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Fault Tolerant Low Cost IMUS for UAVs

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***Abstract*—In this paper a Fault Tolerant system for the attitude estimation of an Unmanned Aerial Vehicle (UAV) using low cost magnetometers, accelerometers, and gyroscopes, implemented in an Inertial Measurement Unit (IMU) is proposed. An approach based on the Unscented Kalman Filter (UKF) for Detection, Isolation and Reconfiguration is investigated in the presence of a Hardware Duplex IMU mounted on board for flight control purposes. A state automaton, with comparative logics, detects if anomalies occurr on one of the two IMUs; then based on a comparison between the measured variables and their UKF estimations, the fault is isolated and identified. The proposed approach is an alternative to standard Hardware Triplex IMU architectures based on triple physical redundancy. At the expense of a higher computational cost and a small delay in isolating faults, the UKF based approach allows to limit the number of IMUs to two, or to guarantee a fail safe behavior in the presence of a double fault, even if contemporary, with Triplex IMUs. Experimental results on the implementation of a multiple IMU platform on a quadrotor flight control board are finally presented.**

***Index Terms*—Inertial Measurements Units; Fault Detection and Isolation; Fault Tolerance; Unscented Kalman Filter; Unmanned Aerial Vehicles; Attitude Estimation.**

I. Introduction

One of the key aspects for a massive diffusion of low cost Unmanned Aerial Vehicles (UAV) for civil applications is safety of flight. Reliability of sensors for flight control is a fundamental factor, especially for unstable aircraft like multirotors that, in the absence of a closed loop control law, cannot be easily piloted. In order to increase sensors system reliability, redundancy can be used in one of its two possible forms, namely *hardware redundancy* and *analytical* or *software redundancy*.

Hardware redundancy means that multiple independent hardware channels, with a procedure to *vote* the healthy measurement, are used. On the other hand, analytical redundancy implies the use of mathematical relations to obtain redundant information in case of fault of some of the measurement devices. In order to achieve analytical sensor redundancy it is necessary to reconstruct one or more measurements, either anlytically or numerically, to simulate the presence of additional sensors, some times called *virtual sensors*.

The use of state estimators to this purpose is a well known possibility, explored in the literature for the solution of Fault Detection, Isolation and Reconfiguration (FDIR) problems [1], [2] , [3], [4], [5]. Well assessed techniques are those based on parity space [6], [7], observer-based methods [8], [9], and approaches based on Kalman filtering and on H_2 or H_∞ theory [10]. Several schemes have been proposed in the literature for the use of estimators in FDI, most of them making use of the so-called *residual*, i.e. the difference between the measured and the estimated outputs.

In the Dedicated Observer Scheme (DOS) [8], different observers are implemented, each of them being driven by a different set of measured outputs. In the event of a fault on a measurement, the corresponding observer will produce inaccurate estimates and therefore fault detection and isolation is made possible.

The Generalized Observer Scheme (GOS) [9] makes use of a bank of observers. In this case, however, each observer is driven by all outputs except one: when an output is faulty, the estimates of all the observer except one are inaccurate.

The simplified detection scheme [11] can be seen as a special DOS case, and makes use of only *one* observer, driven by only one measurement. Therefore, if any other measurement is faulty, the corresponding residual is non-zero. Of course if the sensor used by the observer is faulty, then all the residuals are non-zero and no information can be gathered.

All the above mentioned schemes can be used with any kind of state or output observers including Luenberger observers [12], Kalman filters [13], Unknown Input Observers (UIO) [14].

In this paper we focus on measurement and reconstruction of variables for attitude estimation, including quantities measured by magnetometers, accelerometers, and gyroscopes that are typically fused together, via Kalman Filtering, on Inertial Measurement Units (IMU).

An approach based on the Unscented Kalman Filter (UKF) for Detection, Isolation and Reconfiguration is investigated in the presence of a Duplex IMU mounted on the control board. A state automaton, with comparative logics, detects if anomalies occurr on one of the two IMUs and, based on a comparison between the measured variables and their UKF estimations, can recognize and declare a fault, voting the healthy measurements.

The proposed approach is analysed in the light of a standard Triplex IMU architecture based on pure HW redundancy. At

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the expense of a higher computational cost and a small delay in isolating faults, the proposed approach allows to reduce such a redundancy, adopting a Duplex architecture, or to guarantee a fail safe behavior of the attitude measurement system in the presence of a double fault, even if contemporary in the case of Triplex.

Experimental results are based on the implementation of a triplex IMU platform for flight control. A discussion on *pros and cons* based on a large number of runs is carried out. The numerical burden and implementability of the proposed technique on a low cost flight control computer is also discussed.

