cL}\pi_{cH}}{(P_0\pi_r\pi_d\pi_{cL}\pi_{cH})_R}\sqrt{\frac{T_{t4R}}{T_{t4}}} \tag{8.55}$$

Equation (8.40) applies to the high-pressure compressor of this engine and is

$$\tau_{cH} = 1 + \frac{T_{t4}/T_0}{(T_{t4}/T_0)_R}\frac{(\tau_r\tau_{cL})_R}{\tau_r\tau_{cL}}(\tau_{cH}-1)_R$$

Equations (8.36), (8.37), and (8.38) apply to the high- and low-pressure turbines.

From a power balance between the fan, low-pressure compressor, and low-pressure turbine, we get

$$\eta_{mL}\dot{m}_{4.5}c_{pt}(T_{t4.5}-T_{t5}) = \dot{m}_F c_{pc}(T_{t13}-T_{t2}) + \dot{m}_C c_{pc}(T_{t2.5}-T_{t2})$$

Rewriting this equation in terms of temperature ratios, rearranging into variable and constant terms, and equating the constant to reference values give

$$\frac{\tau_r[(\tau_{cL}-1)+\alpha(\tau_f-1)]}{(T_{t4}/T_0)(1-\tau_{tL})} = \eta_{mL}(1+f)\tau_{tH} = \left\{\frac{\tau_r[\tau_{cL}-1+\alpha(\tau_f-1)]}{(T_{t4}/T_0)(1-\tau_{tL})}\right\}_R$$

Using Eq. (8.53), we substitute for τ_{cL} on the left side of the preceding equation, solve for τ_f, and get

$$\tau_f = 1 + (\tau_{fR}-1)\left[\frac{1-\tau_{tL}}{(1-\tau_{tL})_R}\frac{\tau_\lambda/\tau_r}{(\tau_\lambda/\tau_r)_R}\frac{\tau_{cLR}-1+\alpha_R(\tau_{fR}-1)}{\tau_{cLR}-1+\alpha(\tau_{fR}-1)}\right] \tag{8.56}$$

8.5.5 Solution Scheme

The principal dependent variables for the turbofan engine are π_{tL}, τ_{tL}, α, τ_{cL}, π_{cL}, τ_{cH}, π_{cH}, τ_f, π_f, M_9, and M_{19}. These variables are dependent on each other plus the engine's independent variables—throttle setting and flight condition. The functional interrelationship of the dependent variables can be written as

$$\begin{aligned}
\tau_{cH} &= f_1(\tau_{cL}) & M_{19} &= f_7(\pi_f) \\
\pi_{cH} &= f_2(\tau_{cH}) & M_9 &= f_8(\pi_f, \pi_{cH}, \pi_{tL}) \\
\tau_f &= f_3(\tau_{rL}, \alpha) & \pi_{tL} &= f_9(\tau_{tL}, M_9) \\
\pi_f &= f_4(\tau_f) & \tau_{tL} &= f_{10}(\pi_{tL}) \\
\tau_{cL} &= f_5(\tau_f) & \alpha &= f_{11}(\tau_f, \pi_{cH}, M_{19}) \\
\pi_{cL} &= f_6(\tau_{cL})
\end{aligned}$$

This system of 11 equations is solved by functional iteration, starting with reference quantities as initial values for π_{tL}, τ_{tL}, and τ_f. The following equations are calculated for the 11 dependent variables in the order listed until successive values of τ_{tL} do not change more than a specified amount (say, 0.0001):

$$\tau_{cH} = 1 + \frac{T_{t4}/T_0}{(T_{t4}/T_0)_R}\frac{(\tau_r\tau_{cL})_R}{\tau_r\tau_{cL}}(\tau_{cH}-1)_R \tag{8.57a}$$

$$\pi_{cH} = [1+\eta_{cH}(\tau_{cH}-1)]^{\gamma_c/(\gamma_c-1)} \tag{8.57b}$$

$$\pi_f = [1+(\tau_f-1)\eta_f]^{\gamma_c/(\gamma_c-1)} \tag{8.57c}$$

$$\frac{P_{t19}}{P_0} = \pi_r\pi_d\pi_f\pi_{\text{fn}} \tag{8.57d}$$

$$\text{If} \quad \frac{P_{t19}}{P_0} < \left(\frac{\gamma_c+1}{2}\right)^{\gamma_c/(\gamma_c-1)} \quad \text{then} \quad \frac{P_{t19}}{P_{19}} = \frac{P_{t19}}{P_0}$$

$$\text{else} \quad \frac{P_{t19}}{P_{19}} = \left(\frac{\gamma_c+1}{2}\right)^{\gamma_c/(\gamma_c-1)} \tag{8.57e}$$

$$M_{19} = \sqrt{\frac{2}{\gamma_c-1}\left[\left(\frac{P_{t19}}{P_{19}}\right)^{(\gamma_c-1)/\gamma_c}-1\right]} \tag{8.57f}$$

$$\frac{P_{t9}}{P_0} = \pi_r\pi_d\pi_{cL}\pi_{cH}\pi_b\pi_{tH}\pi_{tL}\pi_n \tag{8.57g}$$

$$\text{If} \quad \frac{P_{t9}}{P_0} < \left(\frac{\gamma_t+1}{2}\right)^{\gamma_t/(\gamma_t-1)} \quad \text{then} \quad \frac{P_{t9}}{P_9} = \frac{P_{t9}}{P_0}$$

