Individual 8

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Introduction to Proof and Problem Solving

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Problem 1. Consider the function

$$f(x) = \begin{cases} x - 2 & x \leq 4 \\ \frac{3}{2}x - 2 & x > 4 \end{cases}.$$

Show that $\lim_{x \to 4} f(x)$ does not exist.

Proof. We will prove $\lim_{x \to 4} f(x)$ does not exist using a proof by contradiction. Suppose that the limit did exist, that $\lim_{x \to 4} f(x) = L_0$ for some L_0. Then for every $\epsilon > 0$, there exists a real number $\delta > 0$ such that for all real numbers $x \in \mathbb{R}$ that satisfy the condition

$$0 < |x - 4| < \delta,$$

the following condition is also satisfied:

$$f(x) - L_0 < \epsilon.$$

□

While working on this proof, I received no external assistance aside from advice from Professor Mehmetaj.