Individual 2

Yu Fan Mei
Introduction to Proof and Problem Solving

September 26, 2024

Problem 1. Set S equal to the set of points in $\mathbb{R}^2$ defined by

$$S = \left\{ \left(\frac{x+1}{x-2}, \frac{5x-1}{x-2} \right) : x \in \mathbb{R} - \{2\} \right\}$$

Similarly, set T equal to the set of points in $\mathbb{R}^2$ defined by

$$T = \{(y+4, 3y+14) : y \in \mathbb{R}\}$$

(a) Prove $S \subseteq T$ using let-variables or prove it is not true using contradiction.

Proof. Let s_0 be any point in S. By definition, there exists a real number x_0 which is not 2 such that

$$s_0 = \left(\frac{x_0+1}{x_0-2}, \frac{5x_0-1}{x_0-2} \right).$$

Set $y_0 = (9-3x_0)/(x_0-2)$. Since $x_0 \neq 2, x_0 - 2 \neq 0$, and $y_0 \in \mathbb{R}$. Then

$$(y_0+4, 3y_0+14) \in T.$$

But

$$\begin{aligned}
(y_0+4, 3y_0+14) &= \left(\frac{9-3x_0}{x_0-2} + 4, 3\left(\frac{9-3x_0}{x_0-2} \right) + 14 \right) \\
&= \left(\frac{x_0+1}{x_0-2}, \frac{5x_0-1}{x_0-2} \right) \\
&= s_0.
\end{aligned}$$

Since $s_0 \in T$, we have $S \subseteq T$. $\square$

(b) Prove $T \subseteq S$ using let-variables or prove it is not true using contradiction.

Proof. Suppose this statement is true. Let t_0 be any ordered pair in T, where

$$t_0 = (y_0 + 4, 3y_0 + 14)$$

Set $y_0 = -3$. We then get $t_0 = (1, 5)$. Let x_0 be any real number that is not 2. By definition, there must be some x_0 such that

$$(1, 5) = \left(\frac{x_0 + 1}{x_0 - 2}, \frac{5x_0 - 1}{x_0 - 2}\right)$$

Examining the first point in the ordered pair, there must be some x_0 that satisfies the following equation:

$$\frac{x_0 + 1}{x_0 - 2} = 1$$

.

Since $x_0 \neq 2$, $x_0 - 2 \neq 0$, and the equation will still hold true when we divide both sides by $x_0 - 2 \neq 0$. Doing this, we get

$$x_0 + 1 = x_0 - 2$$

.

Subtracting x_0 from both sides, we get

$$1 = -2$$

which is not true. So $T \not\subseteq S$.

□

While working on this document, I used no outside help except for Professor Mehmetaj.