$$\text{else} \quad \frac{P_{t9}}{P_9} = \left(\frac{\gamma_t+1}{2}\right)^{\gamma_t/(\gamma_t-1)} \tag{8.57h}$$

$$M_9 = \sqrt{\frac{2}{\gamma_t-1}\left[\left(\frac{P_{t9}}{P_9}\right)^{(\gamma_t-1)/\gamma}-1\right]} \tag{8.57i}$$

$$\alpha = \alpha_R\frac{\pi_{cLR}\pi_{cHR}/\pi_{fR}}{\pi_{cL}\pi_{cH}/\pi_f}\sqrt{\frac{\tau_\lambda/(\tau_r\tau_f)}{[\tau_\lambda/(\tau_r\tau_f)]_R}}\frac{\text{MFP}(M_{19})}{\text{MFP}(M_{19R})} \tag{8.57j}$$

$$\tau_f = 1+(\tau_{fR}-1)\left[\frac{1-\tau_{tL}}{(1-\tau_{tL})_R}\frac{\tau_\lambda/\tau_r}{(\tau_\lambda/\tau_r)_R}\frac{\tau_{cLR}-1+\alpha_R(\tau_{fR}-1)}{\tau_{cLR}-1+\alpha(\tau_{fR}-1)}\right] \tag{8.57k}$$

$$\tau_{cL} = 1+(\tau_f-1)\frac{\tau_{cLR}-1}{\tau_{fR}-1} \tag{8.57l}$$

$$\pi_{cL} = [1+\eta_{cL}(\tau_{cL}-1)]^{\gamma_c/(\gamma_c-1)} \tag{8.57m}$$

$$\tau_{tL} = 1-\eta_{tL}(1-\pi_{tL}^{(\gamma_t-1)/\gamma_t}) \tag{8.57n}$$

$$\pi_{tL} = \pi_{tLR}\sqrt{\frac{\tau_{tL}}{\tau_{tLR}}}\frac{\text{MFP}(M_9R)}{\text{MFP}(M_9)} \tag{8.57o}$$

8.5.6 Summary of Performance Equations—Turbofan Engine with Compressor Stages on Low-Pressure Spool

INPUTS:

Choices
- Flight parameters: M_0, T_0 (K, °R), P_0 (kPa, psia)
- Throttle setting: T_{t4} (K, °R)

Design constants
- π: $\pi_{d\,\max}$, π_b, π_{tH}, π_{AB}, π_n, π_{fn}
- τ: τ_{tH}
- η: η_f, η_{cL}, η_{cH}, η_b, η_{AB}, η_{mH}, η_{mL}
- Gas properties: γ_c, γ_t, c_{pc}, c_{pt} [kJ/(Kg · K), Btu/(Ibm · °R)]
- Fuel: h_{PR} (kJ/kg, Btu/lbm)

Reference conditions
- Flight parameters: M_{0R}, T_{0R} (K, °R), P_{0R} (kPa, psia), τ_{rR}, τ_{rR}
- Throttle setting: T_{t4R} (K, °R)
- Component behavior: π_{dR}, π_{fR}, π_{cLR}, π_{cHR}, π_{tL}, τ_{fR}, τ_{cHR}, τ_{tLR}, α_R, M_{9R}, M_{19R}

OUTPUTS:

- Overall performance: F (N, lbf), $\dot{m}_0$ (kg/s, lbm/s), $f, S\left(\dfrac{\text{mg/s}}{\text{N}}, \dfrac{\text{lbm/h}}{\text{lbf}}\right)\eta_P$, η_T, η_O
- Component behavior: α, π_f, π_{cL}, π_{cH}, π_{tL}, τ_f, τ_{cH}, τ_{tL}, f, M_9, M_{19}, N_{fan}, N_{HPspool}
- Exhaust nozzle pressure: P_0/P_9, P_0/P_{19}

EQUATIONS (in order of calculation):

Equations (8.52a–8.52j)
Set initial values: $\tau_{tL} = \tau_{tLR}$ $\quad \tau_f = \tau_{fR}$ $\quad \tau_{cL} = \tau_{cLR}$ $\quad \pi_{tL} = \pi_{tL\,R}$
Equations (8.57a–8.57o)

$$\dot{m}_0 = \dot{m}_{0R}\frac{1+\alpha}{1+\alpha_R}\frac{P_0\pi_r\pi_d\pi_{cL}\pi_{cH}}{(P_0\pi_r\pi_d\pi_{cL}\pi_{cH})_R}\sqrt{\frac{T_{t4R}}{T_{t4}}} \tag{8.57p}$$

$$f = \frac{\tau_\lambda - \tau_r\tau_{cL}\tau_{cH}}{h_{PR}\eta_b/(c_pT_0) - \tau_\lambda} \tag{8.57q}$$

Equations (8.52z–8.52ag)

$$\left(\frac{N}{N_R}\right)_{\text{HPspool}} = \sqrt{\frac{T_0\tau_r\tau_{cL}}{(T_0\tau_r\tau_{cL})_R}\frac{\pi_{cH}^{(\gamma-1)/\gamma}-1}{\pi_{cHR}^{(\gamma-1)/\gamma}-1}} \tag{8.57r}$$

Equations (8.52ai–8.52ak)

Problems

8.1 The flow in a typical single-spool turbojet engine is choked in two locations. This fact is used in performance analysis to predict the variations of the compressor and turbine with changes in T_{t4} and T_{t2}. As a result, the turbine temperature and pressure ratios are constant for all operating conditions.

