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AER408
Aerospace Guidance & Control Systems

Task (3)
Airplane Simulator Part II

Before you go, please note the following:

- You do not have to use the equations presented in this document, consult the reference you prefer, equations presented here are for illustration
- All angles are in radians through this course !!

Task statement:

a) "*Write a code that calculates the (Aerodynamic & Thrust) (Forces & Moments) acting on an Airplane due to pilots input signals ($\delta_{aileron}$, δ_{rudder}, $\delta_{elevator}$, δ_{thrust}) knowing its stability & control derivatives at nominal flight condition*"

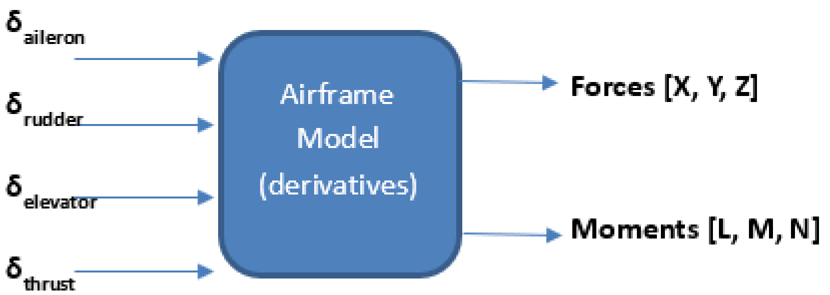
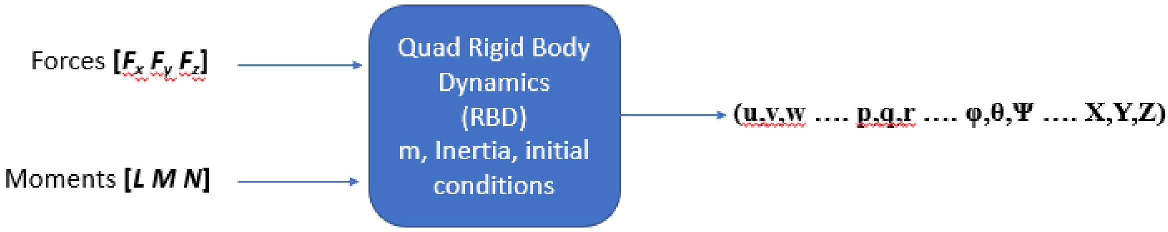


