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Su24 ECE 131A Project

1a

Toss	Probability
10	0.80
50	0.68
100	0.55
500	0.61
1000	0.60

1b.

$S = \{1,3,5\}$

$P = \frac{1}{5} + \frac{1}{5} + \frac{1}{5} = \frac{3}{5} = 0.6$

1c.

As the number of tosses increases, the estimated results approach the mathematical analysis.

1d.

$P = [\frac{2}{7}, \frac{2}{7}, \frac{1}{7}, \frac{1}{7}, \frac{1}{7}]$

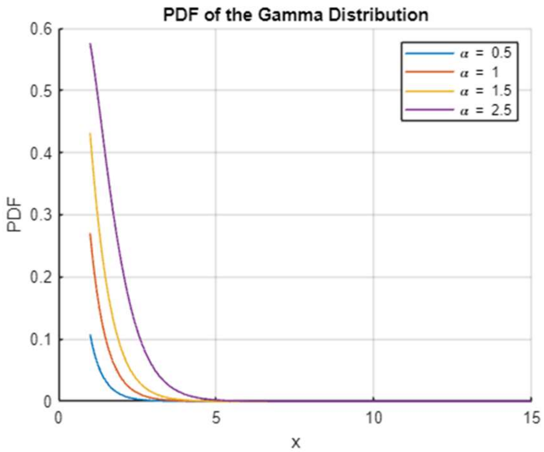
Toss	Probability
10	0.20
50	0.48
100	0.60
500	0.5580
1000	0.5460

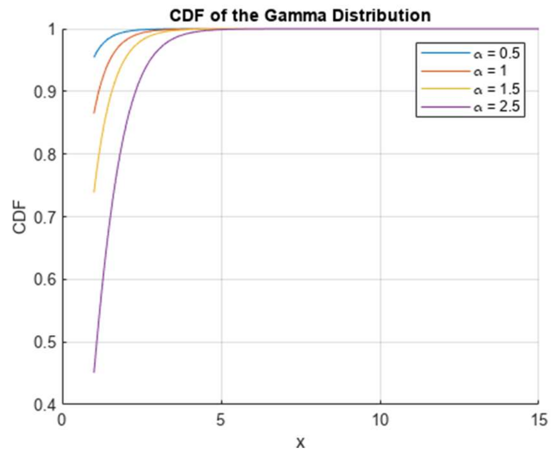
$S = \{1,3,5\}$

$P = \frac{2}{7} + \frac{1}{7} + \frac{1}{7} = \frac{4}{7} = 0.5714$

As the number of tosses increases, the estimated results approach the mathematical analysis.

2a.





2b.

2b $F_X(k) = P(X \le k) = 1-(1-p)^k$

$U = F_X(x) = 1-(1-p)^k$

$1-U = (1-p)^x$

$x = \frac{\log(1-U)}{\log(1-p)}$ $\quad p = \frac{1}{2}$

$x = \frac{\log(1-U)}{\log(1/2)} = \frac{\log(U)}{\log(2)}$

$E(X) = \frac{1}{p} = \frac{1}{1/2} = 2$

2c

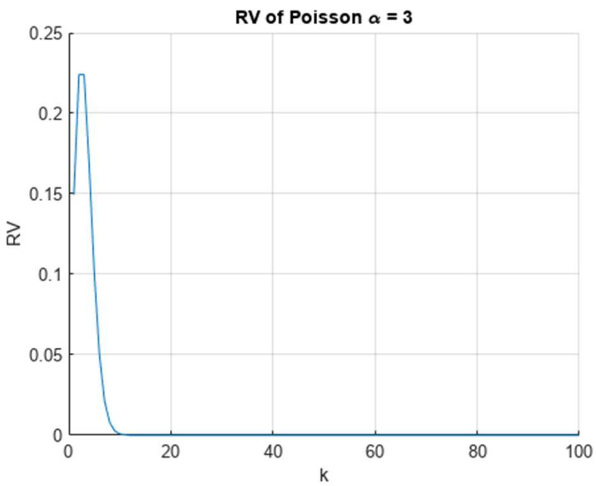
$p(N=k) = \frac{\alpha^k}{k!}e^{-\alpha}$

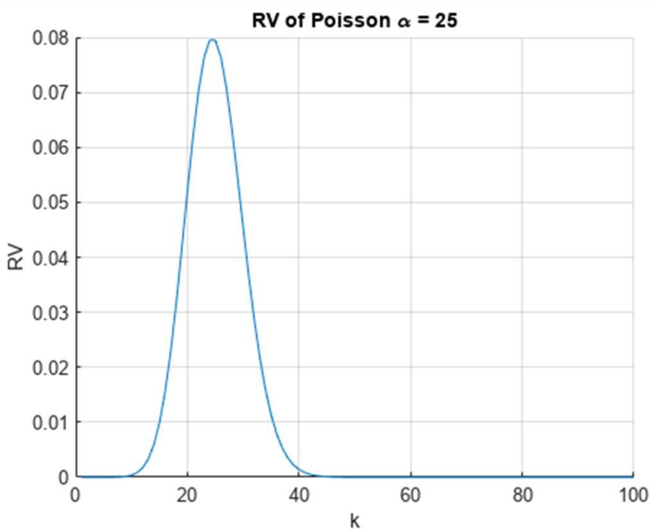
$p(N=0) = \frac{\alpha^0}{0!}e^{-\lambda t} = e^{-\lambda t}$ $\quad f(x) = \lambda e^{-\lambda t}$