II. Model and Problem Statement

For the description of rotational kinematic relationships, two reference frames, originating from the Center of Gravity (CG), are typically used: a body fixed $\mathcal{B} = (X_B Y_B Z_B)$ and an earth fixed frame $\mathcal{E} = (X_E Y_E Z_E)$, such that the Z_E axis is oriented as the gravity vector and the X_E Y_E axes are coincident to X_B Y_B at the initial time t_0 of observation of the aircraft motion.

The orientation of the body fixed frame with respect to the earth fixed frame can be determined using Euler angles describing the amplitude of three sequential rotations to superimpose $(X_B Y_B Z_B)$ to $(X_E Y_E Z_E)$.

To avoid singularities in the definition of the rotation matrices quaternions can be used in the place of Euler angles. With this new orientation descriptor $\vec{q} = [q_1, q_2, q_3, q_4]^T$, the transformation of an arbitrary vector $\vec{z}$ from $\mathcal{E}$ to $\mathcal{B}$, can be written as follows [15]: $\vec{z}^B = \mathbf{C}_{BE}(\vec{q})\,\vec{z}^E$, where $\mathbf{C}_{BE}$ is a quaternion dependent, rotation matrix defined as follows:

$$\mathbf{C}_{BE} = \begin{bmatrix} q_1^2 - q_2^2 - q_3^2 + q_0^2 & 2(q_1q_2 - q_3q_0) & 2(q_1q_3 + q_2q_0) \\ 2(q_1q_2 + q_3q_0) & -q_1^2 + q_2^2 - q_3^2 - q_0^2 & 2(q_2q_3 - q_1q_0) \\ 2(q_1q_3 - q_2q_0) & 2(q_2q_3 + q_1q_0) & -q_1^2 - q_2^2 + q_3^2 + q_0^2 \end{bmatrix} \tag{1}$$

Kinematics of a rigid body rotation can be then described, using $\vec{q}$ quaternions and measured Body frame angular velocities $\vec{\Omega} = [p,\ q,\ r]^T$, as

$$\dot{\vec{q}} = \tfrac{1}{2}\mathbf{M}(\vec{\Omega})\vec{q}; \quad \mathbf{M}(\vec{\Omega}) = \begin{bmatrix} 0 & -p & -q & -r \\ p & 0 & r & -q \\ q & -r & 0 & p \\ r & q & -p & 0 \end{bmatrix} \tag{2}$$

In principle, attitude estimation, in terms of quaternions, could be obtained by integrating eqn.(2) gyroscope measurments, starting from a known initial condition $\vec{q}_0 = \vec{q}(t_0)$. However, the initial attitude is not precisely known, if not measured, and measurements are noisy and affected by biases making the integration endlessly drift.

Another possibility to estimate attitude descriptors, either quaternions or Euler angles, is to use accelerometers and magnetometers measuring accelerations, including gravity, and magnetic fields, including Earth's field, respectively. In a uniform rectilinear steady state aircraft motion, acceleration and magnetic field vectors are univocally related to attitude. Unfortunately, beside usual sensor noise, acceleration measurements are subject to additional vibrations induced noise, and magnetic field measurements are subject to possible magnetic perturbation. Moreover, when maneuvering UAVs, other accelerations are superimposed to the gravity.

Attitude Heading Reference Systems (AHRSs) implement *sensor fusion* and *filtering* algorithms to merge and filter gyroscope, accelerometer and magnetometer measurements in order to achieve an accurate attitude estimation. Roughly speaking, low frequency components of accelerometers and magnetometers measurements can be used to estimate gyroscope biases, while gyroscope information is used to catch the fast dynamics.

To this end, the kinematic model is enlarged with bias dynamics and the following relations can be written, neglecting high frequency noise:

$$\dot{\vec{q}} = \frac{1}{2}\mathbf{M}(\vec{\tilde{\Omega}} - \vec{b})\vec{q}; \quad \dot{\vec{b}} = -\frac{1}{\tau_b}\vec{b} \tag{3}$$

where $\vec{\tilde{\Omega}}$ is the measured value of $\vec{\Omega}$, $\vec{b}$ is the vector of biases, and τ_b is a sufficiently large time constant. As for the other measured outputs, namely acceleration $\vec{a}$ and magnetic field $\vec{M}$, the following output equations, the first one being valid only in steady state conditions, can be used

$$\vec{a} = \mathbf{C}_{BE}(\vec{q})\vec{g}; \quad \vec{M} = \mathbf{C}_{BE}(\vec{q})\vec{M}_E \tag{4}$$

where $\vec{g}$ is the gravity vector and $\vec{M}_E$ is the Earth's magnetic field vector.

Problem 1: **Attitude Estimation Problem.** Given gyroscopes, accelerometers, and magnetometers measurements affected by noise, namely $\vec{\tilde{\Omega}}$, $\vec{\tilde{a}}$ and $\vec{\tilde{M}}$ vectors, with $\vec{\tilde{\Omega}} = \vec{\Omega} - \vec{b}$ also affected by slowly varying biases, find an estimation of the quaternion vector $\vec{\hat{q}}$.