Figure1. Airframe Model

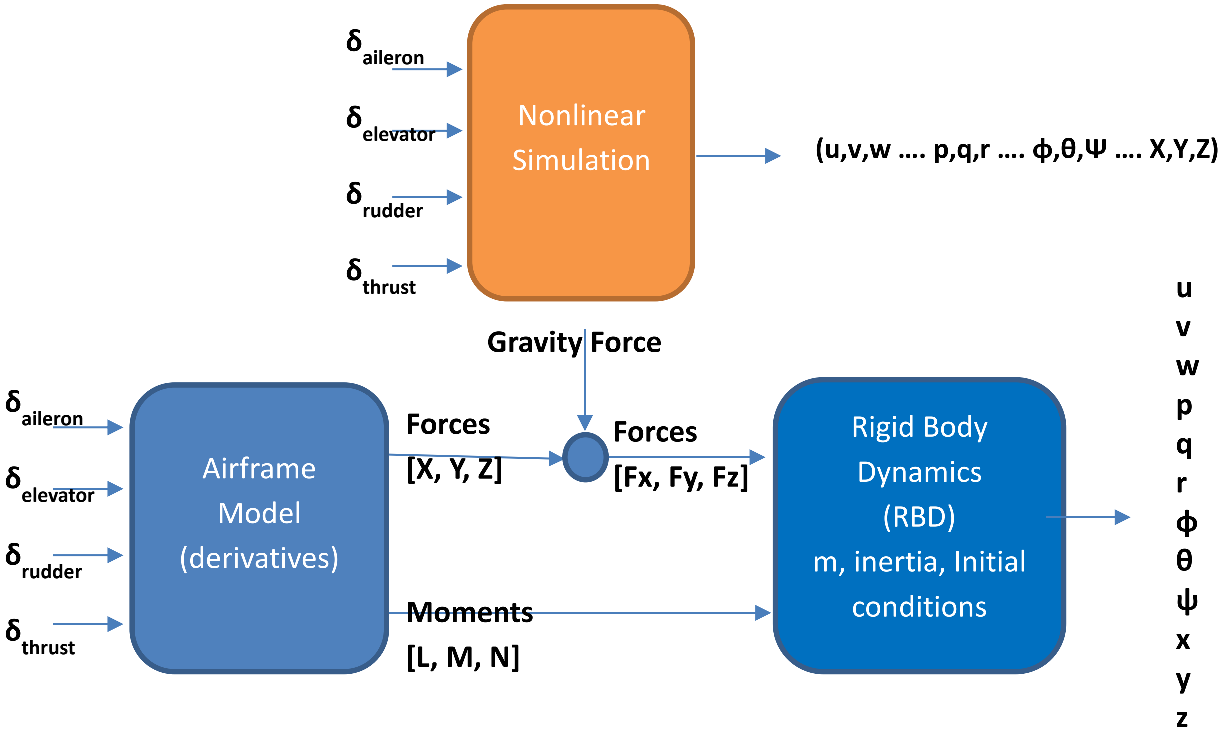
Consult the following document (NASA CR-2144 AIRCRAFT HANDLING QUALITIES DATA) as a reference for the Airplanes' parameters including the (Stability derivatives, Mass, Inertias) at the Reference Flight conditions. Each team will be informed by their corresponding Airplane and Flight condition

b) "*Combine the (Airframe Model) code with the (RBD solver) you built in the previous task in order to build the complete (Airplane Non-linear Flight Simulator)*". The idea is to use the (Pilots inputs) to calculate the Aerodynamic & Thrust (Forces &

Moments) acting on the airplane, and then use these calculated Forces & Moments to solve the (RBD) equations to calculate the new states of he airplane in the next time step, and repeat this procedure at each time step



Note: do not forget to add the (Gravitational Forces) to the (Aerodynamic & Thrust Forces) before using them in the (RBD solver)



Airframe Model + **RBD** = **Non-linear Simulator**

Notes & Hints

Note 1:

The following set of equations represents the **change** in the Aerodynamic & thrust forces & moments, and they are a set of Linear equations function of:

- The stability derivatives
- The perturbation change in the states and the control surfaces deflections from their values at the trim condition

$$\Delta X = \frac{\partial X}{\partial u}\Delta u + \frac{\partial X}{\partial w}\Delta w + \frac{\partial X}{\partial \delta_e}\Delta \delta_e + \frac{\partial X}{\partial \delta_T}\Delta \delta_T$$

$$\Delta Y = \frac{\partial Y}{\partial v}\Delta v + \frac{\partial Y}{\partial p}\Delta p + \frac{\partial Y}{\partial r}\Delta r + \frac{\partial Y}{\partial \delta_r}\Delta \delta_r$$

$$\Delta Z = \frac{\partial Z}{\partial u}\Delta u + \frac{\partial Z}{\partial w}\Delta w + \frac{\partial Z}{\partial \dot{w}}\Delta \dot{w} + \frac{\partial Z}{\partial q}\Delta q + \frac{\partial Z}{\partial \delta_e}\Delta \delta_e + \frac{\partial Z}{\partial \delta_T}\Delta \delta_T$$

$$\Delta L = \frac{\partial L}{\partial v}\Delta v + \frac{\partial L}{\partial p}\Delta p + \frac{\partial L}{\partial r}\Delta r + \frac{\partial L}{\partial \delta_r}\Delta \delta_r + \frac{\partial L}{\partial \delta_a}\Delta \delta_a$$

