

Individual 6

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Introduction to Proof and Problem Solving

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Problem 12. Show the function f mapping \mathbb{Z} into $S = \mathbb{Z}$ is one-to-one or find two integers n_1 and n_2 such that $n_1 \neq n_2$ but $f(n_1) = f(n_2)$, where

$$f(n) = \begin{cases} 0.5n + 3 & \text{if } n \in E \\ 3n - 1 & \text{if } n \in O \end{cases}$$

Proof. We will prove that the function is not one-to-one. Suppose the function is one-to-one. Then for every real number n_1 and n_2 , $f(n_1) \neq f(n_2)$. Set $n_1 = -2$ and $n_2 = 1$. Since n_1 is even,

$$\begin{aligned} f(n_1) &= 0.5n_1 + 3 \\ f(n_1) &= 2. \end{aligned}$$

Since n_2 is odd,

$$\begin{aligned} f(n_2) &= 3n_2 - 1 \\ f(n_2) &= 2. \end{aligned}$$

We can observe that $f(n_1) = f(n_2)$. Additionally, since $n_1 \neq n_2$, this function is not one-to-one.

□

While working on this proof, I received no external assistance aside from advice from Professor Mehmetaj.