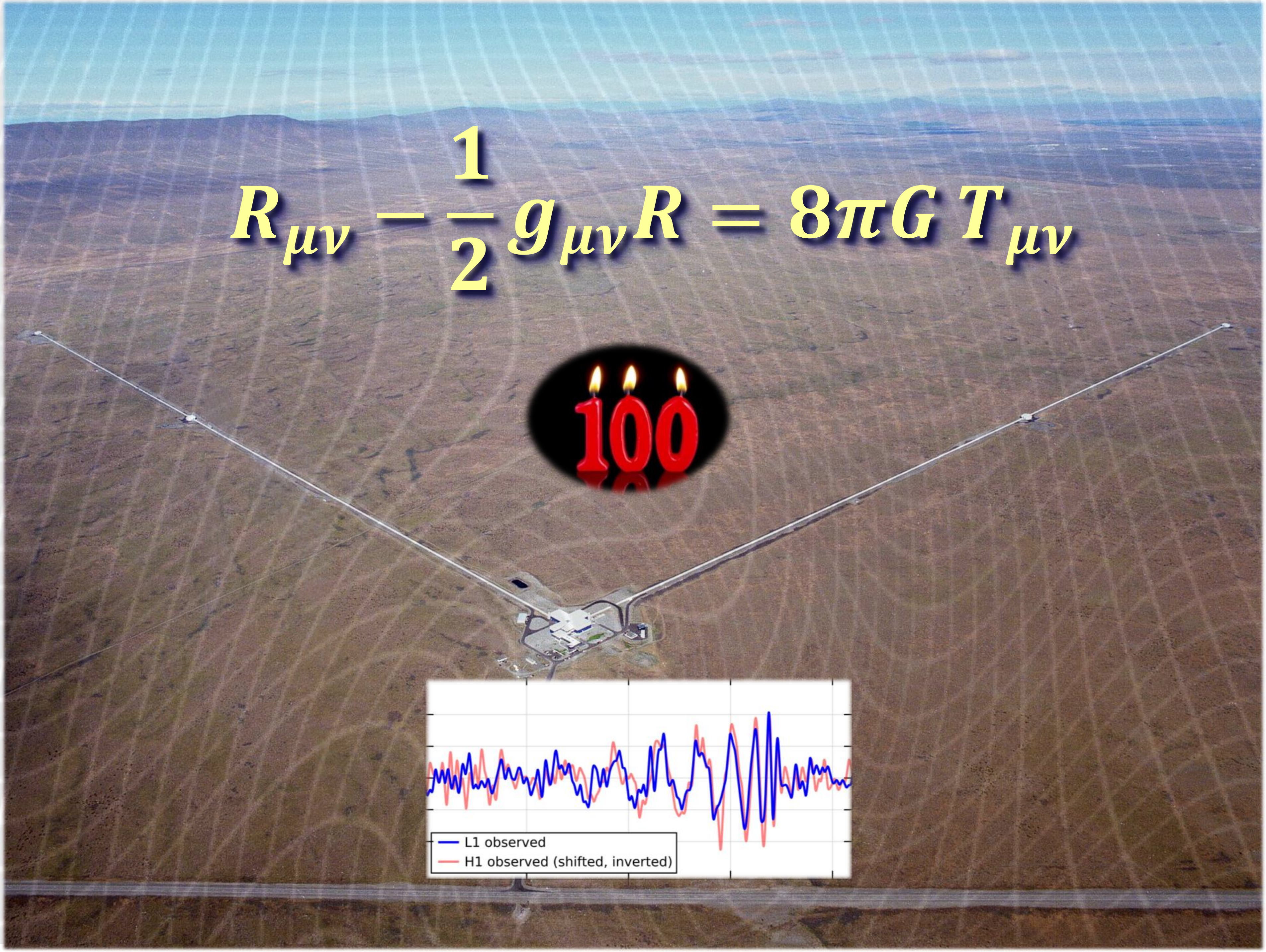
Black Hole fusion made easy

Roberto Emparan
ICREA & UBarcelona

w/ Marina Martínez

CQG 33, 155003 (2016)
arXiv:1603.00712

Iberian Relativity Meeting ERI 2016
Lisboa


$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 8\pi G\, T_{\mu\nu}$
100
L1 observed
H1 observed (shifted, inverted)

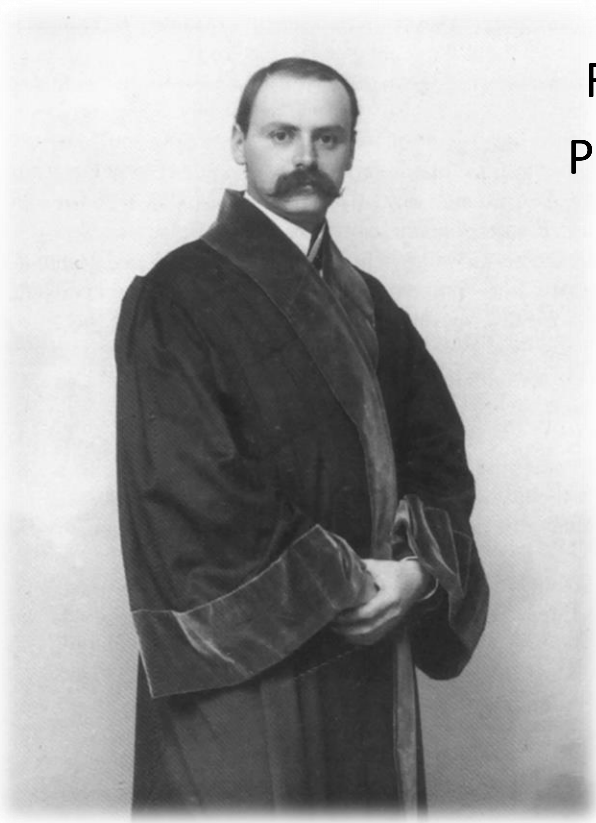
What this talk is all about

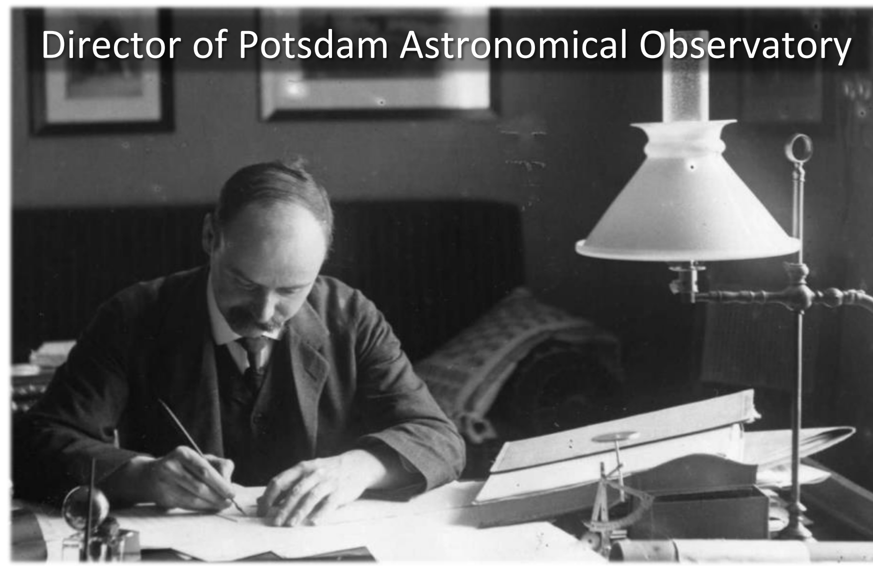
$$ds^2 = -\left(1 - \frac{2GM}{r}\right)dt^2 + \frac{dr^2}{1 - \frac{2GM}{r}} + r^2(d\theta^2 + \sin^2\theta d\phi^2)$$

13 Jan 1916

Frankfurt 9 Oct 1873

Potsdam 11 May 1916





“**mathematics, physics, and astronomy** constitute one knowledge, ... which is only comprehended as a perfect whole”

In 1900 he put astronomical bounds on the curvature radius of space

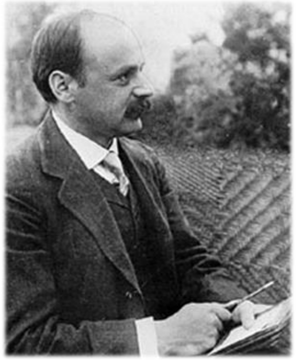
- 64 light-years if hyperbolic
- 1600 light-years if elliptic

1914: volunteers for war

- Belgium: weather station
- France, Russia: artillery trajectories

March 1916: sent back home, ill with pemphigus—dies in May

K Schwarzschild to A Einstein (letter dated **22 December 1915**)

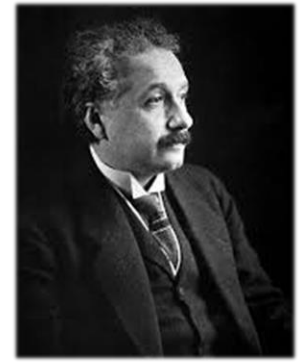


"I made at once by good luck a search for a full solution. A not too difficult calculation gave the following result: ..."

$$ds^2 = \left(1 - \frac{\gamma}{R}\right) dt^2 - \frac{dR^2}{1 - \frac{\gamma}{R}} - R^2(d\theta^2 + \sin^2\theta d\phi^2)$$

A Einstein to K Schwarzschild

(early **January 1915**)



"I had not expected that one could formulate the exact solution of the problem in such a simple way. Next Thursday I shall present the work to the Academy"

Two more articles before he dies:

GR: interior solution for star

Quantum Theory

A Einstein to M Besso

(14 May 1916)

"Schwarzschild is a real loss. He would have been a gem, had he been as decent as he was clever"

Schwarzschild's solution has been rediscovered many times over

J Droste May 1916 (part of PhD thesis under Lorentz – first worked on 1913 *Entwurf* theory)

