Task 1

Autopilot literature review

TEAM MEMBERS

name	sec	bn
Mohammed Ahmed Hassan Ahmed	2	37
Ibrahim Thabet Allam	1	1
Mohammed Hatem Mohammed Saeed	2	39
Mohamed Hassan Gad Ali	2	41
Mohammed Abd El Mawgoud Ghoneam	2	43

Research questions

Autopilot

An autopilot is a system that allows an airplane to navigate without the need for constant manual control by a human operator

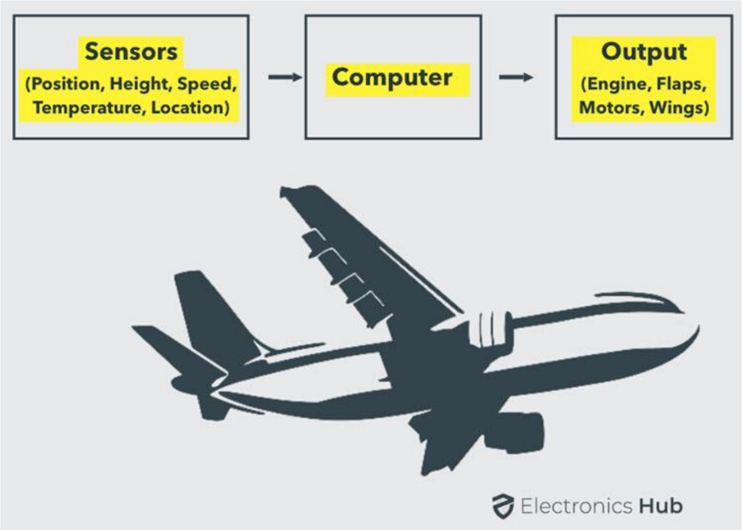
Inputs and outputs of Autopilot system

- inputs
  - Control Inputs:
    1. Radio Control Receivers
    2. MAVLink Data Streams, ie ground control stations or companion computers
  - Sensor inputs:

    GPS, Compass, Airspeed, Rangefinders, IMU
  - Power Management Unit Inputs:
    1. Received Signal Strength Input (RSSI)
    2. Analog Airspeed Sensors

Inputs and outputs of Autopilot system

- Outputs
  1. ESCs (electronic circuit that controls and regulates the speed of an electric motor) for motors
  2. Servos for control surfaces
  3. Telemetry data
  4. Actuators and General Purpose I/O like LEDs, buzzers etc



the role of the pilot in an airplane equipped with an autopilot

The pilot sets the flight plan and turn on the autopilot sometimes the pilot reprograms the autopilot in case if worked incorrectly. autopilot is not smart enough to fly a plane by It self

What is the difference between Autopilot & SAS?

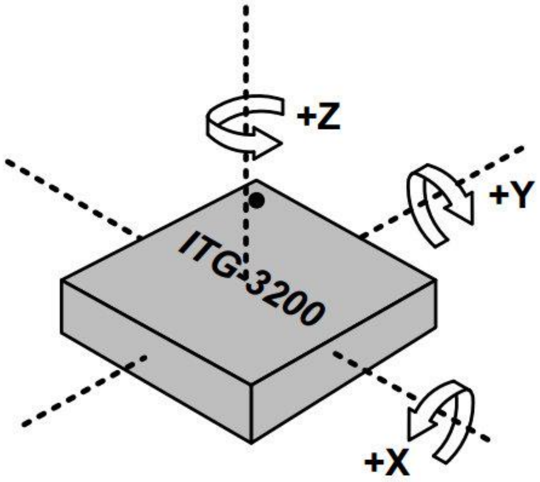
SAS (Stability augmentation system) generally used during low and slow maneuvering where the pilot may be making constant attitude changes in preparation for landing.

