Cairo University Faculty of
Engineering Aerospace
Department

AEROSPACE ENGINEERING

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Task 4

Airframe model

Autopilot -AER 408
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Team 4

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Mohammed Abd El-Mawgoud Ghoneam	2	43
Mohamed Hassan Gad Ali	2	41

Contents

a) Linearization of 12 EOM for a fixed wing A/C

EOM linearization:

EOM

EOM	Airframe derivatives
$\boldsymbol{X - mg\, S_\theta = m(\dot{u} + qw - rv)}$	$\Delta X/m = X_u\,\Delta u + X_w\,\Delta w + X_{\delta_e}\,\Delta\delta_e + X_{\delta_{th}}\,\Delta\delta_{th}$
$\boldsymbol{Y + mg\, C_\theta\, S_\varphi = m(\dot{v} + ru - pw)}$	$\Delta Y/m = Y_\beta\,\Delta\beta + Y_r\,\Delta r + Y_p\,\Delta p + Y_{\delta a}\,\Delta\delta_a + Y_{\delta_r}\,\Delta\delta_r$
$\boldsymbol{Z + mg\, C_\theta\, C_\varphi = m(\dot{w} + pv - qu)}$	$\Delta Z/m = Z_u\,\Delta u + Z_w\,\Delta w + Z_{\dot{w}}\,\Delta\dot{w} + Z_q\,\Delta q + Z_{\delta_e}\,\Delta\delta_e + Z_{\delta_{th}}\,\Delta\delta_{th}$

EOM	Airframe derivatives
$\boldsymbol{L = I_x\dot{p} - I_{xz}\dot{r} + qr(I_z - I_y) - I_{xz}pq}$	$\Delta L/I_{xx} = L_\beta\,\Delta\beta + L_p\,\Delta p + L_r\,\Delta r + L_{\delta_r}\,\Delta\delta_r + L_{\delta_a}\,\Delta\delta_a$
$\boldsymbol{M = I_y\dot{q} + rp(I_x - I_z) - I_{xz}(p^2 - r^2)}$	$\Delta M/I_{yy} = M_u\,\Delta u + M_w\,\Delta w + M_{\dot{w}}\,\Delta\dot{w} + M_q\,\Delta q + M_{\delta_e}\,\Delta\delta_e + M_{\delta_{th}}\,\Delta\delta_{th}$
$\boldsymbol{N = -I_{xz}\dot{p} + I_z\dot{r} + pq(I_y - I_x) + I_{xz}qr}$	$\Delta N/I_{zz} = N_\beta\,\Delta\beta + N_p\,\Delta p + N_r\,\Delta r + N_{\delta_r}\,\Delta\delta_r + N_{\delta_a}\,\Delta\delta_a$

$$\dot{\varphi} = p + q\,S_\varphi\,T_\theta + r\,C_\varphi\,T_\theta$$
$$\dot{\theta} = q\,C_\varphi - r\,S_\varphi$$
$$\dot{\psi} = \left(q\,S_\varphi + r\,C_\varphi\right)\,sec(\theta)$$

$$\begin{bmatrix}\dot{x}\\ \dot{y}\\ \dot{z}\end{bmatrix} = \begin{bmatrix} C_\theta C_\psi & S_\varphi S_\theta C_\psi - C_\varphi S_\psi & C_\psi S_\theta C_\psi + S_\varphi S_\psi \\ C_\theta S_\psi & S_\varphi S_\theta S_\psi + C_\varphi C_\psi & C_\varphi S_\theta S_\psi - S_\varphi C_\psi \\ -S_\theta & S_\varphi C_\theta & C_\varphi C_\theta \end{bmatrix}\begin{bmatrix}u\\ v\\ w\end{bmatrix}$$

1) Let:$(A = A_o + \Delta A)$

$\boldsymbol{u = u_o + \Delta u}$	$\boldsymbol{p = p_o + \Delta p}$	$\boldsymbol{X = X_o + \Delta X}$	$\boldsymbol{L = L_o + \Delta L}$	$\boldsymbol{\varphi = \varphi_o + \Delta\varphi}$
$\boldsymbol{v = v_o + \Delta v}$	$q = q_o + \Delta q$	$Y = Y_o + \Delta Y$	$M = M_o + \Delta M$	$\theta = \theta_o + \Delta\theta$
$\boldsymbol{w = w_o + \Delta w}$	$r = r_o + \Delta r$	$Y = Y_o + \Delta Y$	$N = N_o + \Delta N$	$\psi = \psi_o + \Delta\psi$

And $v_o = p_o = q_o = r_o = \varphi_o = \psi_o = 0$ but $u_o, w_o, \theta_o \neq 0$ for Cruise condition.

2) Longitudinal calculations:

a) $X - mg\,Sin(\theta) = m(\dot{u} + qw - rv)$

i. Recast each variable in terms of a steady-state value and a perturbed value.

$$X_o + \Delta X - mg\,Sin(\theta_o + \Delta\theta)$$
$$= m(\dot{u}_o + \Delta\dot{u} + (q_o + \Delta q)(w_o + \Delta w) - (r_o + \Delta r)(v_o + \Delta v))$$
$$\sin(a+b) = \sin(a)\cos(b) + \sin(b)\cos(a)$$
$$X_o + \Delta X - mg\,(\sin(\theta_o)\cos(\Delta\theta) + \sin(\Delta\theta)\cos(\theta_o))$$
$$= m\big(\Delta\dot{u} + (\Delta q)(w_o + \Delta w) - (\Delta r)(\Delta v)\big)$$

ii. Apply the small-angle assumption to trig functions of perturbed angles

$$\sin(a \to 0) = a\,, \quad \cos(a \to 0) = 1$$
$$X_o + \Delta X - mg\,\sin(\theta_o) - mg\,\cos(\theta_o)\,\Delta\theta$$
$$= m(\Delta\dot{u} + \Delta q * w_o + \Delta q * \Delta w - \Delta r * \Delta v)$$

iii. Assume products of small perturbations are negligible.

$$X_o + \Delta X - mg\,\sin(\theta_o) - mg\,\cos(\theta_o)\,\Delta\theta = m(\Delta\dot{u} + \Delta q * w_o)$$

iv. Remove the steady-state equation from the perturbed equation.

$$\therefore \Delta X - mg\,\cos(\theta_o)\,\Delta\theta = m(\Delta\dot{u} + \Delta q * w_o)$$
$$\therefore \Delta\dot{u} = \frac{\Delta X}{m} - g\,\cos(\theta_o)\,\Delta\theta - \Delta q * w_o \quad , \quad \frac{\Delta X}{m}$$
$$= X_u\,\Delta u + X_w\,\Delta w + X_{\delta_e}\,\Delta\delta_e + X_{\delta_{th}}\,\Delta\delta_{th}$$
$$\therefore \Delta\dot{u} = X_u\,\Delta u + X_w\,\Delta w + X_{\delta_e}\,\Delta\delta_e + X_{\delta_{th}}\,\Delta\delta_{th} - g\,\cos(\theta_o)\,\Delta\theta - \Delta q * w_o$$

b) $Z + mg\,Cos(\theta) * Cos(\varphi) = m(\dot{w} + pv - qu)$

Same as (a)

$$\cos(a+b) = \cos(a)\cos(b) - \sin(a)\sin(b)$$
$$\therefore (1 - Z_{\dot{w}})\Delta\dot{w}$$
$$= Z_u\,\Delta u + Z_w\,\Delta w + (Z_q + u_o)\,\Delta q + Z_{\delta_e}\,\Delta\delta_e + Z_{\delta_{th}}\,\Delta\delta_{th}$$
$$- g\,\sin(\theta_o)\,\Delta\theta$$
$$\therefore \Delta\dot{w} = \frac{Z_u}{(1 - Z_{\dot{w}})}\,\Delta u + \frac{Z_w}{(1 - Z_{\dot{w}})}\,\Delta w + \frac{(Z_q + u_o)}{(1 - Z_{\dot{w}})}\,\Delta q + \frac{Z_{\delta_e}}{(1 - Z_{\dot{w}})}\,\Delta\delta_e$$
$$+ \frac{Z_{\delta_{th}}}{(1 - Z_{\dot{w}})}\,\Delta\delta_{th} - \frac{g\,\sin(\theta_o)}{(1 - Z_{\dot{w}})}\,\Delta\theta$$

c) $M = I_y\dot{q} + rp(I_x - I_z) - I_{xz}(p^2 - r^2)$

$$\therefore \Delta\dot{q} = M_u\,\Delta u + M_w\,\Delta w + M_{\dot{w}} * \left(\frac{Z_u}{(1 - Z_{\dot{w}})}\,\Delta u + \frac{Z_w}{(1 - Z_{\dot{w}})}\,\Delta w + \frac{(Z_q + u_o)}{(1 - Z_{\dot{w}})}\,\Delta q + \frac{Z_{\delta_e}}{(1 - Z_{\dot{w}})}\,\Delta\delta_e + \frac{Z_{\delta_{th}}}{(1 - Z_{\dot{w}})}\,\Delta\delta_{th} - \frac{g\,\sin(\theta_o)}{(1 - Z_{\dot{w}})}\Delta\theta\right) + M_q\,\Delta q + M_{\delta_e}\,\Delta\delta_e + M_{\delta_{th}}\,\Delta\delta_{th}$$

$$\therefore \Delta\dot{q} = \left(M_u + M_{\dot{w}}\frac{Z_u}{(1 - Z_{\dot{w}})}\right)\Delta u + \left(M_w + M_{\dot{w}}\frac{Z_w}{(1 - Z_{\dot{w}})}\right)\Delta w + \left(M_q + M_{\dot{w}}\frac{(Z_q + u_o)}{(1 - Z_{\dot{w}})}\right)\Delta q + \left(M_{\delta_e} + M_{\dot{w}}\frac{Z_{\delta_e}}{(1 - Z_{\dot{w}})}\right)\Delta\delta_e + \left(M_{\delta_{th}} + M_{\dot{w}}\frac{Z_{\delta_{th}}}{(1 - Z_{\dot{w}})}\right)\Delta\delta_{th} - M_{\dot{w}} * \frac{g\,\sin(\theta_o)}{(1 - Z_{\dot{w}})}\Delta\theta$$

d) $\dot{\theta} = q\,C_\varphi - r\,S_\varphi$

$$\therefore \Delta\dot{\theta} = \Delta q$$

3) Lateral calculations:

a) $Y + mg\,C_\theta\,S_\varphi = m(\dot{v} + ru - pw)$

$$\Delta\dot{v} = Y_\beta\,\Delta\beta + Y_r\,\Delta r + Y_p\,\Delta p + Y_{\delta a}\,\Delta\delta_a + Y_{\delta_r}\,\Delta\delta_r + g\,\cos(\theta_o)\,\Delta\emptyset - \Delta r * u_o + w_o * \Delta p$$

$$\Delta\dot{\beta} = \frac{\Delta\dot{v}}{V_{to}}$$

$$\therefore \Delta\dot{\beta} = \frac{Y_\beta}{V_{to}}\,\Delta\beta + \frac{Y_r - u_o}{V_{to}}\,\Delta r + \frac{Y_p + w_o}{V_{to}}\,\Delta p + \frac{Y_{\delta a}}{V_{to}}\,\Delta\delta_a + \frac{Y_{\delta_r}}{V_{to}}\,\Delta\delta_r + \frac{g}{V_{to}}\,\cos(\theta_o)\,\Delta\varphi$$

b) $L = I_x\dot{p} - I_{xz}\dot{r} + qr(I_z - I_y) - I_{xz}pq$

$$eq(1):\quad \Delta\dot{p} - \frac{I_{xz}}{I_x} * \Delta\dot{r} = \frac{\Delta L}{I_x} = L_\beta\,\Delta\beta + L_p\,\Delta p + L_r\,\Delta r + L_{\delta_r}\,\Delta\delta_r + L_{\delta_a}\,\Delta\delta_a$$

c) $N = -I_{xz}\dot{p} + I_z\dot{r} + pq(I_y - I_x) + I_{xz}qr$

$$eq(2): \quad \Delta\dot{r} - \frac{I_{xz}}{I_z} * \Delta\dot{p} = \frac{\Delta N}{I_z}$$

$$= N_\beta\, \Delta\beta + N_p\, \Delta p + N_r\, \Delta r + N_{\delta_r}\, \Delta\delta_r + N_{\delta_a}\, \Delta\delta_a$$

$Solve\ eq(1)\ and\ eq(2)\ in\ (\Delta\dot{\boldsymbol{r}} \quad \Delta\dot{\boldsymbol{p}})\ we\ get:$

$$\therefore \Delta\dot{p} = \left(\frac{\Delta L}{I_x} + \frac{I_{xz}}{I_x} * \frac{\Delta N}{I_z}\right) * G \quad , \quad where \quad G = \frac{I_x I_z}{I_x I_z - (I_{xz})^2}$$

$$\therefore \Delta\dot{r} = \left(\frac{I_{xz}}{I_z} * \frac{\Delta L}{I_x} + \frac{\Delta N}{I_z}\right) * G \quad , \quad where \quad G = \frac{I_x I_z}{I_x I_z - (I_{xz})^2}$$

$$\frac{\Delta L}{I_x} = \sum L_i\, \Delta i \quad , \quad \frac{\Delta N}{I_z} = \sum N_i\, \Delta i$$

$$\therefore \Delta\dot{p} = \sum G * \left(L_i + \frac{I_{xz}}{I_x} * N_i\right) \Delta i = \sum L_i^*\, \Delta i$$

$$= L_\beta^*\, \Delta\beta + L_p^*\, \Delta p + L_r^*\, \Delta r + L_{\delta_r}^*\, \Delta\delta_r + L_{\delta_a}^*\, \Delta\delta_a$$

$$\therefore \Delta\dot{r} = \sum G * \left(N_i + \frac{I_{xz}}{I_z} * L_i\right) \Delta i = \sum N_i^*\, \Delta i$$

$$= N_\beta^*\, \Delta\beta + N_p^*\, \Delta p + N_r^*\, \Delta r + N_{\delta_r}^*\, \Delta\delta_r + N_{\delta_a}^*\, \Delta\delta_a$$

d) $\dot{\varphi} = p + q\, Sin(\varphi)\, Tan(\theta) + r\, Cos(\varphi)\, Tan(\theta)$

$$\dot{\varphi}\cos(\theta) = p\cos(\theta) + q\, Sin(\varphi)\, sin(\theta) + r\, Cos(\varphi)\, sin(\theta)$$

$$\therefore \Delta\dot{\varphi} = \Delta p + \Delta r * \tan(\theta_0)$$

e) $\dot{\psi} = (q\, S_\varphi + r\, C_\varphi)\ sec(\theta)$

$$\therefore \Delta\dot{\psi} = \Delta r * \sec(\theta_0)$$

4) State space Model:

i. Longitudinal

- $\Delta\dot{u} = X_u\,\Delta u + X_w\,\Delta w + X_{\delta_e}\,\Delta\delta_e + X_{\delta_{th}}\,\Delta\delta_{th} - g\,\cos(\theta_o)\,\Delta\theta - \Delta q * w_o$
- $\therefore \Delta\dot{w} = \frac{Z_u}{(1-Z_{\dot{w}})}\,\Delta u + \frac{Z_w}{(1-Z_{\dot{w}})}\,\Delta w + \frac{(Z_q+u_o)}{(1-Z_{\dot{w}})}\,\Delta q + \frac{Z_{\delta_e}}{(1-Z_{\dot{w}})}\,\Delta\delta_e + \frac{Z_{\delta_{th}}}{(1-Z_{\dot{w}})}\,\Delta\delta_{th} - \frac{g\,\sin(\theta_o)}{(1-Z_{\dot{w}})}\Delta\theta$
- $\therefore \Delta\dot{q} = \left(M_u + M_{\dot{w}}\frac{Z_u}{(1-Z_{\dot{w}})}\right)\Delta u + \left(M_w + M_{\dot{w}}\frac{Z_w}{(1-Z_{\dot{w}})}\right)\Delta w + \left(M_q + M_{\dot{w}}\frac{(Z_q+u_o)}{(1-Z_{\dot{w}})}\right)\Delta q + \left(M_{\delta_e} + M_{\dot{w}}\frac{Z_{\delta_e}}{(1-Z_{\dot{w}})}\right)\Delta\delta_e + \left(M_{\delta_{th}} + M_{\dot{w}}\frac{Z_{\delta_{th}}}{(1-Z_{\dot{w}})}\right)\Delta\delta_{th} - M_{\dot{w}} * \frac{g\,\sin(\theta_o)}{(1-Z_{\dot{w}})}\Delta\theta$
- $\therefore \Delta\dot{\theta} = \Delta q$

