# 1
# Introduction

## 1.1 Propulsion

The *Random House College Dictionary* defines *propulsion* as "the act of propelling, the state of being propelled, a propelling force or impulse" and defines the verb *propel* as "to drive, or cause to move, forward or onward."[1] From these definitions, we can conclude that the study of propulsion includes the study of the propelling force, the motion caused, and the bodies involved. Propulsion involves an object to be propelled plus one or more additional bodies, called *propellant.*

The study of propulsion is concerned with vehicles such as automobiles, trains, ships, aircraft, and spacecraft. The focus of this textbook is on the propulsion of aircraft and spacecraft. Methods devised to produce a thrust force for the propulsion of a vehicle in flight are based on the principle of jet propulsion (the momentum change of a fluid by the propulsion system). The fluid may be the gas used by the engine itself (e.g., turbojet), it may be a fluid available in the surrounding environment (e.g., air used by a propeller), or it may be stored in the vehicle and carried by it during the flight (e.g., rocket).

Jet propulsion systems can be subdivided into two broad categories: airbreathing and non-airbreathing. Airbreathing propulsion systems include the reciprocating, turbojet, turbofan, ramjet, turboprop, and turboshaft engines. Non-airbreathing engines include rocket motors, nuclear propulsion systems, and electric propulsion systems. We focus on gas turbine propulsion systems (turbojet, turbofan, turboprop, and turboshaft engines) in this textbook.

The material in this textbook is divided into three parts:

1) Basic concepts and one-dimensional gas dynamics,
2) Analysis and performance of airbreathing propulsion systems, and
3) Analysis of gas turbine engine components.

This chapter introduces the types of airbreathing and rocket propulsion systems and the basic propulsion performance parameters. Also included is an introduction to aircraft and rocket performance. The material on aircraft performance shows the influence of the gas turbine engine on the performance of the aircraft system. This material also permits incorporation of a gas turbine engine design problem such as new engines for an existing aircraft.

Numerous examples are included throughout this book to help students see the application of a concept after it is introduced. For some students, the material on basic concepts and gas dynamics will be a review of material covered in other courses they have already taken. For other students, this may be their first exposure to this material, and it may require more effort to understand.

## 1.2 Units and Dimensions

Since the engineering world uses both the metric SI and English unit system, both will be used in this textbook. One singular distinction exists between the English system and SI—the unit of force is defined in the former but derived in the latter. Newton's second law of motion relates force to mass, length, and time. It states that the sum of the forces is proportional to the rate of change of the momentum ($\boldsymbol{M} = mV$). The constant of proportionality is $1/g_c$:

$$\sum \boldsymbol{F} = \frac{1}{g_c}\frac{\mathrm{d}(mV)}{\mathrm{d}t} = \frac{1}{g_c}\frac{\mathrm{d}\boldsymbol{M}}{\mathrm{d}t} \tag{1.1}$$

The units for each term in the preceding equation are listed in Table 1.1 for both SI and English units. In any unit system, only four of the five items in the table can be specified, and the latter is derived from Eq. (1.1).

As a result of selecting $g_c = 1$ and defining the units of mass, length, and time in SI units, the unit of force is derived from Eq. (1.1) as kilogram-meters per square second ($\mathrm{kg \cdot m/s^2}$), which is called the *newton* (N). In English units, the value of $g_c$ is derived from Eq. (1.1) as

$$g_c = 32.174\,\mathrm{ft \cdot lbm/(lbf \cdot s^2)}$$

Rather than adopt the convention used in many recent textbooks of developing material or use with *only* SI metric units ($g_c = 1$), we will maintain $g_c$ in all our equations. Thus $g_c$ will also show up in the equations for *potential energy* (PE) and *kinetic energy* (KE):

$$\mathrm{PE} = \frac{mgz}{g_c}$$

$$\mathrm{KE} = \frac{mV^2}{2g_c}$$

The total energy per unit mass $e$ is the sum of the specific internal energy $u$, specific kinetic energy ke, and specific potential energy pe:

$$e \equiv u + \mathrm{ke} + \mathrm{pe} = u + \frac{V^2}{2g_c} + \frac{gz}{g_c}$$

There are a multitude of engineering units for the quantities of interest in propulsion. For example, energy can be expressed in the SI unit of *joule*

**Table 1.1 Units and dimensions**

| Unit system | Force | $g_c$ | Mass | Length | Time |
|---|---|---|---|---|---|
| SI | Derived | 1 | Kilogram, kg | Meter, m | Second, s |
| English | Pound-force, lbf | Derived | Pound-mass, lbm | Foot, ft | Second, s |

(1 J = 1 N · m), in British thermal units (Btu), or in foot-pound force (ft · lbf). One must be able to use the available data in the units provided and convert the units when required. Table 1.2 is a unit conversion table provided to help you in your endeavors.

**Table 1.2 Unit conversion table**

| Unit | Conversion |
|---|---|
| Length | 1 m = 3.2808 ft = 39.37 in. |
| | 1 km = 0.621 mile |
| | 1 mile = 5280 ft = 1.609 km |
| | 1 nm = 6080 ft = 1.853 km |
| Area | 1 $m^2$ = 10.764 $ft^2$ |
| | 1 $cm^2$ = 0.155 $in.^2$ |
| Volume | 1 gal = 0.13368 $ft^3$ = 3.785 liter |
| | 1 liter = $10^{-3}$ $m^3$ = 61.02 $in.^3$ |
| Time | 1 h = 3600 s = 60 min |
| Mass | 1 kg = 1000 g = 2.2046 lbm = $6.8521 \times 10^{-2}$ slug |
| | 1 slug = 1 lbf · $s^2$/ft = 32.174 lbm |
| Density | 1 slug/$ft^3$ = 512.38 kg/$m^3$ |
| Force | 1 N = 1 kg · m/$s^2$ |
| | 1 lbf = 4.448 N |
| Energy | 1 J = 1 N · m = 1 kg · $m^2/s^2$ |
| | 1 Btu = 778.16 ft · lbf = 252 cal = 1055 J |
| | 1 cal = 4.186 J |
| | 1 kJ = 0.947813 Btu = 0.23884 kcal |
| Power | 1 W = 1 J/s = 1 kg · $m^2/s^3$ |
| | 1 hp = 550 ft · lbf/s = 2545 Btu/h = 745.7 W |
| | 1 kW = 3412 Btu/h = 1.341 hp |
| Pressure (stress) | 1 atm = 14.696 lb/$in.^2$ or psi = 760 torr = 101,325 Pa |
| | 1 atm = 30.0 inHg = 407.2 $inH_2O$ |
| | 1 ksi = 1000 psi |
| | 1 mmHg = 0.01934 psi = 1 torr |
| | 1 Pa = 1 N/$m^2$ |
| | 1 inHg = 3376.8 Pa |
| Energy per unit mass | 1 kJ/kg = 0.4299 Btu/lbm |
| Specific heat | 1 kJ/(kg · °C) = 0.23884 Btu/(lbm · °F) |
| Temperature | 1 K = 1.8°R |
| | K = 273.15 + °C |
| | °R = 459.69 + °F |
| Temperature change | 1°C = 1.8°F |
| Specific thrust | 1 lbf/(lbm/s) = 9.8067 N/(kg/s) |
| Specific power | 1 hp/(lbm/s) = 1.644 kW/(kg/s) |
| Thrust specific fuel consumption (TSFC) | 1 lbm/(lbf · h) = 28.325 mg/(N · s) |
| Power specific fuel consumption | 1 lbm/(hp · h) = 168.97 mg/(kW · s) |
| Strength/weight ratio ($\sigma/\rho$) | 1 ksi/(slug/$ft^3$) = 144 $ft^2/s^2$ = 13.38 $m^2/s^2$ |

## 1.3 Operational Envelopes and Standard Atmosphere

Each engine type will operate only within a certain range of altitudes and Mach numbers (velocities). Similar limitations in velocity and altitude exist for airframes. It is necessary, therefore, to match airframe and propulsion system capabilities. Figure 1.1 shows the approximate velocity and altitude limits, or *corridor of flight*, within which airlift vehicles can operate. The corridor is bounded by a *lift limit*, a *temperature limit*, and an *aerodynamic force limit.* The lift limit is determined by the maximum level-flight altitude at a given velocity. The temperature limit is set by the structural thermal limits of the material used in construction of the aircraft. At any given altitude, the maximum velocity attained is temperature-limited by aerodynamic heating effects. At lower altitudes, velocity is limited by aerodynamic force loads rather than by temperature.

The operating regions of all aircraft lie within the flight corridor. The operating region of a particular aircraft within the corridor is determined by aircraft design, but it is a very small portion of the overall corridor. Superimposed on the flight corridor in Fig. 1.1 are the operational envelopes of various powered aircraft. The operational limits of each propulsion system are determined by limitations of the components of the propulsion system and are shown in Fig. 1.2.

The analyses presented in this text use the properties of the atmosphere to determine both engine and airframe performance. Since these properties vary with location, season, time of day, etc., we will use the U.S. standard atmosphere[2] to give a known foundation for our analyses. Appendix A gives the properties of the U.S. standard atmosphere, 1976, in both English and SI units. Values of the pressure $P$, temperature $T$, density $\rho$, and speed of sound $a$ are given in dimensionless ratios of the property at altitude to its value at sea level (SL), the reference value. The dimensionless ratios of pressure, temperature, and density

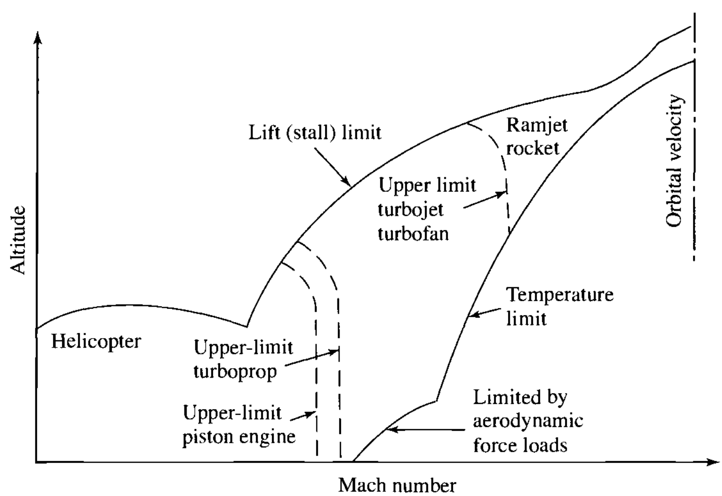


**Fig. 1.1 Flight limits.**

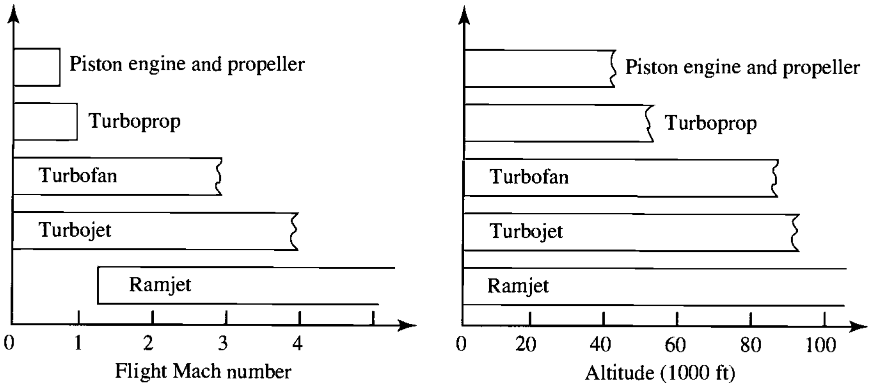


**Fig. 1.2 Engine operational limits.**

are given the symbols $\delta$, $\theta$, and $\sigma$, respectively. These ratios are defined as follows:

$$\delta \equiv \frac{P}{P_{\text{ref}}} \tag{1.2}$$

$$\theta \equiv \frac{T}{T_{\text{ref}}} \tag{1.3}$$

$$\sigma \equiv \frac{\rho}{\rho_{\text{ref}}} \tag{1.4}$$

The reference values of pressure, temperature, and density are given for each unit system at the end of its property table.

For nonstandard conditions such as a hot day, the normal procedure is to use the standard pressure and correct the density, using the perfect gas relationship $\sigma = \delta/\theta$. As an example, we consider a 100°F day at 4-kft altitude. From Appendix A, we have $\delta = 0.8637$ for the 4-kft altitude. We calculate $\theta$, using the 100°F temperature; $\theta = T/T_{\text{ref}} = (100 + 459.7)/518.7 = 1.079$. Note that absolute temperatures must be used in calculating $\theta$. Then the density ratio is calculated using $\sigma = \delta/\theta = 0.8637/1.079 = 0.8005$.

## 1.4 Airbreathing Engines

The turbojet, turbofan, turboprop, turboshaft, and ramjet engine systems are discussed in this part of Chapter 1. The discussion of these engines is in the context of providing thrust for aircraft. The listed engines are not all the engine types (reciprocating, rockets, combination types, etc.) that are used in providing propulsive thrust to aircraft, nor are they used exclusively on aircraft. The thrust of the turbojet and ramjet results from the action of a fluid jet leaving the engine; hence, the name *jet engine* is often applied to these engines. The turbofan, turboprop, and turboshaft engines are adaptations of the turbojet to supply thrust or power through the use of fans, propellers, and shafts.

### 1.4.1 Gas Generator

The "heart" of a gas turbine type of engine is the gas generator. A schematic diagram of a gas generator is shown in Fig. 1.3. The compressor, combustor, and turbine are the major components of the gas generator which is common to the turbojet, turbofan, turboprop, and turboshaft engines. The purpose of a gas generator is to supply high-temperature and high-pressure gas.

### 1.4.2 Turbojet

By adding an inlet and a nozzle to the gas generator, a turbojet engine can be constructed. A schematic diagram of a simple turbojet is shown in Fig. 1.4a, and a turbojet with afterburner is shown in Fig. 1.4b. In the analysis of a turbojet engine, the major components are treated as sections. Also shown in Figs. 1.4a and 1.4b are the station numbers for each section.

The turbojet was first used as a means of aircraft propulsion by von Ohain (first flight August 27, 1939) and Whittle (first flight May 15, 1941). As development proceeded, the turbojet engine became more efficient and replaced some of the piston engines. A photograph of the J79 turbojet with afterburner used in the F-4 Phantom II and B-58 Hustler is shown in Fig. 1.5.

The adaptations of the turbojet in the form of turbofan, turboprop, and turboshaft engines came with the need for more thrust at relatively low speeds. Some characteristics of different turbojet, turbofan, turboprop, and turboshaft engines are included in Appendix B.

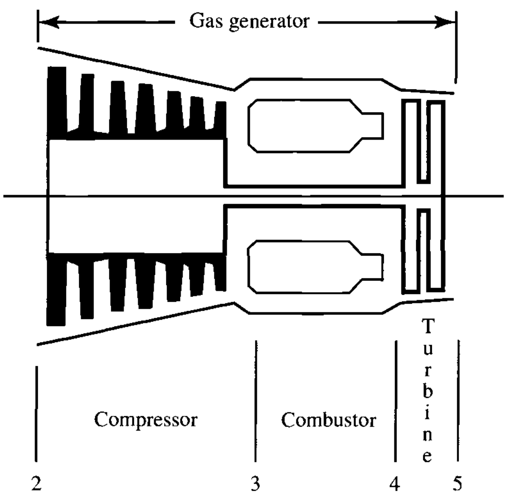


**Fig. 1.3 Schematic diagram of gas generator.**

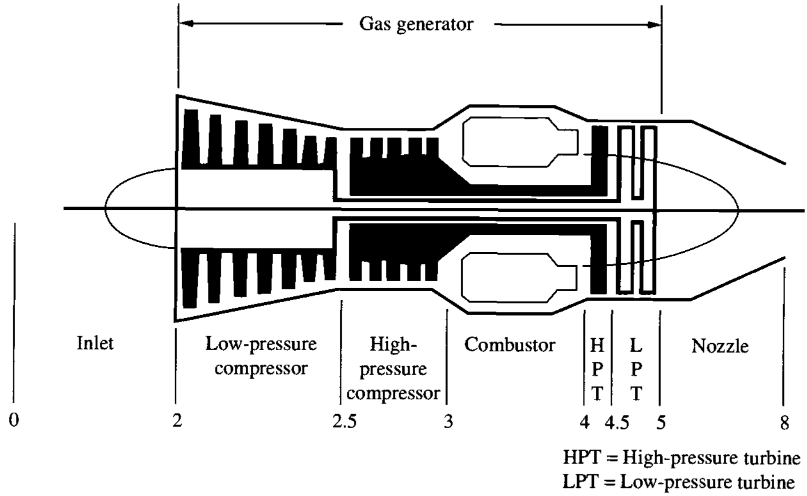


**Fig. 1.4a Schematic diagram of a turbojet (dual axial compressor and turbine).**

The thrust of a turbojet is developed by compressing air in the inlet and compressor, mixing the air with fuel and burning in the combustor, and expanding the gas stream through the turbine and nozzle. The expansion of gas through the turbine supplies the power to turn the compressor. The net thrust delivered by the engine is the result of converting internal energy to kinetic energy.

The pressure, temperature, and velocity variations through a J79 engine are shown in Fig. 1.6. In the compressor section, the pressure and temperature increase as a result of work being done on the air. The temperature of the gas

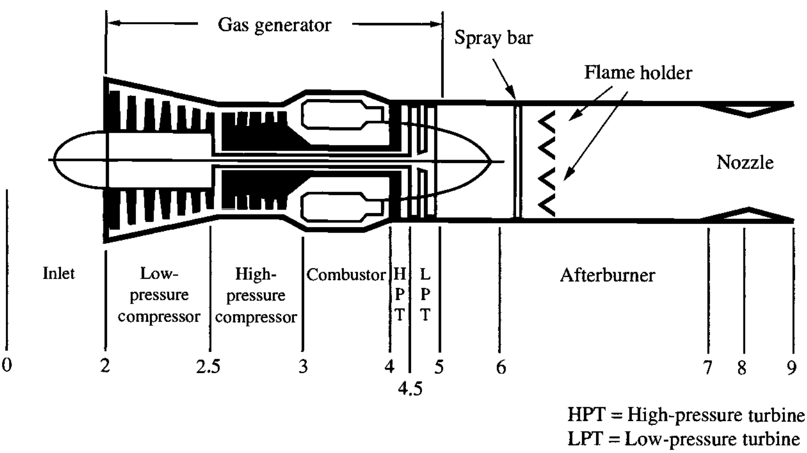


**Fig. 1.4b Schematic diagram of a turbojet with afterburner.**

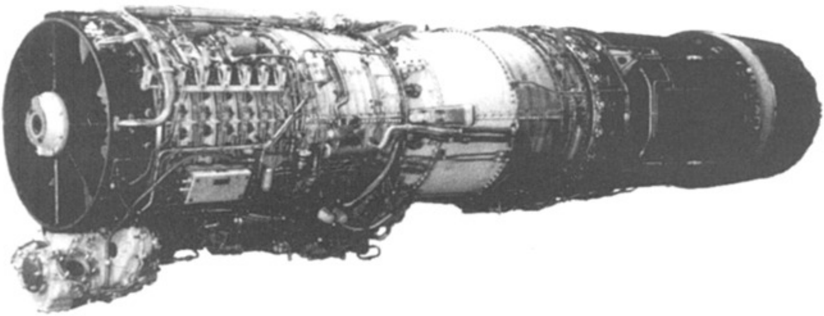

**Fig. 1.5 General Electric J79 turbojet with afterburner. (Courtesy of General Electric Aircraft Engines.)**

is further increased by burning fuel in the combustor. In the turbine section, energy is being removed from the gas stream and converted to shaft power to turn the compressor. The energy is removed by an expansion process that results in a decrease of temperature and pressure. In the nozzle, the gas stream is further expanded to produce a high exit kinetic energy. All the sections of the engine must operate in such a way as to efficiently produce the greatest amount of thrust for a minimum of weight.