(a) Identify the two locations where the flow is choked in the engine.
(b) Describe the basic engineering principle that gives the equation for the lines of constant $T_{t4}//T_{t2}$, sketched in Fig. P8.1 and in Figs. 8.6a and 8.6b.
(c) Sketch the operating line of a typical turbojet on the compressor map of Fig. P8.1.
(d) Describe the advantage of a turbojet engine having a throttle ratio greater than 1.

8.2 If the compressor of a single-spool turbojet has a maximum compressor ratio of 8 with a corrected mass flow rate of 25 kg/s for $T_{t4} = 1800$ K and $\theta_0 = 1$, determine the following (assume $\gamma_c = 1.4$):

(a) Constants K_1 and K_2 of Eqs. (8.17) and (8.18).
(b) Compressor pressure ratio and corrected mass flow rate for $T_{t4} = 1200$ K and $\theta_0 = 1.1$.
(c) The maximum T_{t4} and corresponding compressor pressure ratio and corrected mass flow rate at $\theta_0 = 0.9$ and 1.1 for a throttle ratio of 1.0.
(d) The maximum T_{t4} and corresponding compressor pressure ratio and corrected mass flow rate at $\theta_0 = 0.9$ and 1.1 for a throttle ratio of 1.05.

8.3 The typical operation of the single-spool turbojet engine at maximum throttle for flight conditions where $\theta_0 <$ TR is such that $T_{t4}/T_{t2} =$ constant. Show that π_c, T_{t5}/T_{t2}, P_{t5}/P_{t2}, and $\dot{m}_{c2}$ are also constant.

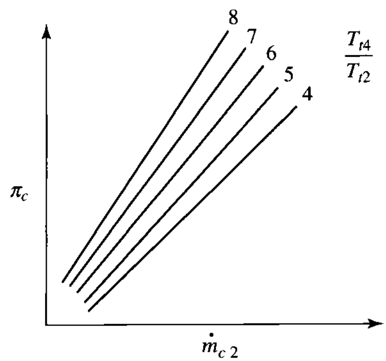

Fig. P8.1

8.4 Show that the total temperature ratios for the compressors of the two-spool turbojet engine [Eqs. (8.42) and (8.40)] can be written as

$$\tau_{cL} = 1 + \frac{T_{t4}/\theta_0}{(T_{t4}/\theta_0)_R}(\tau_{cL} - 1)_R$$

$$\tau_{cH} = 1 + \frac{T_{t4}/\theta_0}{(T_{t4}/\theta_0)_R}\frac{\tau_{cLR}}{\tau_{cL}}(\tau_{cH} - 1)_R$$

8.5 In terms of reference conditions (subscript R), show that the following equations apply for the operation of an *ideal* single-spool turbojet engine:
(a) The compressor pressure ratio is given by

$$\pi_c = \left\{1 + [(\pi_{cR})^{(\gamma-1)/\gamma} - 1]\frac{T_{t4}/T_{t2}}{(T_{t4}/T_{t2})_R}\right\}^{\gamma/(\gamma-1)}$$

(b) The engine mass flow rate is given by

$$\dot{m}_0 = \dot{m}_{0R}\frac{P_{t2}}{P_{t2R}}\frac{\pi_c}{\pi_{cR}}\sqrt{\frac{T_{t4R}}{T_{t4}}}$$

(c) The corrected mass flow rate at station 2 is given by

$$\dot{m}_{c2} = \dot{m}_{c2R}\frac{\pi_c}{\pi_{cR}}\sqrt{\frac{(T_{t4}/T_{t2})_R}{(T_{t4}/T_{t2})}}$$

(d) Given $(T_{t4}//T_{t2})_R = 4$, $\pi_{cR} = 12$, $\dot{m}_{cR} = 200$ lbm/s, and $\gamma = 1.4$, calculate the engine operating line by completing the following table of data; plot the results.

T_{t4}/T_{t2}	4.0	3.6	3.2	2.8	2.4	2.0	1.6
π_c	12						
$\dot{m}_c$ (lbm/s)	200						

8.6 For a ramjet engine with a constant nozzle throat area A_8, develop the performance equations and calculate the performance of the ramjet with the reference data of Problem 7.1.
(a) Show that the engine mass flow rate can be written as

$$\dot{m}_0 = \dot{m}_{0R}\frac{P_0\pi_r\pi_d}{(P_0\pi_r\pi_d)R}\sqrt{\frac{T_{t4R}}{T_{t4}}}\frac{\mathrm{MFP}(M_8)}{\mathrm{MFP}(M_{8R})}$$

where the Mach number at station 8 is determined by the value of P_{t8}/P_0.

(b) Show that the remainder of the performance equations are given by those developed to answer Problem 7.1.

(c) If the ramjet of Problem 7.1 has a reference mass flow rate of 20 kg/s at 15 km and $M_0 = 2$, determine its performance at 20 km, $M_0 = 3$, and $T_{t4} = 1600$ K. Assume $P_9 = P_0$.

8.7 A turbojet engine operated at a flight Mach number of 2.0 and an altitude of 15 km has the following compressor performance with $T_{t4} = 1860$ K: $\pi_c = 8$, $\tau_c = 1.9$, and $\eta_c = 0.9$. Determine τ_c, π_c, and $\dot{m}_2/\dot{m}_{2R}$ for $M_0 = 0.8$, 11-km altitude, and $T_{t4} = 1640$ K. Assume that $\pi_d = 0.88$ at $M_0 = 2$, $\pi_d = 0.98$ at $M_0 = 0.8$, and $\gamma_c = 1.4$.