$$\begin{bmatrix}\Delta\dot{u}\\ \Delta\dot{w}\\ \Delta\dot{q}\\ \Delta\dot{\theta}\end{bmatrix} = \begin{bmatrix} X_u & X_w & -w_o & -g\,\cos(\theta_o) \\ \frac{Z_u}{(1-Z_{\dot{w}})} & \frac{Z_w}{(1-Z_{\dot{w}})} & \frac{(Z_q+u_o)}{(1-Z_{\dot{w}})} & \frac{-g\,\sin(\theta_o)}{(1-Z_{\dot{w}})} \\ M_u + M_{\dot{w}}\frac{Z_u}{(1-Z_{\dot{w}})} & M_w + M_{\dot{w}}\frac{Z_w}{(1-Z_{\dot{w}})} & M_q + M_{\dot{w}}\frac{(Z_q+u_o)}{(1-Z_{\dot{w}})} & -M_{\dot{w}} * \frac{g\,\sin(\theta_o)}{(1-Z_{\dot{w}})} \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix}\Delta u\\ \Delta w\\ \Delta q\\ \Delta\theta\end{bmatrix} + \begin{bmatrix} X_{\delta_e} & X_{\delta_{th}} \\ \frac{Z_{\delta_e}}{(1-Z_{\dot{w}})} & \frac{Z_{\delta_{th}}}{(1-Z_{\dot{w}})} \\ M_{\delta_e} + M_{\dot{w}}\frac{Z_{\delta_e}}{(1-Z_{\dot{w}})} & M_{\delta_{th}} + M_{\dot{w}}\frac{Z_{\delta_{th}}}{(1-Z_{\dot{w}})} \\ 0 & 0 \end{bmatrix} \begin{bmatrix}\Delta\delta_e\\ \Delta\delta_{th}\end{bmatrix}$$

ii. Lateral

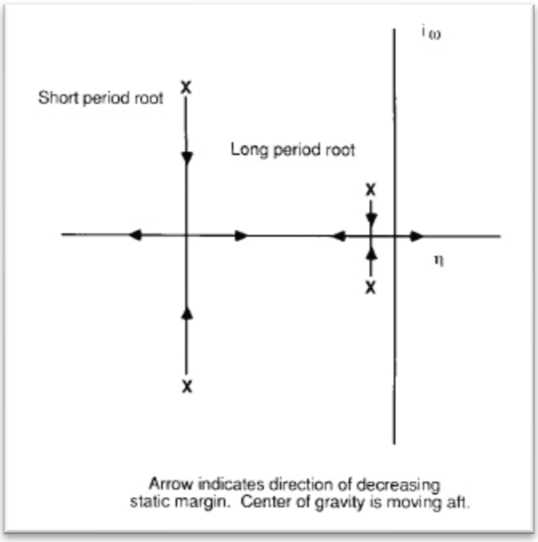
- $\therefore \Delta\dot{\beta} = \frac{Y_\beta}{V_{to}}\,\Delta\beta + \frac{Y_r - u_o}{V_{to}}\,\Delta r + \frac{Y_p + w_o}{V_{to}}\,\Delta p + \frac{Y_{\delta a}}{V_{to}}\,\Delta\delta_a + \frac{Y_{\delta_r}}{V_{to}}\,\Delta\delta_r + \frac{g}{V_{to}}\cos(\theta_o)\,\Delta\varphi$
- $\therefore \Delta\dot{p} = \sum G * \left(L_i + \frac{I_{xz}}{I_x} * N_i\right)\Delta i = \sum L_i^*\,\Delta i = L_\beta^*\,\Delta\beta + L_p^*\,\Delta p + L_r^*\,\Delta r + L_{\delta_r}^*\,\Delta\delta_r + L_{\delta_a}^*\,\Delta\delta_a$
- $\therefore \Delta\dot{r} = \sum G * \left(N_i + \frac{I_{xz}}{I_z} * L_i\right)\Delta i = \sum N_i^*\,\Delta i = N_\beta^*\,\Delta\beta + N_p^*\,\Delta p + N_r^*\,\Delta r + N_{\delta_r}^*\,\Delta\delta_r + N_{\delta_a}^*\,\Delta\delta_a$
- $\therefore \Delta\dot{\varphi} = \Delta p + \Delta r * \tan(\theta_0)$
- $\therefore \Delta\dot{\psi} = \Delta r * \sec(\theta_0)$

$$\begin{bmatrix} \Delta\dot{\beta} \\ \Delta\dot{p} \\ \Delta\dot{r} \\ \Delta\dot{\varphi} \\ \Delta\dot{\psi} \end{bmatrix} = \begin{bmatrix} \frac{Y_\beta}{V_{to}} & \frac{Y_p + w_o}{V_{to}} & \frac{Y_r - u_o}{V_{to}} & \frac{g\cos(\theta_o)}{V_{to}} & 0 \\ L_\beta^* & L_p^* & L_r^* & 0 & 0 \\ N_\beta^* & N_p^* & N_r^* & 0 & 0 \\ 0 & 1 & \tan(\theta_0) & 0 & 0 \\ 0 & 0 & \sec(\theta_0) & 0 & 0 \end{bmatrix} \begin{bmatrix} \Delta\beta \\ \Delta p \\ \Delta r \\ \Delta\varphi \\ \Delta\psi \end{bmatrix} + \begin{bmatrix} \frac{Y_{\delta a}}{V_{to}} & \frac{Y_{\delta_r}}{V_{to}} \\ L_{\delta_a}^* & L_{\delta_r}^* \\ \Delta\delta_a & N_{\delta_r}^* \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \Delta\delta_a \\ \Delta\delta_r \end{bmatrix}$$

b) State the short period & Long period approximations of the Longitudinal dynamics

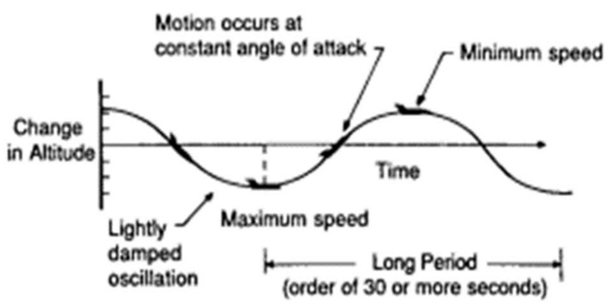
The longitudinal dynamics of the aircraft refer to dynamics in the x-z plane in the longitudinal axis
there are two types of approximations for that system of equations

1. Long period approximation (mode)
2. Short period approximation (mode)



By solving the A matrix from part, a we get 2 pairs of conjugate roots the pair which has small real part has big settling time according to $T_s = \frac{4}{\zeta \omega_n}$
on the other hand with the pair which has bigger real part

Long period approximation (mode)



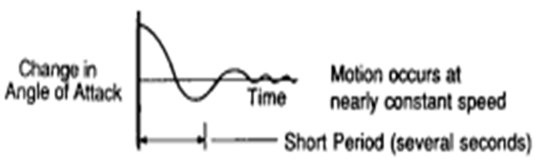
When the aircraft is exposed to some disturbances in the longitudinal direction a change in the altitude is formed due to this disturbance assuming that the angle of attack is constant the approximation to the long period mode is obtained by neglecting the pitching moment equation and assuming that the changes in angle of attack is zero.

$$\triangle\,\alpha = 0 \quad \Delta\dot{w} = 0$$

$$\triangle\,\alpha = \frac{\Delta w}{U_o} \quad \Delta\dot{q} = 0$$

$$\begin{bmatrix}\dot{u}\\ \dot{\theta}\end{bmatrix} = \begin{bmatrix} X_u + \dfrac{w_o Z_u}{Z_q + u_o} & -g\cos(\theta_o) - \dfrac{w_o g sin(\theta_o)}{Z_q + u_o} \\ \dfrac{-Z_u}{Z_q + u_o} & \dfrac{g sin(\theta_o)}{Z_q + u_o}\end{bmatrix}\begin{bmatrix}u\\ \theta\end{bmatrix} + \begin{bmatrix} X_{\delta_e} + \dfrac{w_o Z_{\delta_e}}{Z_q + u_o} & X_{\delta_{th}} + \dfrac{w_o Z_{\delta_{th}}}{Z_q + u_o} \\ \dfrac{-Z_{\delta_e}}{Z_q + u_o} & \dfrac{-Z_{\delta_{th}}}{Z_q + u_o}\end{bmatrix}\begin{bmatrix}\Delta\delta_e\\ \Delta\delta_{th}\end{bmatrix}$$

short period approximation (mode)



The same definition which we applied in the previous mode we will use here but with taking in consideration that the disturbance will damp quickly and but there will be a change in angle of attack and neglecting the change in velocity.

$$\Delta u = 0$$

$$\Delta\dot{\theta} = 0$$

$$M_\alpha = \frac{1}{I_y} * \frac{\partial M}{\partial \alpha} = \frac{1}{I_y} * \frac{\partial M}{\partial \left(\frac{\Delta w}{u_o}\right)} = \frac{u_o}{I_y} * \frac{\partial M}{\partial w} = u_o M_w$$

$$Z_\propto = u_o Z_w$$

$$M_{\dot{\alpha}} = u_o M_{\dot{w}}$$

$$\begin{bmatrix} \dot{\alpha} \\ \dot{q} \end{bmatrix} = \begin{bmatrix} \frac{Z_\alpha}{u_o} & 1 \\ M_\alpha + \frac{M_{\dot{\alpha}}\, Z_\alpha}{u_o} & M_q + M_{\dot{\alpha}} \end{bmatrix} \begin{bmatrix} \alpha \\ q \end{bmatrix} + \begin{bmatrix} Z_{\delta_e} & Z_{\delta_{th}} \\ M_{\delta_e} + \frac{M_\alpha\, Z_{\delta_e}}{u_o} & M_{\dot{\alpha}} + \frac{M_\alpha\, Z_{\delta_{th}}}{u_o} \end{bmatrix} \begin{bmatrix} \Delta\delta_e \\ \Delta\delta_{th} \end{bmatrix}$$

and by solving the characteristic equation of the system and getting its Eigen values we finally reach that;

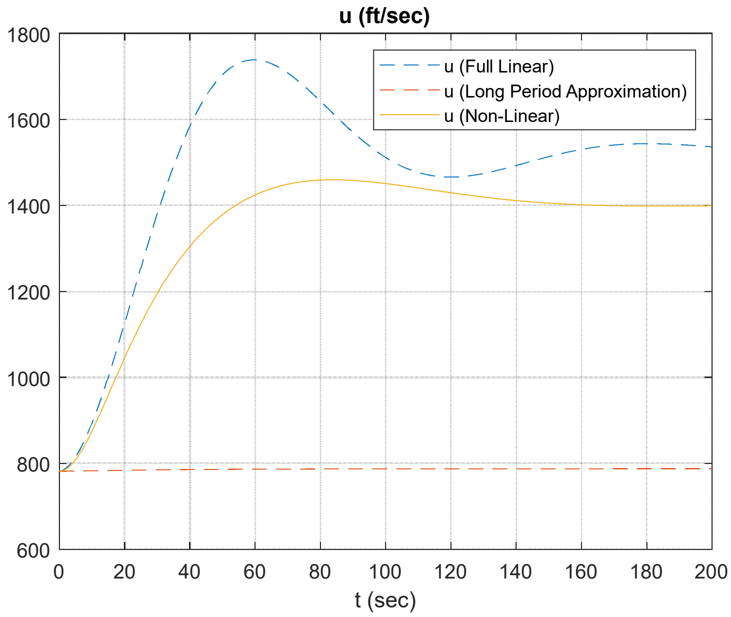
$$w_n = \sqrt{(M_q \frac{Z_\alpha}{u_o} - M_\alpha)}$$

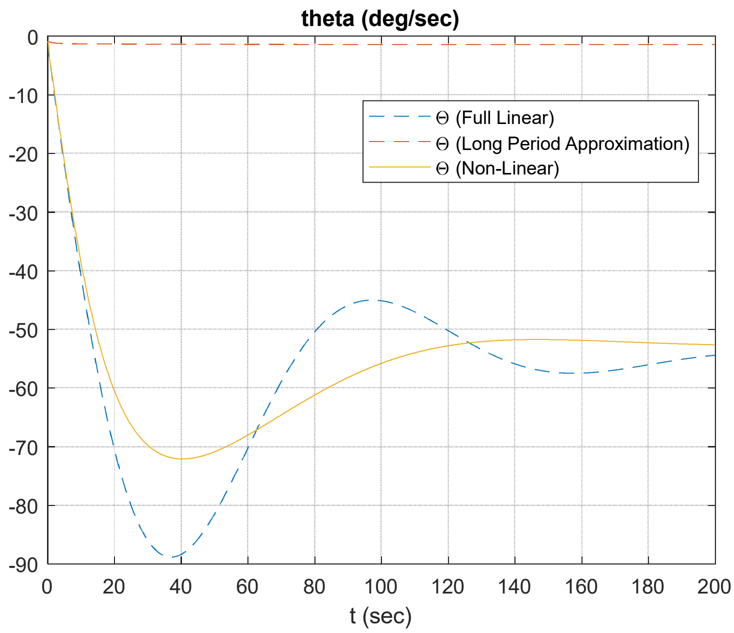
$$\zeta = \frac{\frac{Z_\alpha}{u_o} + M_q + M_{\dot{\alpha}}}{-2 w_n}$$

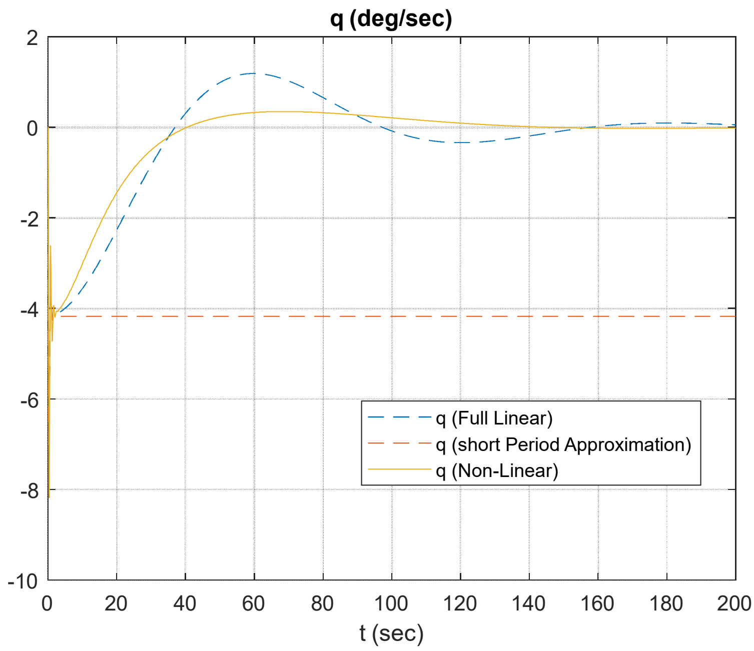
Results of the longitudinal dynamics

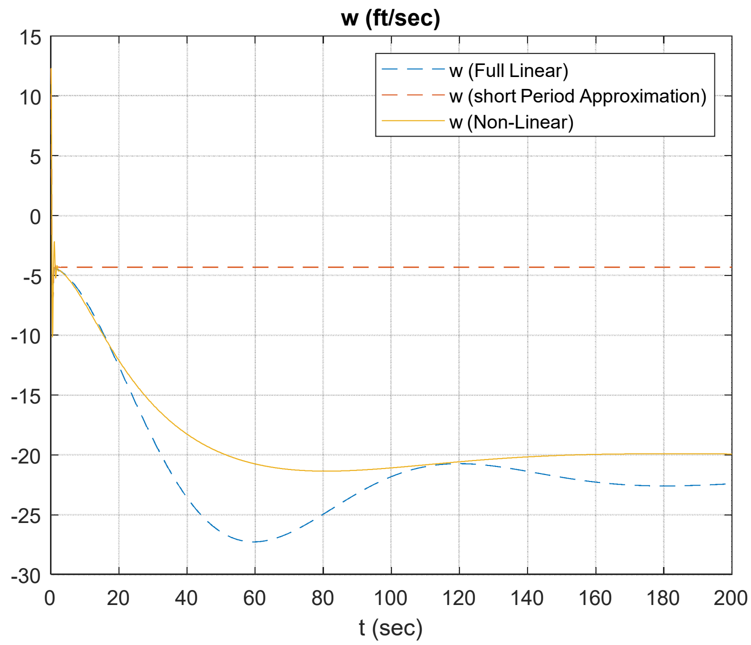
For states $u\ , \theta, w\ , q$

$$\delta_{elevator} = 1$$

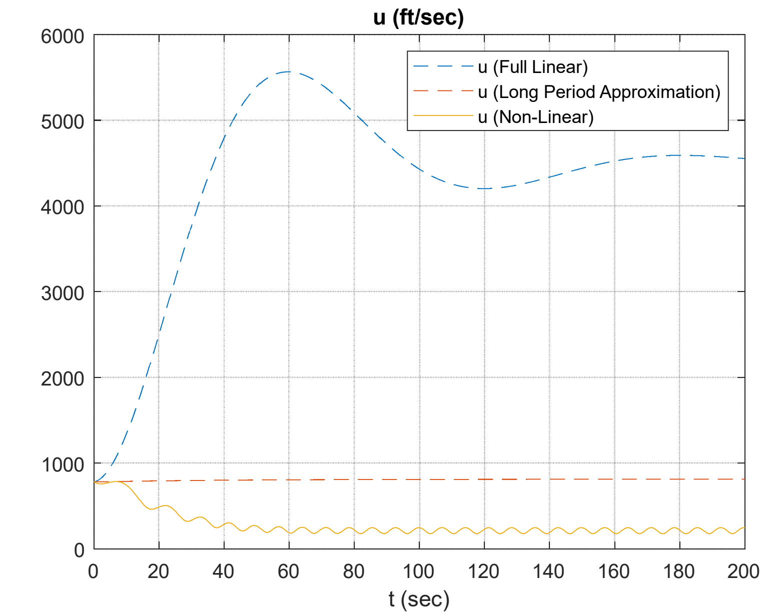

u (ft/sec)
1800
1600
1400
1200
1000
800
600
0
20
40
60
80
100
120
140
160
180
200
t (sec)
u (Full Linear)
u (Long Period Approximation)
u (Non-Linear)

