## The Formulation of Quantum Field Theory in Curved Spacetime

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## Abstract

The usual formulations of quantum field theory in Minkowski spacetime make crucial use of Poincare symmetry, positivity of total energy, and the existence of a unique, Poincare invariant vacuum state. These and other key features of quantum field theory do not generalize straightforwardly to curved spacetime. We discuss the conceptual obstacles to formulating quantum field theory in curved spacetime and how they can be overcome.

Quantum field theory in curved spacetime is a theory wherein matter is treated fully in accord with the principles of quantum field theory, but gravity is treated classically in accord with general relativity. It is not expected to be an exact theory of nature, but it should provide a good approximate description in circumstances where the quantum effects of gravity itself do not play a dominant role. Despite its classical treatment of gravity, quantum field theory in curved spacetime has provided us with some of the deepest insights we presently have into the nature of quantum gravity.

Quantum field theory as usually formulated contains many elements that are very special to Minkowski spacetime. But we know from general relativity that spacetime is not flat, and, indeed there are very interesting quantum field theory phenomena that occur in contexts—such as in the early universe and near black holes—where spacetime cannot be approximated as nearly flat.

It is a relatively simple matter to generalize classical field theory from flat to curved spacetime. That is because there is a clean separation between the field equations and the solutions. The field equations can be straightforwardly generalized to curved spacetime in an entirely local and covariant manner. Solutions to the field equations need not generalize from flat to curved spacetime, but this doesn't matter for the formulation of the theory.

In quantum field theory, "states" are the analogs of "solutions" in classical field theory. However, properties of states—in particular, the existence of a Poincare invariant vacuum state—are deeply embedded in the usual formulations of quantum field theory in Minkowski spacetime. For this reason and a number of other reasons, it is highly nontrivial to generalize the formulation of quantum field theory from flat to curved spacetime.

As a very simple, concrete example of a quantum field theory that illustrates some key features of quantum field theory as well as some of the issues that arise in generalizing the formlulation of quantum field theory to curved spacetime, consider a free, real Klein-Gordon field in flat spacetime,

$$\partial^a \partial_a \phi - m^2 \phi = 0 \ . \tag{1}$$

The usual route towards formulating a quantum theory of  $\phi$  is to decompose it into a series of modes, and then treat each mode by the rules of ordinary quantum mechanics. To avoid technical awkwardness, it is convenient to imagine that the field is in cubic box of side L with periodic boundary conditions. We can then decompose  $\phi$  as a Fourier series in terms

of the modes

$$\phi_{\vec{k}} \equiv L^{-3/2} \int e^{-i\vec{k}\cdot\vec{x}} \phi(t,\vec{x}) \ d^3x \tag{2}$$

where  $\vec{k} = \frac{2\pi}{L}(n_1, n_2, n_3)$ . The Hamiltonian of the system is then given by

$$H = \sum_{\vec{k}} \frac{1}{2} \left( |\dot{\phi}_{\vec{k}}|^2 + \omega_{\vec{k}}^2 |\phi_{\vec{k}}|^2 \right) \tag{3}$$

where

$$\omega_{\vec{k}}^2 = |\vec{k}|^2 + m^2 \ . \tag{4}$$

Thus, a free Klein-Gordon field,  $\phi$ , in flat spacetime is explicitly seen to be simply an infinite collection of decoupled harmonic oscillators. If we take into account the fact that  $\phi$  is real but the modes  $\phi_{\vec{k}}$  are complex, we find that each  $\phi_{\vec{k}}$  is given by the operator expression

$$\phi_{\vec{k}} = \frac{1}{2\omega_{\vec{k}}} (a_{\vec{k}} + a_{-\vec{k}}^{\dagger}) . \tag{5}$$

where  $a_{\vec{k}}$  and  $a_{\vec{k}}^{\dagger}$  satisfy the usual commutation relations

$$[a_{\vec{k}}, a_{\vec{k'}}] = 0 , \quad [a_{\vec{k}}, a_{\vec{k'}}^{\dagger}] = \delta_{\vec{k}\vec{k'}}I$$
 (6)

The Heisenberg field operator  $\phi(t, \vec{x})$  is then formally given by

$$\phi(t, \vec{x}) = L^{-3/2} \sum_{\vec{k}} \frac{1}{2\omega_{\vec{k}}} \left( e^{i\vec{k}\cdot\vec{x} - i\omega_{\vec{k}}t} a_{\vec{k}} + e^{-i\vec{k}\cdot\vec{x} + i\omega_{\vec{k}}t} a_{\vec{k}}^{\dagger} \right). \tag{7}$$

