

# Renormalization group improvement of the effective potential in massive $\phi^4$ theory

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In massive  $\phi^4$  theory two procedures are introduced that use the renormalization group to obtain an improved effective potential. This potential reduces to the ordinary effective potential for small excursions in field space and to the massless effective potential for large fields. Explicit results are given at the two-loop level.

## 1. Introduction

The effective potential (EP) of a relativistic quantum field theory [1,2] is a convenient tool to investigate the vacuum structure of the theory. It is also important in other contexts, like inflation (see e.g. ref. [3]). Since an exact computation of the EP in general involves evaluating an infinite number of arbitrarily complicated Feynman diagrams, it is important to have a sensible approximation scheme. Such a scheme is given by the loop expansion [4]. However the  $n$ -loop contribution will typically involve terms  $\alpha^{n+1} \ln^m(\phi^2/\mu^2)$  with  $0 \leq m \leq n$ , where  $\mu$  is the renormalization scale and  $\alpha$  is the largest appropriately defined coupling in the theory (for details, see e.g. ref. [5]). Since we can make the logarithm small only in the vicinity of one arbitrary subtraction point, it is not sufficient to have small  $\alpha$ , but rather we require  $\alpha \ln(\phi_{\max}/\phi_{\min})$  to be small, where  $\phi_{\min}$  and  $\phi_{\max}$  are the smallest and largest value of the field for which we wish the EP to be valid. This is unsatisfactory for applications where we need to know the EP over a large range in field space involving several orders of magnitude, such as investigations of vacuum stability [5] and inflation.

The renormalization group (RG) comes to rescue and restricts the terms appearing in the EP. Knowledge of the beta and gamma functions and of the EP at the one-, two-, ... loop level provides knowledge of all the leading, next-to-leading, ... logarithmic terms

in the EP [4,5], which then have to be summed up. However, this program has been carried out consistently only in cases where there is no bare mass parameter present. If there are bare masses in the theory, the form of the EP for fields large compared to the masses can usually still be taken from the massless case. However, it is neither immediately clear how to correctly patch together the unimproved potential and the massless large- $\phi$  potential, nor how to define the parameters in the massless potential by the parameters in the unimproved massive potential.

In this letter we study this problem in some detail. After briefly discussing the well-known case of massless  $\phi^4$  theory, we introduce the correct method for summing up the leading, next-to-leading, ... logarithms in the massive case by means of a power expansion in suitably chosen variables. An alternative procedure is also explained that relates the leading, next-to-leading, ... logarithmic contributions to the improved potential by means of differential equations. As an illustration, the improved EP will be given explicitly up to two loops.

## 2. Massless case

In massless  $\phi^4$  theory with

$$\mathcal{L} = \frac{1}{2} \partial_\mu \phi \partial^\mu \phi - V_0(\phi), \quad V_0 = \frac{\lambda}{4!} \phi^4, \quad (1)$$

the one-loop contribution to the (unimproved) EP is

$$V_1 = \frac{\lambda^2 \phi^4}{16(4\pi)^2} \left( \ln \frac{\lambda \phi^2}{2\mu^2} - \frac{3}{2} \right). \quad (2)$$

Here I have used dimensional regularization [6] in conjunction with the  $\overline{\text{MS}}$ -scheme [7], a scheme used throughout this paper. Determining the one-loop RG improved potential [4,5] amounts to solving the renormalization group equation (RGE)

$$\left( \mu \frac{\partial}{\partial \mu} + \beta(\lambda) \frac{\partial}{\partial \lambda} - \gamma(\lambda) \phi \frac{\partial}{\partial \phi} \right) V(\lambda, \phi, \mu) = 0 \quad (3)$$

with the boundary condition

$$V(\phi = \mu) = \frac{\lambda}{4!} \mu^4. \quad (4)$$

Using the lowest order beta and gamma functions<sup>#1</sup>  $\beta(\lambda) = 3\lambda^2/(4\pi)^2$  and  $\gamma(\lambda) = 0$  gives the Coleman–Weinberg result [4]

$$V = \lambda \left[ 4! \left( 1 - \frac{3\lambda}{2(4\pi)^2} \ln \frac{\phi^2}{\mu^2} \right) \right]^{-1} \phi^4. \quad (5)$$

