Calculus of Variations

August 19, 2016

0.0.1 Derivation complement of formula D.6 - D.7

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\begin{aligned}
F[y(x)+\epsilon\eta(x)] &= F[y(x)] + \epsilon\int\{\frac{\partial G}{\partial y}\eta(x)dx + \frac{\partial G}{\partial y'}\eta'(x)\}dx + \mathcal{O}(\epsilon^2) \\
&= F[y(x)] + \epsilon\int\frac{\partial G}{\partial y'}\eta(x)dx + \epsilon\int\frac{\partial G}{\partial y'}\eta'(x)dx + \frac{\partial G}{\partial y'}\eta'(x)\}dx + \mathcal{O}(\epsilon^2) \\
&= F[y(x)] + \epsilon\int\frac{\partial G}{\partial y'}\eta(x)dx + \epsilon[\frac{\partial G}{\partial y'}\eta(x) - \int\frac{d}{dx}(\frac{\partial G}{\partial y'})\eta(x)dx] + \frac{\partial G}{\partial y'}\eta'(x)\}dx + \mathcal{O}(\epsilon^2) \\
&= F[y(x)] + \epsilon\int\frac{\partial G}{\partial y'}\eta(x)dx - \epsilon\int\frac{d}{dx}(\frac{\partial G}{\partial y'})\eta(x)dx + \frac{\partial G}{\partial y'}\eta'(x)\}dx + \mathcal{O}(\epsilon^2) \\
&= F[y(x)] + \epsilon\int\{\frac{\partial G}{\partial y'}\eta(x)dx - \frac{d}{dx}(\frac{\partial G}{\partial y'})\}\eta(x)dx + \mathcal{O}(\epsilon^2)
\end{aligned}
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Note: I am not sure whether the meaning of "$\eta(x)$ must vanish at the boundary of the integral" is $\frac{\partial G}{\partial y'}\eta(x) = 0$

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In [ ]:
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