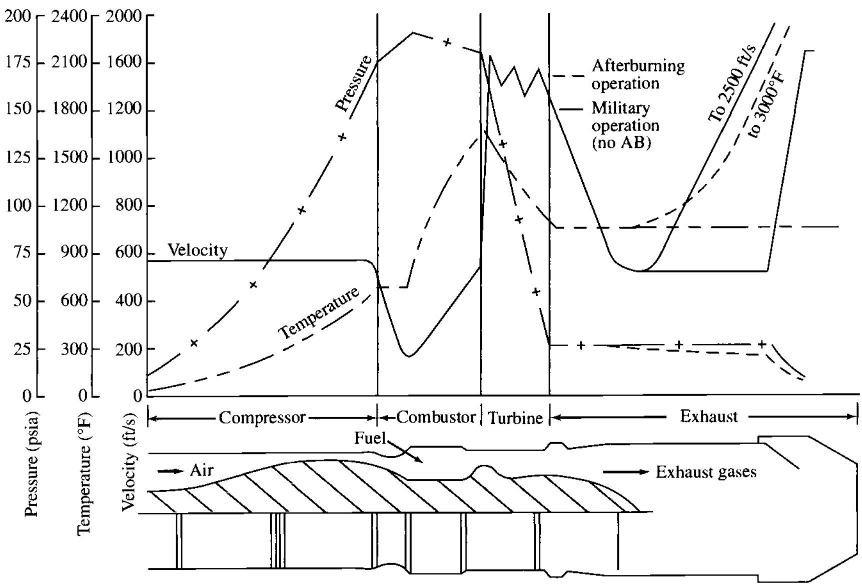


**Fig. 1.6 Property variation through the General Electric J79 afterburning turbojet engine. (Courtesy of General Electric Aircraft Engines.)**

### *1.4.3 Turbofan*

The turbofan engine consists of an inlet, fan, gas generator, and nozzle. A schematic diagram of a turbofan is shown in Fig. 1.7. In the turbofan, a portion of the turbine work is used to supply power to the fan. Generally the turbofan engine is more economical and efficient than the turbojet engine in

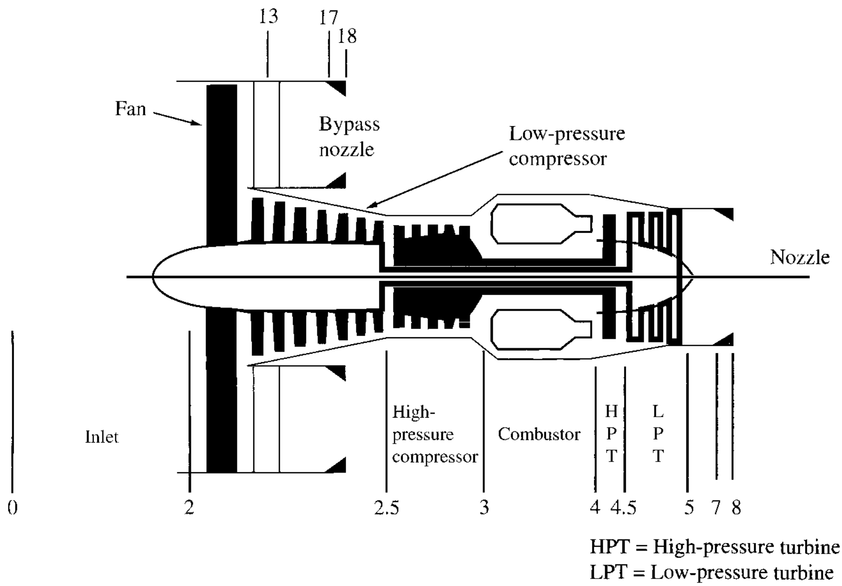


**Fig. 1.7 Schematic diagram of a high-bypass-ratio turbofan.**

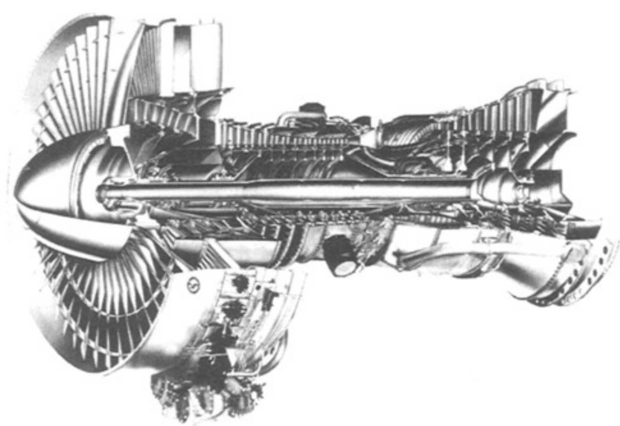

**Fig. 1.8a Pratt & Whitney JT9D turbofan. (Courtesy of Pratt & Whitney.)**

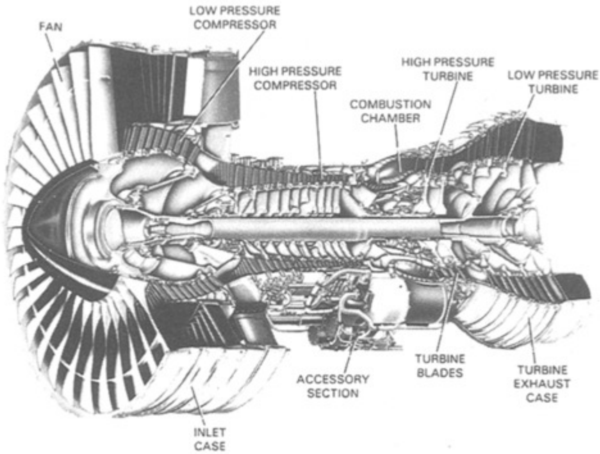


**Fig. 1.8b Pratt & Whitney PW4000 turbofan. (Courtesy of Pratt & Whitney.)**

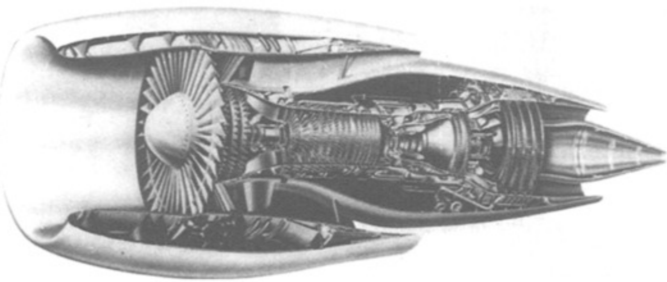

**Fig. 1.8c General Electric CF6 Turbofan. (Courtesy of General Electric Aircraft Engines.)**

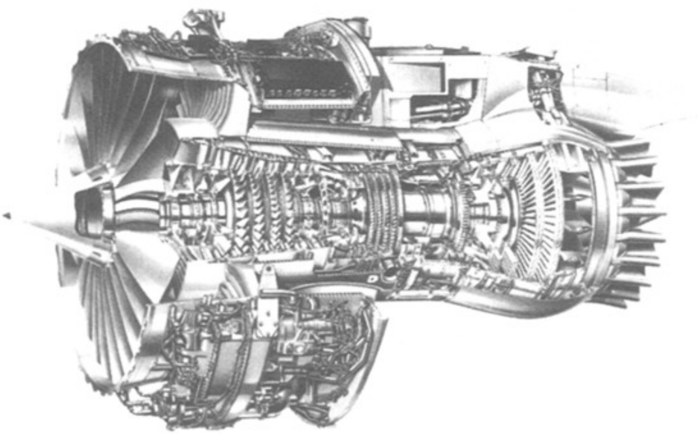

**Fig. 1.8d Rolls-Royce RB-211-524G/H turbofan. (Courtesy of Rolls-Royce.)**

subsonic flight. The *thrust specific fuel consumption* (TSFC, or fuel mass flow rate per unit thrust) is lower for turbofans and indicates a more economical operation. The turbofan also accelerates a larger mass of air to a lower velocity than a turbojet for a higher propulsive efficiency. The frontal area of a turbofan is quite large compared to that of a turbojet, and for this reason more drag and more weight result. The fan diameter is also limited aerodynamically when compressibility effects occur. Several of the current high-bypass-ratio turbofan engines used in subsonic aircraft are shown in Figs. 1.8a–1.8f.

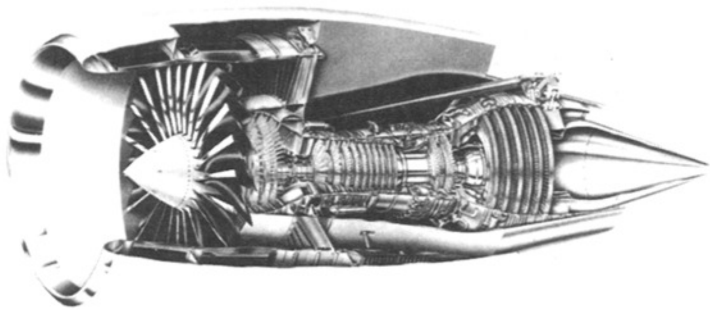

**Fig. 1.8e General Electric GE90 turbofan. (Courtesy of General Electric Aircraft Engines.)**

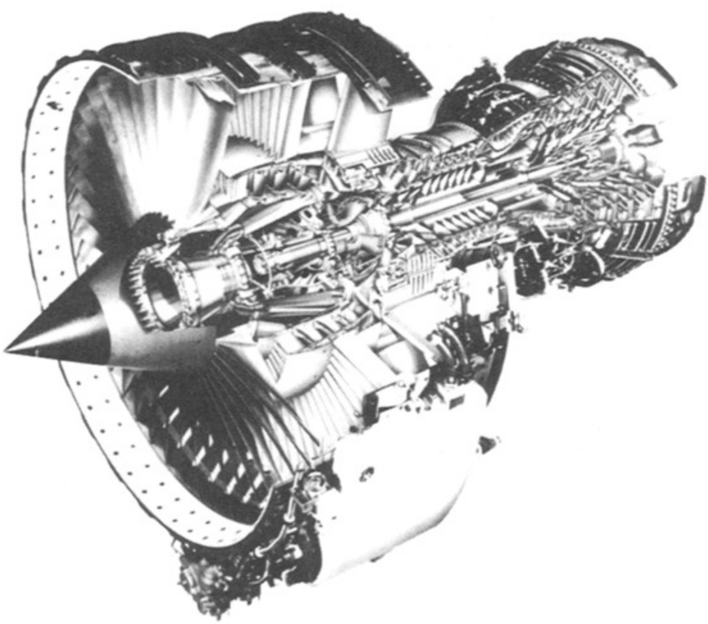

**Fig. 1.8f SNECMA CFM56 turbofan. (Courtesy of SNECMA.)**

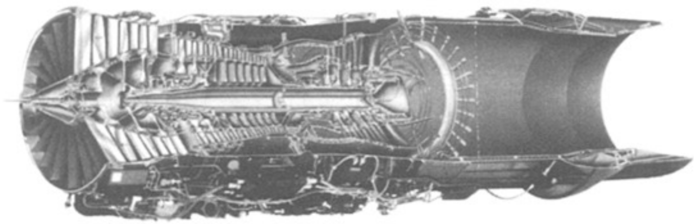

**Fig. 1.9a Pratt & Whitney F100-PW-229 afterburning turbofan. (Courtesy of Pratt & Whitney.)**

Figures 1.9a and 1.9b show the Pratt & Whitney F100 turbofan and the General Electric F110 turbofan, respectively. These afterburning turbofan engines are used in the F15 Eagle and F16 Falcon supersonic fighter aircraft. In this turbofan, the bypass stream is mixed with the core stream before passing through a common afterburner and exhaust nozzle.

### *1.4.4 Turboprop and Turboshaft*

A gas generator that drives a propeller is a turboprop engine. The expansion of gas through the turbine supplies the energy required to turn the propeller. A schematic diagram of the turboprop is shown in Fig. 1.10a. The turboshaft engine is similar to the turboprop except that power is supplied to a shaft rather than a propeller. The turboshaft engine is used quite extensively for supplying power for helicopters. The turboprop engine may find application in vertical takeoff and landing (VTOL) transporters. The limitations and advantages of the turboprop are those of the propeller. For low-speed flight and short-field takeoff, the propeller has a performance advantage. At speeds approaching the speed of sound, compressibility effects set in and the propeller loses its aerodynamic efficiency. Because of the rotation of the propeller, the propeller tip will approach the speed of sound before the vehicle approaches the speed of sound. This compressibility effect when one approaches the speed of sound limits the design of helicopter rotors and propellers. At high subsonic speeds,

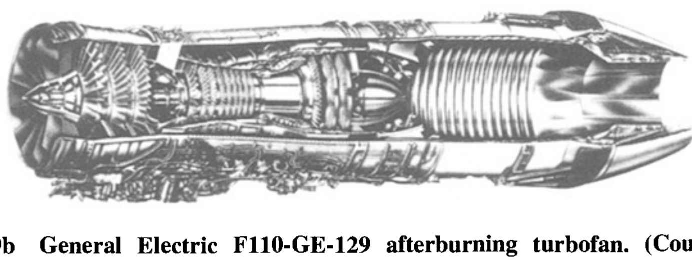

**Fig. 1.9b General Electric F110-GE-129 afterburning turbofan. (Courtesy of General Electric Aircraft Engines.)**

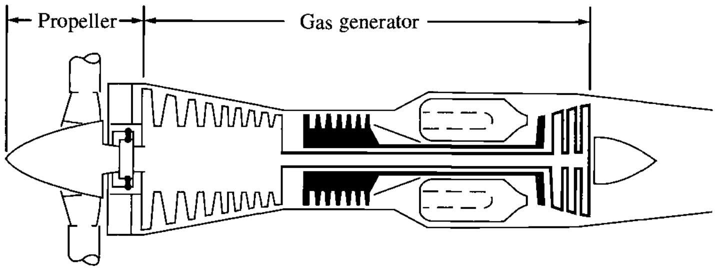


**Fig. 1.10a Schematic diagram of a turboprop.**

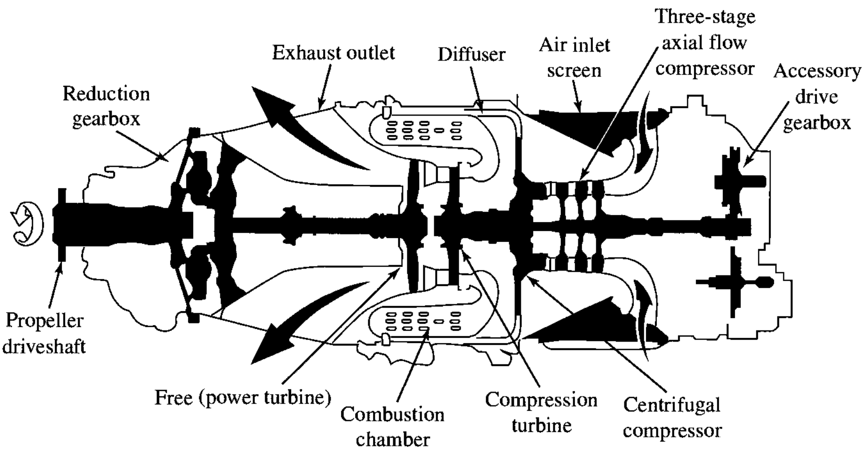


**Fig. 1.10b Canadian Pratt & Whitney PT6 turboshaft. (Courtesy of Pratt & Whitney of Canada.)**

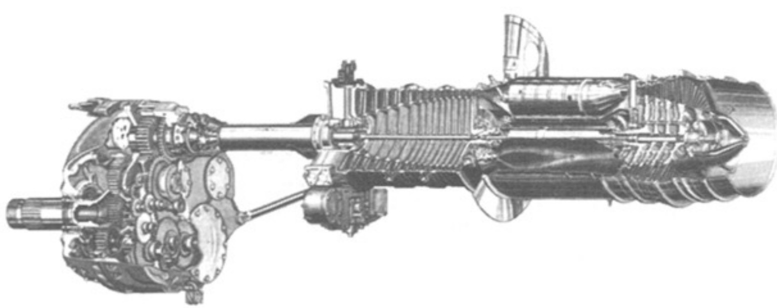

**Fig. 1.10c Allison T56 turboshaft. (Courtesy of Allison Gas Turbine Division.)**

the turbofan engine will have a better aerodynamic performance than the turboprop since the turbofan is essentially a *ducted turboprop.* Putting a duct or shroud around a propeller increases its aerodynamic performance. Examples of a turboshaft engine are the Canadian Pratt & Whitney PT6 (Fig. 1.10b), used in many small commuter aircraft, and the Allison T56 (Fig. 1.10c), used to power the C-130 Hercules and the P-3 Orion.

### 1.4.5 Ramjet

The ramjet engine consists of an inlet, a combustion zone, and a nozzle. A schematic diagram of a ramjet is shown in Fig. 1.11. The ramjet does not have the compressor and turbine as the turbojet does. Air enters the inlet where it is compressed and then enters the combustion zone where it is mixed with the fuel and burned. The hot gases are then expelled through the nozzle, developing thrust. The operation of the ramjet depends on the inlet to decelerate the incoming air to raise the pressure in the combustion zone. The pressure rise makes it possible for the ramjet to operate. The higher the velocity of the incoming air, the greater the pressure rise. It is for this reason that the ramjet operates best at high supersonic velocities. At subsonic velocities, the ramjet is inefficient, and to start the ramjet, air at a relatively higher velocity must enter the inlet.

The combustion process in an ordinary ramjet takes place at low subsonic velocities. At high supersonic flight velocities, a very large pressure rise is developed that is more than sufficient to support operation of the ramjet. Also, if the inlet has to decelerate a supersonic high-velocity airstream to a subsonic velocity, large pressure losses can result. The deceleration process also produces a temperature rise, and at some limiting flight speed, the temperature will approach the limit set by the wall materials and cooling methods. Thus when the temperature increase due to deceleration reaches the limit, it may not be possible to burn fuel in the airstream.

In the past few years, research and development have been done on a ramjet that has the combustion process taking place at supersonic velocities. By using a supersonic combustion process, the temperature rise and pressure loss due to deceleration in the inlet can be reduced. This ramjet with supersonic combustion is known as the *scramjet* (supersonic combustion ramjet). Figure 1.12a shows the schematic of a scramjet engine similar to that proposed for the National

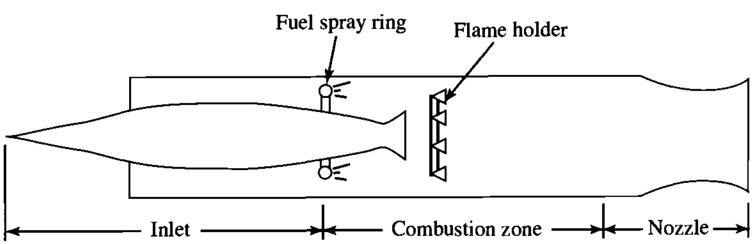


**Fig. 1.11 Schematic diagram of a ramjet.**

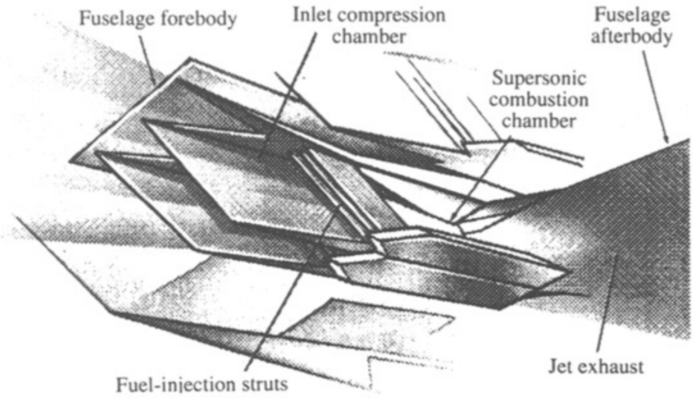


**Fig. 1.12a Schematic diagram of a scramjet.**

AeroSpace Plane (NASP) research vehicle, the X-30 shown in Fig. 1.12b. Further development of the scramjet for other applications (e.g., the Orient Express) will continue if research and development produces a scramjet engine with sufficient performance gains. Remember that since it takes a relative velocity to start the ramjet or scramjet, another engine system is required to accelerate aircraft like the X-30 to ramjet velocities.

