Renormalization-group improved effective Lagrangian for interacting theories in curved spacetime

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Abstract

A method for finding the renormalization group (RG) improved effective Lagrangian for a massive interacting field theory in curved spacetime is presented. As a particular example, the $\lambda \varphi^4$ -theory is considered and the RG improved effective Lagrangian is explicitly found up to second order in the curvature tensors. As a further application, the curvature-induced phase transitions are discussed for both the massive and the massless versions of the theory. The problems which appear when calculating the RG improved effective Lagrangian for gauge theories are discussed, taking as example the asymptotically free SU(2) gauge model.

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1. The concept of effective potential turns out to be very useful in modern particle physics and quantum cosmology. The effective potential formalism combined with the renormalization group (RG) method is even more important in particle physics phenomenology. Specifically, starting from the one-loop effective potential in renormalizable theories one can obtain the RG improved effective potential [1, 2], by summing all the leading (or subleading) logarithms. Thus, starting from a limited range of the background scalars, one can actually extend this range to other values, what renders this formalism very useful in practice [1, 2]. Using the RG effective potential in a modified form [3, 4], estimations on the absolute stability of the electroweak vacuum in the standard model have been recently discussed [3]. From a different side, the effective potential in quantum field theory in curved spacetime has been also widely discussed recently (for a general review, see [5]), starting from Ref. [6], where the possibility of curvature-induced symmetry breaking (or restoration) was realized. The explicit calculations of the one-loop effective potentials for some simple theories, such as the $\lambda \varphi^4$ -theory or scalar electrodynamics have been done on a number of different background spaces (for a list of references, see [5]). On an arbitrary, non-static curved spacetime we need a generalization of the concept of effective potential, i.e., the effective Lagrangian which can be calculated as an expansion of the effective action up to terms of some fixed-order on the curvature tensors. The effective Lagrangian up to quadratic terms in the curvature tensors has been found both for the $\lambda \varphi^4$ -theory [8] and for scalar electrodynamics (see [5], Chap. 5).

Recently [7] the RG improved effective potential in the linear curvature approximation, for an arbitrary, renormalizable massless gauge theory (including GUTs) in curved spacetime has been presented, thus generalizing Coleman-Weinberg's approach [1, 2] corresponding to flat space. The curvature-induced phase transitions, based on the form of this potential, have been discussed in detail and the possibility of such a phase transition in the SU(5) GUT has been shown. This effective potential may be relevant for early universe considerations, in particular, for the discussion of the inflationary universe (for a review and an introduction, see [10, 11]). There, most of the actual studies have been carried out for a flat-space potential.

The next problem which shows up naturally is to generalize the approach of Ref. [7] to the case of massive interacting theories in general curved spacetime. (Note that in such a situation we need the effective Lagrangian up to, at least, second order terms in the curvature tensors). In this work we introduce such a generalization, namely we present an explicit method in order to obtain the RG improved effective Lagrangian for massive interacting theories. Notice that this is quite remarkable, since the RG improved effective Lagrangian gives the result beyond one loop, i.e., it manages to sum all the logarithms of perturbation theory. Note that already two-loop calculations of the β -functions in curved space are very hard to do [9]. As an example, the case of the $\lambda \varphi^4$ -theory is studied in detail,

where we find the RG improved effective Lagrangian up to second order in the curvature tensors.

2. Let us consider the self-interacting scalar field theory with the Lagrangian

$$L_m = \frac{1}{2}g^{\mu\nu}\partial_{\mu}\varphi\partial_{\nu}\varphi - \frac{1}{2}(m^2 - \xi R)\varphi^2 - \frac{\lambda}{24}\varphi^4. \tag{1}$$

In order to have a renormalizable theory in curved spacetime, as usually, we must add to (1) the Lagrangian corresponding to the external gravitational field

$$L_G = \Lambda + \kappa R + a_1 R^2 + a_2 C_{\mu\nu\alpha\beta}^2 + a_3 G + a_4 \square R. \tag{2}$$

then, the whole multiplicatively-renormalizable theory is described by

$$L = L_m + L_G \tag{3}$$

(for details and an introduction, see [5]).