8.8 Calculate the thrust of the turbojet engine in Example 7.1 at $M_0 = 0.8$, 9-km altitude, reduced throttle ($T_{t4} = 1100$ K), and $P_0/P_9 = 1$ for a reference air mass flow rate of 50 kg/s at a reference altitude of 12 km.

8.9 Given a single-spool turbojet engine that has the following reference values.

REFERENCE:

$$\dot{m}_{c2} = 100 \text{ lbm/s}, \quad \pi_c = 6, \quad M_0 = 0, \quad T_{t2} = 518.7^\circ\text{R}$$

$$T_{t4} = 3200^\circ\text{R}, \quad P_0 = 1 \text{ atm}, \quad \pi_d = 0.97, \quad \pi_b = 0.96$$

$$\pi_n = 0.98, \quad \eta_b = 0.995, \quad \eta_m = 0.99, \quad P_0/P_9 = 1$$

$$\eta_c = 0.8725, \quad \pi_t = 0.6098, \quad \tau_t = 0.9024, \quad \eta_t = 0.9051$$

$$f = 0.041895, \quad V_9/a_0 = 2.8876, \quad P_{t9}/P_9 = 3.3390, \quad M_9 = 1.4624$$

$$F/\dot{m}_0 = 104.41 \text{ lbf (lbm/s)}, \quad \gamma_c = 1.4, \quad c_{pc} = 0.24 \text{ Btu/(lbm} \cdot {}^\circ\text{R)}$$

$$\gamma_t = 1.3, \; c_{pt} = 0.296 \text{ Btu/(lbm} \cdot {}^\circ\text{R)}, \quad h_{PR} = 18{,}400 \text{ Btu/lbm}$$

(a) Determine $\dot{m}_0$, P_{t5}/P_{t2}, T_{t5}/T_{t2}, V_9, F, and S at the reference condition.

(b) If this engine has a throttle ratio of 1.0 and is operating at maximum T_{t4} at a flight condition where $\theta_0 = 1.2$, determine π_c, $\dot{m}_{c2}$, P_{t5}/P_{t2}, T_{t5}/T_{t2}, and N_c/N_{cR} at this operating point.

(c) If this engine has a throttle ratio of 1.1 and is operating at maximum T_{t4} at a flight condition where $\theta_0 = 1.2$, determine π_c, $\dot{m}_{c2}$, P_{t5}/P_{t2}, T_{t5}/T_{t2}, and N_c/N_{cR} at this operating point.

(d) Determine the percentage change in performance parameters between parts b and c.

8.10 Given a single-spool turbojet engine that has the following reference values.

REFERENCE:

$$\dot{m}_{c2} = 50\ \text{kg/s}, \quad \pi_c = 8, \; M_0 = 0, \quad T_{t2} = 288.2\ \text{K}$$

$$T_{t4} = 1780\ \text{K}, \quad P_0 = 1\ \text{atm}, \quad \pi_d = 0.97, \quad \pi_b = 0.96$$

$$\pi_n = 0.98, \quad \eta_b = 0.995, \quad \eta_m = 0.99, \quad P_0/P_9 = 1$$

$$\eta_c = 0.8678, \quad \pi_t = 0.5434, \quad \tau_t = 0.8810, \quad \eta_t = 0.9062$$

$$f = 0.040796, \quad V_9/a_0 = 3.0256, \quad P_{t9}/P_9 = 3.9674, \quad M_9 = 1.5799$$

$$F/\dot{m}_0 = 1071\ \text{N/(kg/s)}, \quad \gamma_c = 1.4, \quad c_{pc} = 1.004\ \text{kJ/(kg}\cdot\text{K)}$$

$$\gamma_t = 1.3, \quad c_{pt} = 1.24\ \text{kJ/(kg}\cdot\text{K)}, \quad h_{PR} = 42{,}800\ \text{kJ/kg}$$

(a) Determine $\dot{m}_0$, P_{t5}/P_{t2}, T_{t5}/T_{t2}, V_9, F, and S at the reference condition.

(b) If this engine has a throttle ratio of 1.0 and is operating at maximum T_{t4} at a flight condition where $\theta_0 = 1.4$, determine π_c, $\dot{m}_{c2}$, P_{t5}/P_{t2}, T_{t5}/T_{t2}, and N_c/N_{cR} at this operating point.

(c) If this engine has a throttle ratio of 1.15 and is operating at maximum T_{t4}, at a flight condition where $\theta_0 = 1.4$, determine π_c, $\dot{m}_{c2}$, P_{t5}/P_{t2}, T_{t5}/T_{t2}, and N_c/N_{cR} at this operating point.

(d) Determine the percentage change in performance parameters between parts b and c.

8.11 Calculate the thrust of the afterburning turbojet engine in Example 8.7 at $M_0 = 2.0$, 40-kft altitude, maximum afterburner ($T_{t7} = 3600°\text{R}$), and $P_0/P_9 = 1$ for throttle ratios of 1.0, 1.05, 1.1, 1.15, and 1.2. Comment on the variation in performance with throttle ratio.

8.12 Calculate the thrust and fuel consumption of the afterburning turbojet engine in Example 8.7 at $M_0 = 0.8$, 40-kft altitude, $P_0/P_9 = 1$, a throttle ratio of unity, and a range of T_{t7} from 3600°R down to T_{t5}. Compare your results to those of the PERF computer program.