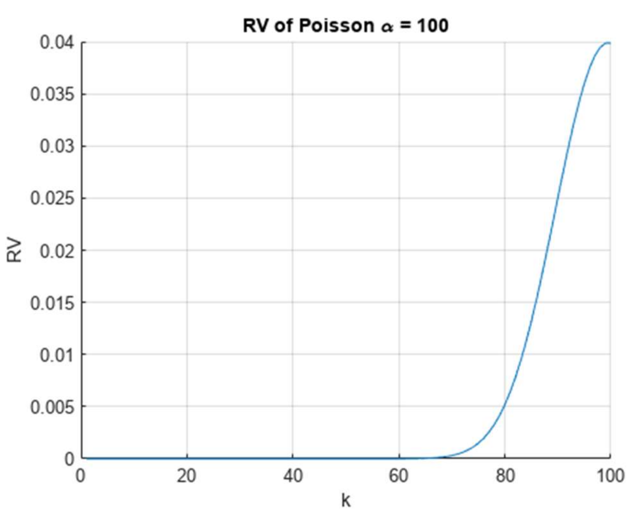
if λ is exponential ~~function~~

where $\lambda t = \alpha$

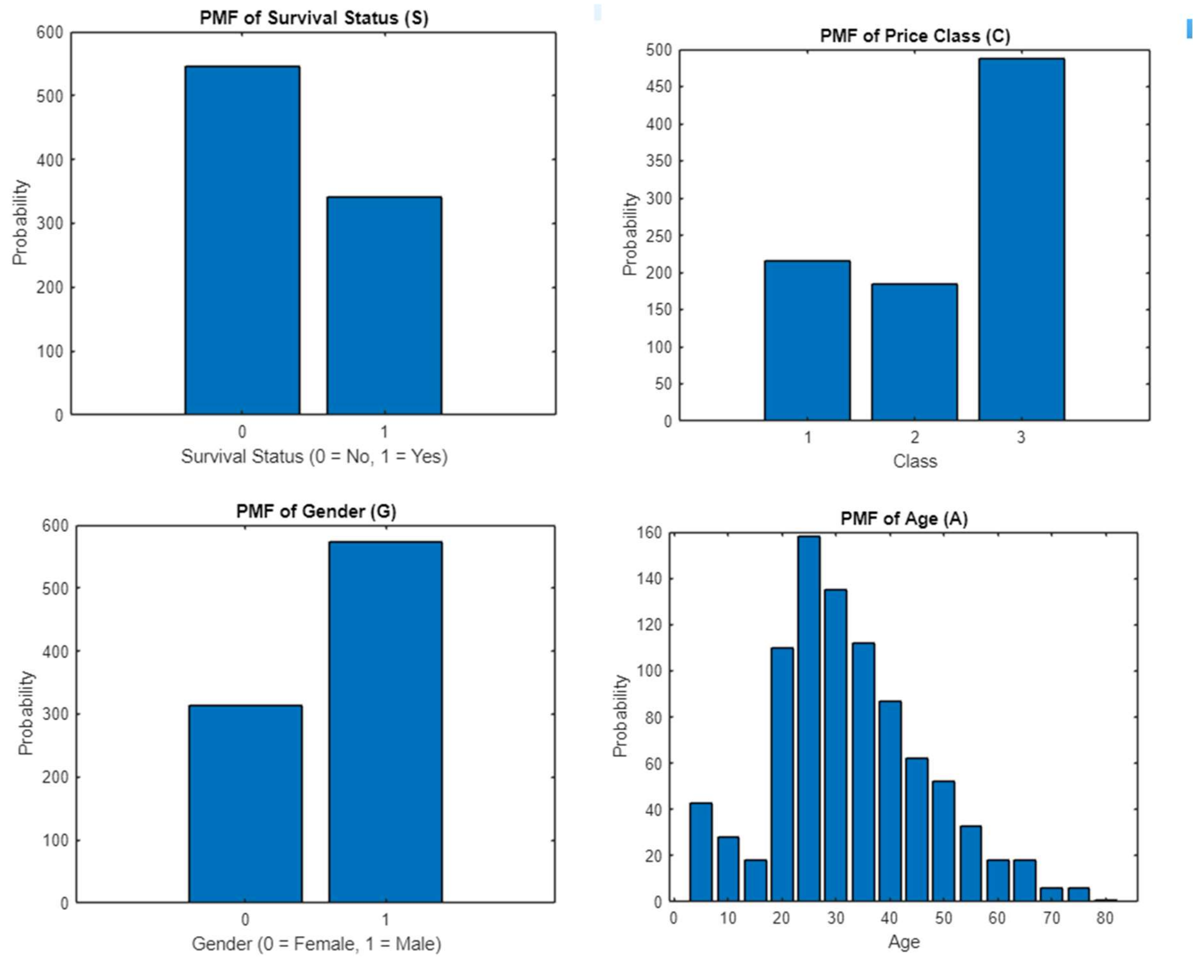
then poisson becomes exponential


RV of Poisson α = 3
0.25
0.2
0.15
0.1
0.05
0
RV
0
20
40
60
80
100
k


RV of Poisson α = 25
0.08
0.07
0.06
0.05
0.04
0.03
0.02
0.01
0
RV
0
20
40
60
80
100
k


RV of Poisson α = 100
0.04
0.035
0.03
0.025
0.02
0.015
0.01
0.005
0
RV
0
20
40
60
80
100
k

3a.