### 1.4.6 Turbojet/Ramjet Combined-Cycle Engine

Two of the Pratt & Whitney J58 turbojet engines (see Fig. 1.13a) are used to power the Lockheed SR71 Blackbird (see Fig. 1.13b). This was the fastest aircraft

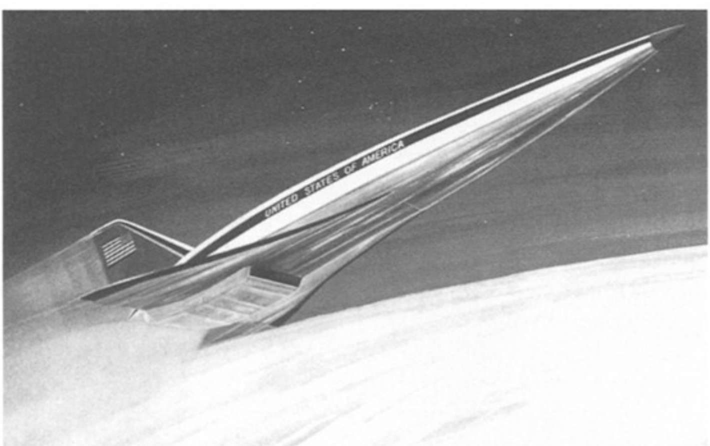


**Fig. 1.12b Conceptual drawing of the X-30. (Courtesy of Pratt & Whitney.)**

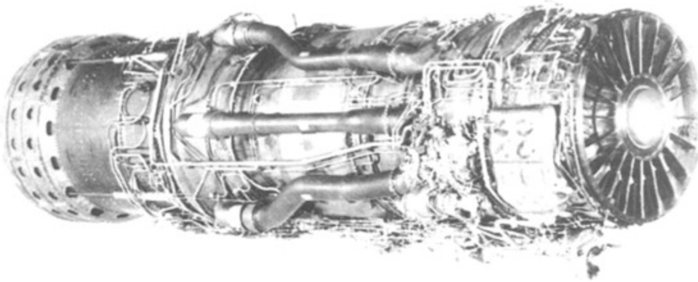

**Fig. 1.13a Pratt & Whitney J58 turbojet. (Courtesy of Pratt & Whitney.)**

(Mach 3+) when it was retired in 1989. The J58 operates as an afterburning turbojet engine until it reaches high Mach level, at which point the six large tubes (Fig. 1.13a) bypass flow to the afterburner. When these tubes are in use, the compressor, burner, and turbine of the turbojet are essentially bypassed, and the engine operates as a ramjet with the afterburner acting as the ramjet's burner.

### *1.4.7 Aircraft Engine Performance Parameters*

This section presents several of the airbreathing engine performance parameters that are useful in aircraft propulsion. The first performance parameter is the thrust of the engine that is available for sustained flight (thrust = drag), accelerated flight (thrust > drag), or deceleration (thrust < drag).

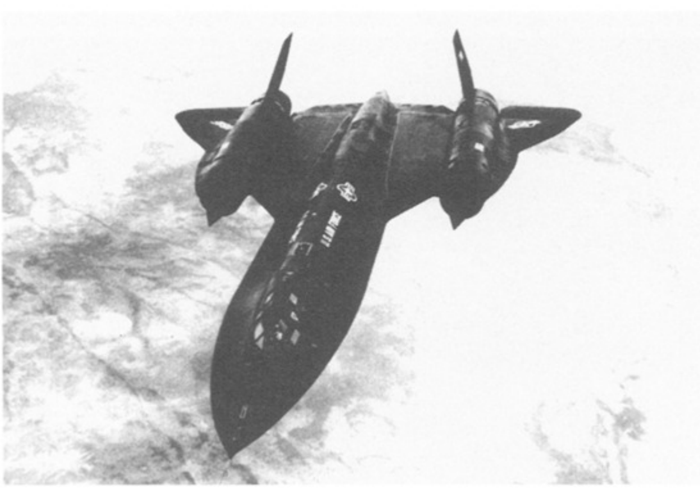

**Fig. 1.13b Lockheed SR71 Blackbird. (Courtesy of Lockheed.)**

As derived in Chapter 4, the uninstalled thrust $F$ of a jet engine (single inlet and single exhaust) is given by

$$F = \frac{(\dot{m}_0 + \dot{m}_f)V_e - \dot{m}_0 V_0}{g_c} + (P_e - P_0)A_e \tag{1.5}$$

where

$\dot{m}_0$, $\dot{m}_f$ = mass flow rates of air and fuel, respectively
$V_0$, $V_e$ = velocities at inlet and exit, respectively
$P_0$, $P_e$ = pressures at inlet and exit, respectively

It is most desirable to expand the exhaust gas to the ambient pressure, which gives $P_e = P_0$. In this case, the uninstalled thrust equation becomes

$$F = \frac{(\dot{m}_0 + \dot{m}_f)V_e - \dot{m}_0 V_0}{g_c} \qquad \text{for } P_e = P_0 \tag{1.6}$$

The installed thrust $T$ is equal to the uninstalled thrust $F$ minus the inlet drag $D_{\text{inlet}}$ and minus the nozzle drag $D_{\text{noz}}$, or

$$T = F - D_{\text{inlet}} - D_{\text{noz}} \tag{1.7}$$

Dividing the inlet drag $D_{\text{inlet}}$ and nozzle drag $D_{\text{noz}}$ by the uninstalled thrust $F$ yields the dimensionless inlet loss coefficient $\phi_{\text{inlet}}$ and nozzle loss coefficient $\phi_{\text{noz}}$, or

$$\begin{aligned} \phi_{\text{inlet}} &= \frac{D_{\text{inlet}}}{F} \\ \phi_{\text{noz}} &= \frac{D_{\text{noz}}}{F} \end{aligned} \tag{1.8}$$

Thus the relationship between the installed thrust $T$ and uninstalled thrust $F$ is simply

$$T = F(1 - \phi_{\text{inlet}} - \phi_{\text{noz}}) \tag{1.9}$$

The second performance parameter is the thrust specific fuel consumption ($S$ and TSFC). This is the rate of fuel use by the propulsion system per unit of thrust produced. The uninstalled fuel consumption $S$ and installed fuel consumption TSFC are written in equation form as

$$S = \frac{\dot{m}_f}{F} \tag{1.10}$$

$$\text{TSFC} = \frac{\dot{m}_f}{T} \tag{1.11}$$

where

$F$ = uninstalled thrust
$S$ = uninstalled thrust specific fuel consumption

$T$ = installed engine thrust
TSFC = installed thrust specific fuel consumption
$\dot{m}_f$ = mass flow rate of fuel

The relation between $S$ and TSFC in equation form is given by

$$S = \text{TSFC}(1 - \phi_{\text{inlet}} - \phi_{\text{noz}}) \tag{1.12}$$

Values of thrust $F$ and fuel consumption $S$ for various jet engines at sea-level static conditions are listed in Appendix B. The predicted variations of uninstalled engine thrust $F$ and uninstalled thrust specific fuel consumption $S$ with Mach number and altitude for an advanced fighter engine[3] are plotted in Figs. 1.14a–1.14d. Note that the thrust $F$ decreases with altitude and the fuel consumption $S$ also decreases with altitude until 36 kft (the start of the isothermal layer of the atmosphere). Also note that the fuel consumption increases with Mach number and that the thrust varies considerably with the Mach number. The predicted partial-throttle performance of the advanced fighter engine is shown at three flight conditions in Fig. 1.14e.

The takeoff thrust of the JT9D high-bypass-ratio turbofan engine is given in Fig. 1.15a vs Mach number and ambient air temperature for two versions. Note the rapid falloff of thrust with rising Mach number that is characteristic of this engine cycle and the constant thrust at a Mach number for temperatures of 86°F and below (this is often referred to as a *flat rating*). The partial-throttle performance of both engine versions is given in Fig. 1.15b for two combinations of altitude and Mach number.

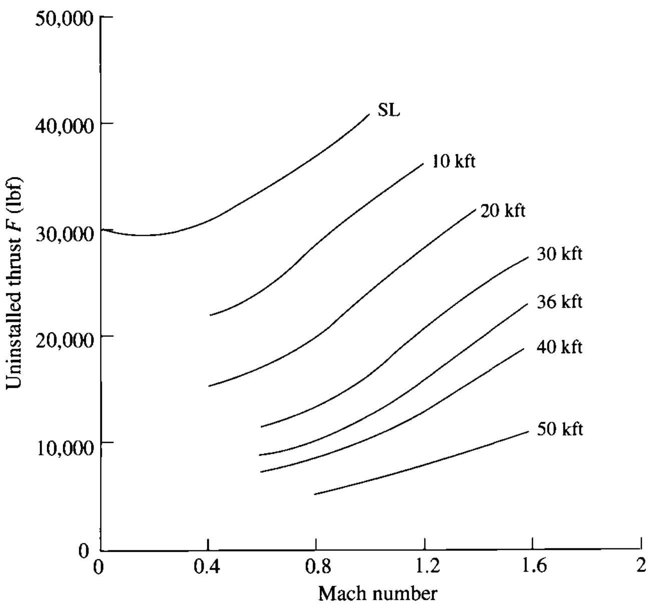


**Fig. 1.14a Uninstalled thrust F of an advanced afterburning fighter engine at maximum power setting, afterburner on. (Extracted from Ref. 3.)**

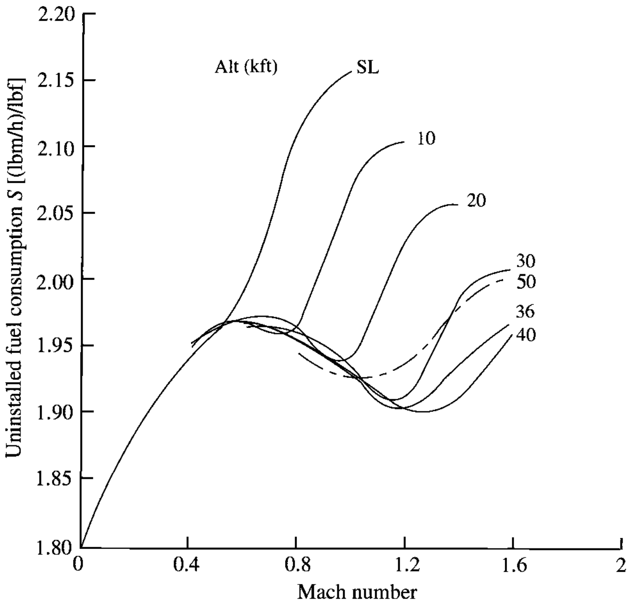


**Fig. 1.14b Uninstalled fuel consumption $S$ of an advanced afterburning fighter engine at maximum power setting, afterburner on. (Extracted from Ref. 3.)**

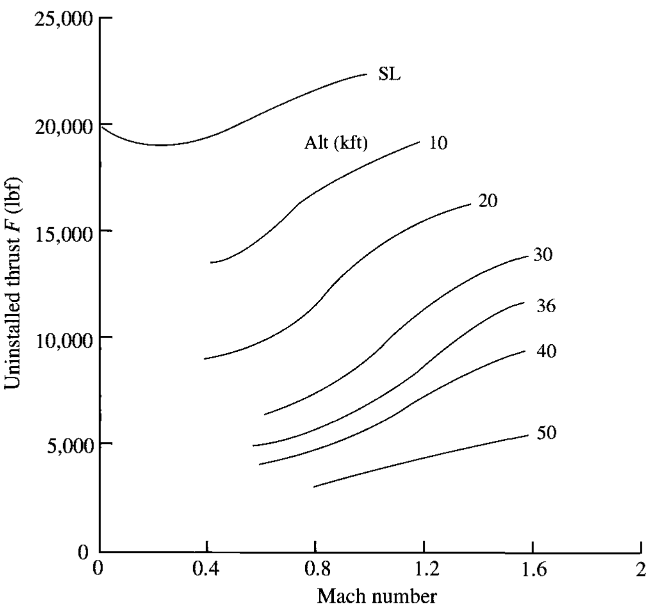


**Fig. 1.14c Uninstalled thrust $F$ of an advanced afterburning fighter engine at military power setting, afterburner off. (Extracted from Ref. 3.)**

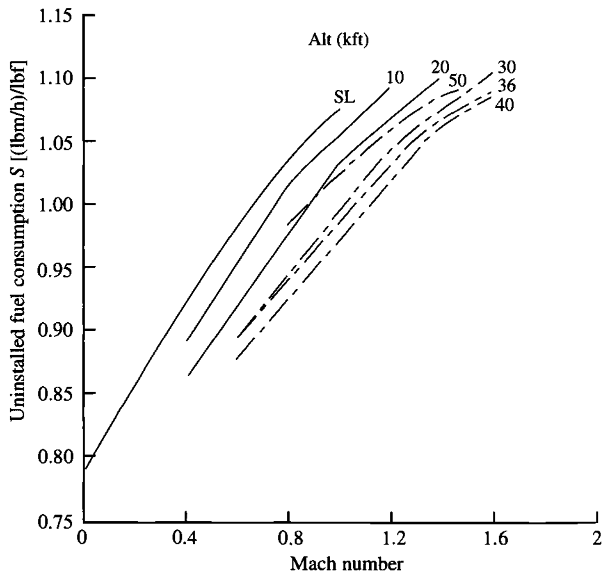


**Fig. 1.14d Uninstalled fuel consumption $S$ of an advanced afterburning fighter engine at military power setting, afterburner off. (Extracted from Ref. 3.)**

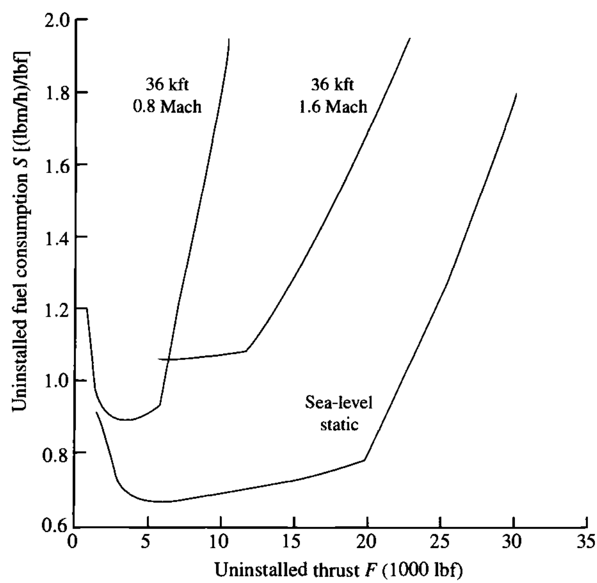


**Fig. 1.14e Partial-throttle performance of an advanced fighter engine. (Extracted from Ref. 3.)**

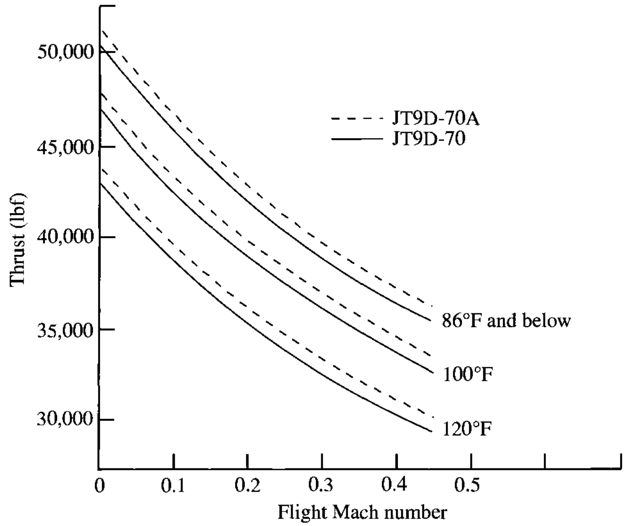


**Fig. 1.15a JT9D-70/-70A turbofan takeoff thrust. (Courtesy of Pratt & Whitney.)**

Although the aircraft gas turbine engine is a very complex machine, the basic tools for modeling its performance are developed in the following chapters. These tools are based on the work of Gordon Oates.[4] They permit performance calculations for existing and proposed engines and generate performance curves similar to Figs. 1.14a–1.14e and Figs. 1.15a and 1.15b.

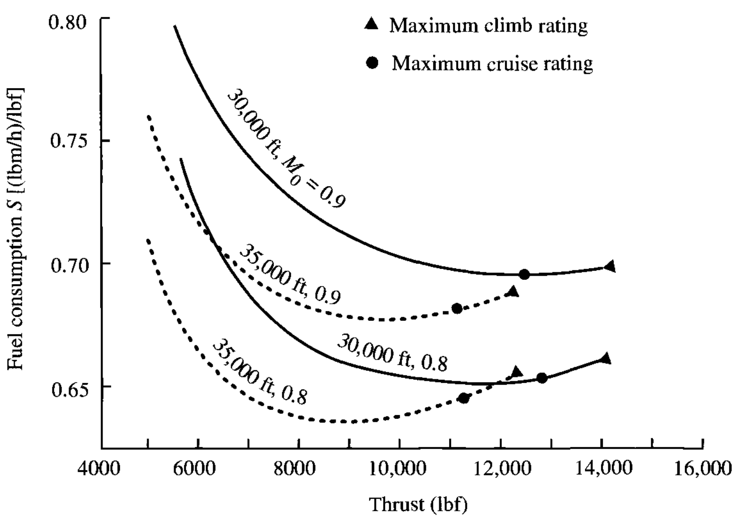


**Fig. 1.15b JT9D-70/-70A turbofan cruise-specific fuel consumption. (Courtesy of Pratt & Whitney.)**

**Table 1.3 Typical aircraft engine thrust installation losses**

| Flight condition: | $M < 1$ | | $M > 1$ | |
|---|---|---|---|---|
| Aircraft type | $\phi_{\text{inlet}}$ | $\phi_{\text{noz}}$ | $\phi_{\text{inlet}}$ | $\phi_{\text{noz}}$ |
| Fighter | 0.05 | 0.01 | 0.05 | 0.03 |
| Passenger/cargo | 0.02 | 0.01 | — | — |
| Bomber | 0.03 | 0.01 | 0.04 | 0.02 |

The value of the installation loss coefficient depends on the characteristics of the particular engine/airframe combination, the Mach number, and the engine throttle setting. Typical values are given in Table 1.3 for guidance.

The thermal efficiency $\eta_T$ of an engine is another very useful engine performance parameter. *Thermal efficiency* is defined as the net rate of organized energy (shaft power or kinetic energy) out of the engine divided by the rate of thermal energy available from the fuel in the engine. The fuel's available thermal energy is equal to the mass flow rate of the fuel $\dot{m}_f$ times the fuel lower-heating value $h_{PR}$. Thermal efficiency can be written in equation form as

$$\eta_T = \frac{\dot{W}_{\text{out}}}{\dot{Q}_{\text{in}}} \tag{1.13}$$

where

$\eta_T$ = thermal efficiency of engine
$\dot{W}_{\text{out}}$ = net power out of engine
$\dot{Q}_{\text{in}}$ = rate of thermal energy released ($\dot{m}_f h_{PR}$)

Note that for engines with shaft power output, $\dot{W}_{\text{out}}$ is equal to this shaft power. For engines with no shaft power output (e.g., turbojet engine), $\dot{W}_{\text{out}}$ is equal to the net rate of change of the kinetic energy of the fluid through the engine. The power out of a jet engine with a single inlet and single exhaust (e.g., turbojet engine) is given by

$$\dot{W}_{\text{out}} = \frac{1}{2g_c}[(\dot{m}_0 + \dot{m}_f)V_e^2 - \dot{m}_0 V_0^2]$$

The propulsive efficiency $\eta_P$ of a propulsion system is a measure of how effectively the engine power $\dot{W}_{\text{out}}$ is used to power the aircraft. *Propulsive efficiency* is the ratio of the aircraft power (thrust times velocity) to the power out of the engine $\dot{W}_{\text{out}}$. In equation form, this is written as

$$\eta_P = \frac{TV_0}{\dot{W}_{\text{out}}} \tag{1.14}$$

where

$\eta_P$ = propulsive efficiency of engine
$T$ = thrust of propulsion system

$V_0$ = velocity of aircraft
$\dot{W}_{out}$ = net power out of engine

For a jet engine with a single inlet and single exhaust and an exit pressure equal to the ambient pressure, the propulsive efficiency is given by

$$\eta_P = \frac{2(1 - \phi_{\text{inlet}} - \phi_{\text{noz}})[(\dot{m}_0 + \dot{m}_f)V_e - \dot{m}_0 V_0]V_0}{(\dot{m}_0 + \dot{m}_f)V_e^2 - \dot{m}_0 V_0^2} \tag{1.15}$$

For the case when the mass flow rate of the fuel is much less than that of air and the installation losses are very small, Eq. (1.15) simplifies to the following equation for the propulsive efficiency:

$$\eta_P = \frac{2}{V_e/V_0 + 1} \tag{1.16}$$

Equation (1.16) is plotted vs the velocity ratio $V_e/V_0$ in Fig. 1.16 and shows that high propulsive efficiency requires the exit velocity to be approximately equal to the inlet velocity. Turbojet engines have high values of the velocity ratio $V_e/V_0$ with corresponding low propulsive efficiency, whereas turbofan engines have low values of the velocity ratio $V_e/V_0$ with corresponding high propulsive efficiency.

