Quantum Fields in Curved Spacetimes and Semiclassical Approaches: A Workshop Summary

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Abstract

I briefly review some of the recent progress in quantum field theory in curved spacetime and other aspects of semiclassical gravity, as reported at the D3 Workshop at GR15.

The study of the behavior of quantum fields in curved spacetime and other semiclassical gravitational phenomena began in earnest in the late 1960's with the study of particle creation effects in cosmology. It then received enormous impetus in the mid 1970's from Hawking's discovery of thermal particle creation effects by black holes. In the late 1970's and early 1980's, a great deal of progress was made in the development of the theory and the exploration of various phenomena. By the 1990's, quantum field theory in curved spacetime had become a mature subject, with its foundational underpinnings well established.

In the past few years, some notable developments have occurred in a number of aspects of quantum field theory in curved spacetime. These developments include: (1) The application of the "microlocal analysis" methods of Hormander to resolve a number of outstanding issues, such as the definition of the stress-energy tensor of a free field as an operator-valued-distribution [1] and the renormalizability of interacting quantum field perturbative expansions [2]. (2) The exploration of issues concerning quantum field theory in non-globally-hyperbolic spacetimes and the analysis of the behavior of quantum fields near various types of Cauchy horizons. (3) The derivation and analysis of some global energy inequalities satisfied by the expected stress-energy tensor of a quantum field. At the D3 workshop at GR15, there were no contributions presented on developments directly related to (1), but developments related to (2) were very well represented in the contributions by Flanagan, Barve, Hiscock, Kay, and Higuchi, and some developments in area (3) were reported in the contribution by Ford.

In addition to research on topics falling within the precise confines of quantum field theory in a curved spacetime, there also was considerable research activity during the past few years in other areas of "semiclassical gravity", involving issues such as pair creation of black holes and black hole thermodynamics. The contributions presented at the D3 workshop by Frolov, Bousso, Spindel, and Manogue provide some representation of a few of these other developments in semiclassical gravity.

In the following, I will provide a "thumbnail sketch" of the 12 oral contributions presented at the D3 workshop. In most cases, references to papers giving a complete exposition of the work will be provided, and the interested reader is strongly advised to consult those references rather than relying on the "sound bites" provided here. Unfortunately, it is not feasible for me to attempt to summarize in any way the nearly 30 contributions accepted for poster presentations at the D3 workshop.

There has been considerable interest in understanding the stability of Cauchy horizons such as the "inner horizon" of the Reissner-Nordstrom and Kerr black holes. The classical "blueshift instability" of these horizons is now well understood. However, there are examples known of spacetimes where such a classical blueshift instability does not occur, but, nevertheless, the Cauchy horizon is unstable when perturbed by a test quantum field. The contribution by E. Flanagan considered two-dimensional spacetimes where the classical blueshift instability does not occur. Flanagan obtained a necessary condition for these Cauchy horizons to be semiclassically unstable. He also showed that the quantum instability of these these horizons could be interpreted as resulting from a "delayed blueshift" instability. Details of this work can be found in [3].

The contribution by S. Barve (reporting on work done in collaboration with T.P. Singh, C. Vaz, and L. Witten) considered the issue of the quantum instability of a Cauchy horizon occurring when a naked singularity arises in a $(1+1)$-dimensional model of the Tolman-Bondi collapse of a ball of dust. It was found that the quantum stress-tensor diverges on the Cauchy horizon in a manner similar to what was previously found to occur in the case of naked singularities produced from the collapse of null dust [4]. Details can be found in [5].

The contribution by W.A. Hiscock noted that the extreme Reissner-Nordstrom solution could be the asymptotic final state of a black hole if magnetic monopoles or other suitable $U(1)$ charges exist in nature. The usual blueshift instability arguments would not apply to the Cauchy horizon of a black hole which asymptotically becomes extreme, since the surface gravity of the Cauchy horizon would vanish. However, Hiscock considered a Reissner-Nordstrom-Vaidya model of the approach to an asymptotically extreme black hole, and found that in this model the Cauchy horizon is singular nevertheless.

A contribution by B.S. Kay (reporting on work done in collaboration with A.R. Borrott) also considered the extreme Reissner-Nordstom spacetime, but concerned itself with the behavior of the quantum field on the black hole event horizon. It is known [6] that in 2 spacetime dimensions, there is a "weak divergence" of stress-energy tensor at the horizon in the vacuum state (and a "strong divergence" for all thermal states at finite temperatures). However, in 4 spacetime dimensions, no divergences occur if one approximates the extreme Reissner-Nordstrom metric by a Robinson-Bertotti metric [7]. (Numerical work on the 4-dimensional Reissner-Nordstrom spacetime itself also indicates the absence of divergences [8].) However, Borrott and Kay have found that in 2 dimensions, the behavior of a quantum field in the vacuum state near the horizon of the extreme Reissner-Nordstrom black hole differs from its behavior in the Robinson-Bertotti spacetime in that Hessling's "quantum equivalence principle" fails for the former but holds for the latter. This suggests that great caution must be exercised in using the Robinson-Bertotti approximation to the extreme Reissner-Nordstrom black hole.

The contribution of A. Higuchi (reporting on work done in collaboration with C.J. Fewster and B.S. Kay) dealt with the construction of quantum field theory in chronology violating—and, thus, non-globally-hyperbolic—

spacetimes. Kay has proposed the condition of "F-locality" as a necessary criterion to be satisfied by a quantum field theory in a non-globally-hyperbolic spacetime [9]. (This condition asserts that every point should have a globally hyperbolic neighborhood $\mathcal{U}$ such that the restriction of the field algebra to $\mathcal{U}$ agrees with what one would obtain by viewing $\mathcal{U}$ as a globally hyperbolic spacetime in its own right.) It is known that F-locality must fail in any spacetime with a compactly generated chronology horizon [10], but some examples (like the "spacelike cylinder") are known of chronology violating spacetimes which admit F-local field algebras. However, the recent work reported by Higuchi on conformal deformations of the 4-dimensional spacelike cylinder provides some evidence that the chronology violating spacetimes which admit F-local field algebras may be non-generic.

