The Hawk-Eye system

The Hawk-Eye system, now common in tennis tournaments, uses six or seven cameras to evaluate the trajectory of a ball and determine if it is in or out. It can be used by players to "challenge" a referee's call. Let us consider the simplified model sketched in Fig. 1.

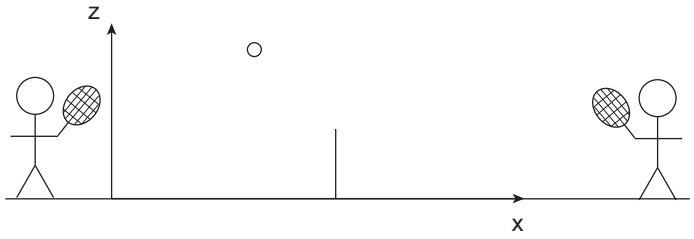

Figure 1: Simplified model for the Hawk-Eye system.

From simultaneous pictures taken by the six or seven cameras at a given time t, let us assume that an image recognition algorithm gives us an estimate of the position $(x(t), z(t))$ of the ball in the x-z plane. For a given hit, one can use pictures at different times to obtain a list of points (x,z) that should belong to the trajectory of the ball. According to Newton's law of dynamics, the trajectory of the ball should be parabolic, that is to say of the form

$$z(x) = ax^2 + bx + c, \tag{1}$$

where a, b and c are scalars. You will find on the course website a file (data.xls) containing a hundred positions of one ball, listed in the following form.

Picture	x (feet)	z (feet)
1	0	2.93
2	0.18	3.03
3	0.36	3.15
...	...	...

The ball is considered out if it touches the ground ($z = 0$) for $x > 78$ feet.

1. Find the best fit for the trajectory according to the least-square approximation and determine whether the ball is in or out. Plot the measured data and the fitting function in one same figure.

2. Try to get this result using less and less pictures. The average error is defined as

$$e = \frac{1}{N}\sqrt{\sum_{i=1}^{N} [f^*(x_i) - z_i]^2} \tag{2}$$

   where N is the number of pictures taken into account, (x_i, z_i) are the measured data from file data.txt and f^* is the fitting function. Plot the average error as a function of N. How many pictures do you need for the average error between your prediction and the measured data to be below 3×10^{-3} feet (≈ 1 mm)?