Upon rescaling  $\mu^2 \rightarrow 2\mu^2 \exp(\frac{3}{2}/\lambda)$  and evaluating the fraction in a geometrical expansion, the first two terms are seen to be identical to the sum of  $V_0$  and  $V_1$  from (1) and (2). Therefore this resummation for the massless case makes perfect sense.

### 3. Power series expansion method

For the massive case, with

$$V_0 = \frac{\lambda}{4!} \phi^4 + \frac{1}{2} m^2 \phi^2, \quad (6)$$

it is unclear with what boundary condition to solve the appropriate RGE

$$\left( \mu \frac{\partial}{\partial \mu} + \beta(\lambda) \frac{\partial}{\partial \lambda} + \gamma_m(\lambda) m^2 \frac{\partial}{\partial m^2} - \gamma(\lambda) \phi \frac{\partial}{\partial \phi} \right) \times V(\lambda, m^2, \phi, \mu) = 0. \quad (7)$$

<sup>#1</sup> The  $n$ -loop contribution to  $\beta(\lambda)$  is proportional to  $\lambda^{n+1}$ , while the  $n$ -loop terms in  $\gamma_m(\lambda)$  (to be defined below) and  $\gamma(\lambda)$  are proportional to  $\lambda^n$ . However, there is no wave function renormalization at the one-loop level in  $\phi^4$  theory and therefore  $\gamma(\lambda) = 0(\lambda^2)$ .  $\beta$ ,  $\gamma$  and  $\gamma_m$  are known at least up to five loops in the  $\overline{\text{MS}}$  scheme [8] which for this purpose is equivalent to the  $\overline{\text{MS}}$  scheme.

To improve the situation it is necessary to better understand the structure of the EP. Using vacuum graphs [9] or tadpoles [10] in a shifted theory to compute the EP, it is not hard to convince oneself that its general structure, as calculated loop by loop, is given by

$$\tilde{V} = a\lambda + \frac{b\lambda}{x} + \sum_{l=1}^{\infty} \lambda^{l+1} \sum_{m=0}^{l-1} x^{m-2} \sum_{n=0}^l y^n a_{l,mn}, \quad (8)$$

where

$$\tilde{V} \equiv \frac{V}{\phi^4}, \quad x \equiv \frac{1}{1 + 2m^2/\lambda\phi^2}, \quad (9) \quad y \equiv \ln \frac{\frac{1}{2}\lambda\phi^2 + m^2}{\mu^2}, \quad (9)$$

and  $L$  labels the  $L$ -loop contribution. Using the same conventions as in (6), we have  $a = -\frac{5}{24}$  and  $b = \frac{1}{4}$ , while the  $a_{l,mn}$  are numbers to be determined.

At tree level it is not clear what function of  $\lambda$  and  $m^2$  to choose as the term constant in  $\phi$ , which we have taken to be zero so far. Indeed, it is straightforward to show that (8), with the  $a_{l,mn}$  calculated loop by loop, fails to obey the RGE (7). However, we can also write the effective potential as a power series in  $\phi$ , where the coefficients are the truncated  $n$ -point functions at zero external momenta [11]:

$$V = - \sum_{n=0}^{\infty} \frac{1}{n!} \Gamma^{(n)}(p_i=0) \phi^n. \quad (10)$$