The thermal and propulsive efficiencies can be combined to give the *overall efficiency* $\eta_O$ of a propulsion system. Multiplying propulsive efficiency by

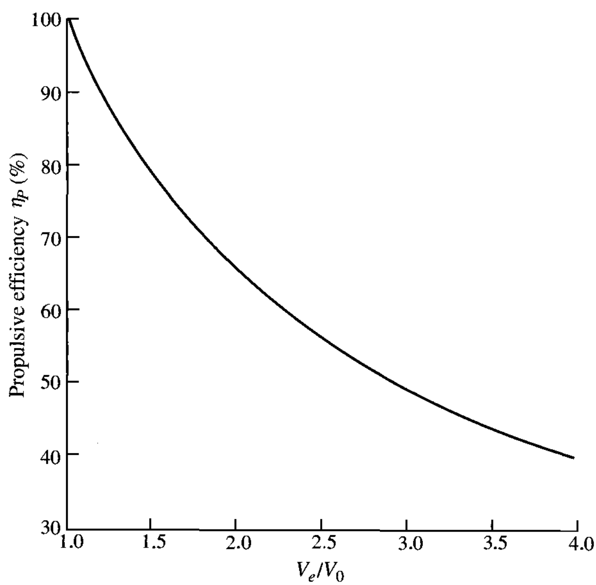


**Fig. 1.16 Propulsive efficiency vs velocity ratio ($V_e/V_0$).**

thermal efficiency, we get the ratio of the aircraft power to the rate of thermal energy released in the engine (the overall efficiency of the propulsion system):

$$\eta_O = \eta_P \eta_T \tag{1.17}$$

$$\eta_O = \frac{TV_0}{\dot{Q}_{\text{in}}} \tag{1.18}$$

Several of the preceding performance parameters are plotted for general types of gas turbine engines in Figs. 1.17a, 1.17b, and 1.17c. These plots can be used to obtain the general trends of these performance parameters with flight velocity for each propulsion system.

Since $\dot{Q}_{\text{in}} = \dot{m}_f h_{PR}$, Eq. (1.18) can be rewritten as

$$\eta_O = \frac{TV_0}{\dot{m}_f h_{PR}}$$

With the help of Eq. (1.11), this equation can be written in terms of the thrust specific fuel consumption as

$$\eta_O = \frac{V_0}{\text{TSFC} \cdot h_{PR}} \tag{1.19}$$

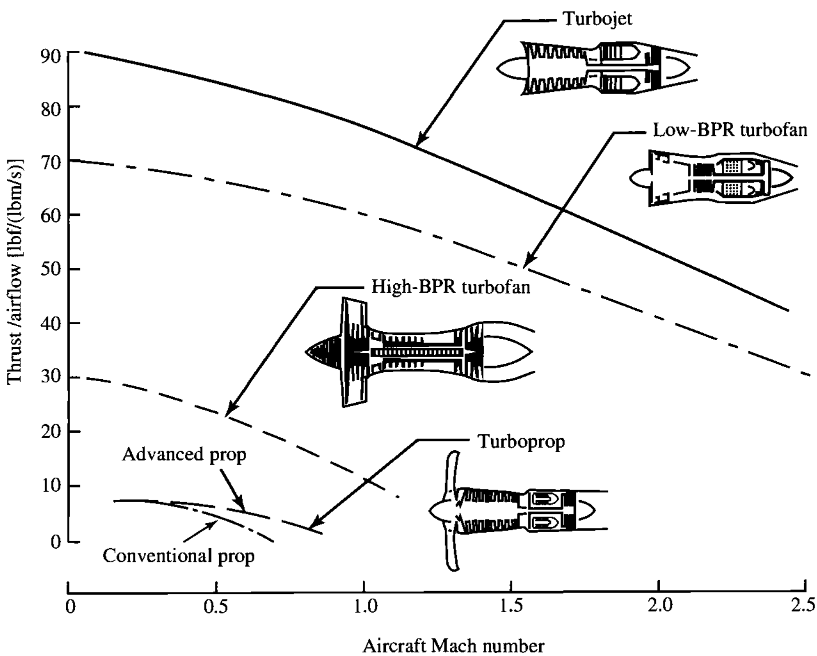


**Fig. 1.17a Specific thrust characteristics of typical aircraft engines. (Courtesy of Pratt & Whitney.)**

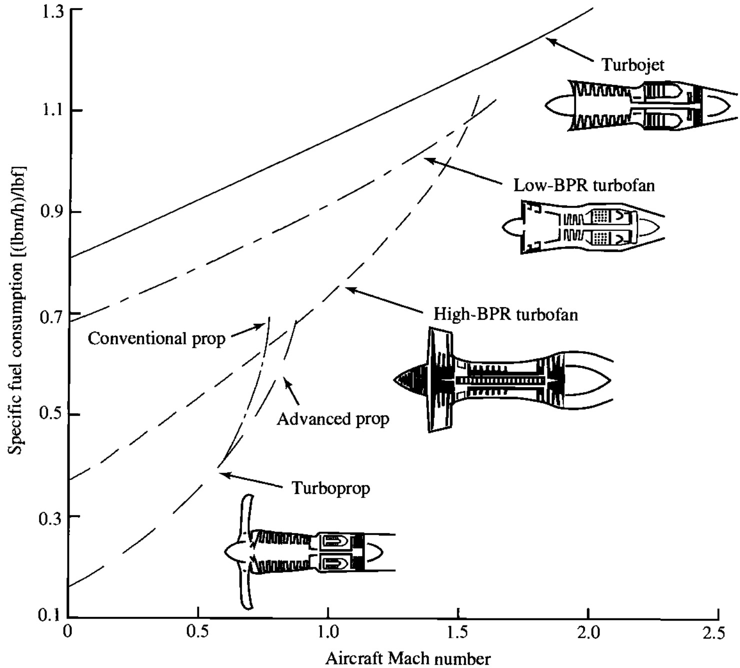


**Fig. 1.17b Thrust-specific fuel consumption characteristics of typical aircraft engines. (Courtesy of Pratt & Whitney.)**

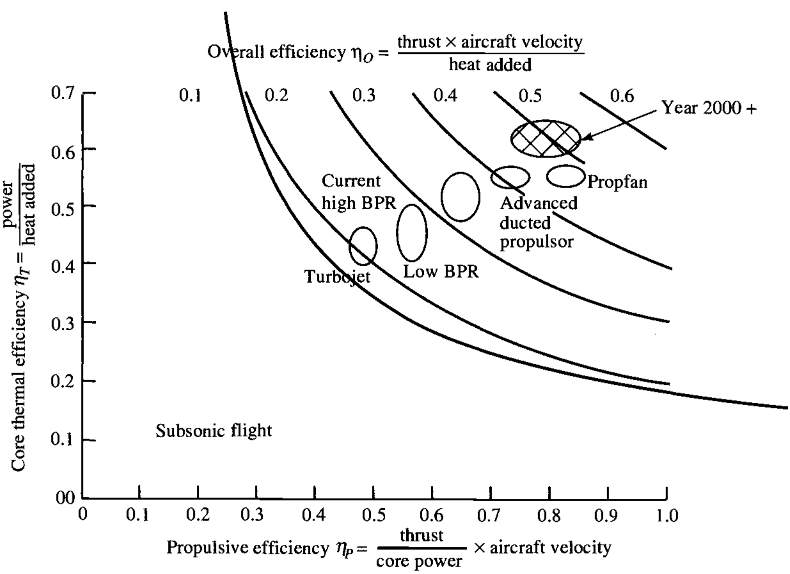


**Fig. 1.17c Efficiency characteristics of typical aircraft engines. (Courtesy of Pratt & Whitney.)**

Using Eqs. (1.17) and (1.19), we can write the following for TSFC:

$$\text{TSFC} = \frac{V_0}{\eta_P \eta_T h_{PR}} \tag{1.20}$$

---

### Example 1.1

An advanced fighter engine operating at Mach 0.8 and 10-km altitude has the following uninstalled performance data and uses a fuel with $h_{PR} = 42{,}800$ kJ/kg:

$$F = 50 \text{ kN} \quad \dot{m}_0 = 45 \text{ kg/s} \quad \text{and} \quad \dot{m}_f = 2.65 \text{ kg/s}$$

Determine the specific thrust, thrust specific fuel consumption, exit velocity, thermal efficiency, propulsive efficiency, and overall efficiency (assume exit pressure equal to ambient pressure).

*Solution*:

$$\frac{F}{\dot{m}_0} = \frac{50\,\text{kN}}{45\,\text{kg/s}} = 1.1111 \text{ kN/(kg/s)} = 1111.1 \text{ m/s}$$

$$S = \frac{\dot{m}_f}{F} = \frac{2.65\,\text{kg/s}}{50\,\text{kN}} = 0.053\,\text{(kg/s)/kN} = 53\,\text{mg/N}\cdot\text{s}$$

$$V_0 = M_0 a_0 = M_0\left(\frac{a_0}{a_{\text{ref}}}\right)a_{\text{ref}} = 0.8(0.8802)340.3 = 239.6 \text{ m/s}$$

From Eq. (1.6) we have

$$V_e = \frac{F g_c + \dot{m}_0 V_0}{\dot{m}_0 + \dot{m}_f} = \frac{50{,}000 \times 1 + 45 \times 239.6}{45 + 2.65} = 1275.6 \text{ m/s}$$

$$\eta_T = \frac{\dot{W}_{\text{out}}}{\dot{Q}_{\text{in}}} = \frac{(\dot{m}_0 + \dot{m}_f)V_e^2 - \dot{m}_0 V_0^2}{2 g_c \dot{m}_f h_{PR}}$$

$$\dot{W}_{\text{out}} = \frac{(\dot{m}_0 + \dot{m}_f)V_e^2 - \dot{m}_0 V_0^2}{2 g_c}$$

$$= \frac{47.65 \times 1275.6^2 - 45 \times 239.6^2}{2 \times 1} = 37.475 \times 10^6 \text{ W}$$

$$\dot{Q}_{\text{in}} = \dot{m}_f h_{PR} = 2.65 \times 42{,}800 = 113.42 \times 10^6 \text{ W}$$

$$\eta_T = \frac{\dot{W}_{\text{out}}}{\dot{Q}_{\text{in}}} = \frac{37.475 \times 10^6}{113.42 \times 10^6} = 33.04\%$$

$$\eta_P = \frac{F V_0}{\dot{W}_{\text{out}}} = \frac{50{,}000 \times 239.6}{37.475 \times 10^6} = 31.97\%$$

$$\eta_O = \frac{F V_0}{\dot{Q}_{\text{in}}} = \frac{50{,}000 \times 239.6}{113.42 \times 10^6} = 10.56\%$$

---

### 1.4.8 Specific Thrust vs Fuel Consumption

For a jet engine with a single inlet and single exhaust and exit pressure equal to ambient pressure, when the mass flow rate of the fuel is much less than that of air and the installation losses are very small, the specific thrust $F/\dot{m}_0$ can be written as

$$\frac{F}{\dot{m}_0} = \frac{V_e - V_0}{g_c} \tag{1.21}$$

Then the propulsive efficiency of Eq. (1.16) can be rewritten as

$$\eta_P = \frac{2}{Fg_c/(\dot{m}_0 V_0) + 2} \tag{1.22}$$

Substituting Eq. (1.22) into Eq. (1.20) and noting that TSFC $= S$, we obtain the following very enlightening expression:

$$S = \frac{Fg_c/\dot{m}_0 + 2V_0}{2\eta_T h_{PR}} \tag{1.23}$$

Aircraft manufacturers desire engines having low thrust specific fuel consumption $S$ and high specific thrust $F/\dot{m}_0$. Low engine fuel consumption can be directly translated into longer range, increased payload, and/or reduced aircraft size. High specific thrust reduces the cross-sectional area of the engine and has a direct influence on engine weight and installation losses. This desired trend is plotted in Fig. 1.18. Equation (1.23) is also plotted in Fig. 1.18 and shows that fuel consumption and specific thrust are directly proportional. Thus the aircraft manufacturers have to make a tradeoff. The line of Eq. (1.23) shifts in the desired direction when there is an increase in the level of technology (increased thermal efficiency) or an increase in the fuel heating value.

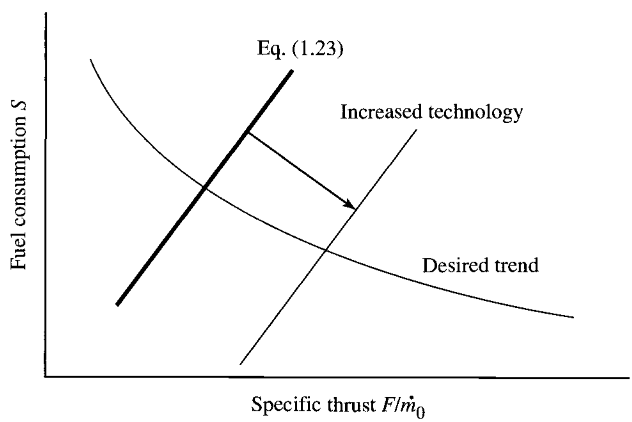


**Fig. 1.18 Relationship between specific thrust and fuel consumption.**

Another very useful measure of merit for the aircraft gas turbine engine is the thrust/weight ratio $F/W$. For a given engine thrust $F$, increasing the thrust/weight ratio reduces the weight of the engine. Aircraft manufacturers can use this reduction in engine weight to increase the capabilites of an aircraft (increased payload, increased fuel, or both) or decrease the size (weight) and cost of a new aircraft under development.

Engine companies expend considerable research and development effort on increasing the thrust/weight ratio of aircraft gas turbine engines. This ratio is equal to the specific thrust $F/\dot{m}_0$ divided by the engine weight per unit of mass flow $W/\dot{m}_0$. For a given engine type, the engine weight per unit mass flow is related to the efficiency of the engine structure, and the specific thrust is related to the engine thermodynamics. The weights per unit mass flow of some existing gas turbine engines are plotted vs specific thrust in Fig. 1.19. Also plotted are lines of constant engine thrust/weight ratio $F/W$.

The engine companies, in conjunction with the U.S. Department of Defense and NASA, are involved in a large research and development effort to increase the engine thrust/weight ratio $F/W$ and decrease the fuel consumption while maintaining engine durability, maintainability, etc. An earlier program was called the *integrated high-performance turbine engine technology* (IHPTET) *initiative* (see Refs. 5 and 6).

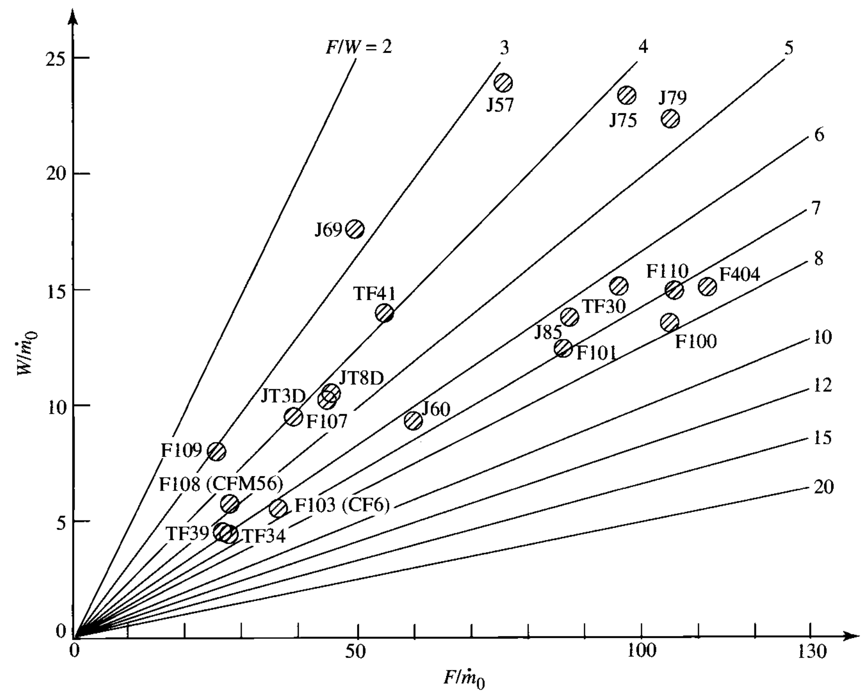


**Fig. 1.19 Engine thrust/weight ratio $F/W$.**

## 1.5 Aircraft Performance

This section on aircraft performance is included so that the reader may get a better understanding of the propulsion requirements of the aircraft.[7] The coverage is limited to a few significant concepts that directly relate to aircraft engines. It is not intended as a substitute for the many excellent references on this subject (see Refs. 8–11).

### 1.5.1 Performance Equation

Relationships for the performance of an aircraft can be obtained from energy considerations (see Ref. 12). By treating the aircraft (Fig. 1.20) as a moving mass and assuming that the installed propulsive thrust $T$, aerodynamic drag $D$, and other resistive forces $R$ act in the same direction as the velocity $V$, it follows that

$$\underbrace{[T-(D+R)]V}_{\substack{\text{rate of}\\ \text{mechanical}\\ \text{energy}\\ \text{input}}} = \underbrace{W\frac{\mathrm{d}h}{\mathrm{d}t}}_{\substack{\text{storage}\\ \text{rate of}\\ \text{potential}\\ \text{energy}}} + \underbrace{\frac{W}{g}\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{V^2}{2}\right)}_{\substack{\text{storage}\\ \text{rate of}\\ \text{kinetic}\\ \text{energy}}} \tag{1.24}$$

Note that the total resistive force $D+R$ is the sum of the drag of the clean aircraft $D$ and any additional drags $R$ associated with such proturberances as landing gear, external stores, or drag chutes.

By defining the energy height $z_e$ as the sum of the potential and kinetic energy terms

$$z_e \equiv h + \frac{V^2}{2g} \tag{1.25}$$

Eq. (1.24) can now be written simply as

$$[T-(D+R)]V = W\frac{\mathrm{d}z_e}{\mathrm{d}t} \tag{1.26}$$

By defining the *weight specific excess power* $P_s$ as

$$P_s \equiv \frac{\mathrm{d}z_e}{\mathrm{d}t} \tag{1.27}$$

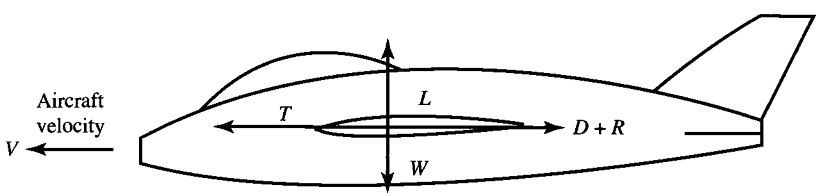


**Fig. 1.20 Forces on aircraft.**

Eq. (1.26) can now be written in its dimensionless form as

$$\frac{T-(D+R)}{W}=\frac{P_s}{V}=\frac{1}{V}\frac{\mathrm{d}}{\mathrm{d}t}\left(h+\frac{V^2}{2g}\right) \tag{1.28}$$

This is a very powerful equation that gives insight into the dynamics of flight, including both the rate of climb $\mathrm{d}h/\mathrm{d}t$ and acceleration $\mathrm{d}V/\mathrm{d}t$.

### *1.5.2 Lift and Drag*

We use the classical aircraft lift relationship

$$L=nW=C_L q S_w \tag{1.29}$$

where $n$ is the load factor or number of $g$ perpendicular to $V$ ($n=1$ for straight and level flight), $C_L$ is the coefficient of lift, $S_w$ is the wing planform area, and $q$ is the dynamic pressure. The dynamic pressure can be expressed in terms of the density $\rho$ and velocity $V$ or the pressure $P$ and Mach number $M$ as

$$q=\frac{1}{2}\rho\frac{V^2}{g_c}=\frac{1}{2}\sigma\rho_{\mathrm{ref}}\frac{V^2}{g_c} \tag{1.30a}$$

or

$$q=\frac{\gamma}{2}PM_0^2=\frac{\gamma}{2}\delta P_{\mathrm{ref}}M_0^2 \tag{1.30b}$$

where $\delta$ and $\sigma$ are the dimensionless pressure and density ratios defined by Eqs. (1.2) and (1.4), respectively, and $\gamma$ is the ratio of specific heats ($\gamma=1.4$ for air). The reference density $\rho_{\mathrm{ref}}$ and reference pressure $P_{\mathrm{ref}}$ of air are their sea-level values on a standard day and are listed in Appendix A.

We also use the classical aircraft drag relationship

$$D=C_D q S_w \tag{1.31}$$

Figure 1.21 is a plot of lift coefficient $C_L$ vs drag coefficient $C_D$, commonly called the *lift-drag polar*, for a typical subsonic passenger aircraft. The drag coefficient curve can be approximated by a second-order equation in $C_L$ written as

$$C_D=K_1C_L^2+K_2C_L+C_{D0} \tag{1.32}$$

where the coefficients $K_1$, $K_2$, and $C_{D0}$ are typically functions of flight Mach number and wing configuration (flap position, etc.).

The $C_{D0}$ term in Eq. (1.32) is the zero lift drag coefficient that accounts for both frictional and pressure drag in subsonic flight and wave drag in supersonic flight. The $K_1$ and $K_2$ terms account for the drag due to lift. Normally $K_2$ is very small and approximately equal to zero for most fighter aircraft.