Autopilot do same functions as SAS in addition it provides more functions ,Autopilot Is more sophisticated than SAS

Role of onboard sensors

- Sensors provide the autopilot computers with data like speed, coordinates, position so the computers can estimate the states and give the correct control actions to the actuators for example

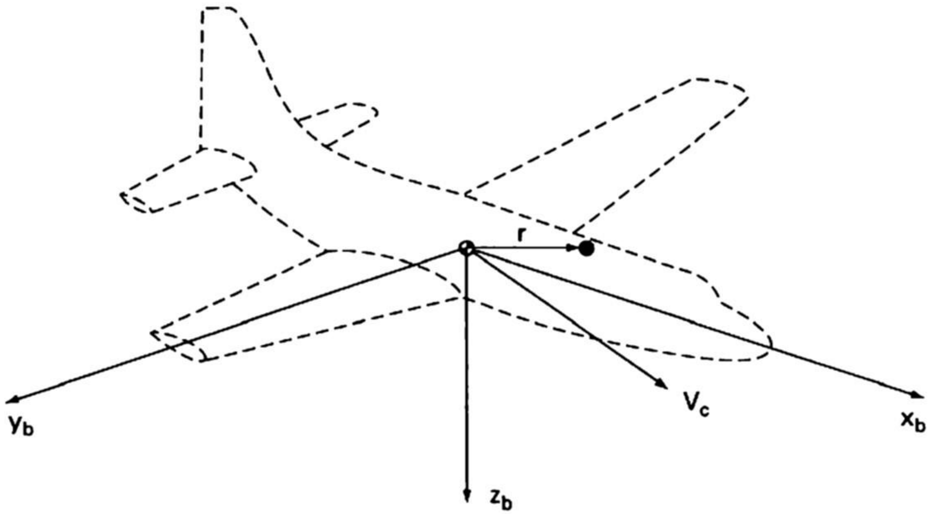
Gyroscope an IMU supplies the autopilot with position data so if the airplane in incorrect position it gives the actuator signal so that it can adjust position



Flight Mechanics review

Assume

- The body is rigid which means the mass is constant and there is no change in geometry (Stress calculations are neglected).
- Fix a body axes at the center of mass of the rigid body.



Flight Mechanics review

- *RBD*
  - calculations of linear momentum:

$$\vec{V} = \vec{V_c} + \frac{d\vec{\boldsymbol{r}}}{dt}$$

$$\delta\vec{F} = \delta m \frac{d\vec{V}}{dt}$$

$$\frac{d\vec{A}}{dt}_I = \frac{d\vec{A}}{dt}_{\boldsymbol{B}} + \omega \times \vec{A}$$

$$\vec{F} = F_x\vec{\imath} + F_y\vec{\jmath} + F_z\vec{k}$$

$$\vec{\omega} = P\vec{\imath} + q\vec{\jmath} + r\vec{k}$$

$$\vec{V_c} = u\boldsymbol{i} + v\boldsymbol{j} + w\boldsymbol{k}$$

Get

$$F_x = m(\dot{u} + qw - rv)$$

$$F_y = m(\dot{v} + ru - pw)$$

$$F_z = m(\dot{w} + pv - qu)$$

Flight Mechanics review

- *RBD*
  - calculations of angular momentum:

$$\vec{V} = \vec{V_c} + \frac{d\vec{\boldsymbol{r}}}{dt}$$

$$\delta\vec{M} = \frac{d}{dt}\delta\vec{H} = \frac{d}{dt}(\vec{r} \times \vec{V})\delta m$$

$$\frac{d\vec{A}}{dt}_I = \frac{d\vec{A}}{dt}_{\boldsymbol{B}} + \omega \times \vec{A}$$

$$\vec{r} = x\boldsymbol{i} + y\boldsymbol{j} + z\boldsymbol{k}$$

$$H_x = pI_x - qI_{xy} - rI_{xz}$$

$$H_y = -pI_{xy} + qI_y - rI_{yz}$$

$$H_z = -pI_{xz} - qI_{yz} + rI_z$$

Get

$$L = \dot{H}_x + qH_z - rH_y$$

$$M = \dot{H}_y + rH_x - pH_z$$

$$N = \dot{H}_z + pH_y - qH_x$$

Flight Mechanics review

- Now we are dealing with an aircraft which means that $I_{xy} = I_{yz} = 0$ $(\boldsymbol{symmetry\ cond.})$

