* Sheet 3:

for 2 streams, unboosted, turbofan engine with a specified OPR, TIT and BPR. find expression for the optimum FPR.

$\pi_c, T_{t4}, \alpha \xrightarrow{\text{find}} \tau_F^*$ $\qquad \pi_F(\pi_t, V_9, V_{19})$.

$$S = \frac{f}{(1+\alpha) F/\dot m_0} \qquad \frac{\partial S}{\partial \tau_F} = \frac{-f}{(1+\alpha)} * \frac{1}{(F/\dot m_0)^2} \frac{\partial}{\partial \tau_F}\left(\frac{F}{\dot m_0}\right) = 0$$

* Assuming fully choked nozzles:

$$F = \dot m_c (1+f)(V_9 - V_0) + \dot m_F (V_{19} - V_0)$$

$$\frac{F}{\dot m_0} = \frac{(1+f)}{(1+\alpha)}\left[\frac{V_9}{V_0} - 1\right] + \frac{\alpha}{1+\alpha}\left[\frac{V_{19}}{V_0} - 1\right], \quad \frac{\partial(\frac{F}{\dot m_0})}{\partial \tau_F} = \left(\frac{1+f}{1+\alpha}\right)\overbrace{\frac{\partial(V_9/V_0)}{\partial \tau_F}} + \frac{\alpha}{(1+\alpha)} \overbrace{\frac{\partial(V_{19}/V_0)}{\partial \tau_F}}$$

$$\therefore \boxed{(1+f)\frac{\partial(V_9/V_0)}{\partial \tau_F} + \alpha \frac{\partial(V_{19}/V_0)}{\partial \tau_F} = 0} \; ***$$

$$M_0^2 = \frac{2}{\gamma_c - 1}[\tau_r - 1]$$

$$\boxed{\left(\frac{V_9}{V_0}\right)^2 = \left(\frac{M_9}{M_0}\right)^2 \frac{\gamma_t T_9}{\gamma_c T_0}} \quad \& \quad \frac{T_9}{T_0} = \frac{T_9}{T_{t9}} \underset{1}{\frac{T_{t9}}{T_{t7}}} * \underset{1}{\frac{T_{t7}}{T_{t5}}} * \overset{\tau_t}{\frac{T_{t5}}{T_{t4}}} \frac{T_{t4}}{T_0} * \left[\frac{Cp_t}{Cp_c} \frac{Cp_c}{Cp_t}\right]$$

$$\boxed{\frac{T_9}{T_0} = \left(\frac{P_9}{P_{t9}}\right)^{\frac{\gamma_n - 1}{\gamma_n}} \tau_t * \tau_\lambda \frac{Cp_c}{Cp_t}} \qquad \boxed{\therefore M_9^2 = \frac{2}{\gamma_n - 1}\left[\left(\frac{P_{t9}}{P_9}\right)^{\frac{\gamma_n-1}{\gamma_n}} - 1\right]} \leftarrow \text{I}$$

$$\therefore \frac{P_{t9}}{P_0} = \underset{1}{\frac{P_{t9}}{P_{t7}}} \underset{1}{\frac{P_{t7}}{P_{t5}}} \underset{\pi_t}{\frac{P_{t5}}{P_{t4}}} \underset{\pi_b}{\frac{P_{t4}}{P_{t3}}} \underbrace{\overset{\pi_{cH}}{\frac{P_{t3}}{P_{t2.5}}} \overset{\pi_F}{\frac{P_{t2.5}}{P_{t2}}}}_{\pi_c} \underset{\pi_{dif}}{\frac{P_{t2}}{P_{t0}}} \underset{\pi_r}{\frac{P_{t0}}{P_0}} \underset{1}{\frac{P_0}{P_9}} = \boxed{[\overbrace{\pi_b \pi_c \pi_{dif} \pi_r}^{\pi_1}] \, \tau_t^{\frac{\gamma_t}{(\gamma_t - 1)e_t}}}$$

$$M_9^2 = \frac{2}{\gamma_n - 1}\left[\pi_1^{\frac{\gamma_n - 1}{\gamma_n}} * \tau_t^{\frac{(\gamma_n-1)\gamma_t}{(\gamma_t-1)\gamma_n e_t}} - 1\right] \quad \text{note that } \frac{P_{t9}}{P_0} = \frac{P_{t9}}{P_9}$$

$$\therefore \frac{T_9}{T_0} = \pi_1^{\frac{1-\gamma_n}{\gamma_n}} \tau_t^{\frac{(1-\gamma_n)\gamma_t}{(\gamma_t-1)\gamma_n e_t}} \tau_t \tau_\lambda \frac{Cp_c}{Cp_t} = \pi_1^{\frac{1-\gamma_n}{\gamma_n}} \tau_t^{1 - \frac{\gamma_t(\gamma_n-1)}{\gamma_n(\gamma_t-1)} \frac{1}{e_t}} \tau_\lambda \frac{Cp_c}{Cp_t}$$

$$\left(\frac{V_9}{V_0}\right)^2 = \frac{2\cancel{(\gamma_c - 1)}\cancel{\gamma_t}}{\cancel{\gamma_c}(\gamma_n - 1) 2 (\tau_r - 1)}\left[\pi_1^{\frac{\gamma_n-1}{\gamma_n}} \tau_t^{\frac{(\gamma_n-1)\gamma_t}{(\gamma_t-1)\gamma_n e_t}} - 1\right] * \left[\pi_1^{\frac{1-\gamma_n}{\gamma_n}} \tau_t^{1 - \frac{\gamma_t(\gamma_n-1)}{\gamma_n(\gamma_t-1)e_t}} \tau_\lambda \cancel{\frac{\gamma_c(\gamma_t-1)}{\gamma_t(\gamma_c-1)}}\right]$$

$$\frac{\dot m_c}{\dot m_c + \dot m_F} = \frac{1}{(1+\alpha)} \qquad \frac{\dot m_F}{\dot m_c + \dot m_F} = \frac{1}{1/\alpha + 1} = \frac{\alpha}{1+\alpha}$$