We will be interested in the study of the effective action for the theory (3). Since it turns out to be impossible to calculate the effective action on curved space-time even in the one-loop approximation, we will study the one-loop effective Lagrangian of the theory, i.e., the expansion of the one-loop effective action (up to quadratic terms in the curvature tensors). Such effective Lagrangian, using some specific renormalization conditions, has been already found in Ref. [8]. It should be noted that this effective Lagrangian (to the order we are interested in), in addition to the terms explicitly written in (3) should also contain the term

$$\Delta L = a_5 \Box \Phi^2, \tag{4}$$

which is a trivial total derivative at the tree level, but which becomes essential at the quantum level.

It is well known that a theory with the action (3) is multiplicatively renormalizable in curved spacetime. Then, the effective action (and also the effective Lagrangian) satisfies the standard RG equation

$$\left(\mu \frac{\partial}{\partial \mu} + \beta_i \frac{\partial}{\partial \lambda_i} - \gamma \Phi \frac{\partial}{\partial \Phi}\right) L_{eff}(\mu, \lambda_i, \Phi) = 0, \tag{5}$$

where $\lambda_i = (\xi, \lambda, m^2, \Lambda, \kappa, a_1, \dots, a_5)$ are all the coupling constants, including also dimensional coupling constants, β_i are the corresponding β -functions and γ is the γ -function of the scalar field. Notice that from the point of view of the background field method Φ is the background scalar field.

A solution of the RG equation can be easily found by the method of the characteristics:

$$L_{eff}(\mu, \lambda_i, \Phi) = L_{eff}(\mu(t), \lambda_i(t), \Phi(t)), \tag{6}$$

where $\Phi(t) = \Phi \exp\left(-\int_0^t \gamma(t') dt'\right)$, $\mu(t) = \mu e^t$, and $\lambda_i(t)$ are the effective coupling constants defined as the solutions of the equation

$$\frac{d\lambda_i(t)}{dt} = \beta_i(\lambda_i(t)), \qquad \lambda_i(0) = \lambda_i. \tag{7}$$

Now, we need the one-loop β -functions of the theory. (These β -functions have been calculated up to two-loop order in flat space [3] and also in curved space [9]).

Explicitly, the one-loop β -functions are

$$\beta_{\lambda} = \frac{3\lambda^{2}}{(4\pi)^{2}}, \quad \beta_{m^{2}} = \frac{\lambda m^{2}}{(4\pi)^{2}}, \quad \gamma = 0, \quad \beta_{\xi} = \frac{\lambda(\xi - 1/6)}{(4\pi)^{2}},$$

$$\beta_{\Lambda} = \frac{m^{4}}{2(4\pi)^{2}}, \quad \beta_{\kappa} = \frac{m^{2}(\xi - 1/6)}{(4\pi)^{2}}, \quad \beta_{a_{1}} = \frac{(\xi - 1/6)^{2}}{2(4\pi)^{2}}, \quad \beta_{a_{2}} = \frac{1}{120(4\pi)^{2}},$$

$$\beta_{a_{3}} = -\frac{1}{360(4\pi)^{2}}, \quad \beta_{a_{4}} = -\frac{\xi - 1/5}{6(4\pi)^{2}}, \quad \beta_{a_{5}} = -\frac{\lambda}{12(4\pi)^{2}}.$$
(8)

Starting from the tree-level effective Lagrangian, we will find the RG improved effective Lagrangian in the following form

$$L_{eff} = \frac{1}{2} g^{\mu\nu} \partial_{\mu} \Phi \partial_{\nu} \Phi - \frac{1}{2} [m^{2}(t) - \xi(t)R] \Phi^{2} - \frac{\lambda(t)}{24} \Phi^{4} + a_{5}(t) \Box \Phi^{2} + \Lambda(t) + \kappa(t)R + a_{1}(t)R^{2} + a_{2}(t)C_{\mu\nu\alpha\beta}^{2} + a_{3}(t)G + a_{4}(t)\Box R.$$
(9)