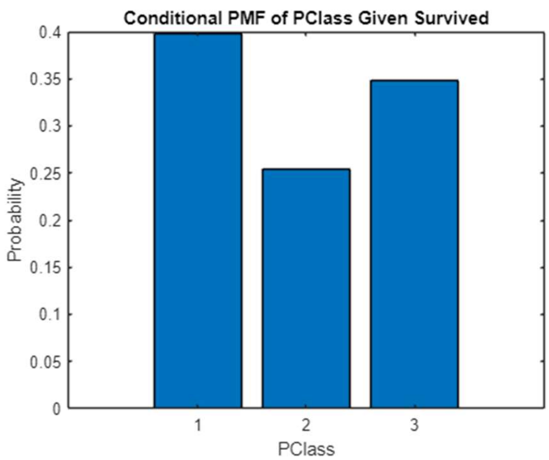


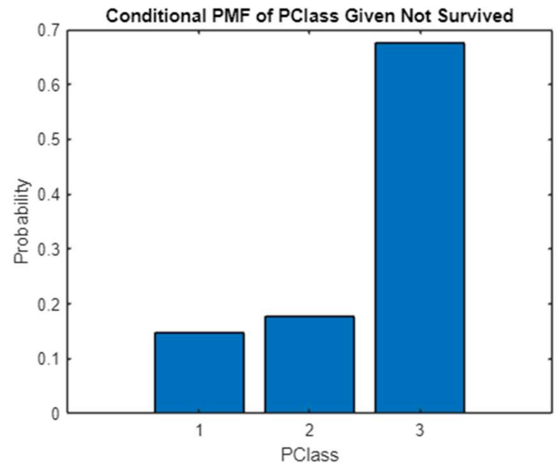
3b.

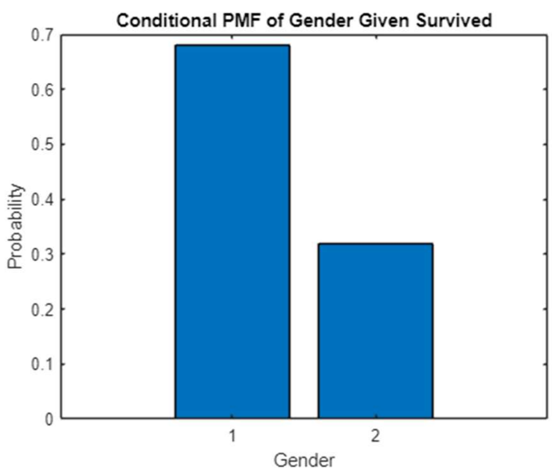
Condition (Survived)	Probability
P(S=1)	0.3856
P(C=1 | S=1)	0.3977
P(C=2 | S=1)	0.2544
P(C=3 | S=1)	0.3480
P(G=0 | S=1)	0.6813
P(G=1 | S=1)	0.3187
P(0≤A<5 | S=1)	0.0965
P(5≤A<10 | S=1)	0.0234
P(10≤A<15 | S=1)	0.0322
P(15≤A<20 | S=1)	0.1199
P(20≤A<25 | S=1)	0.1550
P(25≤A≤<30 | S=1)	0.1491
P(30≤A≤<35 | S=1)	0.1404
P(35≤A≤<40 | S=1)	0.0936

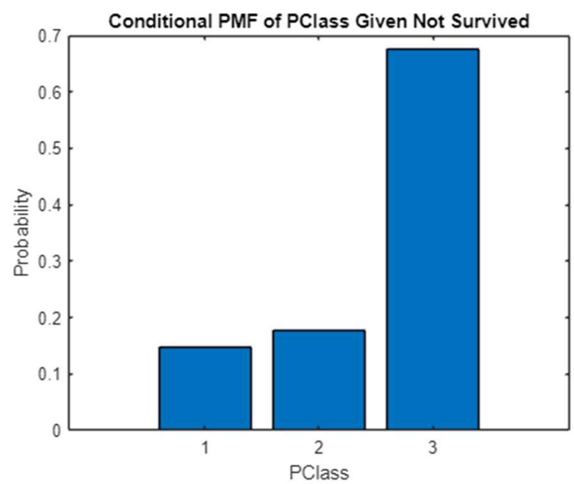
P(40≤A≤<45 | S=1)	0.0614
P(45≤A≤<50 | S=1)	0.0643
P(50≤A≤<55 | S=1)	0.0292
P(55≤A≤<60 | S=1)	0.0205
P(60≤A≤<65 | S=1)	0.0117
P(65≤A≤<70 | S=1)	0
P(70≤A≤<75 | S=1)	0
P(75≤A≤<80 | S=1)	0.0029

