Cairo University Faculty of Engineering Aerospace Department

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Task 4-2

Airframe model

Autopilot -AER 408
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Team 4

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a) Linearization of 12 EOM for a fixed wing A/C

EOM linearization:

EOM

EOM	Airframe derivatives
$X - mg\, S_\theta = m(\dot{u} + qw - rv)$	$\Delta X/m = X_u\, \Delta u + X_w\, \Delta w + X_{\delta_e}\, \Delta\delta_e + X_{\delta_{th}}\, \Delta\delta_{th}$
$Y + mg\, C_\theta\, S_\varphi = m(\dot{v} + ru - pw)$	$\Delta Y/m = Y_\beta\, \Delta\beta + Y_r\, \Delta r + Y_p\, \Delta p + Y_{\delta a}\, \Delta\delta_a + Y_{\delta_r}\, \Delta\delta_r$
$Z + mg\, C_\theta\, C_\varphi = m(\dot{w} + pv - qu)$	$\Delta Z/m = Z_u\, \Delta u + Z_w\, \Delta w + Z_{\dot{w}}\, \Delta\dot{w} + Z_q\, \Delta q + Z_{\delta_e}\, \Delta\delta_e + Z_{\delta_{th}}\, \Delta\delta_{th}$

EOM	Airframe derivatives
$L = I_x\dot{p} - I_{xz}\dot{r} + qr(I_z - I_y) - I_{xz}pq$	$\Delta L/I_{xx} = L_\beta\, \Delta\beta + L_p\, \Delta p + L_r\, \Delta r + L_{\delta_r}\, \Delta\delta_r + L_{\delta_a}\, \Delta\delta_a$
$M = I_y\dot{q} + rp(I_x - I_z) - I_{xz}(p^2 - r^2)$	$\Delta M/I_{yy} = M_u\, \Delta u + M_w\, \Delta w + M_{\dot{w}}\, \Delta\dot{w} + M_q\, \Delta q + M_{\delta_e}\, \Delta\delta_e + M_{\delta_{th}}\, \Delta\delta_{th}$
$N = -I_{xz}\dot{p} + I_z\dot{r} + pq(I_y - I_x) + I_{xz}qr$	$\Delta N/I_{zz} = N_\beta\, \Delta\beta + N_p\, \Delta p + N_r\, \Delta r + N_{\delta_r}\, \Delta\delta_r + N_{\delta_a}\, \Delta\delta_a$

$$\dot{\varphi} = p + q\, S_\varphi\, T_\theta + r\, C_\varphi\, T_\theta$$
$$\dot{\theta} = q\, C_\varphi - r\, S_\varphi$$
$$\dot{\psi} = (q\, S_\varphi + r\, C_\varphi)\, sec(\theta)$$

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \end{bmatrix} = \begin{bmatrix} C_\theta C_\psi & S_\varphi S_\theta C_\psi - C_\varphi S_\psi & C_\psi S_\theta C_\psi + S_\varphi S_\psi \\ C_\theta S_\psi & S_\varphi S_\theta S_\psi + C_\varphi C_\psi & C_\varphi S_\theta S_\psi - S_\varphi C_\psi \\ -S_\theta & S_\varphi C_\theta & C_\varphi C_\theta \end{bmatrix} \begin{bmatrix} u \\ v \\ w \end{bmatrix}$$

1) Let:$(A = A_o + \Delta A)$

$u = u_o + \Delta u$	$p = p_o + \Delta p$	$X = X_o + \Delta X$	$L = L_o + \Delta L$	$\varphi = \varphi_o + \Delta\varphi$
$v = v_o + \Delta v$	$q = q_o + \Delta q$	$Y = Y_o + \Delta Y$	$M = M_o + \Delta M$	$\theta = \theta_o + \Delta\theta$
$w = w_o + \Delta w$	$r = r_o + \Delta r$	$Y = Y_o + \Delta Y$	$N = N_o + \Delta N$	$\psi = \psi_o + \Delta\psi$

And $v_o = p_o = q_o = r_o = \varphi_o = \psi_o = 0$ but $u_o, w_o, \theta_o \neq 0$ for Cruise condition.

2) Longitudinal calculations:

a) $X - mg\,Sin(\theta) = m(\dot{u} + qw - rv)$

i. Recast each variable in terms of a steady-state value and a perturbed value.

$$X_o + \Delta X - mg\,Sin(\theta_o + \Delta\theta)$$
$$= m(\dot{u}_o + \Delta\dot{u} + (q_o + \Delta q)(w_o + \Delta w) - (r_o + \Delta r)(v_o + \Delta v))$$
$$\sin(a+b) = \sin(a)\cos(b) + \sin(b)\cos(a)$$
$$X_o + \Delta X - mg\,(\sin(\theta_o)\cos(\Delta\theta) + \sin(\Delta\theta)\cos(\theta_o))$$
$$= m\big(\Delta\dot{u} + (\Delta q)(w_o + \Delta w) - (\Delta r)(\Delta v)\big)$$

ii. Apply the small-angle assumption to trig functions of perturbed angles

$$\sin(a \to 0) = a\,, \quad \cos(a \to 0) = 1$$
$$X_o + \Delta X - mg\;\sin(\theta_o) - mg\;\cos(\theta_o)\,\Delta\theta$$
$$= m(\Delta\dot{u} + \Delta q * w_o + \Delta q * \Delta w - \Delta r * \Delta v)$$

iii. Assume products of small perturbations are negligible.

$$X_o + \Delta X - mg\;\sin(\theta_o) - mg\;\cos(\theta_o)\,\Delta\theta = m(\Delta\dot{u} + \Delta q * w_o)$$

iv. Remove the steady-state equation from the perturbed equation.

$$\therefore \Delta X - mg\;\cos(\theta_o)\,\Delta\theta = m(\Delta\dot{u} + \Delta q * w_o)$$
$$\therefore \Delta\dot{u} = \frac{\Delta X}{m} - g\;\cos(\theta_o)\,\Delta\theta - \Delta q * w_o \quad , \quad \frac{\Delta X}{m}$$
$$= X_u\,\Delta u + X_w\,\Delta w + X_{\delta_e}\,\Delta\delta_e + X_{\delta_{th}}\,\Delta\delta_{th}$$

$$\therefore \Delta\dot{u} = X_u\,\Delta u + X_w\,\Delta w + X_{\delta_e}\,\Delta\delta_e + X_{\delta_{th}}\,\Delta\delta_{th} - g\;\cos(\theta_o)\,\Delta\theta - \Delta q * w_o$$

b) $Z + mg\,Cos(\theta) * Cos(\varphi) = m(\dot{w} + pv - qu)$

Same as (a)

$$\cos(a+b) = \cos(a)\cos(b) - \sin(a)\sin(b)$$
$$\therefore (1 - Z_{\dot{w}})\Delta\dot{w}$$
$$= Z_u\,\Delta u + Z_w\,\Delta w + (Z_q + u_o)\,\Delta q + Z_{\delta_e}\,\Delta\delta_e + Z_{\delta_{th}}\,\Delta\delta_{th} - g\;\sin(\theta_o)\,\Delta\theta$$

$$\therefore \Delta\dot{w} = \frac{Z_u}{(1 - Z_{\dot{w}})}\,\Delta u + \frac{Z_w}{(1 - Z_{\dot{w}})}\,\Delta w + \frac{(Z_q + u_o)}{(1 - Z_{\dot{w}})}\,\Delta q + \frac{Z_{\delta_e}}{(1 - Z_{\dot{w}})}\,\Delta\delta_e + \frac{Z_{\delta_{th}}}{(1 - Z_{\dot{w}})}\,\Delta\delta_{th} - \frac{g\;\sin(\theta_o)}{(1 - Z_{\dot{w}})}\,\Delta\theta$$

c) $M = I_y\dot{q} + rp(I_x - I_z) - I_{xz}(p^2 - r^2)$

$$\therefore \Delta\dot{q} = M_u\,\Delta u + M_w\,\Delta w + M_{\dot{w}} * \left(\frac{Z_u}{(1-Z_{\dot{w}})}\,\Delta u + \frac{Z_w}{(1-Z_{\dot{w}})}\,\Delta w + \frac{(Z_q+u_o)}{(1-Z_{\dot{w}})}\,\Delta q + \frac{Z_{\delta_e}}{(1-Z_{\dot{w}})}\,\Delta\delta_e + \frac{Z_{\delta_{th}}}{(1-Z_{\dot{w}})}\,\Delta\delta_{th} - \frac{g\,\sin(\theta_o)}{(1-Z_{\dot{w}})}\Delta\theta\right) + M_q\,\Delta q + M_{\delta_e}\,\Delta\delta_e + M_{\delta_{th}}\,\Delta\delta_{th}$$

$$\therefore \Delta\dot{q} = \left(M_u + M_{\dot{w}}\frac{Z_u}{(1-Z_{\dot{w}})}\right)\Delta u + \left(M_w + M_{\dot{w}}\frac{Z_w}{(1-Z_{\dot{w}})}\right)\Delta w + \left(M_q + M_{\dot{w}}\frac{(Z_q+u_o)}{(1-Z_{\dot{w}})}\right)\Delta q + \left(M_{\delta_e} + M_{\dot{w}}\frac{Z_{\delta_e}}{(1-Z_{\dot{w}})}\right)\Delta\delta_e + \left(M_{\delta_{th}} + M_{\dot{w}}\frac{Z_{\delta_{th}}}{(1-Z_{\dot{w}})}\right)\Delta\delta_{th} - M_{\dot{w}} * \frac{g\,\sin(\theta_o)}{(1-Z_{\dot{w}})}\Delta\theta$$

d) $\dot{\theta} = q\,C_\varphi - r\,S_\varphi$

$$\therefore \Delta\dot{\theta} = \Delta q$$

3) Lateral calculations:

a) $Y + mg\,C_\theta\,S_\varphi = m(\dot{v} + ru - pw)$

$$\Delta\dot{v} = Y_\beta\,\Delta\beta + Y_r\,\Delta r + Y_p\,\Delta p + Y_{\delta a}\,\Delta\delta_a + Y_{\delta_r}\,\Delta\delta_r + g\,\cos(\theta_o)\,\Delta\emptyset - \Delta r * u_o + w_o * \Delta p$$

$$\Delta\dot{\beta} = \frac{\Delta\dot{v}}{V_{to}}$$

$$\therefore \Delta\dot{\beta} = \frac{Y_\beta}{V_{to}}\,\Delta\beta + \frac{Y_r - u_o}{V_{to}}\,\Delta r + \frac{Y_p + w_o}{V_{to}}\,\Delta p + \frac{Y_{\delta a}}{V_{to}}\,\Delta\delta_a + \frac{Y_{\delta_r}}{V_{to}}\,\Delta\delta_r + \frac{g}{V_{to}}\,\cos(\theta_o)\,\Delta\varphi$$

b) $L = I_x\dot{p} - I_{xz}\dot{r} + qr(I_z - I_y) - I_{xz}pq$

$$eq(1):\quad \Delta\dot{p} - \frac{I_{xz}}{I_x} * \Delta\dot{r} = \frac{\Delta L}{I_x} = L_\beta\,\Delta\beta + L_p\,\Delta p + L_r\,\Delta r + L_{\delta_r}\,\Delta\delta_r + L_{\delta_a}\,\Delta\delta_a$$

c) $N = -I_{xz}\dot{p} + I_z\dot{r} + pq(I_y - I_x) + I_{xz}qr$

$$eq(2): \quad \Delta\dot{r} - \frac{I_{xz}}{I_z} * \Delta\dot{p} = \frac{\Delta N}{I_z}$$
$$= N_\beta\,\Delta\beta + N_p\,\Delta p + N_r\,\Delta r + N_{\delta_r}\,\Delta\delta_r + N_{\delta_a}\,\Delta\delta_a$$

Solve $eq(1)$ *and* $eq(2)$ *in* $(\Delta\dot{\boldsymbol{r}} \quad \Delta\dot{\boldsymbol{p}})$ *we get*:

$$\therefore \Delta\dot{p} = \left(\frac{\Delta L}{I_x} + \frac{I_{xz}}{I_x} * \frac{\Delta N}{I_z}\right) * G \quad , \quad where \quad G = \frac{I_x I_z}{I_x I_z - (I_{xz})^2}$$

$$\therefore \Delta\dot{r} = \left(\frac{I_{xz}}{I_z} * \frac{\Delta L}{I_x} + \frac{\Delta N}{I_z}\right) * G \quad , \quad where \quad G = \frac{I_x I_z}{I_x I_z - (I_{xz})^2}$$

$$\frac{\Delta L}{I_x} = \sum L_i\,\Delta i \quad , \quad \frac{\Delta N}{I_z} = \sum N_i\,\Delta i$$

$$\therefore \Delta\dot{p} = \sum G * \left(L_i + \frac{I_{xz}}{I_x} * N_i\right)\Delta i = \sum L_i^*\,\Delta i$$
$$= L_\beta^*\,\Delta\beta + L_p^*\,\Delta p + L_r^*\,\Delta r + L_{\delta_r}^*\,\Delta\delta_r + L_{\delta_a}^*\,\Delta\delta_a$$

$$\therefore \Delta\dot{r} = \sum G * \left(N_i + \frac{I_{xz}}{I_z} * L_i\right)\Delta i = \sum N_i^*\,\Delta i$$
$$= N_\beta^*\,\Delta\beta + N_p^*\,\Delta p + N_r^*\,\Delta r + N_{\delta_r}^*\,\Delta\delta_r + N_{\delta_a}^*\,\Delta\delta_a$$

d) $\dot{\varphi} = p + q\,Sin(\varphi)\,Tan(\theta) + r\,Cos(\varphi)\,Tan(\theta)$

$$\dot{\varphi}\cos(\theta) = p\cos(\theta) + q\,Sin(\varphi)\,sin(\theta) + r\,Cos(\varphi)\,sin(\theta)$$

$$\therefore \Delta\dot{\varphi} = \Delta p + \Delta r * \tan(\theta_0)$$

e) $\dot{\psi} = (q\,S_\varphi + r\,C_\varphi)\;sec(\theta)$

$$\therefore \Delta\dot{\psi} = \Delta r * \sec(\theta_0)$$

4) State space Model:

i. Longitudinal

- $\Delta\dot{u} = X_u\,\Delta u + X_w\,\Delta w + X_{\delta_e}\,\Delta\delta_e + X_{\delta_{th}}\,\Delta\delta_{th} - g\,\cos(\theta_o)\,\Delta\theta - \Delta q * w_o$
- $\therefore \Delta\dot{w} = \frac{Z_u}{(1-Z_{\dot{w}})}\,\Delta u + \frac{Z_w}{(1-Z_{\dot{w}})}\,\Delta w + \frac{(Z_q+u_o)}{(1-Z_{\dot{w}})}\,\Delta q + \frac{Z_{\delta_e}}{(1-Z_{\dot{w}})}\,\Delta\delta_e + \frac{Z_{\delta_{th}}}{(1-Z_{\dot{w}})}\,\Delta\delta_{th} - \frac{g\,\sin(\theta_o)}{(1-Z_{\dot{w}})}\Delta\theta$
- $\therefore \Delta\dot{q} = \left(M_u + M_{\dot{w}}\frac{Z_u}{(1-Z_{\dot{w}})}\right)\Delta u + \left(M_w + M_{\dot{w}}\frac{Z_w}{(1-Z_{\dot{w}})}\right)\Delta w + \left(M_q + M_{\dot{w}}\frac{(Z_q+u_o)}{(1-Z_{\dot{w}})}\right)\Delta q + \left(M_{\delta_e} + M_{\dot{w}}\frac{Z_{\delta_e}}{(1-Z_{\dot{w}})}\right)\Delta\delta_e + \left(M_{\delta_{th}} + M_{\dot{w}}\frac{Z_{\delta_{th}}}{(1-Z_{\dot{w}})}\right)\Delta\delta_{th} - M_{\dot{w}} * \frac{g\,\sin(\theta_o)}{(1-Z_{\dot{w}})}\Delta\theta$
- $\therefore \Delta\dot{\theta} = \Delta q$

$$\begin{bmatrix}\Delta\dot{u}\\ \Delta\dot{w}\\ \Delta\dot{q}\\ \Delta\dot{\theta}\end{bmatrix} = \begin{bmatrix} X_u & X_w & -w_o & -g\,\cos(\theta_o) \\ \frac{Z_u}{(1-Z_{\dot{w}})} & \frac{Z_w}{(1-Z_{\dot{w}})} & \frac{(Z_q+u_o)}{(1-Z_{\dot{w}})} & \frac{-g\,\sin(\theta_o)}{(1-Z_{\dot{w}})} \\ M_u + M_{\dot{w}}\frac{Z_u}{(1-Z_{\dot{w}})} & M_w + M_{\dot{w}}\frac{Z_w}{(1-Z_{\dot{w}})} & M_q + M_{\dot{w}}\frac{(Z_q+u_o)}{(1-Z_{\dot{w}})} & -M_{\dot{w}} * \frac{g\,\sin(\theta_o)}{(1-Z_{\dot{w}})} \\ 0 & 0 & 1 & 0 \end{bmatrix}\begin{bmatrix}\Delta u\\ \Delta w\\ \Delta q\\ \Delta\theta\end{bmatrix} + \begin{bmatrix} X_{\delta_e} & X_{\delta_{th}} \\ \frac{Z_{\delta_e}}{(1-Z_{\dot{w}})} & \frac{Z_{\delta_{th}}}{(1-Z_{\dot{w}})} \\ M_{\delta_e} + M_{\dot{w}}\frac{Z_{\delta_e}}{(1-Z_{\dot{w}})} & M_{\delta_{th}} + M_{\dot{w}}\frac{Z_{\delta_{th}}}{(1-Z_{\dot{w}})} \\ 0 & 0 \end{bmatrix}\begin{bmatrix}\Delta\delta_e\\ \Delta\delta_{th}\end{bmatrix}$$

ii. Lateral

- $\therefore \Delta\dot{\beta} = \frac{Y_\beta}{V_{to}}\,\Delta\beta + \frac{Y_r - u_o}{V_{to}}\,\Delta r + \frac{Y_p + w_o}{V_{to}}\,\Delta p + \frac{Y_{\delta a}}{V_{to}}\,\Delta\delta_a + \frac{Y_{\delta_r}}{V_{to}}\,\Delta\delta_r + \frac{g}{V_{to}}\cos(\theta_o)\,\Delta\varphi$
- $\therefore \Delta\dot{p} = \sum G * \left(L_i + \frac{I_{xz}}{I_x} * N_i\right)\Delta i = \sum L_i^*\,\Delta i = L_\beta^*\,\Delta\beta + L_p^*\,\Delta p + L_r^*\,\Delta r + L_{\delta_r}^*\,\Delta\delta_r + L_{\delta_a}^*\,\Delta\delta_a$
- $\therefore \Delta\dot{r} = \sum G * \left(N_i + \frac{I_{xz}}{I_z} * L_i\right)\Delta i = \sum N_i^*\,\Delta i = N_\beta^*\,\Delta\beta + N_p^*\,\Delta p + N_r^*\,\Delta r + N_{\delta_r}^*\,\Delta\delta_r + N_{\delta_a}^*\,\Delta\delta_a$
- $\therefore \Delta\dot{\varphi} = \Delta p + \Delta r * \tan(\theta_0)$
- $\therefore \Delta\dot{\psi} = \Delta r * \sec(\theta_0)$

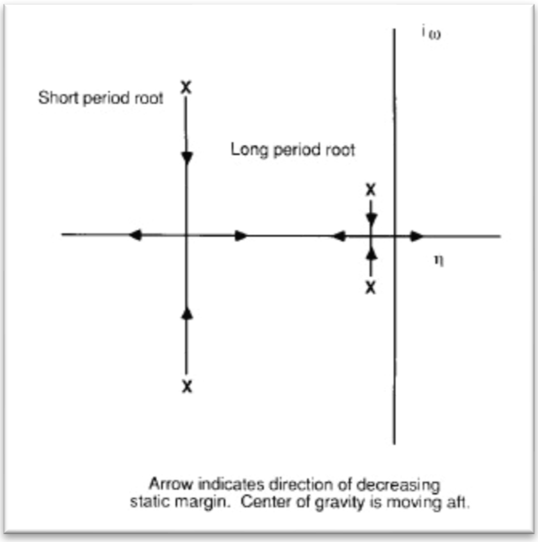
$$\begin{bmatrix} \Delta\dot{\beta} \\ \Delta\dot{p} \\ \Delta\dot{r} \\ \Delta\dot{\varphi} \\ \Delta\dot{\psi} \end{bmatrix} = \begin{bmatrix} \frac{Y_\beta}{V_{to}} & \frac{Y_p + w_o}{V_{to}} & \frac{Y_r - u_o}{V_{to}} & \frac{g\cos(\theta_o)}{V_{to}} & 0 \\ L_\beta^* & L_p^* & L_r^* & 0 & 0 \\ N_\beta^* & N_p^* & N_r^* & 0 & 0 \\ 0 & 1 & \tan(\theta_0) & 0 & 0 \\ 0 & 0 & \sec(\theta_0) & 0 & 0 \end{bmatrix} \begin{bmatrix} \Delta\beta \\ \Delta p \\ \Delta r \\ \Delta\varphi \\ \Delta\psi \end{bmatrix} + \begin{bmatrix} \frac{Y_{\delta a}}{V_{to}} & \frac{Y_{\delta_r}}{V_{to}} \\ L_{\delta_a}^* & L_{\delta_r}^* \\ \Delta\delta_a & N_{\delta_r}^* \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \Delta\delta_a \\ \Delta\delta_r \end{bmatrix}$$

b) Longitudinal dynamics approximations

The longitudinal dynamics of the aircraft refer to dynamics in the x-z plane in the longitudinal axis

there are two types of approximations for that system of equations

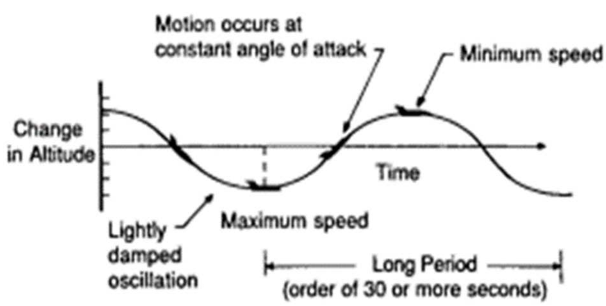
1. Long period approximation (mode)
2. Short period approximation (mode)



By solving the A matrix from part, a we get 2 pairs of conjugate roots the pair which has small real part has big settling time according to $T_s = \frac{4}{\zeta\omega_n}$

on the other hand with the pair which has bigger real part

Long period approximation (mode)



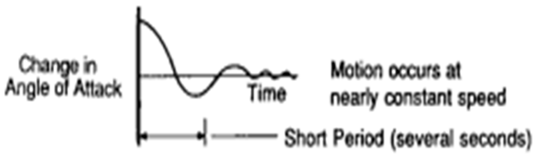
When the aircraft is exposed to some disturbances in the longitudinal direction a change in the altitude is formed due to this disturbance assuming that the angle of attack is constant the approximation to the long period mode is obtained by neglecting the pitching moment equation and assuming that the changes in angle of attack is zero.

$$\triangle \alpha = 0 \quad \Delta\dot{w} = 0$$

$$\triangle \alpha = \frac{\Delta w}{U_o} \quad \Delta\dot{q} = 0$$

$$\begin{bmatrix} \dot{u} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} X_u + \dfrac{w_o Z_u}{Z_q + u_o} & -g\cos(\theta_o) - \dfrac{w_o g sin(\theta_o)}{Z_q + u_o} \\ \dfrac{-Z_u}{Z_q + u_o} & \dfrac{g sin(\theta_o)}{Z_q + u_o} \end{bmatrix} \begin{bmatrix} u \\ \theta \end{bmatrix} + \begin{bmatrix} X_{\delta_e} + \dfrac{w_o Z_{\delta_e}}{Z_q + u_o} & X_{\delta_{th}} + \dfrac{w_o Z_{\delta_{th}}}{Z_q + u_o} \\ \dfrac{-Z_{\delta_e}}{Z_q + u_o} & \dfrac{-Z_{\delta_{th}}}{Z_q + u_o} \end{bmatrix} \begin{bmatrix} \Delta\delta_e \\ \Delta\delta_{th} \end{bmatrix}$$

short period approximation (mode)



The same definition which we applied in the previous mode we will use here but with taking in consideration that the disturbance will damp quickly and but there will be a change in angle of attack and neglecting the change in velocity.

$$\Delta u = 0$$

$$\Delta\dot{\theta} = 0$$

$$M_\alpha = \frac{1}{I_y} * \frac{\partial M}{\partial \alpha} = \frac{1}{I_y} * \frac{\partial M}{\partial \left(\frac{\Delta w}{u_o}\right)} = \frac{u_o}{I_y} * \frac{\partial M}{\partial w} = u_o M_w$$

$$Z_\propto = u_o Z_w$$

$$M_{\dot{\alpha}} = u_o M_{\dot{w}}$$

$$\begin{bmatrix} \dot{\alpha} \\ \dot{q} \end{bmatrix} = \begin{bmatrix} \frac{Z_\alpha}{u_o} & 1 \\ M_\alpha + \frac{M_{\dot{\alpha}}\, Z_\alpha}{u_o} & M_q + M_{\dot{\alpha}} \end{bmatrix} \begin{bmatrix} \alpha \\ q \end{bmatrix} + \begin{bmatrix} Z_{\delta_e} & Z_{\delta_{th}} \\ M_{\delta_e} + \frac{M_\alpha\, Z_{\delta_e}}{u_o} & M_{\dot{\alpha}} + \frac{M_\alpha\, Z_{\delta_{th}}}{u_o} \end{bmatrix} \begin{bmatrix} \Delta\delta_e \\ \Delta\delta_{th} \end{bmatrix}$$

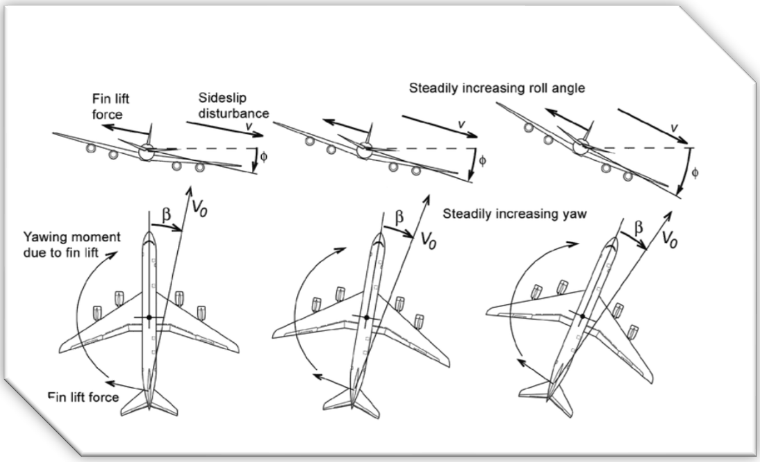
and by solving the characteristic equation of the system and getting its Eigen values we finally reach that;

$$w_n = \sqrt{(M_q \frac{Z_\alpha}{u_o} - M_\alpha)}$$

$$\zeta = \frac{\frac{Z_\alpha}{u_o} + M_q + M_{\dot{\alpha}}}{-2 w_n}$$

Lateral dynamics approximations

3-DOF spiral mode



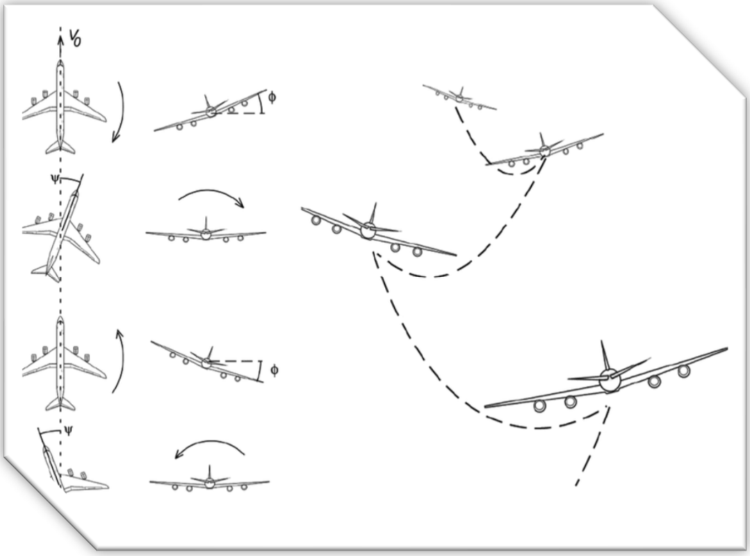
As it is clear in the attached figure how does the airplane behave when it is in the spiral mode.

The side force equation of motion will be neglected and will put$\Delta\beta = 0$.

Therefore, the 3 DOF spiral approximation will be as shown in the following:

$$\begin{bmatrix} \Delta\dot{p} \\ \Delta\dot{r} \\ \Delta\dot{\varphi} \end{bmatrix} = \begin{bmatrix} \grave{L}_p & \grave{L}_r & 0 \\ \grave{N}_p & \grave{N}_r & 0 \\ 1 & \tan(\theta_o) & 0 \end{bmatrix} \begin{bmatrix} \Delta p \\ \Delta r \\ \Delta\varphi \end{bmatrix} + \begin{bmatrix} \acute{L}_{\delta_r} \\ \acute{N}_{\delta_r} \\ 0 \end{bmatrix} [\,\Delta\delta_r]$$

2-DOF Dutch-Roll mode



The combination of the roll and yaw approximations is called the Dutch roll Motion.

In this equation the rolling moment equation is neglected and we put$\Delta\varphi = 0$.