The effective coupling constants are defined by Eqs. (8) to be

$$\lambda(t) = \lambda \left(1 - \frac{3\lambda t}{(4\pi)^2} \right)^{-1}, \quad m^2(t) = m^2 \left(1 - \frac{3\lambda t}{(4\pi)^2} \right)^{-1/3},$$

$$\xi(t) = \frac{1}{6} + \left(\xi - \frac{1}{6} \right) \left(1 - \frac{3\lambda t}{(4\pi)^2} \right)^{-1/3}, \quad \Lambda(t) = \Lambda - \frac{m^4}{2\lambda} \left(1 - \frac{3\lambda t}{(4\pi)^2} \right)^{1/3} + \frac{m^4}{2\lambda},$$

$$\kappa(t) = \kappa - \frac{m^2}{\lambda} \left(\xi - \frac{1}{6} \right) \left[\left(1 - \frac{3\lambda t}{(4\pi)^2} \right)^{1/3} - 1 \right],$$

$$a_1(t) = a_1 - \frac{1}{2\lambda} \left(\xi - \frac{1}{6} \right)^2 \left[\left(1 - \frac{3\lambda t}{(4\pi)^2} \right)^{1/3} - 1 \right],$$

$$a_2(t) = a_2 + \frac{t}{120(4\pi)^2}, \quad a_3(t) = a_3 - \frac{t}{360(4\pi)^2},$$

$$a_4(t) = a_4 + \frac{t}{180(4\pi)^2} + \frac{1}{3\lambda} \left(\xi - \frac{1}{6} \right) \left[\left(1 - \frac{3\lambda t}{(4\pi)^2} \right)^{2/3} - 1 \right],$$

$$a_5(t) = a_5 + \frac{1}{36} \ln \left(1 - \frac{3\lambda t}{(4\pi)^2} \right).$$

$$(10)$$

A few remarks are in order. A convenient choice of the parameter t (compare with flat space [3, 4]) is

$$\mu^{2}(t) = \mu^{2}e^{2t} = m^{2} - \left(\xi - \frac{1}{6}\right)R + \frac{\lambda}{2}\Phi^{2}.$$
 (11)

Such a combination appears naturally in a direct calculation of the one-loop efective Lagrangian. Then, the RG improved effective Lagrangian (9) (summing all the logarithms) in the limit $\lambda \ll 1$, $|\lambda t| \ll 1$, easily reproduces the results of the much more tedious one-loop calculation [8] if the same renormalization conditions are imposed. However, notice that (contrary to what happens in the non-improved case) Eq. (9) is actually valid for all t for which L_{eff} does not diverge (this is the improvement).

If we consider the flat space limit of (9), i.e. R = 0 and also take $\Phi = \text{const.}$, then Eq. (9) reduces to the RG improved effective potential

$$-L_{eff} = V = \frac{\lambda(t)}{24} \Phi^4 + \frac{m^2(t)}{2} \Phi^2 - \Lambda + \frac{m^4}{2\lambda} \left(1 - \frac{3\lambda t}{(4\pi)^2} \right)^{1/3} - \frac{m^4}{2\lambda},\tag{12}$$

with $t = (1/2) \ln[(m^2 + \lambda \Phi^2/2)/\mu^2]$.

This is the standard RG improved effective potential with the effective cosmological constant $\Lambda(t)$ precisely of the form introduced in Ref. [4] (see also the discussion in [3]). Thus we have the natural curved-space interpretation as the effective cosmological constant for the RG improved potential at zero background field.

3. Let us consider now some application of the effective Lagrangian (9). Choosing for simplicity the constant curvature space $R_{\mu\nu} = (R/4)g_{\mu\nu}$ (then, naturally, $\Phi = \text{const.}$), and working in the linear curvature approximation, we get

$$L_{eff} = -\frac{\lambda(t)}{24}\Phi^4 - \frac{1}{2}\left[m^2(t) - \xi(t)R\right]\Phi^2 + \Lambda(t) + \kappa(t)R.$$
 (13)

Already the classical Lagrangian (1) exhibits the possibility of spontaneous symmetry breaking [6]

$$\Phi^2 = \frac{2}{\lambda}(\xi R - m^2),\tag{14}$$

for $\xi R > m^2$.