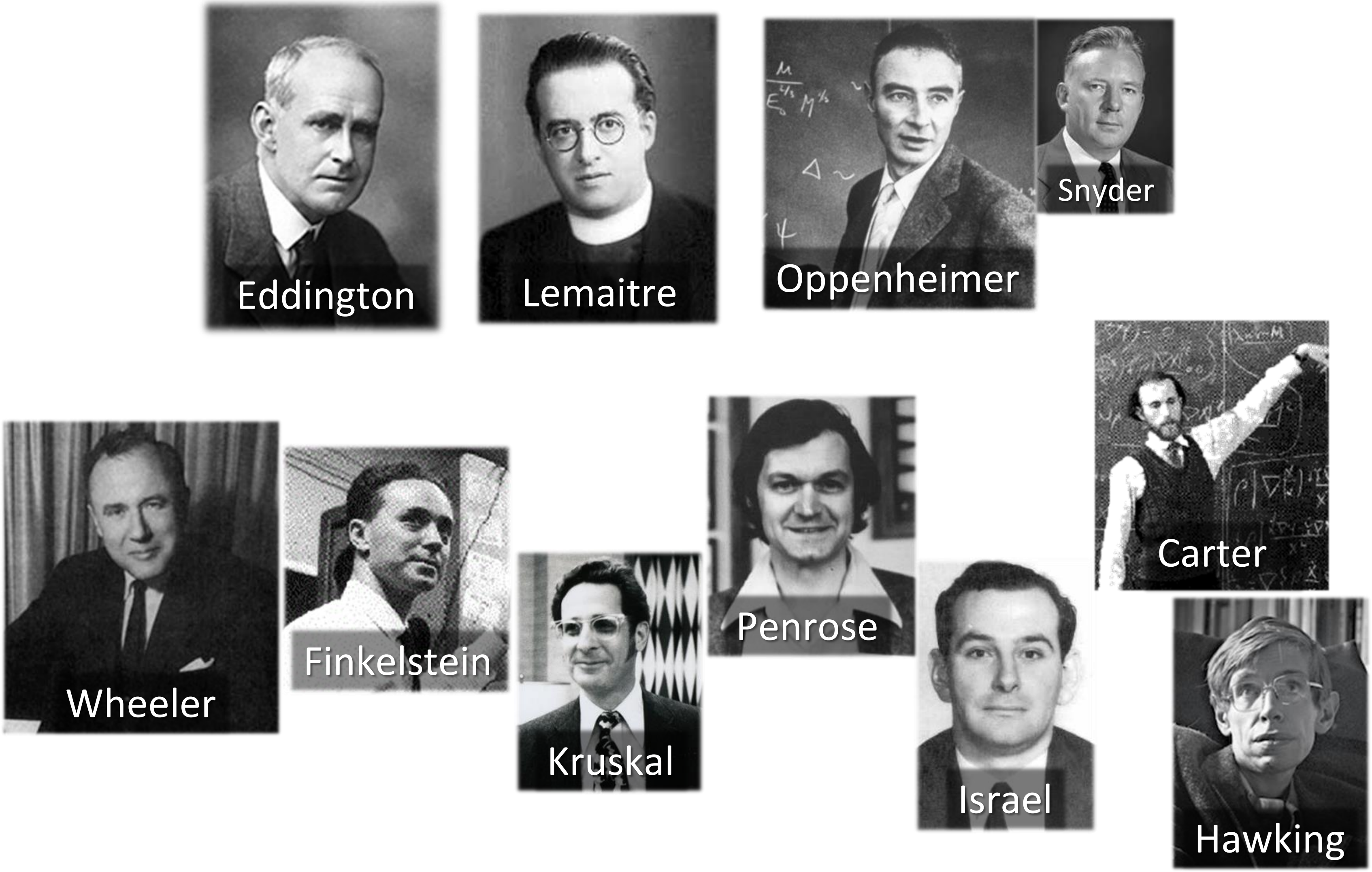
same coordinates

P Painlevé 1921, A Gullstrand 1922

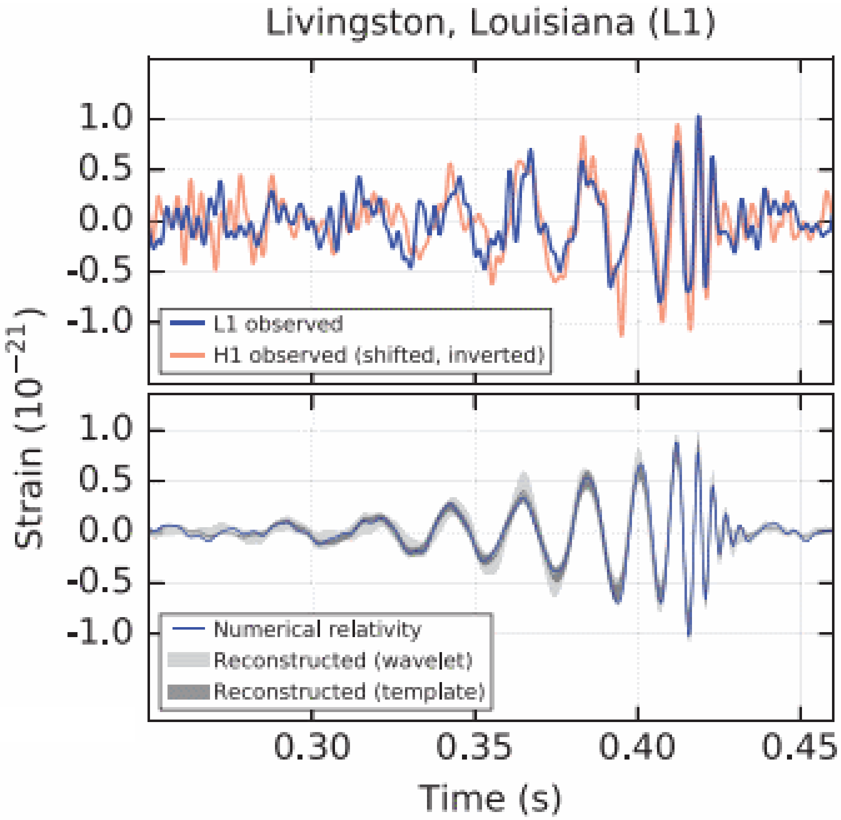
P-G coordinates (didn't realize it was the same as Schw's)

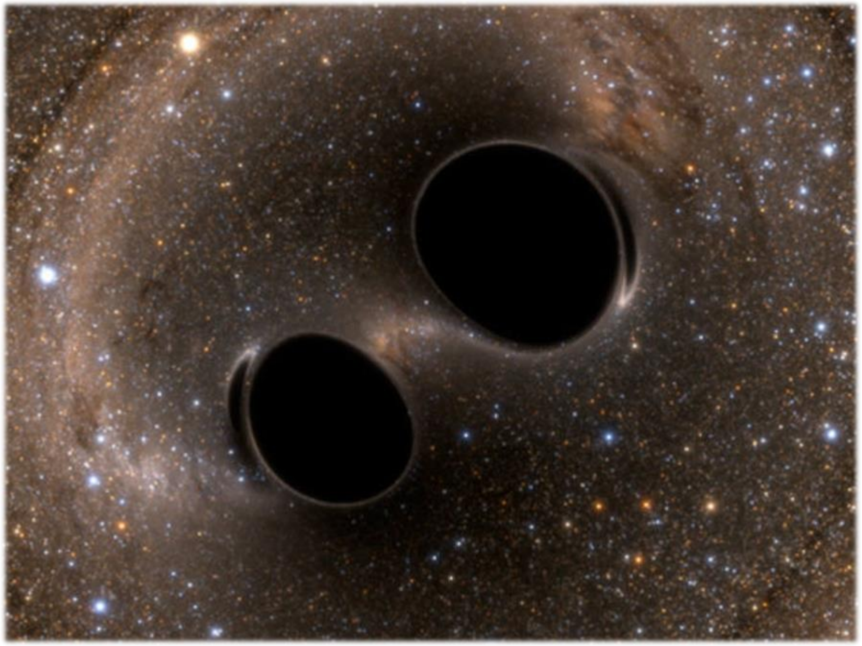
and others

Long, complex, painful path to correct interpretation



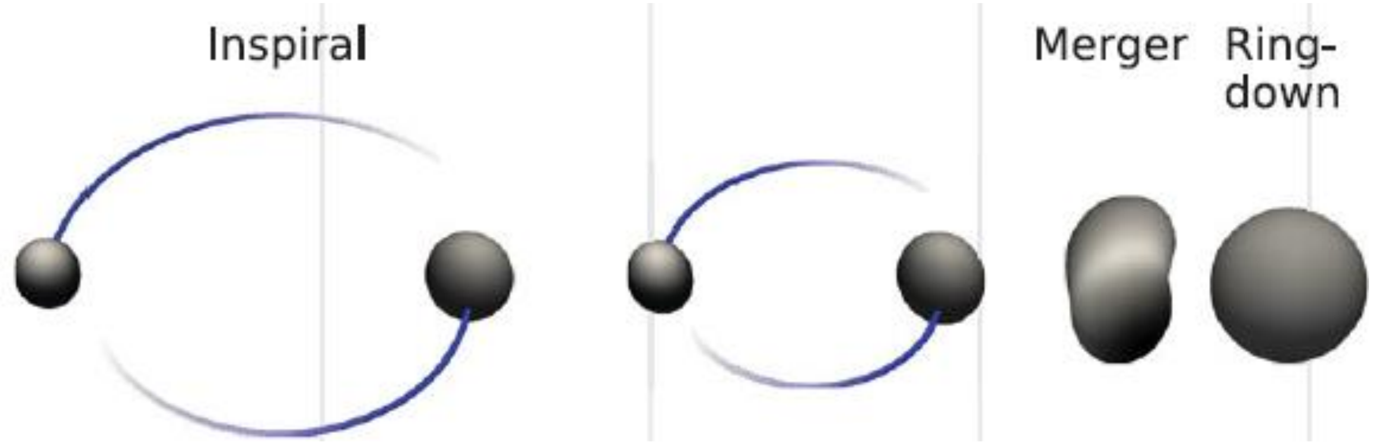
Black holes exist in Nature





Governed by conceptually simple and beautiful equations

$$R_{\mu\nu} = 0$$



Governed by conceptually simple and beautiful equations

$$R_{\mu\nu} = 0$$

but exceedingly hard to solve

Merger is most complicated of all

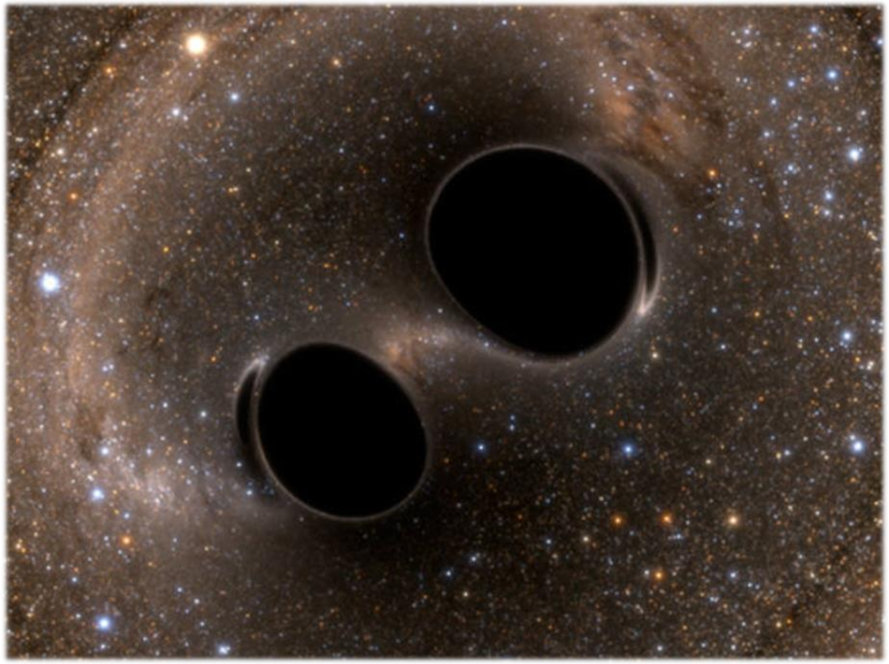
Involves non-linearity of Einstein's equations at its most fiendish

Merger is most complicated of all

Involves non-linearity of Einstein's equations at its most fiendish

or maybe not—not always

This is what you'd *see* (lensing)



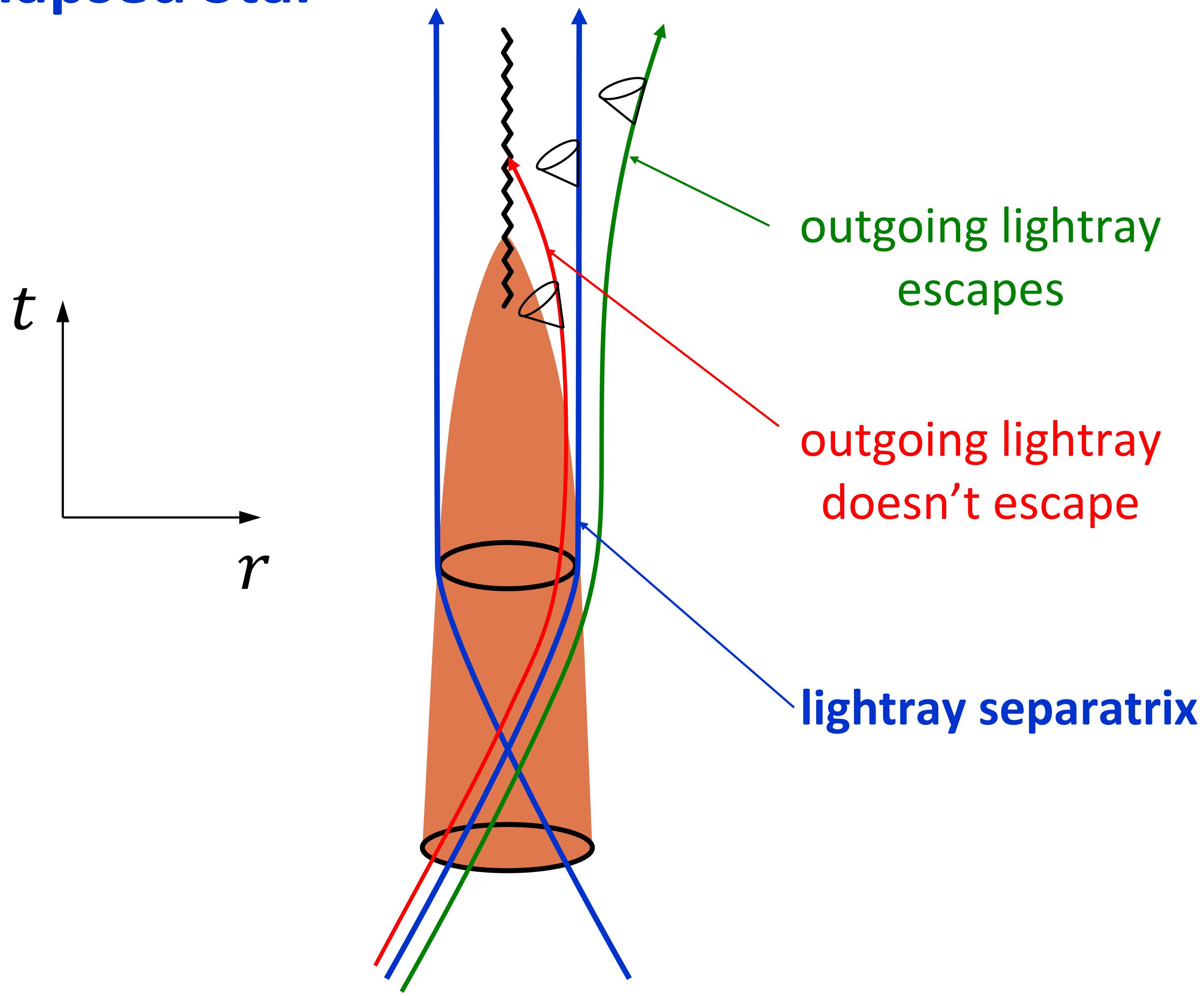
Not a black hole, but its *shadow*

What is a black hole?