Since each summand in (10) is supposed to obey (7), we can throw away the unphysical  $n=0$  term altogether without spoiling (7). Then (8) changes to its subtracted version  $\tilde{V}_{\text{sub}} \equiv V_{\text{sub}}/\phi^4 \equiv [V(\phi) - V(0)]/\phi^4$  or

$$\tilde{V}_{\text{sub}} = a\lambda + \frac{b\lambda}{x} + \sum_{l=1}^{\infty} \lambda^{l+1} \sum_{n=0}^l y^n \left[ \sum_{m=0}^{l-1} x^{m-2} a_{l,mn} - \left( \frac{1-x}{x} \right)^2 \sum_{k=0}^l \binom{n+k}{k} \ln^k(1-x) a_{l,0,k+n} \right]. \quad (11)$$

The RGE for  $\tilde{V}$  (if the correct tree level constant is included, we will return to that point in section 4) and  $\tilde{V}_{\text{sub}}$  in terms of the new variables  $x$  and  $y$  reads

$$\begin{aligned}
& \left[ -2 \frac{\partial}{\partial y} + \frac{\beta}{\lambda} \left( \lambda \frac{\partial}{\partial \lambda} + (1-x)x \frac{\partial}{\partial x} + \frac{\partial}{\partial y} \right) \right. \\
& \quad + \gamma_m (1-x) \left( -x \frac{\partial}{\partial x} + \frac{\partial}{\partial y} \right) \\
& \quad \left. - 2\gamma x \left( (1-x) \frac{\partial}{\partial x} + \frac{\partial}{\partial y} \right) - 4\gamma \right] \\
& \quad \times \tilde{V}_{(sub)}(\lambda, x, y) = 0. \quad (12)
\end{aligned}$$

Defining  $\alpha_k, \beta_k$  and  $\gamma_k$  by

$$\gamma_m \equiv \sum_{k=1}^{\infty} \alpha_k \lambda^k, \quad \beta \equiv \sum_{k=2}^{\infty} \beta_k \lambda^k, \quad \gamma \equiv \sum_{k=1}^{\infty} \gamma_k \lambda^k, \quad (13)$$

and comparing powers of  $\lambda, x, y$  and  $\ln(1-x)$  in the RGE, we get recursion relations for the  $a_{Lmn}$ . Among them are

$$a_{101} = \frac{1}{4}(\alpha_1 - 2\gamma_1)b, \quad (14)$$

$$a_{101} = \frac{1}{2}(4\gamma_1 - \beta_2)a - \frac{1}{2}(\beta_2 - \alpha_1 - 2\gamma_1)b, \quad (15)$$

$$\begin{aligned}
a_{n0n} &= \frac{1}{2n} [(n-2)\beta_2 + 2\alpha_1]a_{n-1,0,n-1}, \\
&\text{for } n \geq 2, \quad (16)
\end{aligned}$$

$$a_{212} = \frac{1}{2}(\beta_2 - \alpha_1 - 2\gamma_1)a_{101}, \quad (17)$$

$$\begin{aligned}
a_{n1n} &= \frac{1}{2n} \{ 2(\beta_2 - \alpha_1 - 2\gamma_1)a_{n-1,0,n-1} \\
&\quad + [(n-1)\beta_2 - 2\gamma_1 + \alpha_1]a_{n-1,1,n-1} \}, \\
&\text{for } n \geq 3, \quad (18)
\end{aligned}$$

$$a_{323} = \frac{1}{6}(\beta_2 - \alpha_1 - 2\gamma_1)a_{212}, \quad (19)$$

$$\begin{aligned}
a_{n2n} &= \frac{1}{2n} [(n\beta_2 - 4\gamma_1)a_{n-1,2,n-1} \\
&\quad + (\beta_2 - \alpha_1 - 2\gamma_1)a_{n-1,1,n-1}], \quad \text{for } n \geq 4. \quad (20)
\end{aligned}$$

It further turns out that

$$a_{nkn} = 0, \quad \text{for } n > k \geq 3. \quad (21)$$

Eqs. (14) and (15) are the only place where the same quantity is given by two relations and indeed they yield the same result  $a_{101} = 1/16(4\pi)^2$ , which also follows from the one-loop result

$$V_1 = \frac{(\frac{1}{2}\lambda\phi^2 + m^2)^2}{4(4\pi)^2} \left( \ln \frac{\frac{1}{2}\lambda\phi^2 + m^2}{\mu^2} - \frac{3}{2} \right). \quad (22)$$

