ASYMPTOTIC FREEDOM OF MASSLESS $\lambda\Phi^4$ THEORIES

P. CASTORINA
Dipartimento di Fisica, Università di Catania, Catania, Italy
and Istituto Nazionale di Fisica Nucleare, Sezione di Catania, Catania, Italy

and

M. CONSOLI [1]
CERN, CH-1211 Geneva 23, Switzerland

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The crucial role of the mass renormalization condition leading to spontaneous symmetry breaking in $\lambda\Phi^4$ theory is discussed. By using the renormalization group equations and a variational estimate of the ground state energy, it is suggested that massless $\lambda\Phi^4$ theory in four dimensions is asymptotically free, not in contradiction with known rigorous results.

The scalar sector of the standard model [1] plays a fundamental role for mass generation. Up to now, there is no experimental evidence for the mechanism underlying spontaneous symmetry breaking, however, stringent, theoretical arguments, constraining the possible structure of massive vector boson interactions [2], strongly suggest their coupling with a scalar sector. For this reason it is worth investigating the internal structure of the self interacting theories as an additional check of the standard model as a consistent quantum field theory.

Clearly the coupling of the Higgs sector to vector bosons and fermions complicates the analysis with respect to a simple self-interacting scalar theory. Nevertheless a pure $\lambda\Phi^4$ theory is an interesting laboratory to get insight about more ambitious theoretical frameworks.

The generally accepted point of view is that $\lambda\Phi^4$ theory, in four space–time dimensions, is "trivial", either non-interacting or else inconsistent [3–5].

A non-perturbative analysis, based on the gaussian approximation to the effective potential, taking into account the crucial role of the mass renormalization condition, was performed in ref. [6]. In that paper it was found that if one sends the cutoff to infinity, with respect to the renormalized mass of the fluctuations of the symmetric vacuum, the theory will develop another scale which corresponds to spontaneous symmetry breaking. Differently from the perturbative analysis of ref. [7], the gaussian approximation, being of variational nature, allows to conclude that spontaneous symmetry breaking is discovered as a sensible result in pure $\lambda\Phi^4$ theory. In perturbation theory this is only true in the presence of gauge bosons, i.e. in scalar electrodynamics.

In ref. [6], it was implicitly assumed that the cutoff regulated $\lambda\Phi^4$ theory was not allowed to take the "continuum" limit, i.e. cutoff to infinity, in a consistent way. However, in a weak coupling regime the cutoff was exponentially decoupled from the spontaneously broken phase thus implying a meaningful physical framework in a low energy region.

Important progress was obtained in ref. [8] by Stevenson and Tarrach. These authors, by applying mass renormalization conditions close to the ones of ref. [6], suggested that the gaussian effective potential of ref. [6] could be renormalized by allowing for a wave function renormalization of the scalar fields.

[1] Permanent address: Dipartimento di Fisica, Università di Catania, Catania, Italy and Istituto Nazionale di Fisica Nucleare, Sezione di Catania, Catania, Italy.

Our aim in this letter is to support their indication of the existence of a non-trivial form of $\lambda\Phi^4$ theory by using a general renormalization group (RG) analysis for the effective potential. By combining the indications of the gaussian effective potential with the RG equations, $\lambda\Phi^4$ theory turns out to be asymptotically free.

As a starting point for our analysis let us consider the hamiltonian density operator ($\lambda>0$)

$$\hat{\mathscr{H}}=\tfrac{1}{2}\hat{\pi}^2+\tfrac{1}{2}(\nabla\hat{\phi})^2+\tfrac{1}{2}m_B^2\hat{\phi}^2+\frac{\lambda}{4!}\hat{\phi}^4\,, \tag{1}$$

which can be variationally evaluated within the class of normalized gaussian wave functionals

$$\Psi_G[\phi]=(\mathrm{Det}\,G)^{-1/4}\times\exp\left(-\tfrac{1}{4}\int d^3x\int d^3y[\phi(x)-F]\times G^{-1}(x,y)[\phi(y)-F]\right), \tag{2}$$

depending on the two parameters F and Ω, respectively associated to the field expectation value

$$\langle\Psi_G|\hat{\phi}|\Psi_G\rangle=F\,, \tag{3}$$

and to the mass of the fluctuations through the relation

$$\langle\Psi_G|\hat{\phi}(x)\hat{\phi}(y)|\Psi_G\rangle=F^2+G(x,y)\,, \tag{4}$$

with

$$G(x,y)=\int\frac{d^3k}{(2\pi)^3}\frac{\exp[ik(x-y)]}{2\sqrt{k^2+\Omega^2}}\,. \tag{5}$$

