Individual 3

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Introduction to Proof and Problem Solving

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Problem 18. Show that the function f mapping $S = \mathbb{R}$ into $T = \mathbb{R}$ is onto T, where

$$f(x) = \begin{cases} x + 3 & x \notin \mathbb{Z} \\ x - 2 & x \in \mathbb{Z} \end{cases}$$

Proof. Let y_0 be an arbitrary real number. We consider two cases.
Case 1: Suppose y_0 is an integer. Set $x_0 = y_0 + 2$. Since integers are closed under addition, x_0 is an integer. Subtracting 2 from both sides, we get

$$x_0 - 2 = y_0.$$

Case 2: Suppose y_0 is not an integer. We know integers are closed under addition. Thus, if y_0 is not an integer, $y_0 - 3$ cannot be an integer. Set $x_0 = y_0 - 3$. Adding 3 to both sides, we get

$$x_0 + 3 = y_0.$$

Since $x_0 \in \mathbb{Z}$ in case 1 and $x_0 \in \mathbb{R} - \mathbb{Z}$ in case 2, x_0 can be any real number and thus $x \in S$. Thus, we have proven $f : S \to T$ is onto T.

□

While working on this proof, I had no external assistance aside from advice from Professor Mehmetaj.