Therefore, the 2 DOF spiral approximation will be as shown in the following:

$$\begin{bmatrix} \Delta\dot{\beta} \\ \Delta\dot{r} \end{bmatrix} = \begin{bmatrix} Y_v & \dfrac{Y_r - 1}{V_{to}} \\ \grave{N}_\beta & \grave{N}_r \end{bmatrix} \begin{bmatrix} \Delta\beta \\ \Delta r \end{bmatrix} + \begin{bmatrix} {Y_{\delta_a}}^{*} & {Y_{\delta_r}}^{*} \\ \acute{N}_{\delta_a} & \acute{N}_{\delta_r} \end{bmatrix} \begin{bmatrix} \Delta\delta_a \\ \Delta\delta_r \end{bmatrix}$$

1-Roll mode

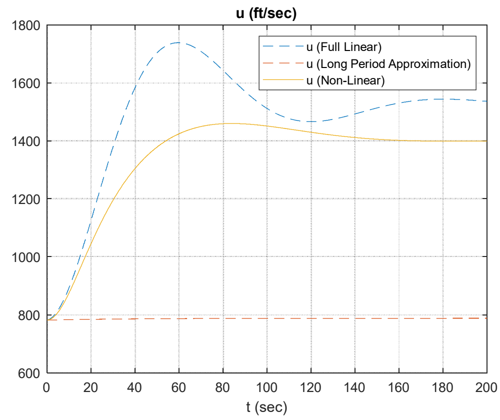
It has been seen that the rolling mode almost corresponds to the pure rolling. Thus it is reasonable to neglect all equations except the rolling moment equation and all perturbations except p. So we will reach the following relation;

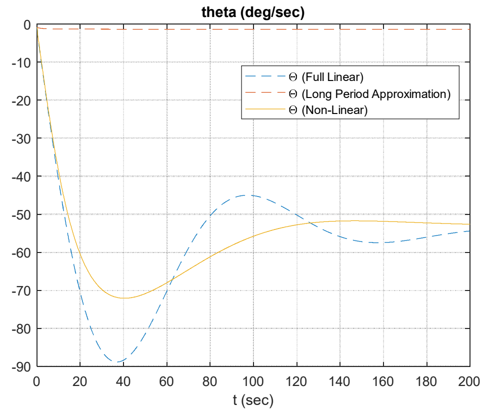
$$\dot{P} - L_p{}'p = L_{\delta a}{}'\delta_a$$

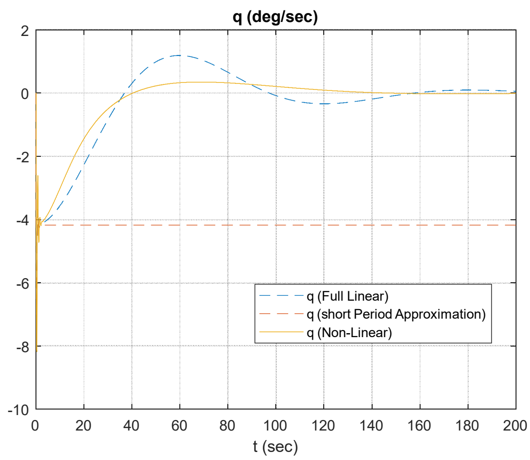
Results of the longitudinal dynamics

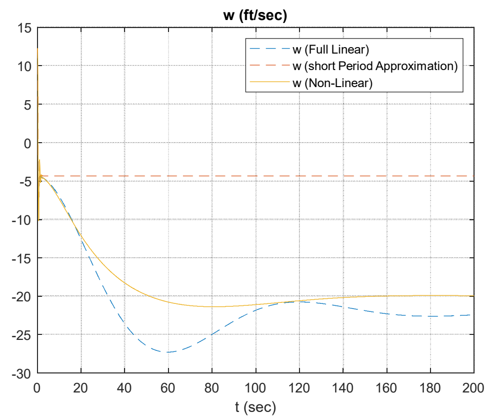
For states u , θ, w , q

$$\delta_{elevator} = 1$$

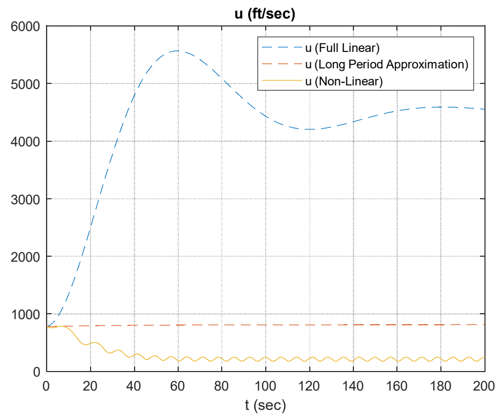


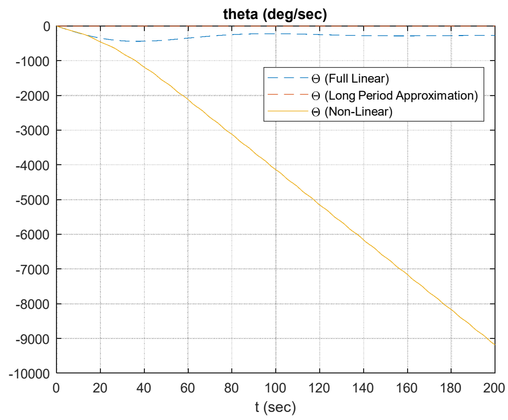


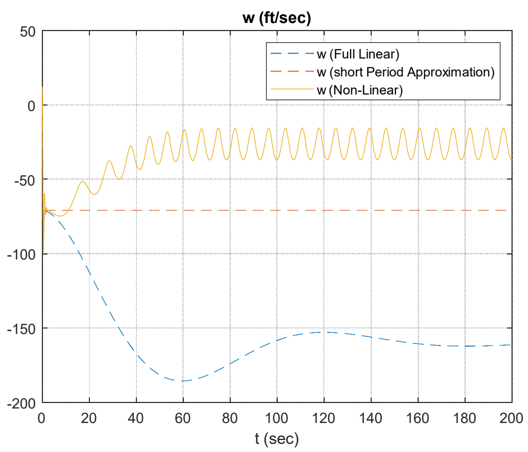


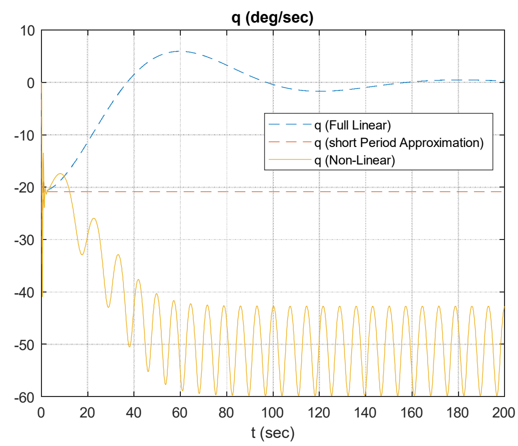


$$\delta_{elevator} = 5$$

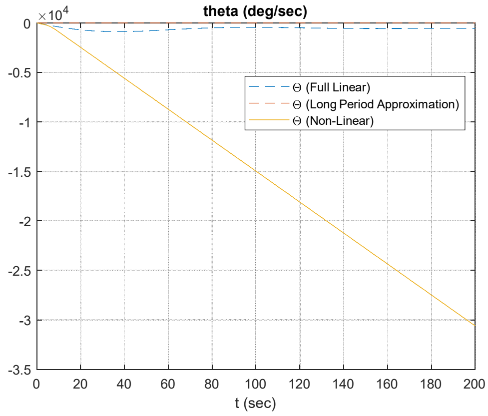


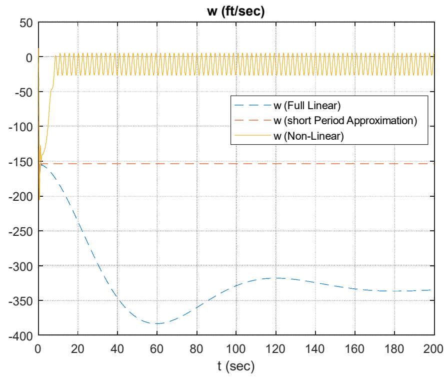


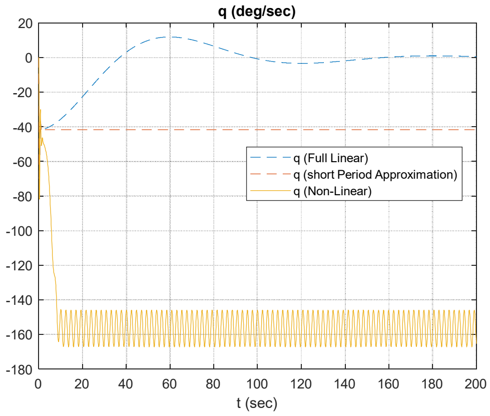


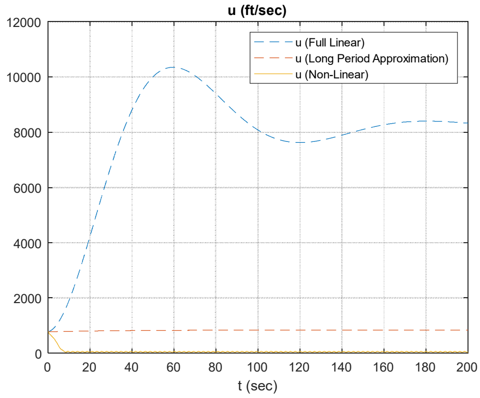


$$\delta_{elevator} = 10$$

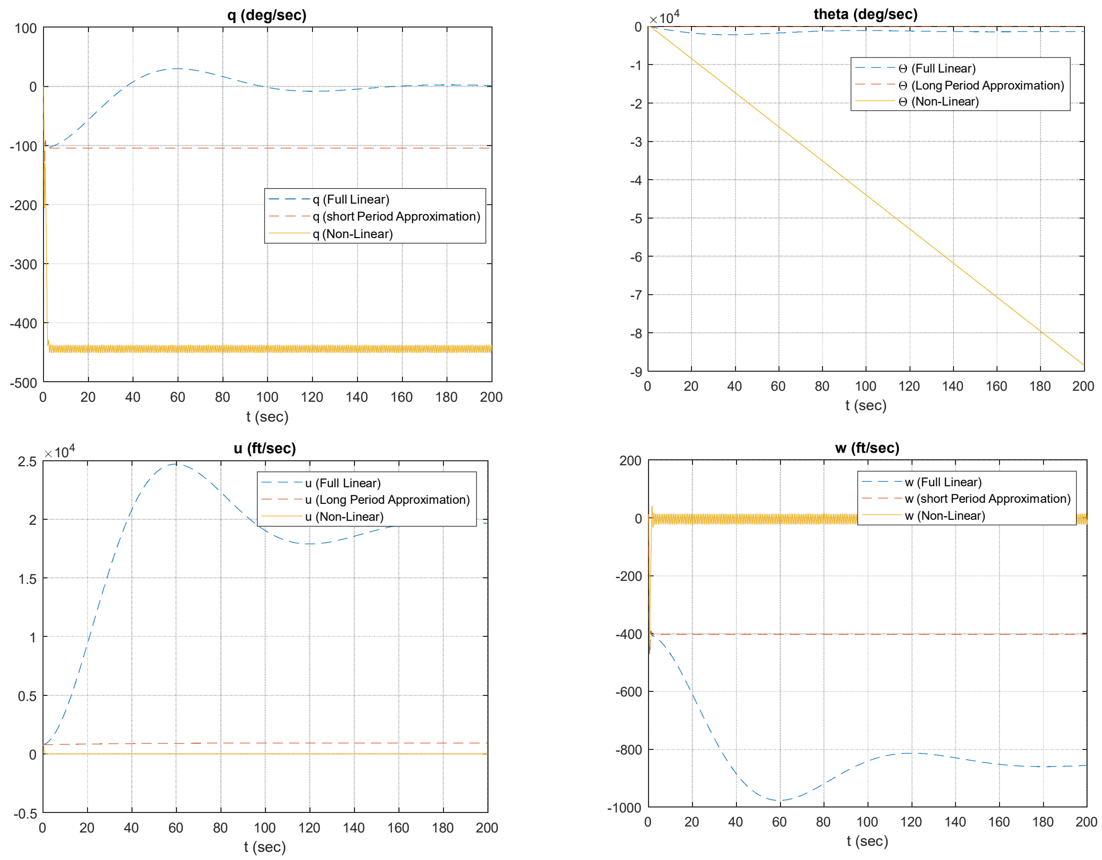




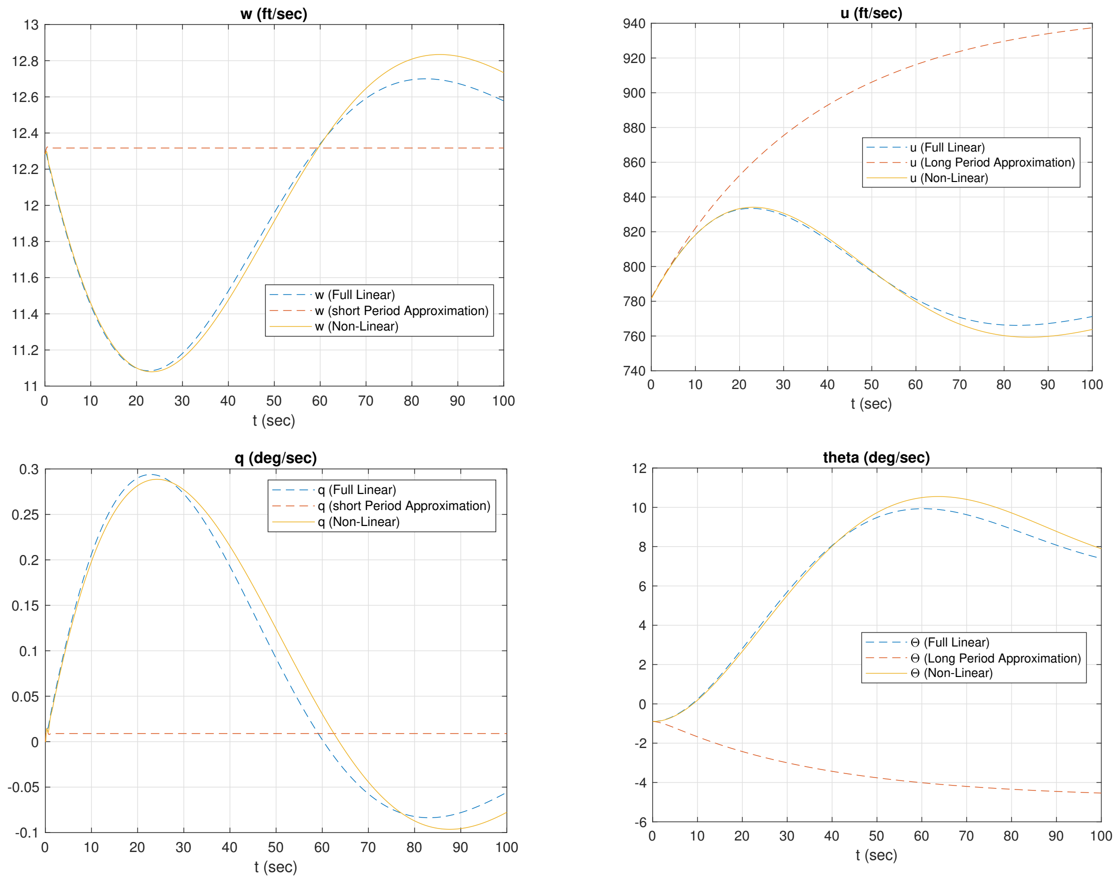




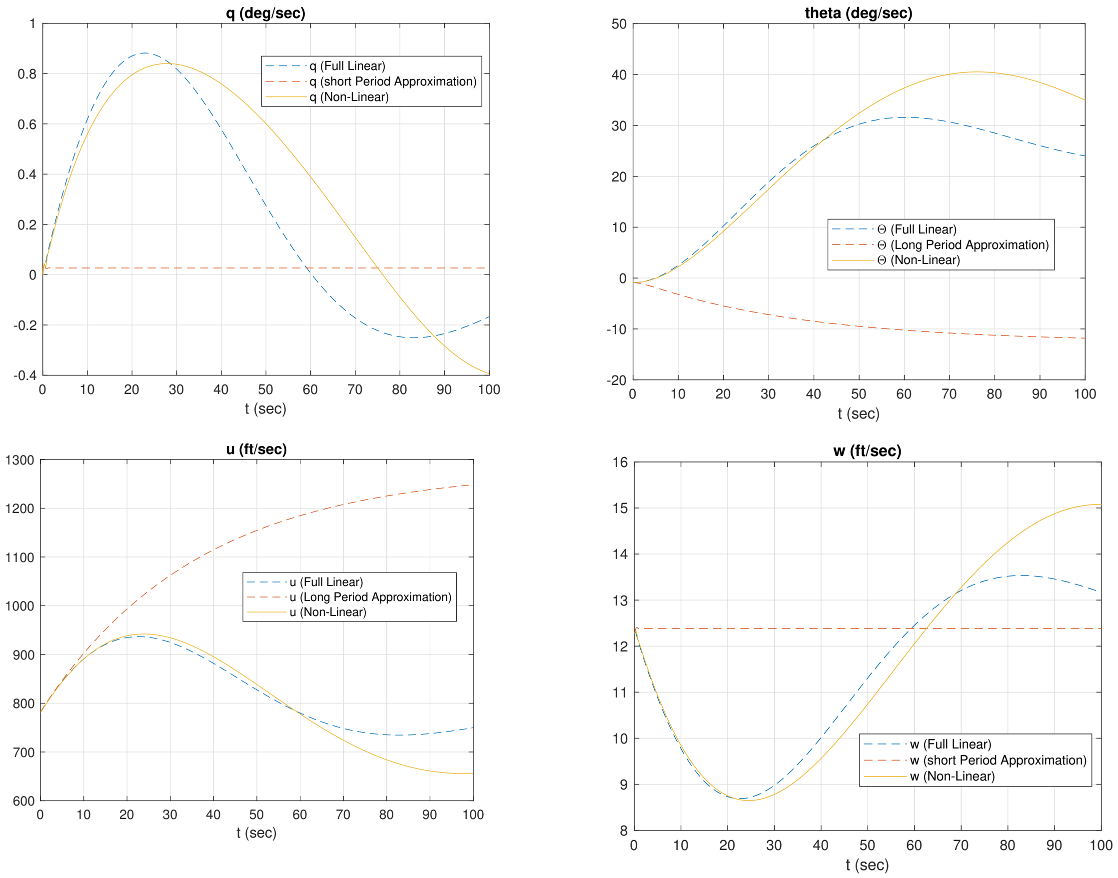
$$\delta_{elevator} = 25$$



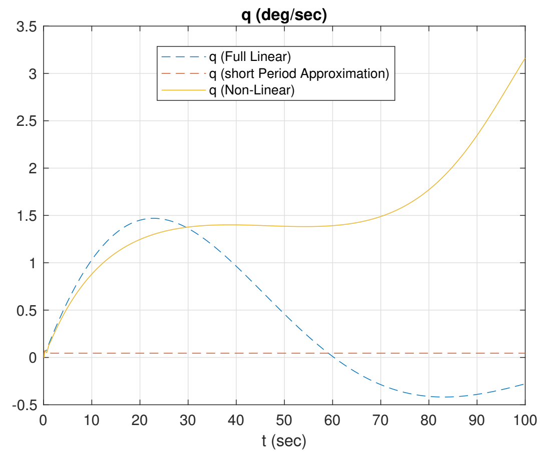
$$\delta_{thrust} = 2000$$

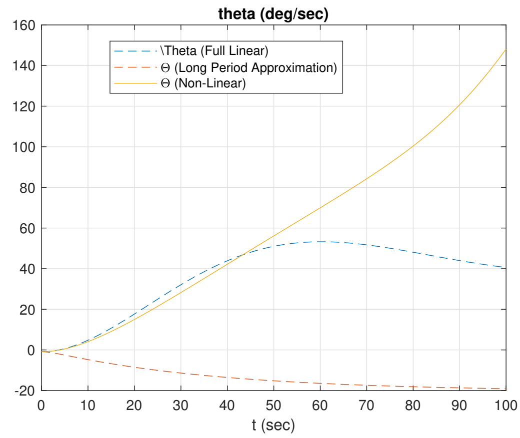


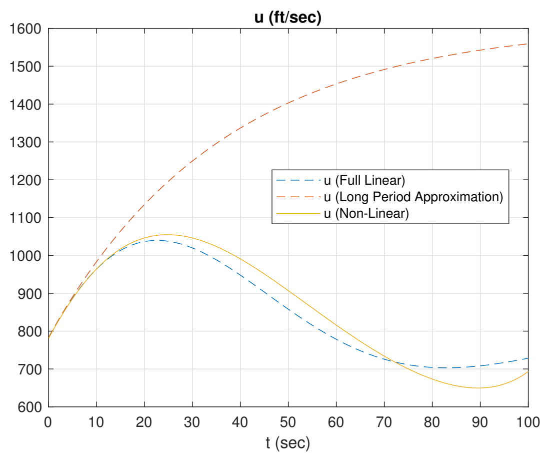
$$\delta_{thrust} = 6000$$

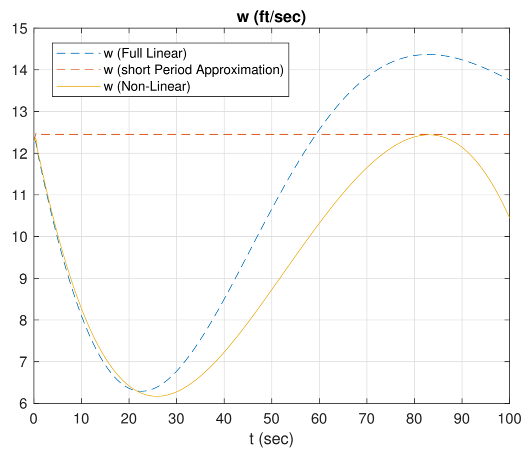


$$\delta_{thrust} = 10000$$



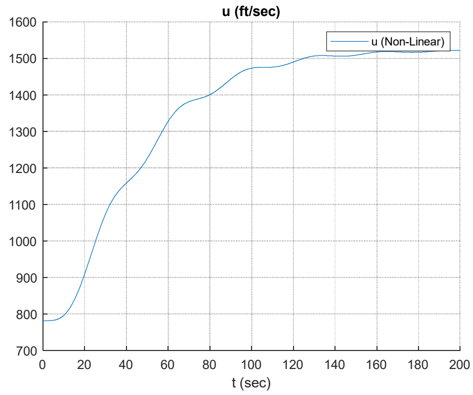


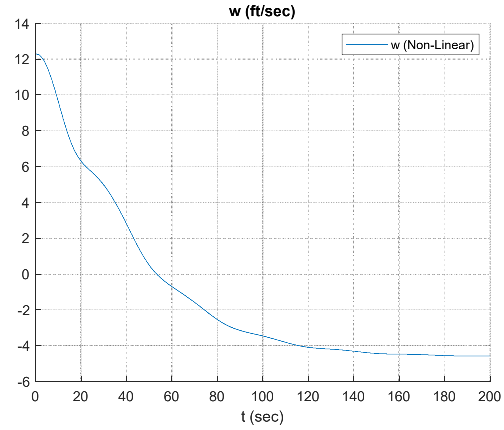


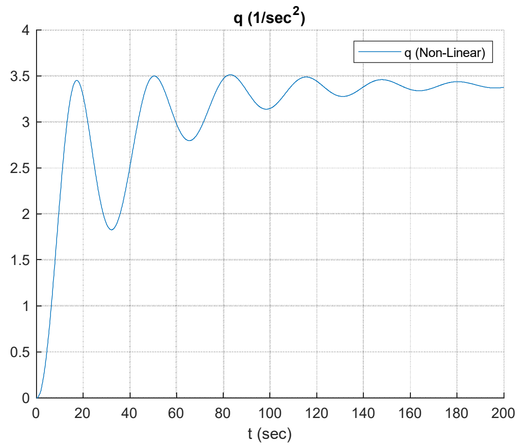


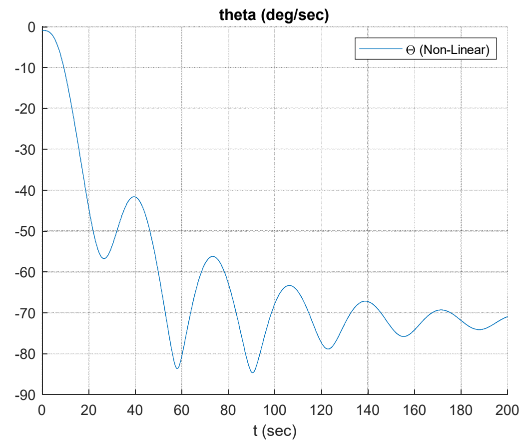
Results of Step response (lateral Dynamics)

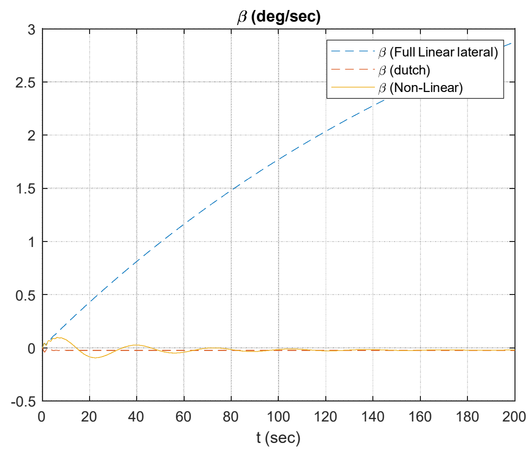
Response due to Aileron ($\delta_a = 1$)

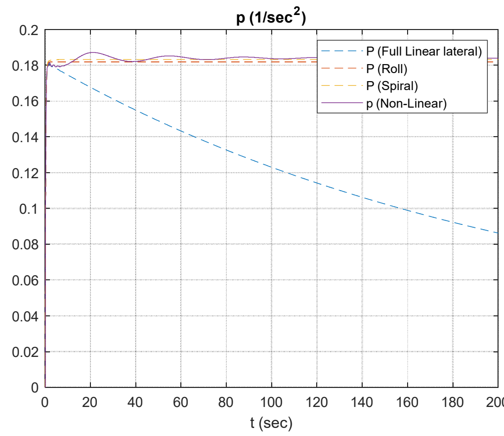


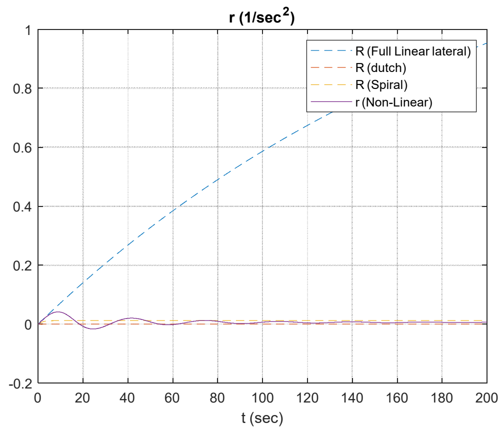


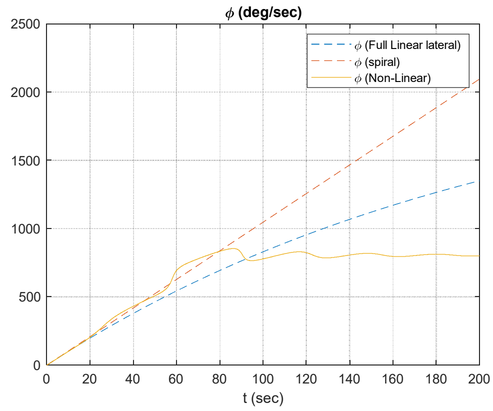


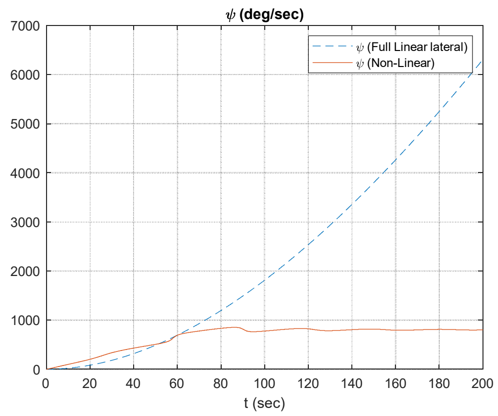




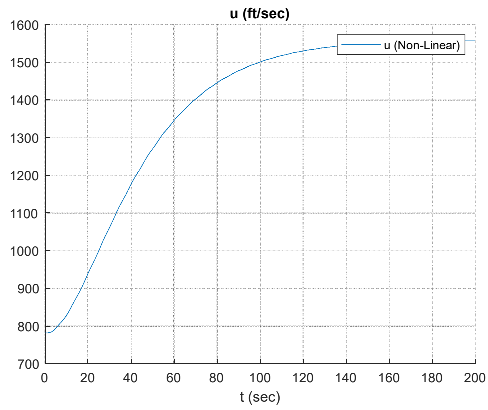


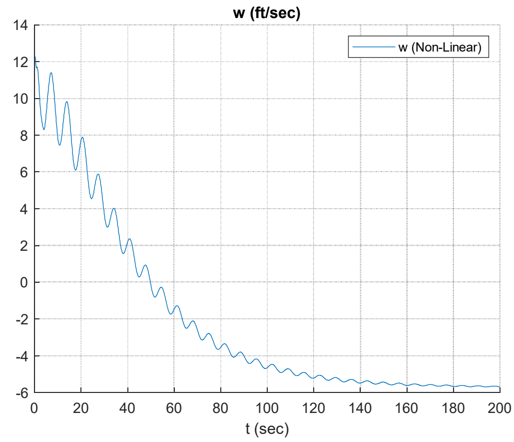


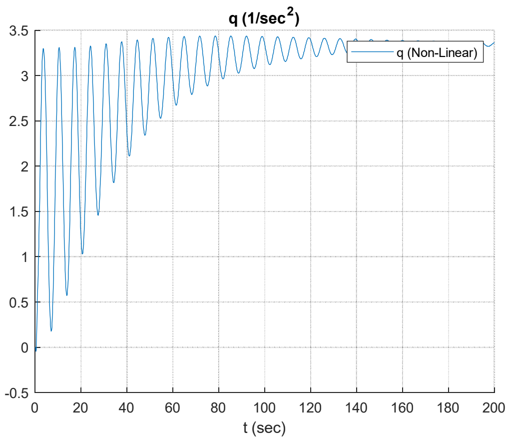




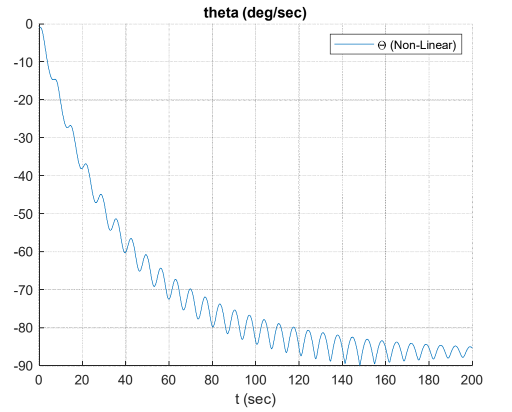
Response due to Aileron ($\delta_a = 5$)



