Individual 4

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Introduction to Proof and Problem Solving

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Problem 12. Write the negation of the statement

$$p \equiv \{\forall n \in \mathbb{Z}, \exists m \in \mathbb{Z} \text{ such that } 4n + 3m = 0 \text{ or } 4n + 3m = 1\}$$

and prove p or $\neg p$ is true. Do some examples.

Solution. The negation of p is

$$\neg p \equiv \{\exists n \in \mathbb{Z}, \forall m \in \mathbb{Z} \text{ such that } 4n + 3m \neq 0 \text{ and } 4n + 3m \neq 1\}.$$

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Proof. We will now prove $\neg p$. Set $n_0 = 2$. Then $4n_0 = 8$. Let m_0 be an arbitrary integer. We will consider cases.
Case 1. Suppose $m_0 \geq -2$. Multiplying by 3, we get $3m_0 \geq -6$. Adding 8 to both sides, we get

$$8 + 3m_0 \geq 2.$$

Substituting $4n_0$ for 8, we get

$$4n_0 + 3m_0 \geq 2.$$

Case 2. Suppose $m_0 \leq -3$. Multiplying both sides by 3, we get $3m_0 \leq -9$. Adding 8 to both sides, we get

$$8 + 3m_0 \leq -1.$$

But since $4n_0 = 8$, we have

$$4n_0 + 3m_0 \leq -1$$

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Since $4n_0 + 3m_0 \neq 0$ and $4n_0 + 3m_0 \neq 1$ for all m_0, $\neg p$ is true.

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While working on this proof, I received no external assistance aside from advice from Professor Mehmetaj.