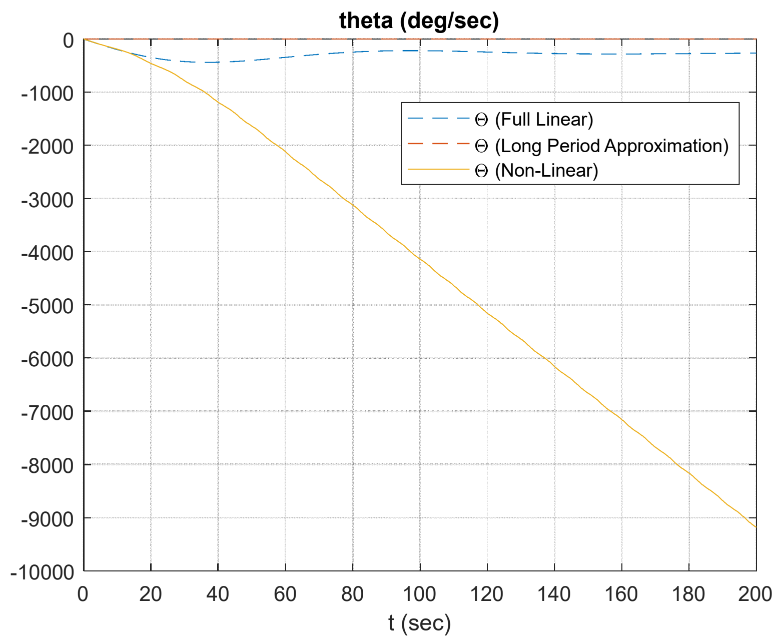

theta (deg/sec)
0
-10
-20
-30
-40
-50
-60
-70
-80
-90
0
20
40
60
80
100
120
140
160
180
200
t (sec)
Θ (Full Linear)
Θ (Long Period Approximation)
Θ (Non-Linear)

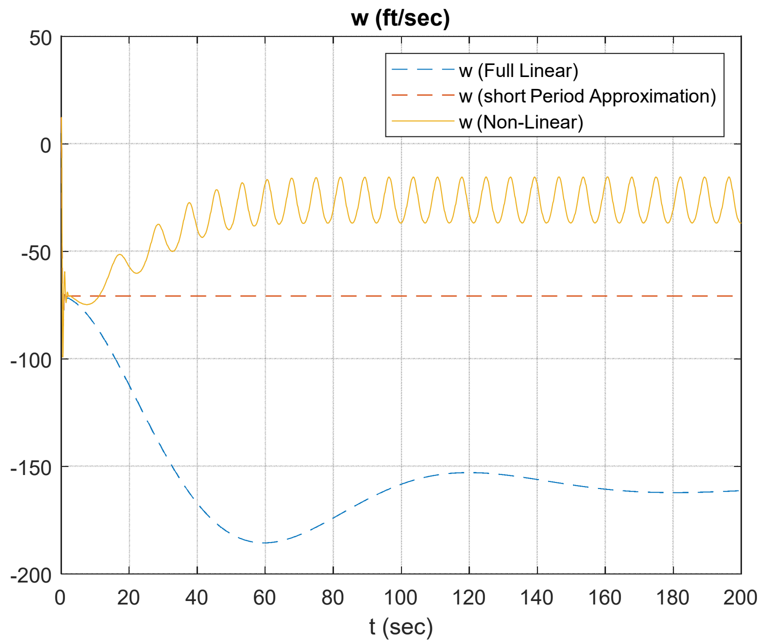

q (deg/sec)
2
0
-2
-4
-6
-8
-10
0
20
40
60
80
100
120
140
160
180
200
t (sec)
q (Full Linear)
q (short Period Approximation)
q (Non-Linear)

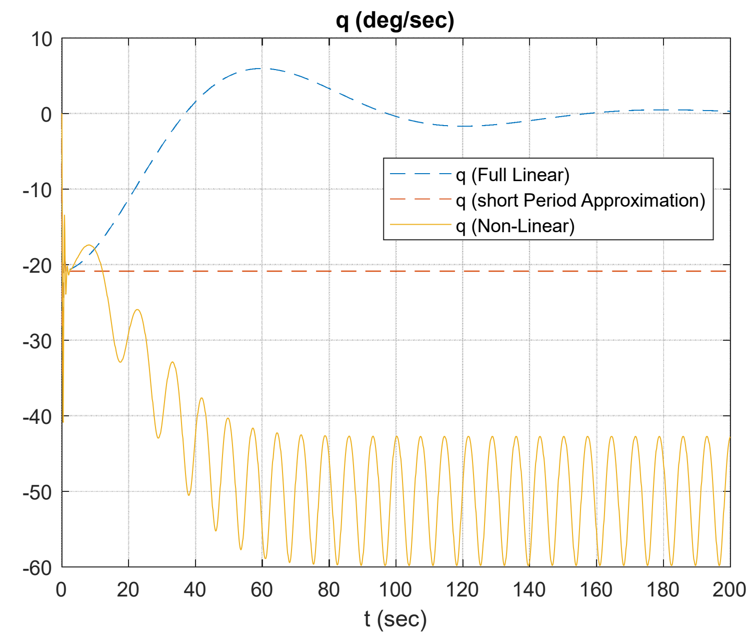

w (ft/sec)
15
10
5
0
-5
-10
-15
-20
-25
-30
0
20
40
60
80
100
120
140
160
180
200
t (sec)
w (Full Linear)
w (short Period Approximation)
w (Non-Linear)

$$\delta_{elevator} = 5$$

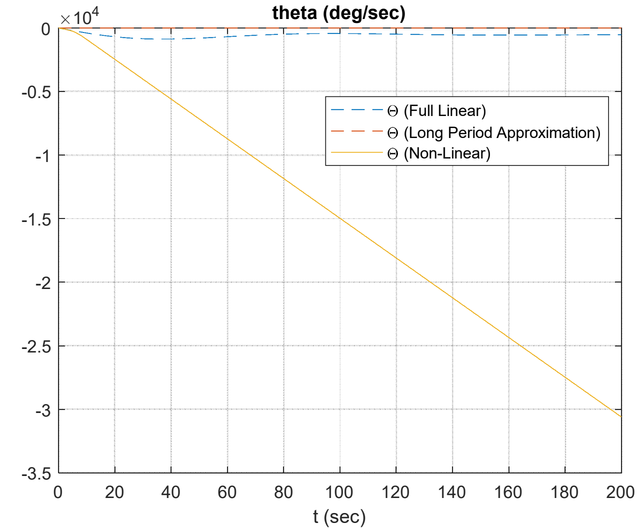


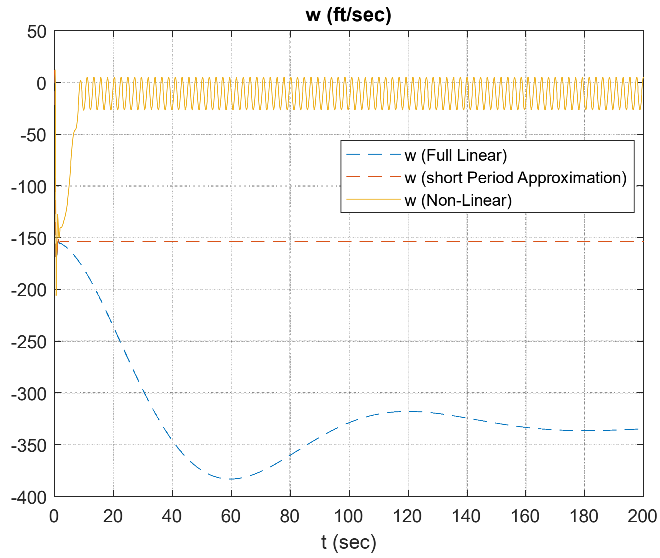


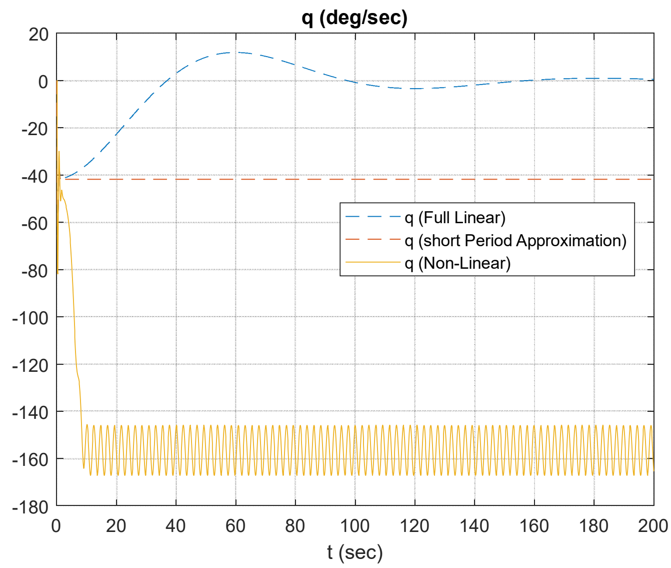




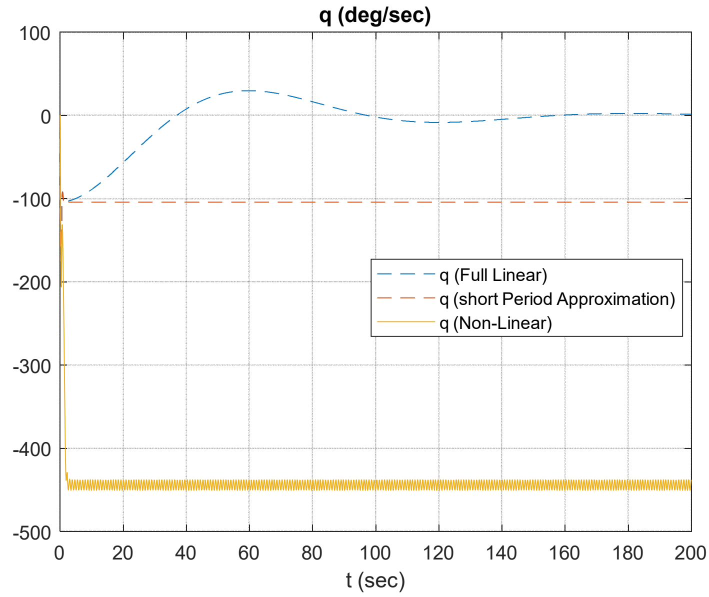
$$\delta_{elevator} = 10$$

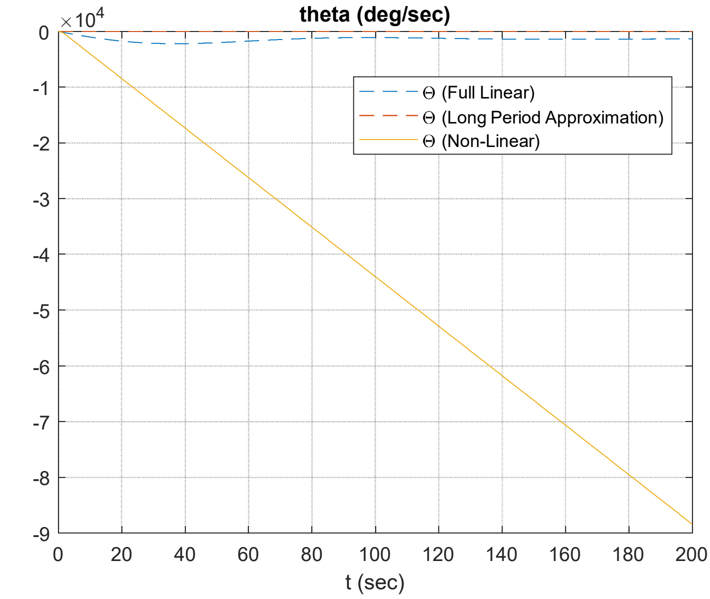

theta (deg/sec)
×10^4
0
-0.5
-1
-1.5
-2
-2.5
-3
-3.5
0
20
40
60
80
100
120
140
160
180
200
t (sec)
Θ (Full Linear)
Θ (Long Period Approximation)
Θ (Non-Linear)

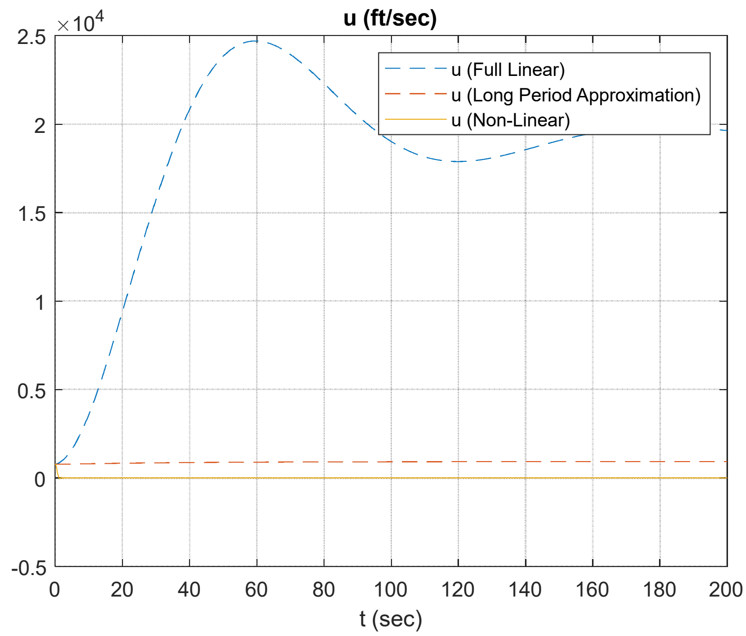

w (ft/sec)
50
0
-50
-100
-150
-200
-250
-300
-350
-400
0
20
40
60
80
100
120
140
160
180
200
t (sec)
w (Full Linear)
w (short Period Approximation)
w (Non-Linear)