In the presence of faults of gyros, accelerometers, and magnetometers, HW (Hardware) redundancy and/or suitable algorithms can be used to guarantee a sufficient level of attitude estimation accuracy solving the following.

Problem 2: **Fault Tolerant Attitude Estimation Problem.** Given multiple gyroscopes, accelerometers, and magnetometers measurements affected by noise, namely $\vec{\tilde{\Omega}}_i$, $\vec{\tilde{a}}_i$ and $\vec{\tilde{M}}_i$ vectors ($i = 2$ Duplex, $i = 3$ Triplex HW), with $\vec{\tilde{\Omega}}_i = \vec{\Omega}_i - \vec{b}_i$, find an estimation of the quaternion vector $\vec{\hat{q}}$ in the presence of one single fault on one of the on-board sensors.

We consider the following models for faults

- **Steps and biases:** systematic constant measurement error arising at a given time instant: modelled with constant or step functions. Biases on gyroscopes are usually present from the beginning and are compensated by the UKF.
- **Drifts:** systematic measurement error increasing in time. Drifts are due to sensor devices faults, often due to thermal drift, or to errors on software buffers. Slow and fast Drifts are modelled with additive ramp signals or sawtooth waveforms simulating a periodic reset of the drift value.
- **Switch to zero:** this fault sets the measurement to zero abruptly and can be due to a loss of communication or a complete breakdown of the sensor.

- **Frozen measurement:** fault freezing the measurement value at the fault time.

III. The Unscented Kalman Filter

Popular sensor fusion and filtering algorithms to solve Problems 1 and 2 are typically KF [16], [17], [18], [19].

Consider the a discrete-time state space model, in the form:

$$\vec{x}_k = f(\vec{x}_{k-1}, \vec{u}_{k-1}, \vec{w}_{k-1}) \quad (5)$$

$$\vec{y}_k = h(\vec{x}_k, \vec{u}_k, \vec{v}_k) \quad (6)$$

where $\vec{x}$ is the state, $\vec{w}$ is the process noise, $\vec{y}$ is the measurement and $\vec{v}$ is the measurement noise.

If $f(\cdot,\cdot,\cdot)$ and $h(\cdot,\cdot,\cdot)$ are linear functions of their arguments, and $\vec{w}$ and $\vec{v}$ are Gaussian noises, an optimal solution to recursive filtering is given KF.

In the case of nonlinear functions, assuming Gaussian and uncorrelated noises, one can resort the EKF that has become a standard technique used in a number of non linear estimation and machine learning applications [20]. In the EKF, the state distribution is approximated by a Gaussian Random Variable (GRV), which is then propagated analytically through the first order linearization of the nonlinear system. This can introduce large errors in the true posterior mean and covariance of the transformed GRV, which may lead to suboptimal performance and sometimes even divergence of the filter.

UKF addresses the approximation issues of the EKF, by using a deterministic sampling approach. The state distribution is again approximated by a GRV, but this is now represented using a minimal set of carefully chosen sample points. These sample points completely capture the true mean and covariance of the GRV, and, when propagated through the nonlinear system, captures the posterior mean and covariance accurately to the 3rd order.

To propagate noise features, the *Unscented Transformation* (UT) algorithm is used. UT is a method to compute the statistics of a random variable which undergoes a nonlinear transformation. Consider $\vec{\xi}$ a random variable (with size n) with expected value $\bar{\vec{\xi}} = E(\vec{\xi})$ and covariance $\mathbf{P}_\xi = E[(\vec{\xi} - E(\vec{\xi}))(\vec{\xi} - E(\vec{\xi}))^T]$. The propagation of $\vec{\xi}$ is obtained by using a nonlinear function $\vec{\eta} = g(\vec{\xi})$. A matrix $\mathbf{X}$ of $2n+1$ sigma vectors $\vec{\xi^i}$ is built to compute the statistics of $\vec{\eta}$, as follows:

$$\vec{\xi^0} = \bar{\vec{\xi}} \quad (7)$$

$$\vec{\xi^i} = \bar{\vec{\xi}} + (\sqrt{(n+\lambda)\mathbf{P}_\xi})_i \quad \forall i = 1, ..., n \quad (8)$$

$$\vec{\xi^i} = \bar{\vec{\xi}} - (\sqrt{(n+\lambda)\mathbf{P}_\xi})_{i-n} \quad \forall i = n+1, ..., 2n \quad (9)$$

$$W_0^{[m]} = \frac{\lambda}{n+\lambda}; \quad W_0^{[c]} = \frac{\lambda}{n+\lambda} + (1 - \alpha^2 + \beta) \quad (10)$$

$$W_i^{[m]} = W_i^{[c]} = \frac{1}{2(n+\lambda)} \quad \forall i = 1, ..., 2n \quad (11)$$

where $\lambda = \alpha^2(n+\kappa) - n$ is a scaling parameter; α determines the spread of the sigma points around $\bar{\vec{\xi}}$ and it is usually set to a small positive value; κ is a secondary scaling parameter usually set to 0; β is used to incorporate prior knowledge of the distribution of $\vec{\xi}$ ($\beta = 2$ for Gaussian distributions); $(\sqrt{(n+\lambda)\mathbf{P}_\xi})_i$ is the i-th row of the lower triangular Cholewsky factorization of matrix $(n+\lambda)\mathbf{P}_\xi$.