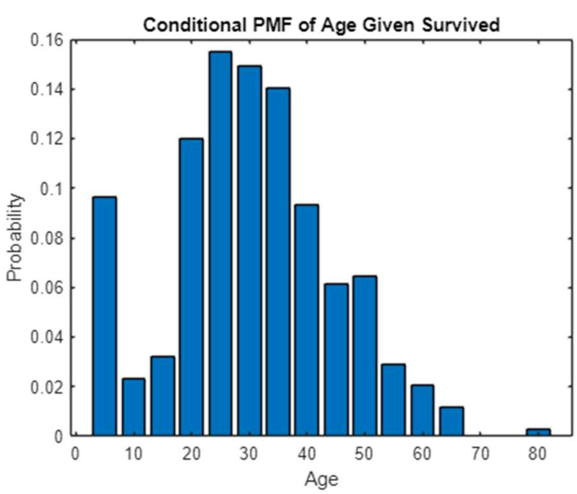
Condition (Not Survived	Probability
P(S=0)	0.6144
P(C=1 | S=0)	0.1468
P(C=2 | S=0)	0.1780
P(C=3 | S=0)	0.6752
P(G=0 | S=0)	0.1486
P(G=1 | S=0)	0.8514
P(0≤A<5 | S=0)	0.0294
P(5≤A<10 | S=0)	0.0294
P(10≤A<15 | S=0)	0.0183
P(15≤A<20 | S=0)	0.1596
P(20≤A<25 | S=0)	0.1945
P(25≤A≤<30 | S=0)	0.1706
P(30≤A≤<35 | S=0)	0.0954
P(35≤A≤<40 | S=0)	0.0954
P(40≤A≤<45 | S=0)	0.0679
P(45≤A≤<50 | S=0)	0.0477
P(50≤A≤<55 | S=0)	0.0294
P(55≤A≤<60 | S=0)	0.0239
P(60≤A≤<65 | S=0)	0.0220
P(65≤A≤<70 | S=0)	0.0092
P(70≤A≤<75 | S=0)	0.0073
P(75≤A≤<80 | S=0)	0


Conditional PMF of PClass Given Survived
Probability
0
0.05
0.1
0.15
0.2
0.25
0.3
0.35
0.4
1
2
3
PClass

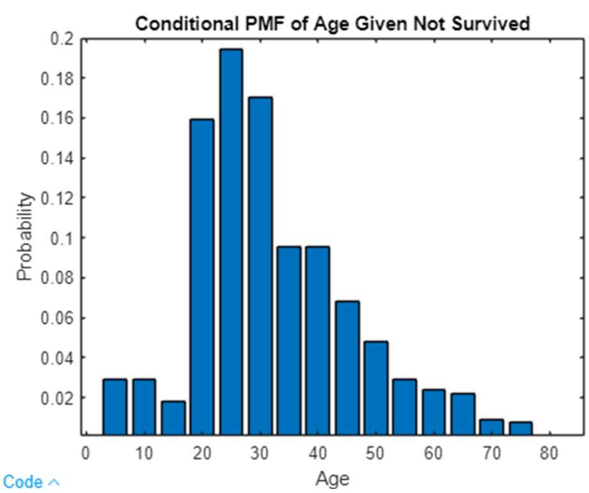

Conditional PMF of PClass Given Not Survived
Probability
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
1
2
3
PClass


Conditional PMF of Gender Given Survived
Probability
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
1
2
Gender


Conditional PMF of PClass Given Not Survived
Probability
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
1
2
3
PClass


Conditional PMF of Age Given Survived
Probability
0
0.02
0.04
0.06
0.08
0.1
0.12
0.14
0.16
0
10
20
30
40
50
60
70
80
Age

Code ^


Conditional PMF of Age Given Not Survived
Probability
0.02
0.04
0.06
0.08
0.1
0.12
0.14
0.16
0.18
0.2
0
10
20
30
40
50
60
70
80
Age

3c.

$$P(S = 1, C = 1, G = 0, A \leq 40) = P(C = 1 \mid S = 0) * P(G = 0 \mid S = 0) * P(A \leq 40 \mid S = 0)$$
$$= 0.2194$$

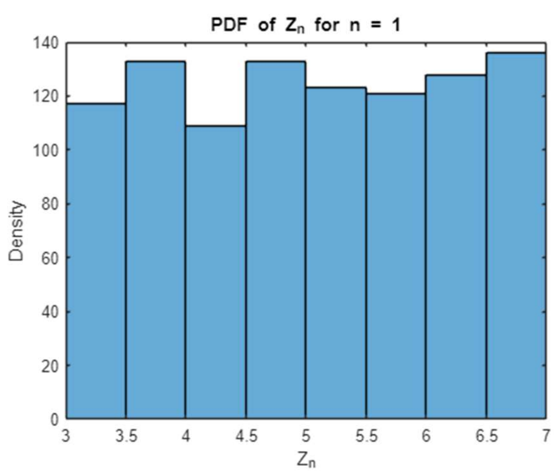
$$P(S = 0, C = 1, G = 0, A \leq 40) = P(C = 1 \mid S = 0) * P(G = 0 \mid S = 0) * P(A \leq 40 \mid S = 0)$$
$$= 0.0173$$

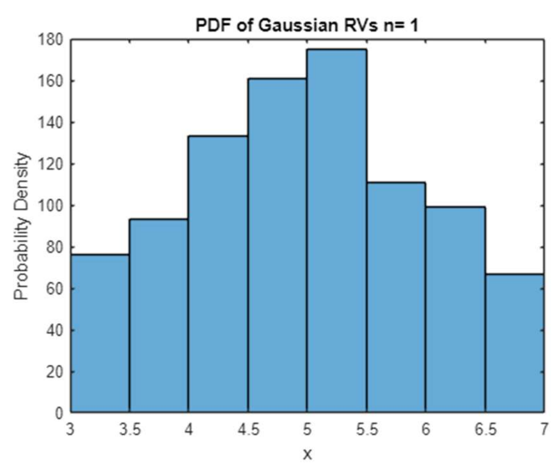
3d.