$$\Delta M = \frac{\partial M}{\partial u}\Delta u + \frac{\partial M}{\partial w}\Delta w + \frac{\partial M}{\partial \dot{w}}\Delta \dot{w} + \frac{\partial M}{\partial q}\Delta q + \frac{\partial M}{\partial \delta_e}\Delta \delta_e + \frac{\partial M}{\partial \delta_T}\Delta \delta_T$$

$$\Delta N = \frac{\partial N}{\partial v}\Delta v + \frac{\partial N}{\partial p}\Delta p + \frac{\partial N}{\partial r}\Delta r + \frac{\partial N}{\partial \delta_r}\Delta \delta_r + \frac{\partial N}{\partial \delta_a}\Delta \delta_a$$

Note: ΔX ΔY ΔZ ΔL ΔM ΔN are the **change** in the forces & moments, i.e. these are not the absolute values of the forces and moments. They should be added to the reference values at the trim condition X_o,Y_o,Z_o,L_o,M_o,N_o to calculate the absolute values X,Y,Z,L,M,N

Similarly, Δu, Δv, Δw,… are the **change** in the states values from their values at the reference condition Δu=u-uo, Δv=v-vo, Δw=w-wo, …..

- inputs and outputs of the (Airframe Model) are perturbations from the reference values
- inputs and outputs of the (RBD are absolute values

So that: ***"Values perturbation values resulting from the (Airframe Model) should be added to the reference values before passing them to the (RBD) and the absolute values resulting from the (RBD) should be converted to perturbation values by subtracting the reference values from them"***

The total forces acting on an airplane are the (Aerodynamic & Thrust forces +Gravity forces)

$$X - mg\sin\theta = m(\dot{u}^E + qw^E - rv^E)$$
$$Y + mg\cos\theta\sin\phi = m(\dot{v}^E + ru^E - pw^E)$$
$$Z + mg\cos\theta\cos\phi = m(\dot{w}^E + pv^E - qu^E)$$

And initially at the reference flight condition the airplane is in an equilibrium state
Equilibrium means: $\sum$ Forces = 0 & $\sum$ Moments = 0
Which means

$$X_0 - mg\ sin\theta_0 = 0\ \rightarrow\ X_0 = mg\ sin\theta_0$$
$$Y_0 - mg\ cos\theta_0\ sin\emptyset_0 = 0 \rightarrow Y_0 = -\ mg\ cos\theta_0\ sin\emptyset_0$$
$$Z_0 - mg\ cos\theta_0\ cos\emptyset_0 = 0 \rightarrow Z_0 = -\ mg\ cos\theta_0\ cos\emptyset_0$$

$$\therefore X = X_0 + \Delta X = \Delta X + mg\ sin\theta_0$$
$$Y = Y_0 + \Delta Y = \Delta Y - mg\ cos\theta_0 sin\emptyset_0$$
$$Z = Z_0 + \Delta Z = \Delta Z - mg\ cos\theta_0 cos\emptyset_0$$

And the total force acting on the airplane (this value is the input which you will give to the RBD)

$$F_x = X - mg\ sin\ \theta = \Delta X + mg\ sin\theta_0 - mgsin\ \theta$$
$$F_y = Y + mg\ cos\theta\ sin\emptyset = \Delta Y - mg\ cos\theta_0\ sin\ \emptyset_0 + mg\ cos\theta\ sin\emptyset$$
$$F_z = Z + mg\ cos\ \theta\ cos\emptyset = \Delta Z - mg\ cos\theta_0\ cos\ \emptyset_0 + mg\ cos\theta\ sin\emptyset$$

Where (θo & Φo) are the pith and roll angles at the reference condition, and (θ & Φ) are their values at any time instant

Note 2:

Consult “Dynamics of Flight, Bernard Etkin” pages 101-103 to review the concept of the **Body axes** of the airplane and its types (principle axes, stability axes, body axes). You should note that the *stability derivatives & Inertias of an airplane have different values and symbols according to the type of the body axes they are represented in.*

Very important: Study the symbols and definitions stated in (NASA CR-2144) **appendices A&B**, then use the tables of the derivatives represented in the **(Body axes)** to extract the derivatives according to your flight condition

