Individual 7

Yu Fan Mei
Introduction to Proof and Problem Solving

October 26, 2024

Lemma 1. For every integer $n > 0$, $n(n+1)$ is even. We will prove this via induction.

Base Case: Set $n = n_0 = 1$. Then $n_0(n_0 + 1) = 2$, which is clearly true.

Inductive Step: Assume there exists integers $n_0 > 0$ and k_0 such that

$$n_0(n_0 + 1) = 2k_0.$$

We want to show that there exists an integer j_0 such that

$$(n_0 + 1)(n_0 + 2) = 2j_0.$$

Set $j_0 = (k_0+n_0+1)$. Since k_0 is an integer and integers are closed under addition, j_0 is also an integer. Unfactoring the left side of the statement above, we get $(n_0+1)(n_0+2) = n_0^2+3n_0+2$. When we substitute $2k + 1$ into this, we're left with $2k_0 + 2n_0 + 2$. When we factor out 2, we are left with

$$2(k_0 + n_0 + 1).$$

We can rewrite this as $2j_0$, which is what we needed to show.

Problem 1. Use induction to prove that $(n^3 - n)(n + 2)$ is divisible by 12 for all $n \geq 1$.

Proof. We will prove problem 1 using induction.

Base Case: Set $n = n_0 = 1$. Then

$$(n^3 - n)(n + 2) = 0.$$

This is clearly true, since 0 can be written as 12(0), and is thus divisible by 12.

Inductive Step: Assume there exists integers $n \geq 1$ and k_0 such that

$$(n^3 - n)(n + 2) = 12k_0.$$

We want to show that this is true for $n+1$, meaning we want to show that there exists an integer j_0 such that

$$((n+1)^3 - (n+1))((n+1)+2) = 12j_0.$$

Set $j_0 =$ something. Then we can see that

$\square$

Problem 2. Problem 6 from Section 3.2.

While working on this proof, I received no external assistance aside from advice from Professor Mehmetaj.