8.13 Using the PERF computer program, find the performance of the afterburning turbojet engine in Example 8.7 for throttle ratios of 1.1 and 1.2 over the same range of Mach numbers and altitudes. Compare these results to those of Example 8.7.

8.14 Calculate the thrust of the high-bypass-ratio turbofan engine in Example 8.8 at $M_0 = 0.8$, 40-kft altitude, and partial throttle ($T_{t4} = 2600°\text{R}$). Assume convergent-only exhaust nozzles. Compare your results to those of the PERF computer program.

8.15 Calculate the thrust of the high-bypass-ratio turbofan engine in Example 8.8 at $M_0 = 0$, sea-level altitude, and increased throttle ($T_{t4} = 3500°\text{R}$). Assume convergent-only exhaust nozzles. Compare your results to those of the PERF computer program.

8.16 Using the PERF computer program, find the performance of the high-bypass-ratio turbofan engine in Example 8.8 for a throttle ratio of 1.0 over the range of Mach numbers from 0 to 0.8 and altitudes of sea level, 20 kft, and 40 kft. Use 3200°R for maximum T_{t4}.

8.17 Early jet aircraft for passenger service used turbojet engines (e.g., Boeing 707) and the newer aircraft use high-bypass-ratio turbofan engines (e.g., Boeing 777). The early turbojets required a much longer takeoff distance than the newer turbofan-powered aircraft. This difference is due mainly to the variation in thrust of these different engine types with Mach number and altitude. To get a better understanding, use the PERF computer program to design two engines and determine their variations in thrust with Mach number and altitude. Consider a turbojet with the component performance of technology level 3 in Table 6.2 (assume type A diffuser, uncooled turbine, and type D nozzle), $T_{t4} = 2500°R$, compressor pressure ratio of 12, and sea-level static thrust of 10,000 lbf. Determine the turbojet's performance for TR = 1 at maximum T_{t4}, Mach 0.8, and 30-kft altitude. Now consider a high-bypass-ratio turbofan with the component performance of technology level 3 in Table 6.2 (assume type A diffuser, uncooled turbine, and type D nozzle), $T_{t4} = 3000°R$, compressor pressure ratio of 22, fan pressure ratio of 1.54, bypass ratio of 5, and sea-level static thrust of 56,000 lbf. Determine the turbofan's performance for TR = 1 at maximum T_{t4}, Mach 0.8, and 30-kft altitude, and compare to the turbojet.

8.18 Develop a set of performance equations for the stationary gas turbine engine with regeneration (see Problem 7.26) depicted in Fig. P8.2 based on the following assumptions:
(a) Flow is choked at engine stations 4 and 4.5.
(b) Constant component efficiencies (η_c, η_b, η_c, η_{rg}, etc.).
(c) Exit pressure equals the ambient pressure ($P_9 = P_0$).
(d) Constant specific heat c_{pc} and ratio of specific heats γ_c from stations 0 to 3.5.
(e) Constant specific heat c_{pt} and ratio of specific heats γ_t from stations 4 to 9.

8.19 Calculate the corrected fuel flow rate [see Eq. (8.7)] and corrected thrust specific fuel consumption [see Eq. (8.6)] for the turbojet engine of Example 8.4 at an altitude of 40 kft and Mach numbers of 0.6, 0.8, 1.0, and 1.2 with maximum T_{t4}. Comment on your results.

8.20 For a single-spool turbojet engine, show that the maximum corrected fuel flow rate is essentially constant for $\theta_0 \leq$ TR. See Eq. (8.27) for a starting point.

8.21 Consider the dual-spool afterburning turbojet engine modeled in Section 8.4 of the textbook. In this development, the area of station 4.5 was modeled as fixed. If this area changes from its reference value, the temperature and pressure ratios of the high- and low-pressure turbines vary from their reference values. As a result, changes occur in the operating

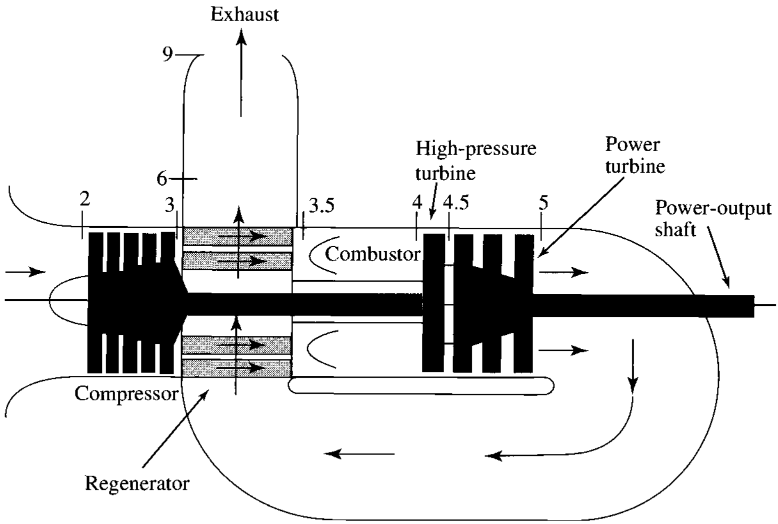

Fig. P8.2

points for both the low- and high-pressure compressors. To better understand this, you are asked to

(a) Redevelop the turbine and compressor relationships influenced by variation in the area at station 4.5. Consider all other assumptions remain valid.

(b) For the reference conditions of Example 8.7, determine the changes in the temperature and pressure ratios of the turbomachinery (compressors and turbines) for a 5% increase in the flow area at station 4.5.