We now investigate the phase structure of the RG improved potential (13). It is nowadays common to think that the very early universe experienced several phase transitions before it reached its present state. Some models of the inflationary universe are based on the temperature phase transition [10, 11]. It is also possible that a phase transition could be induced by the strong gravitational field existing in the very early universe. We will discuss only the possibility of a first order phase transition, where the order parameter, Φ , experiences a quick change for some critical value, R_c , of the curvature. For simplicity, we

will put $\Lambda = \kappa = 0$ in (13), since the general situation is just a rescaling of the constant part of the potential.

One can rewrite (13) as follows

$$-\frac{L_{eff}}{\mu^4} = \frac{V}{\mu^4} = \frac{\lambda(t)x^2}{24} + \frac{1}{2} \left[m^2(t) - \xi(t)y \right] x + \left[\left(1 - \frac{3\lambda t}{(4\pi)^2} \right)^{1/3} - 1 \right] \left[\frac{\widetilde{m}^4}{2\lambda} + \frac{\widetilde{m}^2(\xi - 1/6)y}{\lambda} \right], \tag{15}$$

where

$$x = \frac{\Phi^2}{\mu^2}$$
, $\widetilde{m} = \frac{m}{\mu}$, $y = \frac{R}{\mu^2}$, $t = \frac{1}{2} \ln \left[\frac{\lambda}{2} x + \widetilde{m}^2 - \left(\xi - \frac{1}{6} \right) y \right]$.

4. We can now extend the analysis of critical points, that was performed in Ref. [7] for the massless case, to the general action (15) corresponding to the massive case. Let us recall that the critical parameters, x_c , y_c , corresponding to the first-order phase transition are found from the conditions

$$V(x_c, y_c) = 0, \quad \left. \frac{\partial V}{\partial x} \right|_{x_c, y_c} = 0, \quad \left. \frac{\partial^2 V}{\partial x^2} \right|_{x_c, y_c} > 0.$$
 (16)

For the RG improved potential they lead to some transcendental equations which cannot be solved analytically. We shall be concerned with first-order phase transitions where the order parameter Φ experiences a quick change for some critical value, R_c , of the curvature. Let us again recall first the case of the $\lambda \Phi^4$ theory without mass, where one has [7]

$$\frac{V}{\mu^4} = \frac{\lambda x^2}{4! \left(1 - \frac{3\lambda \ln x}{32\pi^2}\right)} - \frac{1}{2} \epsilon y x \left[\frac{1}{6} + \left(\xi - \frac{1}{6}\right) \left(1 - \frac{3\lambda \ln x}{32\pi^2}\right)^{-1/3} \right],\tag{17}$$

with $x = \Phi^2/\mu^2$, $y = |R|/\mu^2$, and $\epsilon = \text{sgn } R$. The analysis of extrema of (17) yields the following result. One looks for critical points (x_c, y_c) defined by the simultaneous conditions (16). The first two equations (16) yield, for the massless potential (17),

$$y_c = \frac{\epsilon \lambda x_c}{2u \left[1 + (6\xi - 1)u^{-1/3}\right]}, \quad \frac{32\pi^2}{3\lambda} u - \frac{1}{3\left[1 + (6\xi - 1)u^{1/3}\right]} + 1 = 0, \quad u \equiv 1 - \frac{3\lambda \ln x_c}{32\pi^2}. \quad (18)$$

The following models are particularly interesting.

(a) Chaotic inflationary model. For $\lambda = 10^{-13}$ and $\xi = 0$, both for positive $\epsilon = 1$ and negative $\epsilon = -1$ curvature, a critical value appears, which lies close to the pole (x_p) :

$$x_c = \exp\left(-\frac{2}{3}10^{-15}\right) x_p, \quad y_c = -\epsilon 10^{-3} x_c, \quad x_p = \exp\left(\frac{32\pi^2}{3\lambda}\right).$$
 (19)

This point is a minimum of (17), that is, all three equations (16) are indeed satisfied. (Quite on the contrary, the one-loop effective action does not yield any phase transition, the solutions being $x_c = 0$, $y_c = 0$, and $x_c = 1$, $y_c = \epsilon 10^{17} x_c$).

(b) Variable Planck-mass model. For λ we take a typical value corresponding to particle physics models, e.g. $\lambda = 0.05$. For ξ we choose two different values: (b1) $\xi = -10^4$ (which actually corresponds to Ref. [13]) and (b2) $\xi = 1/6$, respectively.