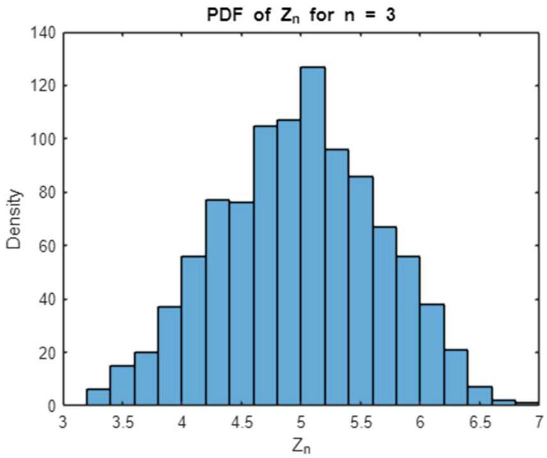
$$P(C = 1, G = 0, A \leq 40 \mid \mathrm{S} = 0) = \frac{P(C = 1 \mid S = 0) * P(G = 0 \mid S = 0) * P(A \leq 40 \mid S = 0)}{P(S = 0)}$$
$$= 0.0281$$

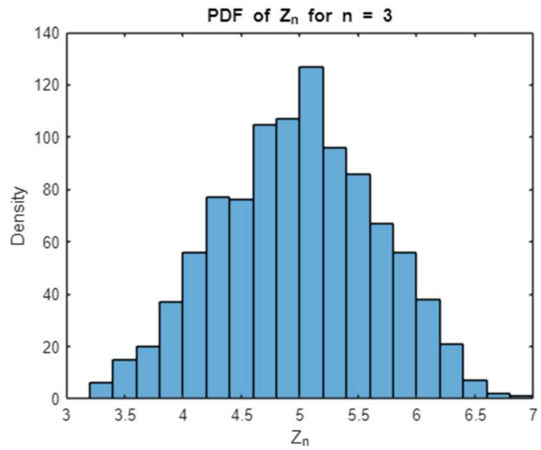
$$P(C = 1, G = 0, A \leq 40 \mid \mathrm{S} = 1) = \frac{P(C = 1 \mid S = 1) * P(G = 0 \mid S = 1) * P(A \leq 40 \mid S = 1)}{P(S = 1)}$$
$$= 0.5691$$

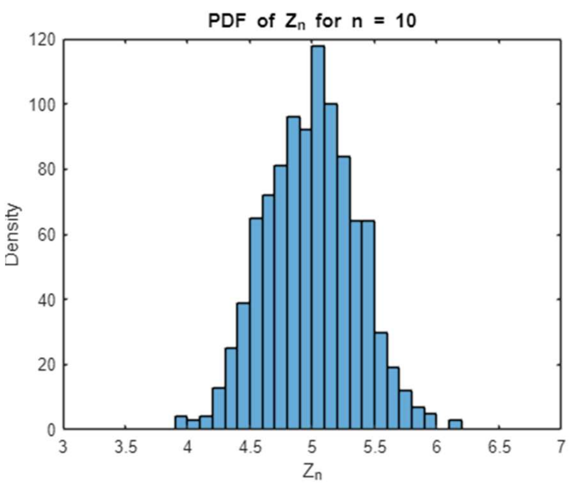
4a.

