

Individual 6

Yu Fan Mei

Introduction to Proof and Problem Solving

October 22, 2024

Problem 12. Show the function f mapping \mathbb{Z} into $S = \mathbb{Z}$ is one-to-one or find two integers n_1 and n_2 such that $n_1 \neq n_2$ but $f(n_1) = f(n_2)$, where

$$f(n) = \begin{cases} 0.5n + 3 & \text{if } n \in E \\ 3n - 1 & \text{if } n \in O \end{cases}$$

Proof. We will prove that the function is not one-to-one. Suppose the function is one-to-one. Then for every real number n_1 and n_2 , $f(n_1) \neq f(n_2)$. Set $n_1 = -2$ and $n_2 = 1$. Since n_1 is even,

$$f(n_1) = 0.5n_1 + 3.$$

Plugging in -2 for n_1 , $f(n_1) = 2$. Let us examine $f(n_2)$ now. Since n_2 is odd,

$$f(n_2) = 3n_2 - 1.$$

Plugging in 1 for n_2 , $f(n_2) = 2$. We can observe that $f(n_1) = f(n_2)$. Additionally, since $n_1 \neq n_2$, this function is not one-to-one.

□

While working on this proof, I received no external assistance aside from advice from Professor Mehmetaj.