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Comment on "Feynman's proof of the Maxwell equations," by F. J. Dyson [Am. J. Phys. 58, 209–211 (1990)]

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Dyson[1] shows that Feynman was able to derive equations for the electromagnetic field and the Lorentz force from Newton's second law of motion using only the commutation relation between position and velocity. These equations for the electromagnetic field are claimed to be equivalent to Maxwell's equations, which are of course Lorentz invariant, even though Newton's law is Galilean invariant. How can this be?

In fact, Dyson only proves the two source-free Maxwell equations,

$$\operatorname{div} \mathbf{H} = 0, \tag{1}$$

$$\operatorname{curl} \mathbf{E} = -\frac{\partial \mathbf{H}}{\partial t}. \tag{2}$$

The remaining two Maxwell equations are claimed to be definitions of charge density ρ and current density $\mathbf{j}$. This provides the clue to the paradox. Can there be another definition of ρ and $\mathbf{j}$ which allows the theory to be Galilean invariant?

Le Bellac and Levy-Leblond[2] studied some time ago Galilean-invariant theories of electromagnetism. (Maxwell's equations are, of course, not Galilean invariant.) Le Bellac and Levy-Leblond show that a Galilean invariant theory of electromagnetism requires that one of the following two hypotheses must be dropped:

(i) the continuity equation $\operatorname{div} \mathbf{j} = -\dfrac{\partial \rho}{\partial t} \neq 0$,

(ii) magnetic forces between electric currents.

So in order to keep Galilean invariance, we can still define ρ by

$$\operatorname{div} \mathbf{E} = 4\pi\rho \tag{3}$$

as usual, but we should take as the new definition of $\mathbf{j}$

$$\operatorname{curl} \mathbf{H} = 4\pi\mathbf{j} \tag{4}$$

with no displacement current. This is the version of Galilean electromagnetism called the magnetic limit in Ref. 2. In particular, only stationary currents satisfying

$$\operatorname{div} \mathbf{j} = 0 \tag{5}$$

are allowed in this version of Galilean-invariant electromagnetism.

It is not very well known that Levy-Leblond studied Galilean-invariant quantum theories of arbitrary spin.[3] He was able to show that a Galilean-invariant theory of a charged spin-1/2 particle has a gyromagnetic ratio $g = 2$. Therefore, the spin and magnetic moment of an electron are not consequences of either relativity or the Dirac equation, contrary to what is claimed in most textbooks.

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I should like to thank David Waxman for pointing out Dyson's article to me and Gabriel Barton for insisting that what I wrote should be intelligible.

[1] F. J. Dyson, "Feynman's proof of the Maxwell equations," Am. J. Phys. **58**, 209–211 (1990).
[2] M. Le Bellac and J-M. Levy-Leblond, Nuovo. Cimento **14** B, 217–234 (1973).
[3] J-M. Levy-Leblond, Commun. Math. Phys. **6**, 286–311 (1967).

Comment on "Feynman's proof of the Maxwell equations," by F. Dyson [Am. J. Phys. 58, 209–211 (1990)]

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Freeman Dyson's statement in his recent interesting and noteworthy paper[1] that the source equations of Maxwell

$$\nabla \cdot \mathbf{E} = \tilde{\rho}/\epsilon \tag{1a}$$

and

$$\nabla \times \mathbf{B} - \frac{\partial \mathbf{E}}{c^2\,\partial t} = \mu \mathbf{J}, \tag{1b}$$

"merely define the external change and current densities, $\tilde{\rho}$ and $\mathbf{J}$," I think diminishes the importance of these equations in establishing the dynamical character of the electromagnetic field.

To justify this opinion, permit me a brief summary of the foundations of classical electromagnetic theory: The relativistic Lagrangian density $\mathscr{L}$ from which Maxwell's