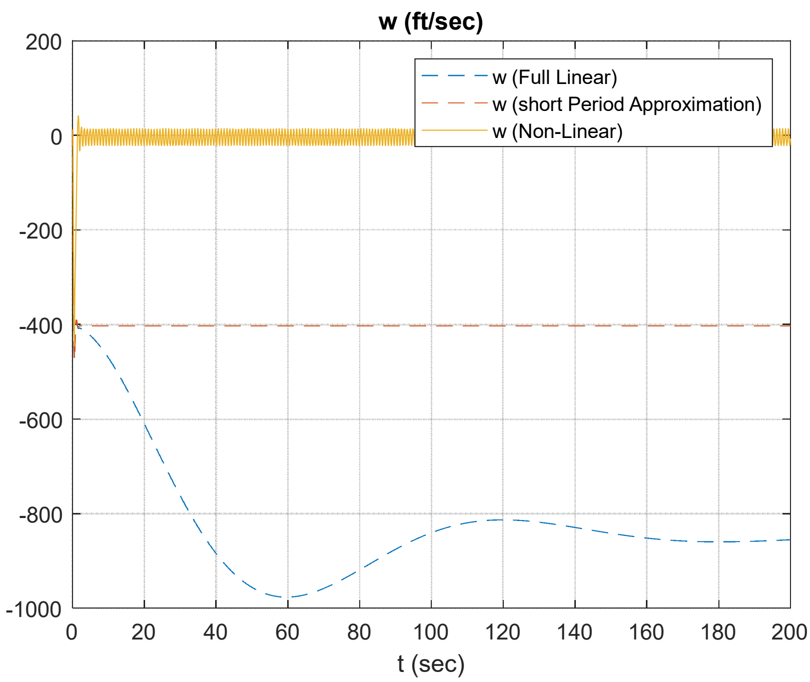

q (deg/sec)
20
0
-20
-40
-60
-80
-100
-120
-140
-160
-180
0
20
40
60
80
100
120
140
160
180
200
t (sec)
q (Full Linear)
q (short Period Approximation)
q (Non-Linear)

$$\delta_{elevator} = 25$$

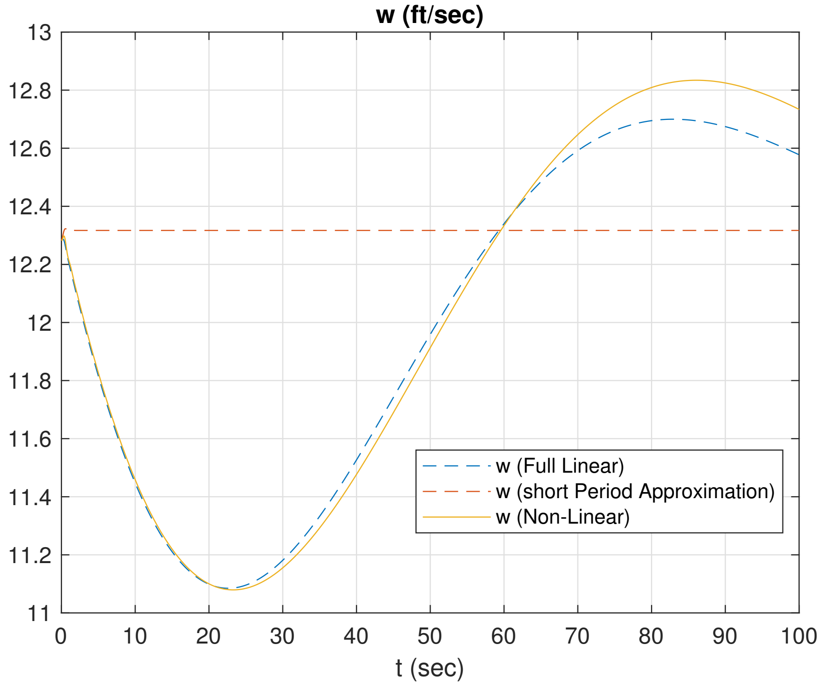


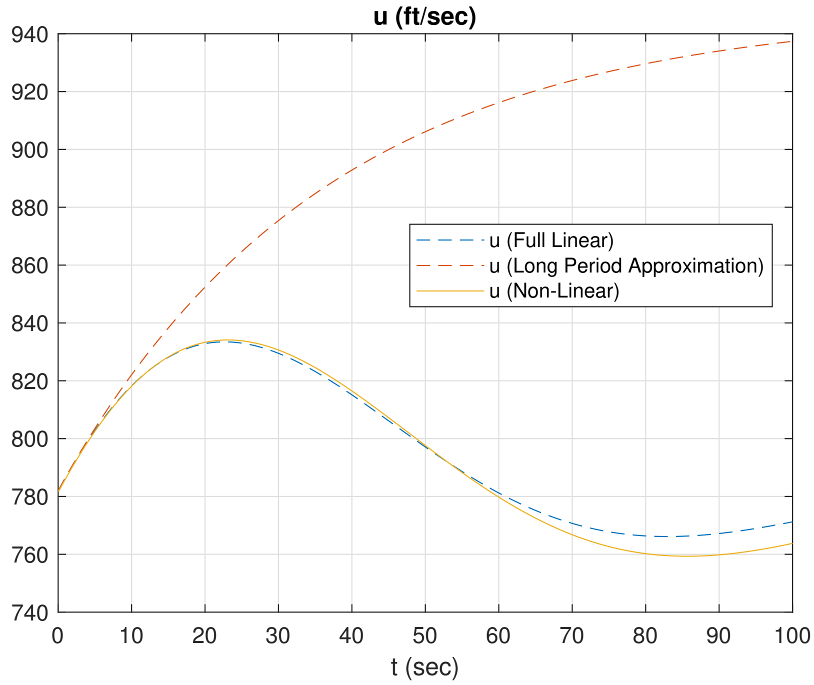


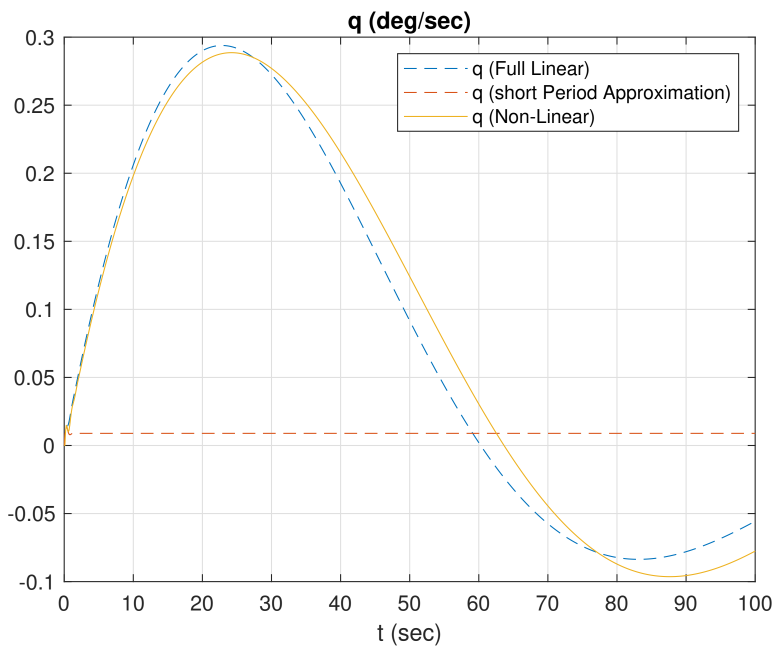

u (ft/sec)
×10^4
2.5
2
1.5
1
0.5
0
-0.5
0 20 40 60 80 100 120 140 160 180 200
t (sec)
u (Full Linear)
u (Long Period Approximation)
u (Non-Linear)

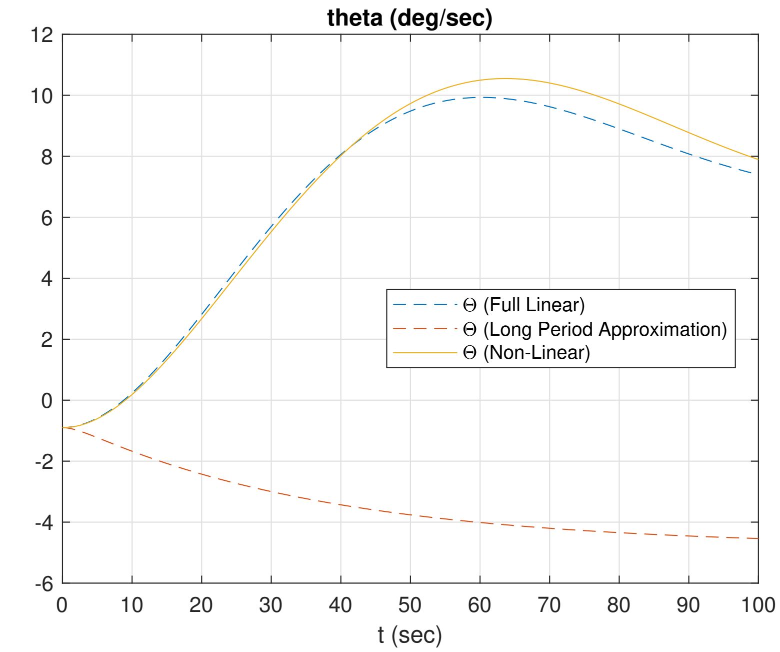

w (ft/sec)
200
0
-200
-400
-600
-800
-1000
0 20 40 60 80 100 120 140 160 180 200
t (sec)
w (Full Linear)
w (short Period Approximation)
w (Non-Linear)

$$\delta_{thrust} = 2000$$

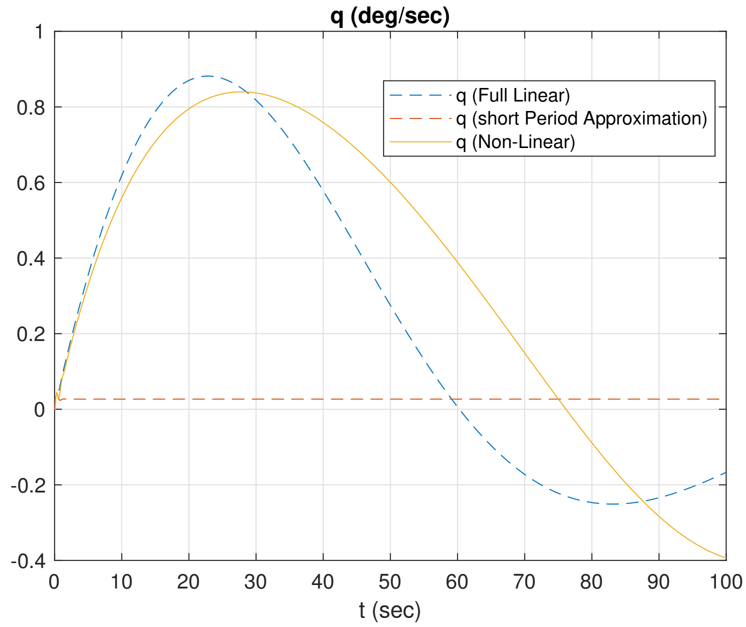


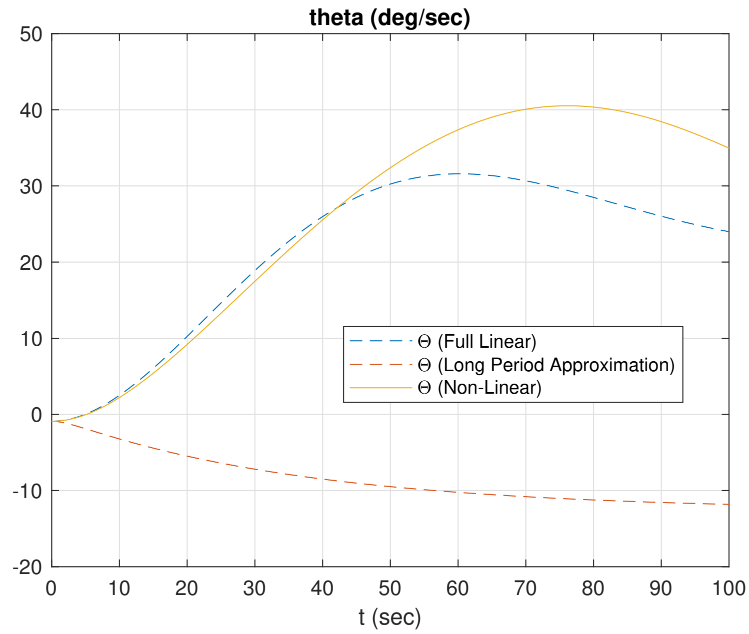


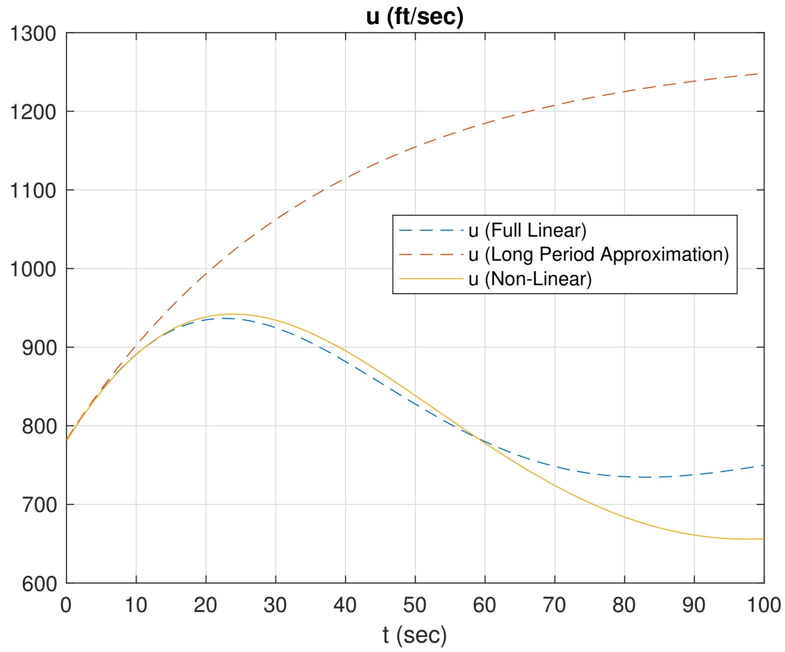

q (deg/sec)
q (Full Linear)
q (short Period Approximation)
q (Non-Linear)
0.3
0.25
0.2
0.15
0.1
0.05
0
-0.05
-0.1
0
10
20
30
40
50
60
70
80
90
100
t (sec)

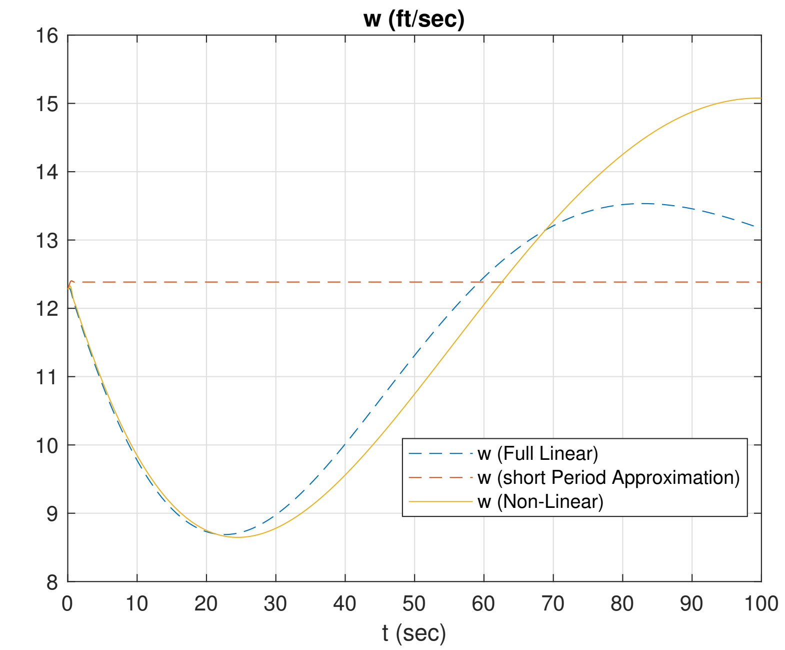

theta (deg/sec)
Θ (Full Linear)
Θ (Long Period Approximation)
Θ (Non-Linear)
12
10
8
6
4
2
0
-2
-4
-6
0
10
20
30
40
50
60
70
80
90
100
t (sec)