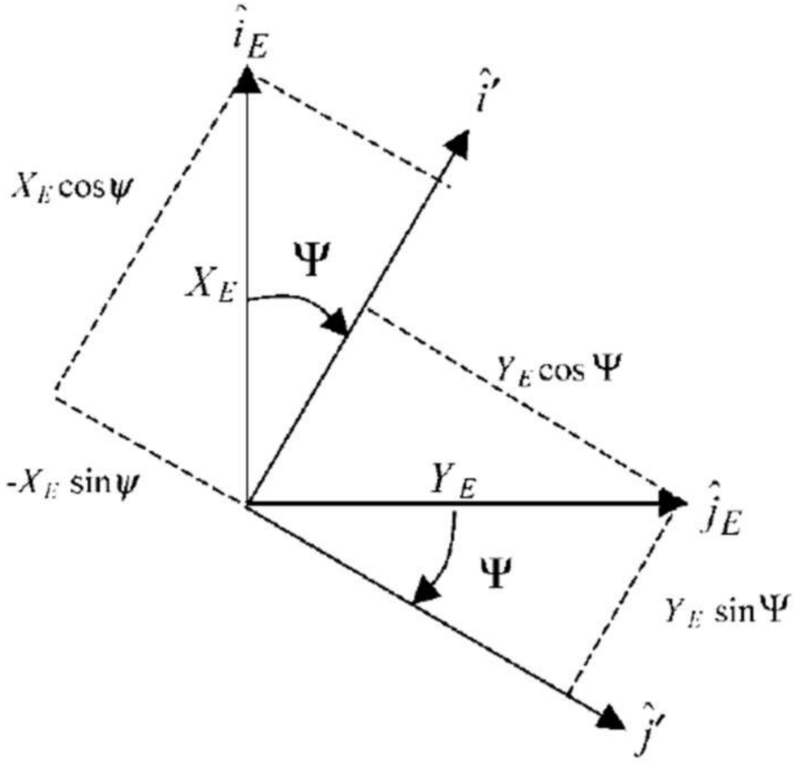
$$L = I_x\dot{p} - I_{xz}\dot{r} + qr\left(I_z - I_y\right) - I_{xz}pq$$
$$M = y\dot{q} + rp(I_x - I_z) + I_{xz}(p^2 - r^2)$$
$$N = -I_{xz}\dot{p} + I_z\dot{r} + pq\left(I_y - I_x\right) + I_{xz}qr$$

Flight Mechanics review

- earth axis to body axis transformation:

$$\begin{bmatrix} X_1 \\ Y_1 \\ Z_1 \end{bmatrix} = \begin{bmatrix} cos\psi & sin\psi & 0 \\ -sin\psi & cos\psi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} X_E \\ Y_E \\ Z_E \end{bmatrix}$$