Thus all the coefficients of the leading logarithms (i.e. all  $a_{Lmn}$  with  $n=L$ ) are known from the tree-level potential (6) and the one-loop results for the beta and gamma functions, i.e.

$$\alpha_1 = \frac{1}{(4\pi)^2}, \quad \beta_2 = \frac{3}{(4\pi)^2}, \quad \gamma_1 = 0. \quad (23)$$

In fact, the leading logarithmic terms can be summed up. The sum of the tree level potential  $V_0$  and the leading logarithms gives for  $\tilde{V}_{sub}$

$$\begin{aligned}
\tilde{V}_{sub} &= \frac{\lambda}{4!} \left( 1 - \frac{1}{2}\beta_2\lambda y \right)^{(4\gamma_1 - \beta_2)/\beta_2} \\
&\quad + \frac{\lambda}{4} \frac{1-x}{x} (1 - \frac{1}{2}\beta_2\lambda y)^{(2\gamma_1 - \alpha_1)/\beta_2} \\
&\quad - \frac{\lambda}{8} \left( \frac{1-x}{x} \right)^2 (1 - \frac{1}{2}\beta_2\lambda y)^{(\beta_2 - 2\alpha_1)/\beta_2} \\
&\quad + \frac{\lambda}{8} \left( \frac{1-x}{x} \right)^2 \\
&\quad \times \{ 1 - \frac{1}{2}\beta\lambda[y + \ln(1-x)] \}^{(\beta_2 - 2\alpha_1)/\beta_2}, \quad (24)
\end{aligned}$$

where the last term corresponds to subtracting the constant out of  $V$ .

The result (24) nicely reintroduces the concept of a running coupling constant and a running mass:

$$V_{sub} = \frac{\hat{\lambda}}{4!} \phi^4 + \frac{1}{2} \hat{m}^2 \phi^2 - \frac{(\hat{m}^2)^2}{2\hat{\lambda}} - (\phi=0), \quad (25)$$

with

$$\begin{aligned}
\hat{\lambda} &= \lambda \left( 1 - \frac{3\lambda}{2(4\pi)^2} \ln \frac{\frac{1}{2}\lambda\phi^2 + m^2}{\mu^2} \right)^{-1}, \\
\hat{m}^2 &= m^2 \left( 1 - \frac{3\lambda}{2(4\pi)^2} \ln \frac{\frac{1}{2}\lambda\phi^2 + m^2}{\mu^2} \right)^{-1/3}. \quad (26)
\end{aligned}$$

For small excursions in field space, (25) reproduces the unimproved result (22), except for the missing  $a_{100}$  term and, of course, the subtracted constant. The  $a_{100}$  term is of higher order in the new scheme. It could be accounted for by rescaling  $\mu$ , thereby also turning on other terms in the expansion. Those however are of higher order as well and anyway receive at this stage unknown contributions from higher loops.

It should be remarked that we have not tried to obtain an effective potential that obeys the RGE with

the one-loop beta and gamma functions. We could change the terms  $a_{L,mn}$  with  $L > n$  so that the one-loop RGE is fulfilled. However, we do not really gain anything by doing so, because these terms receive contributions from higher loops and we have no control of these contributions unless we perform higher loop calculations. Furthermore, this procedure is not unique because from the standpoint of the RG some of those  $a_{L,mn}$  (actually an infinite subset) are free parameters.

We see that after rescaling by  $\mu^2 \rightarrow 2\mu^2/\lambda$ , the massless result (5) gives the correct large  $\phi$  behavior, provided we define  $\lambda$  and  $m^2$  by taking them over from (22). Therefore the result (25) tells us how to correctly patch together the small- $\phi$  behavior described by the sum of (6) and (22) and the large- $\phi$  behavior (5) of the effective potential. This is important because we have to define the parameters in the theory, e.g. by fixing the Higgs mass and the vacuum expectation value of  $\phi$ .