$$\delta_{thrust} = 6000$$

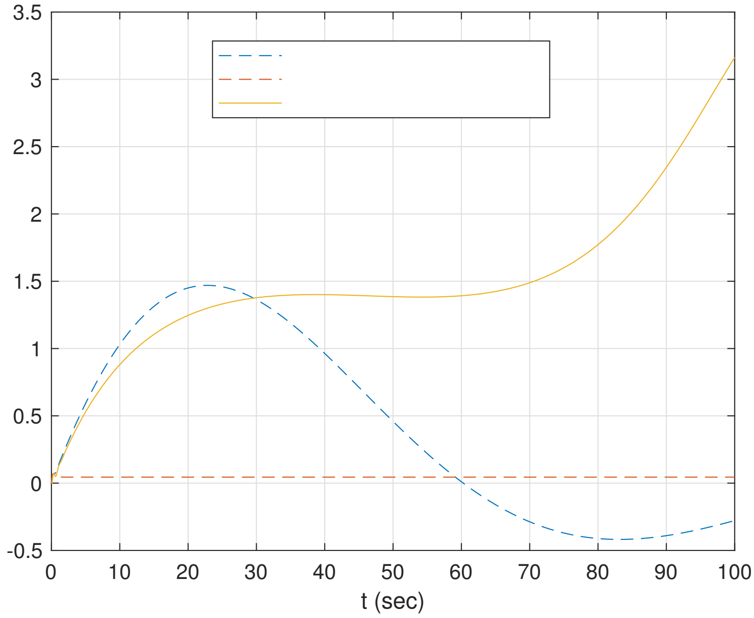


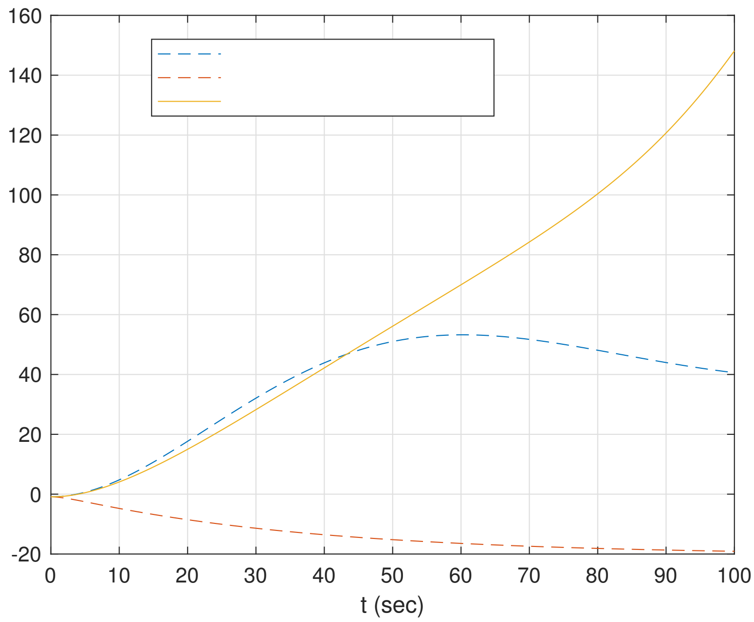


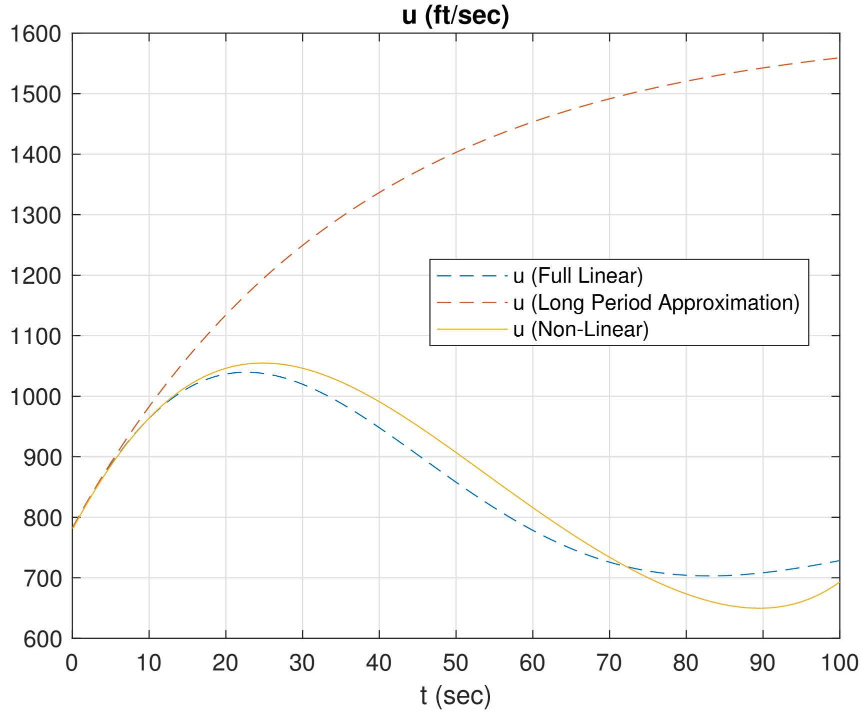


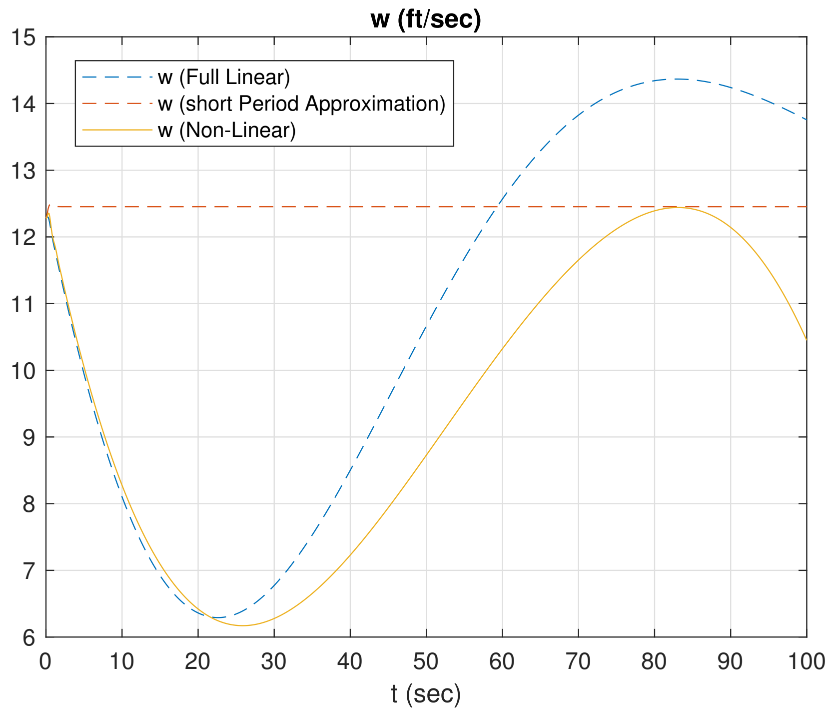


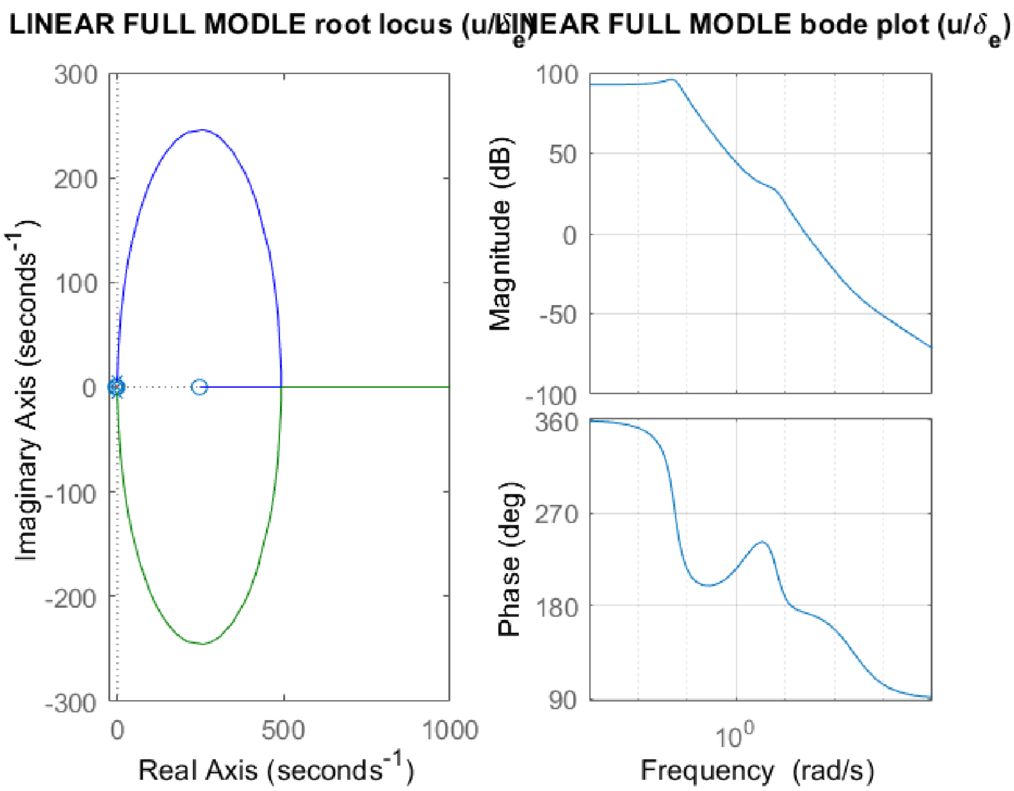
$$\delta_{thrust} = 10000$$

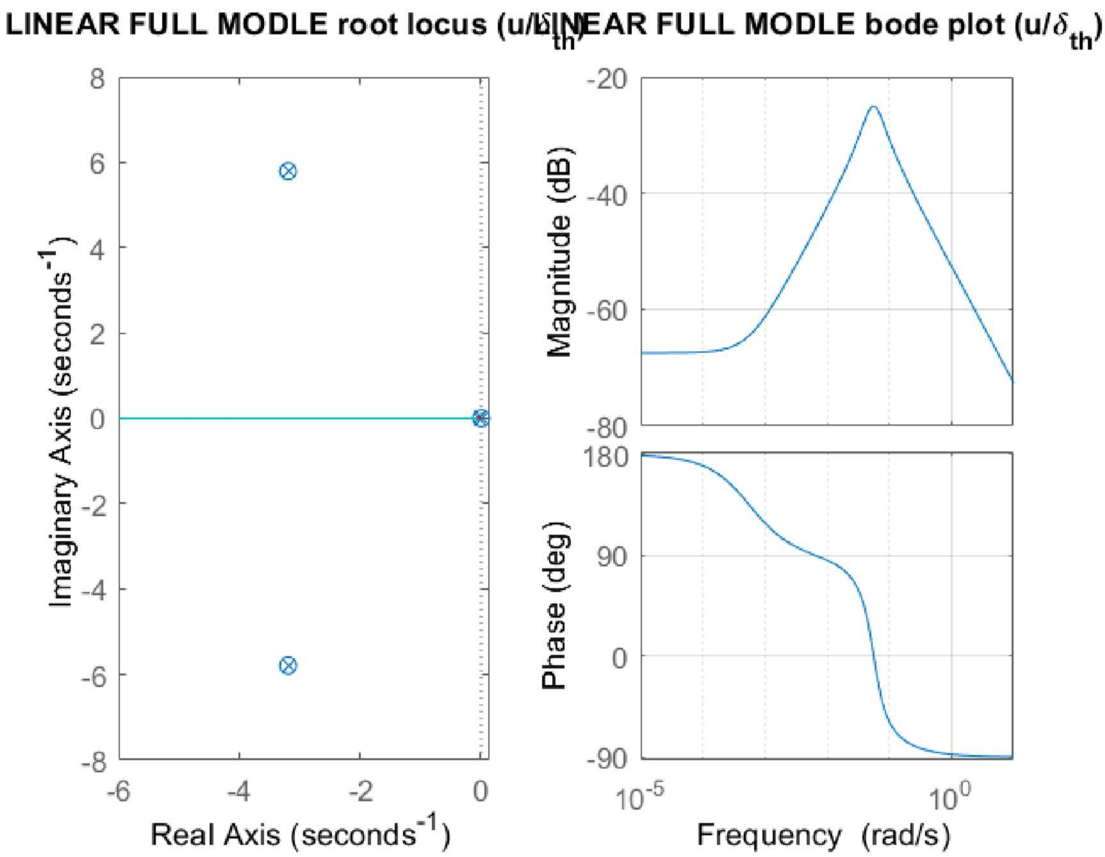

3.5
3
2.5
2
1.5
1
0.5
0
-0.5
0
10
20
30
40
50
60
70
80
90
100
t (sec)

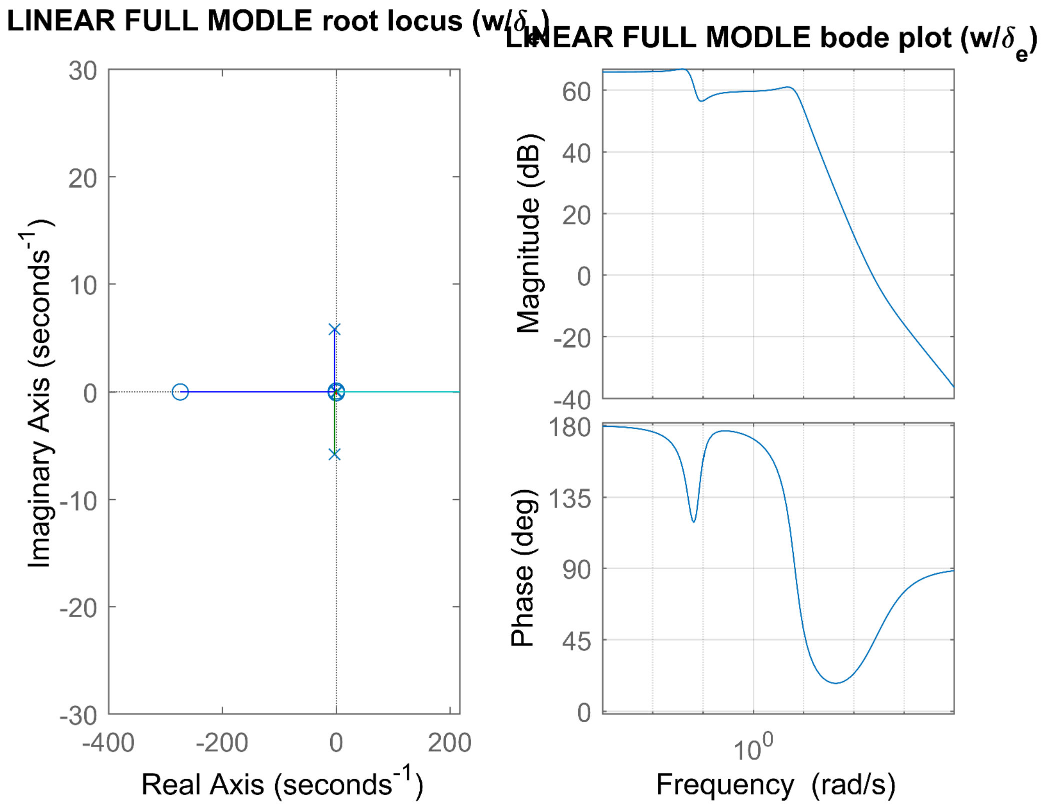

160
140
120
100
80
60
40
20
0
-20
0
10
20
30
40
50
60
70
80
90
100
t (sec)

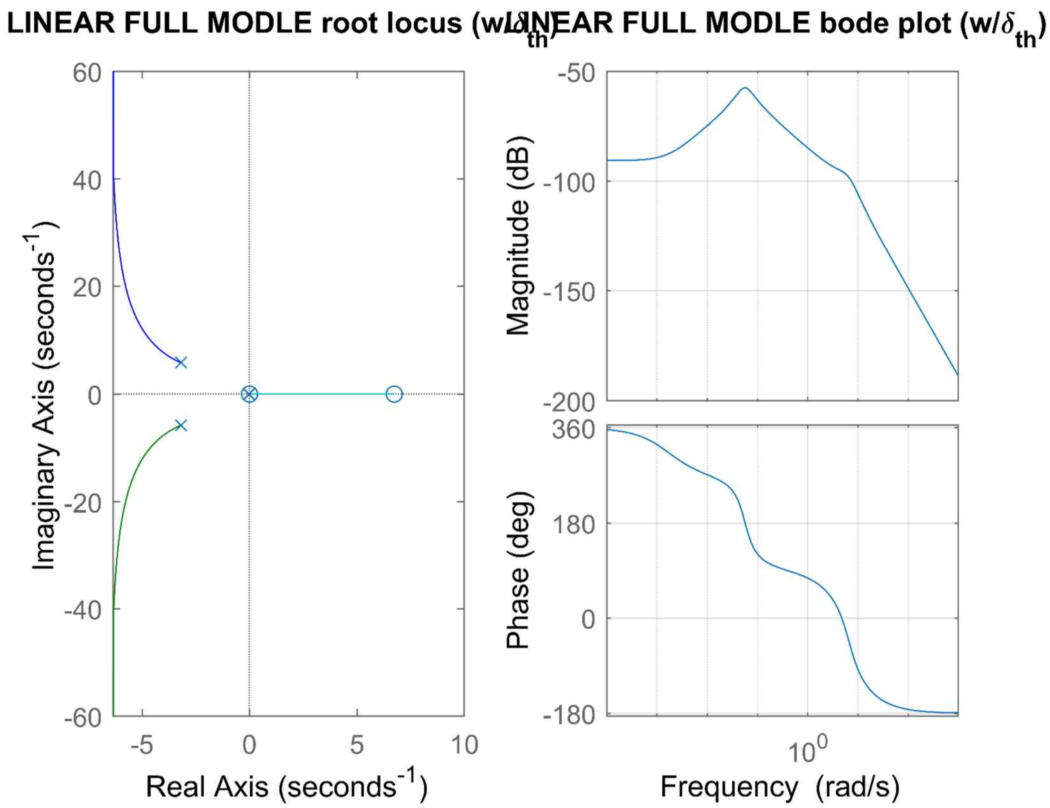

u (ft/sec)
u (Full Linear)
u (Long Period Approximation)
u (Non-Linear)
1600
1500
1400
1300
1200
1100
1000
900
800
700
600
0
10
20
30
40
50
60
70
80
90
100
t (sec)