$$\boldsymbol{F_1} = R_3(\psi)\boldsymbol{F_E}$$

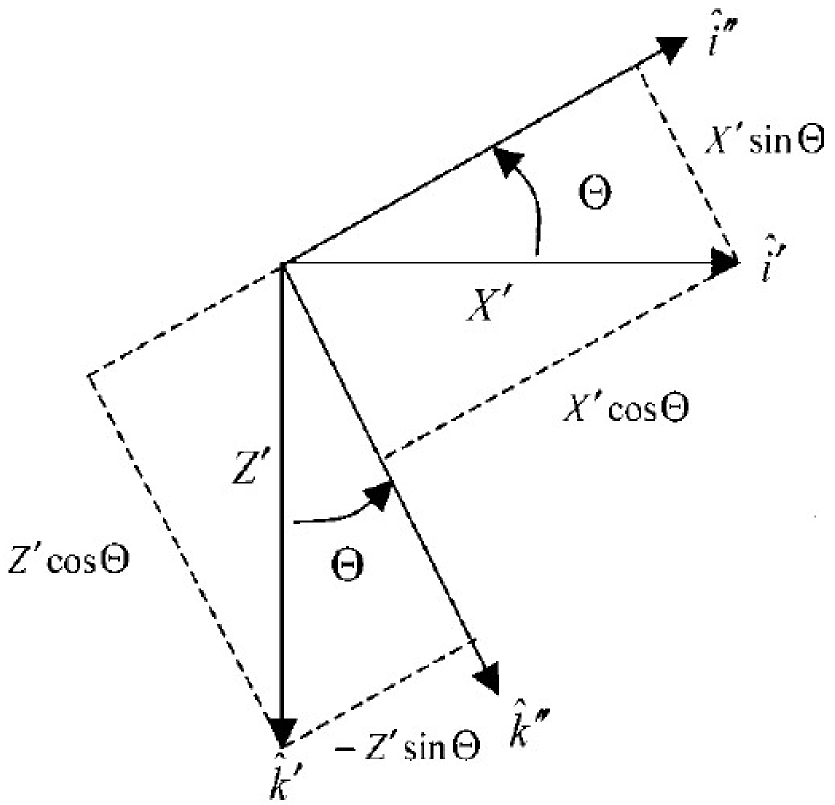


Flight Mechanics review

- earth axis to body axis transformation:

$$\begin{bmatrix} X_2 \\ Y_2 \\ Z_2 \end{bmatrix} = \begin{bmatrix} cos\theta & 0 & -sin\theta \\ 0 & 1 & 0 \\ sin\theta & 0 & cos\theta \end{bmatrix} \begin{bmatrix} X_1 \\ Y_1 \\ Z_1 \end{bmatrix}$$

$$\boldsymbol{F_2} = R_2(\theta)\boldsymbol{F_1} = R_2(\theta)R_3(\psi)\boldsymbol{F_E}$$

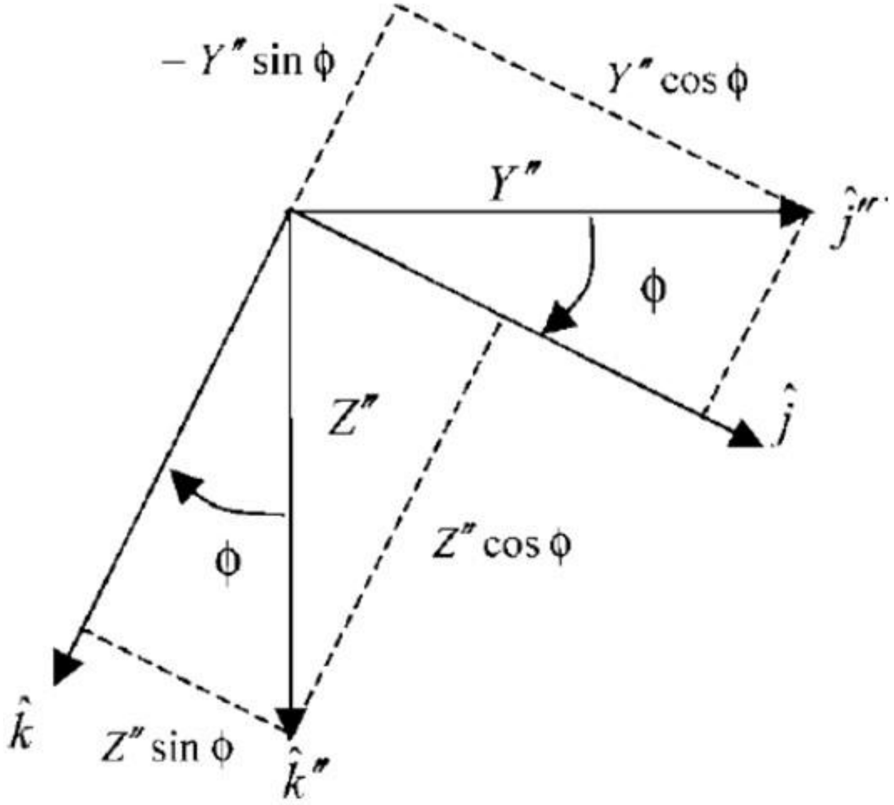


Flight Mechanics review

- earth axis to body axis transformation:

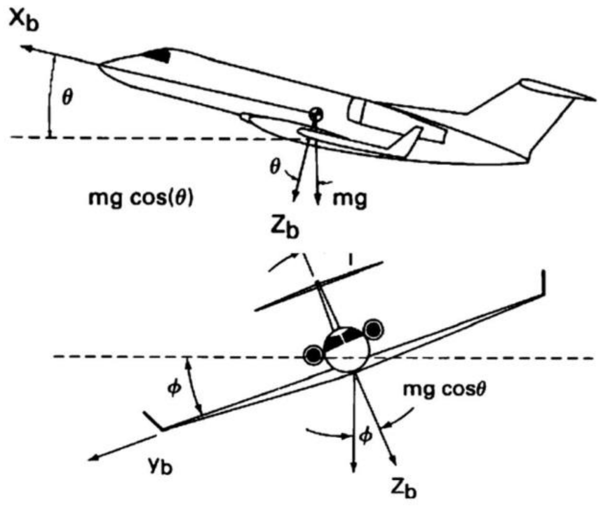
$$\begin{bmatrix} X_B \\ Y_B \\ Z_B \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & cos\phi & sin\phi \\ 0 & -sin\phi & cos\phi \end{bmatrix} \begin{bmatrix} X_2 \\ Y_2 \\ Z_2 \end{bmatrix}$$

$$\boldsymbol{F_B} = R_1(\phi)\boldsymbol{F_2} = R_1(\phi)R_2(\theta)R_3(\psi)\boldsymbol{F_E}$$



Flight Mechanics review

- Let's apply the above transformation on the gravitational force of the airplane as following:



$$\vec{F}_{Gravity_B} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & cos\phi & sin\phi \\ 0 & -sin\phi & cos\phi \end{bmatrix} \begin{bmatrix} cos\theta & 0 & -sin\theta \\ 0 & 1 & 0 \\ sin\theta & 0 & cos\theta \end{bmatrix} \begin{bmatrix} cos\psi & sin\psi & 0 \\ -sin\psi & cos\psi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ mg \end{bmatrix}$$

$$\vec{F}_{Gravity_B} = \begin{bmatrix} -mgsin\theta \\ mgsin\phi cos\theta \\ mgsin\theta cos\phi \end{bmatrix}_B$$

Flight Mechanics review

- Then:

$$-mgsin\theta + F_{T_X} = m(\dot{u} + qw - rv)$$

$$mgsin\phi cos\theta + F_{T_Y} = m(\dot{v} + ru - pw)$$

$$mgsin\theta cos\phi + F_{T_Z} = m(\dot{w} + pv - qu)$$

Flight Mechanics review

- *Also we can get*

$$\begin{Bmatrix} P \\ Q \\ R \end{Bmatrix} = \begin{bmatrix} 1 & 0 & -sin\theta \\ 0 & cos\phi & cos\theta sin\phi \\ 0 & -sin\phi & cos\phi cos\theta \end{bmatrix} \begin{Bmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{Bmatrix}$$

$$\begin{Bmatrix} \dot{X} \\ \dot{Y} \\ \dot{Z} \end{Bmatrix}_E = \begin{bmatrix} C_\theta C_\psi & S_\phi S_\theta C_\psi - C_\phi S_\psi & C_\phi S_\theta C_\psi + S_\phi S_\psi \\ C_\theta S_\psi & S_\phi S_\theta S_\psi + C_\phi C_\psi & C_\phi S_\theta S_\psi - S_\phi C_\psi \\ -S_\theta & S_\theta C_\theta & C_\phi C_\theta \end{bmatrix} \begin{Bmatrix} u \\ v \\ w \end{Bmatrix}_B$$

