Radiometry and Reflectance

To interpret image intensities, we need to understand Radiometric Concepts and Reflectance Properties.

Topics:

(1) Radiometric Concepts

(2) Surface Radiance and Image Irradiance

(3) BRDF: Bidirectional Reflectance Distribution Function

(4) Reflectance Models

(5) Dichromatic Model

From 2D to 3D

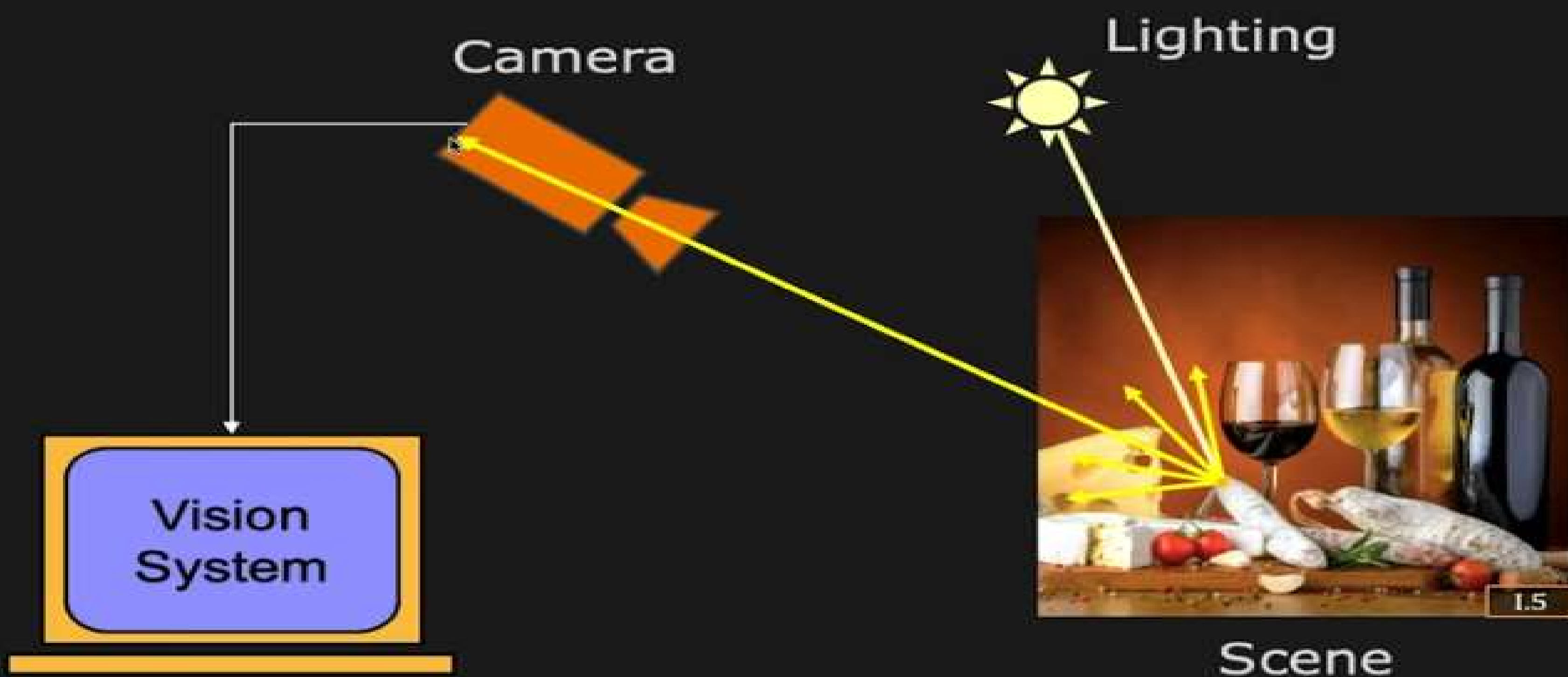


Image Intensity

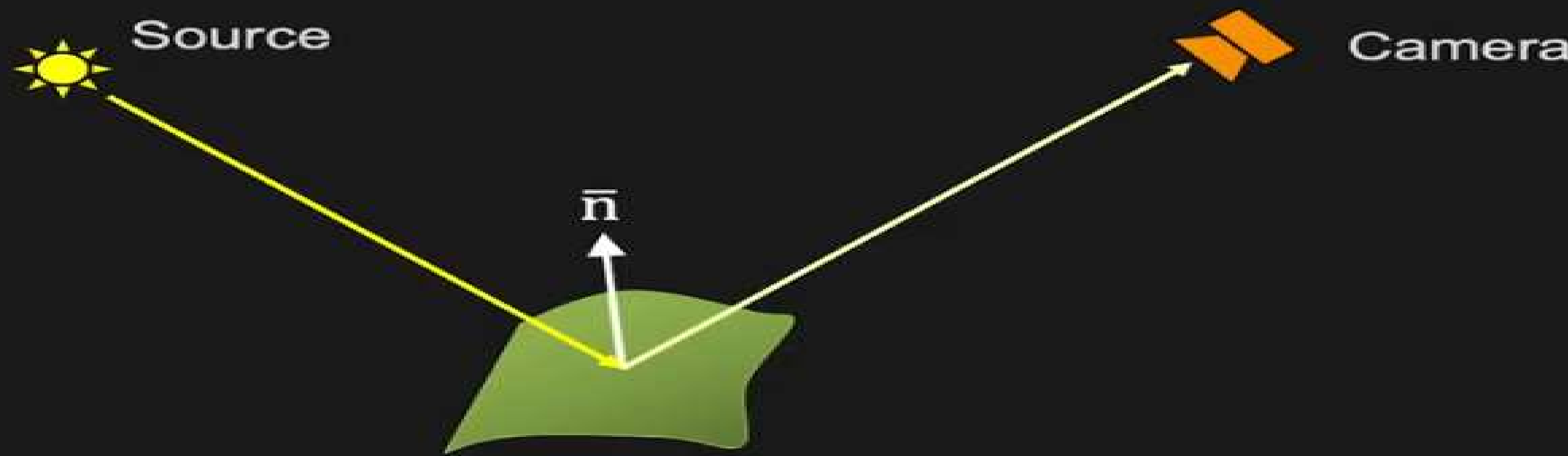
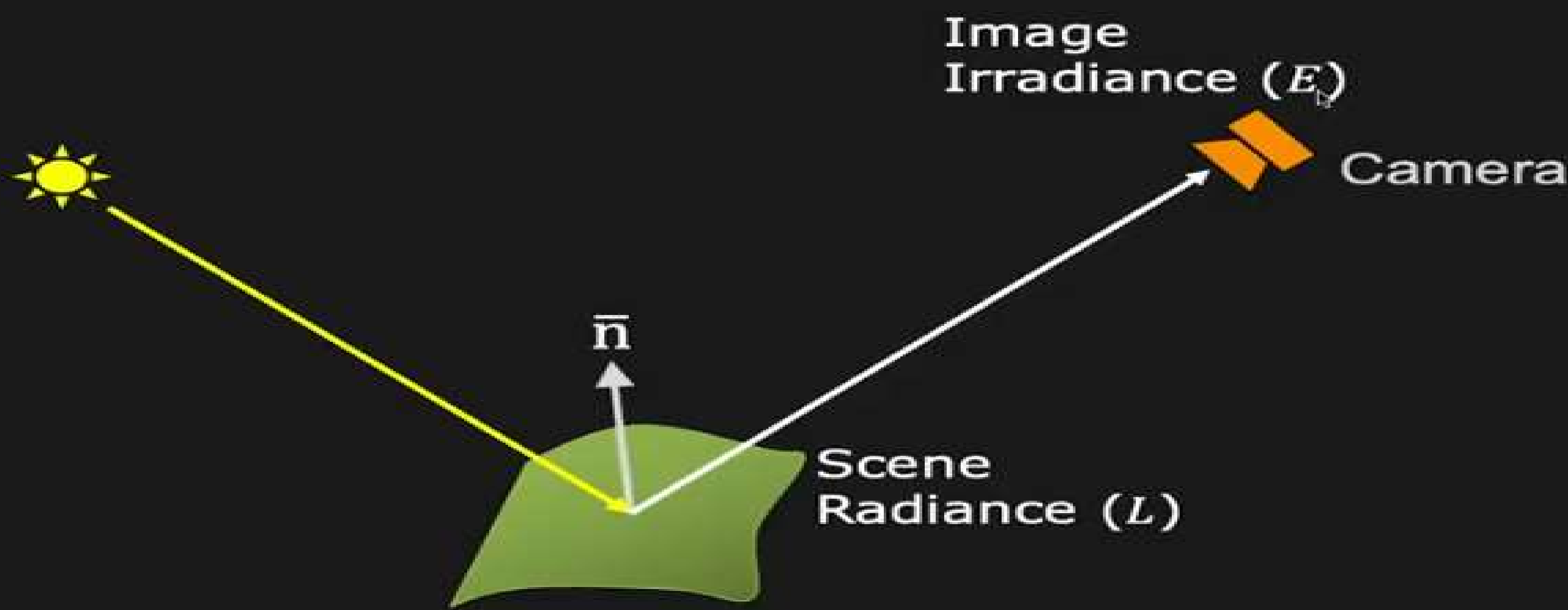