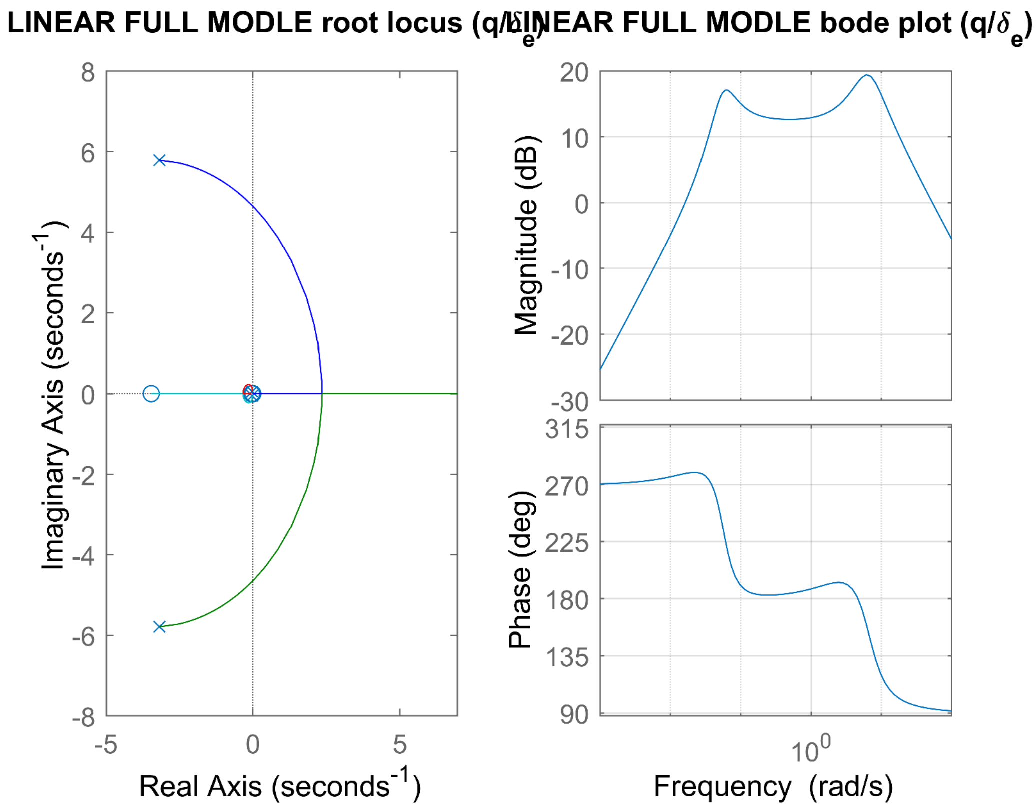

w (ft/sec)
w (Full Linear)
w (short Period Approximation)
w (Non-Linear)
15
14
13
12
11
10
9
8
7
6
0
10
20
30
40
50
60
70
80
90
100
t (sec)

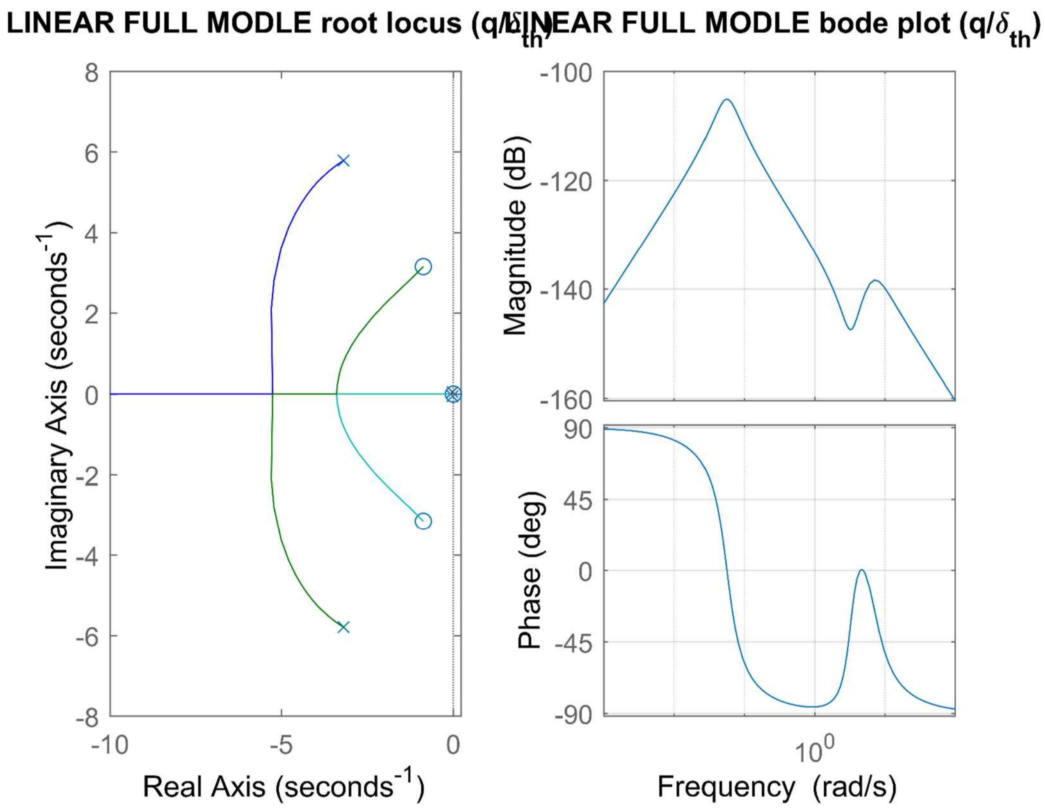

LINEAR FULL MODLE root locus (u/δe)
LINEAR FULL MODLE bode plot (u/δe)
Imaginary Axis (seconds-1)
Real Axis (seconds-1)
300
200
100
0
-100
-200
-300
0
500
1000
Magnitude (dB)
100
50
0
-50
-100
Phase (deg)
360
270
180
90
10^0
Frequency (rad/s)

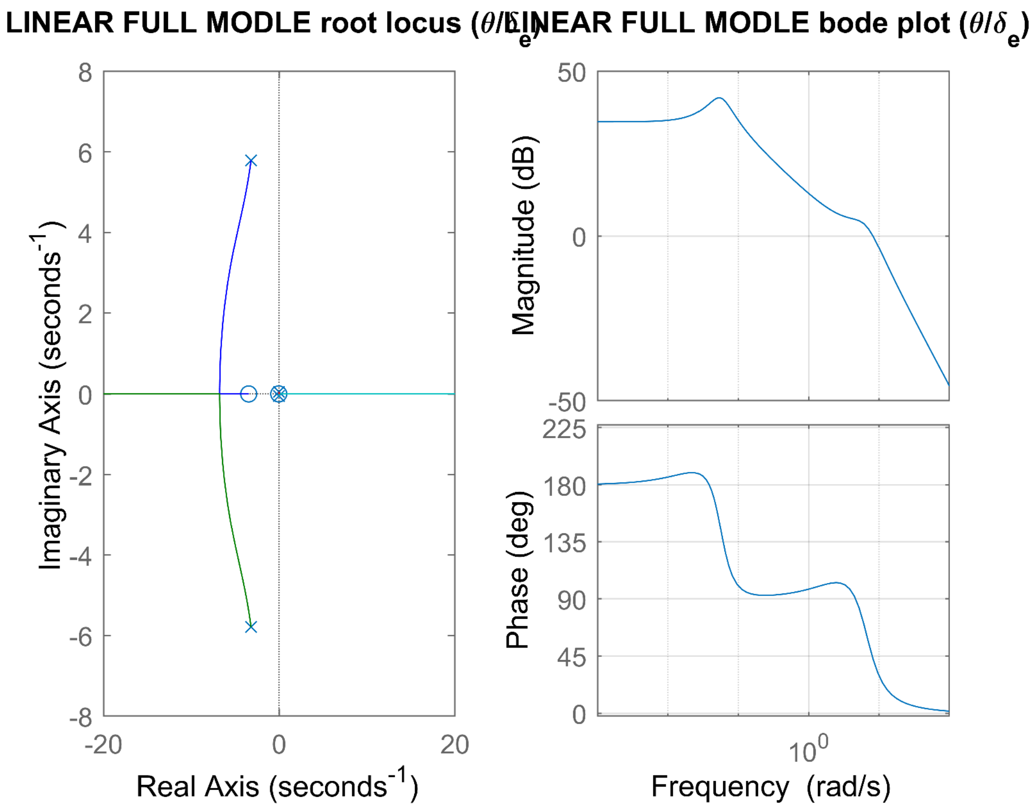

LINEAR FULL MODLE root locus (u/δth)
LINEAR FULL MODLE bode plot (u/δth)
Imaginary Axis (seconds-1)
Real Axis (seconds-1)
8
6
4
2
0
-2
-4
-6
-8
-6
-4
-2
0
Magnitude (dB)
-20
-40
-60
-80
Phase (deg)
180
90
0
-90
10^-5
10^0
Frequency (rad/s)

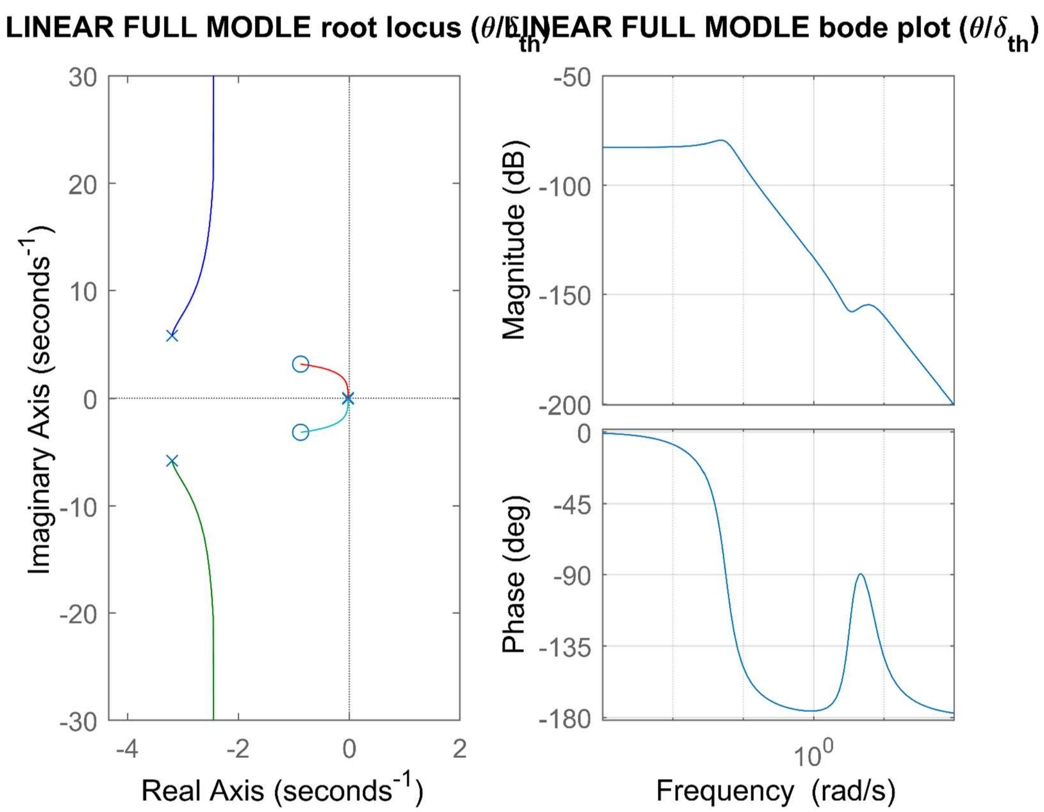

LINEAR FULL MODLE root locus (w/δe)
Imaginary Axis (seconds-1)
Real Axis (seconds-1)
30
20
10
0
-10
-20
-30
-400
-200
0
200
LINEAR FULL MODLE bode plot (w/δe)
Magnitude (dB)
60
40
20
0
-20
-40
Phase (deg)
180
135
90
45
0
10^0
Frequency (rad/s)

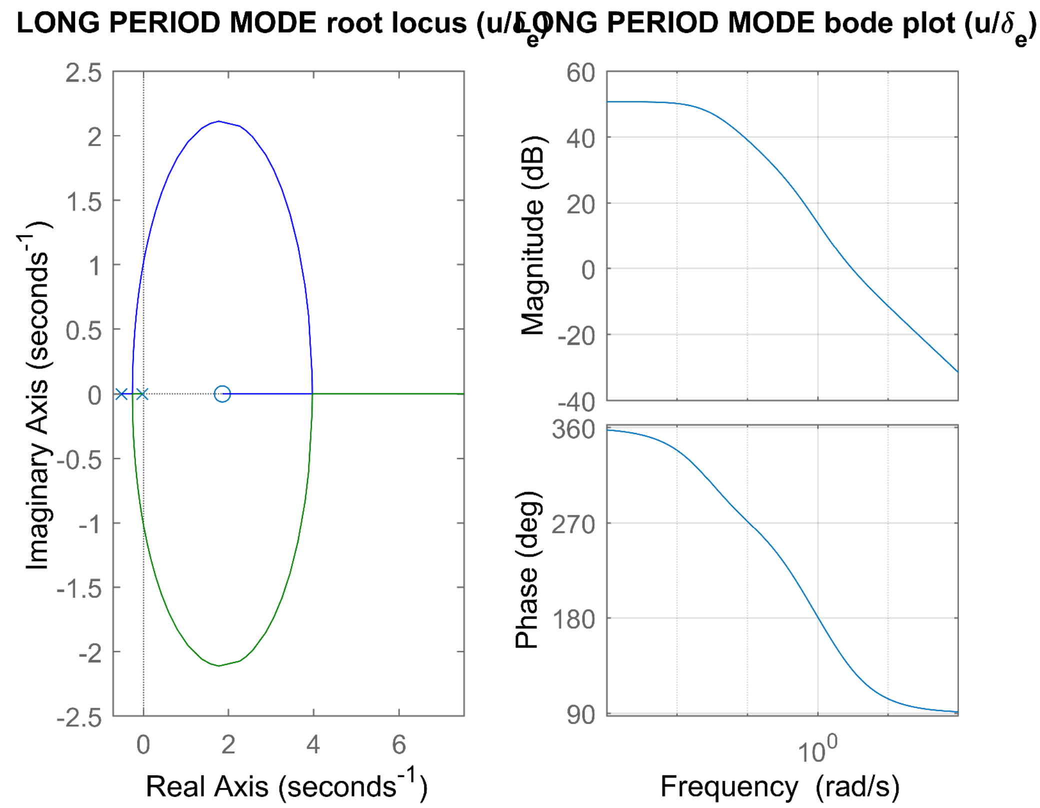

LINEAR FULL MODLE root locus (w/δth)
Imaginary Axis (seconds-1)
Real Axis (seconds-1)
60
40
20
0
-20
-40
-60
-5
0
5
10
LINEAR FULL MODLE bode plot (w/δth)
Magnitude (dB)
-50
-100
-150
-200
Phase (deg)
360
180
0
-180
10^0
Frequency (rad/s)

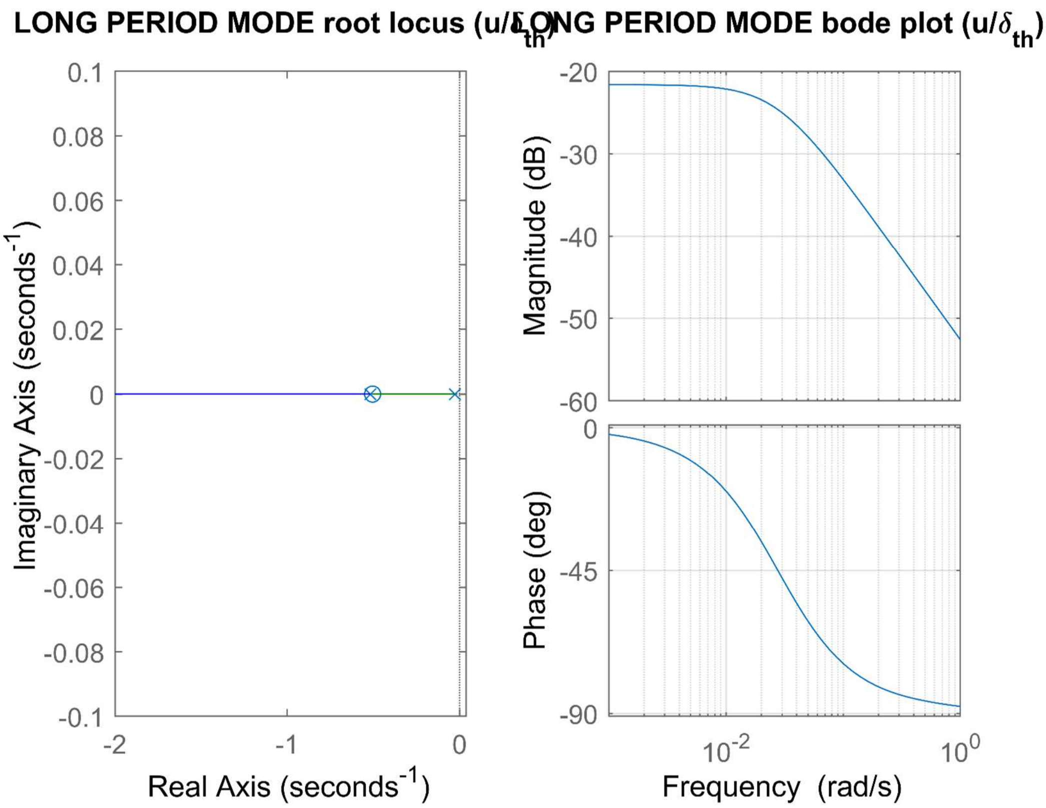

LINEAR FULL MODLE root locus (q/δe)
LINEAR FULL MODLE bode plot (q/δe)
Imaginary Axis (seconds-1)
Real Axis (seconds-1)
Magnitude (dB)
Phase (deg)
Frequency (rad/s)