Sigma vectors are propagated through the nonlinear function $\vec{\eta}_i = g(\vec{\xi^i}) \, \forall i = 0, ..., 2n$ and the expected value and covariance for $\vec{\eta}$ are approximated using a weighted sample mean and covariance of the posterior sigma points:

$$E(\vec{\eta}) \approx \bar{\vec{\eta}} = \sum_{i=0}^{2n} W_i^{[m]} \vec{\eta}_i \,; \quad \mathbf{P}_\eta \approx \sum_{i=0}^{2n} W_i^{[c]} (\vec{\eta}_i - \bar{\vec{\eta}})(\vec{\eta}_i - \bar{\vec{\eta}})^T \quad (12)$$

The simple approach taken with the UT results in approximations that are accurate to the third order for Gaussian inputs for all nonlinearities. For non-Gaussian inputs, approximations are accurate to at least the second-order, and determined by the choice of α and β. The UKF is a straightforward extension of the UT (*Unscented Transformation*) to the recursive estimation, where the state is redefined as the concatenation of the original state and noise variables. The UT sigma point selection scheme is applied to this augmented state, to compute the corresponding sigma matrix $\mathbf{X}_h^a$. No explicit calculation of Jacobians or Hessians is necessary to implement the algorithm.

A. UKF Algorithm for State Estimation

Let be $\vec{x}_k \in R^n$ the state of the system, $\vec{y}_k \in R^p$ the output. Assume that the process and the measurement noises $\vec{w}_k$ and $\vec{v}_k$ are zero-mean white Gaussian $\vec{w}_k\ (0, \mathbf{Q}_k)$, $\vec{v}_k\ (0, \mathbf{R}_k)$. The computation algorithm begins with the initial conditions:

$$\hat{\vec{x}}_0^+ = E[\vec{x}_0]\,; \quad \mathbf{P}_0^+ = E[(\vec{x}_0 - \hat{\vec{x}}_0^+)(\vec{x}_0 - \hat{\vec{x}}_0^+)^T] \quad (13)$$

At the beginning of each iteration, using equations (8) and (9), with $\lambda = 0$, sigma points $\vec{x}_{k-1}^i$ can be calculated as follows:

$$\begin{cases} \vec{x}_{k-1}^0 = & \hat{\vec{x}}_{k-1}^+ \\ \vec{x}_{k-1}^i = & \hat{\vec{x}}_{k-1}^+ + (\sqrt{n\mathbf{P}_{k-1}^+})_i \quad \forall i = 1, 2, ..., n \\ \vec{x}_{k-1}^{i+n} = & \hat{\vec{x}}_{k-1}^+ - (\sqrt{n\mathbf{P}_{k-1}^+})_i \quad \forall i = 1, 2, ..., n \end{cases} \quad (14)$$

where $\sqrt{n\mathbf{P}_{k-1}}$ is the lower triangular Cholewsky factorization of matrix $n\mathbf{P}_{k-1}$ and $(\sqrt{n\mathbf{P}_{k-1}})_i$ is its i-th row. The following step consists of propagation of sigma points by using the nonlinear state equation, to transform the sigma points into $\hat{\vec{x}}_k^i$ vector as:

$$\hat{\vec{x}}_k^i = f(\vec{x}_{k-1}^i, \vec{u}_k)\,. \quad (15)$$

In the prediction step, the a priori estimated state vector $\hat{\vec{x}}_k^-$ is evaluated as an ensemble weighted mean, using equation (12) with $W_i = 1/(2n) \forall i = 0, ..., 2n$:

$$\vec{x}_k^- = \frac{1}{2n} \sum_{i=0}^{2n} t\vec{x}_{k-1}^i \quad (16)$$

The a priori error covariance matrix is computed as eqn. (12):

$$\mathbf{P}_k^- = \frac{1}{2n} \sum_{i=0}^{2n} (\vec{x}_k^i - \hat{\vec{x}}_k^-)(\vec{x}_k^i - \hat{\vec{x}}_k^-)^T + \mathbf{Q}_k \quad (17)$$

