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AER408
Aerospace Guidance & Control Systems

Task (4)
Airplane Equations of Motion Linearization

Task statement:

a) "Linearize the complete set of the equations of motion of fixed wing airplane at a symmetric flight reference condition at which reference angle of attack ($\alpha_o \neq 0$). Decouple the equations into Longitudinal & Lateral dynamics and write them in the state space form"

b) "State the short period & Long period approximations of the Longitudinal dynamics, and the 3-DOF, 2-DOF, 1-DOF approximations of the Lateral dynamics"

c) "Find the step response of your airplane using all the models of the longitudinal dynamics you have obtained (Non-linear simulation, Linear full model, Short period approximation, Long period approximation). Compare & Discuss the results introduced by the different models at each of the following set of inputs respectively (each one at a time)

- Elevator: $\delta_{elevator}$= 1^o, 5^o, 10^o, 25^o
- Thrust: δ_{thrust} = 2000, 6000, 10000 lbs

d) "Find the step response of your airplane using all the models of the lateral dynamics you have obtained (Non-linear simulation, Linear full model, 3-DOF, 2-DOF, 1-DOF). Compare & Discuss the results introduced by the different models at each of the following set of inputs respectively (each one at a time)

- Aileron: $\delta_{aileron}$ = 1^o, 5^o, 10^o, 25^o
- Rudder: δ_{rudder} = 1^o, 5^o, 10^o, 25^o

e) “Extract all the transfer functions of your airplane using all the models of the longitudinal dynamics you have obtained (Linear full model, Short period approximation, Long period approximation). Plot & Compare the Root locus & Bode plot of the different representations of each transfer function

f) “Extract all the transfer functions of your airplane using all the models of the lateral dynamics you have obtained (Linear full model, 3-DOF, 2-DOF, 1-DOF). Plot & Compare the Root locus & Bode plot of the different representations of each transfer function

Notes & Hints

The (step) function in Matlab

When you use the function (step) in Matlab, it gives you the response for a unit step input, i.e. the magnitude of the input is 1

For example, if you are getting the response of the transfer function (u/δe) you are getting the response of (Δu) to a step input in the elevator of magnitude 1, and since the control actions are in (rad) this means that the elevator's deflection is 1rad = 57.32° !! this is very large

So, <u>when you use the function step do not forget to multiply the response by the amplitude of the control action</u> which you write in the excel sheet, and which you get the response of the non-linear simulation code for.

For a control system defined in state-space form, where state matrix **A**, control matrix **B**, output matrix **C**, and direct transmission matrix **D** of state-space equations are known, the command

step(A,B,C,D)

will generate plots of unit-step responses. The time vector is automatically determined when t is not explicitly included in the step commands.

Note that the command step(sys) may be used to obtain the unit-step response of a system. First, we define the system by

sys = tf(num,den)

or

sys = ss(A,B,C,D)

Then, to obtain, for example, the unit-step response, we enter

step(sys)

into the computer.

When step commands have left-hand arguments, such as

```
[y,x,t] = step(num,den,t)
[y,x,t] = step(A,B,C,D,iu)
[y,x,t] = step(A,B,C,D,iu,t)        (3-1)
```

no plot is shown on the screen. Hence, it is necessary to use a plot command to see the response curves. The matrices y and x contain the output and state response of the system, respectively, evaluated at the computation time points t (y has as many columns as outputs and one row for each element in t; x has as many columns as states and one row for each element in t.)

Note in Equation (3–1) that the scalar iu is an index into the inputs of the system and specifies which input is to be used for the response; t is the user-specified time. If the system involves multiple inputs and multiple outputs, the step command, as given by Equation (3–1), produces a series of step response plots one for each input and output combination of

The magnitude of the response from the linear simulation

The response obtained from the transfer function <u>is not the magnitude of the response</u>, but it is the change in the states value from the initial value, i.e. (Δu) not (u), so do not forget to add this response to the initial values of the states to obtain the magnitude of the response for this input, for example $u = u_o + \Delta u$