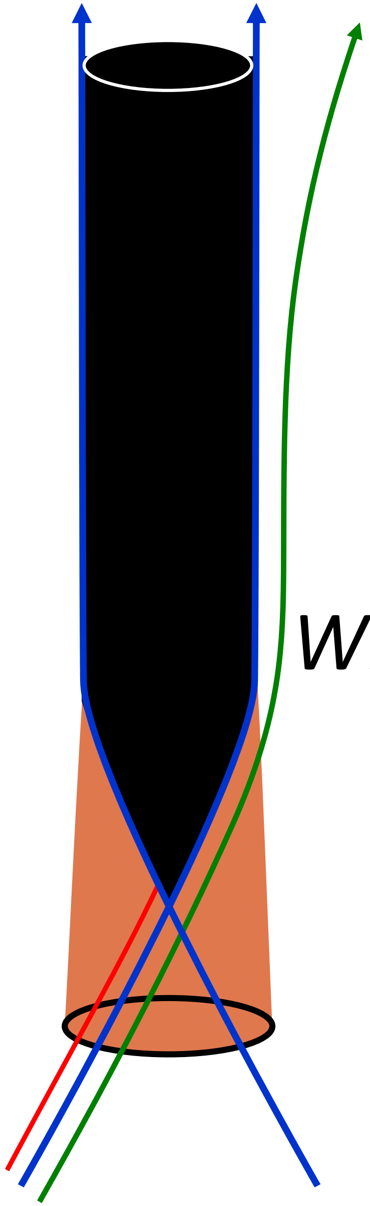
Spacetime region from which not even light can ever escape

Event Horizon

Collapsed Star

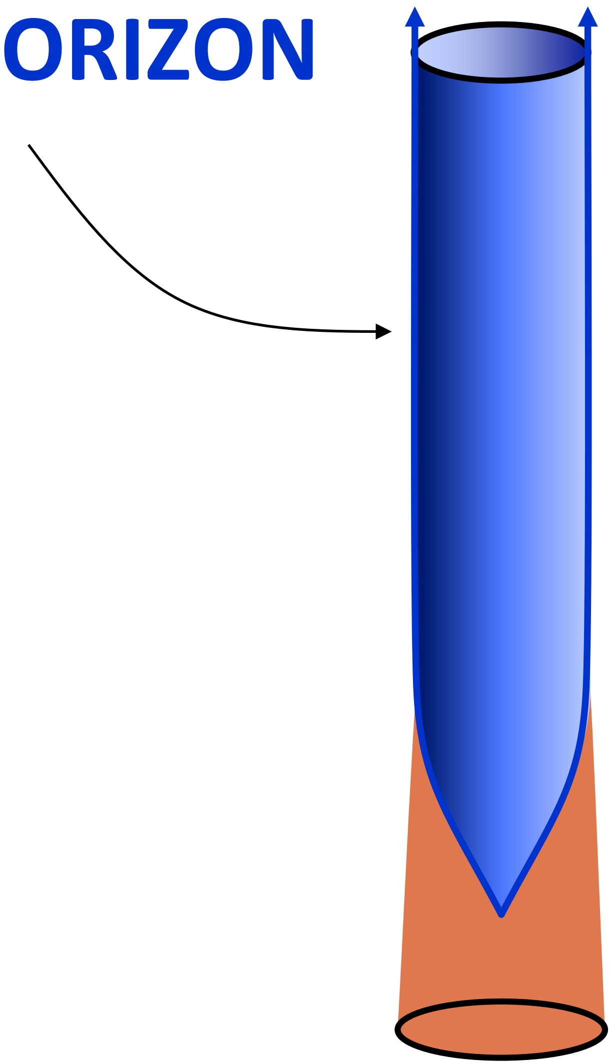


BLACK HOLE



What can never be seen
from outside
(asymptotic infty)

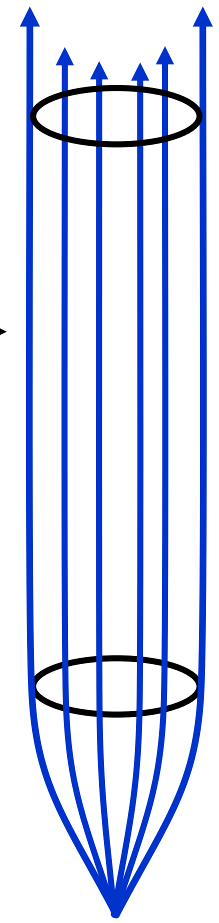
EVENT HORIZON



Null hypersurface

3-dimensional in
4-dimensional spacetime

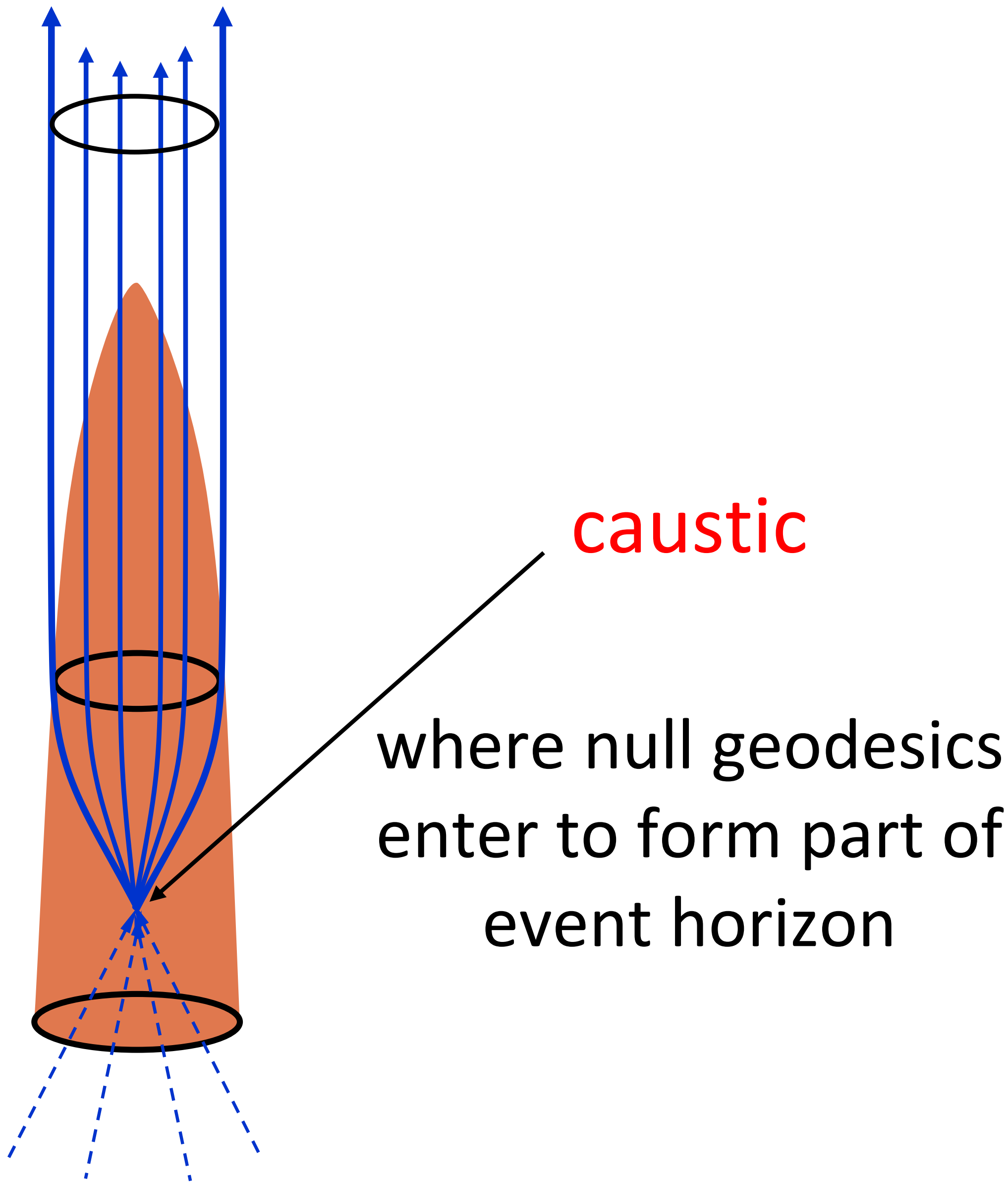
EVENT HORIZON



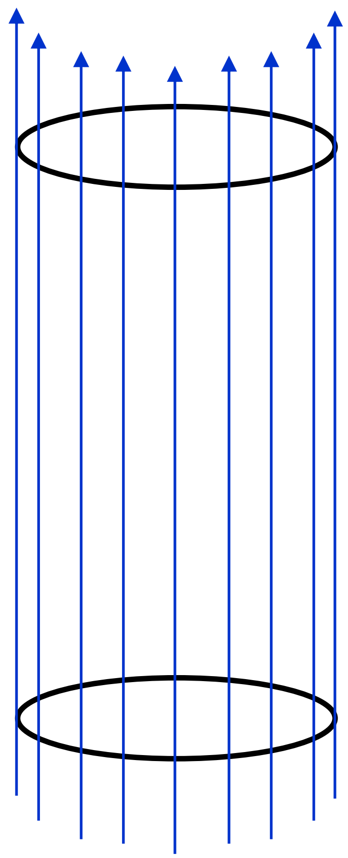
Null hypersurface

<u>made of null geodesics</u>

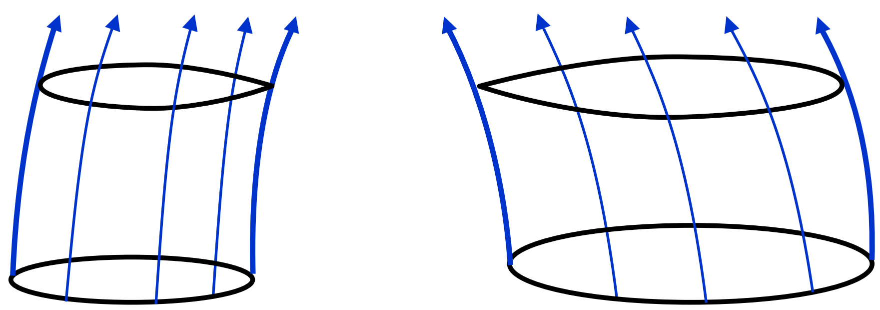
(light rays)


caustic
where null geodesics enter to form part of event horizon

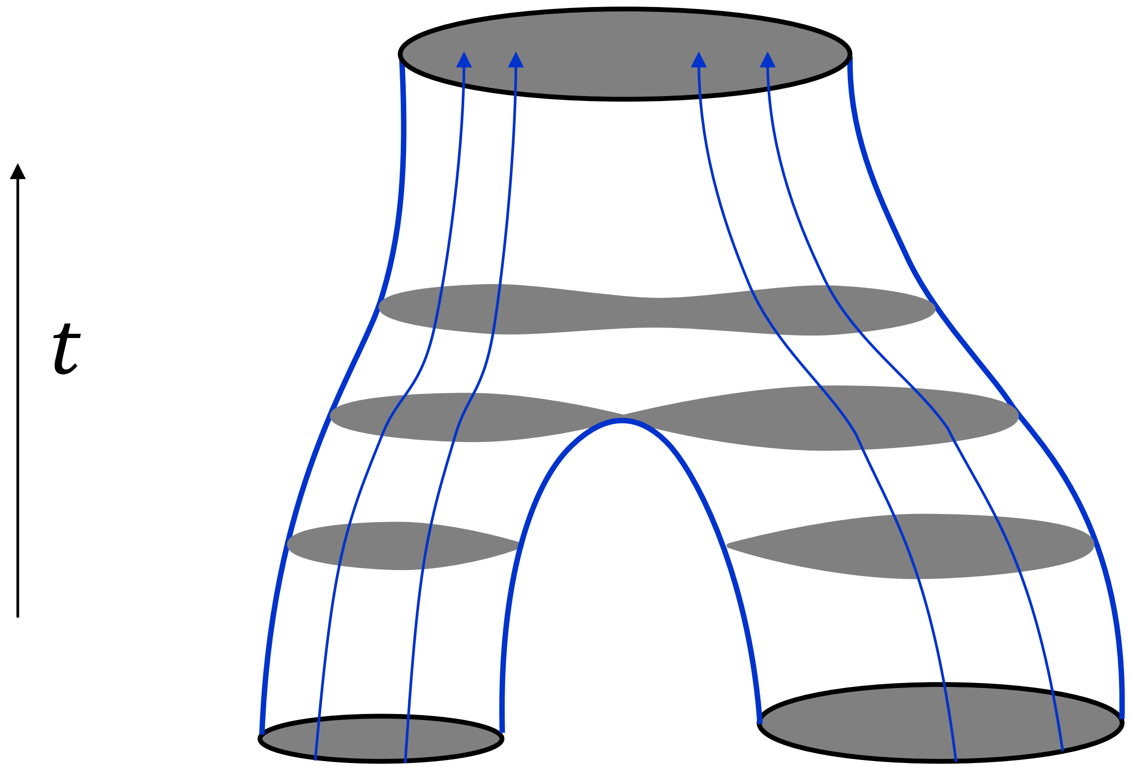
Event horizon is found by tracing a family of light rays in a given spacetime