The a priori error covariance matrix can be used to compute new $2n+1$ sigma points:

$$\begin{cases} \vec{x}_k^0 = & \hat{\vec{x}}_k^- \\ \vec{x}_k^i = & \hat{\vec{x}}_k^- + (\sqrt{n\mathbf{P}_k^-})_i \quad \forall i = 1,2,...,n \\ \vec{x}_k^{i+n} = & \hat{\vec{x}}_k^+ - (\sqrt{n\mathbf{P}_k^-})_i \quad \forall i = 1,2,...,n \end{cases} \tag{18}$$

The new sigma points can be used to compute $\vec{y}_k^i$ vector, by using the measurement equation $\vec{y}_k^i = h(\vec{x}_k^i)$. The predicted output vector is computed as an ensemble mean:

$$\hat{\vec{y}}_k = \frac{1}{2n}\sum_{i=0}^{2n} \vec{y}_k^i \tag{19}$$

The covariance and cross covariance of predicted output, $\mathbf{P}_y$ and cross covariance $\mathbf{P}_{xy}$ respectively, between $\hat{\vec{x}}_k^-$ and $\hat{\vec{y}}_k^-$ are:

$$\mathbf{P}_{y,k} = \frac{1}{2n}\sum_{i=0}^{2n} (\vec{y}_k^i - \hat{\vec{y}}_k)(\vec{y}_k^i - \hat{\vec{y}}_k)^T + \mathbf{R}_k \tag{20}$$

$$\mathbf{P}_{xy,k} = \frac{1}{2n}\sum_{i=0}^{2n} (\vec{x}_k^i - \hat{\vec{x}}_k)(\vec{y}_k^i - \hat{\vec{y}}_k)^T \tag{21}$$

The Kalman gain $\mathbf{K}_k$ is computed as $\mathbf{K}_k = \mathbf{P}_{xy,k}\mathbf{P}_{y,k}^{-1}$, and the state estimation can be updated as:

$$\hat{\vec{x}}_k^+ = \hat{\vec{x}}_k^- + \mathbf{K}_k(\vec{y}_k - \hat{\vec{y}}_k) \tag{22}$$

Finally, error covariance matrix is corrected as follows:

$$\mathbf{P}_k^+ = \mathbf{P}_k^- - \mathbf{K}_k\mathbf{P}_y\mathbf{K}_k^T \tag{23}$$

IV. Fault Tolerant Attitude Estimation

We assume that the UAV is equipped with redunded IMUs, each of them equipped with an UKF giving as output the following set of measurements and estimates at time t: $\tilde{\vec{a}}_i(t)$, $\tilde{\vec{\Omega}}_i(t)$, $\tilde{\vec{M}}_i(t)$ accelerometers, gyros and magnetometers measurement; $\hat{\vec{q}}_i(t)$ quaternions; $\hat{\vec{b}}_i(t)$ gyroscope bias; $\hat{\vec{a}}_i(t)$ accelerations; $\hat{\vec{M}}_i(t)$ magnetometer output estimations.

A so-called triplex fault tolerant system can be completely based on HW redundancy. Generally speaking if $\tilde{\vec{z}}_i$, $i = 1,2,3$, are three independent measurements of $\vec{z}$, the arithmetic mean among the three is $\vec{z}_m = (\tilde{\vec{z}}_1 + \tilde{\vec{z}}_2 + \tilde{\vec{z}}_3)/3$ and the voted measurement can then be assumed as the arithmetic between the nearest two as $\vec{z}_t = (\tilde{\vec{z}}_i + \tilde{\vec{z}}_j)/2$, where $i, j \neq k$,

$$k = \underset{l\in\{1,2,3\}}{\text{argmax}}(|\vec{z}_t - \tilde{\vec{z}}_l|)\,.$$

If the IMU is physically redunded twice (duplex HW), model based software redundancy can be implemented using the UKF reconstruction. A sensor reconfiguration logic has to be implemented to achieve FDIR. This can be formulated in terms of three parallel *event driven* systems for the accelerometers (subscript A), gyroscope (subscript G), and magnetometers (subscript M), respectively, based on the following sets of logical states

$$X_A = \{N_A, A_A, F_{A1}, F_{A2}\}, \quad X_G = \{N_G, A_G, F_{G1}, F_{G2}\}$$
$$X_M = \{N_M, A_M, F_{M1}, F_{M2}\}$$

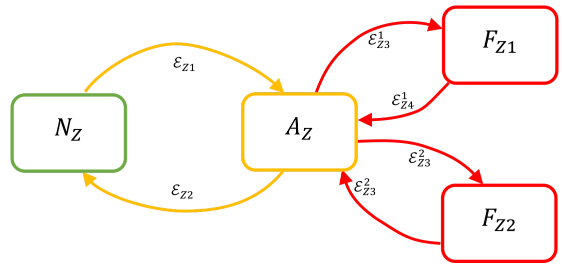

Fig. 1. State transition graph of the state Automaton for Z measurement fault detection

where N_Z is the normal state for sensor $Z = A$, G, M, A_Z, is an alert state activated on the basis of preliminary anomalies of sensor Z, F_{Zj} is a fault state for the IMU unit $j \in \{1,2\}$ based on sensor Z information.