However, this formula does *not* make sense as a definition of  $\phi$  as an operator at each point  $(t, \vec{x})$ . In essence, the contributions from the modes at large  $|\vec{k}|$  do not diminish rapidly enough with  $|\vec{k}|$  for the sum to converge. However, these contributions are rapidly varying in spacetime, so if we "average" the right side of eq.(7) in an appropriate manner over a spacetime region, the sum will converge. More precisely, eq.(7) defines  $\phi$  as an "operator valued distribution", i.e., for any (smooth, compactly supported) "test function", f, the quantity

$$\phi(f) = \int f(t, \vec{x})\phi(t, \vec{x})d^4x \tag{8}$$

is well defined by eq.(7) if the integration is done prior to the summation.

States of the free Klein-Gordon field are given the following interpretation: The state, denoted  $|0\rangle$ , in which all of the oscillators comprising the Klein-Gordon field are in their ground state is interpreted as representing the "vacuum". States of the form  $(a^{\dagger})^n|0\rangle$  are interpreted as ones where a total of n "particles" are present.

In an interacting theory, the state of the field may be such that the field behaves like a free field at early and late times. In that case, one has a particle interpretation at early and late times. The relationship between the early and late time particle descriptions of a state—given by the S-matrix—contains a great deal of the dynamical information about the interacting theory.

The particle interpretation/description of quantum field theory in flat spacetime has been remarkably successful—to the extent that one might easily get the impression that, at a fundamental level, quantum field theory is really a theory of "particles". However, note that even for a free field, the definition and interpretation of the "vacuum" and "particles" depends heavily on the ability to decompose the field into its positive and negative frequency parts (as can be seen explicitly from eq.(7) above). The ability to define this decomposition makes crucial use of the presence of a time translation symmetry in the background Minkowski spacetime. In a generic curved spacetime without symmetries, there is no natural notion of "positive frequency solutions" and, consequently, no natural notion of a "vacuum state" or of "particles".

If one looks more deeply at the usual general formulations of quantum field theory, it can be seen that many other properties that are special to Minkowski spacetime are used in an essential way. This is well illustrated by examining the Wightman axioms, since these axioms abstract the key features of quantum field theory in Minkowski spacetime in a mathematically clear way. We will focus attention on the Wightman axioms below, but a similar discussion would apply to other approaches, including the much less rigorous textbook treatments of quantum field theory.

The Wightman axioms are the following [1]:

- The states of the theory are unit rays in a Hilbert space,  $\mathcal{H}$ , that carries a unitary representation of the Poincare group.
- The 4-momentum (defined by the action of the Poincare group on the Hilbert space) is positive, i.e., its spectrum is contained within the closed future light cone ("spectrum condition").
- There exists a unique, Poincare invariant state ("the vacuum").
- The quantum fields are operator-valued distributions defined on a dense domain  $\mathcal{D} \subset$

 $\mathcal{H}$  that is both Poincare invariant and invariant under the action of the fields and their adjoints.

- The fields transform in a covariant manner under the action of Poincare transformations.
- At spacelike separations, quantum fields either commute or anticommute.

It is obvious that there are serious difficulties with extending the Wightman axioms to curved spacetime, specifically:

- A generic curved spacetime will not possess any symmetries at all, so one certainly cannot require "Poincare invariance/covariance" or invariance under any other type of spacetime symmetry.
- Even for a free quantum field, there exist unitarily inequivalent Hilbert space constructions of the theory. For spacetimes with a noncompact Cauchy surface—and in the absence of symmetries of the spacetime—none appears "preferred".
- In a generic curved spacetime, there is no "preferred" choice of a "vacuum state".
- There is no analog of the spectrum condition in curved spacetime that can be formulated in terms of the "total energy-momentum" of the quantum field.

Thus, of all of the Wightman axioms, only the last one (commutativity or anticommutativity at spacelike separations) generalizes straightforwardly to curved spacetime.