The gaussian effective potential is defined as

$$V_G(F,\Omega)=\langle\Psi|\hat{\mathscr{H}}|\Psi\rangle\,, \tag{6}$$

and depends on the two variational parameters F and Ω as well as on the bare parameters m_B^2 and λ and the ultraviolet cutoff Λ. As shown in ref. [6] the extremum condition

$$\frac{\partial V_G}{\partial\Omega}=0 \tag{7}$$

implies the gap equation ($I_0(\Omega)=G(x,x)$)

$$\Omega^2=m_B^2+\tfrac{1}{2}\lambda F^2+\tfrac{1}{2}\lambda I_0(\Omega)\,, \tag{8}$$

whose solution $\Omega=\Omega(F)$ when inserted in eq. (6), yields, up to an additional constant the effective potential. By eliminating the bare mass m_B^2 through the cutoff independent, renormalized mass $\Omega^2(0)$, we obtain

$$U_G(\Lambda,\lambda,F;\Omega(0))=V_G(F,\Omega(F))-V_G(0,\Omega(0))\,. \tag{9}$$

As shown in ref. [6] and discussed in ref. [8] only in the limit $\Omega(0)/\Lambda\to0$ spontaneous symmetry breaking is recovered. Then the simple choice $\Omega(0)=0$ is not a restriction at all.

The renormalization group equation for the effective potential can be expressed as

$$\left(\Lambda\frac{\partial}{\partial\Lambda}+\Lambda\frac{\partial\lambda}{\partial\Lambda}\frac{\partial}{\partial\lambda}+\Lambda\frac{\lambda F}{\partial\Lambda}\frac{\partial}{\partial F}\right)U_G(\Lambda,\lambda,F)=0\,. \tag{10}$$

The absolute minimum condition

$$\left.\frac{\partial U_G}{\partial F}\right|_{\Lambda,\lambda\ \text{fixed}}=0 \tag{11}$$

provides us with a relation

$$F=\bar{F}(\Lambda,\lambda)\,, \tag{12}$$

which, when inserted in eq.(9), yields the variational estimate for the ground state energy density

$$E_G^0(\Lambda,\lambda)=U_G(\Lambda,\lambda,\bar{F}(\Lambda,\lambda))\,, \tag{13}$$

which, due to eq.(10), is a renormalization group invariant quantity satisfying the equation

$$\left(\Lambda\frac{\partial}{\partial\Lambda}+\beta(\lambda)\frac{\partial}{\partial\lambda}\right)E_G^0(\Lambda,\lambda)=0\,. \tag{14}$$

From the results of ref. [6] we obtain the relations

$$E_G^0(\Lambda,\lambda)=-\frac{\Omega^4(\bar{F})}{128\pi^2}\,, \tag{15}$$

and $\Omega^2(\bar{F})=\tfrac{1}{3}\lambda\bar{F}^2$. For small λ where, as discussed in detail in ref. [6], the gaussian approximation is reliable, one gets

$$\bar{\Omega}^2\simeq\Lambda^2\exp(-16\pi^2/\lambda)\,. \tag{16}$$

This relation, when inserted in eq.(14), yields a non-perturbative estimate of the β-function of the $\lambda\Phi^4$ theory, after taking into account mass renormalization effects, i.e.

$$\beta_G(\lambda) = -\frac{\lambda^2}{8\pi^2} + O(\lambda^3) . \quad (17)$$

One may ask what our result implies for the "true" β-function of the theory. In this case one can use the simple inequality for the true ground state energy

$$E^0 \leqslant E^0_G$$

to show easily that, for small λ, one gets

$$\beta(\lambda) \leqslant \beta_G(\lambda) < 0 , \quad (18)$$

thus confirming the negative sign of the exact β-function.

Differently from the usual perturbative approach which, at $O(\lambda^2)$, ignores the physical value of the renormalized mass, our analysis indicates that $\lambda\Phi^4$ theory is asymptotically free. Our result is consistent with the rigorous results of ref. [5] where it is shown that asymptotic freedom is the only tool to escape triviality in four diemsions. Moreover, the usually accepted point of view that an $O(N)$ invariant self-interacting scalar theory is trivial in the limit $N \to \infty$ [9] should be, in our opinion, reconsidered due to the non-uniformity of the two limits $\Lambda \to \infty$ and $N \to \infty$. Indeed in ref. [10], by analyzing the $O(N)$ continuous symmetry case, it was found that, in a weak coupling regime, the gaussian potential becomes an excellent approximation to the exact effective potential as confirmed from the existence of $N-1$ massless (in the limit of infinite N) bosons, in agreement with the Goldstone theorem. This result is obtained by keeping fixed the " physical Higgs" mass, in the infinite cutoff limit, to set up the scale of the theory, as in the discrete symmetry case analyzed in this paper, thus supporting the validity of our conclusions.

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