The state transition graph for a generic sensor Z is shown in Figure 1. Follows a list of the events driving the state transition of the proposed Automata. The transition from normal N to alert A state (and viceversa) is determined by the occurence that, for a certain number of samples, the outputs of the two IMUs have a distance greater (lower) than a given threshold, considering that, for gyro measurements the estimated bias $\hat{\vec{b}}_{0i}$ at the initial time has to be taken into account

$$\mathcal{E}_{A1}: \quad \|\tilde{\vec{A}}_{1h} - \tilde{\vec{A}}_{2h}\| > \delta_{A1}, \quad \forall h \in [k - k_{A1}, k] \tag{24}$$
$$\mathcal{E}_{A2}: \quad \|\tilde{\vec{A}}_{1h} - \tilde{\vec{A}}_{2h}\| \le \delta_{A2}, \quad \forall h \in [k - k_{A2}, k] \tag{25}$$
$$\mathcal{E}_{G1}: \quad \|\hat{\vec{G}}_{1h} - \hat{\vec{G}}_{2h}\| > \delta_{G1}, \quad \forall h \in [k - k_{G1}, k] \tag{26}$$
$$\mathcal{E}_{G2}: \quad \|\hat{\vec{G}}_{1h} - \hat{\vec{G}}_{2h}\| \le \delta_{G2}, \quad \forall h \in [k - k_{G2}, k] \tag{27}$$
$$\mathcal{E}_{M1}: \quad \|\tilde{\vec{M}}_{1h} - \tilde{\vec{M}}_{2h}\| > \delta_{M1}, \quad \forall h \in [k - k_{M1}, k] \tag{28}$$
$$\mathcal{E}_{M2}: \quad \|\tilde{\vec{M}}_{1h} - \tilde{\vec{M}}_{2h}\| \le \delta_{M2}, \quad \forall h \in [k - k_{M2}, k] \tag{29}$$

where $\hat{\vec{G}}_{ih} = \tilde{\vec{G}}_{ih} + \hat{\vec{b}}_{0i}$, $i = 1,2$

As for the transition from the alert to the faulted state, a fault on the $j-th$ IMU is declared if one of the following conditions holds. The magnetometer measurement has a norm which differs from the norm of the earth magnetic field for a given number of samples.

$$\mathcal{E}^i_{M3}: \quad \|\tilde{\vec{M}}_{ih} - M_E\| > \delta_{M3i}, \quad \forall h \in \left[k - k^i_{M3}, k\right], i = 1,2 \tag{30}$$
$$\mathcal{E}^i_{M4}: \quad \|\tilde{\vec{M}}_{ih} - M_E\| \le \delta_{M4i}, \quad \forall h \in \left[k - k^i_{M4}, k\right], i = 1,2 \tag{31}$$

The norm of the difference between measured and estimated accelerations or angular speeds (including gyro estimated biases) is greater than a given thresholds

$$\mathcal{E}^i_{G3}: \quad \|\tilde{\vec{G}}_{ih} - \hat{\vec{G}}_{ih}\| > \delta_{G3i}, \quad \forall h \in \left[k - k^i_{G3}, k\right], i = 1,2 \tag{32}$$
$$\mathcal{E}^i_{G4}: \quad \|\tilde{\vec{G}}_{ih} - \hat{\vec{G}}_{ih}\| \le \delta_{G4i}, \quad \forall h \in \left[k - k^i_{G4}, k\right], i = 1,2 \tag{33}$$
$$\mathcal{E}^i_{A3}: \quad \|\tilde{\vec{A}}_{ih} - \hat{\vec{A}}_{ih}\| > \delta_{A3i}, \quad \forall h \in \left[k - k^i_{A3}, k\right], i = 1,2 \tag{34}$$
$$\mathcal{E}^i_{A4}: \quad \|\tilde{\vec{A}}_{ih} - \hat{\vec{A}}_{ih}\| \le \delta_{A4i}, \quad \forall h \in \left[k - k^i_{A4}, k\right], i = 1,2 \tag{35}$$

A number of thresholds $\delta_{Z1} \ge \delta_{Z2}$, $\delta_{Z3i} \ge \delta_{Z4i}$, and characteristic discrete times k_{Z1}, k_{Z2}, k_{Z3i}, k_{Z4i}, $(i = 1, 2,$