8.22 Variable area turbojet (VAT) engine: The partial throttle (reduced T_{t4}) performance of the typical turbojet engine (see Fig. 8.8) results in reduced compressor pressure ratio π_c and compressor corrected mass flow rate $\dot{m}_{c2}$. The thermal efficiency of the engine is reduced by the lower compressor pressure ratio. Reduced compressor corrected mass flow rate results in increased installation loss from inlet spillage drag (see Chapter 10). We desire to keep compressor mass flow rate $\dot{m}_{c2}$ and compressor total pressure ratio π_c constant at reduced throttle T_{t4} by varying the areas at stations 4 and 8 in a single-spool turbojet engine.

(a) Starting from Eq. (8.9) and assuming $M_4 = 1$ and π_b and $(1+f)$ are con stants, show that constant $\dot{m}_{c2}$ and π_c requires that the area at station 4 (A_4) be varied in the following way:

$$\frac{A_4}{A_{4R}} = \sqrt{\frac{T_{t4}/T_{t2}}{(T_{t4}/T_{t2})_R}} \tag{VAT.1}$$

(b) Starting with the work balance between the turbine and compressor of Eq (8.13), show that for τ_c constant then the turbine total temperature will vary as

$$\tau_t = 1 - (1 - \tau_{tR})\frac{(T_{t4}/T_{t2})_R}{T_{t4}/T_{t2}} \tag{VAT.2}$$

(c) Using conservation of mass between stations 4 and 8 (with $M_4 = 1$ and $M_8 = 1$), show that

$$\frac{\sqrt{\tau_t}}{\pi_t} = \frac{A_8}{A_4}$$

and the area at station 8 must be varied in the following way:

$$\frac{A_8}{A_{8R}} = \frac{A_8}{A_{4R}}\sqrt{\frac{\tau_t}{\tau_{tR}}}\frac{\pi_{tR}}{\pi_t} = \sqrt{\frac{T_{t4}/T_{t2}}{(T_{t4}/T_{t2})_R}}\sqrt{\frac{\tau_t}{\tau_{tR}}}\frac{\pi_{tR}}{\pi_t} \tag{VAT.3}$$

where, assuming constant turbine efficiency η_t, the turbine total pressure ratio is given by Eq. (8.12b).

(d) Because the corrected mass flow rate into the compressor is constant, show that the engine mass flow rate can be written as

$$\frac{\dot{m}_0}{\dot{m}_{0R}} = \frac{P_0\pi_r\pi_d}{(P_0\pi_r\pi_d)_R}\sqrt{\frac{(T_0\tau_r)_R}{T_0\tau_r}} \tag{VAT.4}$$

Problems for Supporting Material

SM8.1 Calculate the maximum thrust of the afterburning mixed-flow exhaust turbofan engine in Example SM8.1 at $M_0 = 2$ and 40-kft altitude for throttle ratios of 1.1 and 1.2 using the PERF computer program.

SM8.2 Using the PERF computer program, find the performance of the afterburning mixed-flow exhaust turbofan engine in Example 8.10 for throttle ratios of 1.0 and 1.2 over the range of Mach numbers from 0 to 2 and altitude of sea level, 10 kft, 20 kft, 30 kft, 36 kft, 40 kft, and 50 kft. Compare your results to those of Example SM8.1 for a throttle ratio of 1.1.

SM8.3 Calculate the thrust and fuel consumption of the turboprop engine in Example 8.11 at $M_0 = 0.5$, a 6-km altitude, and reduced throttle ($T_{t4} = 1400$ K). Assume a convergent-only exhaust nozzle. Compare your results to those of the PERF computer program.

SM8.4 Using the PERF computer program, find the performance of the turboprop engine in Example SM8.2 at partial throttle at $M_0 = 0.5$ and altitudes of sea level, 3 km, and 6 km. Compare these results to those of Example SM8.2.

Gas Turbine Design Problems

8.D1 Find the best high-bypass-ratio turbofan engine for the HP-1 aircraft from those engines showing promise in Design Problem 7.D1. You are to determine the best engine, sized to meet the required engine thrust at takeoff and/or single engine out (0.45 Mach at 16 kft), and whose fuel consumption is minimum.

Hand-Calculate Engine Performance (HP-1 Aircraft). Using the performance analysis equations for a high-bypass-ratio turbofan engine, hand-calculate the performance of the turbofan engine hand-calculated in Design Problem 7.D1 at the flight condition of 0.83 Mach and 11-km altitude at $T_{t4} = 1500$ K.

Required Thrust at Different Flight Conditions (HP-1 Aircraft). Determine both the required installed thrust and the required uninstalled engine thrust for each of the following flight conditions (assume $\phi_{\text{noz}} = 0.01$):

1) Uninstalled thrust of 267 kN for each engine at sea-level static while at takeoff power setting ($T_{t4} = 1890$ K). Assume $\phi_{\text{inlet}} = 0.05$.
2) Start of Mach 0.83 cruise, 11-km altitude, $P_s = 1.5$ m/s, 95.95% of takeoff weight; $\phi_{\text{inlet}} = 0.01$.
3) Mach 0.45, 5-km altitude, engine out, $P_s = 1.5$ m/s, 88% of takeoff weight; $\phi_{\text{inlet}} = 0.02$. Assume a drag coefficient increment for engine out = 0.0035 (based on wing area).
4) Loiter at 9-km altitude, 64% of takeoff weight; $\phi_{\text{inlet}} = 0.03$.