Event horizon of binary black hole fusion

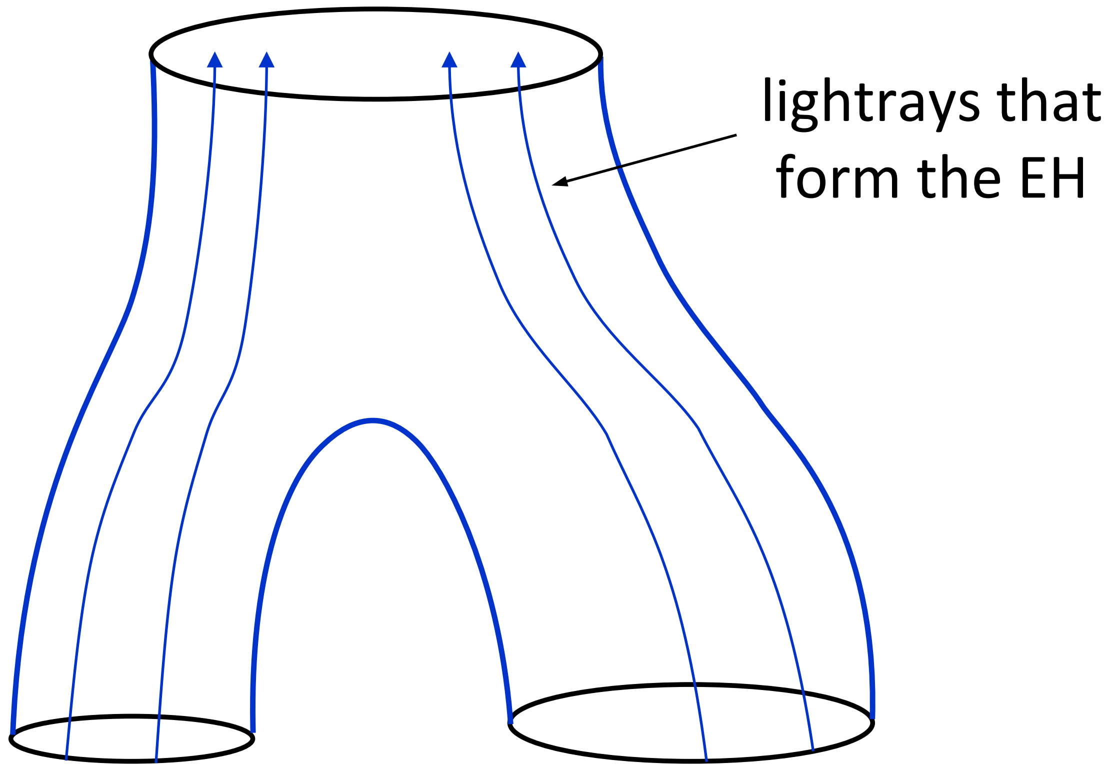


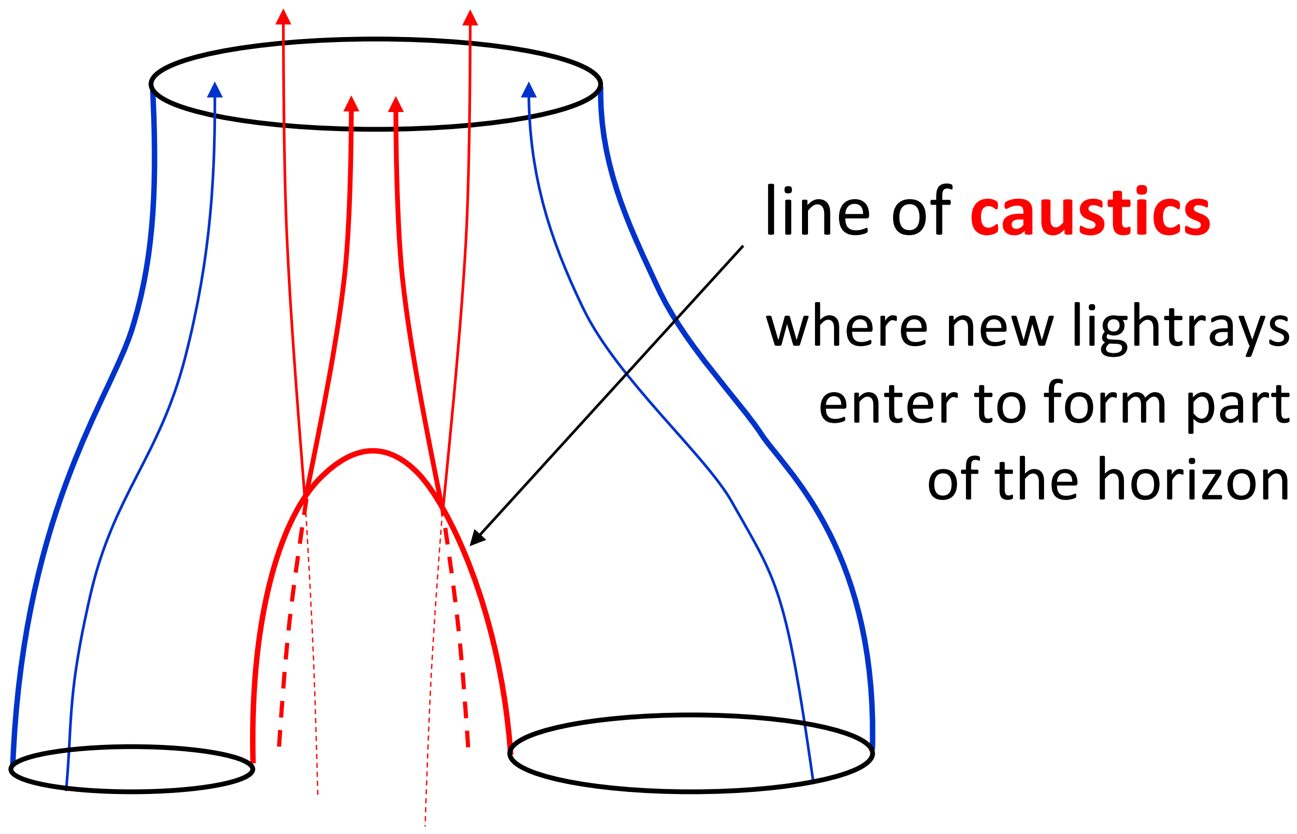
Event horizon of binary black hole fusion



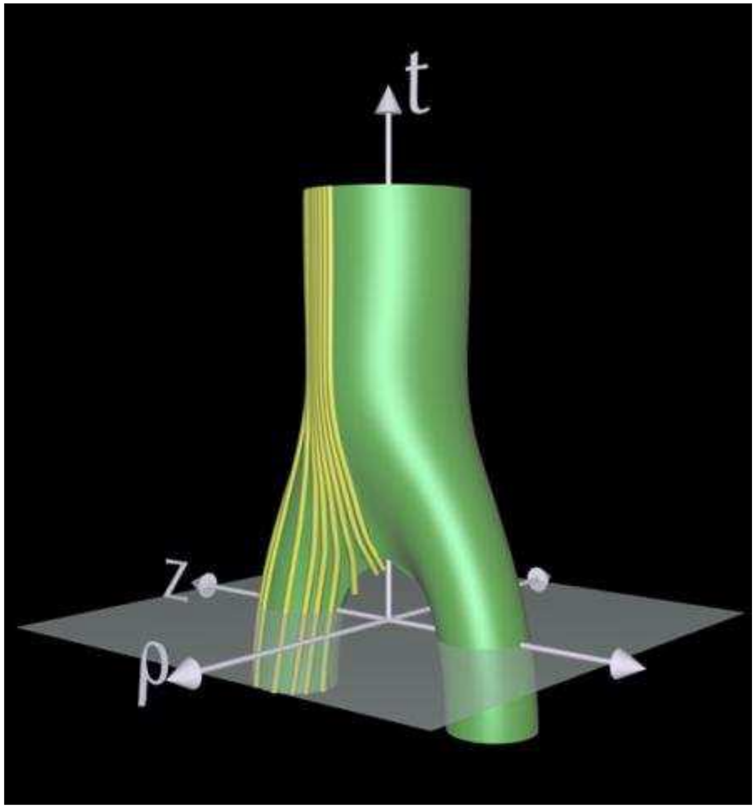
Event horizon of binary black hole fusion

“pants” surface




line of caustics
where new lightrays
enter to form part
of the horizon

Event horizon of binary black hole fusion



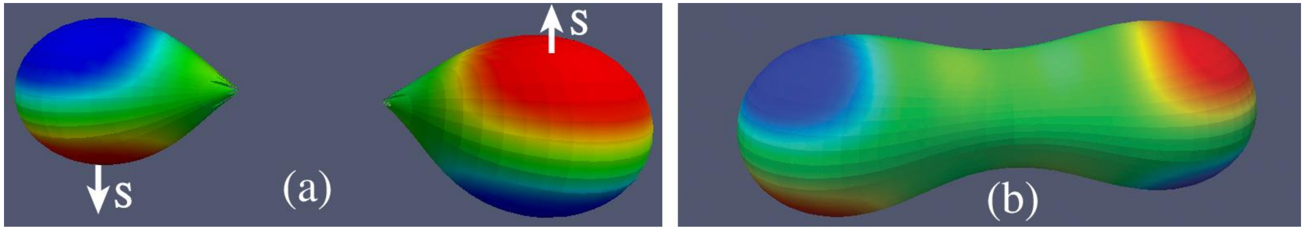
head-on
(axisymmetric)

equal masses

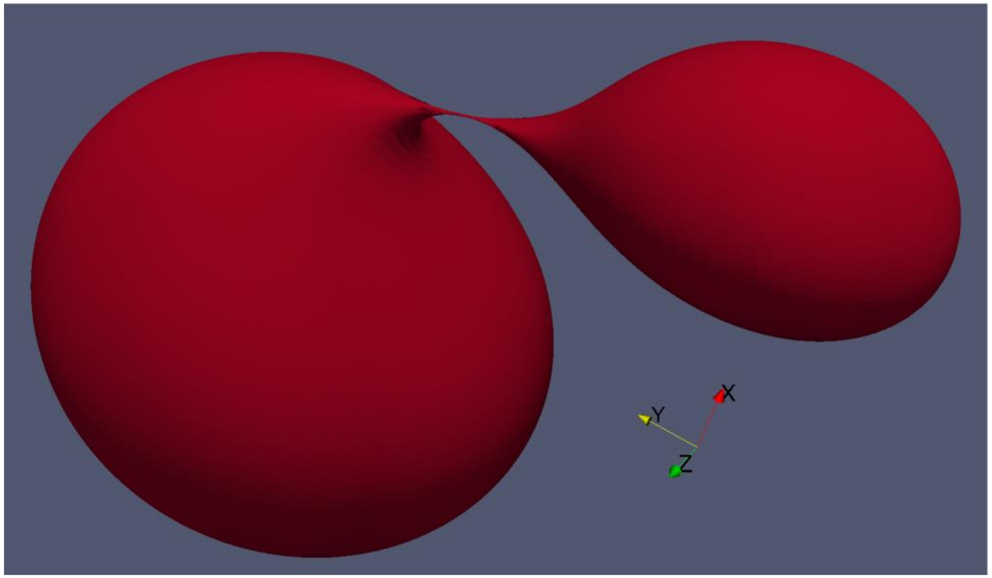
Cover of *Science,* November 10, 1995

Binary Black Hole ***Grand Challenge*** *Alliance* (Matzner et al)

Event horizon of binary black hole fusion



Owen et al, Phys.Rev.Lett. 106 (2011) 151101



Cohen et al, Phys.Rev. D85 (2012) 024031

Surely the fusion of horizons can only be captured with supercomputers

Surely the fusion of horizons can only be captured with supercomputers

or so it’d seem

$\exists$ limiting (but realistic) instance where horizon fusion can be described **exactly**

