Individual 8

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Introduction to Proof and Problem Solving

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Problem 1. Consider the function

$$f(x) = \begin{cases} x - 2 & x \leq 4 \\ \frac{3}{2}x - 2 & x > 4 \end{cases}.$$

Show that $\lim_{x \to 4} f(x)$ does not exist.

Proof. We will prove $\lim_{x \to 4} f(x) \neq$ using cases. Set $\epsilon_0 = 2$. Let δ_0 be an arbitrary real number greater than 0.

We will consider cases:

Case 1: $L_0 < 3$. Set $x = x_0 = 4 + 2\delta/3$. Then, $|x_0 - 4|$ clearly is greater than 0 and less than δ_0:

$$0 < |x_0 - 4| = |4 + \frac{2\delta_0}{3} - 4| = |\frac{2\delta_0}{3}| = \frac{2\delta_0}{3}.$$

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Since $x_0 > 4$,

$$\begin{aligned} |f(x_0) - L_0| &= |\frac{3}{2}x_0 - 2 - L_0| \\ &= |\frac{3}{2}(4 + \frac{2\delta_0}{3}) - 2 - L_0| \\ &= |4 + \delta_0 - L_0|. \end{aligned}$$

Since $L_0 < 3$, we know that $-L_0 > 3$. Adding 4 to both sides, we can see that $4 - L_0 > 7$. So, $|4 + \delta_0 - L_0| = 4 + \delta_0 - L_0 > 7 > \epsilon_0$. So, the limit is not $L_0 < 3$.

Case 2: $L_0 \geq 3$. Set $x = x_0 = 4 - \delta/2$. Then, $|x_0 - 4|$ clearly is greater than 0 and less than δ_0:

$$0 < |x_0 + 4| = |4 - \frac{\delta_0}{2} - 4| = |-\frac{\delta_0}{2}| = \frac{\delta_0}{2}.$$

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Since $x_0 < 4$,

$$\begin{aligned} |f(x_0) - L_0| &= |x_0 - 2 - L_0| \\ &= |4 - \frac{\delta_0}{2} - 2 - L_0 \\ &= |2 - \frac{\delta_0}{2} - L_0. \end{aligned}$$

since $L_0 \geq 3$, we know that $-L_0$

$\square$

While working on this proof, I received no external assistance aside from advice from Professor Mehmetaj.