Computer-Calculated Engine Performance (HP-1 Aircraft). For each of the engines showing promise in Design Problem 7.D1, systematically perform the following analysis:

1) Design the reference engine at sea-level static conditions. Size the engine to provide the required uninstalled thrust (engine size normally will be determined by either the takeoff flight condition or the engine-out flight condition listed previously). Check engine operation at takeoff and make sure that $T_{t3} < 920$ K.
2) Determine the uninstalled fuel consumption at the start of Mach 0.83 cruise, 11-km altitude, and $P_s = 0$. You can do this by input of "% thrust" in the PERF computer program and calculate for the required uninstalled thrust. Assuming cruise climb with constant range factor (RF), calculate the weight at the end of the Mach 0.83 cruise, using the Breguet range equation, Eq. (1.45a or b).
3) Determine the loiter Mach number at 9-km altitude for the current aircraft weight at the start of loiter. Find the engine fuel consumption at start of loiter. Assuming a constant endurance factor (EF), calculate the weight at the end of the 9-km loiter. Some engines may not throttle down to the required uninstalled thrust due to the model used in the computer program, and the uninstalled thrust specific fuel consumption will have to be obtained by *extrapolation*. To extrapolate, use the performance analysis computer

program and iterate on T_{t4} in steps of 25 K to the lowest value giving results. Then plot S vs F as shown in Fig. P8.D1a, and draw a tangent line to obtain the extrapolated value of S at the desired uninstalled thrust.

Fuel Consumption (HP-1 Aircraft). During this preliminary analysis, you can assume that the fuel consumption changes only for the Mach 0.83 cruise and 9-km loiter. For every one of the engines you investigate, you must determine whether it can satisfy the fuel consumption requirements by calculating the amount of fuel consumed during the Mach 0.83 cruise climb and 9-km loiter and adding these to that consumed for other parts of the flight. During your analysis of the engines, make a plot of fuel consumed vs the reference bypass ratio like that shown in Fig. P8.D1b. Starting with one compressor pressure ratio π_{c1}, and a low-bypass-ratio engine, calculate the total fuel consumed for the flight. Now increase the bypass ratio, size the engine, and determine this engine's performance.

Engine Selection (HP-1 Aircraft). Select one of your engines that, according to your criteria, best satisfies the mission requirements. Your criteria *must include* at least the following items (other items may be added based on knowledge gained in other courses and any additional technical sources):

Thrust requirement
Fuel consumption
Aircraft performance
Operating cost (assume 10,000-h engine life and fuel cost of \$1.00/lb)
First cost
Size and weight
Complexity

Determine Engine Thrust vs Mach Number and Altitude (HP-1 Aircraft). For the engine you select, determine and plot the uninstalled thrust F at maximum power vs Mach number at sea level, 5-km altitude, and 11-km altitude and at takeoff power at sea level (see Fig. 8.47). Use $T_{t4} = 1780$ K for maximum power and $T_{t4} = 1890$ K for takeoff power.

Summary. Summarize the final choice for the selected engine including a list of design conditions and choices, performance during the mission, and overall mission performance. Include suggestions, if necessary, for

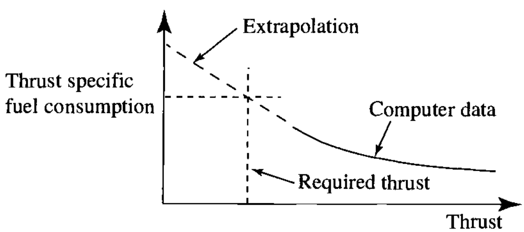

Fig. P8.D1a

overcoming any of the performance shortcomings that may exist in any of the mission legs. In addition, make meaningful comments about the feasibility of building such an engine.

8.D2 Find the best mixed-flow turbofan engine with afterburner for the HF-1 aircraft from those engines showing promise in Design Problem 7.D2. You are to determine the best engine, sized to meet the required engine thrust at takeoff, supercruise, and/or 5-g turns, and whose cruise fuel consumption over the maximum mission (see Design Problem 1.D2) is minimum.

Hand-Calculate Engine Performance (HP-1 Aircraft). Using the performance analysis equations for a mixed-flow turbofan engine with afterburner, hand-calculate the performance of the mixed-flow turbofan engine hand-calculated in Design Problem 7.D2 at the flight condition of 1.6 Mach and 40-kft altitude at $T_{t4} = 3200°\text{R}$ with the afterburner turned off.

Required Thrust at Different Flight Conditions (HP-1 Aircraft). Determine both the required installed thrust and the required uninstalled engine thrust for each of the following flight conditions (assume $\phi_{\text{noz}} = 0.01$):

1) For takeoff, an installed thrust of 23,500 lbf for each engine at sea level, 0.1 Mach while at maximum power setting (afterburner on with $T_{t7} = 3600°\text{R}$). Assume $\phi_{\text{inlet}} = 0.10$.

2) Start of Mach 1.6 supercruise, 40-kft altitude, 92% of takeoff weight W_{TO}; $\phi_{\text{inlet}} = 0.04$.

3) Start of 5-g turn at Mach 1.6, 30-kft altitude, 88% of W_{TO}; $\phi_{\text{inlet}} = 0.04$.

4) Start of 5-g turn at Mach 0.9, 30-kft altitude, 88% of W_{TO}; $\phi_{\text{inlet}} = 0.04$.