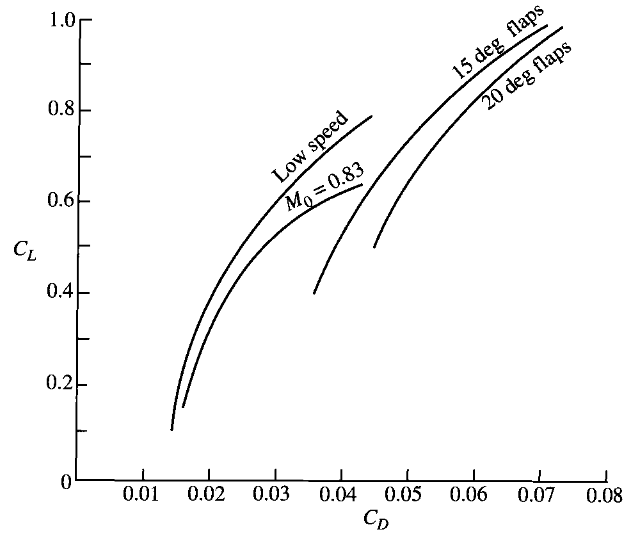


**Fig. 1.21 Typical lift-drag polar.**

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## Example 1.2

For all the examples given in this section on aircraft performance, two types of aircraft will be considered.

*a) Fighter aircraft (HF-1).* An advanced fighter aircraft is approximately modeled after the F-22 Advanced Tactical Fighter shown in Fig. 1.22. For convenience, we will designate our hypothetical fighter aircraft as the HF-1, having the following characteristics:

Maximum gross takeoff weight $W_{TO}$ = 40,000 lbf (177,920 N)
Empty weight = 24,000 lbf (106,752 N)
Maximum fuel plus payload weight = 16,000 lbf (71,168 N)
Permanent payload = 1600 lbf (7117 N, crew plus return armament)
Expended payload = 2000 lbf (8896 N, missiles plus ammunition)
Maximum fuel capacity = 12,400 lbf (55,155 N)
Wing area $S_w$ = 720 ft$^2$ (66.9 m$^2$)
Engine: low-bypass-ratio, mixed-flow turbofan with afterburner
Maximum lift coefficient $C_{L\max}$ = 1.8
Drag coefficients given in Table 1.4

*b) Passenger aircraft (HP-1).* An advanced 253-passenger commercial aircraft approximately modeled after the Boeing 787 is shown in Fig. 1.23. For convenience, we will designate our hypothetical passenger aircraft as the HP-1, having the following characteristics:

Maximum gross takeoff weight $W_{TO}$ = 1,645,760 N (370,000 lbf)
Empty weight = 822,880 N (185,500 lbf)

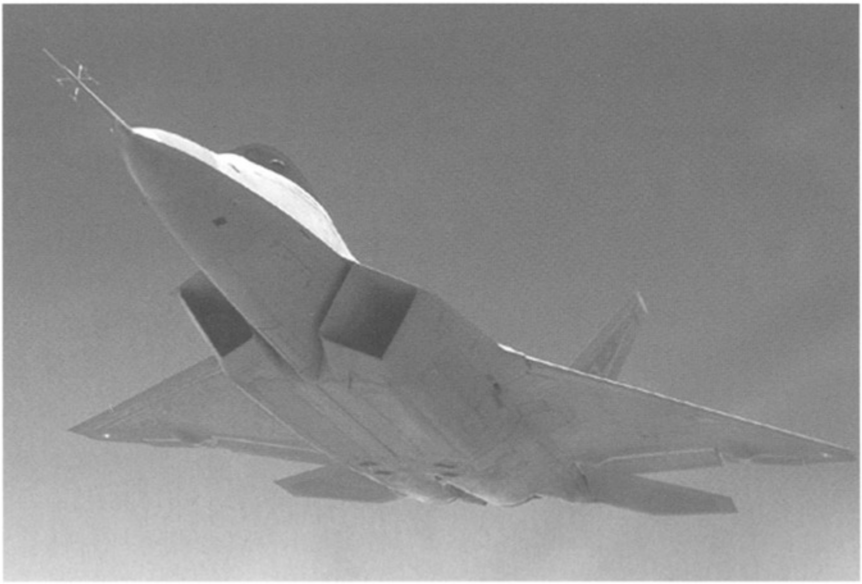

**Fig. 1.22 F-22, Advanced Tactical Fighter. (Photo courtesy of Boeing Defense & Space Group, Military Airplanes Division.)**

Maximum landing weight = 1,356,640 N (305,000 lbf)
Maximum payload = 420,780 N (94,600 lbf, 253 passengers plus 196,000 N of cargo)
Maximum fuel capacity = 716,706 N (161,130 lbf)
Wing area $S_w = 282.5\ \text{m}^2$ ($3040\ \text{ft}^2$)
Engine: high-bypass-ratio turbofan
Maximum lift coefficient $C_{L\max} = 2.0$
Drag coefficients given in Table 1.5.

**Table 1.4 Drag coefficients for hypothetical fighter aircraft (HF-1)**

| $M_0$ | $K_1$ | $K_2$ | $C_{D0}$ |
|---|---|---|---|
| 0.0 | 0.20 | 0.0 | 0.0120 |
| 0.8 | 0.20 | 0.0 | 0.0120 |
| 1.2 | 0.20 | 0.0 | 0.02267 |
| 1.4 | 0.25 | 0.0 | 0.0280 |
| 2.0 | 0.40 | 0.0 | 0.0270 |


**Fig. 1.23 Boeing 787. (Photo courtesy of Boeing.)**

**Table 1.5 Drag coefficients for hypothetical passenger aircraft (HP-1)**

| $M_0$ | $K_1$ | $K_2$ | $C_{D0}$ |
|---|---|---|---|
| 0.00 | 0.056 | −0.004 | 0.0140 |
| 0.40 | 0.056 | −0.004 | 0.0140 |
| 0.75 | 0.056 | −0.008 | 0.0140 |
| 0.83 | 0.056 | −0.008 | 0.0150 |

## Example 1.3

Determine the drag polar and drag variation for the HF-1 aircraft at 90% of maximum gross takeoff weight and the HP-1 aircraft at 95% of maximum gross takeoff weight.

*a) Fighter aircraft (HF-1).* The variation in $C_{D0}$ and $K_1$ with Mach number for the HF-1 are plotted in Fig. 1.24 from the data of Table 1.4. Figure 1.25 shows the drag polar at different Mach numbers for the HF-1 aircraft. Using these drag data and the preceding equations gives the variation in aircraft drag with subsonic Mach number and altitude for level flight ($n = 1$), as shown in Fig. 1.26a. Note that the minimum drag is constant for Mach numbers 0 to 0.8 and then increases. This is the same variation as $C_{D0}$. The variation of drag with load factor $n$ is shown in Fig. 1.26b at two altitudes. The drag increases with increasing

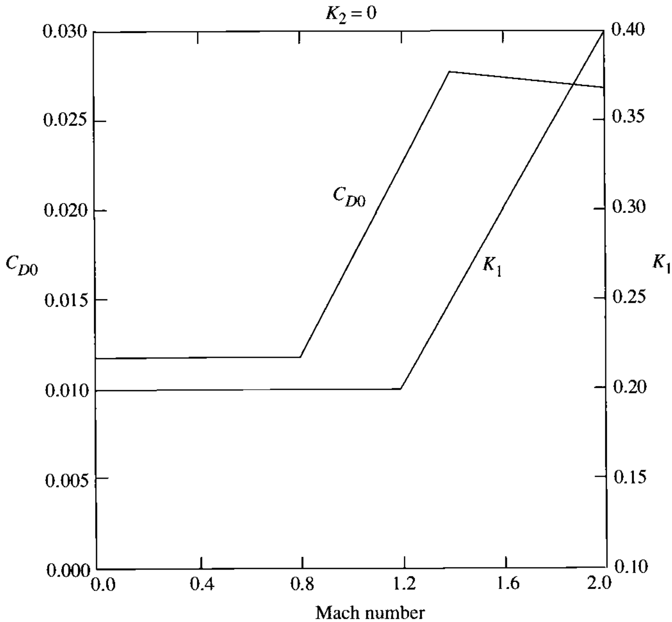


**Fig. 1.24 Values of $K_1$ and $C_{D0}$ for HF-1 aircraft.**

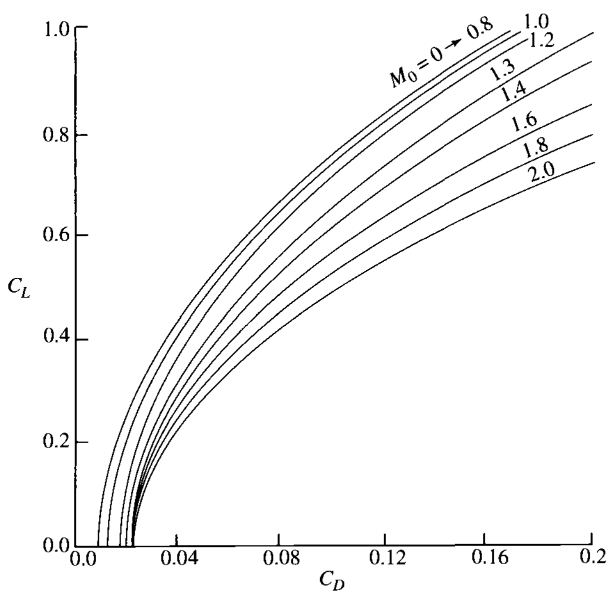


**Fig. 1.25 Lift-drag polar for HF-1 aircraft.**

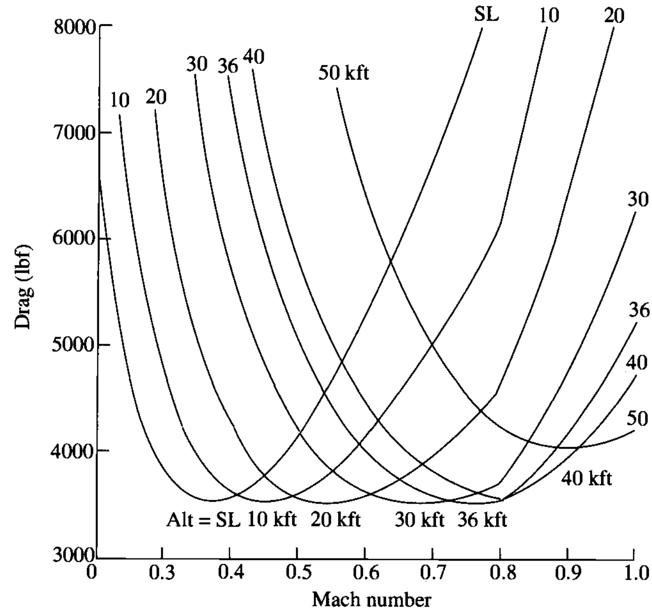


**Fig. 1.26a Drag for level flight ($n = 1$) for HF-1 aircraft.**

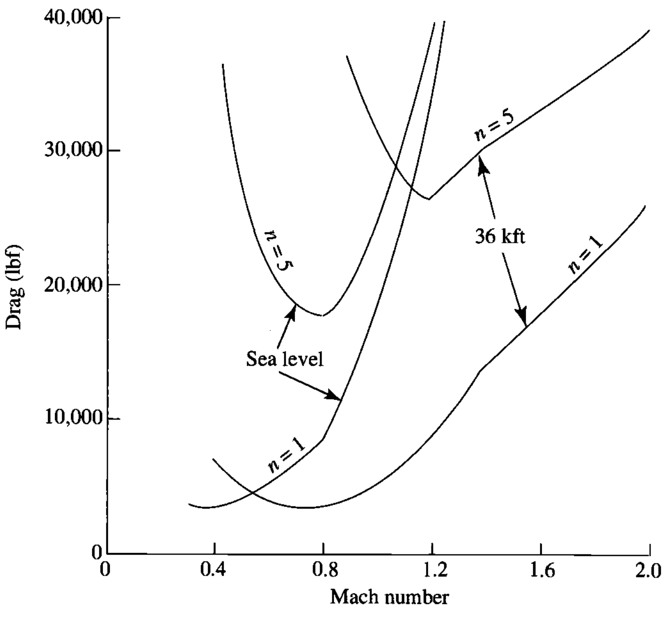


**Fig. 1.26b Drag of HF-1 aircraft at sea level and 36 kft for $n = 1$ and $n = 5$.**

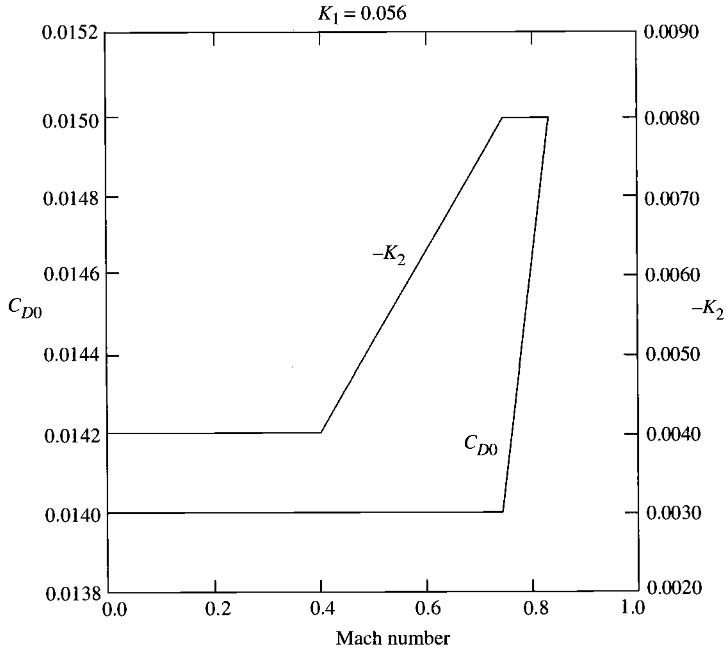


**Fig. 1.27 Values of $K_2$ and $C_{D0}$ for HP-1 aircraft.**

load factor, and there is a flight Mach number that gives minimum drag for a given altitude and load factor.

*b) Passenger aircraft (HP-1).* The variation in $C_{D0}$ and $K_2$ with Mach number for the HP-1 is plotted in Fig. 1.27 from the data of Table 1.5. Figure 1.28 shows the drag polar at different Mach numbers for the HP-1 aircraft. Using these drag data and the preceding equations gives the variation in aircraft drag with subsonic Mach number and altitude for level flight ($n = 1$), as shown in Fig. 1.29. Note that the minimum drag is constant for Mach numbers 0 to 0.75 and then increases. This is the same variation as $C_{D0}$.

## Example 1.4

Calculate the drag at Mach 0.8 and 40-kft altitude of the HF-1 aircraft at 90% of maximum gross takeoff weight with load factors of 1 and 4.

*Solution*: We begin by calculating the dynamic pressure $q$:

$$q = \frac{\gamma}{2}\delta P_{\text{ref}} M_0^2 = 0.7 \times 0.1858 \times 2116 \times 0.8^2 = 176.1 \text{ lbf/ ft}^2$$

From Fig. 1.24 at $M = 0.8$, $C_{D0}= 0.012$, $K_1 = 0.20$, and $K_2 = 0$.

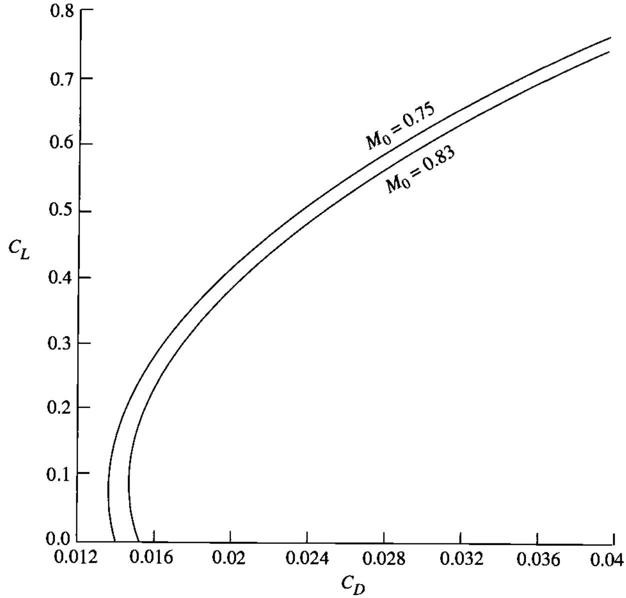


**Fig. 1.28 Lift-drag polar for HP-1 aircraft.**

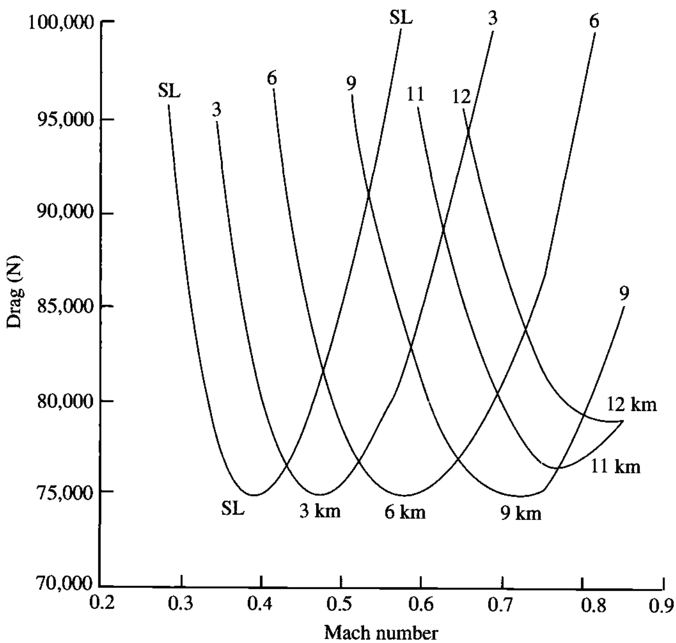


**Fig. 1.29 Drag for level flight ($n$ = 1) for HP-1 aircraft.**

*Case* 1: $n = 1$

$$C_L = \frac{nW}{qS_w} = \frac{1 \times 0.9 \times 40{,}000}{176.1 \times 720} = 0.2839$$

$$C_D = K_1 C_L^2 + K_2 C_L + C_{D0} = 0.2(0.2839^2) + 0.012 = 0.0281$$

$$D = C_D q S_w = 0.0281 \times 176.1 \times 720 = 3563 \text{ lbf}$$

*Case* 2: $n = 4$

$$C_L = \frac{nW}{qS_w} = \frac{4 \times 0.9 \times 40{,}000}{176.1 \times 720} = 1.136$$

$$C_D = K_1 C_L^2 + K_2 C_L + C_{D0} = 0.2(1.136^2) + 0.012 = 0.2701$$

$$D = C_D q S_w = 0.2701 \times 176.1 \times 720 = 34{,}247 \text{ lbf}$$

Note that the drag at $n = 4$ is about 10 times that at $n = 1$.

---

### 1.5.3 Stall, Takeoff, and Landing Speeds

*Stall* is the flight condition when an aircraft's wing loses lift. It is an undesirable condition since vehicle control is lost for a time. During level flight (lift = weight), stall will occur when one tries to obtain a lift coefficient greater than the wing's maximum $C_{L\max}$. The *stall speed* is defined as the level flight speed that corresponds to the wing's maximum lift coefficient, or

$$V_{\text{stall}} = \sqrt{\frac{2g_c}{\rho C_{L\max}} \frac{W}{S_w}} \tag{1.33}$$

To keep away from stall, aircraft are flown at velocities greater than $V_{\text{stall}}$.

Takeoff and landing are two flight conditions in which the aircraft velocity is close to the stall velocity. For safety, the takeoff speed $V_{\text{TO}}$ of an aircraft is typically 20% greater than the stall speed, and the landing speed at touchdown $V_{\text{TD}}$ is 15% greater:

$$\begin{aligned} V_{\text{TO}} &= 1.20 V_{\text{stall}} \\ V_{\text{TD}} &= 1.15 V_{\text{stall}} \end{aligned} \tag{1.34}$$

---

**Example 1.5**

Determine the takeoff speed of the HP-1 at sea level with maximum gross takeoff weight and the landing speed with maximum landing weight.