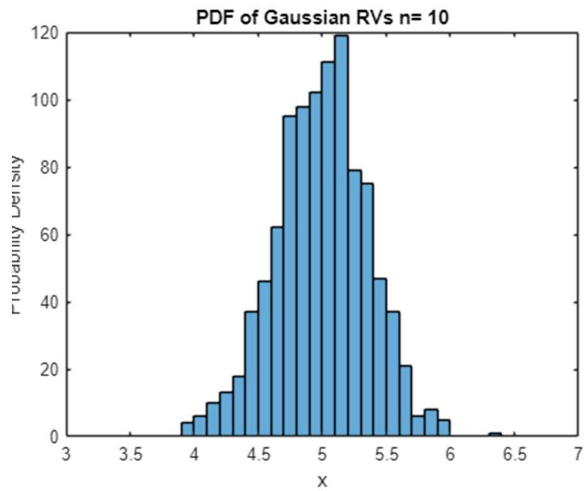


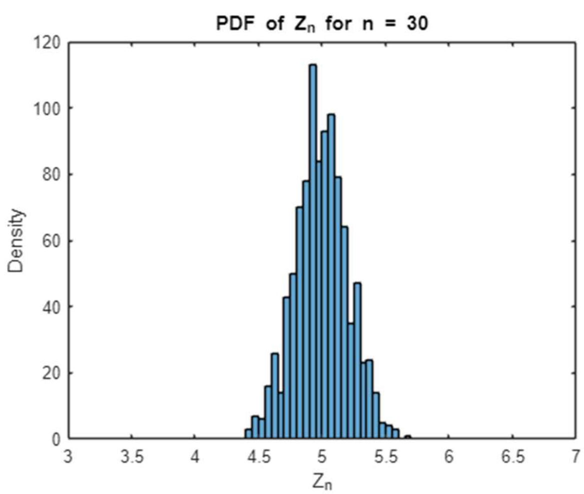


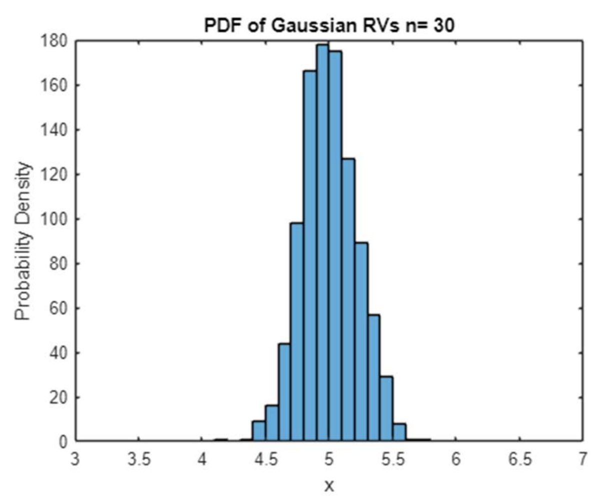

PDF of Z_n for n = 3
Density
Z_n

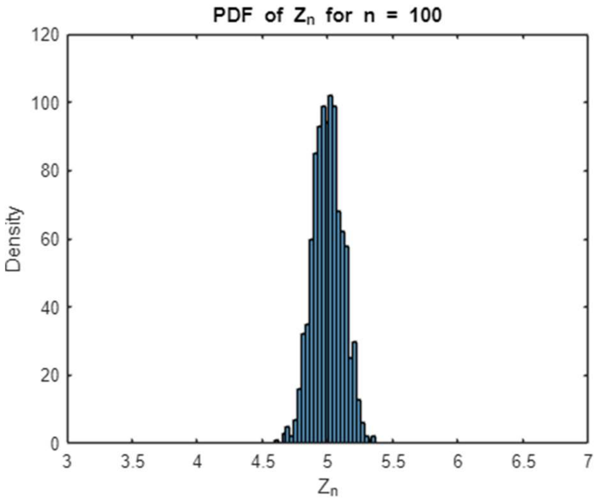

PDF of Z_n for n = 3
Density
Z_n

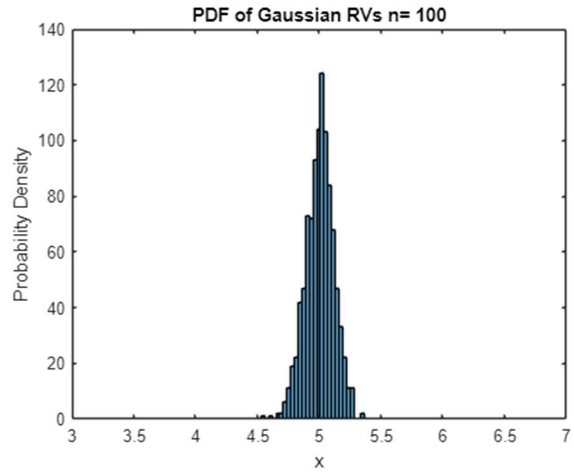

PDF of Z_n for n = 10
Density
Z_n


PDF of Gaussian RVs n= 10
x


PDF of Z_n for n = 30
Density
Z_n


PDF of Gaussian RVs n= 30
Probability Density
x





4b.

$Z_n = \frac{1}{n}\sum_{i=1}^{n} x_i$

$a = 3$
$b = 7$

$S_n = \sum_{i=1}^{n} x_i$

$\sigma^2 =$

$E[x_i] = \mu = \frac{a+b}{2} = 5 \quad VAR(x_i) = \frac{(b-a)^2}{12} = \frac{16}{12} = \frac{4}{3}$

$E[S_n] = n\mu$

$\cancel{E[Z_n] = \frac{1}{\sigma\sqrt{n}} E[S_n - n\mu]}$

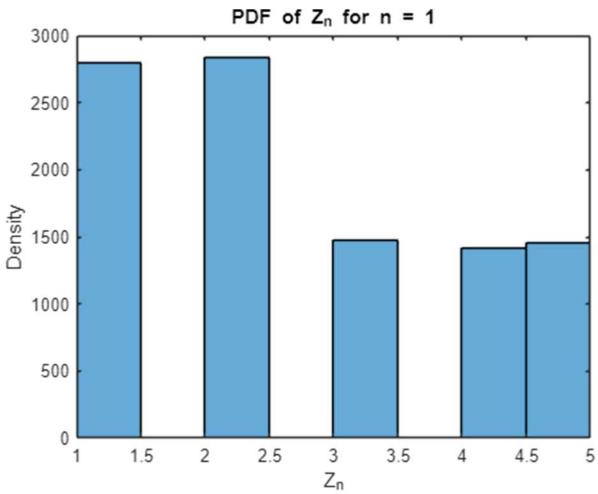
$E[S_n] = n\mu \quad VAR[S_n] = n\sigma^2$