I will now explain in more detail some of the difficulties associated with generalizing the spectrum condition and the existence of a preferred vacuum state to curved spacetime:

Total energy in curved spacetime: The stress energy tensor,  $T_{ab}$ , of a classical field in curved spacetime is well defined and it satisfies local energy-momentum conservation in the sense that  $\nabla^a T_{ab} = 0$ . If  $t^a$  is a vector field on spacetime representing time translations and  $\Sigma$  is a Cauchy surface, one can define the total energy, E, of the field at "time"  $\Sigma$  by

$$E = \int_{\Sigma} T_{ab} t^a n^b d\Sigma \ . \tag{9}$$

Classically, for physically reasonable fields, the stress-energy tensor satisfies the dominant energy condition, so  $T_{ab}t^an^b \geq 0$ . Thus, classically, we have  $E \geq 0$ . However, unless  $t^a$ 

is a Killing field (i.e., unless the spacetime is stationary), E will not be conserved, i.e., independent of choice of Cauchy surface,  $\Sigma$ .

In quantum field theory, it is expected that the stress-energy operator will be well defined as an operator-valued distribution, and it is expected to be conserved,  $\nabla^a T_{ab} = 0$ ; see [2]. However, the definition of  $T_{ab}$  requires spacetime smearing. In Minkowski spacetime, since E is conserved one can, in effect, do "time smearing" without changing the value of E, and there is a unique, well defined notion of total energy. However, in the absence of time translation symmetry, one cannot expect E to be well defined at a "sharp" moment of time. More importantly, it is well known that  $T_{ab}$  cannot satisfy the dominant energy condition in quantum field theory (even if it holds for the corresponding classical theory); locally, energy densities can be arbitrarily negative. It is nevertheless true in Minkowski spacetime that the total energy is positive for physically reasonable states. However, in a curved spacetime without symmetries, there is no reason to expect any "time smeared" version of E to be positive. Furthermore, there are simple examples with time translation symmetry (such as a two-dimensional massless Klein-Gordon field in an  $S^1 \times \mathbf{R}$  universe) where Ecan be computed explicitly and is found to be negative [3]. Thus, it appears hopeless to generalize the spectrum condition to curved spacetime in terms of the positivity of a quantity representing "total energy".

Nonexistence of a "preferred vacuum state" and a notion of "particles": As already noted above, for a free field in Minkowski spacetime, the notion of "particles" and "vacuum" is intimately tied to the notion of "positive frequency solutions", which, in turn relies on the existence of a time translation symmetry. These notions of a (unique) "vacuum state" and "particles" can be straightforwardly generalized to (globally) stationary curved spacetimes. However, there is no natural notion of "positive frequency solutions" in a general, nonstationary curved spacetime.

Nevertheless for a free field on a general curved spacetime, a notion of "vacuum state" can be defined as follows. A state is said to be *quasi-free* if all of its *n*-point correlation functions  $\langle \phi(x_1) \dots \phi(x_n) \rangle$  can be expressed in terms of its 2-point correlation function by the same formula as holds for the ordinary vacuum state in Minkowski spacetime. A state is said to be *Hadamard* if the singularity structure of its 2-point correlation function  $\langle \phi(x_1)\phi(x_2)\rangle$  in the coincidence limit  $x_1 \to x_2$  is the natural generalization to curved spacetime of the singularity structure of  $\langle 0|\phi(x_1)\phi(x_2)|0\rangle$  in Minkowski spacetime (see eq.(13) below). Thus,

in a general curved spacetime, the notion of a quasi-free Hadamard state provides a notion of a "vacuum state", associated to which is a corresponding notion of "particles". The problem is that this notion of vacuum state is highly non-unique. Indeed, for spacetimes with a non-compact Cauchy surface, different choices of quasi-free Hadamard states give rise, in general, to unitarily inequivalent Hilbert space constructions of the theory, so in this case it is not even clear what the correct Hilbert space of states should be. In the absence of symmetries or other special properties of a spacetime, there does not appear to be any preferred choice of quasi-free Hadamard state.

In my view, the quest for a "preferred vacuum state" in quantum field theory in curved spacetime is much like the quest for a "preferred coordinate system" in classical general relativity. After our more than 90 years of experience with classical general relativity, there is a consensus that it is fruitless to seek a preferred coordinate system for general spacetimes, and that the theory is best formulated geometrically, wherein one does not have to specify a choice of coordinate system in order to formulate the theory. Similarly, after our more than 40 years of experience with quantum field theory in curved spacetime, it seems similarly clear to me that it is fruitless to seek a preferred vacuum state for general spacetimes, and that the theory should be formulated in a manner that does not require one to specify a choice specify a choice of state (or representation) to define the theory.