#### 4. Differential equations method

One cannot hope always to be able to sum up explicitly some power series with recursively defined coefficients. It would be much nicer if we could convert the RGE (12) into single equations for the leading, next-to-leading, ... logarithmic contributions to the improved EP. Comparing (8) and (11) shows that for such a procedure to work it is crucial that we manage to avoid having to subtract out the constant term of the potential. But what term do we have to take as the tree level constant? Since we do not want  $\mu$  to appear in  $V_0$ , the only remaining possibility is  $m^4$  times some function of  $\lambda$ . Therefore we try the ansatz

$$\begin{aligned} \tilde{V}_{\text{full}} &= a\lambda \\ &+ \frac{b\lambda}{x} + \sum_{l=1}^{\infty} \lambda^{l+1} \sum_{m=0}^{l-1} x^{m-2} \sum_{n=0}^l y^n a_{l,mn} \\ &+ \left(\frac{1-x}{x}\right)^2 \sum_{k=1}^{\infty} b_k \lambda^k. \end{aligned} \quad (27)$$

The  $b_k$  are to be defined by consistency as we go along (for  $k > 3$  we could absorb the  $b_k$  into the  $a_{L,mn}$ , but then the  $a_{L,mn}$  would no longer be the  $\overline{\text{MS}}$  scheme coefficients).  $\tilde{V}_{\text{full}}$  from (27) can obviously be written as

$$\tilde{V}_{\text{full}} = \sum_{l=1}^{\infty} \lambda^l f_L(x, t), \quad (28)$$

where  $t \equiv \lambda y / (4\pi)^2$  and the  $f_1, f_2, \dots$  sum up the leading, next-to-leading, ... logarithms of the EP. In this expansion the tree level potential becomes formally part of the leading logarithms  $f_1$ .

The RGE can be rewritten in the variables  $\lambda, x$  and  $t$  and applied to (28). Comparing powers of  $\lambda$  leads to the recursion relations

$$\begin{aligned} (4\pi)^2 \sum_{k=1}^l \left( \beta_{k+1} t \frac{\partial}{\partial t} + (1-x)x(\beta_{k+1} - \alpha_k - 2\gamma_k) \frac{\partial}{\partial x} \right. \\ \left. + [(L-k+1)\beta_{k+1} - 4\gamma_k] \right) f_{l-k+1} \\ - 2 \frac{\partial f_l}{\partial t} + \sum_{k=1}^{l-1} [\beta_{k+1}x + \alpha_k(1-x) - 2\gamma_kx] \frac{\partial f_{l-k}}{\partial t} \\ = 0. \end{aligned} \quad (29)$$

The boundary conditions for these equations are given by reexpressing (27) in terms of  $\lambda, x$  and  $t$  and then setting  $t=0$ , where the  $b_k$  can be easily gotten by demanding consistency with the unimproved  $n$ -loop EP. Proceeding in this fashion we get  $b_1 = -\frac{1}{8}$  and

$$\begin{aligned} f_1(x, t) &= \frac{1}{24} (1 - \frac{3}{2}t)^{-1} + \frac{1}{4} \frac{1-x}{x} (1 - \frac{3}{2}t)^{-1/3} \\ &- \frac{1}{8} \left( \frac{1-x}{x} \right)^2 (1 - \frac{3}{2}t)^{1/3} \end{aligned} \quad (30)$$

which agrees with the first three terms in (24). As noted earlier, the last term in (24) subtracts out the constant in  $V$  and we now succeeded to avoid this subtraction.