Image Intensity = f (Illumination,
Surface Orientation,
Surface Reflectance)

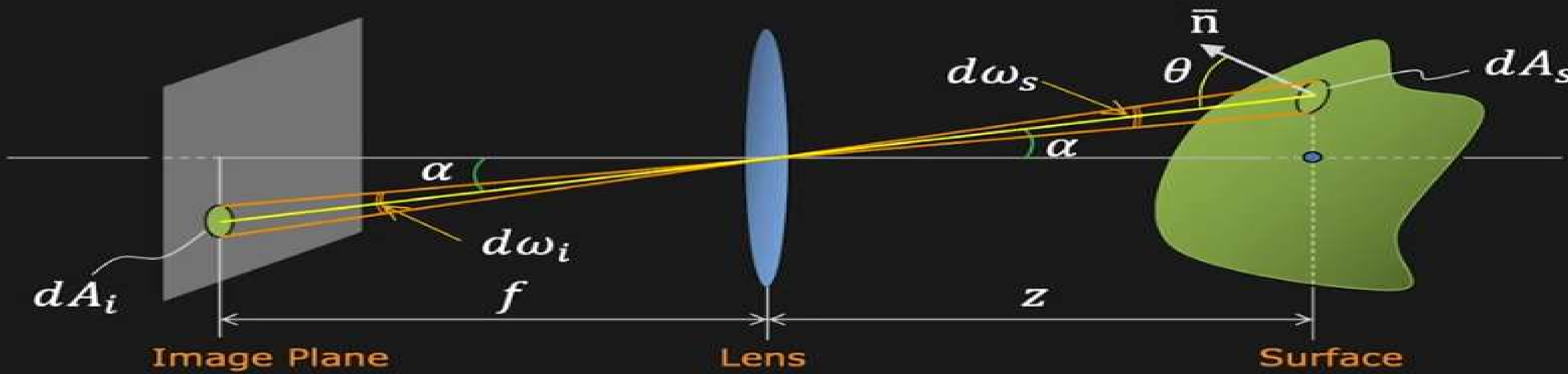
Image intensity understanding is under-constrained!

Scene Radiance and Image Irradiance



What is the relationship between L and E ?

