Individual 6

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Introduction to Proof and Problem Solving

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Problem 12. Show the function f mapping $\mathbb{Z}$ into $S = \mathbb{Z}$ is one-to-one or find two integers n_1 and n_2 such that $n_1 \neq n_2$ but $f(n_1) = f(n_2)$, where

$$f(n) = \begin{cases} 0.5n + 3 & \text{if } n \in E \\ 3n - 1 & \text{if } n \in O \end{cases}$$

Proof. Suppose this is true. Set $n_1 = -2$ and $n_2 = 1$. Since n_1 is even,

$$f(n_1) = 0.5n_1 + 3$$
$$f(n_1) = 2.$$

Since n_2 is odd,

$$f(n_2) = 3n_2 - 1$$
$$f(n_2) = 2.$$

We can observe that $f(n_1) = f(n_2)$. Additionally, since $n_1 \neq n_2$, this function is not one-to-one.

$\square$

While working on this proof, I received no external assistance aside from advice from Professor Mehmetaj.