

# Individual 6

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Introduction to Proof and Problem Solving

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**Problem 12.** Show the function  $f$  mapping  $\mathbb{Z}$  into  $S = \mathbb{Z}$  is one-to-one or find two integers  $n_1$  and  $n_2$  such that  $n_1 \neq n_2$  but  $f(n_1) = f(n_2)$ , where

$$f(n) = \begin{cases} 0.5n + 3 & \text{if } n \in E \\ 3n - 1 & \text{if } n \in O \end{cases}$$

*Proof.* Suppose this is true. Set  $n_1 = -2$  and  $n_2 = 1$ . Since  $n_1$  is even,

$$\begin{aligned} f(n_1) &= 0.5n_1 + 3 \\ f(n_1) &= 2. \end{aligned}$$

Since  $n_2$  is odd

$$\begin{aligned} f(n_2) &= 3n_2 - 1 \\ f(n_2) &= 2 \end{aligned}$$

We can observe that  $f(n_1) = f(n_2)$ . Additionally, since  $n_1 \neq n_2$ , this function is not one-to-one.

□

While working on this proof, I received no external assistance aside from advice from Professor Mehmetaj.