From Appendix A we have $\rho = 1.255\ \mathrm{kg/m^3}$ for sea level. From Example 1.2b we have $C_{L\max} = 2.0$, $W = 1{,}645{,}760$ N, $S_w = 282.5\ \mathrm{m^2}$, and

$$V_{\text{stall}} = \sqrt{\frac{2 \times 1}{1.225 \times 2.0} \frac{1{,}645{,}760}{282.5}} = 69.0\ \mathrm{m/s}$$

Thus

$$V_{\text{TO}} = 1.20 V_{\text{stall}} = 82.8\ \mathrm{m/s}\ (\approx 185\ \mathrm{mph})$$

For landing, $W = 1{,}356{,}640$ N, and

$$V_{\text{stall}} = \sqrt{\frac{2 \times 1}{1.225 \times 2.0} \frac{1{,}356{,}640}{282.5}} = 62.6\ \mathrm{m/s}$$

Thus

$$V_{\text{TD}} = 1.15 V_{\text{stall}} = 72.0\ \mathrm{m/s}\ (\approx 161\ \mathrm{mph})$$

---

### 1.5.4 Fuel Consumption

The rate of change of the aircraft weight $\mathrm{d}W/\mathrm{d}t$ is due to the fuel consumed by the engines. The mass rate of fuel consumed is equal to the product of the installed thrust $T$ and the installed thrust specific fuel consumption. For constant acceleration of gravity $g_0$, we can write

$$\frac{\mathrm{d}W}{\mathrm{d}t} = -\dot{w}_f = -\dot{m}_f \frac{g_0}{g_c} = -T(\text{TSFC})\left(\frac{g_0}{g_c}\right)$$

This equation can be rewritten in dimensionless form as

$$\frac{\mathrm{d}W}{W} = -\frac{T}{W}(\text{TSFC})\left(\frac{g_0}{g_c}\right)\mathrm{d}t \tag{1.35}$$

*1.5.4.1 Estimate of TSFC.* Equation (1.35) requires estimates of installed engine thrust $T$ and installed TSFC to calculate the change in aircraft weight. For many flight conditions, the installed engine thrust $T$ equals the aircraft drag $D$. The value of TSFC depends on the engine cycle, altitude, and Mach number. For preliminary analysis, the following equations (from Ref. 12) can be used to estimate TSFC in units of (lbm/h)/lbf, and $\theta$ is the dimensionless temperature ratio $T/T_{\text{ref}}$:

1) High-bypass-ratio turbofan

$$\text{TSFC} = (0.4 + 0.45 M_0)\sqrt{\theta} \tag{1.36a}$$

2) Low-bypass-ratio, mixed-flow turbofan
Military and lower power settings:

$$\text{TSFC} = (1.0 + 0.35M_0)\sqrt{\theta} \tag{1.36b}$$

Maximum power setting:

$$\text{TSFC} = (1.8 + 0.30M_0)\sqrt{\theta} \tag{1.36c}$$

3) Turbojet
Military and lower power settings:

$$\text{TSFC} = (1.3 + 0.35M_0)\sqrt{\theta} \tag{1.36d}$$

Maximum power setting:

$$\text{TSFC} = (1.7 + 0.26M_0)\sqrt{\theta} \tag{1.36e}$$

4) Turboprop

$$\text{TSFC} = (0.2 + 0.9M_0)\sqrt{\theta} \tag{1.36f}$$

*1.5.4.2 Endurance.* For level unaccelerated flight, thrust equals drag ($T = D$) and lift equals weight ($L = W$). Thus Eq. (1.35) is simply

$$\frac{\mathrm{d}W}{W} = -\frac{C_D}{C_L}(\text{TSFC})\left(\frac{g_0}{g_c}\right)\mathrm{d}t \tag{1.37}$$

We define the endurance factor (EF) as

$$\text{EF} \equiv \frac{C_L}{C_D(\text{TSFC})}\frac{g_c}{g_0} \tag{1.38}$$

Then Eq. (1.37) becomes

$$\frac{\mathrm{d}W}{W} = -\frac{\mathrm{d}t}{\text{EF}} \tag{1.39}$$

Note that the minimum fuel consumption for a time $t$ occurs at the flight condition where the endurance factor is maximum.

For the case when the endurance factor is constant or nearly constant, Eq. (1.39) can be integrated from the initial to final conditions and the following expression obtained for the aircraft weight fraction:

$$\frac{W_f}{W_i} = \exp\left(-\frac{t}{\text{EF}}\right) \tag{1.40a}$$

or

$$\frac{W_f}{W_i} = \exp\left[-\frac{C_D}{C_L}(\mathrm{TSFC})t\frac{g_0}{g_c}\right] \tag{1.40b}$$

*1.5.4.3 Range.* For portions of aircraft flight where distance is important, the differential time d$t$ is related to the differential distance d$s$ by

$$\mathrm{d}s = V\,\mathrm{d}t \tag{1.41}$$

Substituting into Eq. (1.37) gives

$$\frac{\mathrm{d}W}{W} = -\frac{C_D}{C_L}\frac{\mathrm{TSFC}}{V}\frac{g_0}{g_c}\mathrm{d}s \tag{1.42}$$

We define the range factor (RF) as

$$\mathrm{RF} \equiv \frac{C_L}{C_D}\frac{V}{\mathrm{TSFC}}\frac{g_c}{g_0} \tag{1.43}$$

Then Eq. (1.42) can be simply written as

$$\frac{\mathrm{d}W}{W} = -\frac{\mathrm{d}s}{\mathrm{RF}} \tag{1.44}$$

Note that the minimum fuel consumption for a distance $s$ occurs at the flight condition where the range factor is maximum.

For the flight conditions where the RF is constant or nearly constant, Eq. (1.42) can be integrated from the initial to final conditions and the following expression obtained for the aircraft weight fraction:

$$\frac{W_f}{W_i} = \exp\left(-\frac{s}{\mathrm{RF}}\right) \tag{1.45a}$$

or

$$\frac{W_f}{W_i} = \exp\left(-\frac{C_D}{C_L}\frac{\mathrm{TSFC} \times s}{V}\frac{g_0}{g_c}\right) \tag{1.45b}$$

This is called the *Breguet range equation.* For the range factor to remain constant, $C_L/C_D$ and V/TSFC need to be constant. Above 36-kft altitude, the ambient temperature is constant, and a constant velocity $V$ will correspond to constant Mach and constant TSFC for a fixed throttle setting. If $C_L$ is constant, $C_L/C_D$ will remain constant. Since the aircraft weight W decreases during the flight, the altitude must increase to reduce the density of the ambient air and produce the required lift ($L = W$) while maintaining $C_L$ and velocity constant. This flight profile is called a *cruise climb.*

### Example 1.6

Calculate the endurance factor and range factor at Mach 0.8 and 40-kft altitude of hypothetical fighter aircraft HF-1 at 90% of maximum gross takeoff weight and a load factor of 1.

*Solution*:

$$q = \frac{\gamma}{2}\delta P_{\text{ref}} M_0^2 = 0.7 \times 0.1858 \times 2116 \times 0.8^2 = 176.1 \text{ lbf/ft}^2$$

From Fig. 1.24 at $M = 0.8$, $C_{D0} = 0.012$, $K_1 = 0.20$, and $K_2 = 0$:

$$C_L = \frac{nW}{qS_w} = \frac{1 \times 0.9 \times 40{,}000}{176.1 \times 720} = 0.2839$$

$$C_D = K_1 C_L^2 + K_2 C_L + C_{D0} = 0.2(0.2839^2) + 0.012 = 0.0281$$

Using Eq. (1.36b), we have

$$\text{TSFC} = (1.0 + 0.35M_0)\sqrt{\theta} = (1.0 + 0.35 \times 0.8)\sqrt{0.7519} = 1.110\,(\text{lbm/h})/\text{lbf}$$

Thus

$$\text{EF} = \frac{C_L}{C_D(\text{TSFC})}\frac{g_c}{g_0} = \frac{0.2839}{0.0281 \times 1.110}\frac{32.174}{32.174} = 9.102 \text{ h}$$

$$\text{RF} = \frac{C_L}{C_D}\frac{V}{\text{TSFC}}\frac{g_c}{g_0}$$

$$= \frac{0.2839}{0.0281}\frac{0.8 \times 0.8671 \times 1116\,\text{ft/s}}{1.110\,(\text{lbm/h})/\text{lbf}}\frac{3600\,\text{s/h}}{6080\,\text{ft/nm}}\frac{32.174}{32.174}$$

$$= 4170 \text{ nm}$$

### Example 1.7

Determine the variation in endurance factor and range factor for the two hypothetical aircraft HF-1 and HP-1.

*a) Fighter aircraft (HF-1).* The endurance factor is plotted vs Mach number and altitude in Fig. 1.30 for our hypothetical fighter aircraft HF-1 at 90% of maximum gross takeoff weight. Note that the best endurance Mach number (minimum fuel consumption) increases with altitude, and the best fuel consumption occurs at altitudes of 30 and 36 kft. The range factor is plotted vs Mach number and altitude in Fig. 1.31 for the HF-1 at 90% of maximum gross takeoff weight. Note that the best cruise Mach number (minimum fuel consumption) increases with altitude, and the best fuel consumption occurs at an altitude of 36 kft and Mach number of 0.8.

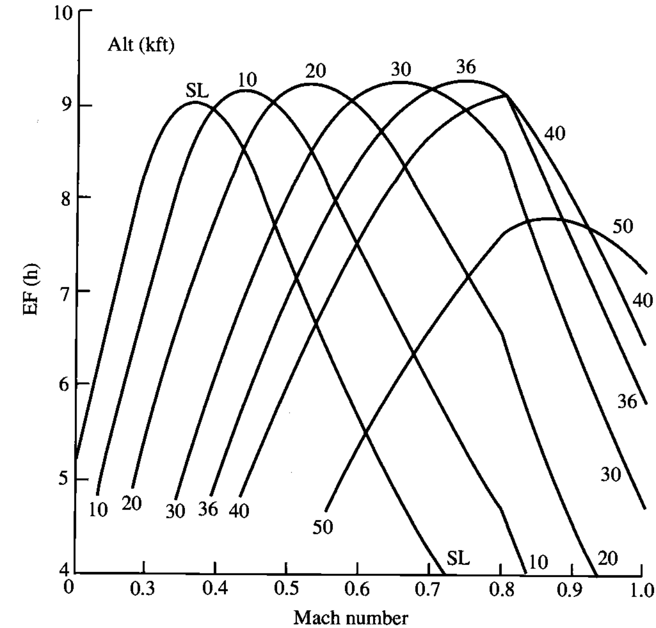


**Fig. 1.30 Endurance factor for HF-1 aircraft.**

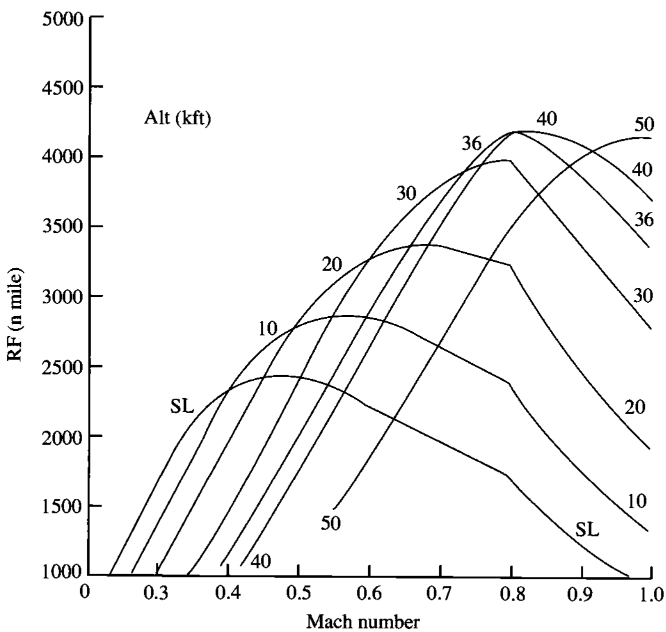


**Fig. 1.31 Range factor for HF-1 aircraft.**

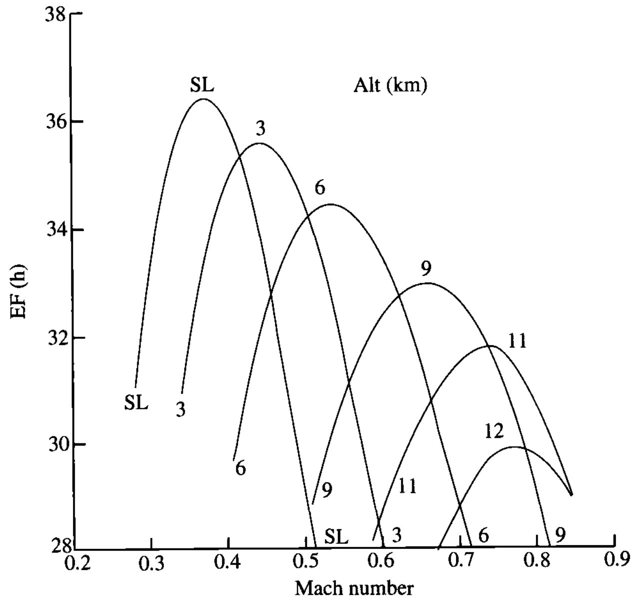


**Fig. 1.32 Endurance factor for HP-1 aircraft.**

*b) Passenger aircraft (HP-1).* The endurance factor is plotted vs Mach number and altitude in Fig. 1.32 for our hypothetical passenger aircraft HP-1 at 95% of maximum gross takeoff weight. Note that the best endurance Mach number (minimum fuel consumption) increases with altitude, and the best fuel consumption occurs at sea level. The range factor is plotted vs Mach number and altitude in Fig. 1.33 for the HP-1 at 95% of maximum gross takeoff weight. Note that the best cruise Mach number (minimum fuel consumption) increases with altitude, and the best fuel consumption occurs at an altitude of 11 km and Mach number of about 0.83.

Since the weight of an aircraft like the HP-1 can vary considerably over a flight, the variation in range factor with cruise Mach number was determined for 95 and 70% of maximum gross takeoff weight (MGTOW) at altitudes of 11 and 12 km and is plotted in Fig. 1.34. If the HP-1 flew at 0.83 Mach and 12-km altitude, the range factors at 95% MGTOW and at 70% MGTOW are about the same. However, if the HP-1 flew at 0.83 Mach and 11-km altitude, the range factor would decrease with aircraft weight, and the aircraft's range would be less than that of the HP-1 flown at 0.83 Mach and 12-km altitude. One can see from this discussion that the proper cruise altitude can dramatically affect an aircraft's range.

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*1.5.4.4 Maximum $C_L/C_D$.* For flight conditions requiring minimum fuel consumption, the optimum flight condition can be approximated by that

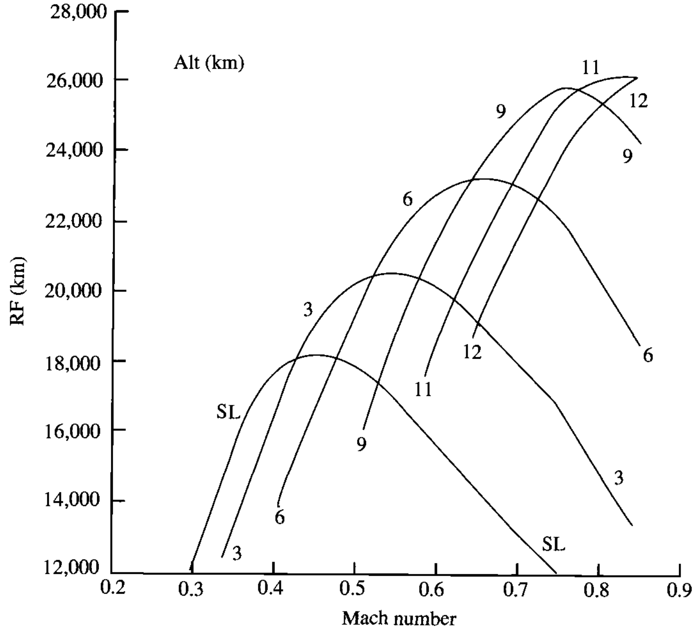


**Fig. 1.33 Range factor for HP-1 aircraft for various altitudes.**

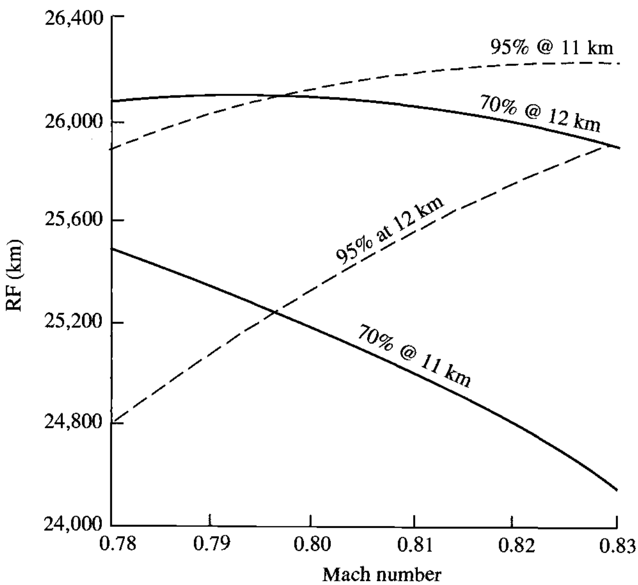


**Fig. 1.34 Range factor for HP-1 aircraft at 70 and 95% MGTOW.**

corresponding to maximum $C_L/C_D$. From Eq. (1.32), the maximum $C_L/C_D$ (minimum $C_D/C_L$) can be found by taking the derivative of the following expression, setting it equal to zero, and solving for the $C_L$ that gives minimum $C_D/C_L$:

$$\frac{C_D}{C_L} = K_1 C_L + K_2 + \frac{C_{D0}}{C_L} \tag{1.46}$$

The lift coefficient that gives maximum $C_L/C_D$ (minimum $C_D/C_L$) is

$$C_L^* = \sqrt{\frac{C_{D0}}{K_1}} \tag{1.47}$$

and maximum $C_L/C_D$ is given by

$$\left(\frac{C_L}{C_D}\right)^* = \frac{1}{2\sqrt{C_{D0}K_1} + K_2} \tag{1.48}$$

The drag $D$, range factor, endurance factor, and $C_L/C_D$ vs Mach number at an altitude are plotted in Fig. 1.35 for the HF-1 aircraft and in Fig. 1.36 for the HP-l. Note that the maximum $C_L/C_D$ occurs at Mach 0.8 for the HF-1 and at Mach 0.75 for the HP-1, the same Mach numbers where drags are minimum. The endurance factor is a maximum at a substantially lower Mach number than that corresponding to $(C_L/C_D)^*$ for the HF-1 due to the high TSFC and its increase with Mach number [see Eq. (1.36b)]. The endurance factor for

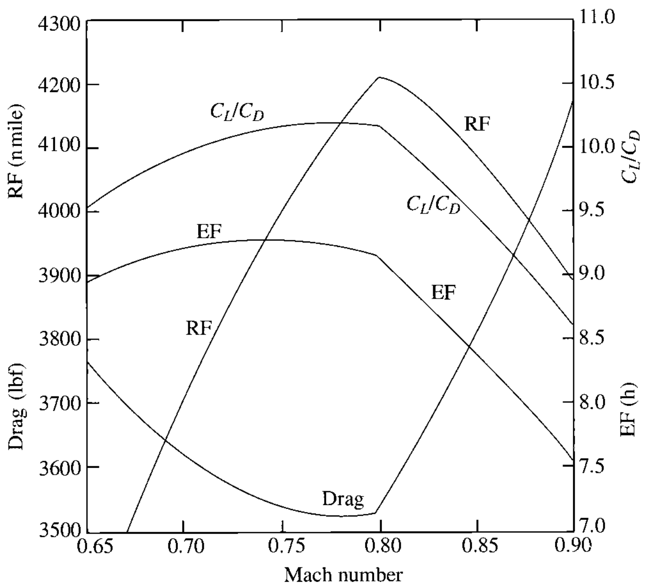


**Fig. 1.35 Comparison of drag $C_L/C_D$, endurance factor, and range factor for the HF-1 at 36-kft altitude.**

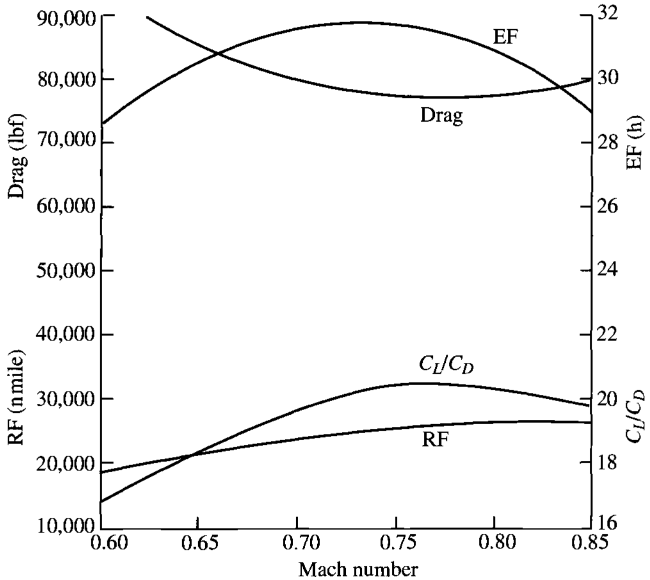


**Fig. 1.36 Comparison of drag $C_L/C_D$, endurance factor, and range factor for the HP-1 at 11-km altitude.**

the HP-1 is a maximum at the same Mach number that $C_L/C_D$ is maximum due to the lower TSFC of the high-bypass-ratio turbofan engine [see Eq. (1.36a)].