Nevertheless, many of the above difficulties can be resolved in an entirely satisfactory manner:

- The difficulties that arise from the existence of unitarily inequivalent Hilbert space constructions of quantum field theory in curved spacetime can be overcome by formulating the theory via the algebraic framework. The algebraic approach also fits in very well with the viewpoint naturally arising in quantum field theory in curved spacetime that the fundamental observables in quantum field theory are the local quantum fields themselves.
- The difficulties that arise from the lack of an appropriate notion of the total energy of the quantum field can be overcome by replacing the spectrum condition by a "microlocal spectrum condition" that restricts the singularity structure of the expectation values of the correlation functions of the local quantum fields in the coincidence limit.

• Many aspects of the requirement of Poincare invariance of the quantum fields can be replaced by the requirement that the quantum fields be locally and covariantly constructed out of the metric.

I will now explain these resolutions in more detail:

The algebraic approach: In the algebraic approach, instead of starting with a Hilbert space of states and then defining the field observables as operators on this Hilbert space, one starts with a \*-algebra,  $\mathcal{A}$ , of field observables. A state,  $\omega$ , is simply a linear map  $\omega : \mathcal{A} \to \mathbf{C}$  that satisfies the positivity condition  $\omega(A^*A) \geq 0$  for all  $A \in \mathcal{A}$ . The quantity  $\omega(A)$  is interpreted as the expectation value of the observable A in the state  $\omega$ .

If  $\mathcal{H}$  is a Hilbert space which carries a representation,  $\pi$ , of  $\mathcal{A}$ , and if  $\Psi \in \mathcal{H}$  then the map  $\omega : \mathcal{A} \to \mathbf{C}$  given by

$$\omega(A) = \langle \Psi | \pi(A) | \Psi \rangle \tag{10}$$

defines a state on  $\mathcal{A}$  in the above sense. Conversely, given a state,  $\omega$ , on  $\mathcal{A}$ , we can use it to obtain a Hilbert space representation of  $\mathcal{A}$  by the following procedure, known as the Gelfand-Naimark-Segal (GNS) construction. First, we use  $\omega$  to define a (pre-)inner-product on  $\mathcal{A}$  by

$$(A_1, A_2) = \omega(A_1^* A_2). \tag{11}$$

By factoring by zero-norm vectors and completing this space, we get a Hilbert space  $\mathcal{H}$ , which carries a natural representation,  $\pi$ , of  $\mathcal{A}$ . The vector  $\Psi \in \mathcal{H}$  corresponding to  $I \in \mathcal{A}$  then satisfies  $\omega(A) = \langle \Psi | \pi(A) | \Psi \rangle$  for all  $A \in \mathcal{A}$ .

Thus, the algebraic approach is not very different from the usual Hilbert space approach in that every state in the algebraic sense corresponds to a state in the Hilbert space sense and vice-versa. The key difference is that, by adopting the algebraic approach, one may simultaneously consider all states arising in all Hilbert space constructions of the theory without having to make a particular choice of representation at the outset. It is particularly important to proceed in this manner in, e.g., studies of phenomena in the early universe, so as not to prejudice in advance which states might be present.

The microlocal spectrum condition: Microlocal analysis provides a refined characterization of the singularities of a distribution by examining the decay properties of its Fourier transform. More precisely, let D be a distribution on a manifold, M, and let  $(x, k_a)$  be a point in the cotangent bundle of M. If D has the property that it can be multiplied by a smooth function, f, of compact support with  $f(x) \neq 0$ , such that the Fourier transform of fD decays more rapidly than any inverse power of |k| in a neighborhood of the direction in Fourier transform space given by  $k_a$ , then D is said to be nonsingular at  $(x, k_a)$ . If D does not satisfy this property, then  $(x, k_a)$  is said to lie in the wavefront set [4], WF(D), of D. In the case of quantum field theory in curved spacetime, the wavefront set can be used to characterize the singular behavior of the distributions  $\omega[\phi_1(x_1) \dots \phi_n(x_n)]$  (as a subset of the cotangent bundle of  $M \times \cdots \times M$ , where M is the spacetime manifold).

Now, for free fields in Minkowski spacetime, the positivity of total energy is directly related to the choice of positive frequency solutions in the decomposition (7). This, in turn, is directly related to the "locally positive frequency character" of the singular behavior of the n-point correlation functions  $\omega[\phi(x_1)...\phi(x_n)]$  in the coincidence limit. Consequently, it can be shown that in Minkowski spacetime, the spectrum condition (positivity of total energy) is equivalent to a microlocal spectrum condition that restricts the wavefront set of  $\omega[\phi_1(x_1)...\phi_n(x_n)]$ . This microlocal spectrum condition can be generalized straightforwardly to curved spacetime. In this manner, it is possible to impose the requirement that states have a "locally positive frequency character" even in spacetimes where there is no natural global notion of "positive frequency" (i.e., no global notion of Fourier transform).

