

Individual 4

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Introduction to Proof and Problem Solving

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Problem 12. (1) Consider the statement

$$p \equiv \{\forall M \in \mathbb{R}, \exists K \in \mathbb{R} \text{ s.t. } \forall x > K, f(x) > M\}.$$

(a) Write $\neg p$.

(b) Consider the function f from \mathbb{R} into \mathbb{R} defined by

$$f(x) = \begin{cases} \frac{1}{x} & x \neq 0 \\ 0 & x = 0 \end{cases}.$$

Does f satisfy p or $\neg p$? Prove your answer.

Solution. (a) The negation of p is

$$\neg p \equiv \{\exists M \in \mathbb{R} \text{ such that } \forall K \in \mathbb{R}, \exists x > K \text{ such that } f(x) \leq M\}$$

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Proof. We will prove that f satisfies $\neg p$. Set $M_0 = 2$. Let K_0 be any real number. Set $x_0 = |K_0| + 3$. Since $|K_0| \geq 0$, we know that $|K_0| + 2 > 0$. Adding 1 to both sides, we get $|K_0| + 3 > 1$, or $x_0 > 1$.

We know that $2 > 1$. Since $x_0 > 1$, multiplying the left hand side of the inequality by x_0 gives us $2x_0 > 1$. Diving both sides by x_0 , we get

$$\begin{aligned} 2 &> \frac{1}{x_0} \\ M_0 &> \frac{1}{x_0}. \\ M_0 &\geq \frac{1}{x_0}. \end{aligned}$$

Thus, the negation of p is true.

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While working on this proof, I received no external assistance aside from advice from Professor Mehmetaj.