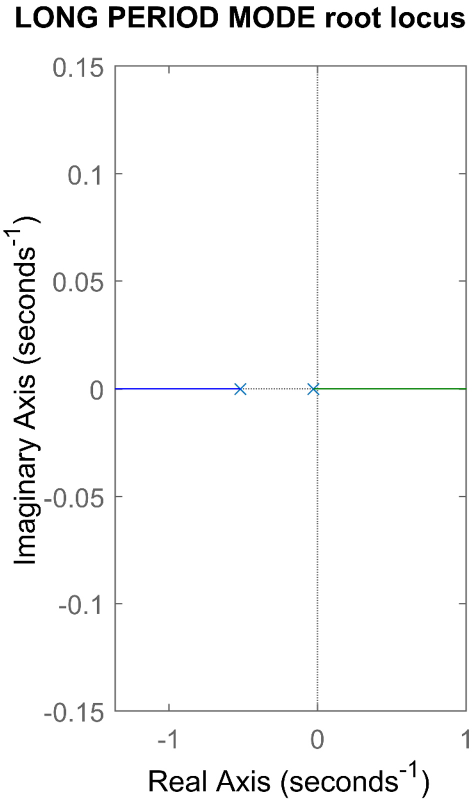

LINEAR FULL MODLE root locus (q/δth)
LINEAR FULL MODLE bode plot (q/δth)
Imaginary Axis (seconds-1)
Real Axis (seconds-1)
Magnitude (dB)
Phase (deg)
Frequency (rad/s)

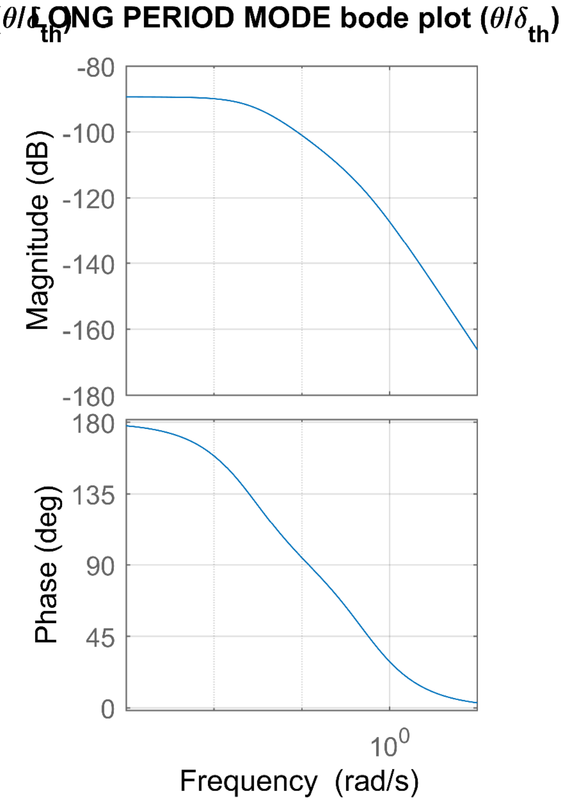

LINEAR FULL MODLE root locus (θ/δe)
LINEAR FULL MODLE bode plot (θ/δe)
Imaginary Axis (seconds-1)
Real Axis (seconds-1)
Magnitude (dB)
Phase (deg)
Frequency (rad/s)

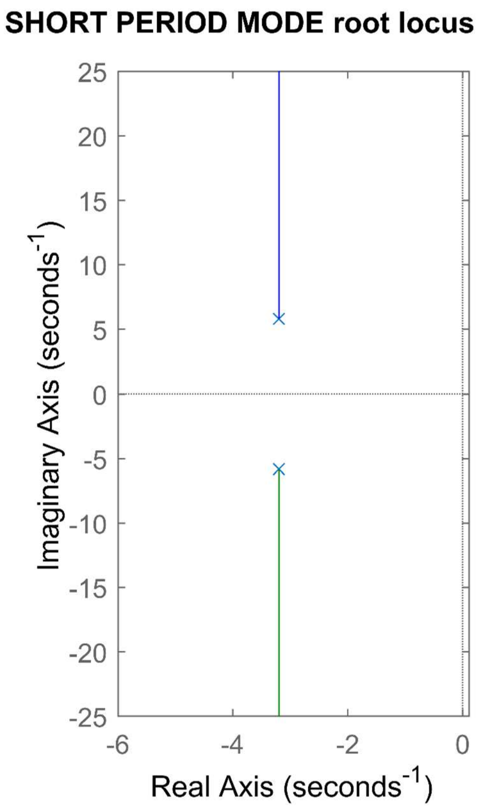

LINEAR FULL MODLE root locus (θ/δth)
LINEAR FULL MODLE bode plot (θ/δth)
Imaginary Axis (seconds-1)
Real Axis (seconds-1)
Magnitude (dB)
Phase (deg)
Frequency (rad/s)

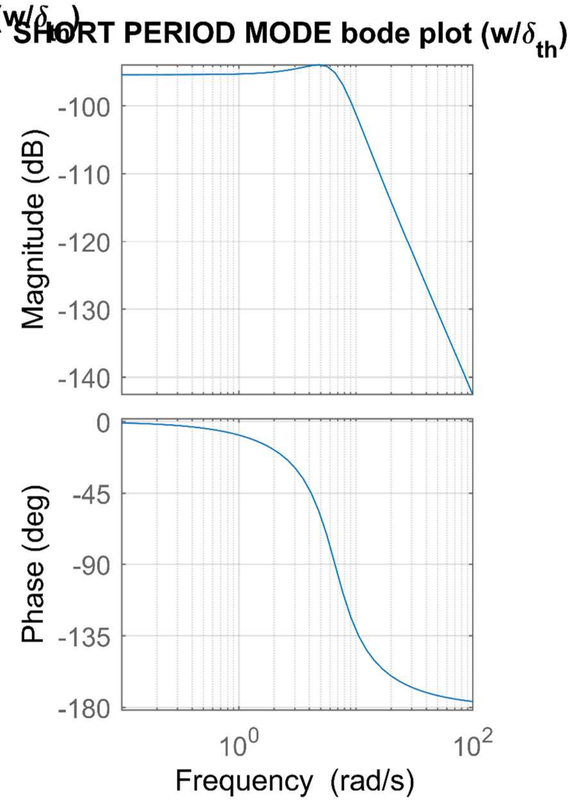

LONG PERIOD MODE root locus (u/δe)
LONG PERIOD MODE bode plot (u/δe)
Imaginary Axis (seconds^-1)
Real Axis (seconds^-1)
Magnitude (dB)
Phase (deg)
Frequency (rad/s)

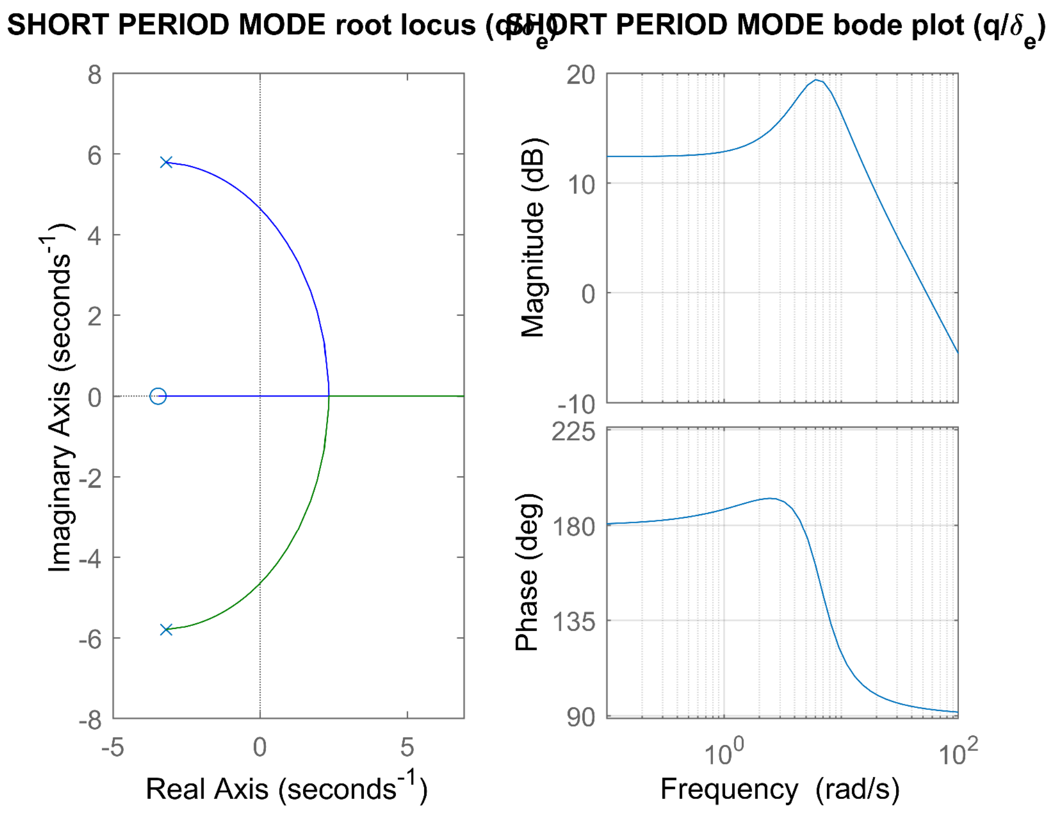

LONG PERIOD MODE root locus (u/δth)
LONG PERIOD MODE bode plot (u/δth)
Imaginary Axis (seconds^-1)
Real Axis (seconds^-1)
Magnitude (dB)
Phase (deg)
Frequency (rad/s)

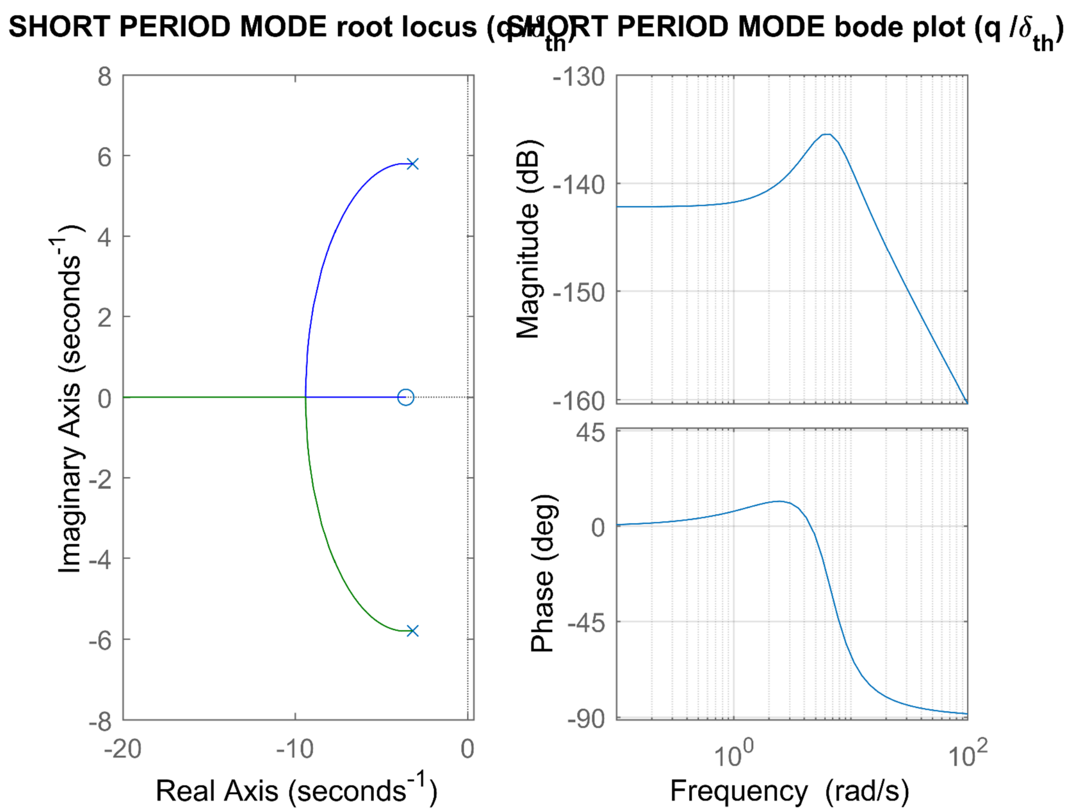

LONG PERIOD MODE root locus (θ/δth)
Imaginary Axis (seconds-1)
Real Axis (seconds-1)
0.15
0.1
0.05
0
-0.05
-0.1
-0.15
-1
0
1

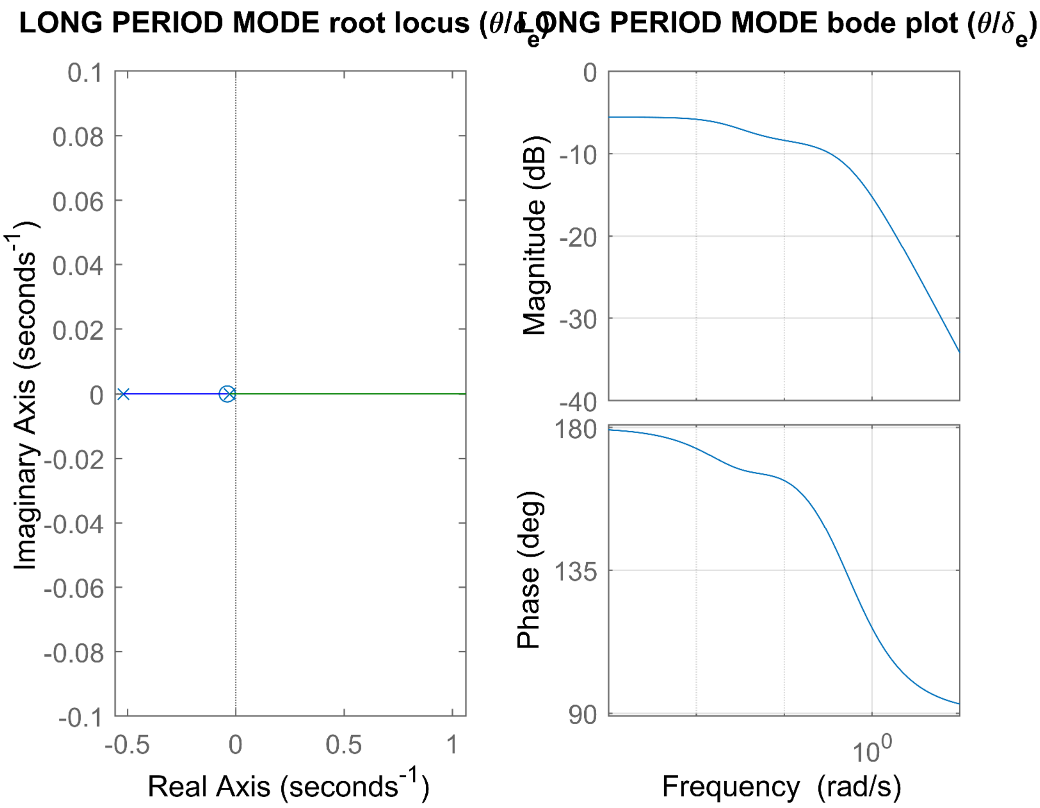

LONG PERIOD MODE bode plot (θ/δth)
Magnitude (dB)
Phase (deg)
Frequency (rad/s)
-80
-100
-120
-140
-160
-180
180
135
90
45
0
10^0


SHORT PERIOD MODE root locus (w/δth)
Imaginary Axis (seconds-1)
Real Axis (seconds-1)
25
20
15
10
5
0
-5
-10
-15
-20
-25
-6
-4
-2
0


SHORT PERIOD MODE bode plot (w/δth)
Magnitude (dB)
Phase (deg)
Frequency (rad/s)
-100
-110
-120
-130
-140
0
-45
-90
-135
-180
10^0
10^2


SHORT PERIOD MODE root locus (q/δe)
SHORT PERIOD MODE bode plot (q/δe)
Imaginary Axis (seconds-1)
Real Axis (seconds-1)
Magnitude (dB)
Phase (deg)
Frequency (rad/s)


SHORT PERIOD MODE root locus (q/δth)
SHORT PERIOD MODE bode plot (q /δth)
Imaginary Axis (seconds-1)
Real Axis (seconds-1)
Magnitude (dB)
Phase (deg)
Frequency (rad/s)


LONG PERIOD MODE root locus (θ/δ_e)
LONG PERIOD MODE bode plot (θ/δ_e)
Imaginary Axis (seconds^{-1})
Real Axis (seconds^{-1})
0.1
0.08
0.06
0.04
0.02
0
-0.02
-0.04
-0.06
-0.08
-0.1
-0.5
0
0.5
1
Magnitude (dB)
Phase (deg)
-10
-20
-30
-40
180
135
90
10^0
Frequency (rad/s)

Codes

```
clc; clear; close all;
```

Inputs

Forces, Moments and Inertia

```
[Mass, g, I, invI, timeSpan, dt, ICs, ICs_dot0, Vt0, ...
    dControl, SD_Long, SD_Lat, initialGravity] = Input("NT-33A_4.xlsx");
steps = (timeSpan(2) - timeSpan(1))/dt;
Result = NaN(12, steps);
Result(:,1) = ICs;
time_V = linspace(0, timeSpan(2), steps+1);
```