$\dot m_0 \rightarrow$ $\rightarrow \dot m_F$, $\rightarrow \dot m_c$

$$\therefore \boxed{\left(\frac{V_9}{V_0}\right)^2 = \overbrace{\left(\frac{\tau_\lambda}{(\tau_r - 1)} \frac{(\gamma_t - 1)}{(\gamma_n - 1)} \pi_1^{\frac{1-\gamma_n}{\gamma_n}}\right)}^{A} * \left[\pi_1^{\frac{\gamma_n - 1}{\gamma_n}} \tau_t - \tau_t^{1 - \frac{\gamma_t(\gamma_n-1)}{\gamma_n(\gamma_t-1)e_t}}\right]}$$

Similarly:

$$\boxed{\left(\frac{V_{19}}{V_0}\right)^2 = \overbrace{\left(\frac{\tau_r}{\tau_r - 1} \pi_2^{\frac{1-\gamma_c}{\gamma_c}}\right)}^{B} \left[\pi_2^{\frac{\gamma_c - 1}{\gamma_c}} \tau_F - \tau_F^{1 - e_F}\right]}$$

notings:
$\tau_\lambda \equiv \tau_r$
$\gamma_t \equiv \gamma_F$
$\gamma_c = \gamma_F$

$\pi_1 = \pi_b \pi_c \pi_{dif} \pi_r$
$\pi_2 = \pi_{Fdif} \pi_r$

$$\frac{\partial (V_9/V_0)^2}{\partial \tau_F} = 2 \frac{\partial(V_9/V_0)}{\partial \tau_F}(V_9/V_0) \Rightarrow \frac{\partial(V_9/V_0)}{\partial \tau_F} = \frac{1}{2(V_9/V_0)} \frac{\partial (V_9/V_0)^2}{\partial \tau_F}$$

$$\therefore \frac{\partial(V_9/V_0)^2}{\partial \tau_F} = A\left\{\pi_1^{\frac{\gamma_n-1}{\gamma_n}} \frac{\partial \tau_t}{\partial \tau_F} - \left(1 - \frac{\gamma_t(\gamma_n-1)}{\gamma_n(\gamma_t-1)e_t}\right) \tau_t^{-\frac{\gamma_t(\gamma_n-1)}{\gamma_n(\gamma_t-1)e_t}} * \frac{\partial \tau_t}{\partial \tau_F}\right\}$$

$$\tau_t \Leftrightarrow \tau_F$$

$$\boxed{\therefore \tau_t = 1 - \frac{\tau_r/\tau_\lambda}{\eta_m(1+f)}\left[(\tau_c - 1) + \alpha(\tau_F - 1)\right]} \rightarrow \frac{\partial \tau_t}{\partial \tau_F} = \frac{-\alpha \tau_r/\tau_\lambda}{\eta_m(1+f)}$$

$$\frac{\partial(V_9/V_0)}{\partial \tau_F} = \frac{-\alpha \tau_r/\tau_\lambda A^{1/2}}{2\eta_m(1+f)}\left[\pi_1^{\frac{\gamma_n-1}{\gamma_n}} \tau_t - \tau_t^{1 - \frac{\gamma_t(\gamma_n-1)}{\gamma_n(\gamma_t-1)e_t}}\right]^{-1}\left[\pi_1^{\frac{\gamma_n-1}{\gamma_n}} - \left(1 - \frac{\gamma_t(\gamma_n-1)}{\gamma_n(\gamma_t-1)e_t}\right) * \tau_t^{-\frac{\gamma_t(\gamma_n-1)}{\gamma_n(\gamma_t-1)e_t}}\right]^{1/2} \; *①$$

$\Phi_1(\tau_t)$

$$\frac{\partial(V_{19}/V_0)^2}{\partial \tau_F} = B\left[\pi_2^{\frac{\gamma_c-1}{\gamma_c}} - (1 - e_F)\tau_F^{-e_F}\right]$$

$$\therefore \frac{\partial(V_{19}/V_0)}{\partial \tau_F} = \frac{\left(\pi_2^{\frac{\gamma_c-1}{\gamma_c}} - (1-e_F)\tau_F^{-e_F}\right) B^{1/2}}{2\left[\pi_2^{\frac{\gamma_c-1}{\gamma_c}} \tau_F - \tau_F^{1-e_F}\right]^{1/2}} = \Phi_2(\tau_F) \; **$$

$$\therefore \boxed{(1+f)\,\Phi_1(\tau_t) + \alpha\,\Phi_2(\tau_F) = 0}$$