Individual 9

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Introduction to Proof and Problem Solving

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Problem 1. Show that

$$C((0,1]) = C((0,1))$$

by showing that the function

$$f(x) = \begin{cases} x & \text{if } x \neq \frac{1}{n} \text{ for any } n \in \mathbb{Z}^+ \\ \frac{1}{n+1} & \text{if } x = \frac{1}{n} \text{ for some } n \in \mathbb{Z}^+ \end{cases}$$

is one-to-one from $(0,1]$ onto $(0,1)$. It might help to graph the function.

Proof. In order to prove these two sets have the same cardinality, we will prove that the function $f : (0,1] \to (0,1)$ is one-to-one and onto. Let x_0 be any real number such that $x_0 \in (0,1]$

□

While working on this proof, I received no external assistance aside from advice from Professor Mehmetaj.