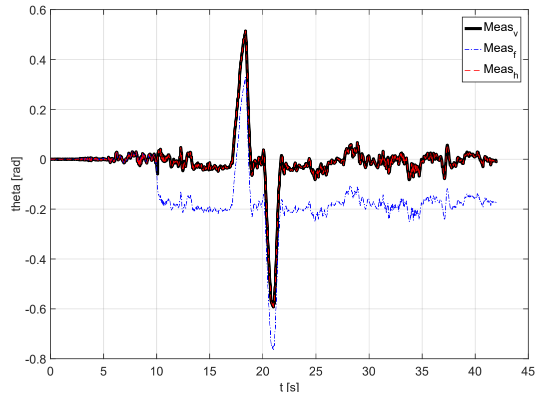

Fig. 2. Bias on a_x: θ estimated by the healthy IMU and the faulted IMU compared with the voted measurement (Fault at t=10s)

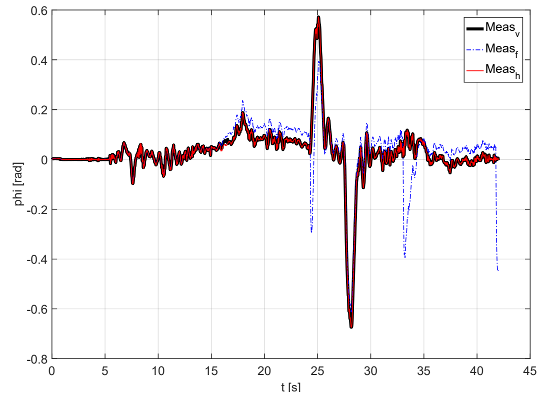

Fig. 3. Drift on Ω_p: ϕ estimated by the healthy IMU and the faulted IMU compared with the voted measurement (Fault at t=15s)

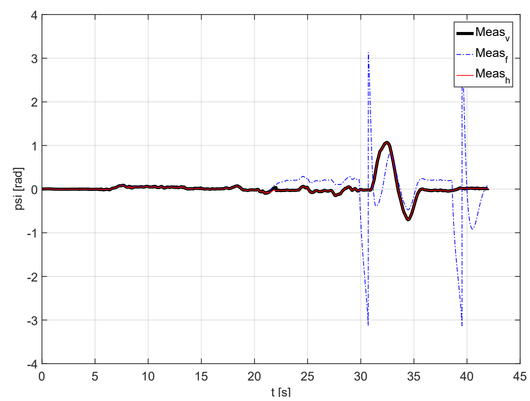

Fig. 4. Drift on Ω_r: ψ estimated by the healthy IMU and the faulted IMU compared with the voted measurement (Fault at t=21s)

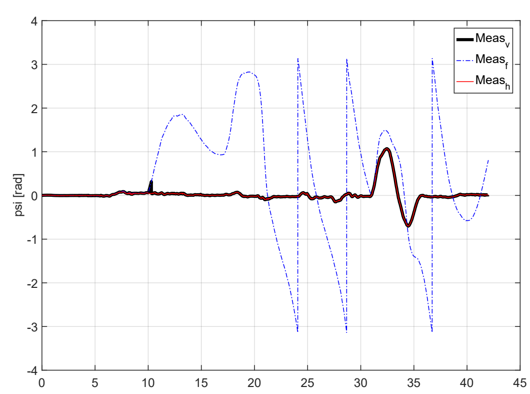

Fig. 5. Set to zero on M_x: ψ estimated by the healthy IMU and the faulted IMU compared with the voted measurement (Fault at t=10s)

$Z = A, G, M$) have to be chosen to tune the detection and isolation system. A fault is declared for the $i-th$ IMU, if the one or more states of the three Automata is F_{Zi} (Z = A, G and/or M). Healty measurements are then acquired from IMU $j \neq i$.

V. Experimental Results and Real Time Implementation

In order to validate the proposed procedure a number of UAV flight have been performed collecting measurements from three different synchronized IMUs sampled at $200Hz$. Three sequential maneuvers have been performed during flight to prove that no false alarms occur during transients.

Four different fault scenarios have been produced on the basis of the experimental measurements, namely:

- $B_Z(t_f, A)$ A bias of amplitude A is added to measured data on Z sensor at time t_f.
- $D_Z(t_f, S)$ A drift with slope S is added $\forall\ t \geq t_f$ to measured data on Z sensor. Tests have been performed considering two possible slopes: a slow and a fast drift.
- $Z_Z(t_f)$: measured data on Z sensor is set to zero $\forall\ t \geq t_f$.
- $F_Z(t_f)$: measured data on Z sensor is frozen at time t_f.

Detection and isolation performance are shown in tables I and II. The detection time estimation t_d is obtained averaging on 20 runs with different fault times t_f; e_{max} is the maximum displacement between estimation and output. The following figures show comparisons between the healty (subscript h), the faulted (subscript f), and the voted (subscript v) signals.

On the other hand to prove the possibility to implement the procedure in real time, it has been implemented on a Raspberry Pi used on-board during flight, connected with two low cost IMU boards, based on ARM Cortex M0 CPU, through USB controller. Beside guidance and control algorithms, two UKF filters were launched in parallel on the same Raspberry Pi to perform attitude estimation. A third thread was devoted to implement the detection and isolation algorithm and vote the healthy measurement to be used by the control system.