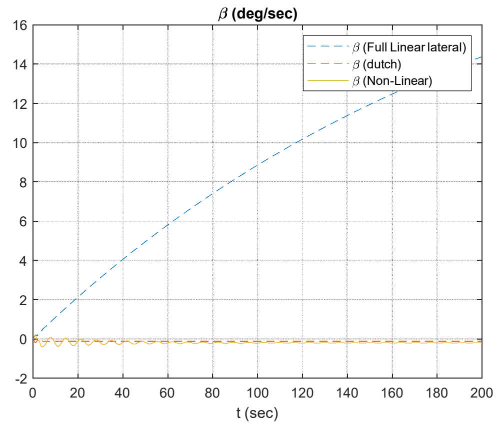




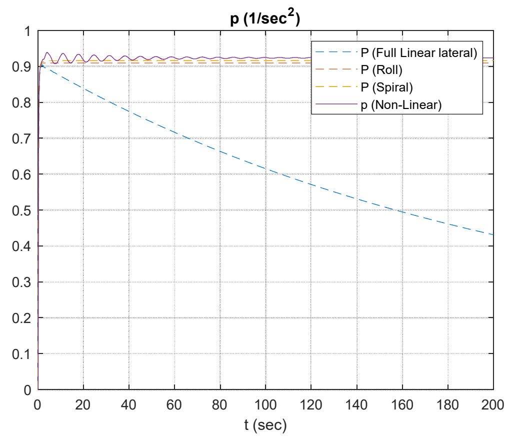
q (1/sec^2)
q (Non-Linear)
t (sec)



theta (deg/sec)
Θ (Non-Linear)
t (sec)

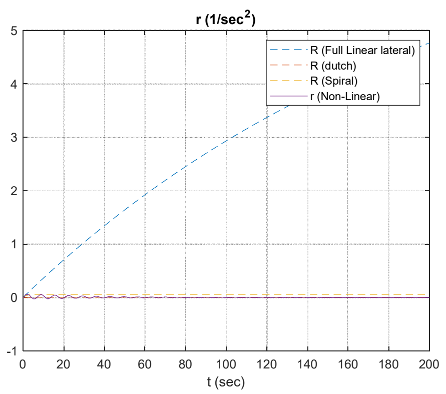


β (deg/sec)
β (Full Linear lateral)
β (dutch)
β (Non-Linear)
t (sec)

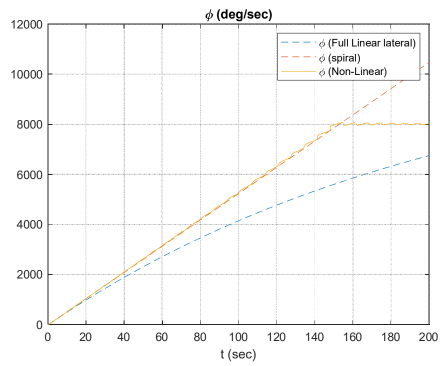


p (1/sec^2)
P (Full Linear lateral)
P (Roll)
P (Spiral)
p (Non-Linear)
t (sec)

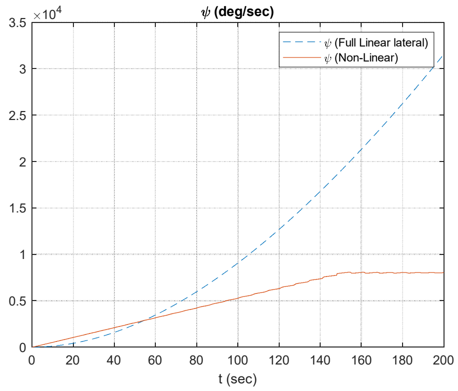
‘



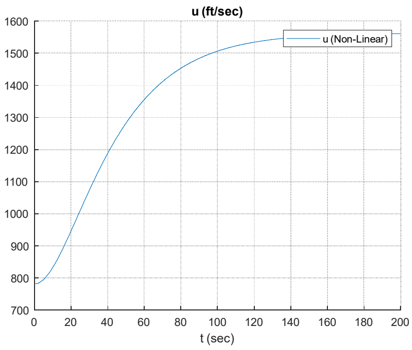
r (1/sec^2)
R (Full Linear lateral)
R (dutch)
R (Spiral)
r (Non-Linear)
t (sec)

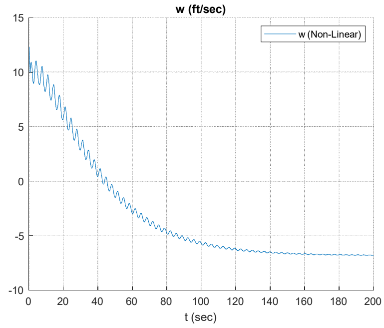


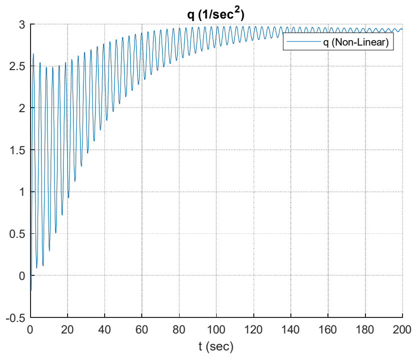
ϕ (deg/sec)
ϕ (Full Linear lateral)
ϕ (spiral)
ϕ (Non-Linear)
t (sec)

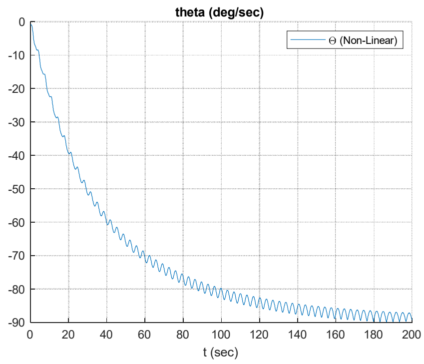


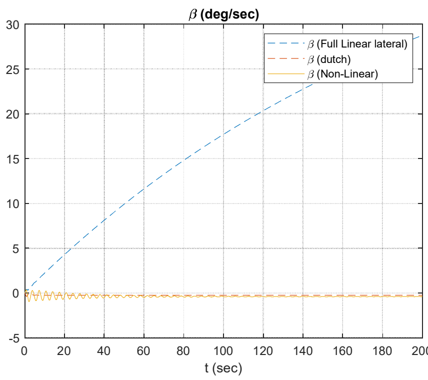
Response due to Rudder ($\delta_a = 10$)

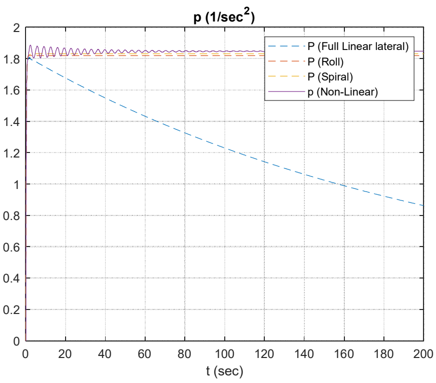


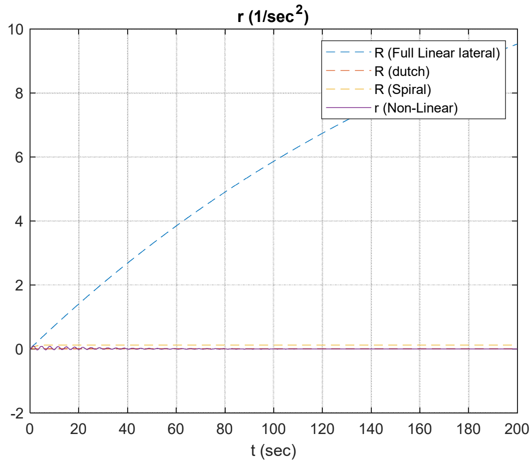


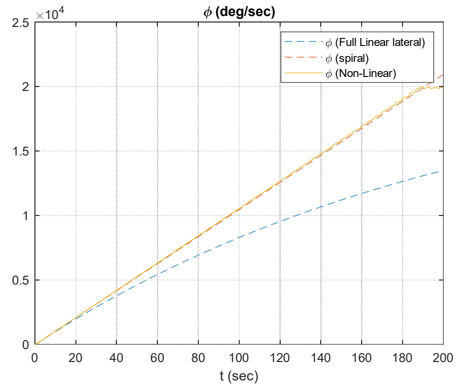


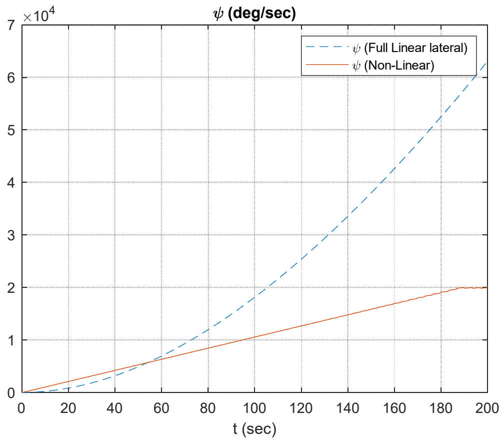




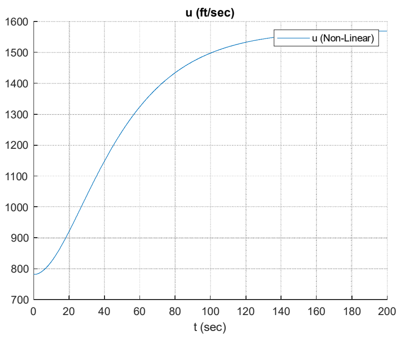


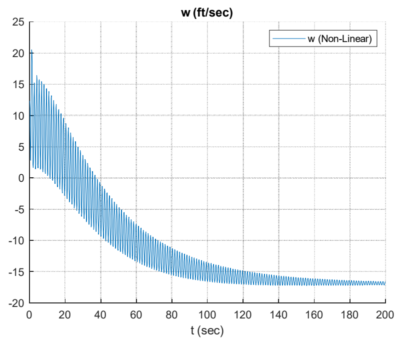


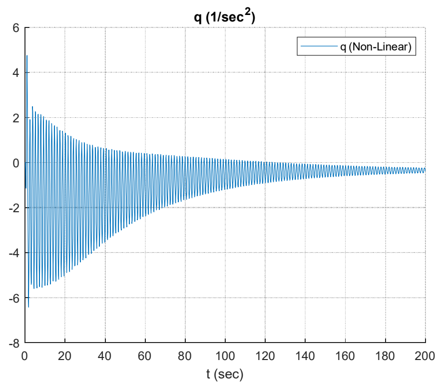


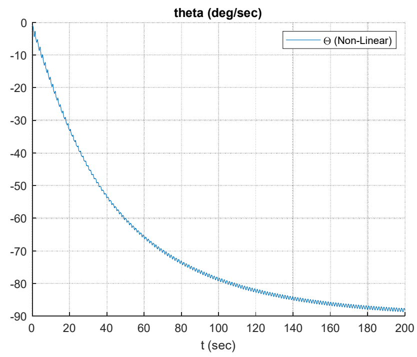


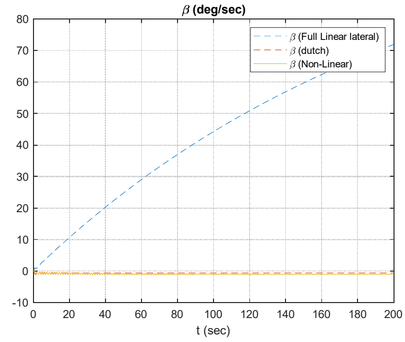
Response due to Rudder ($\delta_a = 25$)

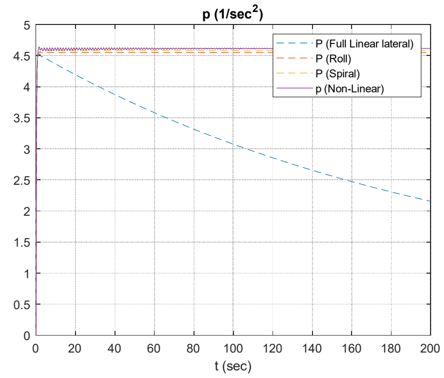


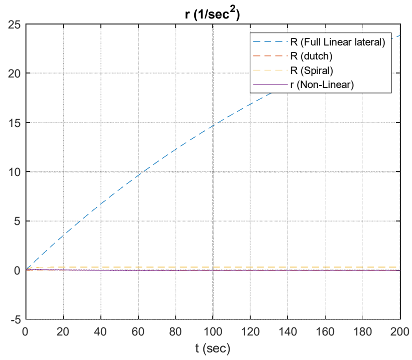


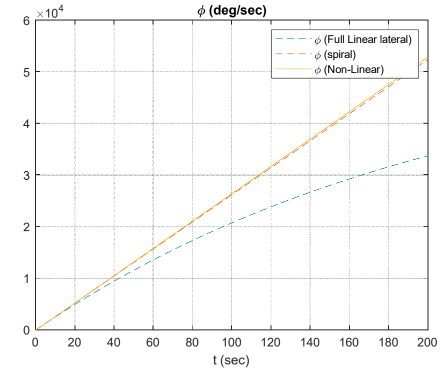


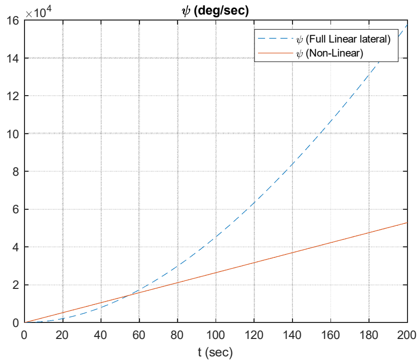




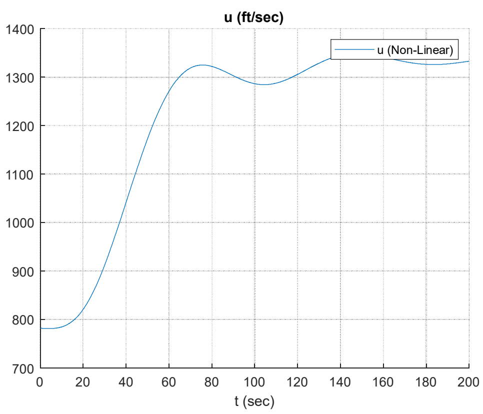


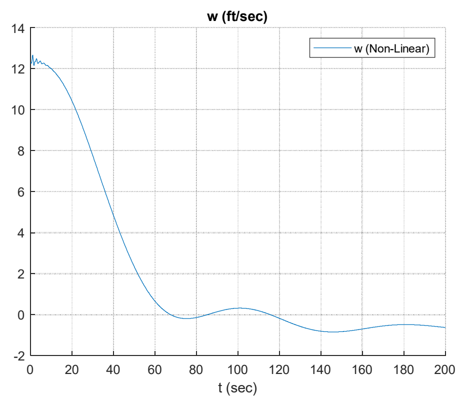


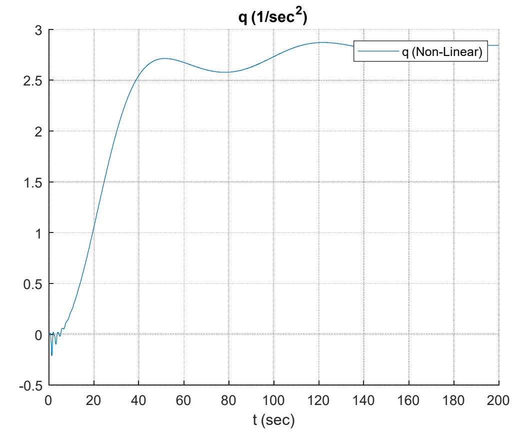


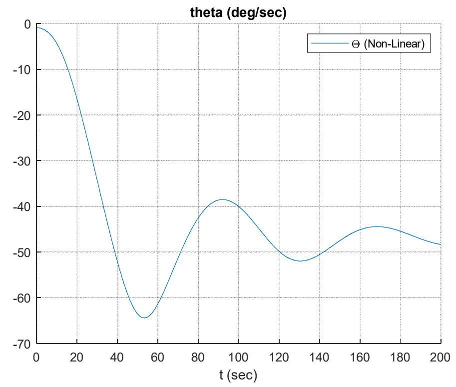


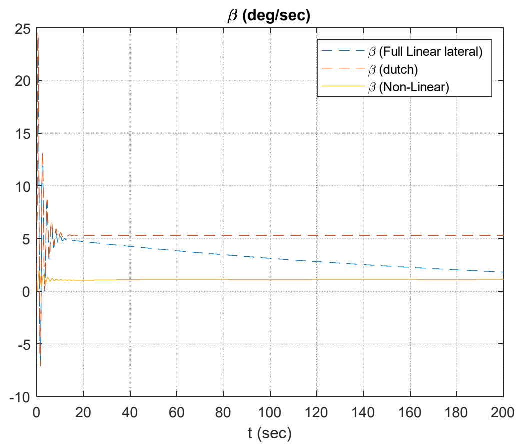
Response due to Rudder ($\delta_r = 1$)

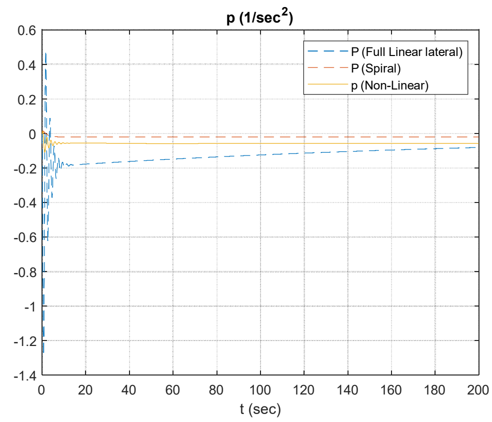


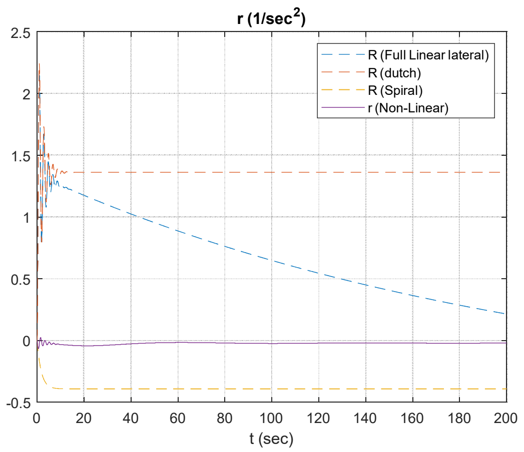


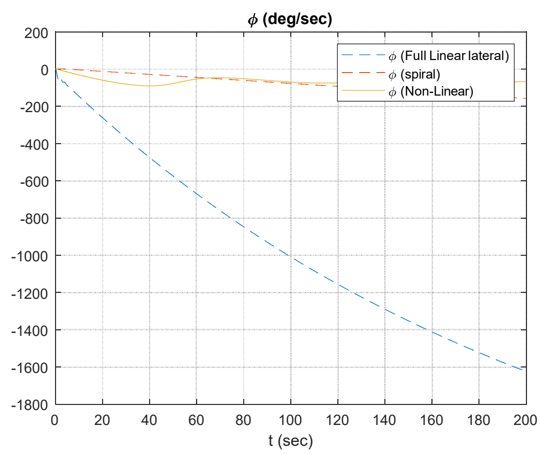

q (1/sec^2)
q (Non-Linear)
t (sec)

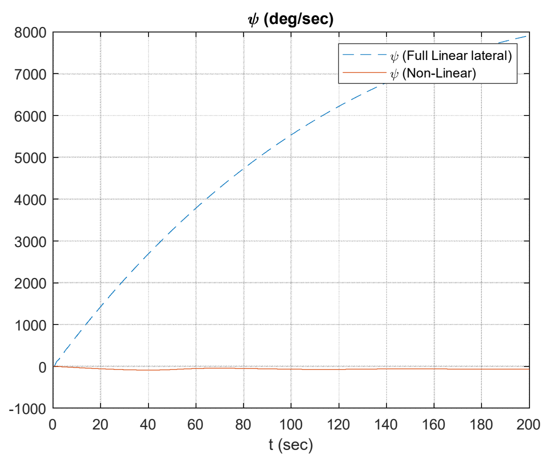

theta (deg/sec)
Θ (Non-Linear)
t (sec)


β (deg/sec)
β (Full Linear lateral)
β (dutch)
β (Non-Linear)
t (sec)

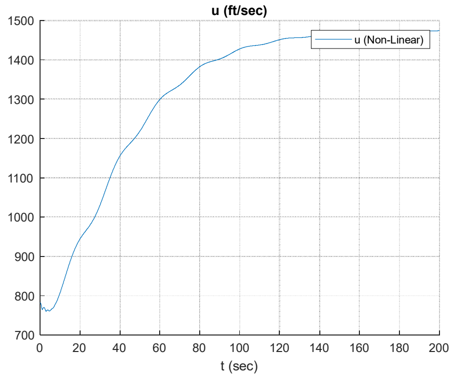

p (1/sec^2)
P (Full Linear lateral)
P (Spiral)
p (Non-Linear)
t (sec)

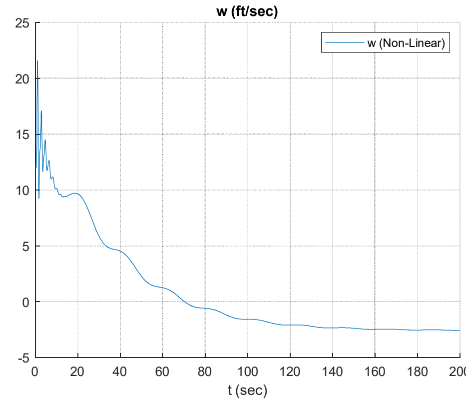

r (1/sec^2)
R (Full Linear lateral)
R (dutch)
R (Spiral)
r (Non-Linear)
t (sec)

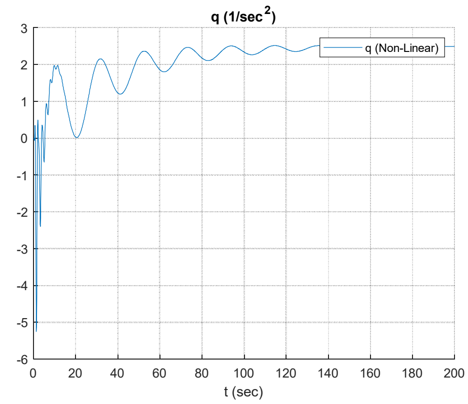

ϕ (deg/sec)
ϕ (Full Linear lateral)
ϕ (spiral)
ϕ (Non-Linear)
t (sec)

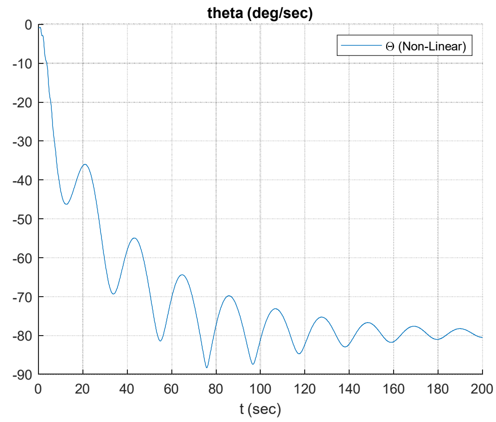

ψ (deg/sec)
ψ (Full Linear lateral)
ψ (Non-Linear)
t (sec)

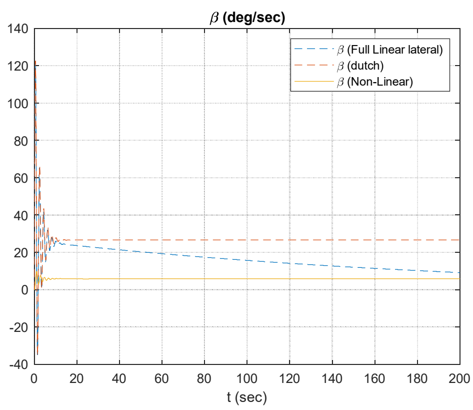
Response due to Rudder ($\delta_r = 5$)

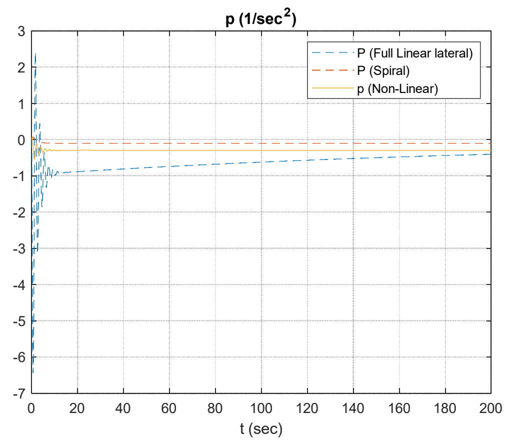


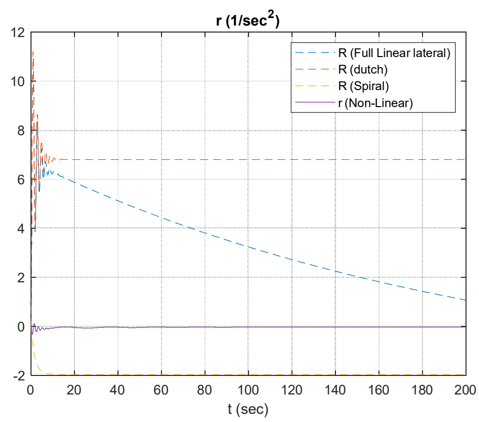


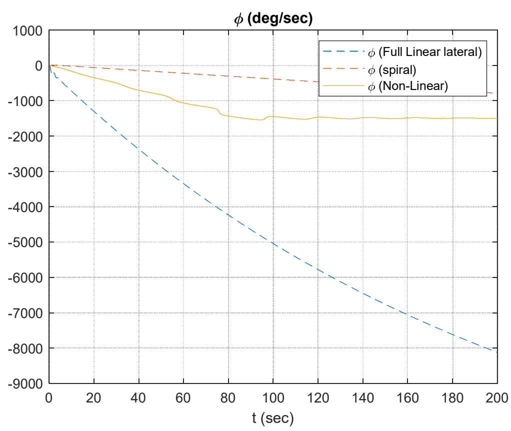


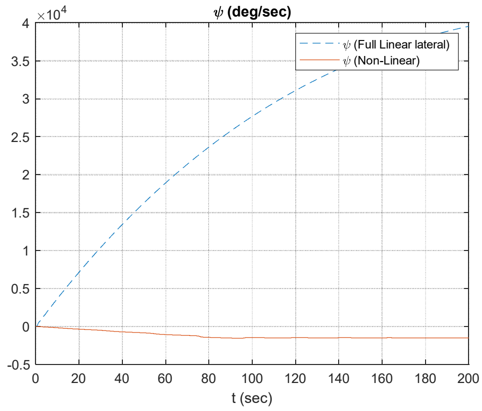




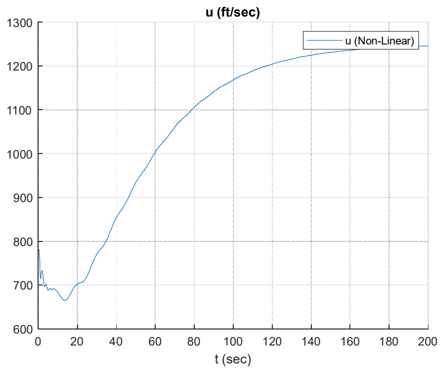


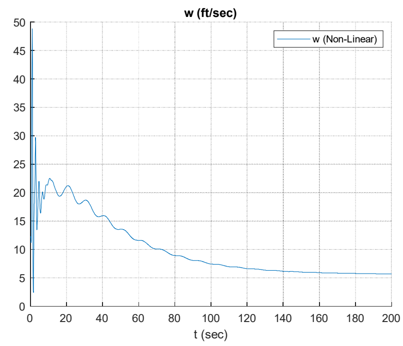


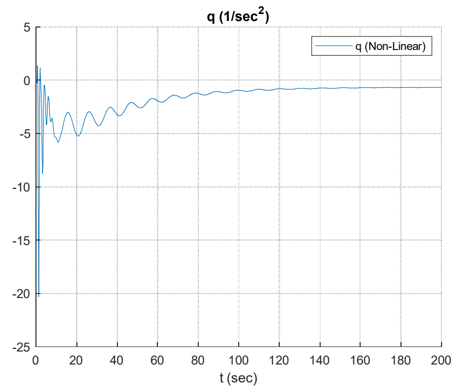


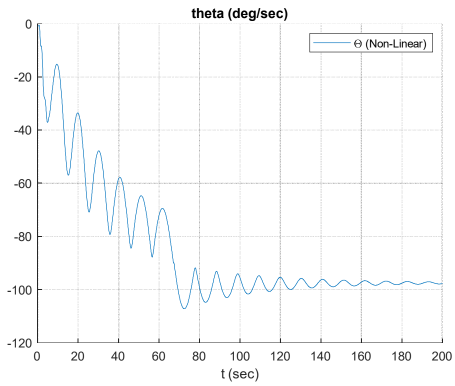


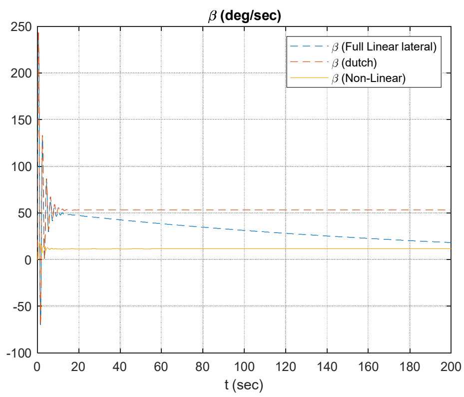
Response due to Rudder ($\delta_r = 10$)