Scene Radiance and Image Irradiance

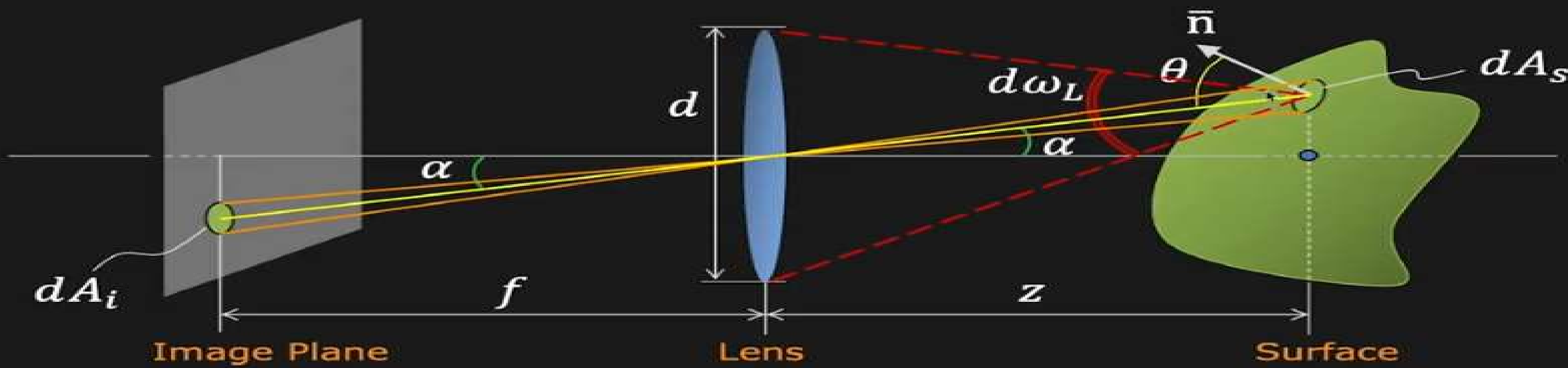


Solid Angles: $d\omega_i = d\omega_s$

$$\frac{dA_i \cos\alpha}{(f/\cos\alpha)^2} = \frac{dA_s \cos\theta}{(z/\cos\alpha)^2} \Rightarrow \boxed{\frac{dA_s}{dA_i} = \frac{\cos\alpha}{\cos\theta}\left(\frac{z}{f}\right)^2}$$

Equation (1)

Scene Radiance and Image Irradiance

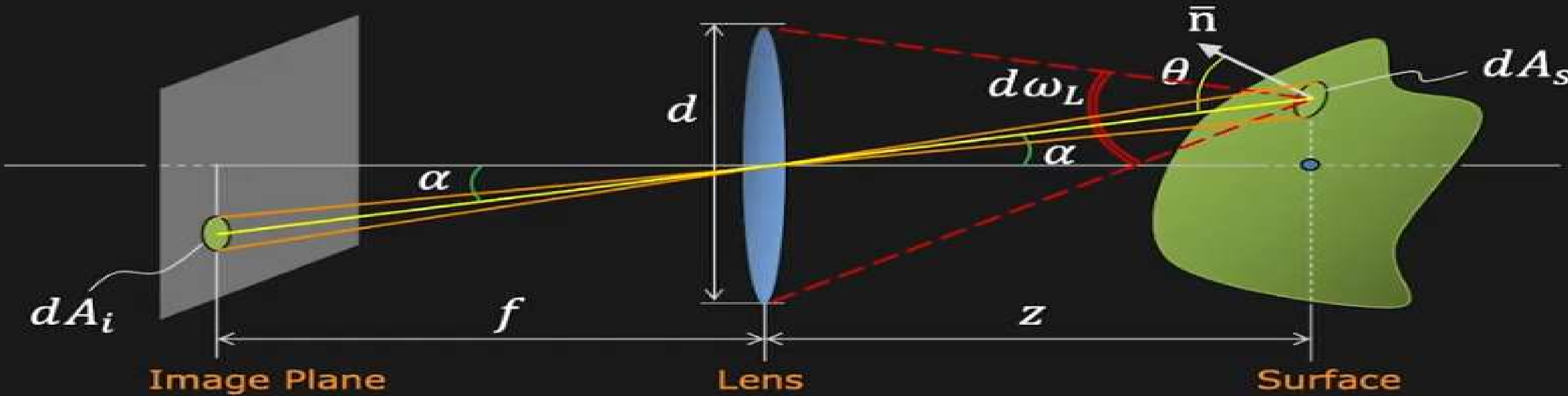


Solid Angle subtended by the lens:

$$\boxed{d\omega_L = \frac{\pi d^2}{4} \frac{\cos\alpha}{(z/\cos\alpha)^2}}$$

Equation (2)