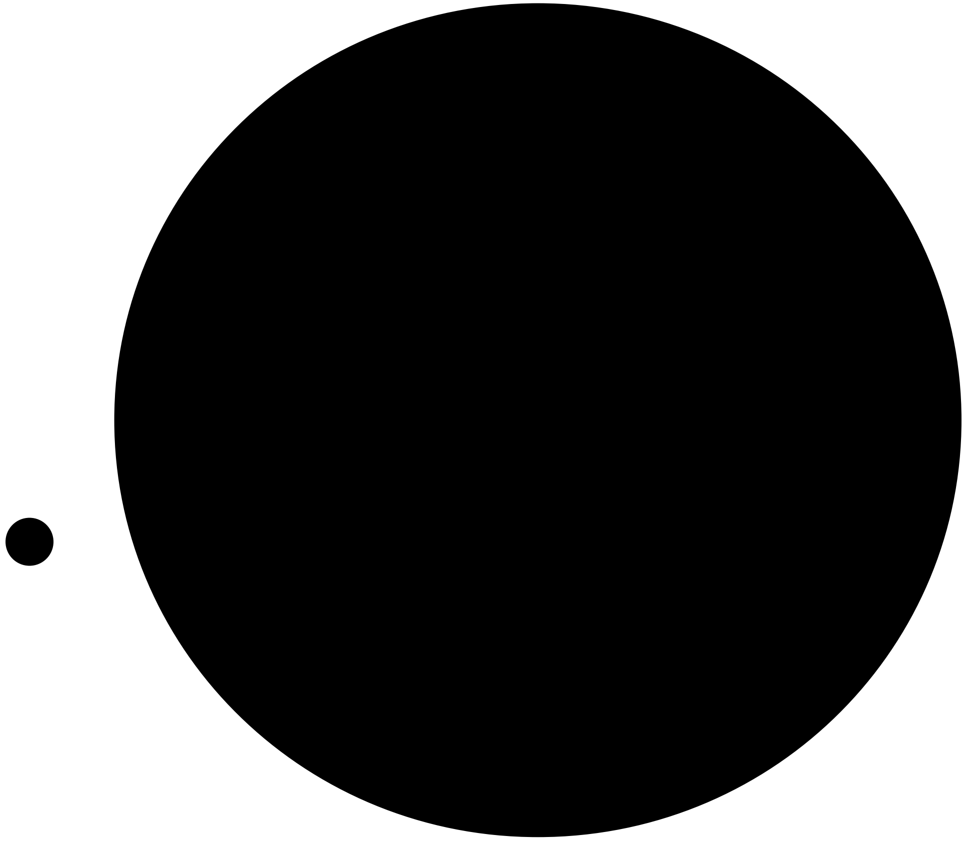
It involves only elementary ideas and techniques

Equivalence Principle (1907)

Schwarzschild solution & Null geodesics (1916)

Notion of Event Horizon (1950s/1960s)

Extreme-Mass-Ratio (EMR) merger



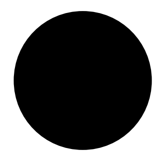
$$m \ll M$$

EMR mergers in the Universe

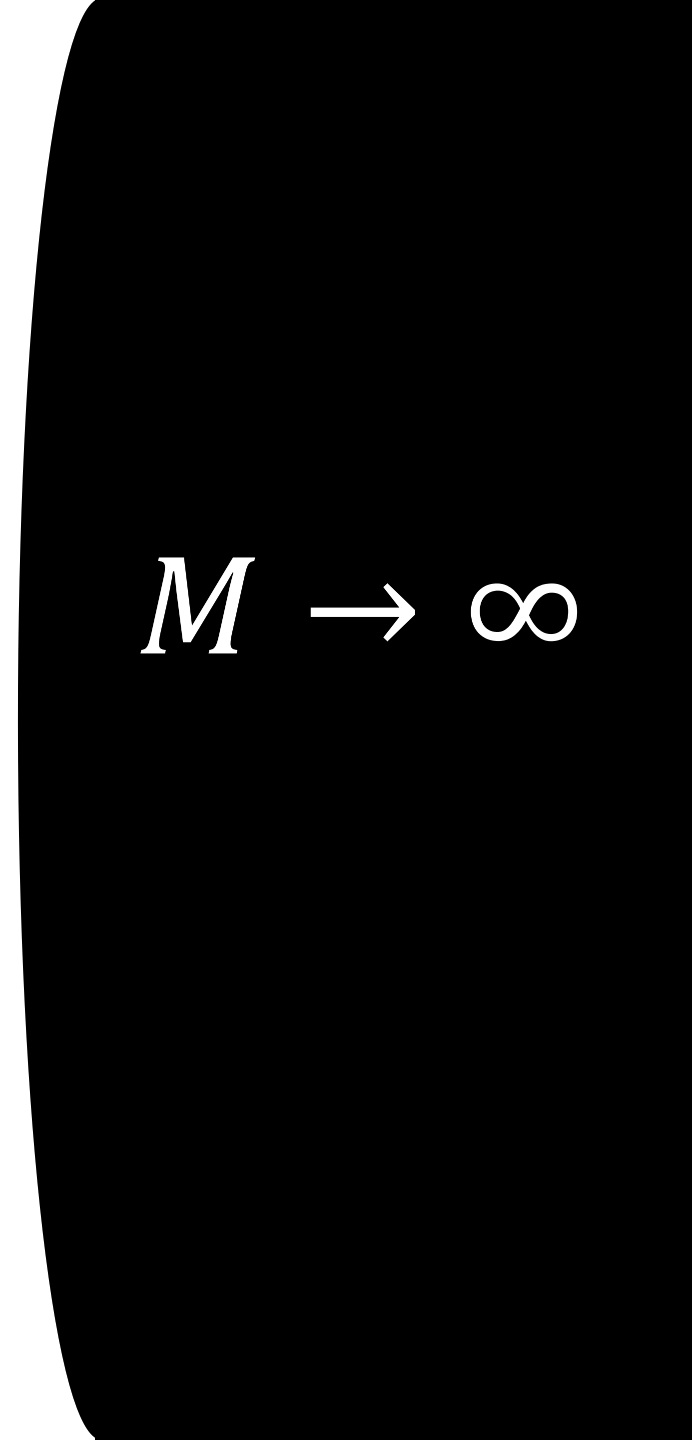
$\frac{m}{M} \simeq \frac{1}{30}$ (or even less) may be detected with LIGO

$\frac{m}{M} \simeq 10^{-4} - 10^{-8}$ or less may be detected with LISA

Fusion of horizons involves scales $\sim m$

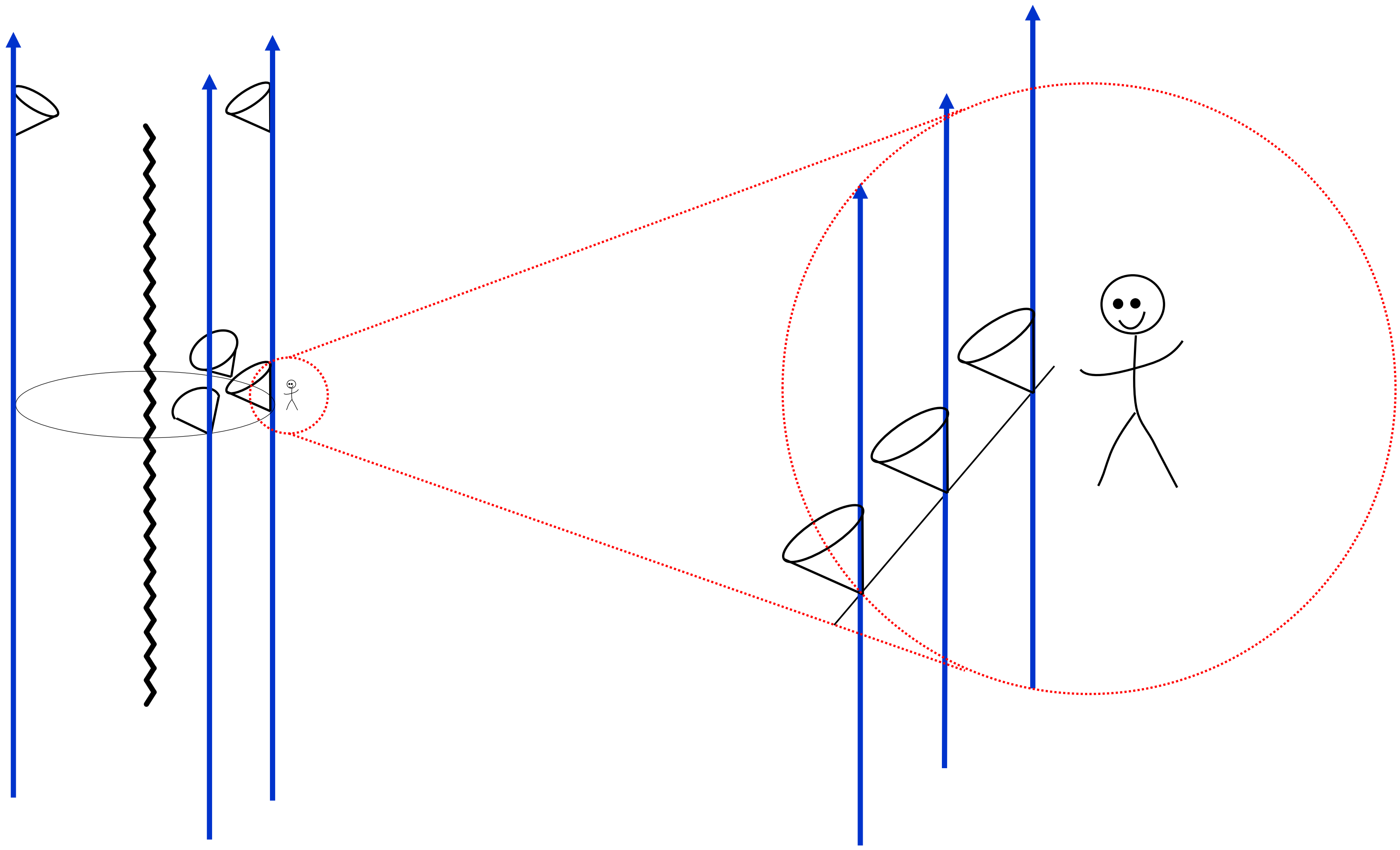


m finite



$$\boxed{M \to \infty}$$

Very large black hole / Very close to the horizon



Very close to a Black Hole

Horizon well approximated by null plane in Minkowski space

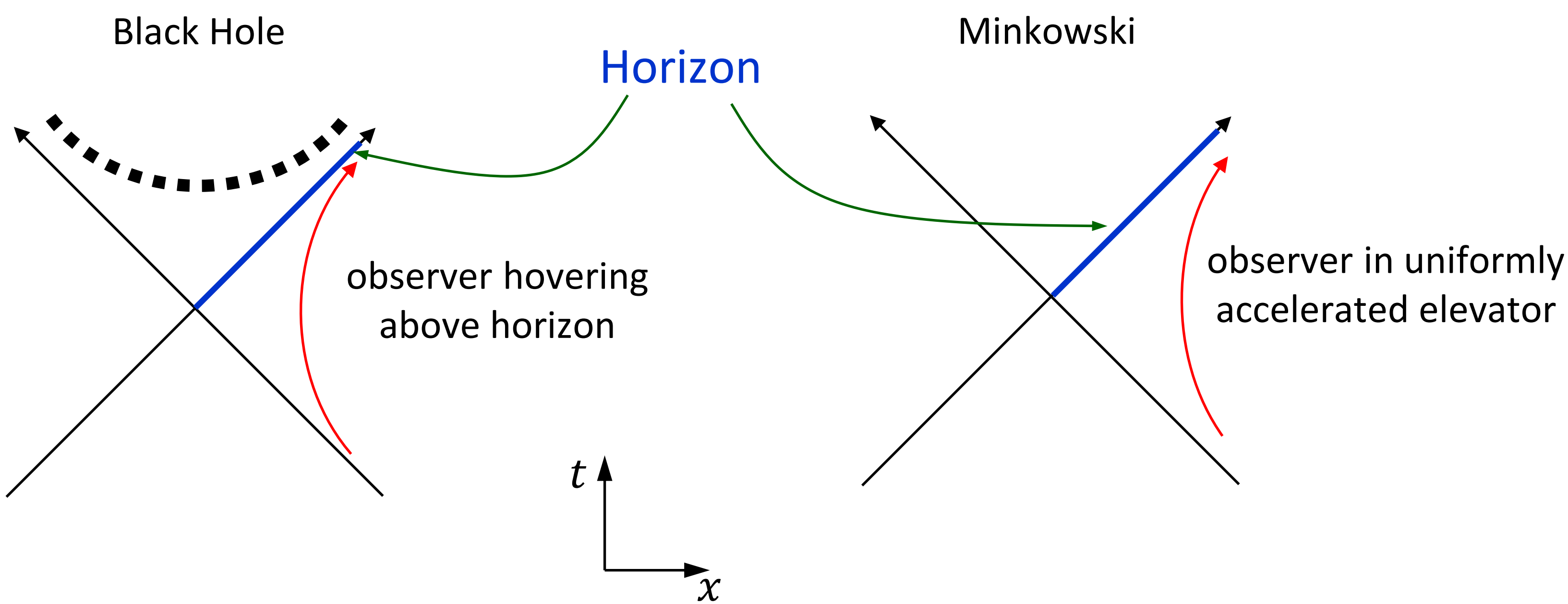
This follows from
the Equivalence Principle