LONGITUDINAL DIMENSIONAL DERIVATIVES

(BODY AXIS SYSTEM)

F/C #	1	2	3	4	5	6	7	8	9	10
H	SL	SL	SL	SL	20 K	20 K	20 K	40 K	40 K	40 K
M	.198	.249	.450	.650	.500	.650	.800	.700	.800	.900
XU *	-.0209	-.0108	-.00499	-.00777	-.00247	-.00280	-.00643	.00187	-.00276	-.0200
ZU *	-.202	-.150	-.0807	-.126	-.0679	-.0832	-.0941	-.0696	-.0650	-.0424
MU *	.000117	.000181	.000146	-.000199	.000247	.885E-4	-.000222	.000259	.000193	-.623E-4
XW	.122	.106	.0743	.0345	.0782	.0482	.0253	.0263	.0389	.0159
ZW	-.512	-.613	-.736	-.963	-.433	-.539	-.624	-.292	-.317	-.401
MW	-.00177	-.00193	-.00262	-.00239	-.00170	-.00190	-.00153	-.00101	-.00105	-.00190
ZWD	.0334	.0338	.0257	.0293	.0157	.0156	.0144	.00704	.00656	.00614
ZQ	-6.22	-7.58	-10.4	-12.8	-6.39	-8.09	-9.98	-4.32	-5.16	-6.71
MWD	-.000246	-.000240	-.000221	-.000228	-.000125	-.000155	-.000212	-.905E-4	-.000116	-.000160
MQ	-.357	-.437	-.699	-.925	-.421	-.535	-.659	-.284	-.339	-.401
XDE	.959	.971	1.18	0.	2.02	1.15	0.	1.93	1.44	.781
ZDE	-6.42	-9.73	-21.8	-32.4	-16.9	-26.4	-32.7	-15.1	-17.9	-18.6
MDE	-.378	-.574	-1.40	-2.07	-1.09	-1.69	-2.09	-.970	-1.16	-1.22
XDTH	.570E-4	.570E-4	.505E-4	.505E-4	.505E-4	.505E-4	.505E-4	.505E-4	.505E-4	.505E-4
ZDTH	-.249E-5	-.249E-5	-.220E-5	-.220E-5	-.220E-5	-.220E-5	-.220E-5	-.220E-5	-.220E-5	-.220E-5
MDTH	.310E-6	.310E-6	.302E-6	.302E-6	.302E-6	.302E-6	.302E-6	.302E-6	.302E-6	.302E-6

LATERAL-DIRECTIONAL DIMENSIONAL DERIVATIVES

(BODY AXIS SYSTEM)

F/C #	1	2	3	4	5	6	7	8	9	10
H	SL	SL	SL	SL	20 K	20 K	20 K	40 K	40 K	40 K
M	.198	.249	.450	.650	.500	.650	.800	.700	.800	.900
YV	-.0890	-.0997	-.143	-.197	-.0822	-.104	-.120	-.0488	-.0558	-.0606
YB	-19.7	-27.8	-71.7	-143.	-42.6	-70.4	-99.4	-33.1	-43.2	-52.8
LB'	-1.33	-1.63	-3.19	-5.45	-2.05	-2.96	-4.12	-1.45	-3.05	-1.32
NB'	.168	.247	.810	1.82	.419	.923	1.62	.404	.598	.971
LP'	-.975	-1.10	-1.12	-1.47	-.652	-.804	-.974	-.404	-.465	-.459
NP'	-.166	-.125	-.0706	-.0214	-.0701	-.0531	-.0157	-.0366	-.0316	.00284
LR'	.327	.198	.379	.256	.376	.317	.292	.312	.388	.280
NR'	-.217	-.229	-.246	-.344	-.140	-.193	-.232	-.0963	-.115	-.141
Y*CA	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
L'CA	.227	.318	.229	.372	.128	.210	.310	.0964	.143	.186
N'CA	.0264	.0300	.0285	.0371	.0177	.0199	.0127	.00875	.00775	-.00611
Y*CR	.0148	.0182	.0226	.0213	.0131	.0142	.0124	.00777	.00729	.00464
L'CR	.0636	.110	.254	.318	.148	.211	.183	.115	.153	.100
N'CR	-.151	-.233	-.614	-.970	-.391	-.616	-.922	-.331	-.475	-.442