	Scenarios with biases			Scenarios with slow drifts			Scenarios with fast drifts		
X	A	$t_d[s]$	$e_{max}[rad]$	S	$t_d[s]$	$e_{max}[rad]$	S	$t_d[s]$	$e_{max}[rad]$
a_x	0.2	0.092	0.066 (θ)	0.2 1/s	0.610	0.055 (θ)	1.0 1/s	0.205	0.062 (θ)
a_y	0.2	0.094	0.073 (ϕ)	0.2 1/s	1.071	0.125 (θ)	1.0 1/s	0.209	0.070 (ϕ)
a_z	0.2	0.077	0.007 (θ)	0.2 1/s	0.421	0.007 (ψ)	1.0 1/s	0.167	0.007 (θ)
Ω_p	1.74 $\frac{rad}{s}$	0.267	0.048 (ϕ)	0.2 $\frac{rad}{s^2}$	2.787	0.012 (θ)	1.0 $\frac{rad}{s^2}$	0.999	0.019 (ϕ)
Ω_q	1.74 $\frac{rad}{s}$	0.245	0.048 (θ)	0.2 $\frac{rad}{s^2}$	2.643	0.012 (θ)	1.0 $\frac{rad}{s^2}$	0.954	0.020 (θ)
Ω_r	1.74 $\frac{rad}{s}$	0.336	0.154 (ψ)	0.2 $\frac{rad}{s^2}$	2.075	0.026 (ψ)	1.0 $\frac{rad}{s^2}$	0.918	0.071 (ψ)
M_x	0.2	0.107	0.010 (θ)	0.2 1/s	0.734	0.013 (ψ)	1.0 1/s	0.234	0.011 (θ)
M_y	0.2	0.098	0.043 (ψ)	0.2 1/s	0.650	0.060 (ψ)	0.2 1/s	0.650	0.047 (ψ)
M_z	0.2	0.153	0.009 (θ)	0.2 1/s	0.802	0.010 (θ)	0.2 1/s	0.256	0.009 (θ)

TABLE I

	Scenarios $Z_X(t_f)$		Scenarios $F_X(t_f)$	
X	$t_d[s]$	$e_{max}[rad]$	$t_d[s]$	$e_{max}[rad]$
a_x	0.076	0.122 (θ)	0.313	0.086 (θ)
a_y	0.184	0.152 (ϕ)	0.243	0.073 (ϕ)
a_z	0.060	0.020 (θ)	0.437	0.005 (θ)
Ω_p	1.285	0.037 (ϕ)	0.566	0.040 (ϕ)
Ω_q	0.846	0.034 (θ)	0.963	0.034 (θ)
Ω_r	0.587	0.109 (ψ)	0.888	0.124 (ψ)
M_x	1.339	0.188 (ψ)	0.365	0.008 (θ)
M_y	0.231	0.040 (ψ)	0.435	0.044 (ψ)
M_z	0.075	0.028 (θ)	0.570	0.006 (θ)

TABLE II

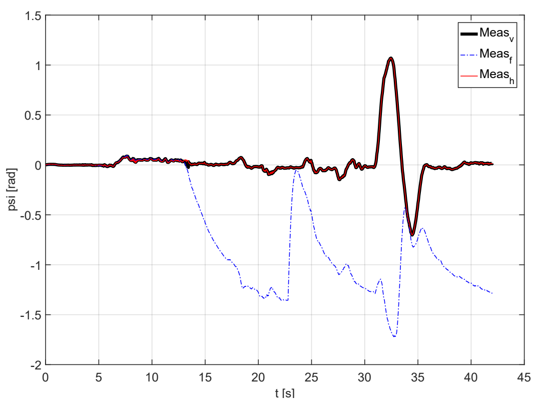

Fig. 6. Drift on M_y: ψ estimated by the healthy IMU and the faulted IMU compared with the voted measurement (Fault at t=12s)

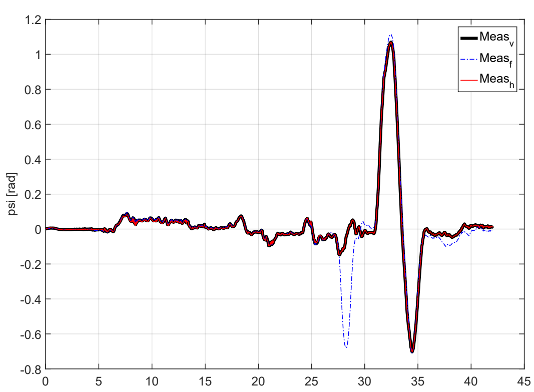

Fig. 7. Set to zero on M_z: ψ estimated by the healthy IMU and the faulted IMU compared with the voted measurement (Fault at t=26s)

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