Local and covariant fields: It is often said that in special relativity one has invariance under "special coordinate transformations" (i.e., Poincare transformations), whereas in general relativity, one has invariance under "general coordinate transformations" (i.e., all diffeomorphisms). However, this is quite misleading. By explicitly incorporating the flat spacetime metric into the formulation of special relativity, it can easily be seen that special relativity can be formulated in as "generally covariant" a manner as general relativity, but the act of formulating special relativity in a generally covariant manner does not provide one with any additional symmetries or other useful conditions. The true meaning of "general covariance" is that the theory is constructed in a local manner from the spacetime metric and other dynamical fields, with no non-dynamical background structure (apart from manifold structure, and choices of space and time orientations and spin structure) playing any role in the formulation of the theory. This is the proper generalization of the notion of Poincare invariance to general relativity.

In the present context, we wish to impose the requirement that in an arbitrarily small

neighborhood of a point x, the quantum fields  $\Phi$  under consideration "be locally and covariantly constructed out of the spacetime geometry" in that neighborhood. In order to formulate this requirement, it is essential that quantum field theory in curved spacetime be formulated for all (globally hyperbolic) curved spacetimes—so that we can formulate the notion that "nothing happens" to the fields near x when we vary the metric in an arbitrary manner away from point x. The notion of a local and covariant field may then be formulated as follows [5]: Suppose that we have a causality preserving isometric embedding  $i: M \to \mathcal{O}' \subset M'$  of a spacetime  $(M, g_{ab})$  into a region  $\mathcal{O}'$ , of a spacetime  $(M', g'_{ab})$ . We require that this embedding induce a natural isomorphism of the quantum field algebra  $\mathcal{A}(M')$  of the spacetime  $(M, g_{ab})$  and the subalgebra of the quantum field algebra  $\mathcal{A}(M')$  associated with region  $\mathcal{O}'$ . We further demand that under this isomorphism, each quantum field  $\Phi(f)$  on M be taken into the corresponding quantum field  $\Phi(i^*f)$  in  $\mathcal{O}'$ .

In what sense is this condition a replacement for Poincare covariance? In the case of Minkowski spacetime, we can isometrically embed all of Minkowski spacetime into itself by a Poincare transformation. The above condition then provides us with an action of the Poincare group on the field algebra of Minkowski spacetime and also requires each quantum field in Minkowski spacetime to transform covariantly under Poincare transformations. Thus, the above condition contains much of the essential content of Poincare invariance, but it is applicable to arbitrary curved spacetimes without symmetries.

Let us now take stock of where things stand on the generalization of the basic principles of quantum field theory—as expressed by the Wightman axioms—to curved spacetime.

- The axiom that requires states to lie in a Hilbert space that carries a unitary representation of the Poincare group is satisfactorily replaced by formulating theory via the algebraic approach and requiring that the quantum fields be local and covariant.
- The spectrum condition is satisfactorily replaced by the microlocal spectrum condition.
- The axiom stating that quantum fields are operator-valued distributions defined on a dense domain that is Poincare invariant and invariant under the action of the fields and their adjoints is satisfactorily replaced by the GNS construction in the algebraic approach and the local and covariant field condition.

- The axiom that the fields transform in a covariant manner under the action of Poincare transformations is satisfactorily replaced by the local and covariant field condition.
- As previously noted, the condition that at spacelike separations quantum fields either commute or anticommute generalizes straightforwardly to curved spacetime.

Thus, the only Wightman axiom that does not admit a satisfactory generalization to curved spacetime based on the ideas described above is the existence of a unique, Poincare invariant state ("the vacuum"). This axiom plays a key role in the proofs of the PCT and spin-statistics theorem and many other results, so one would lose a great deal of the content of quantum field theory if one simply omitted this axiom. In particular, vacuum expectation values of products of fields play an important role in many arguments, and it is crucial that these "c-number" quantities share the symmetries of the fields. On the other hand, we have already argued that it is hopeless to define a unique "preferred state" in generic spacetimes. Furthermore, states are inherently global in character and cannot share the "local and covariant" property of fields.

