

Individual XXXX

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Introduction to Proof and Problem Solving

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Problem 1. 1. Consider the function $f : \mathbb{R} \rightarrow \mathbb{R}$ defined by

$$f(x) = \begin{cases} \frac{1}{1-x} & x \neq 1 \\ 0 & x = 1 \end{cases}.$$

Is f continuous at 1? Prove your claim.

Proof. We will prove that $f(x)$ is not continuous at $x = 1$ by using contradiction to show that the limit does not exist. Suppose the limit did exist, that there existed a number L_0 such that $\lim_{x \rightarrow 1} f(x) = L_0$. Then for every $\epsilon > 0$, there exists a real number $\delta > 0$ such that for all real numbers $x \in \mathbb{R}$ that satisfy the condition

$$0 < |x - 1| < \delta,$$

the following condition is also satisfied:

$$f(x) - L_0 < \epsilon.$$

Set $\epsilon_0 = b$. Let δ_0 be any real number greater than 0. Set x_1

□

While working on this proof, I received no external assistance aside from advice from Professor Mehmetaj.