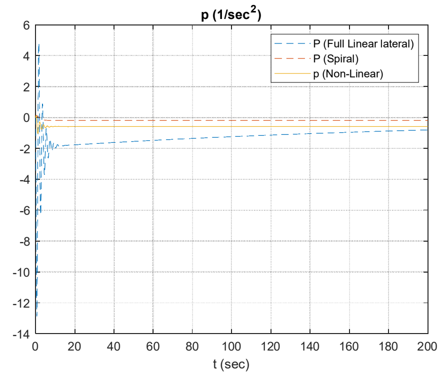


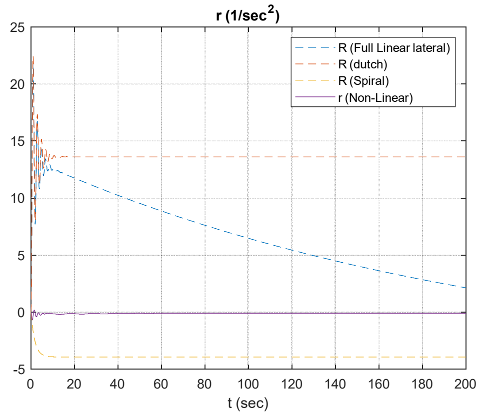


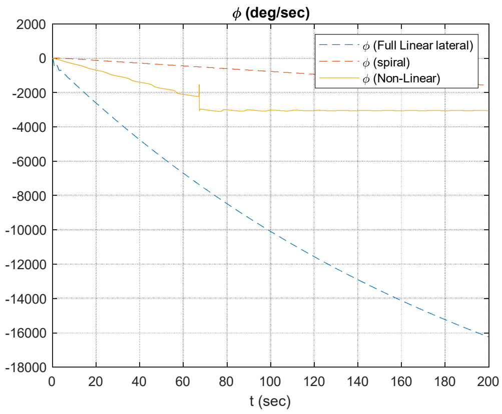


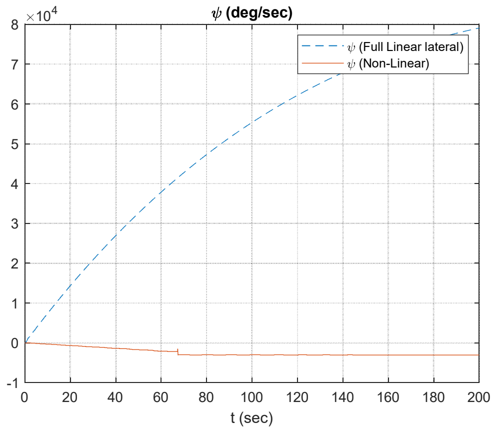




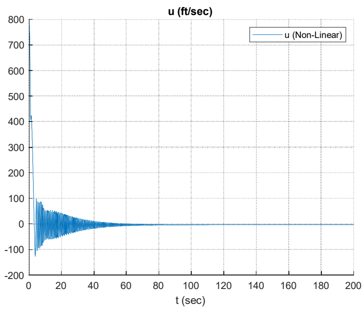


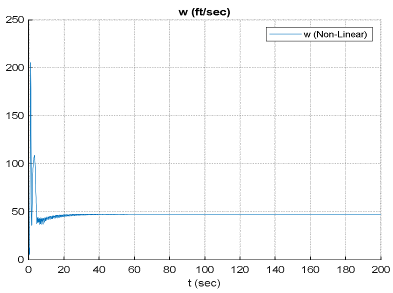


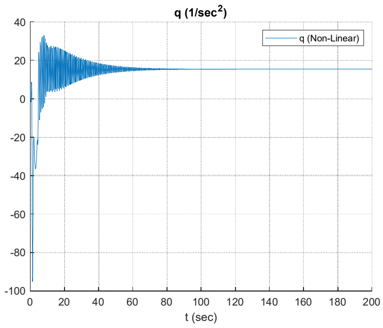


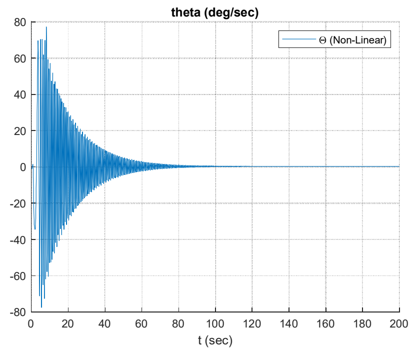


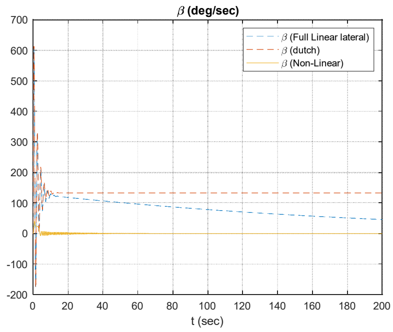
Response due to Rudder ($\delta_r = 25$)

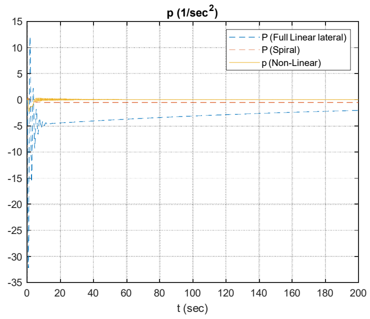


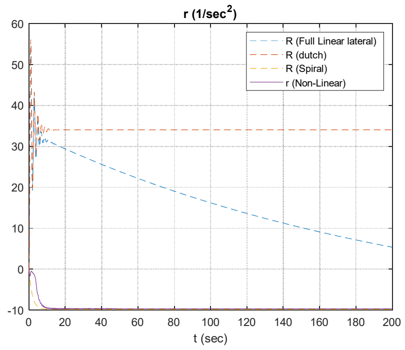


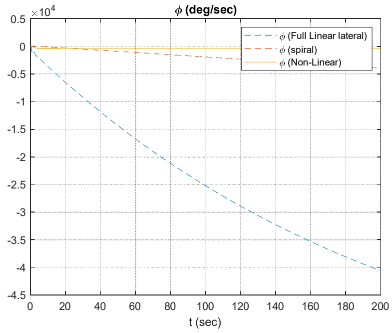


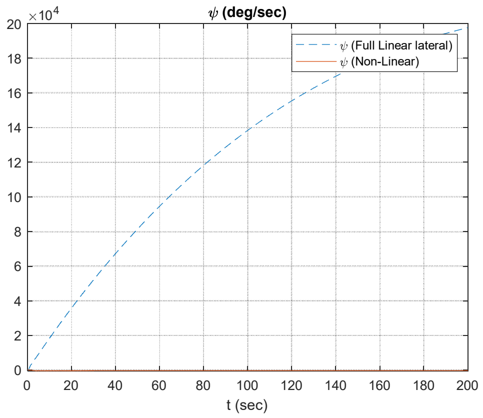












codes

```
clc; clear; close all;
```

Inputs

Forces, Moments and Inertia

```
[Mass, g, I, invI, timeSpan, dt, ICs, ICs_dot0, Vt0, ...
    dControl, SD_Long, SD_Lat,SD_Lat_dash, initialGravity] = Input("NT-
33A_4.xlsx");
steps = (timeSpan(2) - timeSpan(1))/dt;
Result = NaN(12, steps);
Result(:,1) = ICs;
time_V = linspace(0, timeSpan(2), steps+1);
```

Solving

```
%profile on;
dForces = [0 ; 0; 0];
dMoments = [0 ; 0; 0];

for i =1:steps

    Result(:, i+1) = RBDSolver(Result(:, i), dt, (initialGravity + dForces),
dMoments, Mass, I, invI, g);

    [dF, dM] = airFrame(SD_Long, SD_Lat, dControl, ICs, ICs_dot0, Result(:, i+1)
,Vt0, ...
        (initialGravity + dForces), dMoments, Mass, I, invI, g);


    dForces = vpa(dF');
    dMoments = vpa(dM');

end
%profile viewer
```

Rearranging Results

```
u = Result(1,:);

v = Result(2,:);

w = Result(3,:);

p = Result(4,:);

q = Result(5,:);

r = Result(6,:);

phi = Result(7,:);

theta = Result(8,:);

psi = Result(9,:);

x = Result(10,:);

y = Result(11,:);

z = Result(12,:);



beta_deg=asin(v/Vt0)*180/pi;

alpha_deg=atan(w./u)*180/pi;

p_deg=p*180/pi;

q_deg=q*180/pi;
```

```
r_deg=r*180/pi;

phi_deg=phi*180/pi;

theta_deg=theta*180/pi;

psi_deg=psi*180/pi;
```

Longitudenal Full Linear Model

```
XU=SD_Long(1);

ZU=SD_Long(2);

MU=SD_Long(3);

XW=SD_Long(4);

ZW=SD_Long(5);

MW=SD_Long(6);

ZWD=SD_Long(7);

ZQ=SD_Long(8);

MWD=SD_Long(9);

MQ=SD_Long(10);

XDE=SD_Long(11);

ZDE=SD_Long(12);

MDE=SD_Long(13);

XDTH=SD_Long(14);

ZDTH=SD_Long(15);

MDTH=SD_Long(16);

u0 = ICs(1);

v0 = ICs(2);

w0 = ICs(3);

q0 = ICs(5);

theta0 = ICs(8);

A_long=[XU XW -w0 -g*cos(theta0)
```

```
    ZU/(1-ZWD) ZW/(1-ZWD) (ZQ+u0)/(1-ZWD) -g*sin(theta0)/(1-ZWD)

    MU+MWD*ZU/(1-ZWD) MW+MWD*ZW/(1-ZWD) MQ+MWD*(ZQ+u0)/(1-ZWD) -
MWD*g*sin(theta0)/(1-ZWD)

    0 0 1 0];

B_long=[XDE XDTH

    ZDE/(1-ZWD) ZDTH/(1-ZWD)

    MDE+MWD*ZDE/(1-ZWD) MDTH+MWD*ZDTH/(1-ZWD)

    0 0];

C_long=eye(4);

D_long=zeros(4,2);



% Two Inputs - Four Output Each
LongSS = ss(A_long, B_long, C_long, D_long);
```

APPROXIMATE
PHUGOID MODE (LONG PERIOD MODE)

```
A_phug=[XU -g

    -ZU/(u0+ZQ) 0];

B_phug=[XDE XDTH

    -ZDE/(ZQ+u0) -ZDTH/(ZQ+u0)];

C_phug=eye(2);D_phug=zeros(2,2);



PHUG_SS=ss(A_phug,B_phug,C_phug,D_phug);
```

SHORT PERIOD MODE

```
A_short=[ZW/(1-ZWD) (ZQ+u0)/(1-ZWD)

     (MW+ZW*MWD/(1-ZWD)) (MQ+MWD*(ZQ+u0)/(1-ZWD))];

B_short=[ZDE/(1-ZWD) ZDTH/(1-ZWD)

    MDE+MWD*ZDE/(1-ZWD) MDTH+MWD*ZDTH/(1-ZWD)];

C_short=eye(2);D_short=zeros(2,2);
```

```
SHORT_SS=ss(A_short,B_short,C_short,D_short);
```

Longitudinal Full Linear Model Step Response

Due to delta_elevetor or delta_thrust

```
dControl_long = dControl(3:4); % dE, dTh
opt = stepDataOptions;
opt.StepAmplitude = dControl_long;
[res, ~, ~] = step(LongSS, time_V, opt);
res_dE  =res(:,:,1);
res_dTh =res(:,:,2);
```

Longitudinal Approximate Models Step Response

Due to delta_elevetor or delta_thrust

```
dControl_long = dControl(3:4); % dE, dTh
opt = stepDataOptions;
opt.StepAmplitude = dControl_long;
[APPres_PH, ~,~] = step(PHUG_SS, time_V, opt);
[APPres_SH, ~,~] = step(SHORT_SS, time_V, opt);

APPres_dE=zeros (length(time_V),4);
APPres_dE(:,[1,4])=APPres_PH(:,:,1);
APPres_dE(:,[2,3])=APPres_SH(:,:,1);

APPres_dTH=zeros (length(time_V),4);
APPres_dTH(:,[1,4])=APPres_PH(:,:,2);
APPres_dTH(:,[2,3])=APPres_SH(:,:,2);
```

u response Full Linear - Approximate - Non Linear

```
figure(1)


if(dControl_long(1) ~= 0)
    plot(time_V, res_dE(:, 1) + u0, '--', 'DisplayName', 'u (Full Linear)');  %
Full Linear Model
    hold on
    plot(time_V, APPres_dE(:, 1) + u0, '--', 'DisplayName', 'u (Long Period
Approximation)');  % Approximate (long period Mode)
elseif(dControl_long(2) ~= 0)
    plot(time_V, res_dTh(:, 1) + u0, '--', 'DisplayName', 'u (Full Linear)');  %
Full Linear Model
    hold on
    plot(time_V, APPres_dTH(:, 1) + u0, '--', 'DisplayName', 'u (Long Period
Approximation)');  % Approximate (long period Mode)
end

hold on
plot(time_V, u, '-', 'DisplayName', 'u (Non-Linear)');                % Non-
Linear Model
title('u (ft/sec)'); xlabel('t (sec)');
legend('show');
grid on
```

w response Full Linear - Approximate - Non Linear

```
figure(2)
```

```
if(dControl_long(1) ~= 0)
    plot(time_V, res_dE(:, 2) + w0, '--', 'DisplayName', 'w (Full Linear)');  %
Full Linear Model
    hold on
    plot(time_V, APPres_dE(:, 2) + w0, '--', 'DisplayName', 'w (short Period
Approximation)');  % Approximate (short period Mode)
elseif(dControl_long(2) ~= 0)
    plot(time_V, res_dTh(:, 2) + w0, '--', 'DisplayName', 'w (Full Linear)');  %
Full Linear Model
    hold on
    plot(time_V, APPres_dTH(:, 2) + w0, '--', 'DisplayName', 'w (short Period
Approximation)');  % Approximate (short period Mode)
end

hold on
```

q response Full Linear - Approximate - Non Linear

```
figure(3)

if(dControl_long(1) ~= 0)
    q_ = res_dE(:, 3) + q0;
    plot(time_V, q_*180/pi, '--', 'DisplayName', 'q (Full Linear)');  % Full Linear
Model
    hold on
    q_APP = APPres_dE(:, 3) + q0;
    plot(time_V, q_APP*180/pi, '--', 'DisplayName', 'q (short Period
Approximation)');  % Approximate (short period Mode)

elseif(dControl_long(2) ~= 0)
    q_ = res_dTh(:, 3) + q0;
    plot(time_V, q_*180/pi, '--', 'DisplayName', 'q (Full Linear)');  % Full Linear
Model
    hold on
    q_APP = APPres_dTH(:, 3) + q0;
    plot(time_V, q_APP*180/pi, '--', 'DisplayName', 'q (short Period
Approximation)');  % Approximate (short period Mode)
end

hold on
plot(time_V, q_deg , '-', 'DisplayName', 'q (Non-Linear)');                  % Non-
Linear Model
title('q (1/sec^2)'); xlabel('t (sec)');
legend('show');
grid on

plot(time_V, w, '-', 'DisplayName', 'w (Non-Linear)');                  % Non-
Linear Model
title('w (ft/sec)'); xlabel('t (sec)');
legend('show');
grid on
```

theta response Full Linear - Approximate - Non Linear

```
figure(4)

if(dControl_long(1) ~= 0)
    theta_ = (res_dE(:, 4) + theta0)*180/pi;
    plot(time_V, theta_, '--', 'DisplayName', '\Theta (Full Linear)');  % Full
Linear Model
    hold on
    theta_APP = (APPres_dE(:, 4) + theta0)*180/pi;
    plot(time_V, theta_APP, '--', 'DisplayName', '\Theta (Long Period
Approximation)');  % Full Linear Model
elseif(dControl_long(2) ~= 0)
```

```
    theta_ = (res_dTh(:, 4) + theta0)*180/pi;
    plot(time_V, theta_, '--', 'DisplayName', '\Theta (Full Linear)');  % Full
Linear Model
    hold on
    theta_APP = (APPres_dTH(:, 4) + theta0)*180/pi;
    plot(time_V, theta_APP, '--', 'DisplayName', '\Theta (Long Period
Approximation)');  % Full Linear Model
end

hold on
plot(time_V, theta_deg, '-', 'DisplayName', '\Theta (Non-Linear)');
% Non-Linear Model
title('theta (deg/sec)'); xlabel('t (sec)');
legend('show');
grid on
```

Lateral Full Linear Model

```
U0=u0; W0=w0; TH0=theta0; psi0=ICs(9); phi0=ICs(7);

Vto = sqrt(ICs(1)^2 + ICs(2)^2 + ICs(3)^2);    % Vto

% stability derivatives Lateral motion

Yp=0;
Yr=0;
YDa_star=SD_Lat_dash(9);
YDr_star=SD_Lat_dash(10);
Yb=SD_Lat_dash(2);
YDa=SD_Lat_dash(9)*Vto;
YDr=SD_Lat_dash(10)*Vto;

Lbd=SD_Lat_dash(3);
Lpd=SD_Lat_dash(5);
Lrd=SD_Lat_dash(7);
LDrd=SD_Lat_dash(13);
LDad=SD_Lat_dash(11);

Nbd=SD_Lat_dash(4);
Npd=SD_Lat_dash(6);
Nrd=SD_Lat_dash(8);
NDrd=SD_Lat_dash(14);
NDad=SD_Lat_dash(12);


A_Lat=[Yb/Vto (Yp+W0)/Vto (Yr-U0)/Vto g*cos(TH0)/Vto 0;...
       Lbd Lpd Lrd 0 0;...
       Nbd Npd Nrd 0 0;...
       0 1 tan(TH0) 0 0;...
       0 0 1/cos(TH0) 0 0];
B_Lat=[YDa_star YDr_star;...
       LDad LDrd;...
       NDad NDrd;...
       0 0;0 0];
C_Lat=eye(5); D_Lat=zeros(5,2);


Lateral_SS = ss(A_Lat,B_Lat,C_Lat,D_Lat);
Lateral_TF = tf(Lateral_SS);

B_DA_L = Lateral_TF(1,1);
B_DR_L = Lateral_TF(1,2);

P_DA_L = Lateral_TF(2,1);
P_DR_L = Lateral_TF(2,2);
```

```
R_DA_L = Lateral_TF(3,1);
R_DR_L = Lateral_TF(3,2);

PHI_DA_L = Lateral_TF(4,1);
PHI_DR_L = Lateral_TF(4,2);

PSI_DA_L = Lateral_TF(5,1);
PSI_DR_L = Lateral_TF(5,2);
```

3DOF Spiral Mode Approximation

```
A_Spiral = [Lpd Lrd 0;...
            Npd Nrd 0;...
            1 tan(TH0) 0];
B_Spiral = [LDad LDrd;NDad NDrd;0 0];
C_Spiral = eye(3); D_Spiral = zeros(3,2);

Spiral_SS = ss(A_Spiral,B_Spiral,C_Spiral,D_Spiral);
Spiral_TF = tf(Spiral_SS);

P_DA_S= Spiral_TF(1, 1);
P_DR_S = Spiral_TF(1, 2);

R_DA_S = Spiral_TF(2, 1);
R_DR_S = Spiral_TF(2, 2);

PHI_DA_S = Spiral_TF(3, 1);
PHI_DR_S = Spiral_TF(3, 2);
```

2DOF Dutch Mode Approximation

```
A_Dutch = [Yb/Vto (Yr-U0)/Vto-tan(TH0)*(Yp+W0)/Vto;Nbd Nrd-tan(TH0)*Npd];

B_Dutch = [YDa_star YDr_star;NDad NDrd];

C_Dutch = eye(2); D_Dutch = zeros(2,2);



Dutch_SS = ss(A_Dutch, B_Dutch, C_Dutch, D_Dutch);

Dutch_TF = tf(Dutch_SS);



B_DA_D = Dutch_TF(1, 1);

B_DR_D = Dutch_TF(1, 2);



R_DA_D = Dutch_TF(2, 1);

R_DR_D = Dutch_TF(2, 2);
```

1DOF Roll Approximation

```
A_Roll = Lpd;
```

```
B_Roll = LDad;

C_Roll = eye(1); D_Roll = zeros(1, 1);



Roll_SS = ss(A_Roll, B_Roll, C_Roll, D_Roll);

Roll_TF = tf(Roll_SS);



P_DA_R = Roll_TF(1 ,1);
```

Lateral Full Linear Model Step Response

Due to delta_elevetor or delta_thrust

```
dControl_latr = dControl(1:2); % dA, dR
opt = stepDataOptions;
opt.StepAmplitude = dControl_latr;
[Lat_res, ~, ~] = step(Lateral_SS, time_V, opt);
Lat_res_dA  =Lat_res(:,:,1);
Lat_res_dR =Lat_res(:,:,2);
```

Lateral Approximate Models Step Response

```
dControl_latr = dControl(1:2); % dA, dR
opt = stepDataOptions;
opt.StepAmplitude = dControl_latr;

[Spir_res, ~,~] = step(Spiral_SS, time_V, opt);
[Dutch_res, ~,~] = step(Dutch_SS, time_V, opt);

dC1= dControl(1); % dA, dR
opt = stepDataOptions;
opt.StepAmplitude = dC1;
[Roll_res, ~,~] = step(Roll_SS, time_V, opt);
```

Spiral

```
Spir_res_dA=zeros (length(time_V),5);

Spir_res_dA(:,[2,3,4])=Spir_res(:,:,1);



Spir_res_dR=zeros (length(time_V),5);

Spir_res_dR(:,[2,3,4])=Spir_res(:,:,2);
```

Dutch

```
Dutch_res_dA=zeros (length(time_V),5);

Dutch_res_dA(:,[1,3])=Dutch_res(:,:,1);
```

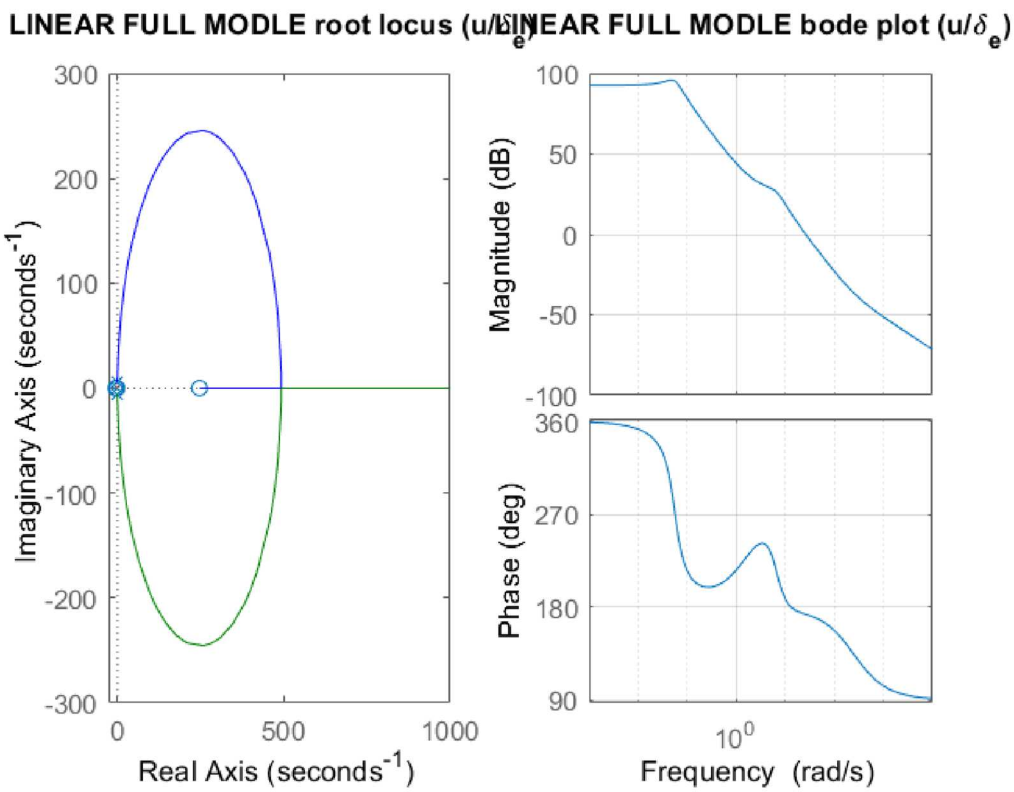
```
Dutch_res_dR=zeros (length(time_V),5);

Dutch_res_dR(:,[1,3])=Dutch_res(:,:,2);
```

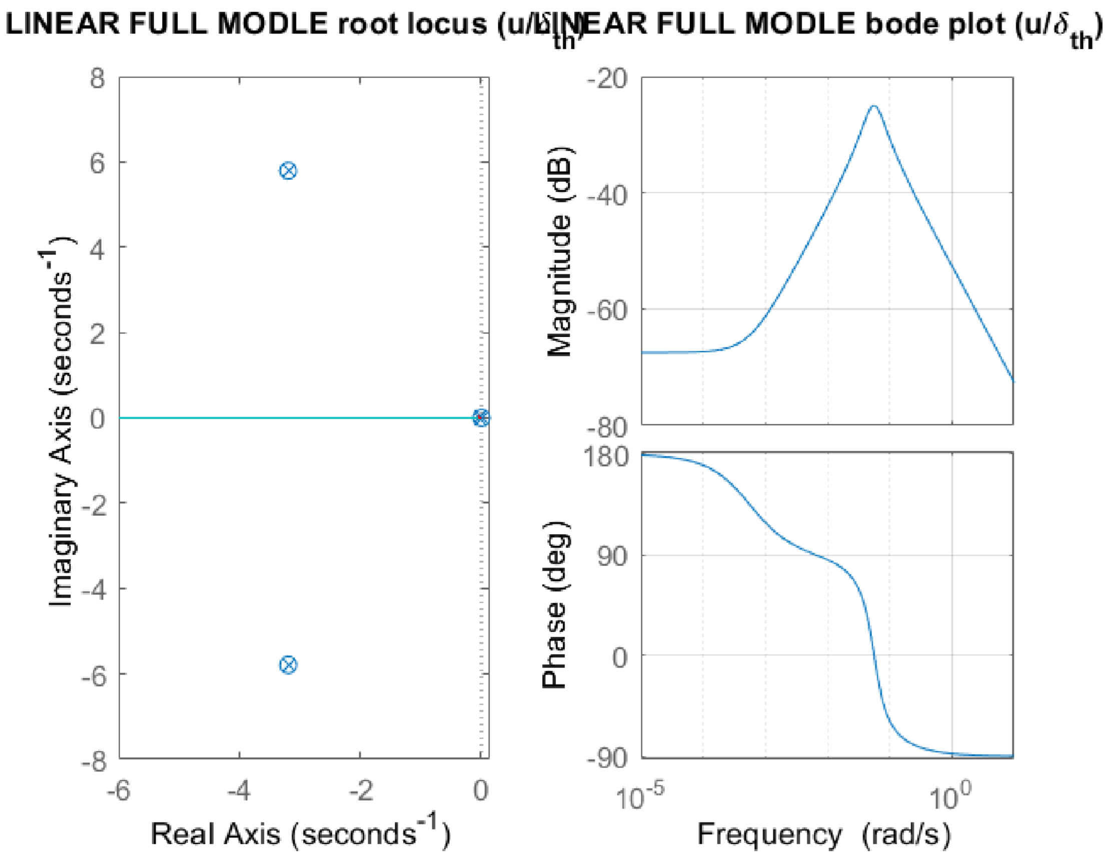
Roll

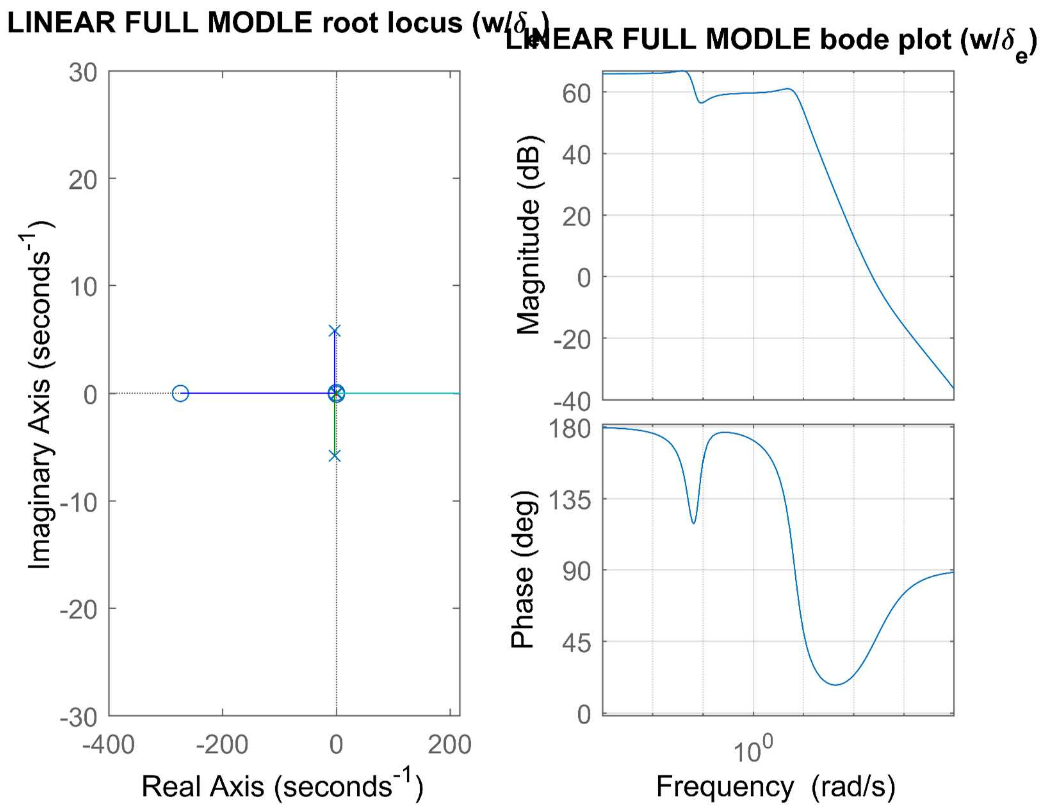
```
Roll_res_dA=zeros (length(time_V),5);

Roll_res_dA(:,2)=Roll_res;

figure(5)

beta0=v0/Vt0;

if(dControl_latr(1) ~= 0)
    beta_ = (Lat_res_dA(:, 1) + beta0)*180/pi;
    plot(time_V, beta_, '--', 'DisplayName', '\beta (Full Linear lateral)');  %
Full Lateral Linear Model
    hold on
    beta_d = (Dutch_res_dA(:, 1) + beta0)*180/pi;
    plot(time_V, beta_d, '--', 'DisplayName', '\beta (dutch)');  % spiral Linear
Model
elseif(dControl_latr(2) ~= 0)
    beta_ = (Lat_res_dR(:, 1) + beta0)*180/pi;
    plot(time_V, beta_, '--', 'DisplayName', '\beta (Full Linear lateral)');  %
Full Lateral Linear Model
    hold on
    beta_d = (Dutch_res_dR(:, 1) + beta0)*180/pi;
    plot(time_V, beta_d, '--', 'DisplayName', '\beta (dutch)');  % spiral Linear
Model
end

hold on
plot(time_V, beta_deg, '-', 'DisplayName', '\beta (Non-Linear)');
% Non-Linear Model
title('\beta (deg/sec)'); xlabel('t (sec)');
legend('show');
grid on
figure(6)

beta0=v0/Vt0;

if(dControl_latr(1) ~= 0)
    P_ = Lat_res_dA(:, 2) ;
    plot(time_V, P_, '--', 'DisplayName', 'P (Full Linear lateral)');  % Full
Lateral Linear Model
    hold on
    P_r = Roll_res_dA(:, 2);
    plot(time_V, P_r, '--', 'DisplayName', 'P (Roll)');  % spiral Linear Model
    hold on
    P_s = Spir_res_dA(:, 2);
    plot(time_V, P_s, '--', 'DisplayName', 'P (Spiral)');  % spiral Linear Model
elseif(dControl_latr(2) ~= 0)
    P_ = Lat_res_dR(:, 2) ;
    plot(time_V, P_, '--', 'DisplayName', 'P (Full Linear lateral)');  % Full
Lateral Linear Model
    hold on
    P_s = Spir_res_dR(:, 2);
    plot(time_V, P_s, '--', 'DisplayName', 'P (Spiral)');  % spiral Linear Model
end
```