Computer-Calculated Engine Performance (HP-1 Aircraft). Develop a reference mixed-flow turbofan engine with afterburner based on the data used in Design Problem 7.D2. For the afterburner, use $\gamma_{\text{AB}} = 1.3$, $c_{p\text{AB}} = 0.296\ \text{Btu/(lbm}\cdot°\text{R)}$, $\pi_{\text{AB}} = 0.96$, $\eta_{\text{AB}} = 0.97$, and $T_{t7} = 3600°\text{R}$.

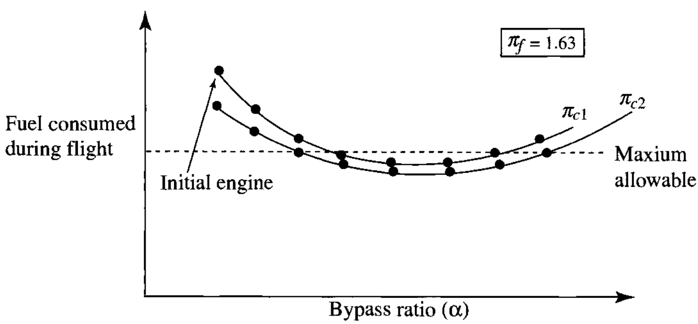

Fig. P8.D1b

For each of the engines showing promise in Design Problem 7.D2, systematically perform the following analysis:

1) Design the reference engine at sea-level static conditions. Size the engine to provide the required uninstalled thrust (engine size will normally be determined by the takeoff flight condition, by the start of supercruise flight condition, or by the 5-g turn at Mach 1.6 listed previously). Check engine operation at takeoff, and make sure that $T_{t3} < 1600°\text{R}$ and that the compressor pressure is within a specified limit.

2) Determine uninstalled fuel consumption at the start of the Mach 1.6 supercruise, 40-kft altitude. You can do this by input of "% thrust" in the PERF computer program and calculate for the required uninstalled thrust. Your engine should be able to deliver the required thrust with the afterburner off. Assuming cruise climb with constant range factor (RF), calculate the weight at the end of the Mach 1.6 supercruise, using the Breguet range equation, Eq. (1.45a or b).

3) Calculate the aircraft weight at the start of the Mach 0.9 cruise and the corresponding best cruise altitude (maximum C_L/C_D) and aircraft drag. Calculate the uninstalled required thrust at start of Mach 0.9 cruise, assuming $\phi_{\text{inlet}} = 0.05$ and $\phi_{\text{noz}} = 0.01$. Now determine the uninstalled fuel consumption at the start of Mach 0.9 cruise. Assuming cruise climb with constant range factor (RF), calculate the weight at the end of the Mach 0.9 cruise, using the Breguet range equation, Eq. (1.45a,b).

4) Determine the loiter Mach number at 30-kft altitude for the current aircraft weight at the start of loiter. Find the engine fuel consumption at the start of loiter. Assuming constant endurance factor (EF), calculate the weight at the end of the 30-kft loiter. Some engines may not throttle down to the required uninstalled thrust due to the model used in the computer program, and the uninstalled thrust specific fuel consumption will have to be obtained by *extrapolation*. To extrapolate, use the performance analysis computer program and iterate on T_{t4} in steps of 50°R to the lowest value giving results. Then plot S vs F as shown in Fig. P8.D1a, and draw a tangent line to obtain the extrapolated value of S at the desired uninstalled thrust.

Fuel Consumption (HP-1 Aircraft). During this preliminary analysis, you can assume that the fuel consumption changes only for the Mach 1.6 supercruise, Mach 0.9 cruise, and 30-kft loiter. For every one of the engines you investigate, you must determine whether it can satisfy the fuel consumption requirements by calculating the amount of fuel consumed during the Mach 1.6 supercruise, Mach 0.9 cruise, and 30-kft loiter and adding these values to that consumed for other parts of the maximum mission. During your analysis of the engines, make a plot of fuel consumed vs the reference bypass ratio like that shown in Fig. P8.D1b. Starting with one compressor pressure ratio π_{c1} and a low-bypass-ratio engine, calculate the total fuel consumed for the mission. Now increase the bypass ratio, size the engine, and determine this engine's performance.

Engine Selection (HP-1 Aircraft). Select one of your engines that, according to your criteria, best satisfies the mission requirements. Your criteria *must include* at least the following items (other items may be added based on knowledge gained in other courses and any additional technical sources):

Thrust required
Fuel consumption
Aircraft performance
Operating cost (assume 10,000-h engine life and fuel cost of $1.00/lb)
First cost
Size and weight
Complexity

Determine Engine Thrust vs Mach Number and Altitude (HP-1 Aircraft). For the engine you select, determine and plot the uninstalled thrust F and thrust specific fuel consumption at both maximum power (afterburner on) and military power (afterburner off) vs Mach number at altitudes of sea level, 10 kft, 20 kft, 30 kft, 36 kft, and 40 kft (see Figs. 8.65, 8.66, 8.67, and 8.68). Use $T_{t4\max} = 3250°\text{R}$ and the throttle ratio TR for your engine. Also determine and plot the partial-throttle performance (see Fig. 8.69) of your engine at sea-level static, 1.6 Mach and 40 kft, 1.6 Mach and 30 kft, and 0.9 Mach and 30 kft.

Summary. Summarize the final choice for the selected engine including a list of design conditions and choices, performance during the mission, and overall mission performance. Include suggestions, if necessary, for overcoming any of the performance shortcomings that may exist in any of the mission legs. In addition, make meaningful comments about the feasibility of building such an engine.