Individual 7

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Introduction to Proof and Problem Solving

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Lemma 1. We will prove via induction that for every integer $n > 0$, $n(n+1)$ is even.

Base Case: Set $n = n_0 = 1$. Then $n_0(n_0 + 1) = 2$, which is clearly true.

Inductive Step: Assume there exists integers $n_0 > 0$ and k_0 such that

$$n_0(n_0 + 1) = 2k_0.$$

We want to show that there exists an integer j_0 such that

$$(n_0 + 1)(n_0 + 2) = 2j_0.$$

Set $j_0 = (k_0 + n_0 + 1)$. Then (top to bottom).

Problem 1. Use induction to prove that $(n^3 - n)(n + 2)$ is divisible by 12 for all $n \geq 1$.

Proof. We will prove problem 1 using induction.

Base Case: Set $n = n_0 = 1$. Then

$$(n^3 - n)(n + 2) = 0.$$

This is clearly true, since 0 can be written as 12(0), and is thus divisible by 12.

Inductive Step: Assume there exists integers $n \geq 1$ and k_0 such that

$$(n^3 - n)(n + 2) = 12k_0.$$

We want to show that this is true for $n + 1$, meaning we want to show that there exists an integer j_0 such that

$$((n + 1)^3 - (n + 1))((n + 1) + 2) = 12j_0.$$

Set $j_0 =$ something. Then we can see that

□

Problem 2. Problem 6 from Section 3.2.

While working on this proof, I received no external assistance aside from advice from Professor Mehmetaj.