The Mach number for an altitude giving a maximum range factor is called the *best cruise Mach* (BCM). The best cruise Mach normally occurs at a little higher Mach than that corresponding to $(C_L/C_D)^*$. This is because the velocity term in the range factor normally dominates over the increase in TSFC with Mach number. As a first approximation, many use the Mach number corresponding to $(C_L/C_D)^*$ for the best cruise Mach.

---

### Example 1.8

Calculate the Mach giving maximum $C_L/C_D$ at 20-kft altitude for the HF-1 aircraft at 90% of maximum gross takeoff weight and a load factor of 1.

*Solution:* From Fig. 1.24 at $M_0 < 0.8$, $C_{D0} = 0.012$, $K_1 = 0.20$, and $K_2 = 0$:

$$C_L^* = \sqrt{\frac{C_{D0}}{K_1}} = \sqrt{\frac{0.012}{0.2}} = 0.2449$$

$$q = \frac{W}{C_L S_w} = \frac{0.9 \times 40{,}000}{0.2449 \times 720} = 204.16\,\text{lbf/ft}^2$$

$$M_0 = \sqrt{\frac{q}{(\gamma/2)\delta P_{\text{ref}}}} = \sqrt{\frac{204.16}{0.7 \times 0.4599 \times 2116}} = 0.547$$

---

*1.5.4.5 Accelerated flight.* For flight conditions when thrust $T$ is greater than drag $D$, an expression for the fuel consumption can be obtained by first noting from Eq. (1.28) that

$$\frac{T}{W} = \frac{P_s}{V[1 - (D+R)/T]}$$

We define the ratio of drag $D + R$ to thrust $T$ as

$$u \equiv \frac{D+R}{T} \tag{1.49}$$

The preceding equation for thrust to weight becomes

$$\frac{T}{W} = \frac{P_s}{V(1-u)} \tag{1.50}$$

Now Eq. (1.35) can be rewritten as

$$\frac{\mathrm{d}W}{W} = -\frac{\mathrm{TSFC}}{V(1-u)}\frac{g_0}{g_c}P_s\,\mathrm{d}t$$

Since $P_s\,\mathrm{d}t = \mathrm{d}z_e$, the preceding equation can be expressed in its most useful forms as

$$\frac{\mathrm{d}W}{W}\frac{\mathrm{TSFC}}{V(1-u)}\frac{g_0}{g_c}\mathrm{d}z_e = -\frac{\mathrm{TSFC}}{V(1-u)}\frac{g_0}{g_c}\mathrm{d}\left(h + \frac{V^2}{2g}\right) \tag{1.51}$$

The term $1 - u$ represents the fraction of engine power that goes to increasing the aircraft energy $z_e$, and $u$ represents that fraction that is lost to aircraft drag $D + R$. Note that this equation applies for cases when $u$ is not unity. When $u$ is unity, either Eq. (1.39) or Eq. (1.44) is used.

To obtain the fuel consumption during an acceleration flight condition, Eq. (1.51) can be easily integrated for known flight paths (values of $V$ and $z_e$) and known variation of $\mathrm{TSFC}/[V(1-u)]$ with $z_e$.

### *1.5.5 Aerospace Vehicle Design—A Team Effort*

Aeronautical and mechanical engineers in the aerospace field do many things, but for the most part their efforts all lead to the design of some type of aerospace vehicle. The design team for a new aircraft may be divided into four principal groups: aerodynamics, propulsion, structures, and flight mechanics. The design of a vehicle calls on the extraordinary talents of engineers in each group. Thus the design is a team effort. A typical design team is shown in Fig. 1.37. The chief engineer serves as the referee and integrates the efforts of everyone into the vehicle design. Figure 1.38 shows the kind of aircraft design that might result if any one group were able to dominate the others.

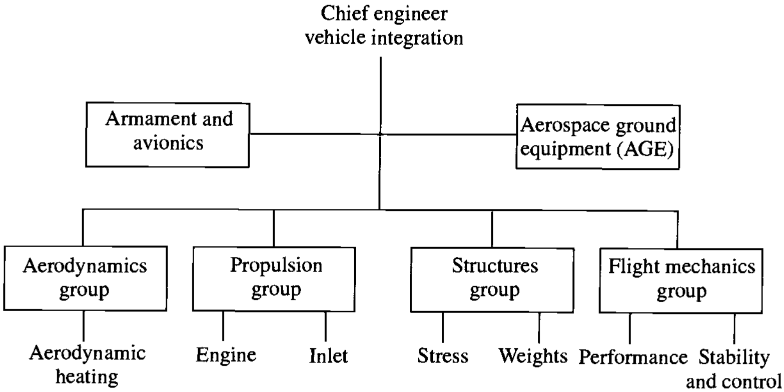


**Fig. 1.37 Organization of a typical vehicle design team.**

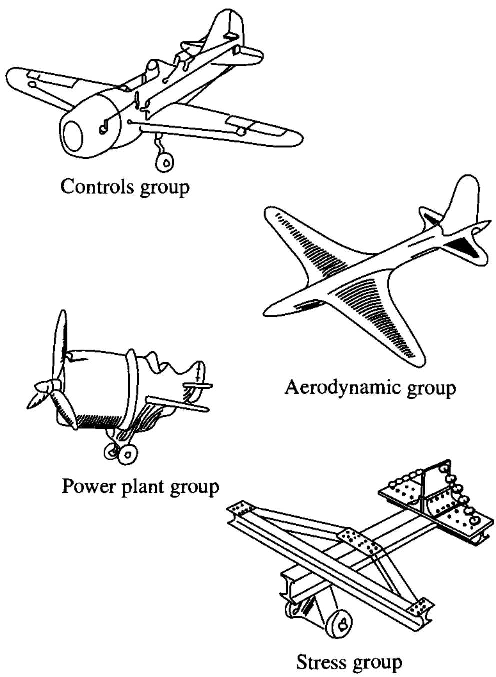


**Fig. 1.38 Aircraft designs.**

## 1.6 Rocket Engines

Non-airbreathing propulsion systems are characterized by the fact that they carry both fuel and the oxidizer within the aerospace vehicle. Such systems thus may be used anywhere in space as well as in the atmosphere. Figure 1.39

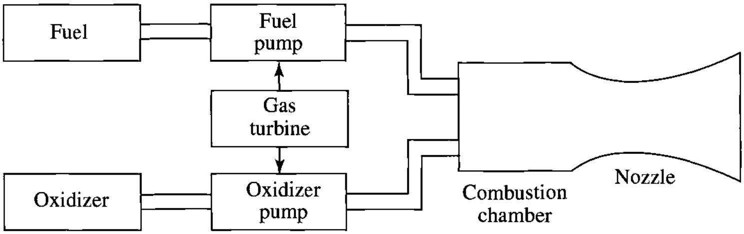


**Fig. 1.39 Liquid-propellant rocket motor.**

shows the essential features of a liquid-propellant rocket system. Two propellants (an oxidizer and a fuel) are pumped into the combustion chamber where they ignite. The nozzle accelerates the products of combustion to high velocities and exhausts them to the atmosphere or space.

A solid-propellant rocket motor is the simplest of all propulsion systems. Figure 1.40 shows the essential features of this type of system. In this system, the fuel and oxidizer are mixed together and cast into a solid mass called the *grain*. The grain, usually formed with a hole down the middle called the *perforation*, is firmly cemented to the inside of the combustion chamber. After ignition, the grain burns radially outward, and the hot combustion gases pass down the perforation and are exhausted through the nozzle.

The absence of a propellant feed system in the solid-propellant rocket is one of its major advantages. Liquid rockets, on the other hand, may be stopped and later restarted, and their thrust may be varied somewhat by changing the speed of the fuel and oxidizer pumps.

### *1.6.1 Rocket Engine Thrust*

A natural starting point in understanding the performance of a rocket is the examination of the static thrust. Application of the momentum equation developed in Chapter 2 will show that the static thrust is a function of the propellant flow rate $\dot{m}_p$, the exhaust velocity $V_e$ and pressure $P_e$, the exhaust area $A_e$, and the ambient pressure $P_a$. Figure 1.41 shows a schematic of a stationary rocket

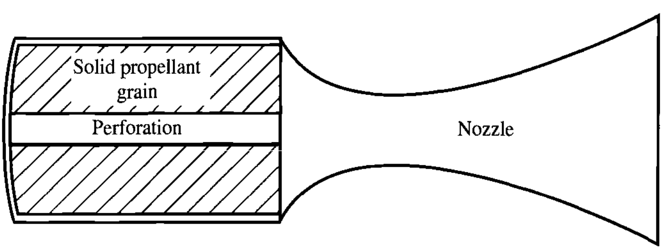


**Fig. 1.40 Solid-propellant rocket motor.**

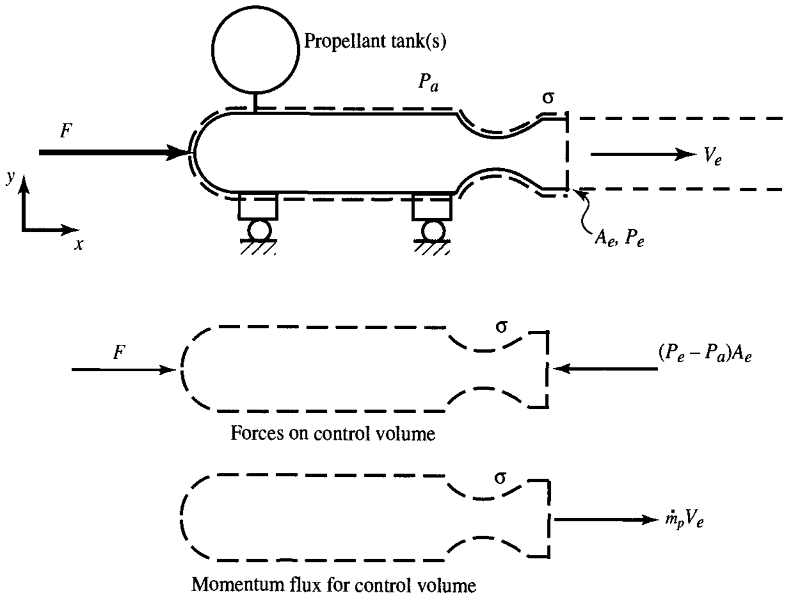


**Fig. 1.41 Schematic diagram of static rocket engine.**

to be considered for analysis. We assume the flow to be one-dimensional, with a steady exit velocity $V_e$ and propellant flow rate $\dot{m}_p$. About this rocket we place a control volume $\sigma$ whose control surface intersects the exhaust jet perpendicularly through the exit plane of the nozzle. Thrust acts in the direction opposite to the direction of $V_e$. The reaction to the thrust $F$ necessary to hold the rocket and control volume stationary is shown in Fig. 1.41.

The momentum equation applied to this system gives the following:

1) Sum of forces acting on the outside surface of the control volume:

$$\sum F_x = F - (P_e - P_a)A_e$$

2) The net rate of change of momentum for the control volume:

$$\Delta(\text{momentum}) = \dot{M}_{\text{out}} = \frac{\dot{m}_p V_e}{g_c}$$

Since the sum of the forces acting on the outside of the control volume is equal to the net rate of change of the momentum for the control volume, we have

$$F - (P_e - P_a)A_e = \frac{\dot{m}_p V_e}{g_c} \tag{1.52}$$

If the pressure in the exhaust plane $P_e$ is the same as the ambient pressure $P_a$, the thrust is given by $F = \dot{m}_p V_e / g_c$. The condition $P_e = P_a$ is called *on-design* or *optimum expansion* because it corresponds to maximum thrust for the given chamber conditions. It is convenient to define an *effective exhaust velocity* $C$ such that

$$C \equiv V_e + \frac{(P_e - P_a)A_e g_c}{\dot{m}_p} \tag{1.53}$$

Thus the static thrust of a rocket can be written as

$$F = \frac{\dot{m}_p C}{g_c} \tag{1.54}$$

### 1.6.2 Specific Impulse

The *specific impulse* $I_{sp}$ for a rocket is defined as the thrust per unit of propellant weight flow:

$$I_{sp} \equiv \frac{F}{\dot{w}_p} = \frac{F}{\dot{m}_p} \frac{g_c}{g_0} \tag{1.55}$$

where $g_0$ is the acceleration due to gravity at sea level. The unit of $I_{sp}$ is the second. From Eqs. (1.54) and (1.55), the specific impulse can also be written as

$$I_{sp} = \frac{C}{g_0} \tag{1.56}$$

---

**Example 1.9**

Find the specific impulse of the space shuttle main engine (SSME) shown in Fig. 1.42a that produces 470,000 lbf in a vacuum with a propellant weight flow of 1030 lbf/s. By using Eq. (1.55), we find that the SSME has a specific impulse $I_{sp}$ of 456 s (= 470,000/1030) in vacuum.

An estimate of the variation in thrust with altitude for the space shuttle main engine is shown in Fig. 1.42b. The typical specific impulses for some rocket engines are listed in Table 1.6. Other performance data for rocket engines are contained in Appendix C.

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### 1.6.3 Rocket Vehicle Acceleration

The mass of a rocket vehicle varies a great deal during flight due to the consumption of the propellant. The velocity that a rocket vehicle attains during powered flight can be determined by considering the vehicle in Fig. 1.43.

The figure shows an accelerating rocket vehicle in a gravity field. At some time, the mass of the rocket is $m$ and its velocity is $V$. In an infinitesimal time $dt$, the rocket exhausts an incremental mass $dm_p$ with an exhaust velocity $V_e$

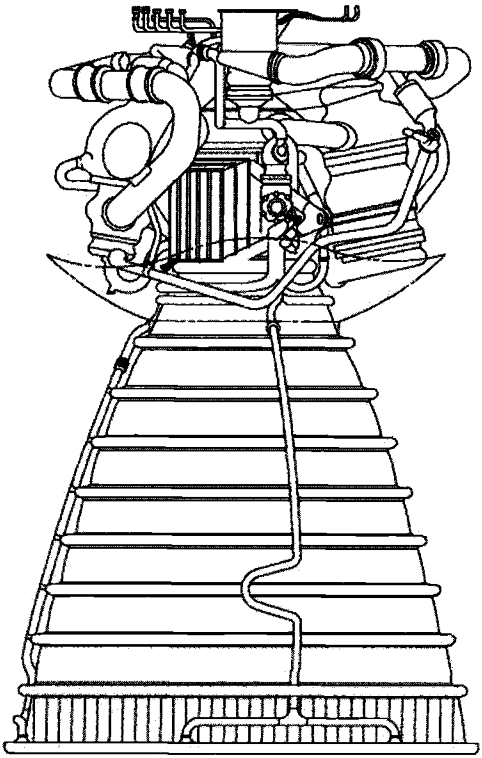

**Fig. 1.42a Space shuttle main engine (SSME).**

relative to the rocket as the rocket velocity changes to $V + \mathrm{d}V$. The net change in momentum of the control volume $\sigma$ is composed of the momentum out of the rocket at the exhaust plus the change of the momentum of the rocket. The momentum out of the rocket in the $V$ direction is $-V_e \, \mathrm{d}m_p$, and the change in the momentum of the rocket in the $V$ direction is $m \, \mathrm{d}V$. The forces acting on the control volume $\sigma$ are composed of the net pressure force, the drag $D$, and the gravitational force. The sum of these forces in the $V$ direction is

$$\sum F_V = (P_e - P_a)A_e - D - \frac{mg}{g_c} \cos\theta$$

The resultant impulse on the rocket $\left(\sum F_V\right)\mathrm{d}t$ must equal the momentum change of the system $\Delta(\text{momentum}) = (-V_e \, \mathrm{d}m_p + m \, \mathrm{d}V)/g_c$. Thus

$$\left[(P_e - P_a)A_e - D - \frac{mg}{g_c} \cos\theta\right]\mathrm{d}t = \frac{-V_e \ \mathrm{d}m_p + m \ \mathrm{d}V}{g_c}$$

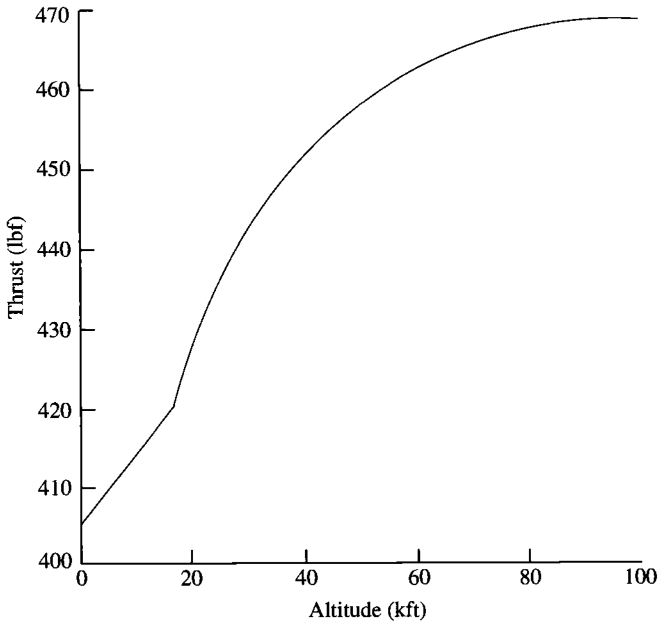


**Fig. 1.42b Rocket thrust variation with altitude.**

From the preceding relationship, the momentum change of the rocket ($m \, dV$) is

$$\frac{m \, dV}{g_c} = \left[(P_e - P_a)A_e - D - \frac{mg}{g_c} \cos\theta\right]dt + \frac{V_e \, dm_p}{g_c} \tag{1.57}$$

Since $dm_p = \dot{m}_p \, dt = -(dm/dt)dt$, then Eq. (1.57) can be written as

$$\frac{m \, dV}{g_c} = \left[(P_e - P_a)A_e + \frac{\dot{m}_p V_e}{g_c} - D - \frac{mg}{g_c}\cos\theta\right]dt$$

**Table 1.6 Ranges of specific impulse $I_{sp}$ for typical rocket engines**

| Fuel/oxidizer | $I_{sp}$, s |
|---|---|
| Solid propellant | 250 |
| Liquid $O_2$: kerosene (RP) | 310 |
| Liquid $O_2$: $H_2$ | 410 |
| Nuclear fuel: $H_2$ propellant | 840 |

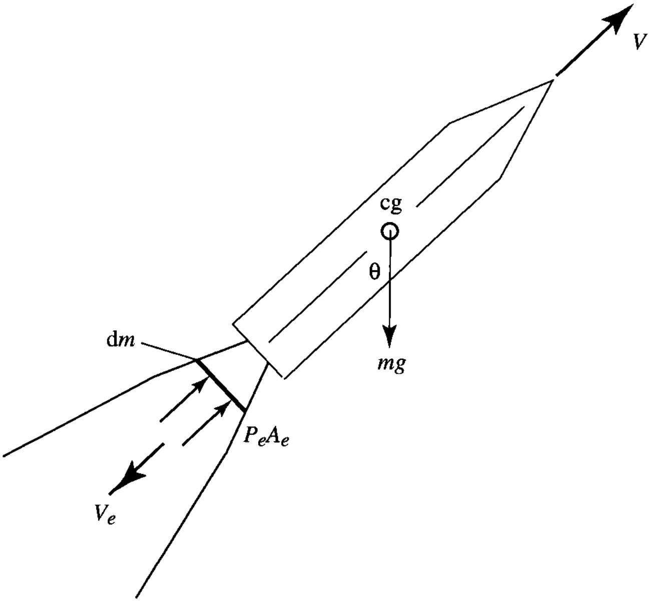


**Fig. 1.43 Rocket vehicle in flight.**

By using Eq. (1.53), this relationship becomes

$$\frac{m\,\mathrm{d}V}{g_c} = \left(\frac{\dot{m}_p}{g_c}C - D - \frac{mg}{g_c}\cos\theta\right)\mathrm{d}t$$

or

$$\mathrm{d}V = -C\frac{\mathrm{d}m}{m} - \frac{Dg_c}{m}\mathrm{d}t - g\cos\theta\,\mathrm{d}t \tag{1.58}$$

The velocity of a rocket along its trajectory can be determined from the preceding equation if $C$, $D$, $g$, and $\theta$ are known.