In case (b1), the critical point corresponding to (17) is obtained for

$$x_c = e^{2/3} x_p, \quad y_c = -5 \cdot 10^{-5} x_c,$$
 (20)

both for positive and for negative curvature. (For the one-loop action, the only solution is again the trivial one $x_c = y_c = 0$).

In case (b2), the critical point for the RG improved action (massless case) is at

$$x_c = e x_p, \quad y_c = -\epsilon \, 50 \, x_c, \tag{21}$$

which is not consistent with our approximation $x_c >> y_c$. (For the one-loop effective action $x_c = y_c = 0$ is again the only solution).

Guided by the results of the analysis above corresponding to the massless case —and in order to simplify the more involved one of the massive case— it is natural to start by rescaling the variables and the potential in (15) as follows:

$$\bar{x} = x \exp\left(-\frac{32\pi^2}{3\lambda}\right), \quad \bar{y} = y \exp\left(-\frac{32\pi^2}{3\lambda}\right), \quad \bar{m}^2 = \tilde{m}^2 \exp\left(-\frac{32\pi^2}{3\lambda}\right),$$

$$\bar{u} = -\frac{3\lambda}{32\pi^2}\bar{t}, \quad \bar{t} = \ln\left[\frac{\lambda}{2}\bar{x} + \bar{m}^2 - \left(\xi - \frac{1}{6}\right)\bar{y}\right], \quad \bar{V} = \frac{V}{\mu^4} \exp\left(-\frac{64\pi^2}{3\lambda}\right), \quad (22)$$

so that we obtain the simplified expression

$$\bar{V} = \frac{\lambda \bar{x}^2}{24\bar{u}} + \frac{\bar{x}}{2} \left\{ \bar{m}^2 \bar{u}^{-1/3} - \bar{y} \left[\frac{1}{6} + \left(\xi - \frac{1}{6} \right) \bar{u}^{-1/3} \right] \right\} + \frac{\bar{m}^2}{\lambda} \left(\bar{u}^{1/3} - 1 \right) \left[\frac{\bar{m}^2}{2} + \left(\xi - \frac{1}{6} \right) \bar{y} \right]. \tag{23}$$

How does the introduction of mass change the results given above corresponding to the massless case? A careful analysis of (23) leads to the following conclusions. The changes in the equations for the critical points can be absorbed by a shift in the y-variable proportional to the mass, namely

$$\bar{y} \longrightarrow \bar{y} - \frac{\bar{m}^2}{\xi - 1/6}.$$
 (24)

This is also valid for the second case (b1) above, but certainly not for the third case (b2). Let us recall, though, that this is the only situation in which we do not obtain a critical point consistent with the approximation in which we are working, already for the massless model. This fact is made no better through the introduction of mass. Again, concerning the first two cases, and for reasonable values of the mass (i.e., $\bar{m}^2 < 1$) we do not get a substantial change of the situation described for the massless case. However, the nontrivial

critical points do not show up any more for higher values of the mass ($\bar{m}^2 > 1$, recall the rescaling (22)). Thus we see that for positive $\xi > 1/6$ a non-zero (positive) mass leads to a smaller critical curvature. Also, for negative \bar{m}^2 what one gets is an increase of the critical curvature, which gets very large near $\xi = 1/6$.

Notice, moreover, that in order to include higher-order terms in the above study one has to do already a very tedious numerical analysis for different values of the parameters.

The higher-order terms in the effective Lagrangian may be important in some different contexts. For example, starting from the RG improved effective potential, taking into account second-order curvature terms, we may expand it on Φ . In this case the typical form of the potential will be

$$V = \Lambda_{eff} + m_{eff}^2 \Phi^2 + \mathcal{O}(\Phi^4), \tag{25}$$

where m_{eff}^2 includes all the curvature terms up to second order. Then, it is easy to estimate the influence of the second-order curvature terms on the symmetry breaking or restoration of the theory, for different types of constant curvature spaces, and to compare it with the corresponding results for the non-improved effective potential [8].