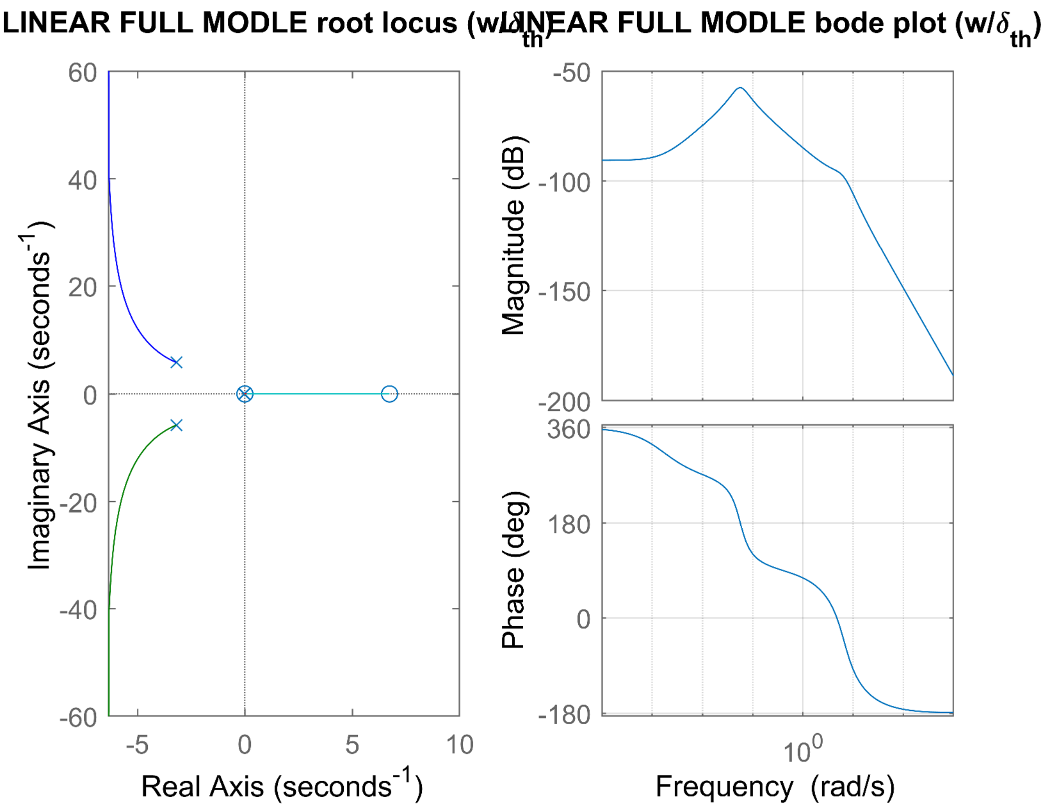
```
hold on
plot(time_V, p, '-', 'DisplayName', 'p (Non-Linear)');                    % Non-
Linear Model
title('p (1/sec^2)'); xlabel('t (sec)');
legend('show');
grid on

figure(7)


if(dControl_latr(1) ~= 0)
    R_ = Lat_res_dA(:, 3) ;
    plot(time_V, R_, '--', 'DisplayName', 'R (Full Linear lateral)');  % Full
Lateral Linear Model
    hold on
    R_d = Dutch_res_dA(:, 3);
    plot(time_V, R_d, '--', 'DisplayName', 'R (dutch)');  % spiral Linear Model
    hold on
    R_s = Spir_res_dA(:, 3);
    plot(time_V, R_s, '--', 'DisplayName', 'R (Spiral)');  % spiral Linear Model
elseif(dControl_latr(2) ~= 0)
    R_ = Lat_res_dR(:, 3) ;
    plot(time_V, R_, '--', 'DisplayName', 'R (Full Linear lateral)');  % Full
Lateral Linear Model
    hold on
    R_d = Dutch_res_dR(:, 3);
    plot(time_V, R_d, '--', 'DisplayName', 'R (dutch)');  % spiral Linear Model
    hold on
    R_s = Spir_res_dR(:, 3);
    plot(time_V, R_s, '--', 'DisplayName', 'R (Spiral)');  % spiral Linear Model
end

hold on
plot(time_V, r, '-', 'DisplayName', 'r (Non-Linear)');                    % Non-
Linear Model
title('r (1/sec^2)'); xlabel('t (sec)');
legend('show');
grid on
figure(8)


if(dControl_latr(1) ~= 0)
    phi_ = (Lat_res_dA(:, 4) + phi0)*180/pi;
    plot(time_V, phi_, '--', 'DisplayName', '\phi (Full Linear lateral)');  % Full
Lateral Linear Model
    hold on
    phi_s = (Spir_res_dA(:, 4) + phi0)*180/pi;
    plot(time_V, phi_s, '--', 'DisplayName', '\phi (spiral)');  % spiral Linear
Model
elseif(dControl_latr(2) ~= 0)
      phi_ = (Lat_res_dR(:, 4) + phi0)*180/pi;
    plot(time_V, phi_, '--', 'DisplayName', '\phi (Full Linear lateral)');  % Full
Linear Model
    hold on
    phi_s = (Spir_res_dR(:, 4) + phi0)*180/pi;
    plot(time_V, phi_s, '--', 'DisplayName', '\phi (spiral)');  % Full Linear Model
end

hold on
plot(time_V, phi_deg, '-', 'DisplayName', '\phi (Non-Linear)');                  %
Non-Linear Model
title('\phi (deg/sec)'); xlabel('t (sec)');
legend('show');
grid on
figure(9)


if(dControl_latr(1) ~= 0)
    psi_ = (Lat_res_dA(:, 5) + psi0)*180/pi;
```

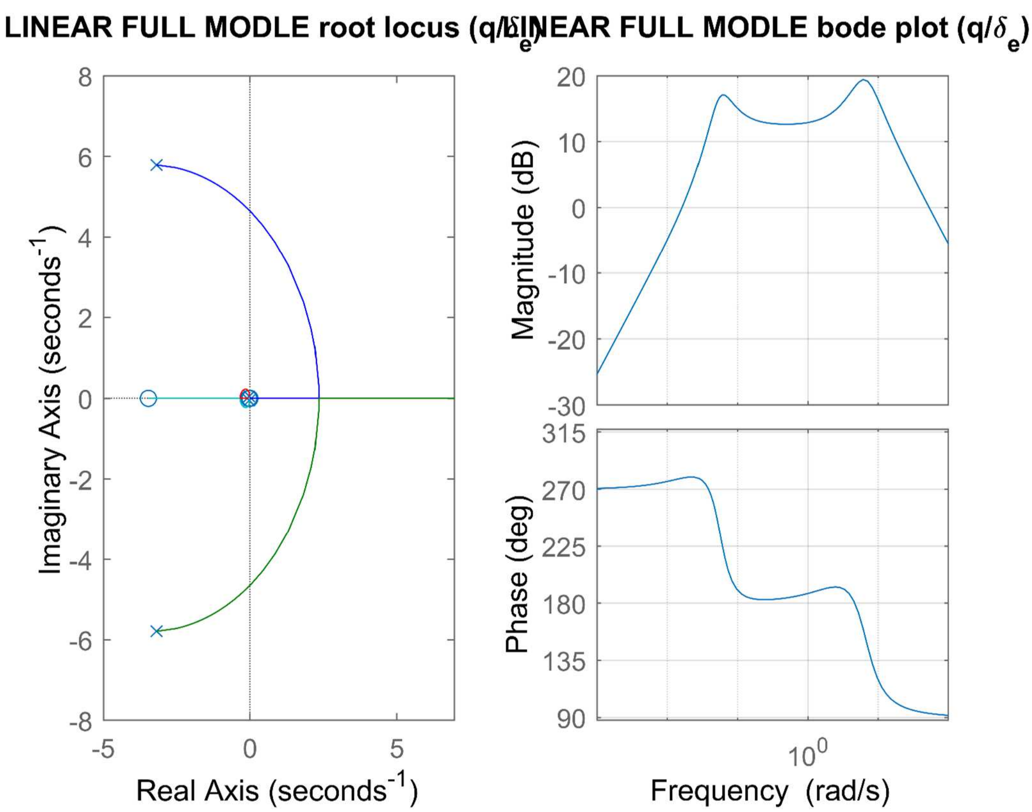
```
    plot(time_V, psi_, '--', 'DisplayName', '\psi (Full Linear lateral)');  % Full
Linear Model

elseif(dControl_latr(2) ~= 0)
      psi_ = (Lat_res_dR(:, 5) + psi0)*180/pi;
    plot(time_V, psi_, '--', 'DisplayName', '\psi (Full Linear lateral)');  % Full
Linear Model

end

hold on
plot(time_V, phi_deg, '-', 'DisplayName', '\psi (Non-Linear)');                    %
Non-Linear Model
title('\psi (deg/sec)'); xlabel('t (sec)');
legend('show');
grid on
export_figure(max(double(get(groot,'Children')))+[-8:0], '', {'t1',
't2','t3','t4','t5','t6','t7','t8','t9'}, 300, {'emf',
'emf','emf','emf','emf','emf','emf','emf','emf'})
```

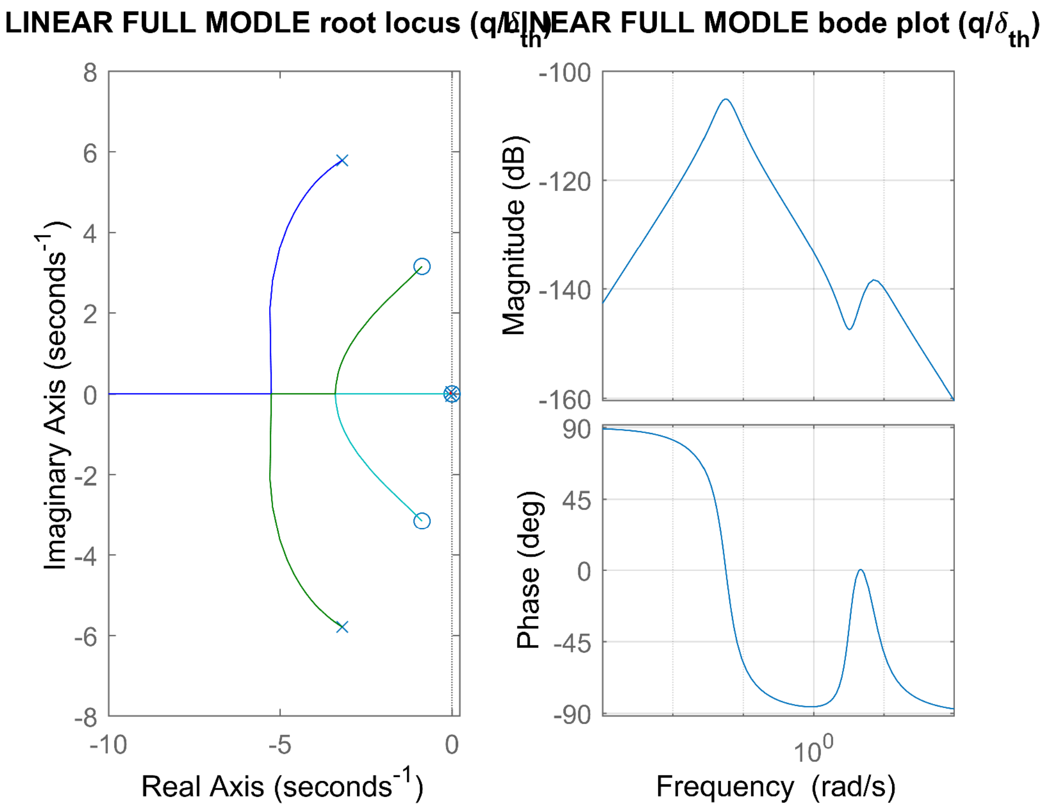


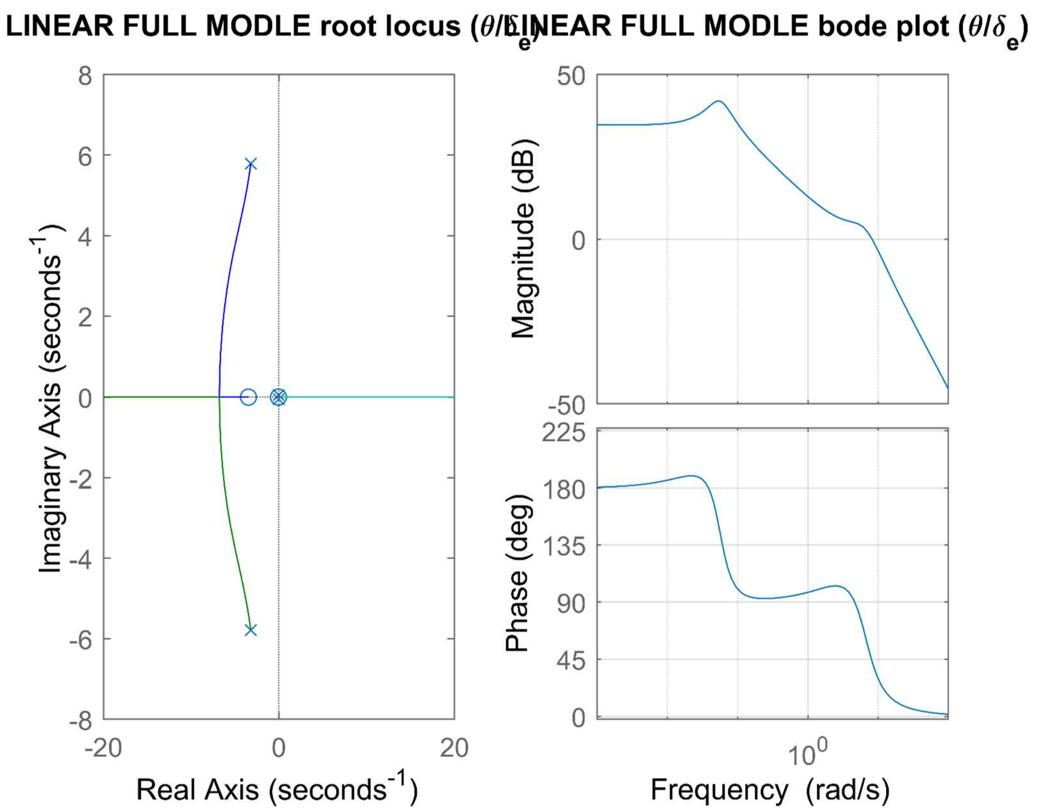

LINEAR FULL MODLE root locus (u/δ_{th})
LINEAR FULL MODLE bode plot (u/δ_{th})
Imaginary Axis (seconds^{-1})
Real Axis (seconds^{-1})
Magnitude (dB)
Phase (deg)
Frequency (rad/s)

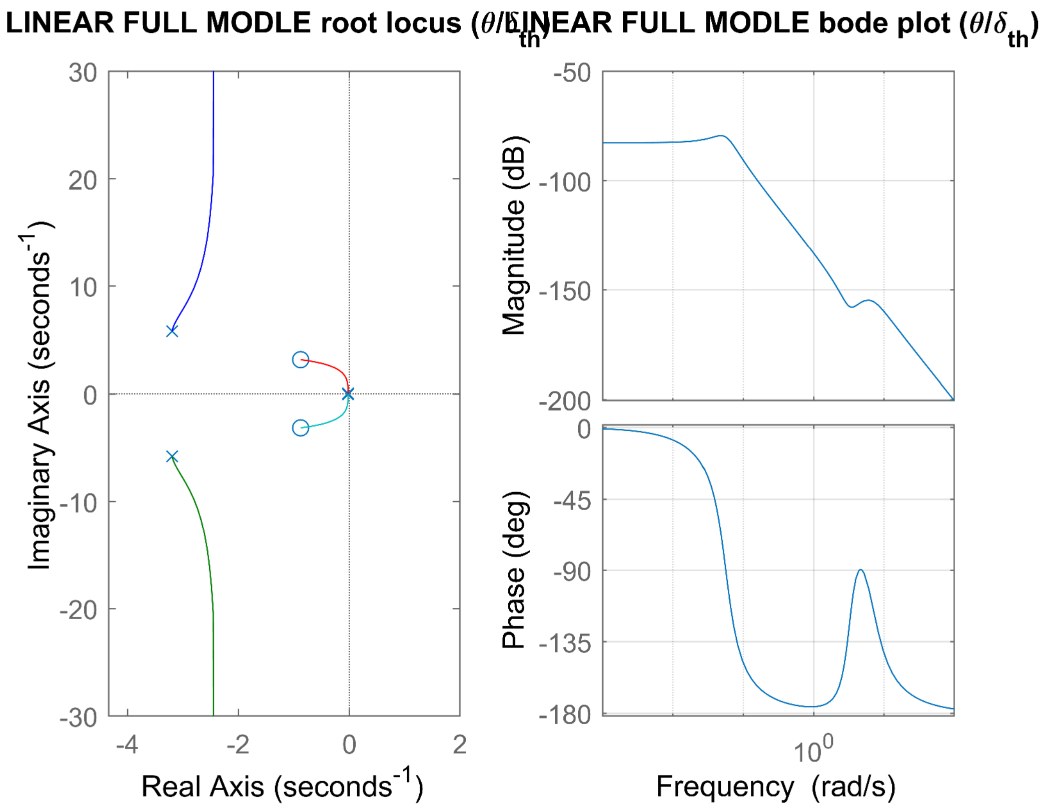

LINEAR FULL MODLE root locus (w/δ_e)
LINEAR FULL MODLE bode plot (w/δ_e)
Imaginary Axis (seconds^{-1})
Real Axis (seconds^{-1})
Magnitude (dB)
Phase (deg)
Frequency (rad/s)

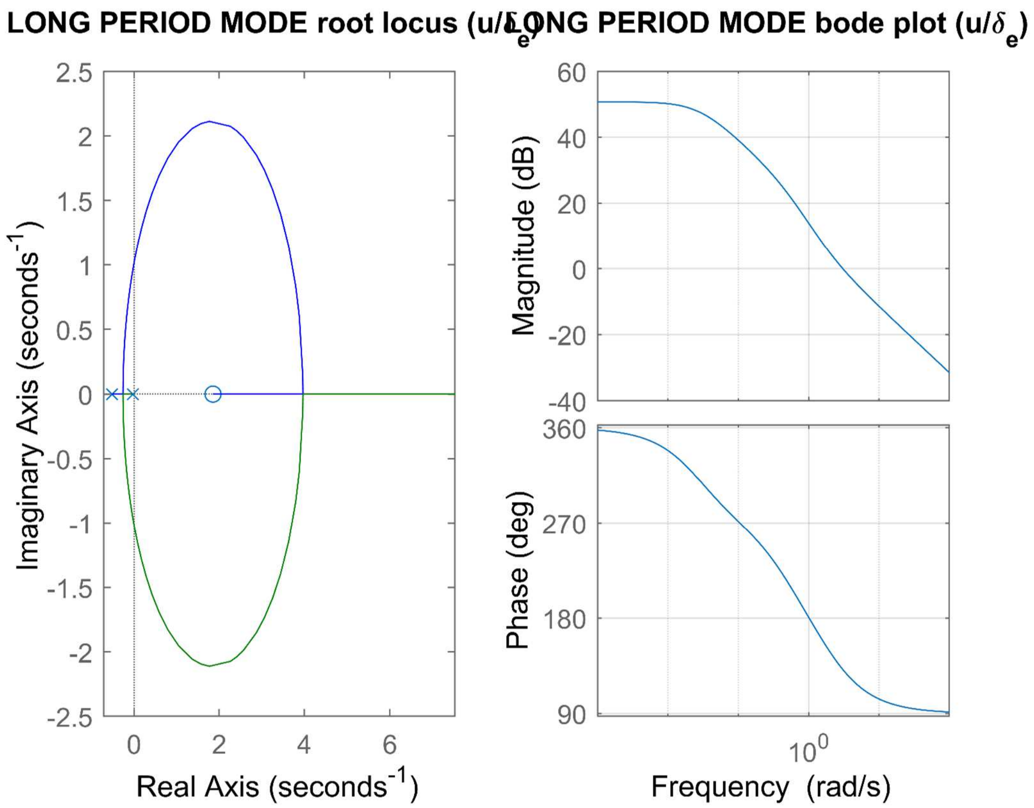

LINEAR FULL MODLE root locus (w/δth)
LINEAR FULL MODLE bode plot (w/δth)
Imaginary Axis (seconds-1)
Real Axis (seconds-1)
Magnitude (dB)
Phase (deg)
Frequency (rad/s)

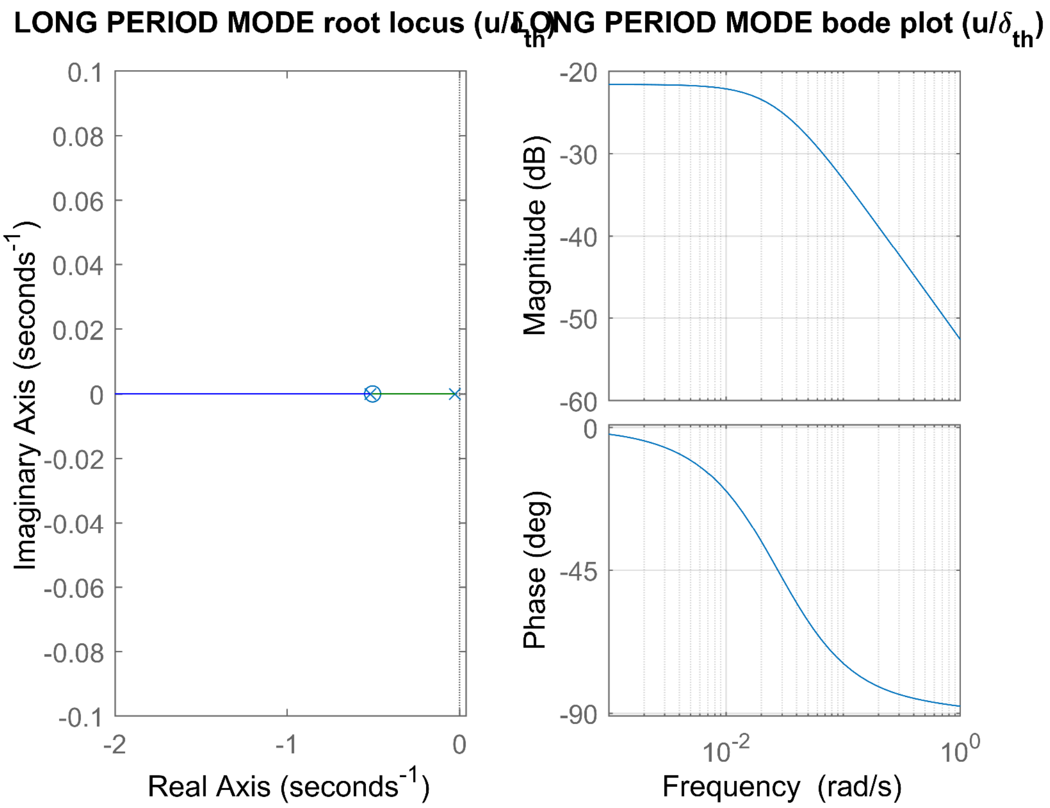

LINEAR FULL MODLE root locus (q/δe)
LINEAR FULL MODLE bode plot (q/δe)
Imaginary Axis (seconds-1)
Real Axis (seconds-1)
Magnitude (dB)
Phase (deg)
Frequency (rad/s)

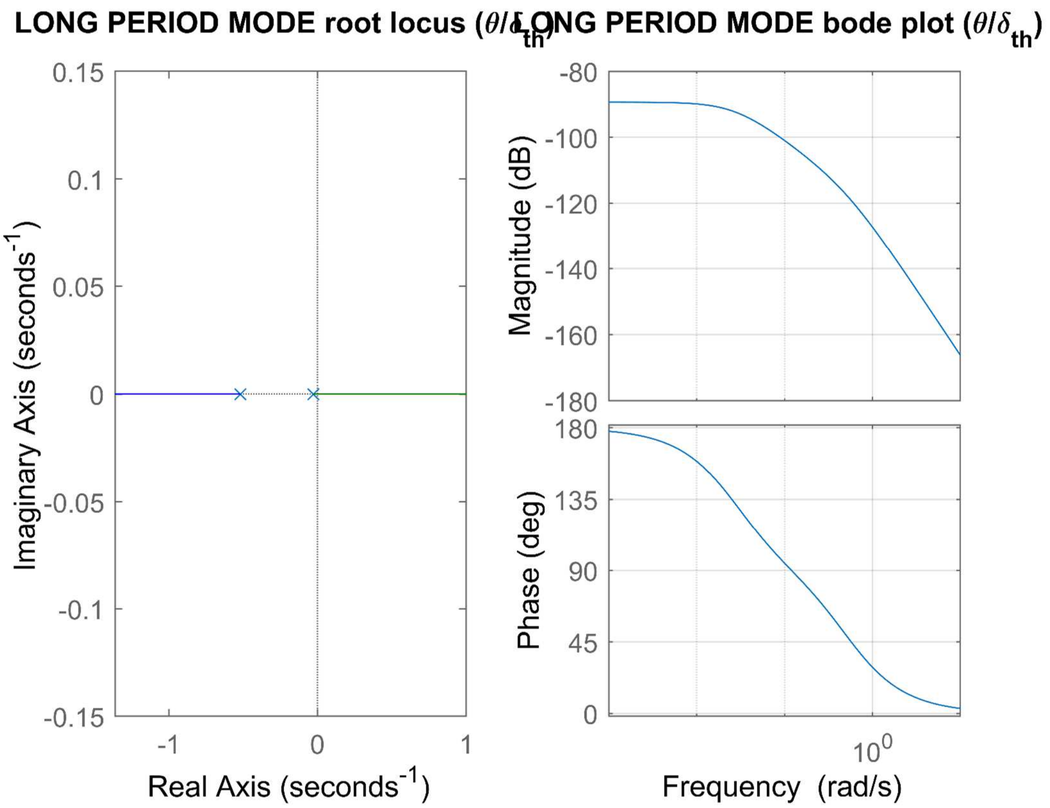

LINEAR FULL MODLE root locus (q/δth)
LINEAR FULL MODLE bode plot (q/δth)
Imaginary Axis (seconds-1)
Real Axis (seconds-1)
Magnitude (dB)
Phase (deg)
Frequency (rad/s)

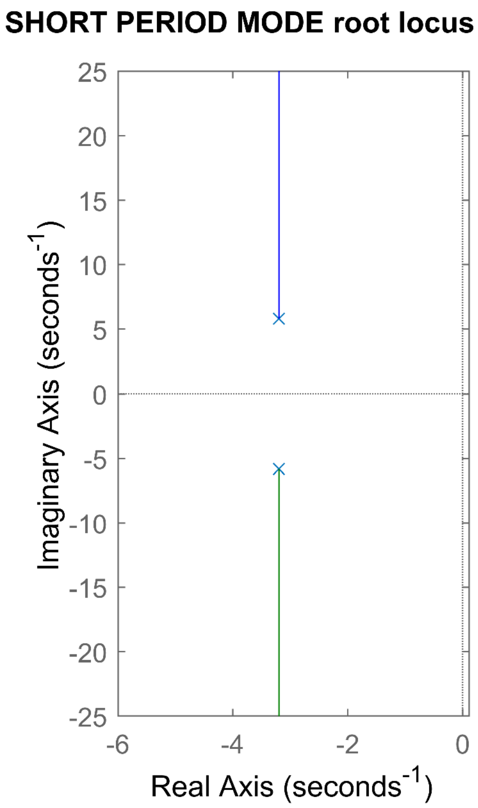

LINEAR FULL MODLE root locus (θ/δe)
LINEAR FULL MODLE bode plot (θ/δe)
Imaginary Axis (seconds-1)
Real Axis (seconds-1)
Magnitude (dB)
Phase (deg)
Frequency (rad/s)