At short enough scales, geometry is
equivalent to flat Minkowski space

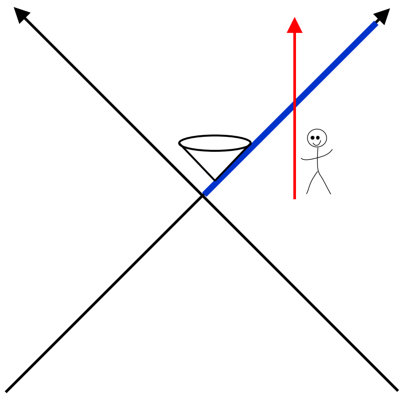
Curvature effects become small,
but horizon remains

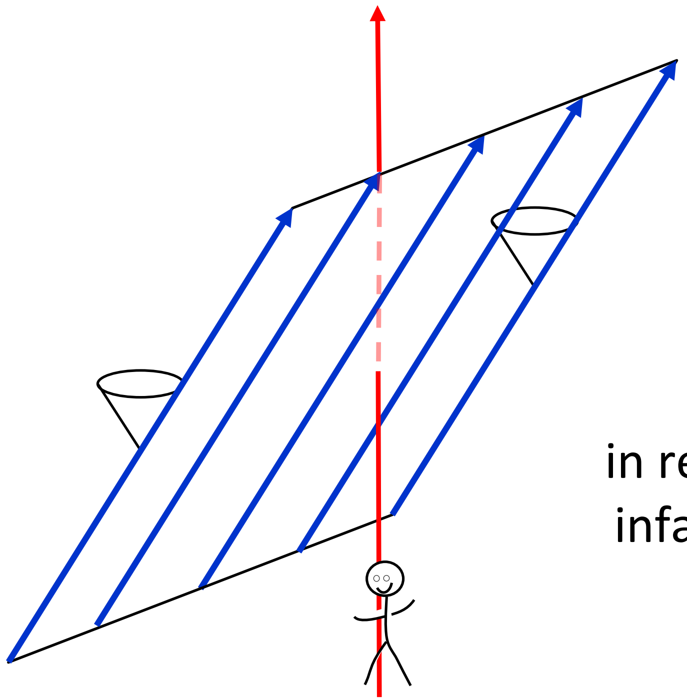
Locally gravity is equivalent to acceleration

Locally black hole horizon is equivalent to acceleration horizon



Falling into very large bh = crossing a null plane in Minkowski space





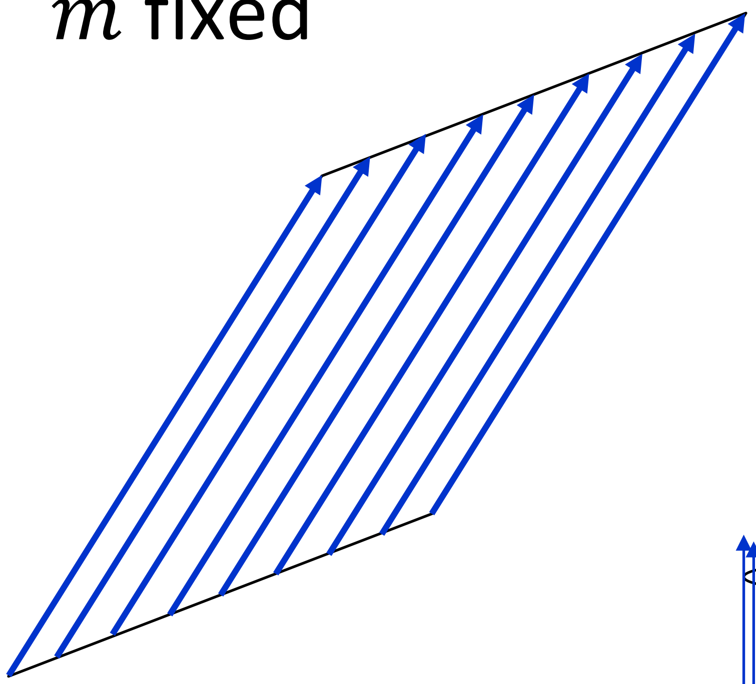
in rest-frame of infalling object

Small Black Hole falling into a Large Black Hole

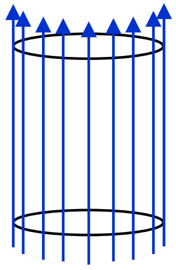
$M \to \infty$

m fixed

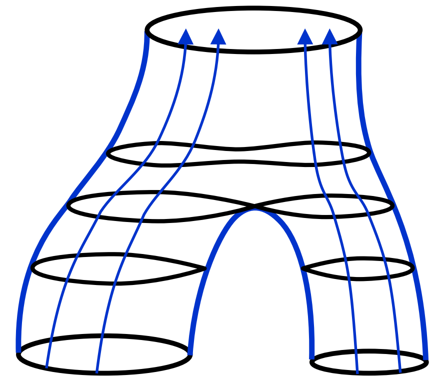
Both are made of lightrays

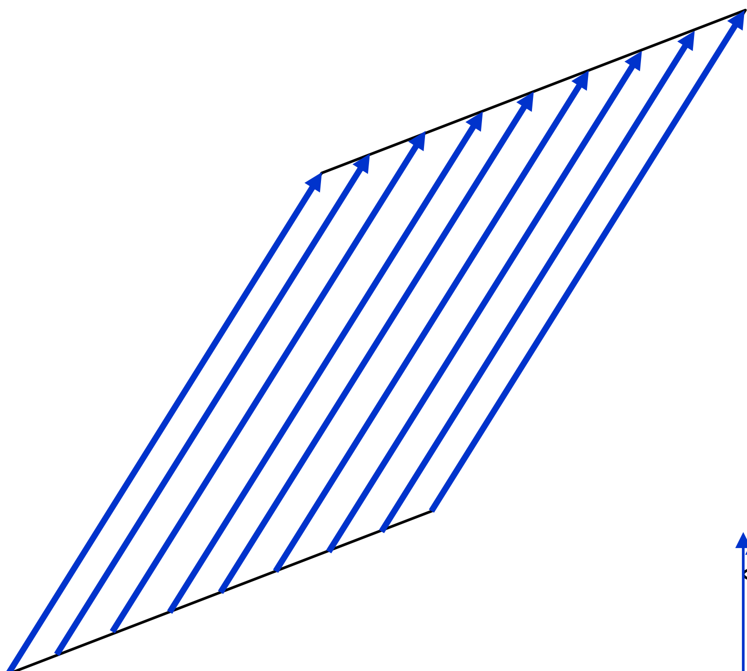


in rest frame of small black hole

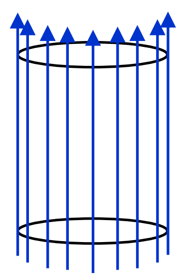


Somehow, lightrays must merge to form a "pants-like" surface





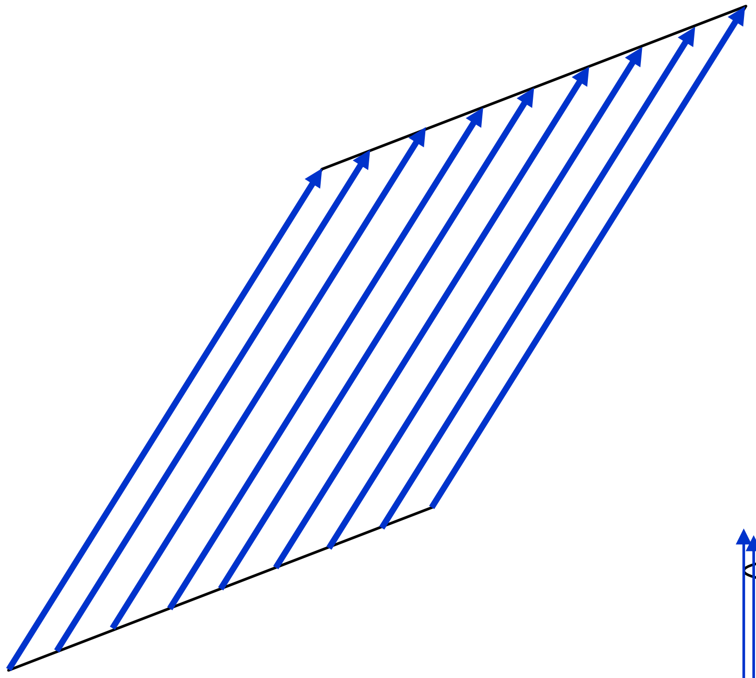
"oversized leg"



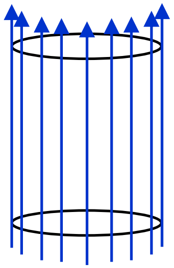
"thin leg"

How?

EH is a family of lightrays in spacetime



Curvature of small black hole is not zero: Schwarzschild solution with mass m



To find the “pants” surface:

Trace a family of null geodesics in the

Schwarzschild solution

that approach a null plane at infinity

All the equations you need to solve

$$t_q(r) = \int \frac{r^3 dr}{(r-1)\sqrt{r(r^3 - q^2(r-1))}}$$

$$\phi_q(r) = \int \frac{q dr}{\sqrt{r(r^3 - q^2(r-1))}}$$

with appropriate final conditions:

null plane at infinity

$q =$ impact parameter of lightrays at infinity

All the equations you need to solve

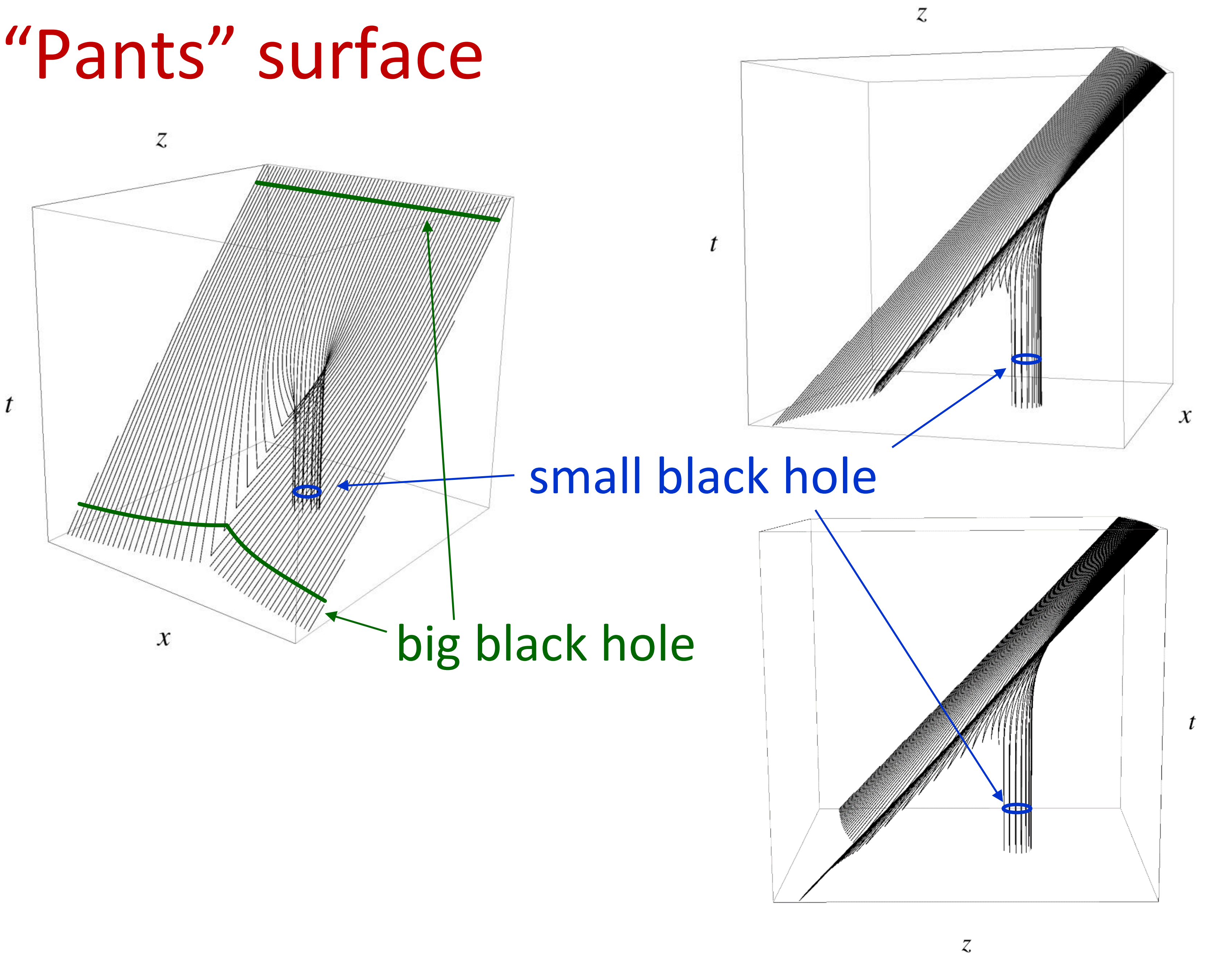
$$t_q(r) = \int \frac{r^3\,dr}{(r-1)\sqrt{r(r^3-q^2(r-1))}}$$