Solving

```
%profile on;
dForces = [0 ; 0; 0];
dMoments = [0 ; 0; 0];

for i =1:steps
    i
    Result(:, i+1) = RBDSolver(Result(:, i), dt, (initialGravity + dForces),
dMoments, Mass, I, invI, g);

    [dF, dM] = airFrame(SD_Long, SD_Lat, dControl, ICs, ICs_dot0, Result(:, i+1)
,Vt0, ...
        (initialGravity + dForces), dMoments, Mass, I, invI, g);


    dForces = vpa(dF');
    dMoments = vpa(dM');

end
```

Rearranging Results

```
u = Result(1,:);

v = Result(2,:);

w = Result(3,:);

p = Result(4,:);

q = Result(5,:);

r = Result(6,:);

phi = Result(7,:);

theta = Result(8,:);

psi = Result(9,:);

x = Result(10,:);

y = Result(11,:);
```

```
z = Result(12,:);


beta_deg=asin(v/Vt0)*180/pi;

alpha_deg=atan(w./u)*180/pi;

p_deg=p*180/pi;

q_deg=q*180/pi;

r_deg=r*180/pi;

phi_deg=phi*180/pi;

theta_deg=theta*180/pi;

psi_deg=psi*180/pi;
```

Longitudenal Full Linear Model

```
XU=SD_Long(1);

ZU=SD_Long(2);

MU=SD_Long(3);

XW=SD_Long(4);

ZW=SD_Long(5);

MW=SD_Long(6);

ZWD=SD_Long(7);

ZQ=SD_Long(8);

MWD=SD_Long(9);

MQ=SD_Long(10);

XDE=SD_Long(11);

ZDE=SD_Long(12);

MDE=SD_Long(13);

XDTH=SD_Long(14);

ZDTH=SD_Long(15);

MDTH=SD_Long(16);
```

```
u0 = ICs(1);

w0 = ICs(3);

q0 = ICs(5);

theta0 = ICs(8);

A_long=[XU XW -w0 -g*cos(theta0)

    ZU/(1-ZWD) ZW/(1-ZWD) (ZQ+u0)/(1-ZWD) -g*sin(theta0)/(1-ZWD)

    MU+MWD*ZU/(1-ZWD) MW+MWD*ZW/(1-ZWD) MQ+MWD*(ZQ+u0)/(1-ZWD) -
MWD*g*sin(theta0)/(1-ZWD)

    0 0 1 0];

B_long=[XDE XDTH

    ZDE/(1-ZWD) ZDTH/(1-ZWD)

    MDE+MWD*ZDE/(1-ZWD) MDTH+MWD*ZDTH/(1-ZWD)

    0 0];

C_long=eye(4);

D_long=zeros(4,2);



% Two Inputs - Four Output Each
LinearLongSS = ss(A_long, B_long, C_long, D_long);
LinearLongTF = tf(LinearLongSS);
```

APPROXIMATE

PHUGOID MODE (LONG PERIOD MODE)

```
A_phug=[XU -g*cos(theta0)

    ZU/(u0+ZQ) g*sin(theta0)];

B_phug=[XDE XDTH

    ZDE/(ZQ+u0) ZDTH/(ZQ+u0)];

C_phug=eye(2);D_phug=zeros(2,2);



PHUG_SS = ss(A_phug,B_phug,C_phug,D_phug);

PHUG_TF = tf(PHUG_SS);
```

```
Long_U_DTH = '';
```

SHORT PERIOD MODE

```
A_short=[ZW/(1-ZWD) (ZQ+u0)/(1-ZWD)

    (MW+ZW*MWD/(1-ZWD)) (MQ+MWD*(ZQ+u0)/(1-ZWD))];

B_short=[ZDE/(1-ZWD) ZDTH

    MDE+MWD*ZDE/(1-ZWD) MDTH+MWD*ZDTH/(1-ZWD)];

C_short=eye(2);D_short=zeros(2,2);



SHORT_SS=ss(A_short,B_short,C_short,D_short);
```

Plotting Longitudinal Full Linear Model Step Response

Due to delta_elevetor or delta_thrust

```
dControl_long = dControl(3:4); % dE, dTh
opt = stepDataOptions;
opt.StepAmplitude = dControl_long;
[res, ~, ~] = step(LinearLongSS, time_V, opt);
res_dE  =res(:,:,1);
res_dTh =res(:,:,2);
```

Plotting Longitudinal Long & Short Period Approximation Model Step Response

Due to delta_elevetor or delta_thrust

```
dControl_long = dControl(3:4); % dE, dTh
opt = stepDataOptions;
opt.StepAmplitude = dControl_long;

[APPres_PH, ~,~] = step(PHUG_SS, time_V, opt);
[APPres_SH, ~,~] = step(SHORT_SS, time_V, opt);

APPres_dE = zeros (length(time_V),4);
APPres_dE(:,[1,4]) = APPres_PH(:,:,1);
APPres_dE(:,[2,3]) = APPres_SH(:,:,1);

APPres_dTH = zeros (length(time_V),4);
APPres_dTH(:,[1,4]) = APPres_PH(:,:,2);
APPres_dTH(:,[2,3]) = APPres_SH(:,:,2);
```

u response Full Linear - Approximate - Non Linear

```
figure(1)

if(dControl_long(1) ~= 0)
    plot(time_V, res_dE(:, 1) + u0, '--', 'DisplayName', 'u (Full Linear)');  %
Full Linear Model
    hold on
    plot(time_V, APPres_dE(:, 1) + u0, '--', 'DisplayName', 'u (Long Period
Approximation)');  % Approximate (long period Mode)
elseif(dControl_long(2) ~= 0)
```

```
    plot(time_V, res_dTh(:, 1) + u0, '--', 'DisplayName', 'u (Full Linear)');  %
Full Linear Model
    hold on
    plot(time_V, APPres_dTH(:, 1) + u0, '--', 'DisplayName', 'u (Long Period
Approximation)');  % Approximate (long period Mode)
end

hold on
plot(time_V, u, '-', 'DisplayName', 'u (Non-Linear)');                      % Non-
Linear Model
title('u (ft/sec)'); xlabel('t (sec)');
legend('show');
grid on
```

w response Full Linear - Approximate - Non Linear

```
figure(2)


if(dControl_long(1) ~= 0)
    plot(time_V, res_dE(:, 2) + w0, '--', 'DisplayName', 'w (Full Linear)');  %
Full Linear Model
    hold on
    plot(time_V, APPres_dE(:, 2) + w0, '--', 'DisplayName', 'w (short Period
Approximation)');  % Approximate (short period Mode)
elseif(dControl_long(2) ~= 0)
    plot(time_V, res_dTh(:, 2) + w0, '--', 'DisplayName', 'w (Full Linear)');  %
Full Linear Model
    hold on
    plot(time_V, APPres_dTH(:, 2) + w0, '--', 'DisplayName', 'w (short Period
Approximation)');  % Approximate (short period Mode)
end

hold on
plot(time_V, w, '-', 'DisplayName', 'w (Non-Linear)');                      % Non-
Linear Model
title('w (ft/sec)'); xlabel('t (sec)');
legend('show');
grid on
```

q response Full Linear - Approximate - Non Linear

```
figure(3)


if(dControl_long(1) ~= 0)
    q_ = res_dE(:, 3) + q0;
    plot(time_V, q_*180/pi, '--', 'DisplayName', 'q (Full Linear)');  % Full Linear
Model
    hold on
    q_APP = APPres_dE(:, 3) + q0;
    plot(time_V, q_APP*180/pi, '--', 'DisplayName', 'q (short Period
Approximation)');  % Approximate (short period Mode)

elseif(dControl_long(2) ~= 0)
    q_ = res_dTh(:, 3) + q0;
    plot(time_V, q_*180/pi, '--', 'DisplayName', 'q (Full Linear)');  % Full Linear
Model
    hold on
    q_APP = APPres_dTH(:, 3) + q0;
    plot(time_V, q_APP*180/pi, '--', 'DisplayName', 'q (short Period
Approximation)');  % Approximate (short period Mode)
end

hold on
```

```
plot(time_V, q_deg , '-', 'DisplayName', 'q (Non-Linear)');          % Non-
Linear Model
title('q (deg/sec)'); xlabel('t (sec)');
legend('show');
grid on
```

theta response Full Linear - Approximate - Non Linear

```
figure(4)

if(dControl_long(1) ~= 0)
    theta_ = (res_dE(:, 4) + theta0)*180/pi;
    plot(time_V, theta_, '--', 'DisplayName', '\Theta (Full Linear)');  % Full
Linear Model
    hold on
    theta_APP = (APPres_dE(:, 4) + theta0)*180/pi;
    plot(time_V, theta_APP, '--', 'DisplayName', '\Theta (Long Period
Approximation)');  % Full Linear Model
elseif(dControl_long(2) ~= 0)
    theta_ = (res_dTh(:, 4) + theta0)*180/pi;
    plot(time_V, theta_, '--', 'DisplayName', '\Theta (Full Linear)');  % Full
Linear Model
    hold on
    theta_APP = (APPres_dTH(:, 4) + theta0)*180/pi;
    plot(time_V, theta_APP, '--', 'DisplayName', '\Theta (Long Period
Approximation)');  % Full Linear Model
end

hold on
plot(time_V, theta_deg, '-', 'DisplayName', '\Theta (Non-Linear)');
% Non-Linear Model
title('theta (deg/sec)'); xlabel('t (sec)');
legend('show');
grid on
```

Transfer Function codes

```
% the aerodynamic forces and moments can be expressed as a function of all the
motion variables [Nelson] page 63
clc
clear All
close all
%  Excel Sheets Data
% global aircraft_derivatives_dimensions
filename_density_L = 'NT-33A_4.xlsx'; %%put here the location of your excel sheet

aircraft_data=xlsread(filename_density_L,'B2:B61');%% here B2:B61 means read the
excel sheet from cell B2 to cell B61

%%in the next step we will read from the vector(aircraft_data) but take care of the
order the values in excel sheet is arranged
% initial conditions
s0 = aircraft_data(4:15);

% control actions values
% da = aircraft_data(57);
% dr = aircraft_data(58);
% de = aircraft_data(59);
% dth = aircraft_data(60);
dc = [ aircraft_data(57:59) * pi/180 ; aircraft_data(60)];
```

```
% gravity, mass % inertia
m = aircraft_data(51);
g = aircraft_data(52);
Ixx = aircraft_data(53);
Iyy = aircraft_data(54);
Izz = aircraft_data(55);
Ixz = aircraft_data(56);    Ixy=0;  Iyz=0;
I = [Ixx , -Ixy , -Ixz ;...
    -Ixy , Iyy , -Iyz ;...
    -Ixz , -Iyz , Izz];
invI=inv(I);

% stability derivatives Longitudinal motion
SD_Long = aircraft_data(21:36);
XU=SD_Long(1);
ZU=SD_Long(2);
MU=SD_Long(3);
XW=SD_Long(4);
ZW=SD_Long(5);
MW=SD_Long(6);
ZWD=SD_Long(7);
ZQ=SD_Long(8);
MWD=SD_Long(9);
MQ=SD_Long(10);
XDE=SD_Long(11);
ZDE=SD_Long(12);
MDE=SD_Long(13);
XDTH=SD_Long(14);
ZDTH=SD_Long(15);
MDTH=SD_Long(16);

U0=s0(1); W0=s0(3); TH0=s0(8);
% stability derivatives Lateral motion
SD_Lat_dash = aircraft_data(37:50);

% initial gravity force
mg0 = m*g * [ sin(s0(8)) ; -cos(s0(8))*sin(s0(7)) ; -cos(s0(8))*cos(s0(7)) ];
```

linearized set of Longitudinal equations

```
A_longt=[XU XW -W0 -g*cos(TH0)

    ZU/(1-ZWD) ZW/(1-ZWD) (ZQ+U0)/(1-ZWD) -g*sin(TH0)/(1-ZWD)

    MU+MWD*ZU/(1-ZWD) MW+MWD*ZW/(1-ZWD) MQ+MWD*(ZQ+U0)/(1-ZWD) -MWD*g*sin(TH0)/(1-
ZWD)

    0 0 1 0];

B_longt=[XDE XDTH

    ZDE/(1-ZWD) ZDTH/(1-ZWD)

    MDE+MWD*ZDE/(1-ZWD) MDTH+MWD*ZDTH/(1-ZWD)

    0 0];

C_longt=eye(4); D_longt=zeros(4,2);
```

```
LONGT_SS=ss(A_longt,B_longt,C_longt,D_longt);

LONGT_TF=tf(LONGT_SS);



U_DE_F=LONGT_TF(1,1);

U_DTH_F=LONGT_TF(1,2);

W_DE_F=LONGT_TF(2,1);

W_DTH_F=LONGT_TF(2,2);

Q_DE_F=LONGT_TF(3,1);

Q_DTH_F=LONGT_TF(3,2);

THETA_DE_F=LONGT_TF(4,1);

THETA_DTH_F=LONGT_TF(4,2);
```

PHUGOID MODE (LONG PERIOD MODE)

```
A_phug=[XU -g*cos(TH0)

    ZU/(U0+ZQ) g*sin(TH0)];

B_phug=[XDE XDTH

    ZDE/(ZQ+U0) ZDTH/(ZQ+U0)];

C_phug=eye(2);D_phug=zeros(2,2);



PHUG_SS=ss(A_phug,B_phug,C_phug,D_phug);

PHUG_TF=tf(PHUG_SS);



U_DE_PH=PHUG_TF(1,1);

U_DTH_PH=PHUG_TF(1,2);

THETA_DE_PH=PHUG_TF(2,1);

THETA_DTH_PH=PHUG_TF(2,2);
```

SHORT PERIOD MODE

```
A_short=[ZW/(1-ZWD) (ZQ+U0)/(1-ZWD)

    (MW+ZW*MWD/(1-ZWD)) (MQ+MWD*(ZQ+U0)/(1-ZWD))];

B_short=[ZDE/(1-ZWD) ZDTH

    MDE+MWD*ZDE/(1-ZWD) MDTH+MWD*ZDTH/(1-ZWD)];

C_short=eye(2);D_short=zeros(2,2);



SHORT_SS=ss(A_short,B_short,C_short,D_short);

SHORT_TF=tf(SHORT_SS);



W_DE_PH=SHORT_TF(1,1);

W_DTH_PH=SHORT_TF(1,2);

Q_DE_PH=SHORT_TF(2,1);

Q_DTH_PH=SHORT_TF(2,2);
```

root locus and bode plots

full linear model

```
%u/de
figure()
subplot(1, 2, 1);
rlocus(LONGT_TF(1,1))
title('LINEAR FULL MODLE root locus (u/\delta_{e}) ')
subplot(1, 2, 2);
bode(LONGT_TF(1,1))
title('LINEAR FULL MODLE bode plot (u/\delta_{e}) ')
grid on

%u/d_th
figure()
subplot(1, 2, 1);
rlocus(LONGT_TF(1,2))
title('LINEAR FULL MODLE root locus (u/\delta_{th}) ')
subplot(1, 2, 2);
bode(LONGT_TF(1,2))
title('LINEAR FULL MODLE bode plot (u/\delta_{th}) ')
grid on

%w/de
figure()
subplot(1, 2, 1);
rlocus(LONGT_TF(2,1))
title('LINEAR FULL MODLE root locus (w/\delta_{e}) ')
subplot(1, 2, 2);
bode(LONGT_TF(2,1))
```

```
title('LINEAR FULL MODLE bode plot (w/\delta_{e}) ')
grid on

%w/d_th
figure()
subplot(1, 2, 1);
rlocus(LONGT_TF(2,2))
title('LINEAR FULL MODLE root locus (w/\delta_{th}) ')
subplot(1, 2, 2);
bode(LONGT_TF(2,2))
title('LINEAR FULL MODLE bode plot (w/\delta_{th}) ')
grid on

%q/de
figure()
subplot(1, 2, 1);
rlocus(LONGT_TF(3,1))
title('LINEAR FULL MODLE root locus (q/\delta_{e}) ')
subplot(1, 2, 2);
bode(LONGT_TF(3,1))
title('LINEAR FULL MODLE bode plot (q/\delta_{e}) ')
grid on

%q/d_th
figure()
subplot(1, 2, 1);
rlocus(LONGT_TF(3,2))
title('LINEAR FULL MODLE root locus (q/\delta_{th}) ')
subplot(1, 2, 2);
bode(LONGT_TF(3,2))
title('LINEAR FULL MODLE bode plot (q/\delta_{th}) ')
grid on

%theta/de
figure()
subplot(1, 2, 1);
rlocus(LONGT_TF(4,1))
title('LINEAR FULL MODLE root locus (\theta/\delta_{e}) ')
subplot(1, 2, 2);
bode(LONGT_TF(4,1))
title('LINEAR FULL MODLE bode plot (\theta/\delta_{e}) ')
grid on

%theta/d_th
figure()
subplot(1, 2, 1);
rlocus(LONGT_TF(4,2))
title('LINEAR FULL MODLE root locus (\theta/\delta_{th}) ')
subplot(1, 2, 2);
bode(LONGT_TF(4,2))
title('LINEAR FULL MODLE bode plot (\theta/\delta_{th}) ')
grid on
```