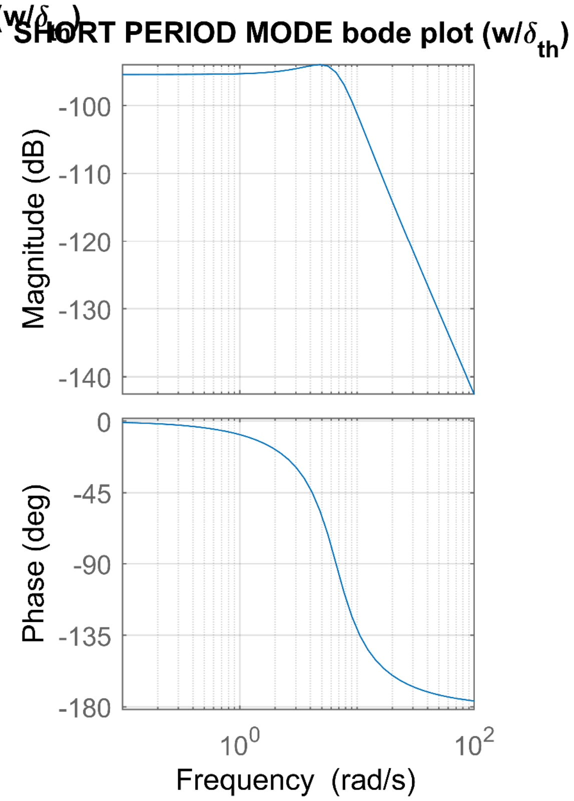

LINEAR FULL MODLE root locus (θ/δth)
Imaginary Axis (seconds-1)
Real Axis (seconds-1)
30
20
10
0
-10
-20
-30
-4
-2
0
2
LINEAR FULL MODLE bode plot (θ/δth)
Magnitude (dB)
-50
-100
-150
-200
Phase (deg)
0
-45
-90
-135
-180
10^0
Frequency (rad/s)

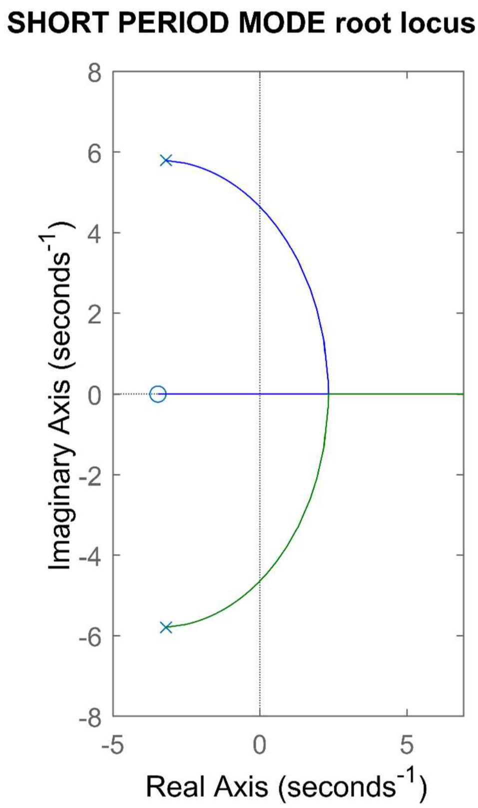

LONG PERIOD MODE root locus (u/δe)
Imaginary Axis (seconds-1)
Real Axis (seconds-1)
2.5
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
-2.5
0
2
4
6
LONG PERIOD MODE bode plot (u/δe)
Magnitude (dB)
60
40
20
0
-20
-40
Phase (deg)
360
270
180
90
10^0
Frequency (rad/s)

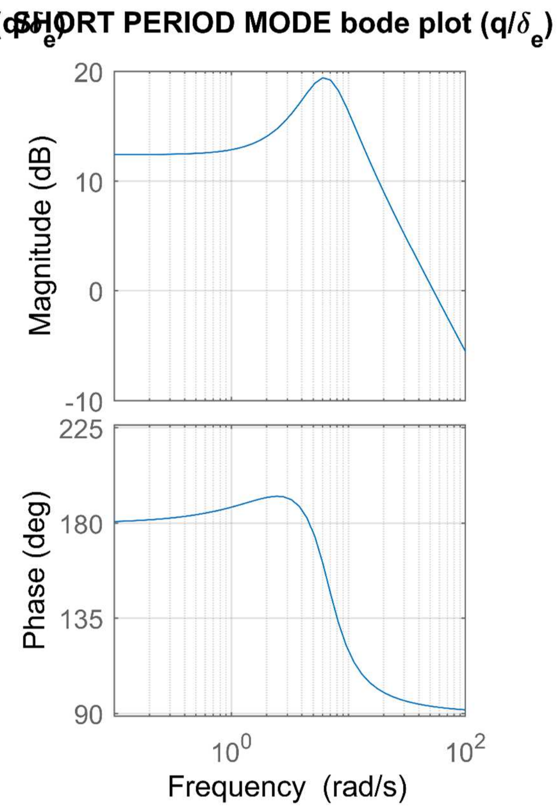

LONG PERIOD MODE root locus (u/δ_{th})
LONG PERIOD MODE bode plot (u/δ_{th})
Imaginary Axis (seconds^{-1})
Real Axis (seconds^{-1})
Magnitude (dB)
Phase (deg)
Frequency (rad/s)

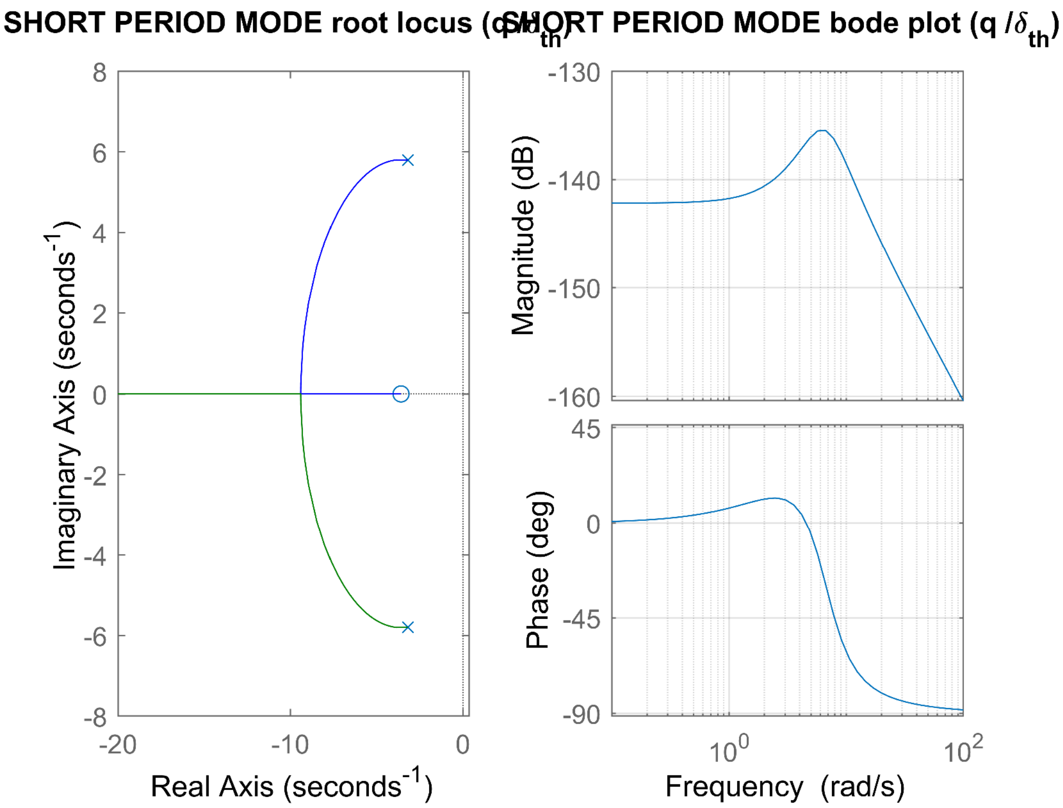

LONG PERIOD MODE root locus (θ/δ_{th})
LONG PERIOD MODE bode plot (θ/δ_{th})
Imaginary Axis (seconds^{-1})
Real Axis (seconds^{-1})
Magnitude (dB)
Phase (deg)
Frequency (rad/s)

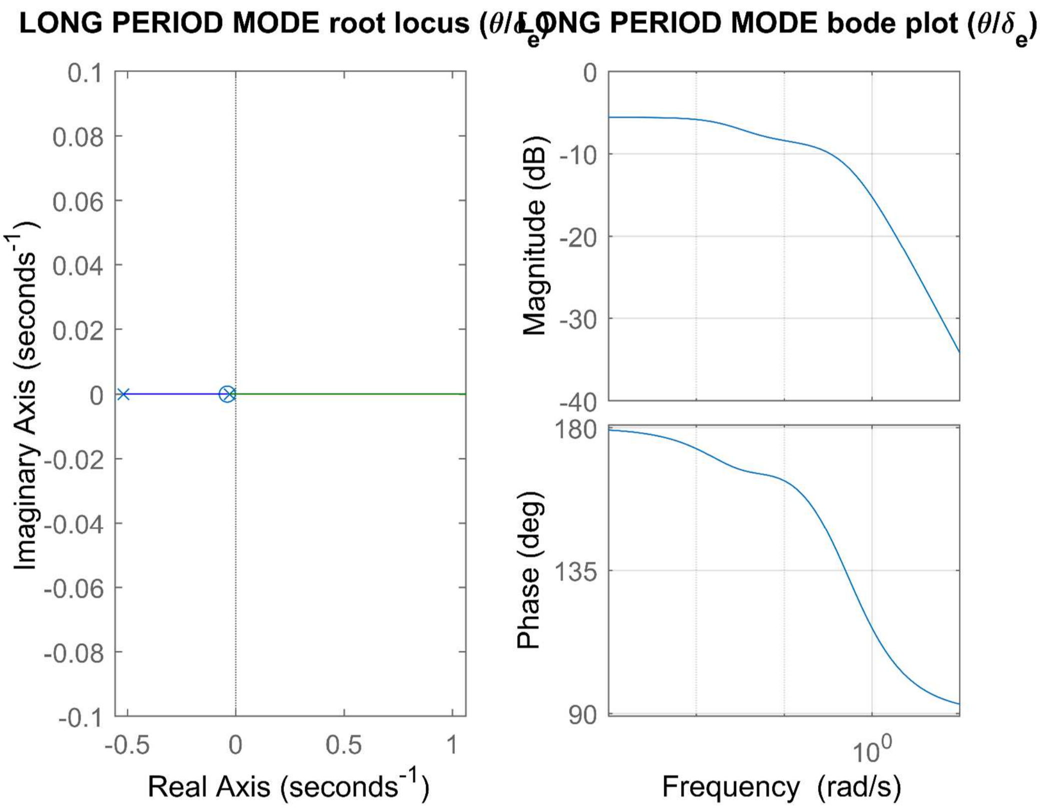

SHORT PERIOD MODE root locus (w/δ
Imaginary Axis (seconds-1)
Real Axis (seconds-1)
25
20
15
10
5
0
-5
-10
-15
-20
-25
-6
-4
-2
0


SHORT PERIOD MODE bode plot (w/δth)
Magnitude (dB)
-100
-110
-120
-130
-140
Phase (deg)
0
-45
-90
-135
-180
10^0
10^2
Frequency (rad/s)

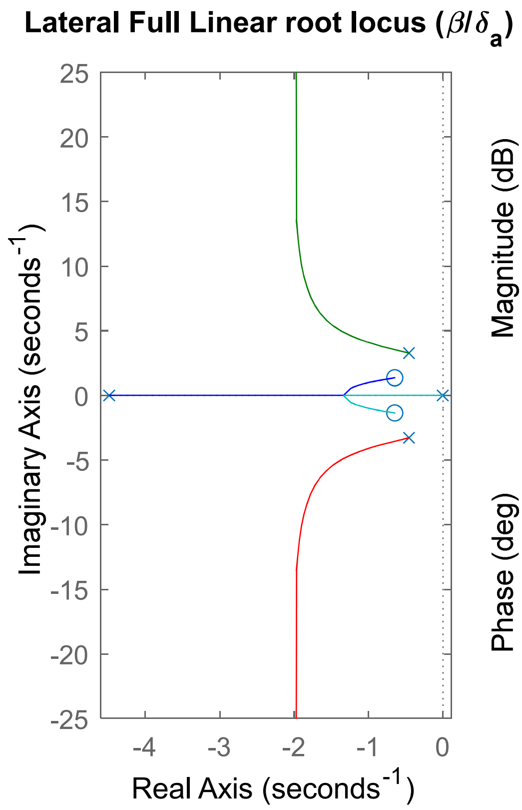

SHORT PERIOD MODE root locus (q/δe)
Imaginary Axis (seconds-1)
Real Axis (seconds-1)
8
6
4
2
0
-2
-4
-6
-8
-5
0
5

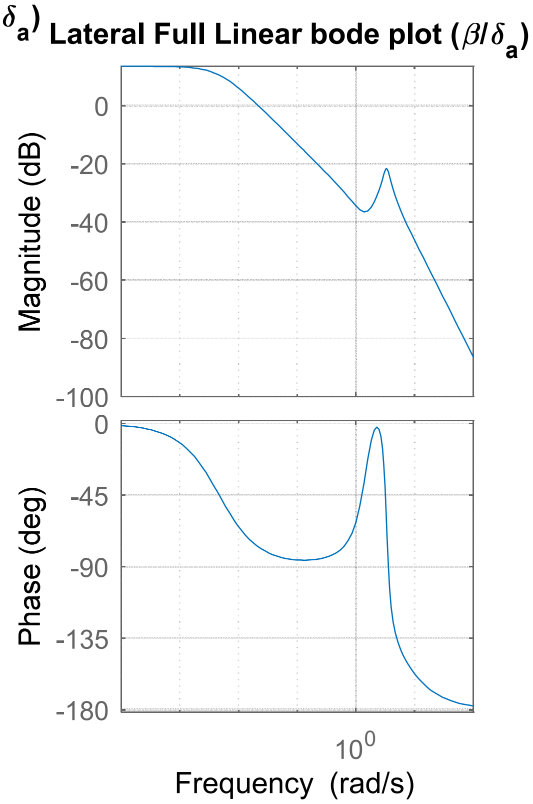

SHORT PERIOD MODE bode plot (q/δe)
Magnitude (dB)
20
10
0
-10
Phase (deg)
225
180
135
90
10^0
10^2
Frequency (rad/s)

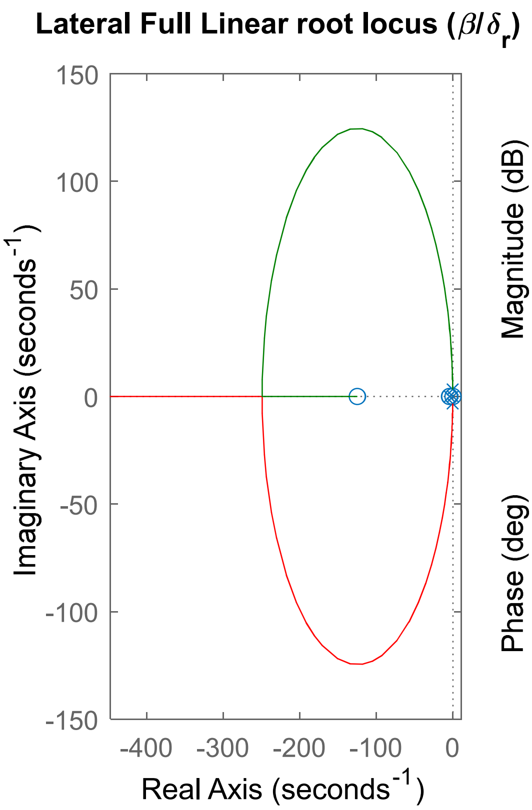

SHORT PERIOD MODE root locus (q/δth)
SHORT PERIOD MODE bode plot (q /δth)
Imaginary Axis (seconds-1)
Real Axis (seconds-1)
8
6
4
2
0
-2
-4
-6
-8
-20
-10
0
Magnitude (dB)
-130
-140
-150
-160
Phase (deg)
45
0
-45
-90
10^0
10^2
Frequency (rad/s)

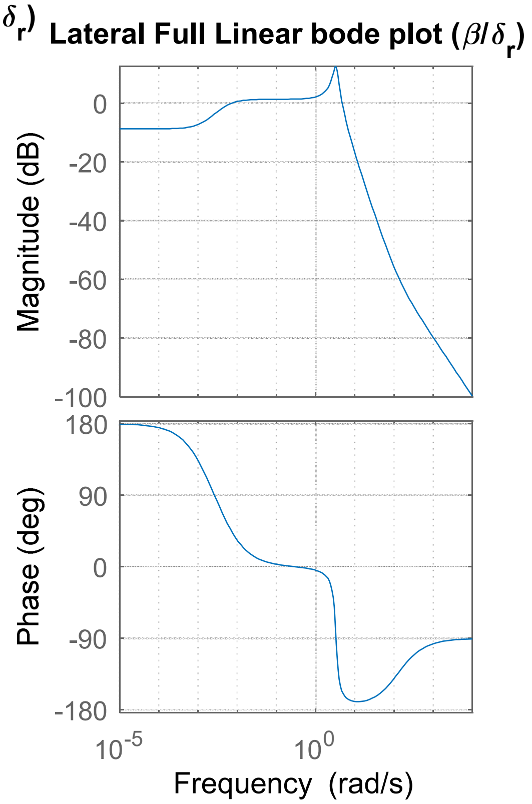

LONG PERIOD MODE root locus (θ/δe)
LONG PERIOD MODE bode plot (θ/δe)
Imaginary Axis (seconds-1)
Real Axis (seconds-1)
0.1
0.08
0.06
0.04
0.02
0
-0.02
-0.04
-0.06
-0.08
-0.1
-0.5
0
0.5
1
Magnitude (dB)
0
-10
-20
-30
-40
Phase (deg)
180
135
90
10^0
Frequency (rad/s)

Transfer Function graphs (Lateral Dynamics)

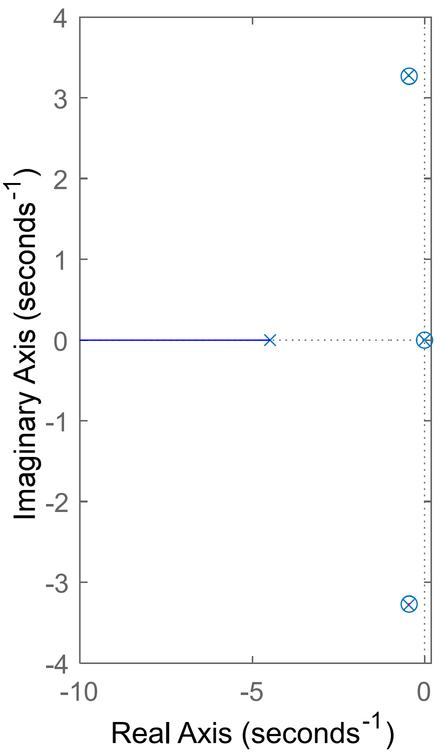




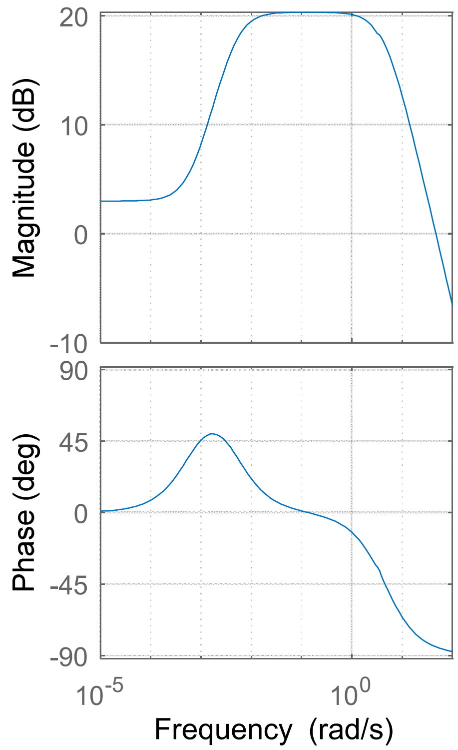




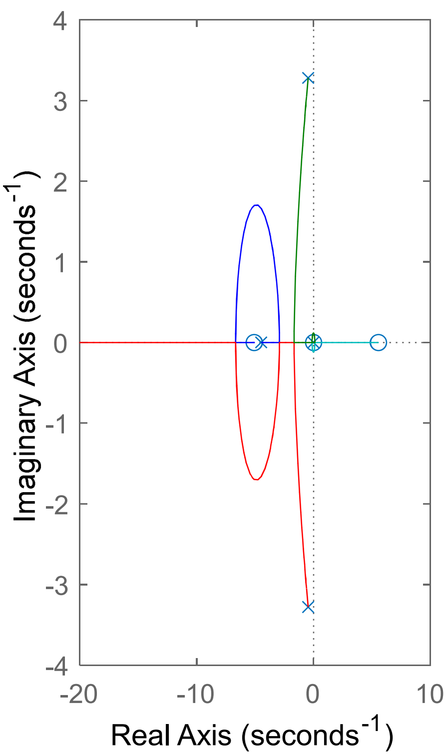
Lateral Full Linear root locus (p/δ_a)



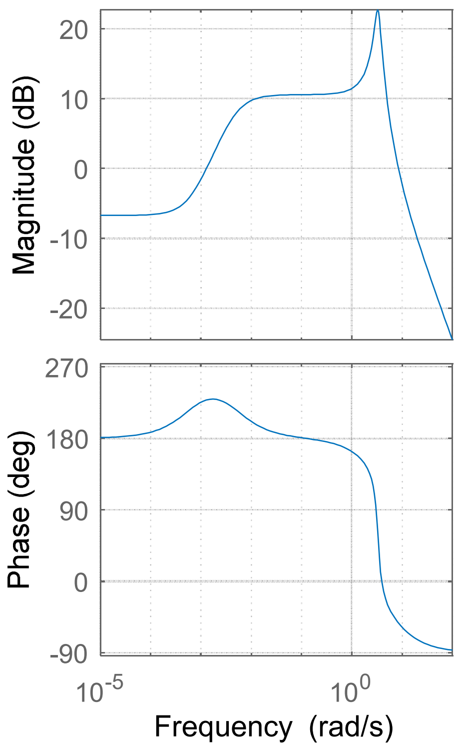
Lateral Full Linear bode plot (p/δ_a)

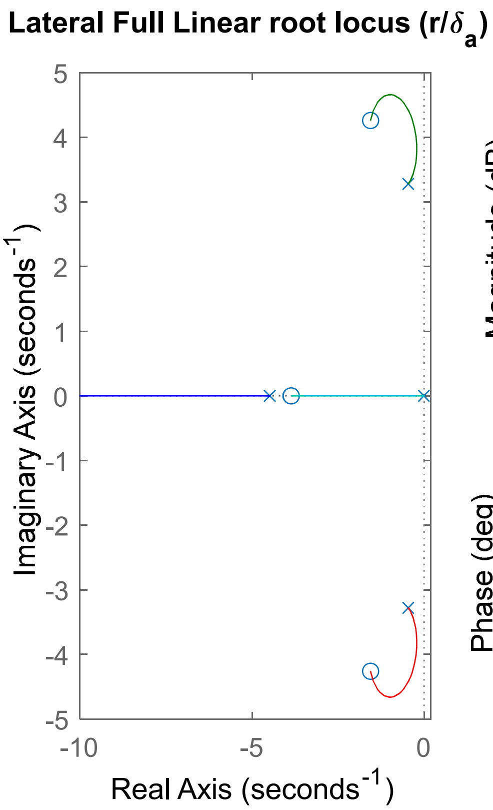


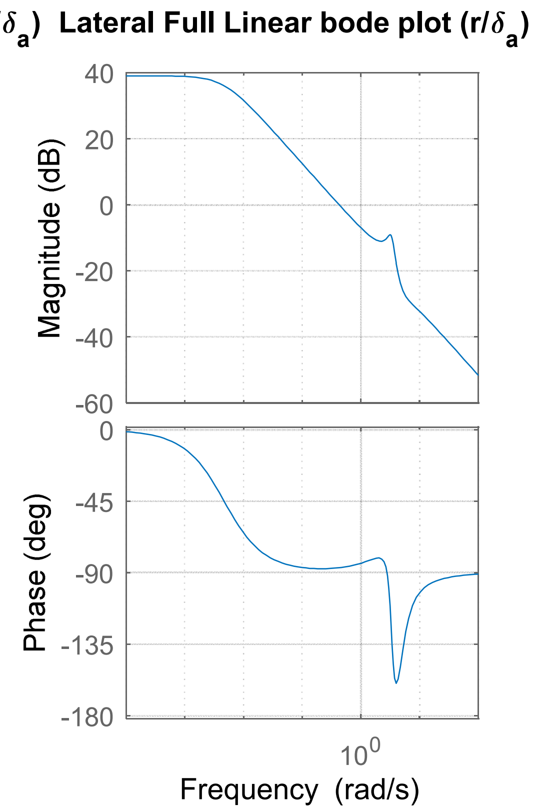
Lateral Full Linear root locus (p/δ_r)

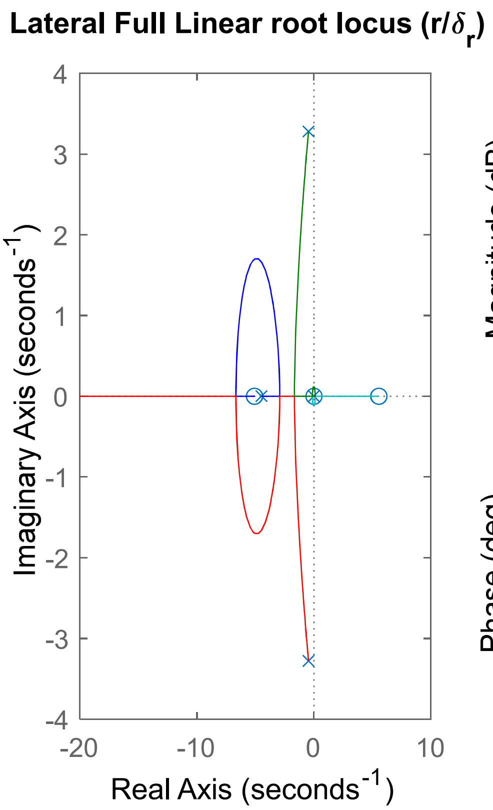


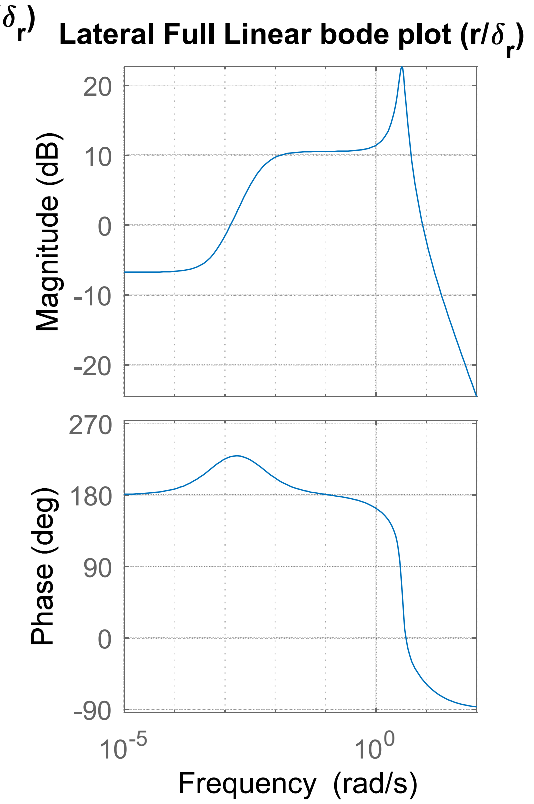
Lateral Full Linear bode plot (p/δ_r)



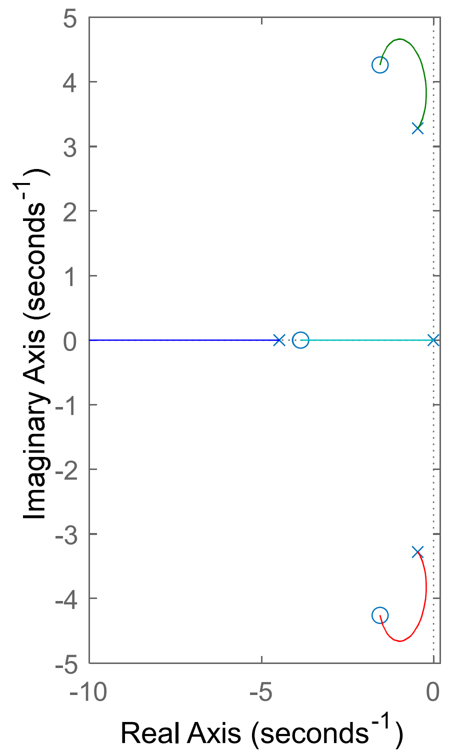

Lateral Full Linear root locus (r/δa)
Imaginary Axis (seconds-1)
Real Axis (seconds-1)
5
4
3
2
1
0
-1
-2
-3
-4
-5
-10
-5
0


Lateral Full Linear bode plot (r/δa)
Magnitude (dB)
40
20
0
-20
-40
-60
Phase (deg)
0
-45
-90
-135
-180
10^0
Frequency (rad/s)

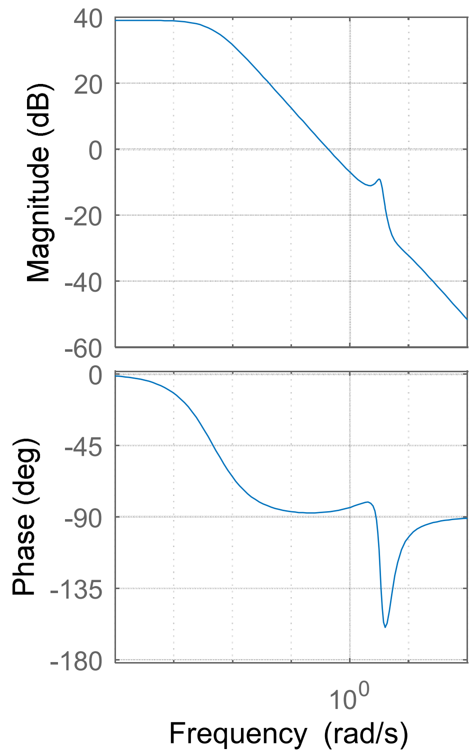

Lateral Full Linear root locus (r/δr)
Imaginary Axis (seconds-1)
Real Axis (seconds-1)
4
3
2
1
0
-1
-2
-3
-4
-20
-10
0
10