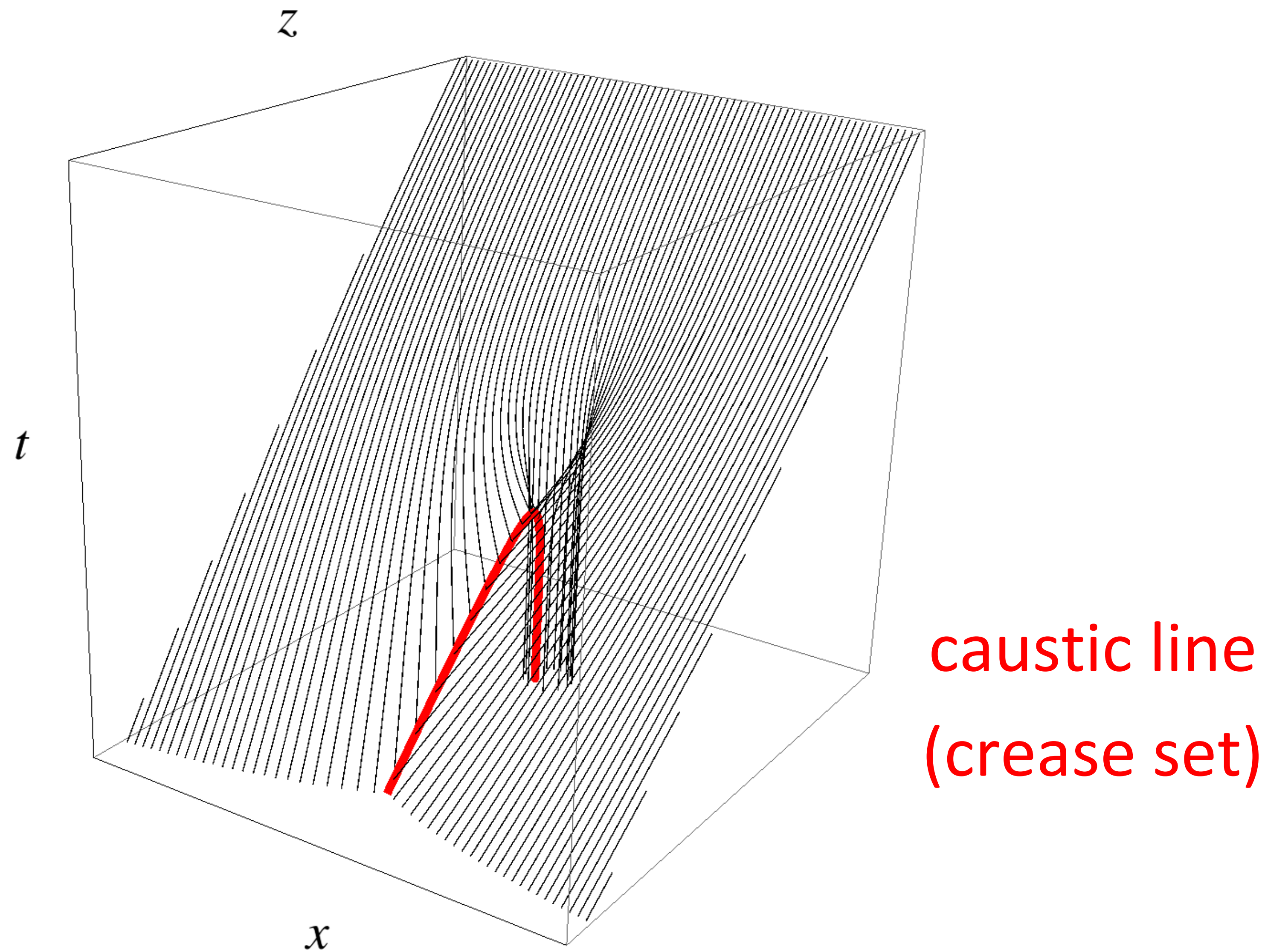
$$\phi_q(r) = \int \frac{q\,dr}{\sqrt{r(r^3-q^2(r-1))}}$$

elliptic integrals

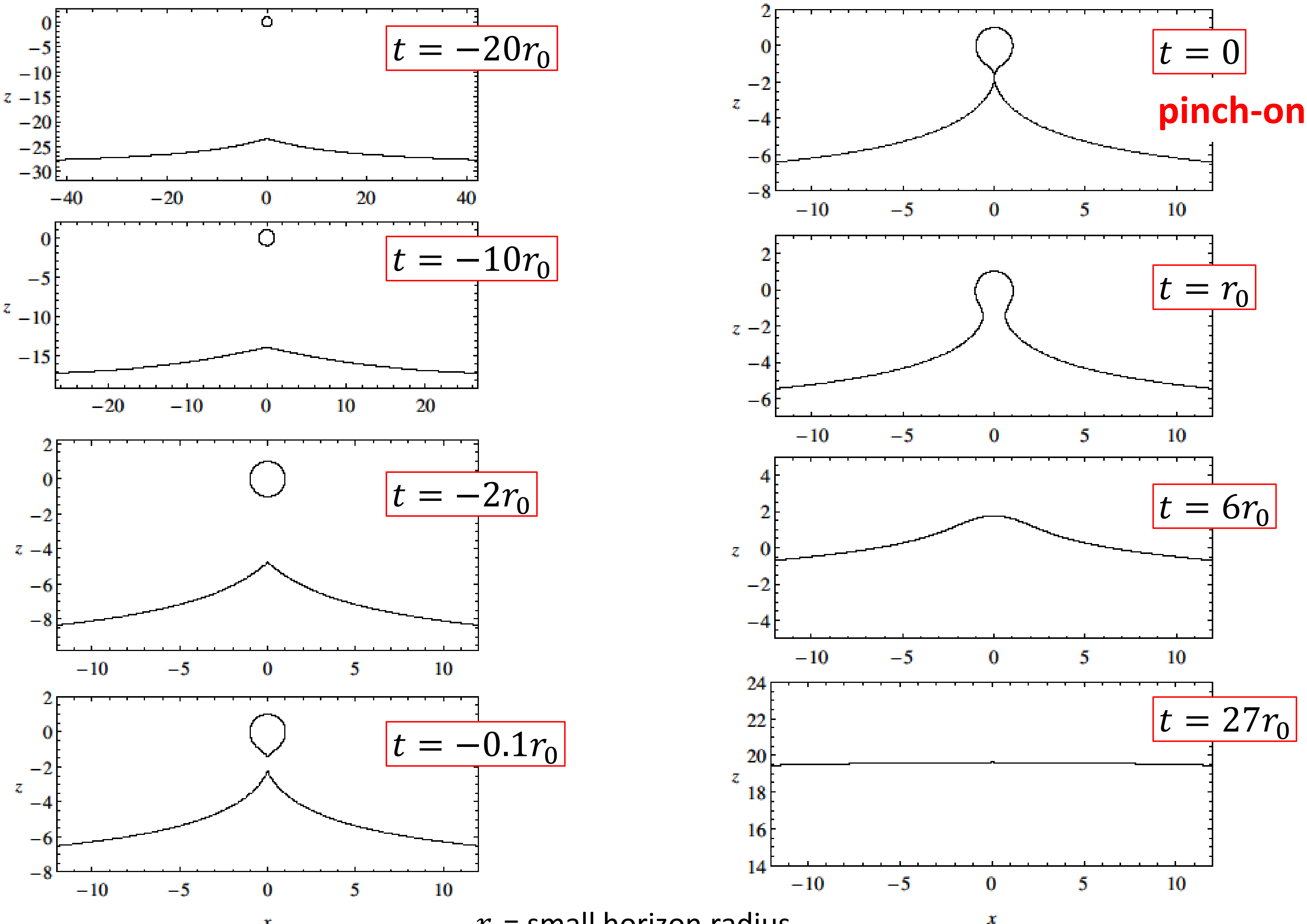
not very nice, but explicit

“Pants” surface



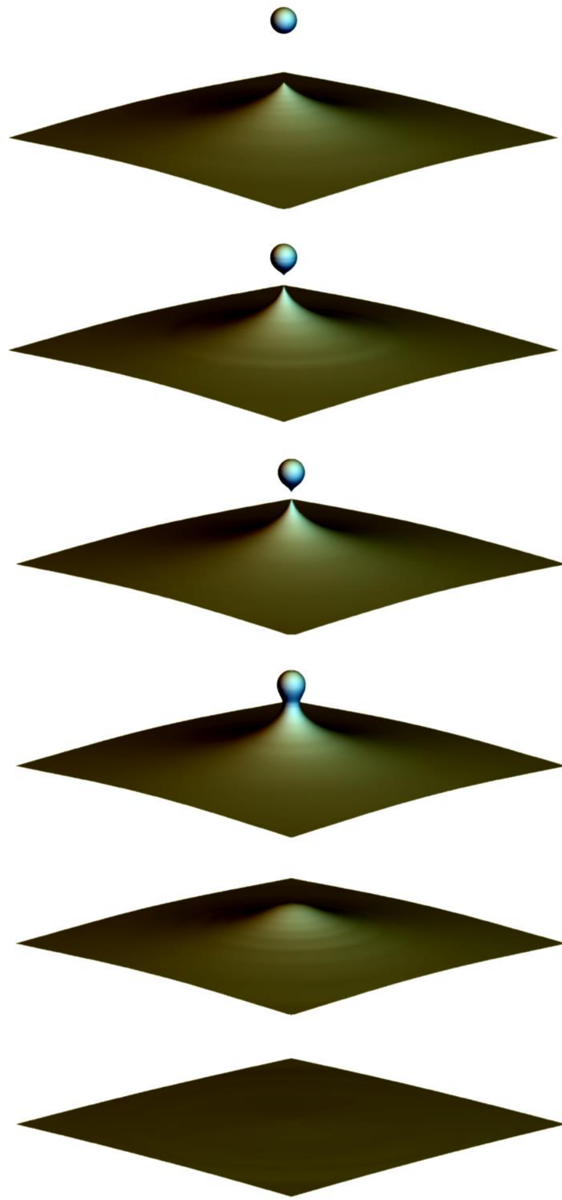

z
t
x
caustic line
(crease set)