The behavior of a quantum field near a chronology horizon was the subject of a contribution by B.S. Kay reporting on work done with C.R. Cramer. (This contribution was scheduled to be presented by Cramer, but Cramer was unable to attend the meeting.) Although a general theorem [10] establishes that the stress-tensor of a quantum field must always be singular on a compactly generated chronology horizon, explicit examples are known of states on 2- and 4-dimensional Misner spacetime for which the stress-energy tensor does not diverge as one approaches the chronology horizon. These examples were re-examined by Cramer and Kay using image sum techniques in order to gain insight into how and why the singularity predicted by [10] occurs nevertheless. Details of this work can be found in [11].

The contribution by L.H. Ford (reporting on work done in collaboration with M.J. Pfenning) was concerned with restrictions on the energy density of a quantum field in curved spacetime. It is well known that none of the local (pointwise) energy conditions of classical field theory apply to the expected stress-energy of a quantum field: For any point p in spacetime, one can find states that make the energy density at p be arbitrarily negative. However, in flat spacetime, a number of global restrictions occur. In particular, there exist "quantum inequalities", which, roughly speaking, state that a geodesic observer cannot observe a time averaged energy density more negative than $-\hbar/t^4$, where t denotes the "sampling time". The main purpose of the present work by Ford and Pfenning was to generalize the quantum inequalities to static curved spacetimes. It was shown that for short sampling times, quantum inequalities exist and take the form of the flat spacetime result plus subdominant, spacetime-dependent corrections. Furthermore, they showed that the average energy density measured along the worldline of a static observer is bounded from below by the vacuum energy density. Details of this work can be found in [12].

Quantum field theory on a stationary spacetime containing an "ergoregion" but no black hole (as would occur for a solution describing a sufficiently rapidly rotating relativistic star) was considered in the contribution by G. Kang. Such spacetimes are known to be classically unstable, and the presence of unstable modes poses some difficulties for formulating the canonical quantization of a scalar field if one tries to use the same procedures that are applicable in stationary spacetimes without ergoregions. Kang presented a prescription for quantizing a scalar field in the presence of classically unstable modes, and showed how the ergoregion instability persists in the quantum theory. Details of this work can be found in [13]

V.P. Frolov reported on work he and his collaborators have done during the past few years with the aim of accounting for the Bekenstein-Hawking formula, $S = A/4$, for the entropy of a black hole in the context of Sakharov's theory of induced gravity. In Sakharov's proposal, the dynamical aspects of gravity arise from the collective excitations of massive fields. Constraints are then placed on these massive fields to cancel divergences and ensure that the effective cosmological constant vanishes. In the induced gravity model explicitly considered by Frolov, the Bekenstein-Hawking entropy is then explained as arising from the ordinary statistical mechanical entropy of a thermally excited gas of the heavy constituents. Details of this work can be found in [14].

The contribution of R. Bousso (reporting on research done in collaboration with S.W. Hawking) concerned the quantum behavior of Schwarzschild black holes in de Sitter spacetime. A Schwarzschild black hole in an asymptotically flat spacetime will evaporate via Hawking radiation, and the same should be true for a "small" Schwarzschild black hole in de Sitter spacetime, since the temperature of the black hole will be larger than that of the cosmological horizon. However, when the black hole is of maximal mass (corresponding to the Narai solution), the black hole and cosmological temperatures are equal, so equilibrium should be possible. However, one would expect this equilibrium to be unstable, since if the black hole loses mass it should become hotter and, hence, should continue to evaporate. Bousso and Hawking analyzed this issue and found that stability actually occurs for a certain class of metric perturbations, but that unstable modes exist for perturbations outside of this class. Furthermore, they argued that these unstable modes should be excited if black holes are produced by spontaneous pair creation. Details of this work can be found in [15].

P. Spindel reported on research (done in collaboration with C. Gabriel, S. Massar, and R. Parentani) concerning the Unruh effect. In Unruh's original model of an accelerating detector, the internal states of the detector were treated quantum mechanically, but the acceleration of the world line of the detector was treated as being classically imposed. The work reported by Spindel provided a fully quantum field theoretic model of an Unruh detector in which one has charged fields of masses M and m which are placed in a uniform electric field (which provides the acceleration) and these massive fields are allowed to interact via coupling to yet another field. Thermal effects are then investigated by comparing the ratio of populations of the two species of massive particles. In the limit where M and m go to infinity with $(M - m)$ and the acceleration, a, fixed, and in the limit where the charge of the field which couples the two massive fields goes to zero, the ratio of these populations was found to correspond to a thermal distribution at the Unruh temperature $a/2\pi$. Details of this work can be found in [16].

The final oral presentation of the workshop was given by C.A. Manogue, presenting work (done in collaboration with T. Dray) on a new dimensional reduction scheme. In this approach, one starts with the momentum space Dirac equation in 10 dimensions, described in terms of 2-component spinors over the octonions. The choice of a preferred octonion unit is then used to effectively reduce the 10 spacetime dimensions to 4 without resorting to compactification. The preferred octonion unit also singles out 3 quaternionic subalgebras of the octonions, which are interpreted as corresponding to the 3 generations of leptons, whose massless particles naturally have just a single helicity. Details of this work are given in [17].

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