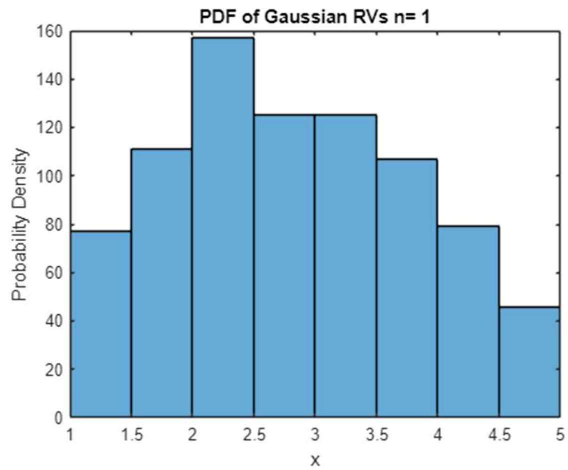
$E[Z_n] = E\left[\frac{1}{n}S_n\right] = \frac{1}{n}n\mu = \mu$

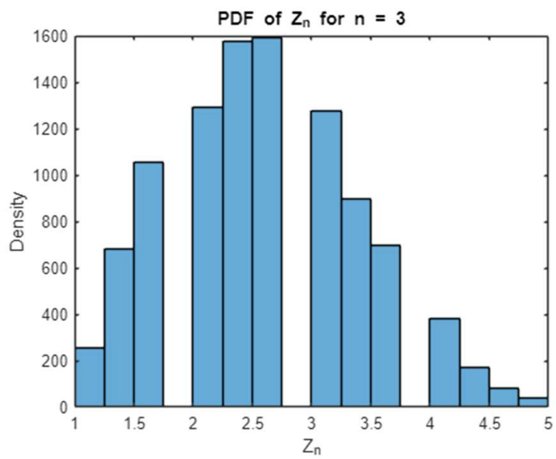
$\cancel{E}$

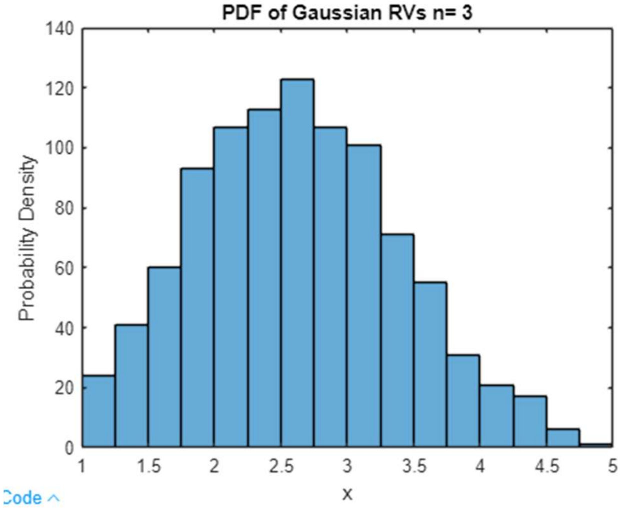
$VAR\left[\frac{1}{n}S_n\right] = \frac{1}{n^2}VAR[S_n] = \frac{1}{n^2}n\sigma^2 = \frac{\sigma^2}{n} \quad \frac{4/3}{n}$

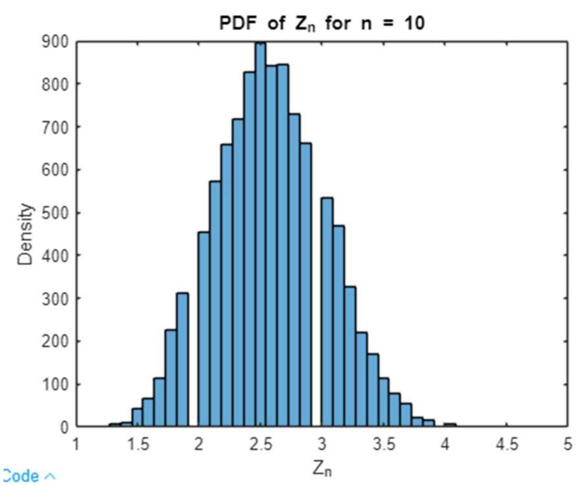
4d.

