

# Individual 6

Yu Fan Mei

Introduction to Proof and Problem Solving

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**Problem 12.** Show the function  $f$  mapping  $\mathbb{Z}$  into  $S = \mathbb{Z}$  is one-to-one or find two integers  $n_1$  and  $n_2$  such that  $n_1 \neq n_2$  but  $f(n_1) = f(n_2)$ , where

$$f(n) = \begin{cases} 0.5n + 3 & \text{if } n \in E \\ 3n - 1 & \text{if } n \in O \end{cases}$$

*Proof.* We will prove that the function is not one-to-one. Suppose the function is one-to-one. Then for every real number  $n_1$  and  $n_2$ ,  $f(n_1) \neq f(n_2)$ . Set  $n_1 = -2$  and  $n_2 = 1$ . Since  $n_1$  is even,

$$f(n_1) = 0.5n_1 + 3.$$

Plugging in -2 for  $n_1$ ,  $f(n_1) = 2$ . Let's examine  $f(n_2)$  now. Since  $n_2$  is odd,

$$f(n_2) = 3n_2 - 1.$$

Plugging in 1 for  $n_2$ ,  $f(n_2) = 2$ . We can observe that  $f(n_1) = f(n_2)$ . Additionally, since  $n_1 \neq n_2$ , this function is not one-to-one.

□

While working on this proof, I received no external assistance aside from advice from Professor Mehmetaj.