Sequence of constant-time slices



r_0= small horizon radius

Sequence of constant-time slices



$t = -2r_0$

$t = -0.1\, r_0$

$t = 0$: **pinch-on**

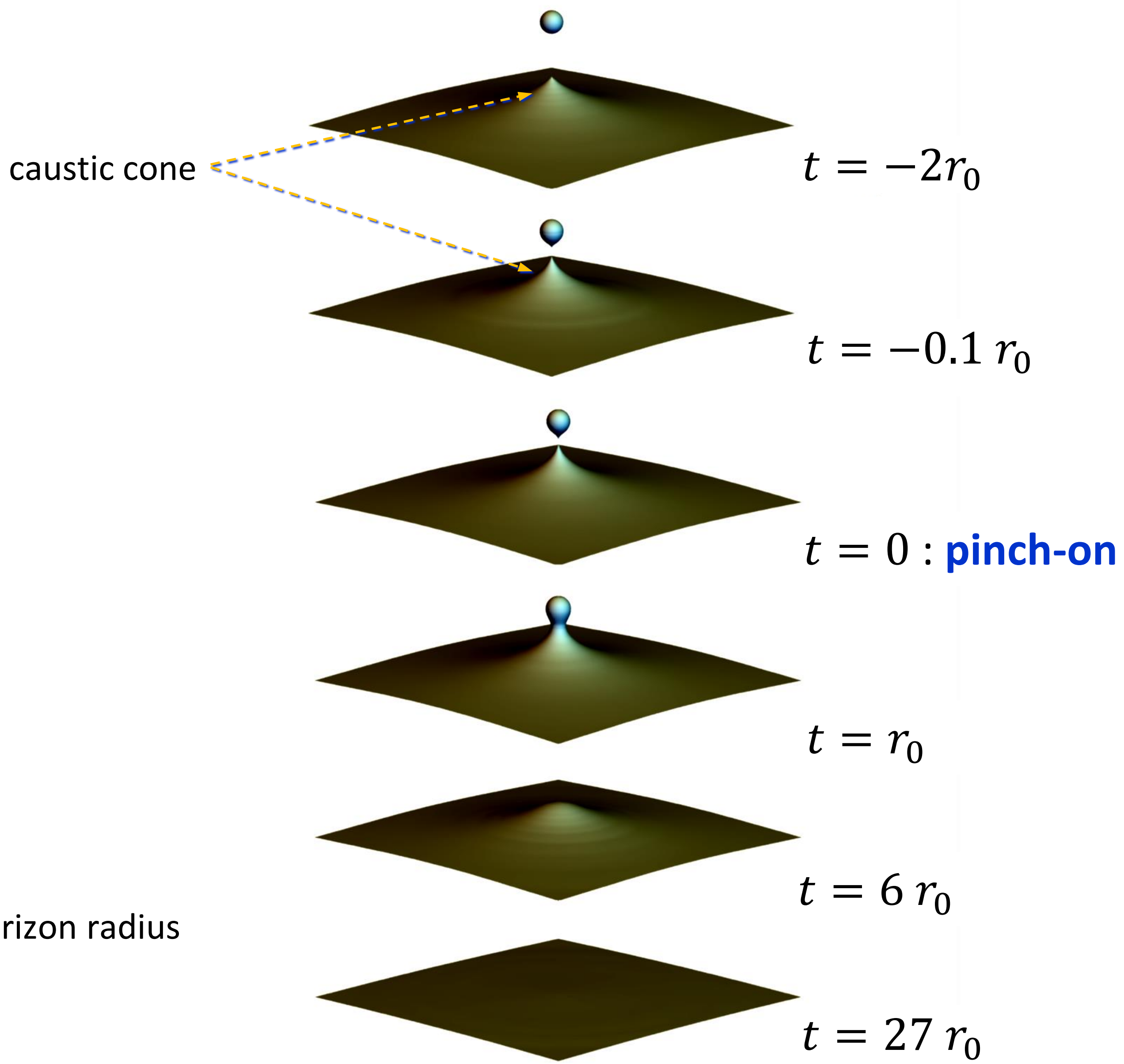
$t = r_0$

$t = 6\, r_0$

$t = 27\, r_0$

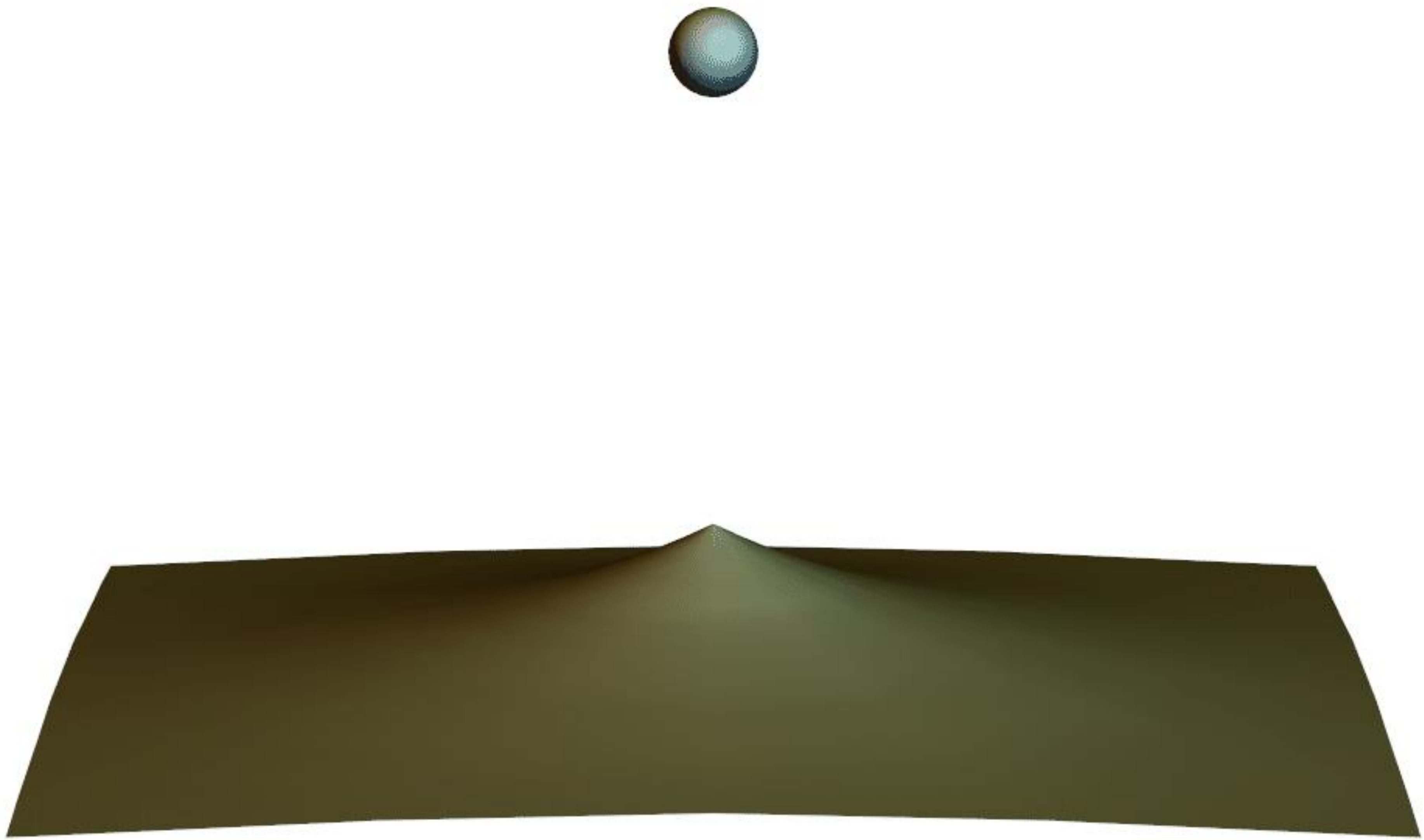
r_0= small horizon radius

Sequence of constant-time slices

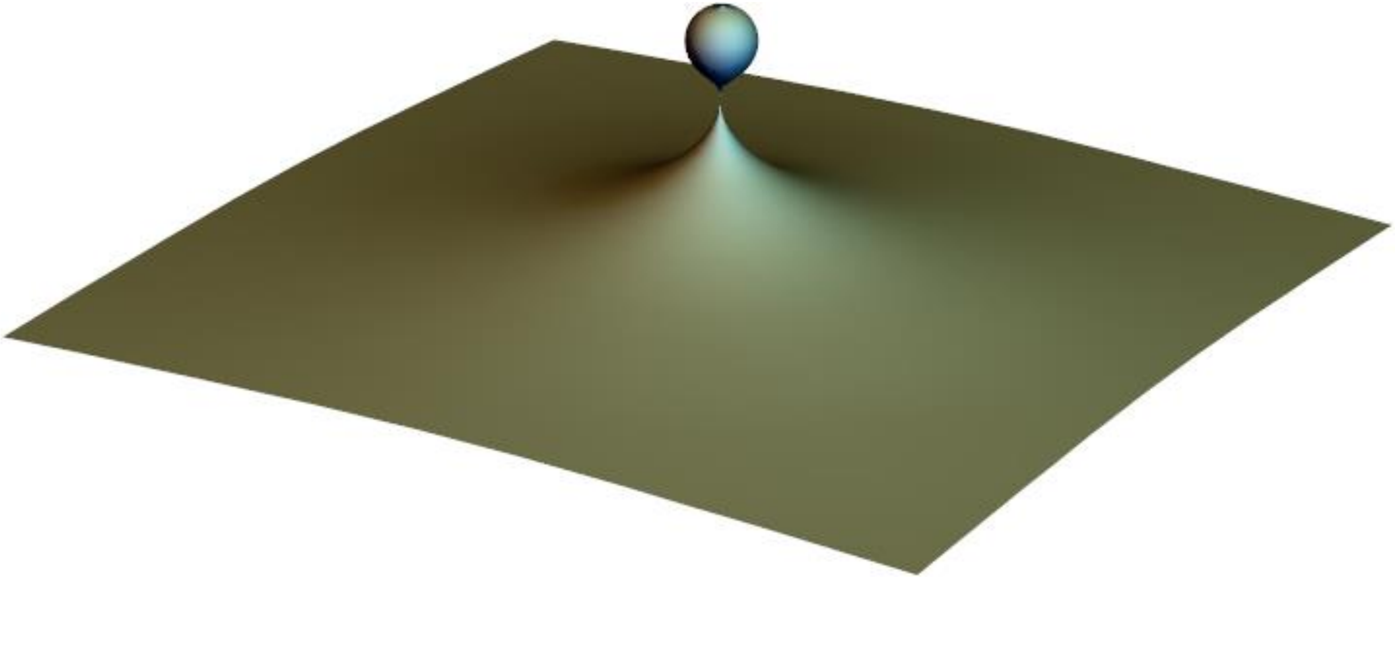


r_0= small horizon radius

made with *Mathematica* on a laptop computer

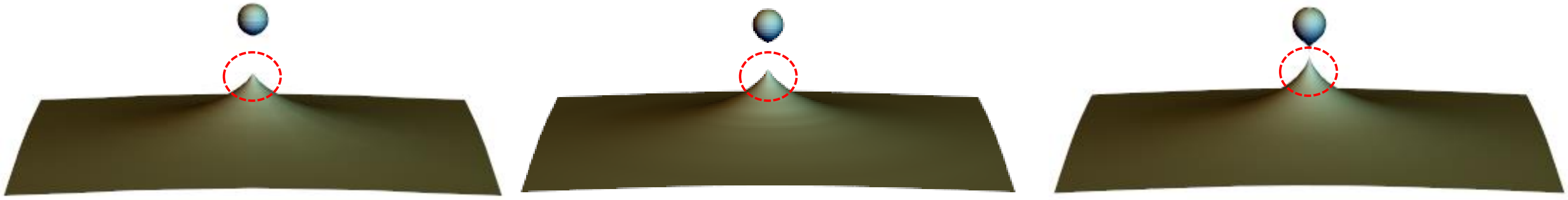


Pinch-on: Criticality



Pinch-on: Criticality

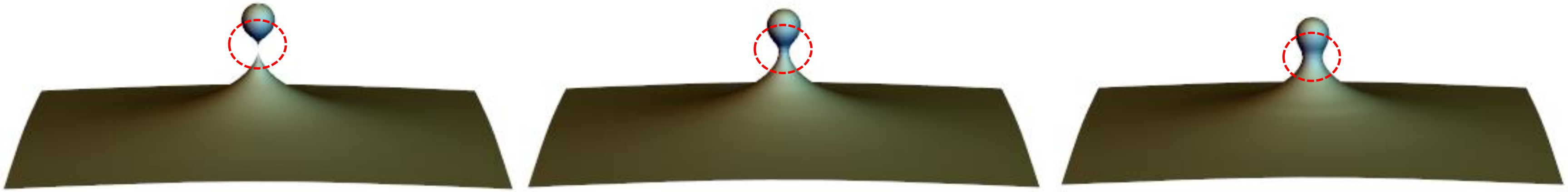
Opening angles of cones $\sim |t|^{1/2}$



same behavior in 5D: universal?

Pinch-on: Criticality

Throat growth $\sim t$



same behavior in 5D: universal?

Gravitational waves?

When $M \to \infty$ the radiation zone is pushed out to infinity

No gravitational waves in this region

GWs reappear if we introduce corrections for small $\frac{m}{M}$

Change Schwarzschild $\rightarrow$ Kerr

Fusion of **any Black Hole binary in the Universe**

to leading order in $\frac{m}{M} \ll 1$

Final remarks

Can we *observe* this?

Maybe not

Then, what is it good for?

Fusion of Black Hole Event Horizons
is a signature phenomenon of
General Relativity

Equivalence Principle allows to
capture and *understand* it easily in
a (realistic) limit

Exact result:

- Benchmark for detailed numerical studies

- First step in expansion in $\frac{m}{M} \ll 1$

(matched asymptotic expansion)

Equivalence Principle magic:

Get 2 black holes out of a geometry with only 1

This could have been done (at least) 50 years ago!