Even  $f_2$  can be computed explicitly: With  $b_2 = 1/(4(4\pi)^2)$  from demanding consistency with the unimproved two-loop contribution <sup>#2</sup>

<sup>#2</sup> The numerical coefficients in ref. [10] cannot be trusted. In ref. [12] different renormalization conditions are imposed. However, the leading logarithms are scheme independent and coincide with (31), in contrast to ref. [10]. The result in ref. [9] for the massless case also supports (31). Further, the result of recent work [13] about the  $O(N)$  symmetric theory reduces to (31) for  $N=1$ . I have determined (22) and (31) using vacuum graphs in a shifted theory [9].

$$\tilde{V}_2 = \lambda^3 \left[ \frac{1}{x^2} \left( \frac{y^2}{32(4\pi)^4} - \frac{y}{16(4\pi)^4} + a_{200} \right) + \frac{1}{x} \left( \frac{y^2}{16(4\pi)^4} - \frac{y}{4(4\pi)^4} + a_{210} \right) \right] \quad (31)$$

( $a_{200}$  and  $a_{210}$  are of no concern here), we get with

$$\alpha_2 = -\frac{5}{6(4\pi)^4}, \quad \beta_3 = -\frac{17}{3(4\pi)^4}, \quad \gamma_2 = \frac{1}{12(4\pi)^4} \quad (32)$$

the result

$$\begin{aligned} f_2(x, t) = & \frac{1}{(4\pi)^2} \left[ \left( -\frac{3}{32} - \frac{1}{144}t + \frac{25}{432} \ln(1 - \frac{3}{2}t) \right. \right. \\ & + \frac{1}{16} \ln\{1 + x[(1 - \frac{3}{2}t)^{-2/3} - 1]\} \left. \right) (1 - \frac{3}{2}t)^{-2} \\ & + \left( -\frac{3}{16} + \frac{1}{9}t + \frac{25}{216} \ln(1 - \frac{3}{2}t) \right. \\ & + \frac{1}{8} \ln\{1 + x[(1 - \frac{3}{2}t)^{-2/3} - 1]\} \left. \right) \\ & \times (1 - \frac{3}{2}t)^{-4/3} \frac{1-x}{x} \\ & + \left( \frac{5}{32} - \frac{19}{144}t + \frac{25}{432} \ln(1 - \frac{3}{2}t) \right. \\ & + \frac{1}{16} \ln\{1 + x[(1 - \frac{3}{2}t)^{-2/3} - 1]\} \left. \right) \\ & \times (1 - \frac{3}{2}t)^{-2/3} \left. \left( \frac{1-x}{x} \right)^2 \right]. \quad (33) \end{aligned}$$

I have checked this result with several  $a_{L,mn}$  obtained with the power expansion method and found agreement.

## 5. Summary

Two methods were introduced to compute the renormalization group improved potential in massive  $\phi^4$  theory loop by loop. After the crucial step of understanding the general structure (8) of the effective potential in the theory, we found the leading, next-to-leading, ... logarithms to be completely determined by one-, two-, ... loop calculations, as in the massless case. The leading logarithms could be summed up to give (25). Another method was introduced that does not rely on being able to sum up a power series involving coefficients gotten through

complicated recursion relations. After introducing a suitably chosen constant into the tree-level potential, the leading, next-to-leading, ... logarithms can be obtained by solving one differential equation for each. The contributions of the leading and next-to-leading logarithms were given explicitly in (30) (agreeing with the power series result (25)) and (33).

The problem treated here is closely related to effective field theory considerations (see e.g. refs. [14–17]). The scheme adopted in this paper has the same effect a physical (mass dependent) scheme would have in explicitly decoupling [15] the heavy modes at small field strength. Alternatively one could pursue an effective field theory approach and integrate out [16] the heavy modes to get the effective potential at small field strength and then match onto the low-energy theory. In other words, in this paper we exploit the fact that the renormalization group matches the theory at different scales without changing the particle content [17].

Although the differential equations method can be extended [18] to the  $O(N)$  symmetric theory which contains Goldstone bosons, it is not known to me if the outlined ideas can be successfully applied to more realistic theories, where complications might arise because of fermions, gauge fields, or through the presence of several scales [19] and coupling constants. It would further be desirable to extend the methods to the case of finite temperature.

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