► ***Flow past Joukowski airfoil:***

Air data: *free stream velocity "V_∞" and angle of attack "α"*

Airfoil geometric characteristics: *chord "c", maximum thickness "t_{max}", maximum camber "C_{max}"*

Circle parameters: $b = c/4$, $e = (t_{max}/c)/1.3$, $\beta = 2*(C_{max}/c)$, $a = b(1+e)/\cos\beta$, $x_0 = -be$, $y_0 = a\beta$

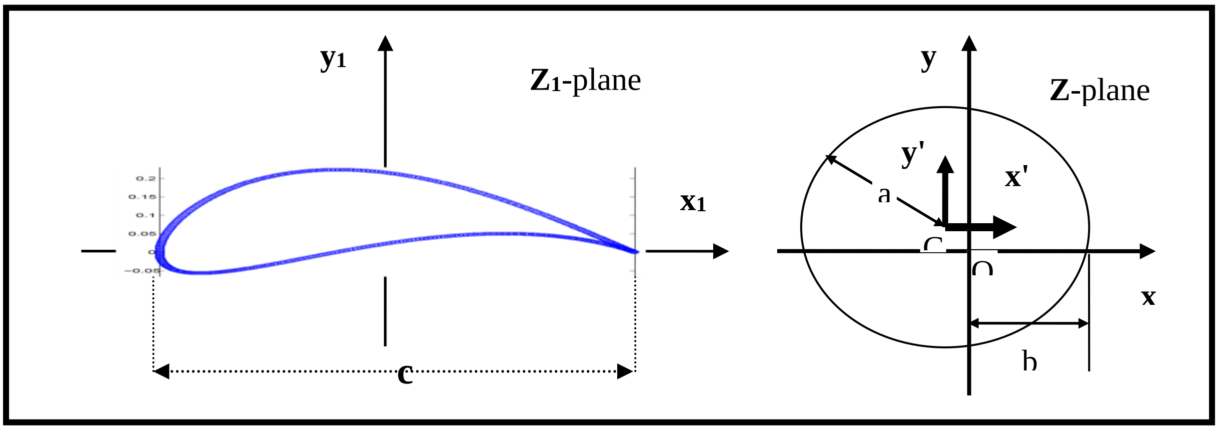
Airfoil coordinates: $x_1 = 2\,b\cos\theta$, $y_1 = 2\,b\,e\,(1-\cos\theta)\sin\theta + 2\,b\,\beta\sin^2\theta$

with $r = b\,[1 + e\,(1-\cos\theta) + \beta\sin\theta]$

In the Z' plane(*where the center of the circle of radius "a" is at the origin*)

$$Z' = x' + iy' = r'\,e^{i\theta'} = r'\cos\theta' + i\,r'\sin\theta'$$

In the Z plane: $Z = x + iy = r\,e^{i\theta} = r\cos\theta + i\,r\sin\theta$



Relation between Z, Z' and Z_1 : $Z = z_0 + Z'$ where $z_0 = x_0 + i\,y_0$ & $\mathbf{Z_1 = Z + b^2 / Z}$

where $x_0 = -\,be$, $y_0 = a\,\beta$

The velocity components in Z' plan are :-

$$v'_r = V_\infty\,(1 - a^2/r'^2)\cos(\theta'-\alpha) \quad \& \quad v'_\theta = -V_\infty\,[\sin(\theta'-\alpha)\left(1 + a^2/r'^2\right) + 2(a/r')\sin(\alpha+\beta)]$$

The velocity components in Z_1 plan are :-

$$\frac{dW}{dZ_1} = \frac{(A+iB)}{(C+iD)} = u_1 + iv_1$$

where

$$A = (v'_r\cos\theta' - v'_\theta\sin\theta') \qquad \& \qquad B = -(v'_r\sin\theta' + v'_\theta\cos\theta')$$

$$C = 1 - \frac{b^2}{r^2}\cos(2\theta) \qquad \& \qquad D = \frac{b^2}{r^2}\sin(2\theta)$$

The velocity "V_1" at any point in the Z_1 plan:-

$$V_1^2 = u_1^2 + v_1^2 = \frac{A^2 + B^2}{C^2 + D^2}$$

The velocity "V_1" on the surface of the airfoil:-

$$V_1^2 = \frac{v'^2_\theta}{1 - 2\dfrac{b^2}{r^2}\cos(2\theta) + \dfrac{b^4}{r^4}}$$

The pressure $p + \tfrac{1}{2}\rho V^2 = p_\infty + \tfrac{1}{2}\rho V_\infty^2$ *and* $c_p = 1 - \left(\dfrac{V}{V_\infty}\right)^2$

The lift "L" and the lift coefficient "C_L"

$$L = \rho V_\infty \Gamma = 4\pi\rho V_\infty^2\,a\sin(\alpha+\beta) \qquad \& \qquad C_L = 2\pi(1+e)\sin(\alpha+\beta)$$