Note 3:

The Lateral-Directional derivatives given in the table are **(dashed)**, these are not the values need to be used in the (forces & moments equations), check the **appendix B** in the report to find the relation between the (dashed & undashed derivatives) to calculate the (undashed ones)

$$L'_\beta = (L_\beta + I_{xz}N_\beta/I_x)G \qquad 1/\text{sec}^2$$

$$L'_p = (L_p + I_{xz}N_p/I_x)G \qquad 1/\text{sec}$$

$$L'_r = (L_r + I_{xz}N_r/I_x)G \qquad 1/\text{sec}$$

$$L_{\delta_r}' = (L_{\delta_r} + I_{xz}N_{\delta_r}/I_x)G \qquad 1/\text{sec}^2$$

$$L_{\delta_a}' = (L_{\delta_a} + I_{xz}N_{\delta_a}/I_x)G \qquad 1/\text{sec}^2$$

$$N'_\beta = (N_\beta + I_{xz}L_\beta/I_z)G \qquad 1/\text{sec}^2$$

$$N'_p = (N_p + I_{xz}L_p/I_z)G \qquad 1/\text{sec}$$

$$N'_r = (N_r + I_{xz}L_r/I_z)G \qquad 1/\text{sec}$$

$$N_{\delta_r}' = (N_{\delta_r} + I_{xz}L_{\delta_r}/I_z)G \qquad 1/\text{sec}^2$$

$$N_{\delta_a}' = (N_{\delta_a} + I_{xz}L_{\delta_a}/I_z)G \qquad 1/\text{sec}^2$$

$$G = \frac{1}{1 - \frac{I_{xz}^2}{I_x I_z}}$$

Note 4:

Please do not put the value of $\Theta_0 = 0$,as the pitch angle is the summation of the angle of attack and the climb angle or flight path angle

$$\theta = \alpha + \gamma$$

$$\therefore \theta_0 = \alpha_0 + \gamma_0$$

You will find the values of α_0 & $\gamma_0$in the tables in NASA report in the flight condition table. Also, the initial values of the velocities (u_0, v_0, w_0) should be calculated from the value of the total speed along with the angle of attack and the side slip angle

In the excel sheet these values are calculated and provided to you ready to be used

F/C #

H(FT)

M(-)

VTO(FPS)

VTO(KTAS)

VTO(KCAS)

W(LBS)

C.G.(MGC)

IX (SLUG-FT SQ)

IY (SLUG-FT SQ)

IZ (SLUG-FT SQ)

IXZ(SLUG-FT SQ)

EPSI(LCN(DEG)

Q(PSF)

QC(PSF)

ALPHA(DEG)

GAMMA(DEG)

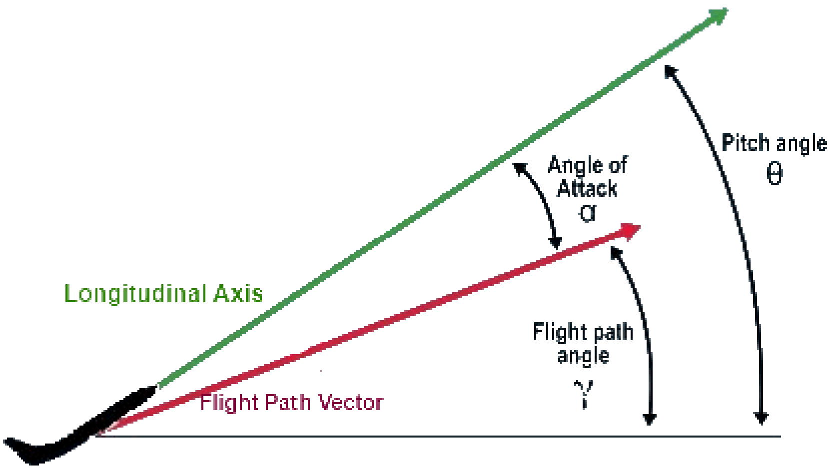
LXP(FT)