Lateral Full Linear bode plot (r/δr)
Magnitude (dB)
20
10
0
-10
-20
Phase (deg)
270
180
90
0
-90
10^-5
10^0
Frequency (rad/s)

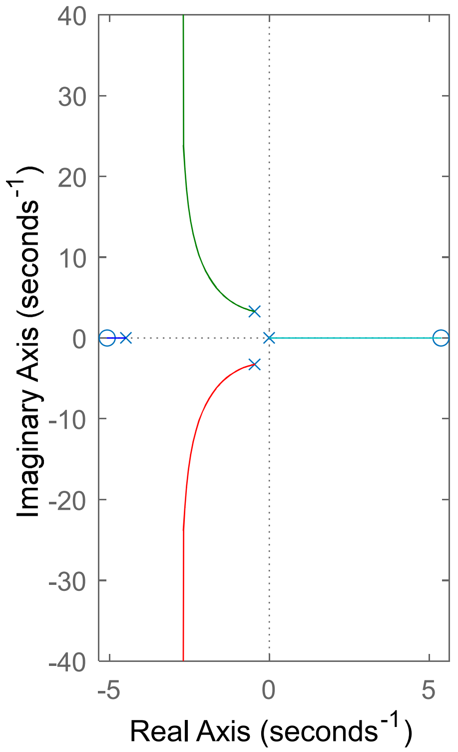
Lateral Full Linear root locus (ϕ/δ_a)



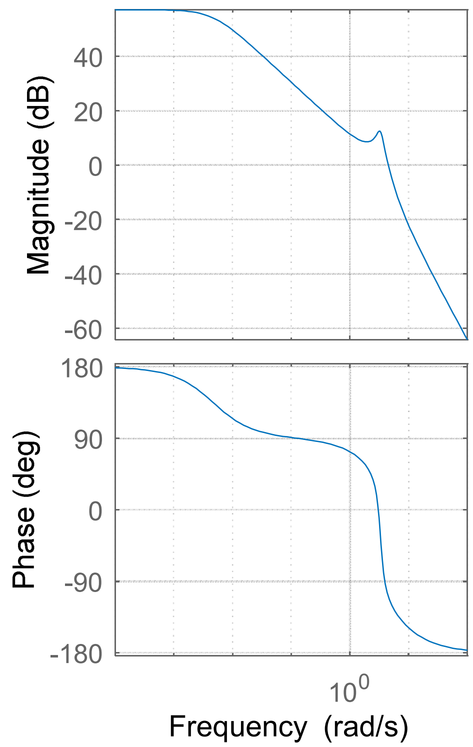
Lateral Full Linear bode plot (ϕ/δ_a)

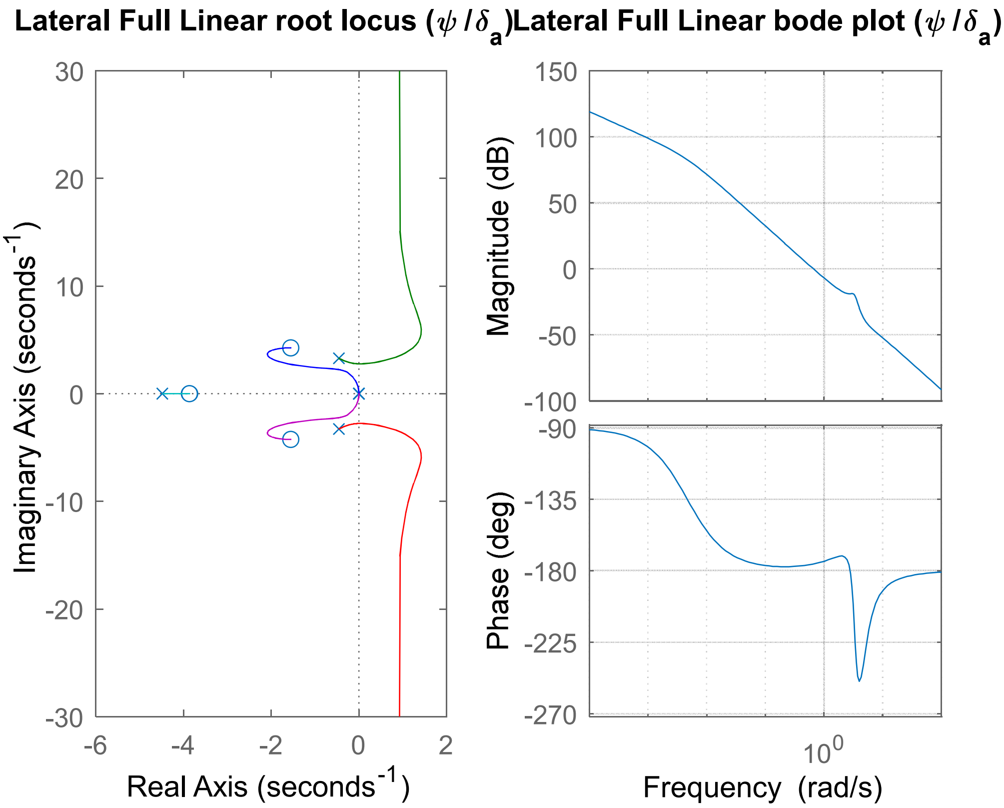


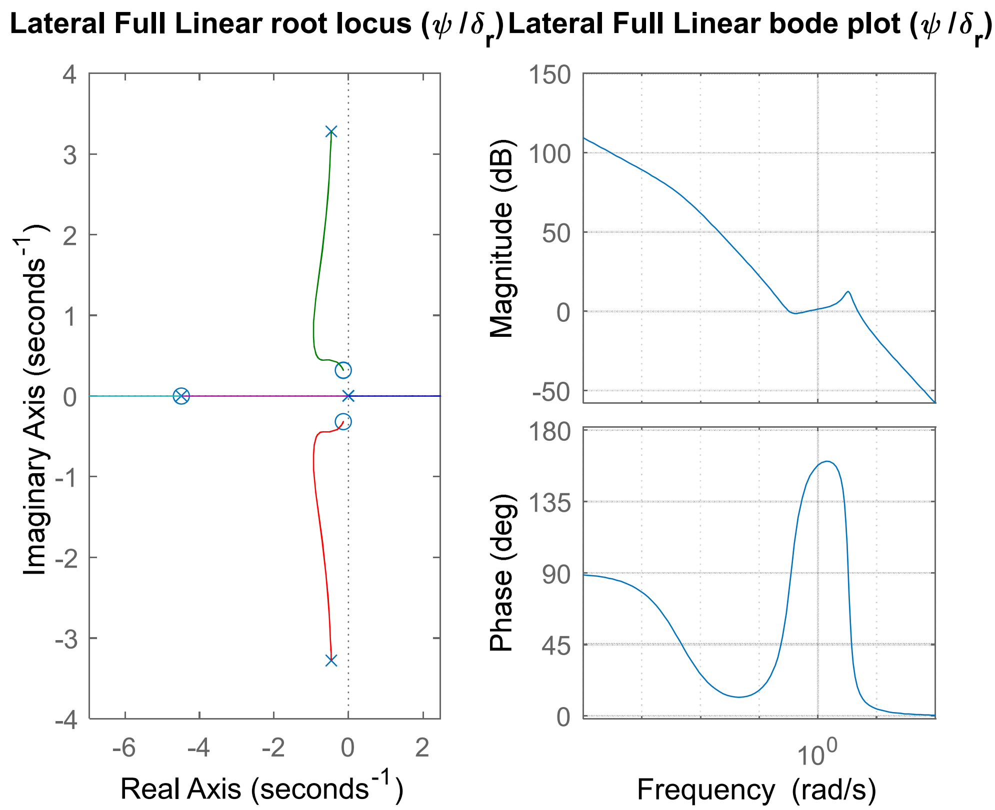
Lateral Full Linear root locus (ϕ/δ_r)

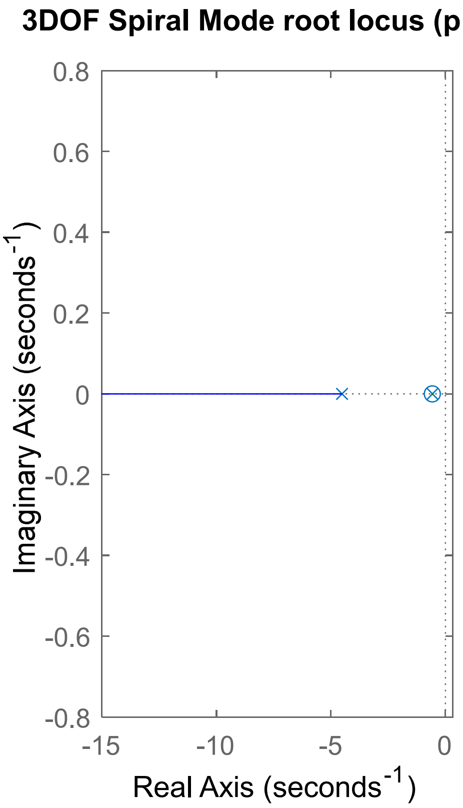


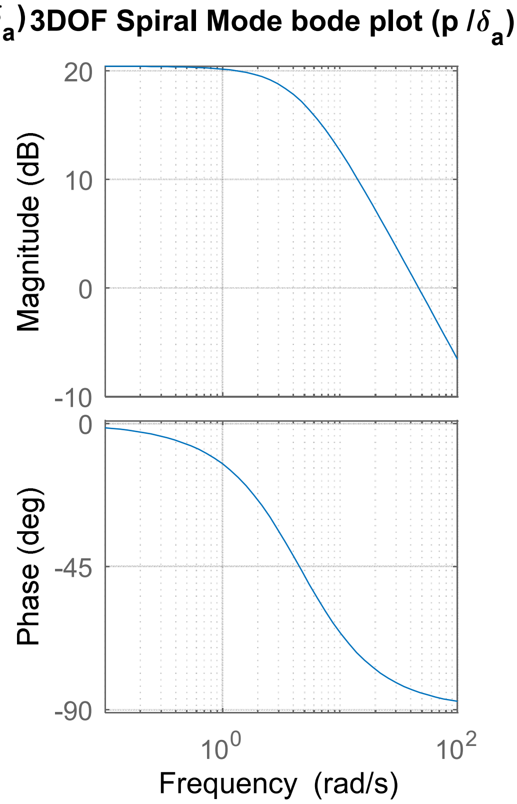
Lateral Full Linear bode plot (ϕ/δ_r)

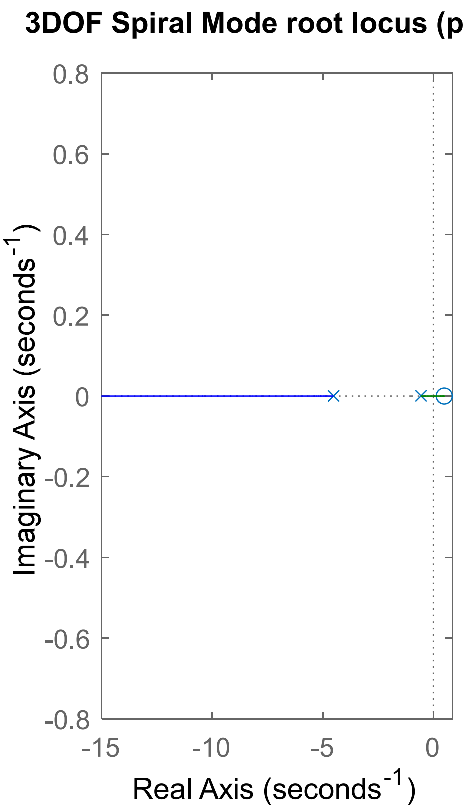


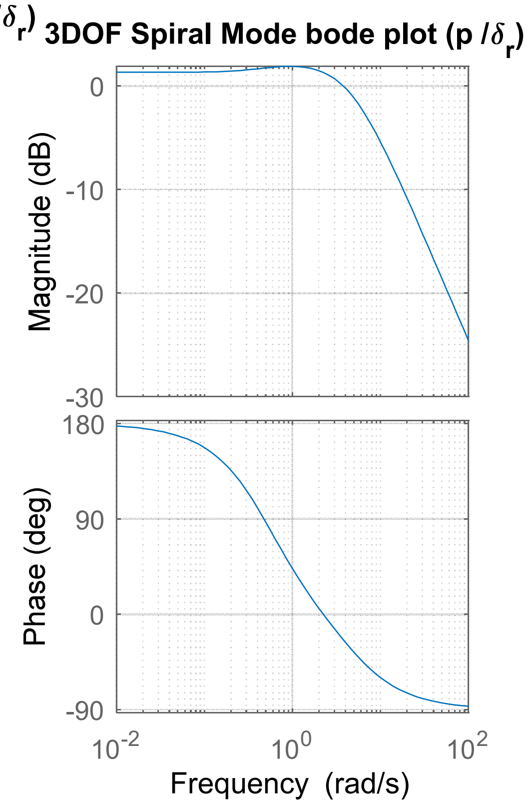

Lateral Full Linear root locus (ψ/δ_a)
Lateral Full Linear bode plot (ψ/δ_a)
Imaginary Axis (seconds^{-1})
Real Axis (seconds^{-1})
Magnitude (dB)
Phase (deg)
Frequency (rad/s)

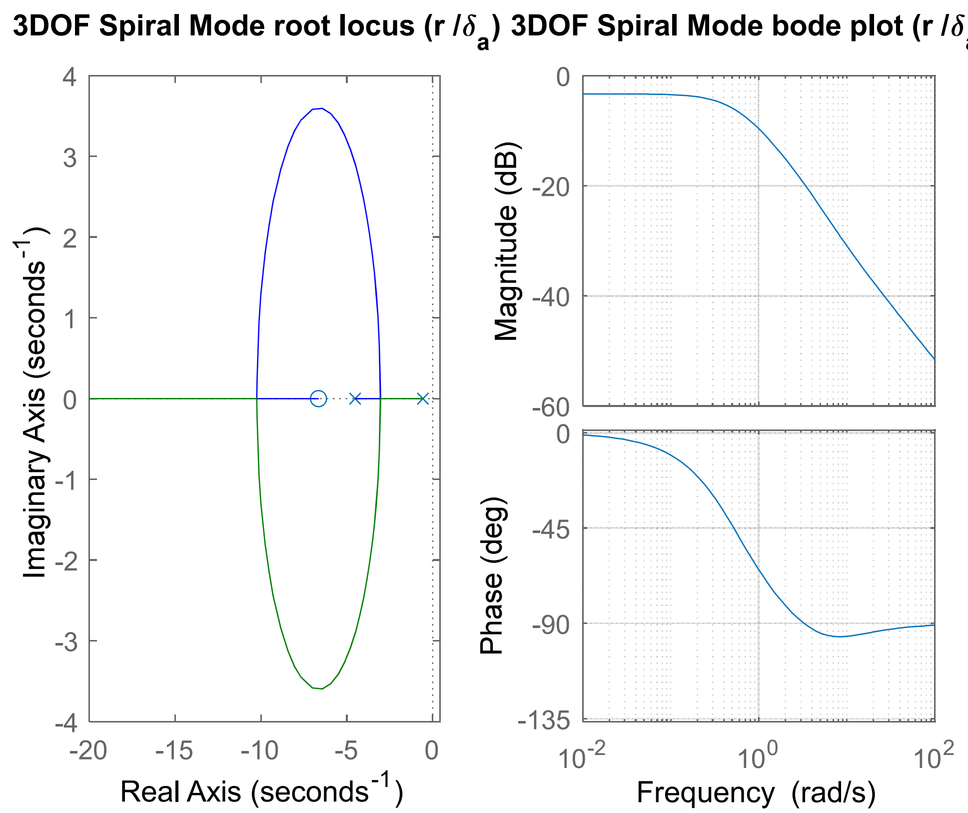

Lateral Full Linear root locus (ψ/δ_r)
Lateral Full Linear bode plot (ψ/δ_r)
Imaginary Axis (seconds^{-1})
Real Axis (seconds^{-1})
Magnitude (dB)
Phase (deg)
Frequency (rad/s)

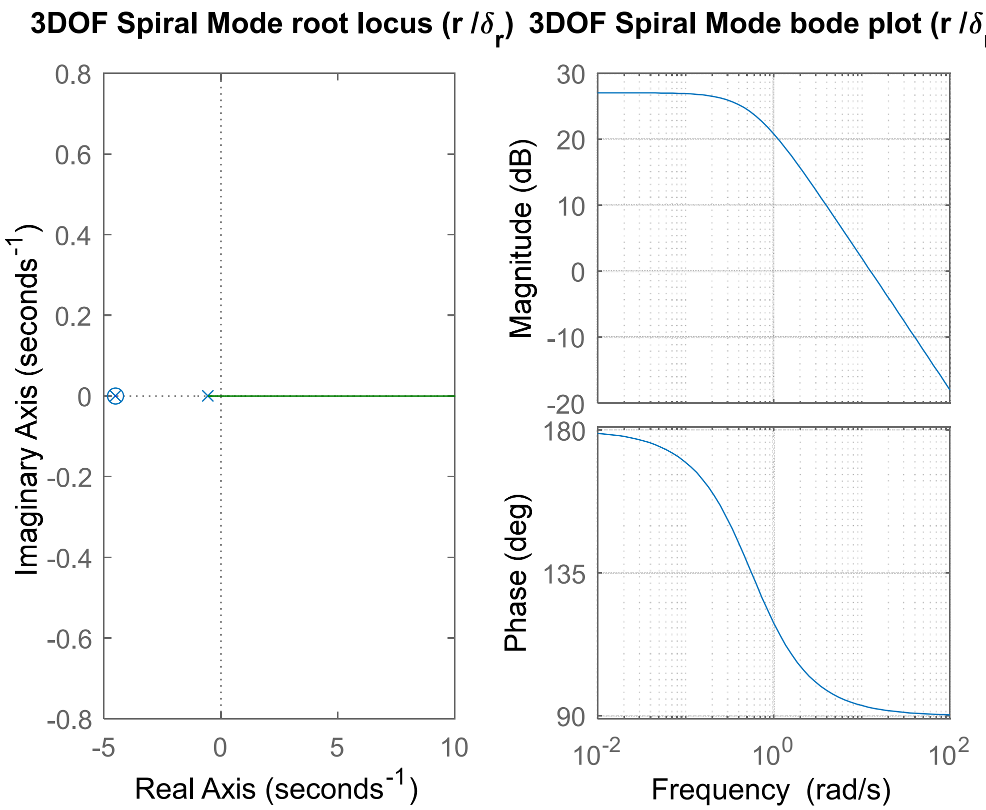

3DOF Spiral Mode root locus (p /δa)
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
Imaginary Axis (seconds-1)
-15
-10
-5
0
Real Axis (seconds-1)


3DOF Spiral Mode bode plot (p /δa)
20
10
0
-10
Magnitude (dB)
0
-45
-90
Phase (deg)
10^0
10^2
Frequency (rad/s)

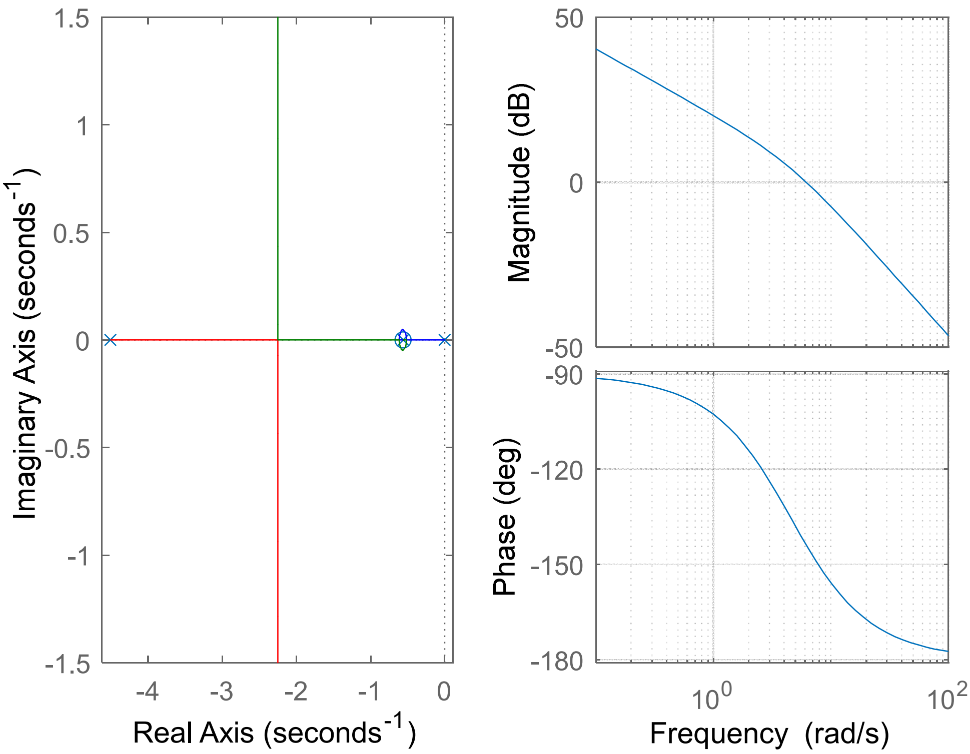

3DOF Spiral Mode root locus (p /δr)
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
Imaginary Axis (seconds-1)
-15
-10
-5
0
Real Axis (seconds-1)


3DOF Spiral Mode bode plot (p /δr)
0
-10
-20
-30
Magnitude (dB)
180
90
0
-90
Phase (deg)
10^-2
10^0
10^2
Frequency (rad/s)

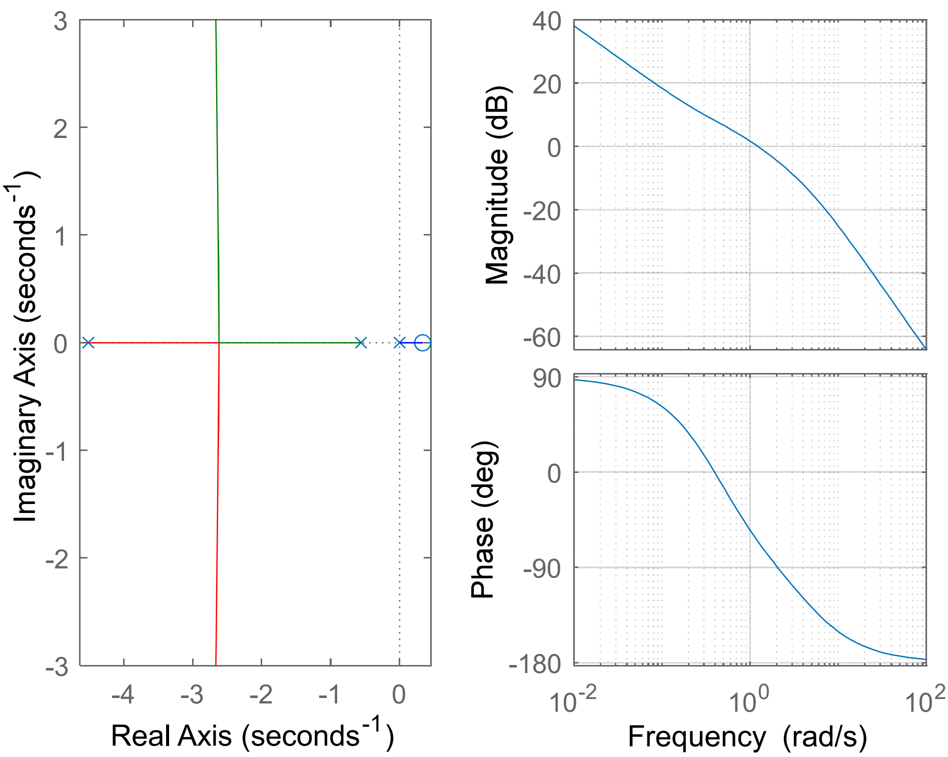

3DOF Spiral Mode root locus (r/δ_a)
3DOF Spiral Mode bode plot (r/δ_a)
Imaginary Axis (seconds^{-1})
Real Axis (seconds^{-1})
Magnitude (dB)
Phase (deg)
Frequency (rad/s)

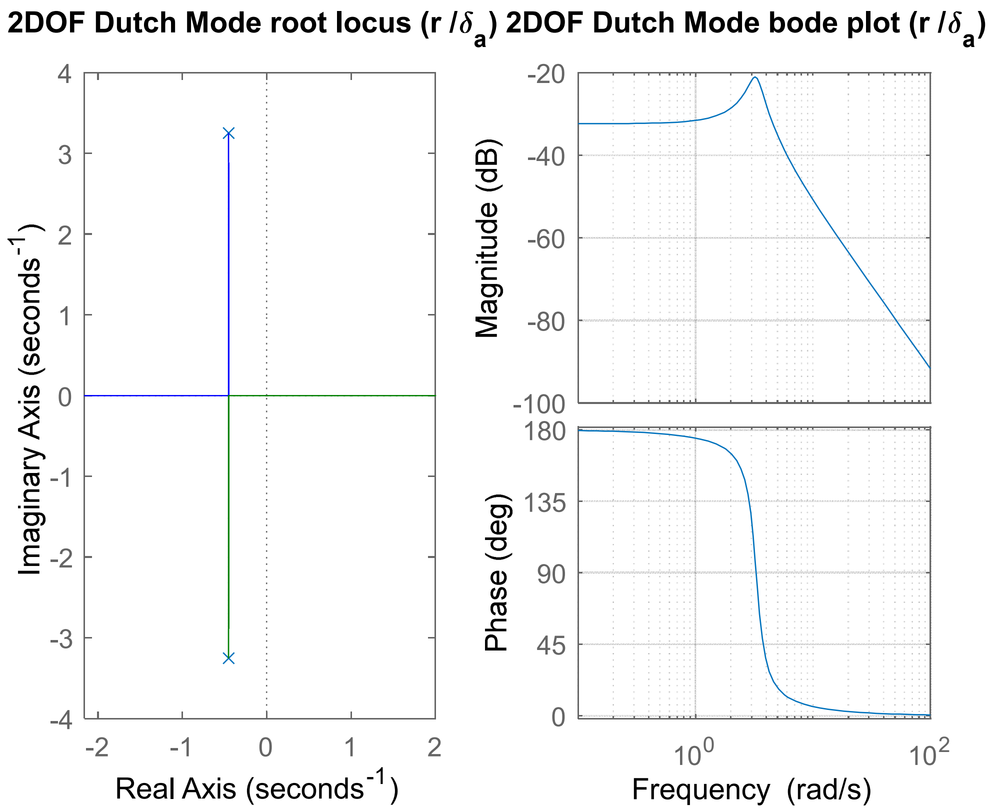

3DOF Spiral Mode root locus (r/δ_r)
3DOF Spiral Mode bode plot (r/δ_r)
Imaginary Axis (seconds^{-1})
Real Axis (seconds^{-1})
Magnitude (dB)
Phase (deg)
Frequency (rad/s)

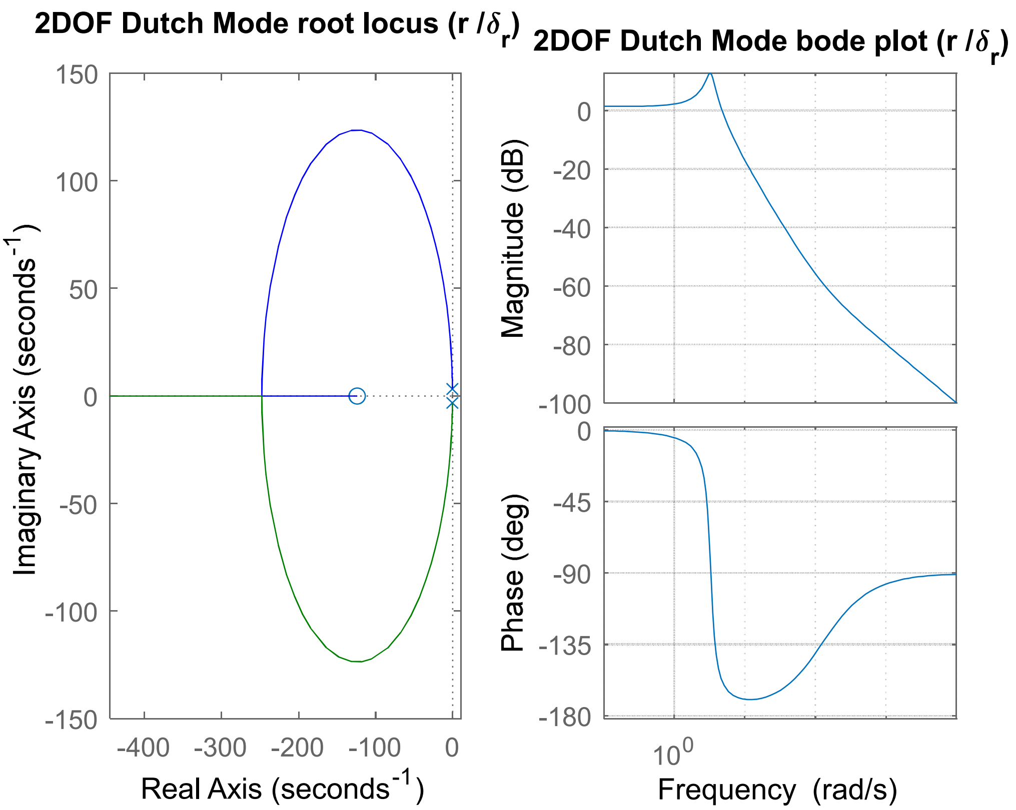
3DOF Spiral Mode root locus (ϕ/δ_a) 3DOF Spiral Mode bode plot (ϕ/δ_a)

