Individual XXXX

Yu Fan Mei
Time Series Analysis

September 27, 2025

Problem 1. Let $Y_t = -t + 2t^2 + \frac{1}{2}Z_t$, where Z_t is a zero mean stationary process with autocovariance function γ_k. Find the expectation and autocovariance of Y_t.

Before we prove this, we will need a lemma.

Proof. Part a. The expected value $E(Y_t)$ is

$$
\begin{aligned}
E(Y_t) &= E\left(-t + 2t^2 + \frac{1}{2}Z_t\right) \\
&= -t + 2t^2 + \frac{1}{2}E(Z_t) \\
&= -t + 2t^2 + \frac{1}{2}(0) \\
&= -t + 2t^2
\end{aligned}
$$

□

Proof. The autocovariance function is defined as

$$
\begin{aligned}
\gamma_k &= Cov(Y_t, Y_{t+k}) \\
&= E\left[(Y_t - E(Y_t))(Y_{t+k} - E(Y_{t+k}))\right] \\
&= E\left[\left(-t + 2t^2 + \frac{1}{2}Z_t - (-t + 2t^2)\right)\left(-(t+k) + 2(t+k)^2 + \frac{1}{2}Z_{t+k} - (-(t+k) + 2(t+k)^2)\right)\right] \\
&= E\left[\left(\frac{1}{2}Z_t\right)\left(\frac{1}{2}Z_{t+k}\right)\right] \\
&= \frac{1}{4}E(Z_t Z_{t+k}) \\
&= \frac{1}{4}\gamma_k
\end{aligned}
$$

□

Problem L. et $Z_t = W_t^2$, where $W_t \sim N(0, 1)$ is a zero mean stationary process with autocovariance function γ_k. Find the variance of Z_t.

Proof. The variance of Z_t is defined as

$$\begin{aligned} Var(Z_t) &= E(Z_t^2) - [E(Z_t)]^2 \\ &= E(W_t^4) - [E(W_t^2)]^2 \\ &= 3 - 1^2 \\ &= 2. \end{aligned}$$

where we used the fact that for a standard normal variable, $E(W_t^4) = 3$ and $Var(W_t) = 1$. $\square$

While working on this proof, I received no external assistance aside from advice from Professor Mehmetaj.