LZP(FT)

ITH(DEG)

XI(DEG)

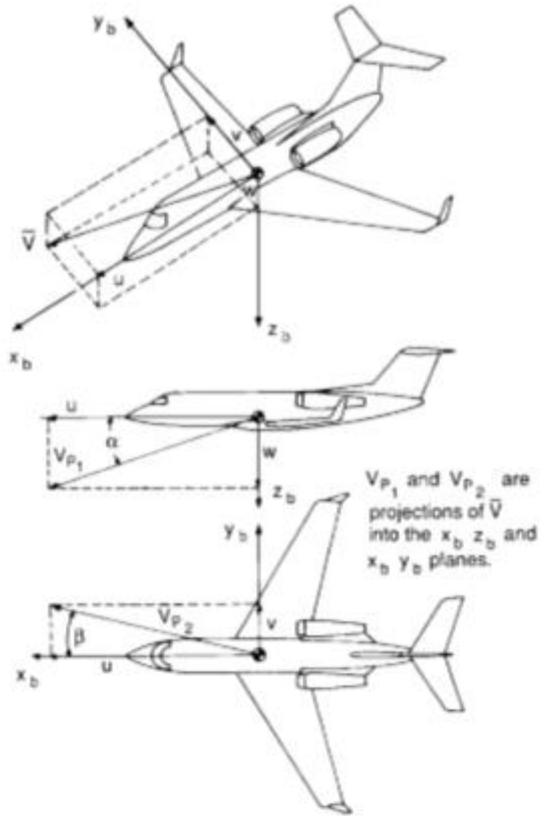
LTH(FT)



$$\alpha = \tan^{-1}\frac{w}{u}$$

and

$$\beta = \sin^{-1}\frac{v}{V}$$



Note 5:

For the sake of validation, you should test your simulator against the “Benchmark test” in which certain maneuvers of the (Boeing 747 / flight condition 5) are performed, you are asked to run your simulator with the same data and publish the results in the same way as presented in the Benchmark test document

The following lines of code will make the plotting for you, copy them in your code

```
figure
plot3(x,-y,-z);
title('Trajectory')

figure
subplot(4,3,1)
plot(time_V,u)
title('u (ft/sec)')
xlabel('time (sec)')

subplot(4,3,2)
plot(time_V,beta_deg)
title('\beta (deg)')
xlabel('time (sec)')
```

```
subplot(4,3,3)
plot(time_V,alpha_deg)
title('\alpha (deg)')
xlabel('time (sec)')

subplot(4,3,4)
plot(time_V,p_deg)
title('p (deg/sec)')
xlabel('time (sec)')

subplot(4,3,5)
plot(time_V,q_deg)
title('q (deg/sec)')
xlabel('time (sec)')

subplot(4,3,6)
plot(time_V,r_deg)
title('r (deg/sec)')
xlabel('time (sec)')

subplot(4,3,7)
plot(time_V,phi_deg)
title('\phi (deg)')
xlabel('time (sec)')

subplot(4,3,8)
plot(time_V,theta_deg)
title('\theta (deg)')
xlabel('time (sec)')

subplot(4,3,9)
plot(time_V,psi_deg)
title('\psi (deg)')
xlabel('time (sec)')

subplot(4,3,10)
plot(time_V,P(1,:))
title('x (ft)')
xlabel('time (sec)')

subplot(4,3,11)
plot(time_V,P(2,:))
title('y (ft)')
xlabel('time (sec)')

subplot(4,3,12)
plot(time_V,P(3,:))
title('z (ft)')
xlabel('time (sec)')
```