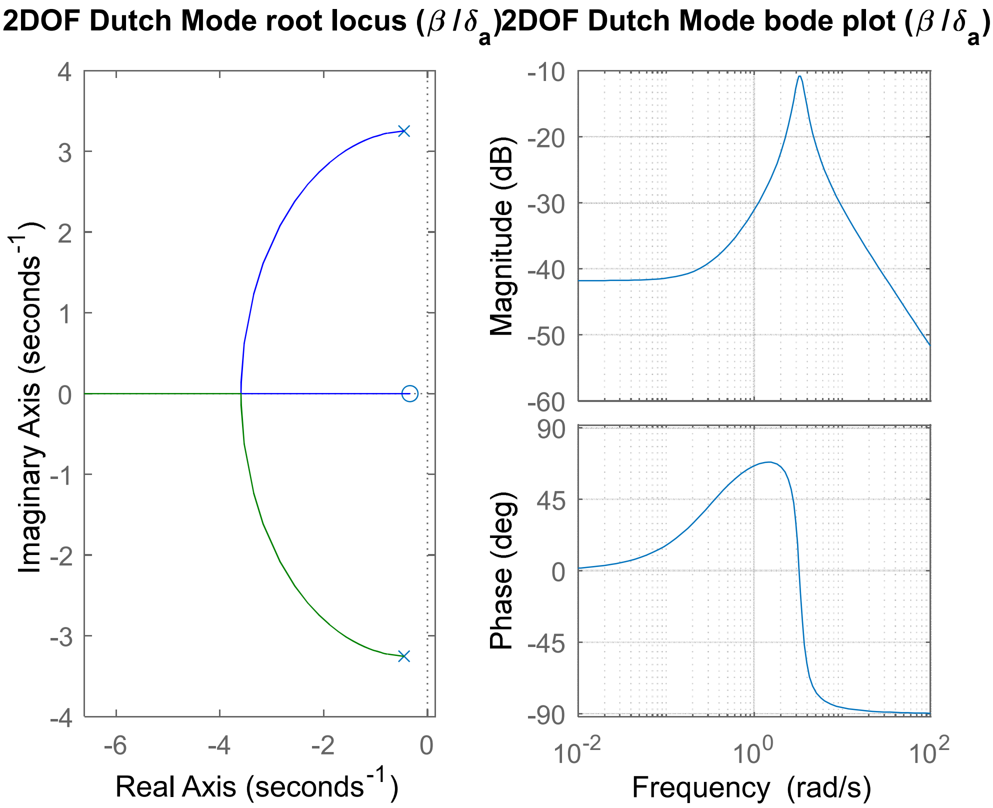


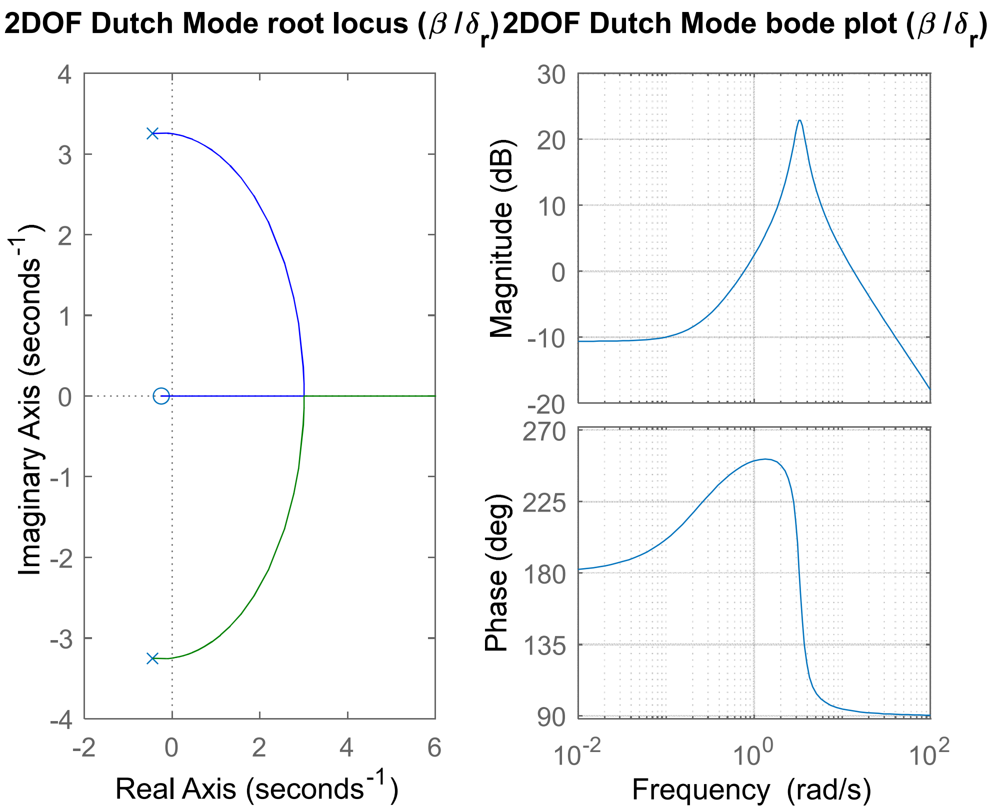
3DOF Spiral Mode root locus (ϕ/δ_r) 3DOF Spiral Mode bode plot (ϕ/δ_r)

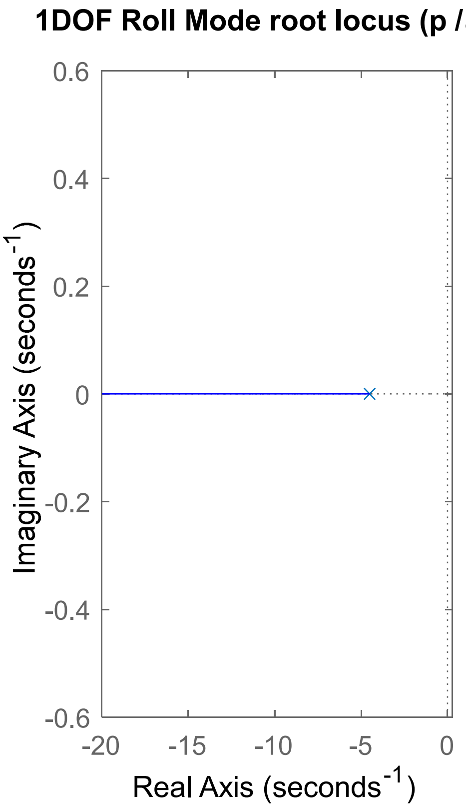


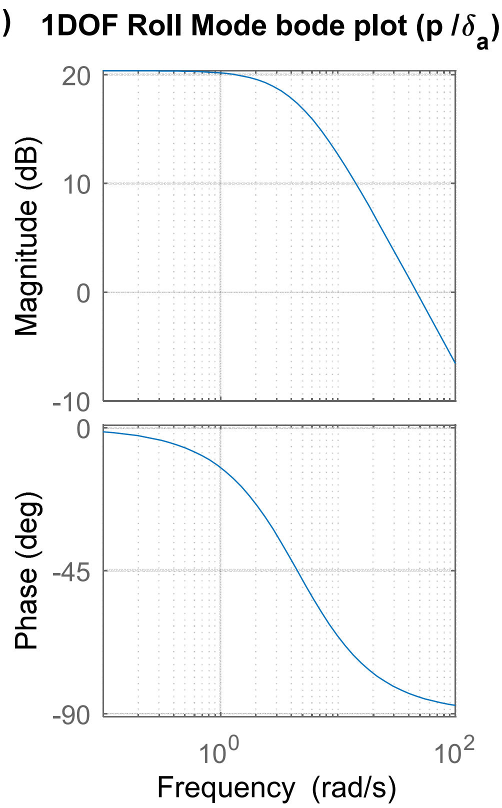

2DOF Dutch Mode root locus (r /δa)
2DOF Dutch Mode bode plot (r /δa)
Imaginary Axis (seconds-1)
Real Axis (seconds-1)
Magnitude (dB)
Phase (deg)
Frequency (rad/s)


2DOF Dutch Mode root locus (r /δr)
2DOF Dutch Mode bode plot (r /δr)
Imaginary Axis (seconds-1)
Real Axis (seconds-1)
Magnitude (dB)
Phase (deg)
Frequency (rad/s)


2DOF Dutch Mode root locus (β/δ_a)
2DOF Dutch Mode bode plot (β/δ_a)
Imaginary Axis (seconds^{-1})
Real Axis (seconds^{-1})
Magnitude (dB)
Phase (deg)
Frequency (rad/s)
4
3
2
1
0
-1
-2
-3
-4
-6
-4
-2
0
-10
-20
-30
-40
-50
-60
90
45
0
-45
-90
10^{-2}
10^{0}
10^{2}


2DOF Dutch Mode root locus (β/δ_r)
2DOF Dutch Mode bode plot (β/δ_r)
Imaginary Axis (seconds^{-1})
Real Axis (seconds^{-1})
Magnitude (dB)
Phase (deg)
Frequency (rad/s)
4
3
2
1
0
-1
-2
-3
-4
-2
0
2
4
6
30
20
10
0
-10
-20
270
225
180
135
90
10^{-2}
10^{0}
10^{2}





Transfer Function codes (Lateral Dynamics)

```
% the aerodynamic forces and moments can be expressed as a function of all the
motion variables [Nelson] page 63
clc
clear All
close all
%  Excel Sheets Data
% global aircraft_derivatives_dimensions
filename_density_L = 'NT-33A_4.xlsx'; %%put here the location of your excel sheet

aircraft_data=xlsread(filename_density_L,'B2:B61');%% here B2:B61 means read the
excel sheet from cell B2 to cell B61

%%in the next step we will read from the vector(aircraft_data) but take care of the
order the values in excel sheet is arranged
% initial conditions
s0 = aircraft_data(4:15);

% control actions values
% da = aircraft_data(57);
% dr = aircraft_data(58);
% de = aircraft_data(59);
% dth = aircraft_data(60);
dc = [ aircraft_data(57:59) * pi/180 ; aircraft_data(60)];

% gravity, mass % inertia
m = aircraft_data(51);
g = aircraft_data(52);
```

```
Ixx = aircraft_data(53);
Iyy = aircraft_data(54);
Izz = aircraft_data(55);
Ixz = aircraft_data(56);    Ixy=0;  Iyz=0;
I = [Ixx , -Ixy , -Ixz ;...
    -Ixy , Iyy , -Iyz ;...
    -Ixz , -Iyz , Izz];
invI=inv(I);

% stability derivatives Longitudinal motion
SD_Long = aircraft_data(21:36);
XU=SD_Long(1);
ZU=SD_Long(2);
MU=SD_Long(3);
XW=SD_Long(4);
ZW=SD_Long(5);
MW=SD_Long(6);
ZWD=SD_Long(7);
ZQ=SD_Long(8);
MWD=SD_Long(9);
MQ=SD_Long(10);
XDE=SD_Long(11);
ZDE=SD_Long(12);
MDE=SD_Long(13);
XDTH=SD_Long(14);
ZDTH=SD_Long(15);
MDTH=SD_Long(16);

U0=s0(1); W0=s0(3); TH0=s0(8);
Vto = sqrt(s0(1)^2 + s0(2)^2 + s0(3)^2);    % Vto

% stability derivatives Lateral motion
SD_Lat_dash = aircraft_data(37:50);

Yp=0;
Yr=0;
YDa_star=SD_Lat_dash(9);
YDr_star=SD_Lat_dash(10);
Yb=SD_Lat_dash(2);
YDa=SD_Lat_dash(9)*Vto;
YDr=SD_Lat_dash(10)*Vto;

Lbd=SD_Lat_dash(3);
Lpd=SD_Lat_dash(5);
Lrd=SD_Lat_dash(7);
LDrd=SD_Lat_dash(13);
LDad=SD_Lat_dash(11);

Nbd=SD_Lat_dash(4);
Npd=SD_Lat_dash(6);
Nrd=SD_Lat_dash(8);
NDrd=SD_Lat_dash(14);
NDad=SD_Lat_dash(12);

% initial gravity force
mg0 = m*g * [ sin(s0(8)) ; -cos(s0(8))*sin(s0(7)) ; -cos(s0(8))*cos(s0(7)) ];
```

liniearized set of Longitudinal equs

```
A_longt=[XU XW -W0 -g*cos(TH0)

    ZU/(1-ZWD) ZW/(1-ZWD) (ZQ+U0)/(1-ZWD) -g*sin(TH0)/(1-ZWD)

    MU+MWD*ZU/(1-ZWD) MW+MWD*ZW/(1-ZWD) MQ+MWD*(ZQ+U0)/(1-ZWD) -MWD*g*sin(TH0)/(1-
ZWD)
```

```
    0 0 1 0];

B_longt=[XDE XDTH

    ZDE/(1-ZWD) ZDTH/(1-ZWD)

    MDE+MWD*ZDE/(1-ZWD) MDTH+MWD*ZDTH/(1-ZWD)

    0 0];

C_longt=eye(4); D_longt=zeros(4,2);


LONGT_SS=ss(A_longt,B_longt,C_longt,D_longt);

LONGT_TF=tf(LONGT_SS);


U_DE_F=LONGT_TF(1,1);

U_DTH_F=LONGT_TF(1,2);

W_DE_F=LONGT_TF(2,1);

W_DTH_F=LONGT_TF(2,2);

Q_DE_F=LONGT_TF(3,1);

Q_DTH_F=LONGT_TF(3,2);

THETA_DE_F=LONGT_TF(4,1);

THETA_DTH_F=LONGT_TF(4,2);
```

PHUGOID MODE (LONG PERIOD MODE)

```
A_phug=[XU -g*cos(TH0)

    -ZU/(U0+ZQ) g*sin(TH0)];

B_phug=[XDE XDTH

    -ZDE/(ZQ+U0) -ZDTH/(ZQ+U0)];

C_phug=eye(2);

D_phug=zeros(2,2);


PHUG_SS=ss(A_phug,B_phug,C_phug,D_phug);
```

```
PHUG_TF=tf(PHUG_SS);



U_DE_PH=PHUG_TF(1,1);

U_DTH_PH=PHUG_TF(1,2);

THETA_DE_PH=PHUG_TF(2,1);

THETA_DTH_PH=PHUG_TF(2,2);
```

SHORT PERIOD MODE

```
A_short=[ZW/(1-ZWD) (ZQ+U0)/(1-ZWD)

    (MW+ZW*MWD/(1-ZWD)) (MQ+MWD*(ZQ+U0)/(1-ZWD))];

B_short=[ZDE/(1-ZWD) ZDTH

    MDE+MWD*ZDE/(1-ZWD) MDTH+MWD*ZDTH/(1-ZWD)];

C_short=eye(2);

D_short=zeros(2,2);



SHORT_SS=ss(A_short,B_short,C_short,D_short);

SHORT_TF=tf(SHORT_SS);



W_DE_PH=SHORT_TF(1,1);

W_DTH_PH=SHORT_TF(1,2);

Q_DE_PH=SHORT_TF(2,1);

Q_DTH_PH=SHORT_TF(2,2);
```

Liniearized set of Lateral Equation

```
A_Lat=[Yb/Vto (Yp+W0)/Vto (Yr-U0)/Vto g*cos(TH0)/Vto 0;...
       Lbd Lpd Lrd 0 0;...
       Nbd Npd Nrd 0 0;...
       0 1 tan(TH0) 0 0;...
       0 0 1/cos(TH0) 0 0];
B_Lat=[YDa_star YDr_star;...
       LDad LDrd;...
       NDad NDrd;...
       0 0;0 0];
C_Lat=eye(5); D_Lat=zeros(5,2);
```

```
Lateral_SS = ss(A_Lat,B_Lat,C_Lat,D_Lat);
Lateral_TF = tf(Lateral_SS);

B_DA_L = Lateral_TF(1,1);
B_DR_L = Lateral_TF(1,2);

P_DA_L = Lateral_TF(2,1);
P_DR_L = Lateral_TF(2,2);

R_DA_L = Lateral_TF(3,1);
R_DR_L = Lateral_TF(3,2);

PHI_DA_L = Lateral_TF(4,1);
PHI_DR_L = Lateral_TF(4,2);

PSI_DA_L = Lateral_TF(5,1);
PSI_DR_L = Lateral_TF(5,2);
```

3DOF Spiral Mode Approximation

```
A_Spiral = [Lpd Lrd 0;...
            Npd Nrd 0;...
            1 tan(TH0) 0];
B_Spiral = [LDad LDrd;NDad NDrd;0 0];
C_Spiral = eye(3); D_Spiral = zeros(3,2);

Spiral_SS = ss(A_Spiral,B_Spiral,C_Spiral,D_Spiral);
Spiral_TF = tf(Spiral_SS);

P_DA_S= Spiral_TF(1, 1);
P_DR_S = Spiral_TF(1, 2);

R_DA_S = Spiral_TF(2, 1);
R_DR_S = Spiral_TF(2, 2);

PHI_DA_S = Spiral_TF(3, 1);
PHI_DR_S = Spiral_TF(3, 2);
```

2DOF Dutch Mode Approximation

```
A_Dutch = [Yb/Vto (Yr-U0)/Vto-tan(TH0)*(Yp+W0)/Vto;Nbd Nrd-tan(TH0)*Npd];

B_Dutch = [YDa_star YDr_star;NDad NDrd];

C_Dutch = eye(2); D_Dutch = zeros(2,2);



Dutch_SS = ss(A_Dutch, B_Dutch, C_Dutch, D_Dutch);

Dutch_TF = tf(Dutch_SS);



R_DA_D = Dutch_TF(1, 1);

R_DR_D = Dutch_TF(1, 2);
```

```
B_DA_D = Dutch_TF(2, 1);

B_DR_D = Dutch_TF(2, 2);
```

1DOF Roll Approximation

```
A_Roll = Lpd;

B_Roll = LDad;

C_Roll = eye(1); D_Roll = zeros(1, 1);



Roll_SS = ss(A_Roll, B_Roll, C_Roll, D_Roll);

Roll_TF = tf(Roll_SS);



P_DA_R = Roll_TF(1 ,1);
```

root locus and bode plots

Longtuidnal Full linear modle

```
%u/de
figure()
subplot(1, 2, 1);
rlocus(LONGT_TF(1,1))
title('Longitudinal LINEAR FULL MODLE root locus (u/\delta_{e}) ')
subplot(1, 2, 2);
bode(LONGT_TF(1,1))
title('Longitudinal LINEAR FULL MODLE bode plot (u/\delta_{e}) ')
grid on

%u/d_th
figure()
subplot(1, 2, 1);
rlocus(LONGT_TF(1,2))
title('Longitudinal LINEAR FULL MODLE root locus (u/\delta_{th}) ')
subplot(1, 2, 2);
bode(LONGT_TF(1,2))
title('Longitudinal LINEAR FULL MODLE bode plot (u/\delta_{th}) ')
grid on

%w/de
figure()
subplot(1, 2, 1);
rlocus(LONGT_TF(2,1))
title('Longitudinal LINEAR FULL MODLE root locus (w/\delta_{e}) ')
subplot(1, 2, 2);
bode(LONGT_TF(2,1))
title('Longitudinal LINEAR FULL MODLE bode plot (w/\delta_{e}) ')
grid on

%w/d_th
figure()
subplot(1, 2, 1);
```

```
rlocus(LONGT_TF(2,2))
title('Longitudinal LINEAR FULL MODLE root locus (w/\delta_{th}) ')
subplot(1, 2, 2);
bode(LONGT_TF(2,2))
title('Longitudinal LINEAR FULL MODLE bode plot (w/\delta_{th}) ')
grid on

%q/de
figure()
subplot(1, 2, 1);
rlocus(LONGT_TF(3,1))
title('Longitudinal LINEAR FULL MODLE root locus (q/\delta_{e}) ')
subplot(1, 2, 2);
bode(LONGT_TF(3,1))
title('Longitudinal LINEAR FULL MODLE bode plot (q/\delta_{e}) ')
grid on

%q/d_th
figure()
subplot(1, 2, 1);
rlocus(LONGT_TF(3,2))
title('Longitudinal LINEAR FULL MODLE root locus (q/\delta_{th}) ')
subplot(1, 2, 2);
bode(LONGT_TF(3,2))
title('Longitudinal LINEAR FULL MODLE bode plot (q/\delta_{th}) ')
grid on

%theta/de
figure()
subplot(1, 2, 1);
rlocus(LONGT_TF(4,1))
title('LINEAR FULL MODLE root locus (\theta/\delta_{e}) ')
subplot(1, 2, 2);
bode(LONGT_TF(4,1))
title('LINEAR FULL MODLE bode plot (\theta/\delta_{e}) ')
grid on

%theta/d_th
figure()
subplot(1, 2, 1);
rlocus(LONGT_TF(4,2))
title('Longitudinal LINEAR FULL MODLE root locus (\theta/\delta_{th}) ')
subplot(1, 2, 2);
bode(LONGT_TF(4,2))
title('Longitudinal LINEAR FULL MODLE bode plot (\theta/\delta_{th}) ')
grid on
```

long period mode (approximate)

```
%u/de
figure()
subplot(1, 2, 1);
rlocus(PHUG_TF(1,1))
title('LONG PERIOD MODE root locus (u/\delta_{e}) ')
subplot(1, 2, 2);
bode(PHUG_TF(1,1))
title('LONG PERIOD MODE bode plot (u/\delta_{e}) ')
grid on

%u/d_th
figure()
subplot(1, 2, 1);
rlocus(PHUG_TF(1,2))
title('LONG PERIOD MODE root locus (u/\delta_{th}) ')
subplot(1, 2, 2);
bode(PHUG_TF(1,2))
title('LONG PERIOD MODE bode plot (u/\delta_{th}) ')
```

```
grid on

%theta/de
figure()
subplot(1, 2, 1);
rlocus(PHUG_TF(2,1))
title('LONG PERIOD MODE root locus (\theta/\delta_{e}) ')
subplot(1, 2, 2);
bode(PHUG_TF(2,1))
title('LONG PERIOD MODE bode plot (\theta/\delta_{e}) ')
grid on

%theta/d_th
figure()
subplot(1, 2, 1);
rlocus(PHUG_TF(2,2))
title('LONG PERIOD MODE root locus (\theta/\delta_{th}) ')
subplot(1, 2, 2);
bode(PHUG_TF(2,2))
title('LONG PERIOD MODE bode plot (\theta/\delta_{th}) ')
grid on
```

short period mode (approximate)

```
%w/de
figure()
subplot(1, 2, 1);
rlocus(SHORT_TF(1,1))
title('SHORT PERIOD MODE root locus (w/\delta_{e}) ')
subplot(1, 2, 2);
bode(SHORT_TF(1,1))
title('SHORT PERIOD MODE bode plot (w/\delta_{e}) ')
grid on

%w/d_th
figure()
subplot(1, 2, 1);
rlocus(SHORT_TF(1,2))
title('SHORT PERIOD MODE root locus (w/\delta_{th}) ')
subplot(1, 2, 2);
bode(SHORT_TF(1,2))
title('SHORT PERIOD MODE bode plot (w/\delta_{th}) ')
grid on

%q/de
figure()
subplot(1, 2, 1);
rlocus(SHORT_TF(2,1))
title('SHORT PERIOD MODE root locus (q/\delta_{e}) ')
subplot(1, 2, 2);
bode(SHORT_TF(2,1))
title('SHORT PERIOD MODE bode plot (q/\delta_{e}) ')
grid on

%q/d_th
figure()
subplot(1, 2, 1);
rlocus(SHORT_TF(2,2))
title('SHORT PERIOD MODE root locus (q /\delta_{th}) ')
subplot(1, 2, 2);
bode(SHORT_TF(2,2))
title('SHORT PERIOD MODE bode plot (q /\delta_{th}) ')
grid on
```

Lateral Full Linear Mode

beta/da

```
figure()

subplot(1, 2, 1);

rlocus(B_DA_L)

title('Lateral Full Linear root locus (\beta/\delta_{a}) ')
subplot(1, 2, 2);
bode(B_DA_L)
title('Lateral Full Linear bode plot (\beta/\delta_{a}) ')
grid on

% beta/dr
figure()
subplot(1, 2, 1);
rlocus(B_DR_L)
title('Lateral Full Linear root locus (\beta/\delta_{r}) ')
subplot(1, 2, 2);
bode(B_DR_L)
title('Lateral Full Linear bode plot (\beta/\delta_{r}) ')
grid on

% p/da
figure()
subplot(1, 2, 1);
rlocus(P_DA_L)
title('Lateral Full Linear root locus (p/\delta_{a}) ')
subplot(1, 2, 2);
bode(P_DA_L)
title('Lateral Full Linear bode plot (p/\delta_{a}) ')
grid on

% p/dr
figure()
subplot(1, 2, 1);
rlocus(P_DR_L)
title('Lateral Full Linear root locus (p/\delta_{r}) ')
subplot(1, 2, 2);
bode(P_DR_L)
title('Lateral Full Linear bode plot (p/\delta_{r}) ')
grid on


% r/da
figure()
subplot(1, 2, 1);
rlocus(R_DA_L)
title('Lateral Full Linear root locus (r/\delta_{a}) ')
subplot(1, 2, 2);
bode(R_DA_L)
title('Lateral Full Linear bode plot (r/\delta_{a}) ')
grid on

% r/dr
figure()
subplot(1, 2, 1);
rlocus(P_DR_L)
title('Lateral Full Linear root locus (r/\delta_{r}) ')
subplot(1, 2, 2);
bode(P_DR_L)
title('Lateral Full Linear bode plot (r/\delta_{r}) ')
grid on

% phi/da
figure()
subplot(1, 2, 1);
```

```
rlocus(R_DA_L)
title('Lateral Full Linear root locus (\phi /\delta_{a}) ')
subplot(1, 2, 2);
bode(R_DA_L)
title('Lateral Full Linear bode plot (\phi /\delta_{a}) ')
grid on

% phi/dr
figure()
subplot(1, 2, 1);
rlocus(PHI_DR_L)
title('Lateral Full Linear root locus (\phi /\delta_{r}) ')
subplot(1, 2, 2);
bode(PHI_DR_L)
title('Lateral Full Linear bode plot (\phi /\delta_{r}) ')
grid on

% psi/da
figure()
subplot(1, 2, 1);
rlocus(PSI_DA_L)
title('Lateral Full Linear root locus (\psi /\delta_{a}) ')
subplot(1, 2, 2);
bode(PSI_DA_L)
title('Lateral Full Linear bode plot (\psi /\delta_{a}) ')
grid on

% psi/dr
figure()
subplot(1, 2, 1);
rlocus(PSI_DR_L)
title('Lateral Full Linear root locus (\psi /\delta_{r}) ')
subplot(1, 2, 2);
bode(PSI_DR_L)
title('Lateral Full Linear bode plot (\psi /\delta_{r}) ')
grid on
```

3DOF Spiral Mode Approximation

```
% p/da
figure()
subplot(1, 2, 1);
rlocus(P_DA_S)
title('3DOF Spiral Mode root locus (p /\delta_{a}) ')
subplot(1, 2, 2);
bode(P_DA_S)
title('3DOF Spiral Mode bode plot (p /\delta_{a}) ')
grid on

% p/dr
figure()
subplot(1, 2, 1);
rlocus(P_DR_S)
title('3DOF Spiral Mode root locus (p /\delta_{r}) ')
subplot(1, 2, 2);
bode(P_DR_S)
title('3DOF Spiral Mode bode plot (p /\delta_{r}) ')
grid on

% r/da
figure()
subplot(1, 2, 1);
rlocus(R_DA_S)
title('3DOF Spiral Mode root locus (r /\delta_{a}) ')
subplot(1, 2, 2);
bode(R_DA_S)
title('3DOF Spiral Mode bode plot (r /\delta_{a}) ')
```

```
grid on

% r/dr
figure()
subplot(1, 2, 1);
rlocus(R_DR_S)
title('3DOF Spiral Mode root locus (r /\delta_{r}) ')
subplot(1, 2, 2);
bode(R_DR_S)
title('3DOF Spiral Mode bode plot (r /\delta_{r}) ')
grid on


% phi/da
figure()
subplot(1, 2, 1);
rlocus(PHI_DA_S)
title('3DOF Spiral Mode root locus (\phi /\delta_{a}) ')
subplot(1, 2, 2);
bode(PHI_DA_S)
title('3DOF Spiral Mode bode plot (\phi /\delta_{a}) ')
grid on

% phi/dr
figure()
subplot(1, 2, 1);
rlocus(PHI_DR_S)
title('3DOF Spiral Mode root locus (\phi /\delta_{r}) ')
subplot(1, 2, 2);
bode(PHI_DR_S)
title('3DOF Spiral Mode bode plot (\phi /\delta_{r}) ')
grid on
```

2DOF Dutch Mode Approximation

```
% r/da
figure()
subplot(1, 2, 1);
rlocus(R_DA_D)
title('2DOF Dutch Mode root locus (r /\delta_{a}) ')
subplot(1, 2, 2);
bode(R_DA_D)
title('2DOF Dutch Mode bode plot (r /\delta_{a}) ')
grid on

% r/dr
figure()
subplot(1, 2, 1);
rlocus(R_DR_D)
title('2DOF Dutch Mode root locus (r /\delta_{r}) ')
subplot(1, 2, 2);
bode(R_DR_D)
title('2DOF Dutch Mode bode plot (r /\delta_{r}) ')
grid on

% beta/da
figure()
subplot(1, 2, 1);
rlocus(B_DA_D)
title('2DOF Dutch Mode root locus (\beta /\delta_{a}) ')
subplot(1, 2, 2);
bode(B_DA_D)
title('2DOF Dutch Mode bode plot (\beta /\delta_{a}) ')
grid on

% beta/dr
figure()
```

```
subplot(1, 2, 1);
rlocus(B_DR_D)
title('2DOF Dutch Mode root locus (\beta /\delta_{r}) ')
subplot(1, 2, 2);
bode(B_DR_D)
title('2DOF Dutch Mode bode plot (\beta /\delta_{r}) ')
grid on
```

1DOF Roll Mode Approximation

```
% p/da
figure()
subplot(1, 2, 1);
rlocus(P_DA_R)
title('1DOF Roll Mode root locus (p /\delta_{a}) ')
subplot(1, 2, 2);
bode(P_DA_R)
title('1DOF Roll Mode bode plot (p /\delta_{a}) ')
grid on
export_figure(max(double(get(groot,'Children')))+[-36:0], '', {'tf1',
'tf2','tf3','tf4','tf5','tf6','tf7','tf8','tf9','tf10',
'tf11','tf12','tf13','tf14','tf15','tf16','tf17','tf18','tf19','tf20'
,'tf21','tf22','tf23','tf24','tf25','tf26','tf27','tf28','tf29',
'tf30','tf31','tf32','tf33','tf34','tf35','tf36','tf37'}, 600, {'emf',
'emf','emf','emf','emf','emf','emf','emf','emf','emf','emf',
'emf','emf','emf','emf','emf','emf','emf','emf','emf','emf',
'emf','emf','emf','emf','emf','emf','emf','emf','emf','emf',
'emf','emf','emf','emf','emf','emf'})
```