In the absence of drag and gravity, integration of Eq. (1.58) gives the following, assuming constant effective exhaust velocity $C$:

$$\Delta V = C\,\ell n\frac{m_t}{m_f} \tag{1.59}$$

where $\Delta V$ is the change in velocity, $m_i$ is the initial mass of the rocket system, and $m_f$ is the final mass. Equation (1.59) can be solved for the mass ratio as

$$\frac{m_i}{m_f} = \exp\frac{\Delta V}{C} \tag{1.60}$$

### Example 1.10

We want to estimate the mass ratio (final to initial) of an $H_2$-$O_2$ ($C = 4000$ m/s) rocket for an Earth orbit ($\Delta V = 8000$ m/s), neglecting drag and gravity. Using Eq. (1.59), we obtain $m_f/m_t = e^{-2} = 0.132$, or a single-stage rocket would be about 13% payload and structure and 87% propellant.

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## Problems

**1.1** Calculate the uninstalled thrust for Example 1.1, using Eq. (1.6).

**1.2** Develop the following analytical expressions for a turbojet engine:

(a) When the fuel flow rate is very small in comparison with the air mass flow rate, the exit pressure is equal to ambient pressure, and the installation loss coefficients are zero, then the installed thrust $T$ is given by

$$T = \frac{\dot{m}_0}{g_c}(V_e - V_0)$$

(b) For the preceding conditions, the thrust specific fuel consumption is given by

$$\text{TSFC} = \frac{Tg_c/\dot{m}_0 + 2V_0}{2\eta_T h_{PR}}$$

(c) For $V_0 = 0$ and 500 ft/s, plot the preceding equation for TSFC [in (lbm/h)/lbf] vs specific thrust $T/\dot{m}_0$ [in lbf/(lbm/s)] for values of specific thrust from 0 to 120. Use $\eta_T = 0.4$ and $h_{PR} = 18{,}400$ Btu/lbm.

(d) Explain the trends.

**1.3** Repeat 1.2c, using SI units. For $V_0 = 0$ and 150 m/s, plot TSFC [in (mg/s)/N] vs specific thrust $T/\dot{m}_0$ [in N/(kg/s)] for values of specific thrust from 0 to 1200. Use $\eta_T = 0.4$ and $h_{PR} = 42{,}800$ kJ/kg.

**1.4** A J57 turbojet engine is tested at sea-level, static, standard-day conditions ($P_0 = 14.696$ psia, $T_0 = 518.7°$R, and $V_0 = 0$). At one test point, the thrust is 10,200 lbf while the airflow is 164 lbm/s and the fuel flow is 8520 lbm/h. Using these data, estimate the exit velocity $V_e$ for the case of exit pressure equal to ambient pressure ($P_0 = P_e$).

**1.5** The thrust for a turbofan engine with separate exhaust streams is equal to the sum of the thrust from the engine core $F_C$ and the thrust from the bypass stream $F_B$. The bypass ratio of the engine $\alpha$ is the ratio of the mass flow through the bypass stream to the core mass flow, or $\alpha \equiv \dot{m}_B/\dot{m}_C$. When the exit pressures are equal to the ambient pressure, the thrusts of the core and bypass stream are given by

$$F_C = \frac{1}{g_c}[(\dot{m}_C + \dot{m}_f)V_{Ce} - \dot{m}_C V_0]$$

$$F_B = \frac{\dot{m}_B}{g_c}(V_{Be} - V_0)$$

where $V_{Ce}$ and $V_{Be}$ are the exit velocities from the core and bypass, respectively, $V_0$ is the inlet velocity, and $\dot{m}_f$ is the mass flow rate of fuel burned in the core of the engine.

Show that the specific thrust and thrust specific fuel consumption can be expressed as

$$\frac{F}{\dot{m}_0} = \frac{1}{g_c}\left(\frac{1 + \dot{m}_f/\dot{m}_C}{1+\alpha}V_{Ce} + \frac{\alpha}{1+\alpha}V_{Be} - V_0\right)$$

$$S = \frac{\dot{m}_f}{F} = \frac{\dot{m}_f/\dot{m}_C}{(F/\dot{m}_0)(1+\alpha)}$$

where $\dot{m}_0 = \dot{m}_C + \dot{m}_B$.

**1.6** The CF6 turbofan engine has a rated thrust of 40,000 lbf at a fuel flow rate of 13,920 lbm/h at sea-level static conditions. If the core airflow rate is 225 lbm/s and the bypass ratio is 6.0, what are the specific thrust [lbf/(lbm/s)] and thrust specific fuel consumption [(lbm/h)/lbf]?

**1.7** The JT9D high-bypass-ratio turbofan engine at maximum static thrust ($V_0 = 0$) on a sea-level, standard day ($P_0 = 14.696$ psia, $T_0 = 518.7°$R) has the following data: the air mass flow rate through the core is 247 lbm/s, the air mass flow rate through the fan bypass duct is 1248 lbm/s, the exit velocity from the core is 1190 ft/s, the exit velocity from the bypass duct is 885 ft/s, and the fuel flow rate into the combustor is 15,750 lbm/h. Estimate the following for the case of exit pressures equal to ambient pressure ($P_0 = P_e$):

(a) The thrust of the engine

(b) The thermal efficiency of the engine (heating value of jet fuel is about 18,400 Btu/lbm)

(c) The propulsive efficiency and thrust specific fuel consumption of the engine

**1.8** Repeat Problem 1.7, using SI units.

**1.9** One advanced afterburning fighter engine, whose performance is depicted in Figs. 1.14a–1.14e, is installed in the HF-1 fighter aircraft. Using the aircraft drag data of Fig. 1.26b, determine and plot the variation of weight specific excess power ($P_s$ in feet per second) vs flight Mach number for level flight ($n = 1$) at 36-kft altitude. Assume the installation losses are constant with values of $\phi_{\text{inlet}} = 0.05$ and $\phi_{\text{noz}} = 0.02$.

**1.10** Determine the takeoff speed of the HF-1 aircraft.

**1.11** Determine the takeoff speed of the HP-1 aircraft at 90% of maximum gross takeoff weight.

**1.12** Derive Eqs. (1.47) and (1.48) for maximum $C_L/C_D$. Start by taking the derivative of Eq. (1.46) with respect to $C_L$ and finding the expression for the lift coefficient that gives maximum $C_L/C_D$.

**1.13** Show that for maximum $C_L/C_D$, the corresponding drag coefficient $C_D$ is given by

$$C_D = 2C_{D0} + K_2\sqrt{\frac{C_{D0}}{K_1}}$$

**1.14** An aircraft with a wing area of 800 ft$^2$ is in level flight ($n = 1$) at maximum $C_L/C_D$. Given that the drag coefficients for the aircraft are $C_{D0} = 0.02$, $K_2 = 0$, and $K_1 = 0.2$, find
(a) The maximum $C_L/C_D$ and the corresponding values of $C_L$ and $C_D$
(b) The flight altitude [use Eqs. (1.29) and (1.30b)] and aircraft drag for an aircraft weight of 45,000 lbf at Mach 0.8
(c) The flight altitude and aircraft drag for an aircraft weight of 35,000 lbf at Mach 0.8
(d) The range for an installed engine thrust specific fuel consumption rate of 0.8 (lbm/h)/lbf, if the 10,000-lbf difference in aircraft weight between parts b and c is due only to fuel consumption

**1.15** An aircraft weighing 110,000 N with a wing area of 42 m$^2$ is in level flight ($n = 1$) at the maximum value of $C_L/C_D$. Given that the drag coefficients for the aircraft are $C_{D0} = 0.03$, $K_2 = 0$, and $K_1 = 0.25$, find the following:
(a) The maximum $C_L/C_D$ and the corresponding values of $C_L$ and $C_D$
(b) The flight altitude [use Eqs. (1.29) and (1.30b)] and aircraft drag at Mach 0.5
(c) The flight altitude and aircraft drag at Mach 0.75

**1.16** The Breguet range equation [Eq. (1.45b)] applies for a cruise climb flight profile with constant RF. Another range equation can be developed for a level cruise flight profile with varying RF. Consider the case where we keep $C_L$, $C_D$, and TSFC constant and vary the flight velocity with aircraft weight by the expression

$$V = \sqrt{\frac{2g_c W}{\rho C_L S_w}}$$

Using the subscripts $i$ and $f$ for the initial and final flight conditions, respectively, show the following:

(a) Substitution of this expression for flight velocity into Eq. (1.42) gives

$$\frac{dW}{\sqrt{W}} = -\frac{\sqrt{W_i}}{RF_i} ds$$

(b) Integration of the preceding between the initial $i$ and final $f$ conditions gives

$$\frac{W_f}{W_i} = \left[1 - \frac{s}{2(RF_i)}\right]^2$$

(c) For a given weight fraction $W_f/W_i$, the maximum range $s$ for this level cruise flight corresponds to starting the flight at the maximum altitude (minimum density) and maximum value of $\sqrt{C_L}/C_D$.

(d) For the drag coefficient equation of Eq. (1.32), maximum $\sqrt{C_L}/C_D$ corresponds to $C_L = (1/6K_1)(\sqrt{12K_1 C_{D0} + K_2^2} - K_2)$.

**1.17** An aircraft begins a cruise at a wing loading $W/S_w$ of 100 lbf/ft$^2$ and Mach 0.8. The drag coefficients are $K_1 = 0.056$, $K_2 = -0.008$, and $C_{D0} = 0.014$, and the fuel consumption TSFC is constant at 0.8 (lbm/h)/lbf. For a weight fraction $W_f/W_i$ of 0.9, determine the range and other parameters for two different types of cruise.

(a) For a cruise climb (maximum $C_L/C_D$) flight path, determine $C_L$, $C_D$, initial and final altitudes, and range.

(b) For a level cruise (maximum $\sqrt{C_L}/C_D$) flight path, determine $C_L$, $C_D$, altitude, initial and final velocities, and range.

**1.18** An aircraft weighing 70,000 lbf with a wing area of 1000 ft$^2$ is in level flight ($n = 1$) at 30-kft altitude. Using the drag coefficients of Fig. 1.24 and the TSFC model of Eq. (1.36b), find the following:

(a) The maximum $C_L/C_D$ and the corresponding values of $C_L$, $C_D$, and Mach number (Note: Since the drag coefficients are a function of Mach number and it is an unknown, you must first guess a value for the Mach number to obtain the drag coefficients. Try a Mach number of 0.8 for your first guess.)

(b) The $C_L$, $C_D$, $C_L/C_D$, range factor, endurance factor, and drag for flight Mach numbers of 0.74, 0.76, 0.78, 0.80, 0.81, and 0.82

(c) The best cruise Mach (maximum RF)
(d) The best loiter Mach (maximum EF)

**1.19** An aircraft weighing 200,000 N with a wing area of 60 $m^2$ is in level flight ($n = 1$) at 9-km altitude. Using the drag coefficients of Fig. 1.24 and TSFC model of Eq. (1.36b), find the following:
(a) The maximum $C_L/C_D$ and the corresponding values of $C_L$, $C_D$, and Mach number (Note: Since the drag coefficients are a function of the Mach number and it is an unknown, you must first guess a value for the Mach number to obtain the drag coefficients. Try a Mach number of 0.8 for your first guess.)
(b) The $C_L$, $C_D$, $C_L/C_D$, range factor, endurance factor, and drag for flight Mach numbers of 0.74, 0.76, 0.78, 0.80, 0.81, and 0.82
(c) The best cruise Mach (maximum RF)
(d) The best loiter Mach (maximum EF)

**1.20** What is the specific impulse in seconds of the JT9D turbofan engine in Problem 1.7?

**1.21** A rocket motor is fired in place on a static test stand. The rocket exhausts 100 lbm/s at an exit velocity of 2000 ft/s and pressure of 50 psia. The exit area of the rocket is 0.2 $ft^2$. For an ambient pressure of 14.7 psia, determine the effective exhaust velocity, the thrust transmitted to the test stand, and the specific impulse.

**1.22** A rocket motor under static testing exhausts 50 kg/s at an exit velocity of 800 m/s and pressure of 350 kPa. The exit area of the rocket is 0.02 $m^2$. For an ambient pressure of 100 kPa, determine the effective exhaust velocity, the thrust transmitted to the test stand, and the specific impulse.

**1.23** The propellant weight of an orbiting space system amounts to 90% of the system gross weight. Given that the system rocket engine has a specific impulse of 300 s, determine:
(a) The maximum attainable velocity if all the propellant is burned and the system's initial velocity is 7930 m/s
(b) The propellant mass flow rate, given that the rocket engine thrust is 1,670,000 N

**1.24** A chemical rocket motor with a specific impulse of 400 s is used in the final stage of a multistage launch vehicle for deep-space exploration. This final stage has a mass ratio (initial to final) of 6, and its single rocket motor is first fired while it orbits the Earth at a velocity of 26,000 ft/s. The final stage must reach a velocity of 36,700 ft/s to escape the Earth's gravitational field. Determine the percentage of fuel that must be used to perform this maneuver (neglect gravity and drag).

## Gas Turbine Design Problems

**1.D1** *Background (HP-1 aircraft).* You are to determine the thrust and fuel consumption requirements of the two engines for the hypothetical passenger aircraft, the HP-1. The twin-engine aircraft will cruise at 0.83 Mach and be capable of the following requirements:

1) Takeoff at maximum gross takeoff weight $W_{TO}$ from an airport at 1.6-km pressure altitude on a hot day (38°C) uses a 3650-m (12-kft) runway. The craft is able to maintain a 2.4% single-engine climb gradient in the event of engine failure at liftoff.
2) It transports 253 passengers and luggage (90 kg each) over a still-air distance of 11,120 km (6000 n mile). It has 30 min of fuel in reserve at end (loiter).
3) It attains an initial altitude of 11 km at beginning of cruise ($P_s = 1.5$ m/s).
4) The single-engine craft cruises at 5-km altitude at 0.45 Mach ($P_s = 1.5$ m/s).

All of the data for the HP-1 contained in Example 1.2 apply. Preliminary mission analysis of the HP-1 using the methods of Ref. 12 for the 11,120-km flight with 253 passengers and luggage (22,770-kg payload) gives the preliminary fuel use shown in Table P1.D1.

Analysis of takeoff indicates that each engine must produce an installed thrust of 214 kN on a hot day (38°C) at 0.1 Mach and 1.6-km pressure altitude. To provide for reasonable-length landing gear, the maximum diameter of the engine inlet is limited to 2.2 m. Based on standard design practice (see Chapter 10), the maximum mass flow rate per unit area is given by

$$\frac{\dot{m}}{A} = 231.8\frac{\delta_0}{\sqrt{\theta_0}} \quad (\mathrm{kg/s})/\mathrm{m}^2$$

**Table P1.D1**

| Description | Distance, km | Fuel used, kg |
|---|---|---|
| Taxi | | 200[a] |
| Takeoff | | 840[a] |
| Climb and acceleration | 330 | 5,880[a] |
| Cruise | 10,650 | 50,240 |
| Descent | 140 | 1,090[a] |
| Loiter (30 min at 9-km altitude) | | 2,350 |
| Land and taxi | | 600[a] |
| | 11,120 | 61,200 |

[a]These fuel consumptions can be considered to be constant.

Thus on a hot day (38°C) at 0.1 Mach and 1.6-km pressure altitude, $\theta = (38 + 273.1)/288.2 = 1.079$, $\theta_0 = 1.079 \times 1.002 = 1.081$, $\delta = 0.8256$, $\delta_0 = 0.8256 \times 1.007 = 0.8314$, and the maximum mass flow through the 2.2-m-diam inlet is 704.6 kg/s.

*Calculations (HP-1 Aircraft).*

1) If the HP-1 starts out the cruise at 11 km with a weight of 1,577,940 N, find the allowable TSFC for the distance of 10,650 km for the following cases:
   (a) Assume the aircraft performs a cruise climb (flies at a constant $C_D/C_L$). What is its altitude at the end of the cruise climb?
   (b) Assume the aircraft cruises at a constant altitude of 11 km. Determine $C_D/C_L$ at the start and end of cruise. Using the average of these two values, calculate the allowable TSFC.
2) Determine the loiter (endurance) Mach numbers for altitudes of 10, 9, 8, 7, and 6 km when the HP-1 aircraft is at 64% of $W_{TO}$.
3) Determine the aircraft drag at the following points in the HP-1 aircraft's 11,120-km flight based on the fuel consumptions just listed:
   (a) Takeoff, $M = 0.23$, sea level
   (b) Start of cruise, $M = 0.83$, 11 km
   (c) End of cruise climb, $M = 0.83$, altitude = ? ft
   (d) End of 11-km cruise, $M = 0.83$, 11 km
   (e) Engine out (88% of $W_{TO}$), $M = 0.45$, 5 km

**1.D2** *Background (HF-1 Aircraft).* You are to determine the thrust and fuel consumption requirements of the two engines for the hypothetical fighter aircraft HF-1. This twin-engine fighter will supercruise at 1.6 Mach and will be capable of the following requirements:

1) Takeoff at maximum gross takeoff weight $W_{TO}$ from a 1200-ft (366-m) runway at sea level on a standard day.
2) Supercruise at 1.6 Mach and 40-kft altitude for 250 nm (463 km) at 92% of $W_{TO}$.
3) Perform 5-$g$ turns at 1.6 Mach and 30-kft altitude at 88% of $W_{TO}$.
4) Perform 5-$g$ turns at 0.9 Mach and 30-kft altitude at 88% of $W_{TO}$.
5) Perform the maximum mission listed in the following.

All of the data for the HF-1 contained in Example 1.2 apply. Preliminary mission analysis of the HF-1 using the methods of Ref. 12 for the maximum mission gives the preliminary fuel use shown in Table P1.D2:

Analysis of takeoff indicates that each engine must produce an installed thrust of 23,500 lbf on a standard day at 0.1 Mach and sea-level altitude. To provide for optimum integration into the airframe, the maximum area of the engine inlet is limited to 5 ft$^2$. Based on standard design practice (see Chapter 10), the maximum mass flow rate per unit

**Table P1.D2**

| Description | Distance, nm | Fuel used, lbm |
|---|---|---|
| Warmup, taxi, takeoff | | 700[a] |
| Climb and acceleration to 0.9 Mach and 40 kft | 35 | 1,800[a] |
| Accelerate from 0.9 to 1.6 Mach | 12 | 700[a] |
| Supercruise at 1.6 Mach and 40 kft | 203 | 4,400 |
| Deliver payload of 2000 lbf | 0 | 0[a] |
| Perform one 5-$g$ turn at 1.6 Mach and 30 kft | 0 | 1,000[a] |
| Perform two 5-$g$ turns at 0.9 Mach and 30 kft | 0 | 700[a] |
| Climb to best cruise altitude and 0.9 Mach | 23 | 400[a] |
| Cruise climb at 0.9 Mach | 227 | 1,600 |
| Loiter (20 min at 30-kft altitude) | | 1,100 |
| Land | | 0[a] |
| | 500 | 12,400 |

[a]These fuel consumptions can be considered to be constant.

area for subsonic flight conditions is given by

$$\frac{\dot{m}}{A} = 47.5 \frac{\delta_0}{\sqrt{\theta_0}} \quad (\mathrm{lbm/s})/\mathrm{ft}^2$$

Thus at 0.1 Mach and sea-level standard day, $\theta = 1.0$, $\theta_0 = 1.002$, $\delta = 1.0$, $\delta_0 = 1.007$, and the maximum mass flow through the 5-ft$^2$ inlet is 238.9 lbm/s. For supersonic flight conditions, the maximum mass flow rate per unit area is simply the density of the air $\rho$ times its velocity $V$.

*Calculations (HF-1 Aircraft).*

1) If the HF-1 starts the supercruise at 40 kft with a weight of 36,800 lbf, find the allowable TSFC for the distance of 203 nm for the following cases:
   (a) Assume the aircraft performs a cruise climb (flies at a constant $C_D/C_L$). What is its altitude at the end of the cruise climb?
   (b) Assume the aircraft cruises at a constant altitude of 40 kft. Determine $C_D/C_L$ at the start and end of cruise. Using the average of these two values, calculate the allowable TSFC.
2) Find the best cruise altitude for the subsonic return cruise at 0.9 Mach and 70.75% of $W_{TO}$.
3) Determine the loiter (endurance) Mach numbers for altitudes of 32, 30, 28, 26, and 24 kft when the HF-1 aircraft is at 67% of $W_{TO}$.

4) Determine the aircraft drag at the following points in the HF-1 aircraft's maximum mission based on the fuel consumptions just listed:
(a) Takeoff, $M = 0.172$, sea level
(b) Start of supercruise, $M = 1.6$, 40 kft
(c) End of supercruise climb, $M = 1.6$, altitude = ? ft
(d) End of 40-kft supercruise, $M = 1.6$, 40 kft
(e) Start of subsonic cruise, $M = 0.9$, altitude = best cruise altitude
(f) Start of loiter, altitude = 30 kft