Flight Mechanics review

- Classification of equations
  - Kinetics equations

$$-mgsin\theta + F_{T_X} = m(\dot{u} + qw - rv)$$

$$mgsin\phi cos\theta + F_{T_Y} = m(\dot{v} + ru - pw)$$

$$mgsin\theta cos\phi + F_{T_Z} = m(\dot{w} + pv - qu)$$

$$L = I_x\dot{p} - I_{xz}\dot{r} + qr\left(I_z - I_y\right) - I_{xz}pq$$
$$M = y\dot{q} + rp(I_x - I_z) + I_{xz}(p^2 - r^2)$$
$$N = -I_{xz}\dot{p} + I_z\dot{r} + pq\left(I_y - I_x\right) + I_{xz}qr$$

Flight Mechanics review

- Classification of equations
  - Kinematics equations:

$$\begin{Bmatrix} P \\ Q \\ R \end{Bmatrix} = \begin{bmatrix} 1 & 0 & -sin\theta \\ 0 & cos\phi & cos\theta sin\phi \\ 0 & -sin\phi & cos\phi cos\theta \end{bmatrix} \begin{Bmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{Bmatrix}$$

$$\begin{Bmatrix} \dot{X} \\ \dot{Y} \\ \dot{Z} \end{Bmatrix}_E = \begin{bmatrix} C_\theta C_\psi & S_\phi S_\theta C_\psi - C_\phi S_\psi & C_\phi S_\theta C_\psi + S_\phi S_\psi \\ C_\theta S_\psi & S_\phi S_\theta S_\psi + C_\phi C_\psi & C_\phi S_\theta S_\psi - S_\phi C_\psi \\ -S_\theta & S_\theta C_\theta & C_\phi C_\theta \end{bmatrix} \begin{Bmatrix} u \\ v \\ w \end{Bmatrix}_B$$

Flight Mechanics review

- **The additional EOM of airplanes:**

→ The effect of the control surfaces: δ_e , δ_a , δ_r , δ_T

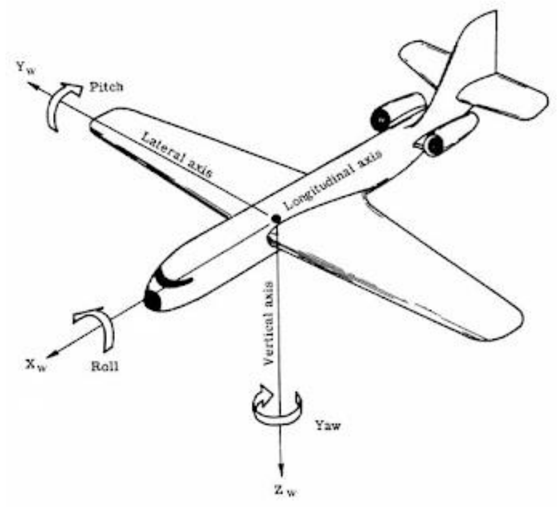
i- The change in forces:

- $\Delta X = m\left(\frac{\partial X}{\partial u}\,\Delta u + \frac{\partial X}{\partial w}\,\Delta w + \frac{\partial X}{\partial \delta_e}\,\Delta\delta_e + \frac{\partial X}{\partial \delta_T}\,\Delta\delta_T\right)$
- $\Delta Y = m\left(\frac{\partial Y}{\partial v}\,\Delta v + \frac{\partial Y}{\partial \beta}\,\Delta\beta + \frac{\partial Y}{\partial \delta_a}\,\Delta\delta_a + \frac{\partial Y}{\partial \delta_r}\,\Delta\delta_r\right)$
- $\Delta Z = m\left(\frac{\partial Z}{\partial u}\,\Delta u + \frac{\partial Z}{\partial w}\,\Delta w + \frac{\partial Z}{\partial \dot{w}}\,\Delta\dot{w} + \frac{\partial Z}{\partial q}\,\Delta q + \frac{\partial Z}{\partial \delta_e}\,\Delta\delta_e + \frac{\partial Z}{\partial \delta_T}\,\Delta\delta_T\right)$

