1.2 Probability Theory

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0.0.1 Derivation complement of equation 1.54

$$
\begin{aligned}
lnp(\mathbf{x}|\mu,\sigma^2) &= ln\prod_{n=1}^{N}\frac{1}{(2\pi\sigma^2)^{\frac{1}{2}}}exp\{-\frac{1}{2\sigma^2}(X_n-\mu)^2\} \\
&= ln[(2\pi\sigma^2)^{-\frac{N}{2}}exp\{-\frac{1}{2\sigma^2}\sum_{n=1}^{N}(X_n-\mu)^2\}] \\
&= -\frac{N}{2}ln(2\pi\sigma^2)-\frac{1}{2\sigma^2}\sum_{n=1}^{N}(x_n-\mu)^2 \\
&= -\frac{1}{2\sigma^2}\sum_{n=1}^{N}(x_n-\mu)^2-\frac{N}{2}ln\sigma^2 ln(2\pi)
\end{aligned}
$$

0.0.2 Derivation complement of equation 1.55

$$
\frac{\partial lnp(\mathbf{x}|\mu,\sigma^2)}{\partial\mu}=\frac{1}{\sigma^2}\sum_{n=1}^{N}(x_n-\mu)=0\rightarrow\mu_{ML}=\frac{1}{N}\sum_{n=1}^{N}x_n
$$

0.0.3 Derivation complement of equation 1.56

$$
\frac{\partial lnp(\mathbf{x}|\mu,\sigma^2)}{\partial\sigma^2}=-\frac{1}{2}\sum_{n=1}^{N}(X_n-\mu)^2\cdot(-2\cdot\frac{1}{\sigma^3})-\frac{N}{2}\cdot\frac{2\sigma}{\sigma^2}=0\rightarrow\sigma^2_{ML}=\frac{1}{N}\sum_{n=2}^{N}(x_n-\mu_{ML})^2
$$

0.0.4 Derivation complement of equation 1.63

$$
\frac{\partial p(\mathbf{t}|\mathbf{x},w,\beta)}{\partial\beta}=-\frac{1}{2}\sum_{n=1}^{N}\{y(x_nm,\mathbf{w}-t_n)^2+\frac{N}{2\beta}=0\rightarrow\frac{1}{\beta_{ML}}=\frac{1}{N}\sum_{n=1}^{N}\{y(x_n,\mathbf{w}_{ML})-t_n\}^2\}
$$

0.0.5 Comprehension about equation 1.68

I can understand the equation 1.68 intuitively, but I wanna make clear how to give a rigorous derivation. In a fully Bayesian approach, the data set x is observed(i.e. It is already known), so the unkown variables are w and the target value t.Thus we can simplify (1.68) by eliminating variable x

$$
p(t)=\int p(t|\mathbf{w})p(\mathbf{w})d\mathbf{w}=\int p(t,\mathbf{w})d\mathbf{w}
$$

This is easy to understand, we can model the joint distributrion $\int p(t, \mathbf{w})d\mathbf{w}$ and then marginalize it to obtain the probability $p(t)$

Substituting the data variable x, these equation will look like this:

$$p(t) = \int p(t|\mathbf{w})p(\mathbf{w})d\mathbf{w} = \int p(t, \mathbf{w})d\mathbf{w}$$

Reference

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URL https://www.zhihu.com/question/44532390/answer/99662629
SourceZhihu

0.0.6 Derivation complement from 1.69 to 1.72

I totally didn't understand the equation 1.69-1.72 at the first, such many mathematical symbols showing up in the meantime scare the shit out of me, but I believe I can figure it out for the next time!