Another interesting question is connected with the back reaction problem. One can use the RG improved effective Lagrangian to study Einstein equations with quantum corrections of interacting matter fields. However, this analysis involves (again) tedious numerical calculations, owing to the high non-linearity of the problem.

5. Finally, let us discuss the generalization of the above method to multiple-mass cases. As an explicit example we shall consider the massive gauge theory based on the gauge group SU(2) with one multiplet of scalars (φ^a , a=1,2,3), taken in the adjoint representation of SU(2), and one or two multiplets of spinors taken in the adjoint representation too (for details of this model in an asymptotically free regime, see [12]). This theory is asymptotically free for all coupling constants, and for the case of one spinor multiplet it is also asymptotically conformally invariant.

In principle, one can apply the above procedure in order to get the RG improved effective potential of the theory. However, now some of the effective masses are different:

$$m_B^2 = m^2 - \left(\xi - \frac{1}{6}\right) R + c_1 \lambda \Phi^2 + c_2 g^2 \Phi^2, \quad M_F^2 = h^2 \Phi^2 + c_3 R, \quad M_{\mu\nu}^2 \simeq \delta_{\mu\nu} (g^2 \Phi^2 + c_4 \lambda \Phi^2) + c_5 R_{\mu\nu},$$
(26)

where c_1, \ldots, c_5 are some constants, and $\Phi^2 = \varphi^a \varphi^a$. It turns out that it is impossible to choose t in (11) for the theory under discussion in the same unique way as for the $\lambda \varphi^4$ -theory above. Of course, choosing $t = \ln(\Phi/\mu)$ we always will have the correct behavior of L_{eff} at large Φ but not at all scales. The question is, however, if we are able to construct the

more exact, RG improved effective Lagrangian, not just its asymptotic form. The answer is certainly positive.

Indeed, working in the asymptotically free regime

$$\lambda(t) = k_1 g^2(t), \quad h^2(t) = k_2 g^2(t), \quad g^2(t) = g^2 \left[1 + \frac{a^2 g^2 t}{(4\pi)^2} \right]^{-1},$$

where k_1, k_2 and a^2 are given in [12], one may construct the RG improved L_{eff} as above, and then choose $t = \frac{1}{2} \ln(h^2 \Phi^2/\mu^2)$. Then, one has (for simplicity, we write the RG improved effective potential in the linear curvature approximation only)

$$V_{eff} = \frac{1}{4!} \Phi^4 f^4(t) k_1 g^2(t) - \frac{1}{2} R \Phi^2 f^2(t) \left[\frac{1}{6} + \left(\xi - \frac{1}{6} \right) \left(1 + \frac{a^2 g^2 t}{(4\pi)^2} \right)^{(5k_1/3 + 8k_2 - 12)/a^2} \right] - \frac{1}{2} \Phi^2 f^2(t) m^2 \left(1 + \frac{a^2 g^2 t}{(4\pi)^2} \right)^{(5k_1/3 + 8k_2 - 12)/a^2} - \Lambda(t) - \kappa(t) R,$$
(27)

where

$$f(t) = \left(1 + \frac{a^2 g^2 t}{(4\pi)^2}\right)^{(6-4k_2)/a^2}, \quad \Lambda(t) = \Lambda + \frac{3m^4}{2a^2 g^2 A} \left[\left(1 + \frac{a^2 g^2 t}{(4\pi)^2}\right)^A - 1 \right],$$

$$\kappa(t) = \kappa + \frac{3m^2(\xi - 1/6)}{a^2g^2A} \left[\left(1 + \frac{a^2g^2t}{(4\pi)^2} \right)^A - 1 \right], \quad A = \left(\frac{5}{3}\kappa_1 + 8\kappa_2 - 12 \right) \frac{1}{a^2} + 1.$$

This potential is very similar (forgetting about the mass, cosmological and Newton terms) to the one of Ref. [7] corresponding to the massless case. However, this means that we get L_{eff} (or V_{eff}) only in the region $h^2\Phi^2 > m^2$. In order to obtain the RG improved effective Lagrangian we have to find it in other regions. This is not so straightforward and demands some careful considerations (using, for instance, the multiscale RG of Einhorn and Jones [2]), that will be presented in future work.

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