Flight Mechanics review

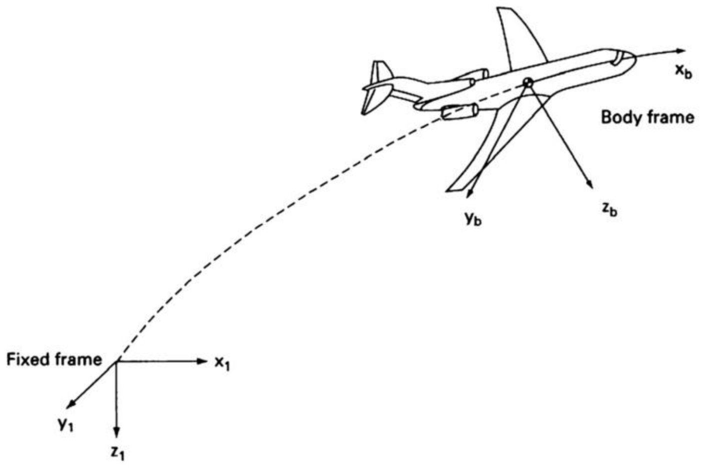
ii- The change in moments:

- $\Delta L = I_{xx}\left(\frac{\partial L}{\partial \beta}\,\Delta\beta + \frac{\partial L}{\partial p}\,\Delta p + \frac{\partial L}{\partial r}\,\Delta r + \frac{\partial L}{\partial \delta_r}\,\Delta\delta_r + \frac{\partial L}{\partial \delta_a}\,\Delta\delta_a\right)$
- $\Delta M = I_{yy}\left(\frac{\partial M}{\partial u}\,\Delta u + \frac{\partial M}{\partial w}\,\Delta w + \frac{\partial M}{\partial \dot{w}}\,\Delta\dot{w} + \frac{\partial M}{\partial q}\,\Delta q + \frac{\partial M}{\partial \delta_e}\,\Delta\delta_e + \frac{\partial M}{\partial \delta_T}\,\Delta\delta_T\right)$
- $\Delta N = I_{zz}\left(\frac{\partial N}{\partial \beta}\,\Delta\beta + \frac{\partial N}{\partial p}\,\Delta p + \frac{\partial N}{\partial r}\,\Delta r + \frac{\partial N}{\partial \delta_r}\,\Delta\delta_r + \frac{\partial N}{\partial \delta_a}\,\Delta\delta_a\right)$



Flight Mechanics review

- The EOM's are coupled, nonlinear and first order DE.
- Difference bet. Body & inertial Axes
  - Body axes: they are a set of axes which are fixed at the body in its translational and rotations.
  - Inertial axes: they are a set of axes which are fixed at a specified position on the ground.



Flight Mechanics review

- **Pitch angle**: it’s the angle between the body y-axis and the horizon.
- **AOA**: the angle at which the chord of an aircraft's wing meets the relative wind.
- **Sideslip angle**: it’s formed due to lateral deviation between the plane and the direction of the flow.
- **Yaw angle (heading)**: it’s formed due to the rotation about z-axis.

numerical solving algorithms for ODE

One-step methods, like:

- Euler-Cauchy method
- Improved Euler method
- Raunge-Kutta method
- Backward Euler method

Multistep methods, like:

- Adams-Bash forth method
- Adams-Moulton method

Other methods

- Predector –coeector methods
- Exponential integrator methods

The chosen method for solving the aircraft EOM

We choose to use Raunge-Kutta method since its one of the most accurate one step methods used in the numerical solutions, in addition it's the method used in the MATLAB ODE45 algorithm.

4th order Runge-Kutta

$$k_1 = \Delta t f(t_n, x_n)$$

$$k_2 = \Delta t f\left(t_n + \frac{1}{2}\Delta t, x_n + \frac{1}{2}k_1\right)$$

$$k_3 = \Delta t f\left(t_n + \frac{1}{2}\Delta t, x_n + \frac{1}{2}k_2\right)$$

$$k_4 = \Delta t f(t_n + \Delta t, x_n + k_3)$$

$$x_{n+1} = x_n + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$$