What is the appropriate replacement in curved spacetime of the requirement that there exist a Poincare invariant state in Minkowski spacetime? Hollands and I have recently proposed [6] that the appropriate replacement is the existence of an operator product expansion of the quantum fields. An operator product expansion (OPE) is a short-distance asymptotic formula for products of quantum fields near point y in terms of quantum fields defined at y, i.e., formulae of the form

$$\phi^{(i_1)}(x_1)\cdots\phi^{(i_n)}(x_n) \sim \sum_{(j)} C_{(j)}^{(i_1)\dots(i_n)}(x_1,\dots,x_n;y)\,\phi^{(j)}(y) \tag{12}$$

for all  $i_1, \ldots, i_n$ , which hold as asymptotic relations as  $\{x_1, \ldots, x_n\} \to y$ . The simplest example of an OPE is that for a product of two free Klein-Gordon fields in curved spacetime. We have

$$\phi(x_1)\phi(x_2) = H(x_1, x_2)\mathbf{1} + \phi^2(y) + \dots$$
(13)

where H is a locally and covariantly constructed Hadamard distribution (see, e.g., [7] or [8] for the precise form of H) and "..." has higher scaling degree than the other terms (i.e., it goes to zero more rapidly in the limit  $x_1, x_2 \to y$ ). An OPE exists for free fields in curved spacetime and Hollands [9] has shown that it holds order-by-order in perturbation theory

for renormalizable interacting fields in curved spacetime. The requirement that an operator product expansion exists and satisfies a list of suitable properties [6] appears to provide an appropriate replacement for the requirement of the existence of a Poincare invariant state. In particular, the distributional coefficients of the identity element in OPE expansions play much of the role played by "vacuum expectation values" in Minkowski spacetime quantum field theory.

By elevating the existence of an OPE to a fundamental status, Hollands and I have been led to the following viewpoint on quantum field theory in curved spacetime: The background structure,  $\mathcal{M}$ , of quantum field theory in curved spacetime is the spacetime  $(M, g_{ab})$ , together with choices of time orientation, spacetime orientation, and spin structure. For each  $\mathcal{M}$ , we have an algebra  $\mathcal{A}(\mathcal{M})$  of local field observables. In traditional algebraic approaches to quantum field theory,  $\mathcal{A}(\mathcal{M})$  would contain the full information about the quantum field theory. However, in our approach,  $\mathcal{A}(\mathcal{M})$ , is essentially nothing more than the free \*-algebra generated by the list of quantum fields  $\phi^{(i)}(f)$  (including "composite fields"), though it may be factored by relations that arise from the OPE.

In our viewpoint, all of the nontrivial information about the quantum field theory is contained in its OPE, eq.(12). The distributions  $C_{(j)}^{(i_1)\dots(i_n)}(x_1,\dots,x_n;y)$  appearing in eq.(12) are required to satisfy a list of properties, which include locality and covariance and an "associativity" property. States are positive linear maps on the algebra that satisfy the OPE relations as well as microlocal spectrum conditions.

Spin-statistics and PCT theorems have been proven within this framework [6]. Interestingly, the PCT theorem relates processes in a given spacetime to processes (involving charge conjugate fields) in a spacetime with the opposite time orientation, e.g., it relates processes involving particles in an expanding universe to processes involving antiparticles in a contracting universe.

Renormalized perturbation theory can be carried out within this framework [10], [11], [12]. For a free field, composite fields (Wick powers)—such as  $\phi^2$  and  $T_{ab}$ —and all time-ordered products of fields can be defined in local and covariant manner. (However, "normal ordering" cannot be used to define composite fields.) The definition of time-ordered-products is unique up to "renormalization ambiguities" of the type expected from Minkowski spacetime analyses, but with additional local curvature ambiguities (which also occur in the definition of the composite fields). Theories that are renormalizable in Minkowski spacetime remain

renormalizable in curved spacetime. Renormalization group flow can be defined in terms of the behavior of the quantum field theory under scaling of the spacetime metric,  $g_{ab} \to \lambda^2 g_{ab}$  [13]. Additional renormalization conditions can be imposed so that, in perturbation theory, the stress-energy tensor of the interacting field is conserved (for an arbitrary covariant interaction) [2].

In summary, the attempt to describe quantum field phenomena in curved spacetime has directly led to a viewpoint where symmetries and notions of "vacuum" and "particles" play no fundamental role. The theory is formulated in a local and covariant manner in terms of the quantum fields. This formulation is very well suited to investigation of quantum field effects in the early universe. In addition, the definition of composite fields, such as the stress-energy tensor, is intimately related to the OPE, and thus arises naturally in this framework. It is my hope that quantum field theory in curved spacetime will continue to provide us with deep insights into the nature of quantum phenomena in strong gravitational fields and into the nature of quantum field theory itself.

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