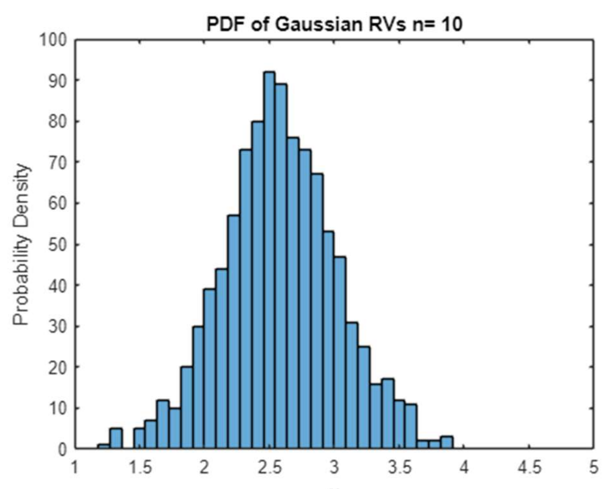

PDF of Z_n for n = 1
Density
Z_n

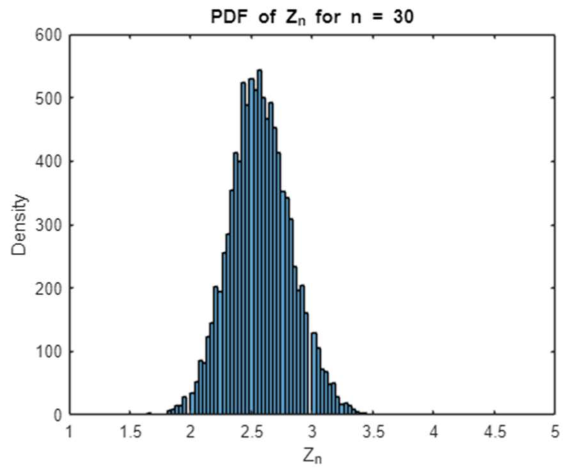

PDF of Gaussian RVs n= 1
Probability Density
x

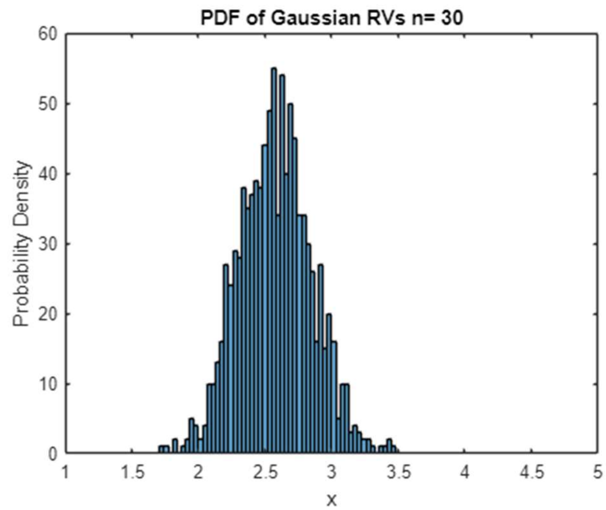

PDF of Z_n for n = 3
Density
Z_n

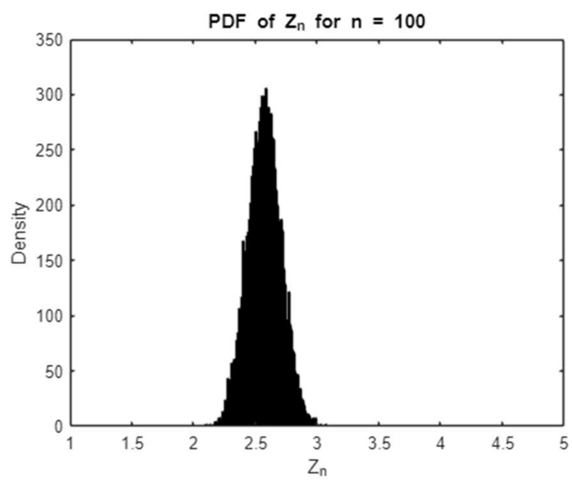

PDF of Gaussian RVs n= 3
Probability Density
x

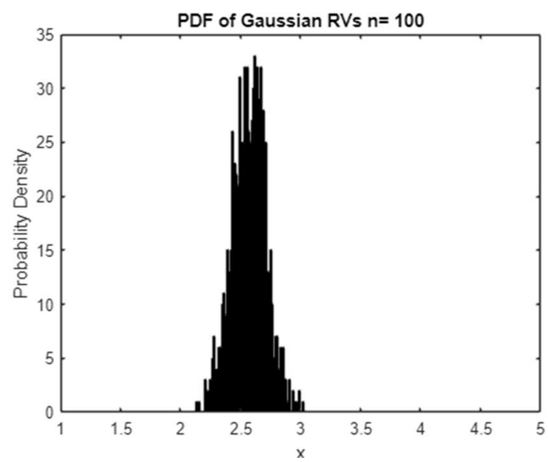

PDF of Z_n for n = 10
Density
Z_n


PDF of Gaussian RVs n= 10
Probability Density
x









$P = [2/7, 2/7, 1/7, 1/7, 1/7]$

$E[x] = \sum_{i=1}^{n} x_i P(X=x_i)$

$= 1 \cdot \frac{2}{7} + 2 \cdot \frac{2}{7} + 3 \cdot \frac{1}{7} + 4 \cdot \frac{1}{7} + 5 \cdot \frac{1}{7} = \frac{18}{7} = 2.57$

$\cancel{VAR}(x) = E(x^2) = \sum_{i=1}^{n} x^2 P(x=x_i)$

$E[x^2] = 1^2 \frac{2}{7} + 2^2 \frac{2}{7} + 3^2 \frac{1}{7} + 4^2 \frac{1}{7} + 5^2 \frac{1}{7}$

$E(x^2) = \frac{60}{7} = 8.57$

$VAR(x) = E(x^2) - (E(x))^2 = 8.57 - 2.57^2$

$= 1.96$

$E[Z_n] = \mu = 2.57$

$VAR[Z_n] = \frac{\sigma^2}{n} = \frac{1.96}{n}$ $\quad \sigma = \sqrt{\frac{1.96}{n}}$