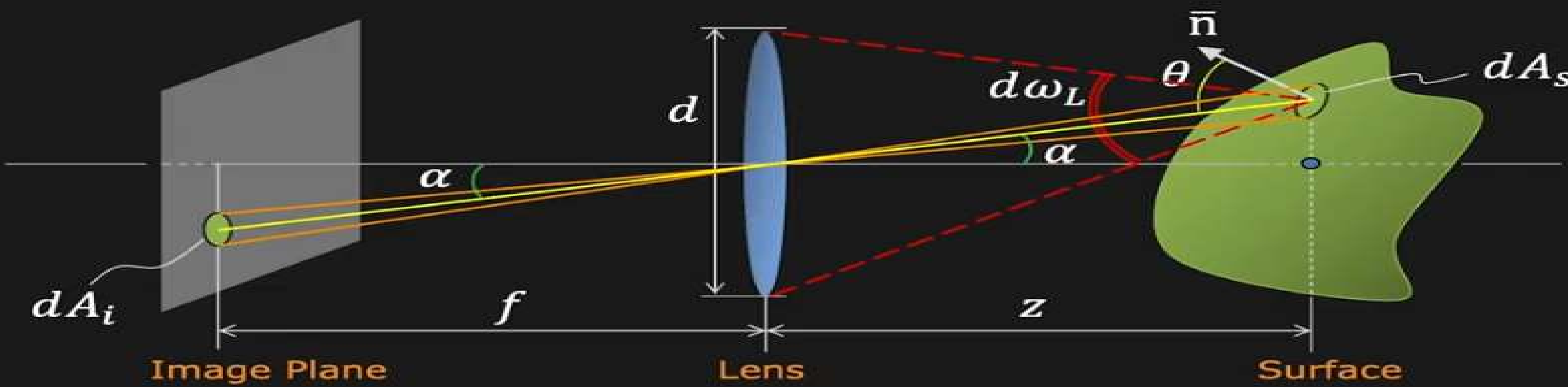
Scene Radiance and Image Irradiance



Energy Conservation:

Flux received by lens from dA_s = Flux projected onto dA_i

Scene Radiance and Image Irradiance



Scene Radiance:

$$L = \frac{d^2\Phi}{(dA_s \cos\theta)\, d\omega_L}$$

Flux received by lens from dA_s

$$\boxed{d^2\Phi = L\,(dA_s \cos\theta)\, d\omega_L}$$

Equation (3)

Scene Radiance and Image Irradiance

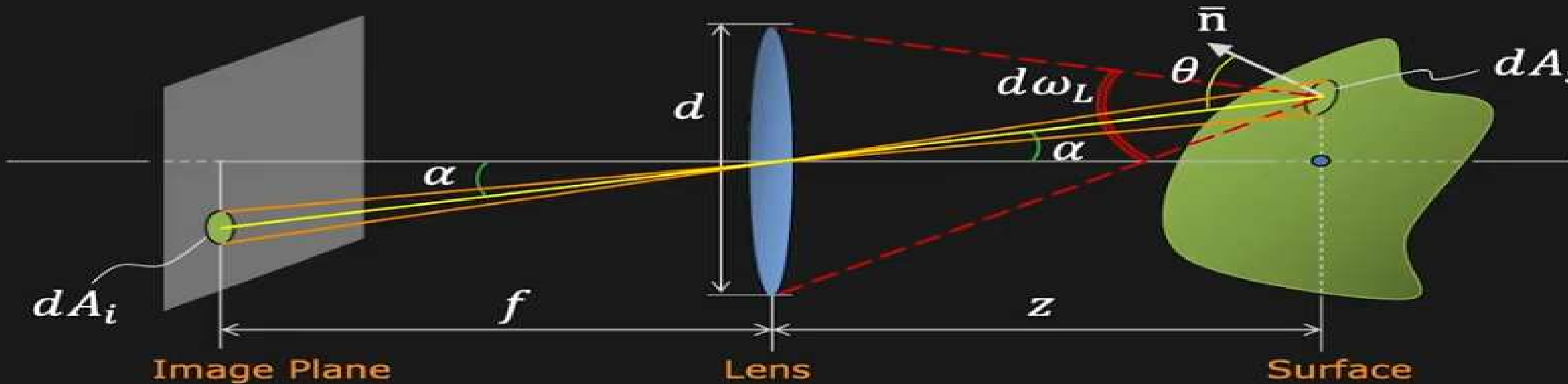


Image Irradiance: Flux projected onto dA_i

$$E = \frac{d\Phi}{dA_i} \Rightarrow \boxed{d\Phi = E\, dA_i}$$

Equation (4)

Scene Radiance and Image Irradiance

Equation (1)

$$\frac{dA_s}{dA_i} = \frac{\cos\alpha}{\cos\theta}\left(\frac{z}{f}\right)^2$$

Equation (2)

$$d\omega_L = \frac{\pi d^2}{4}\frac{\cos\alpha}{(z/\cos\alpha)^2}$$

Equation (3)

$$d^2\Phi = L\,(dA_s\cos\theta)\,d\omega_L$$

Equation (4)

$$d\Phi = E\,dA_i$$

$$\boxed{E = L\frac{\pi}{4}\left(\frac{d}{f}\right)^2\cos^4\alpha}$$

Scene Radiance and Image Irradiance

$$E = L\frac{\pi}{4}\left(\frac{d}{f}\right)^2 \cos^4\alpha$$

Image Irradiance

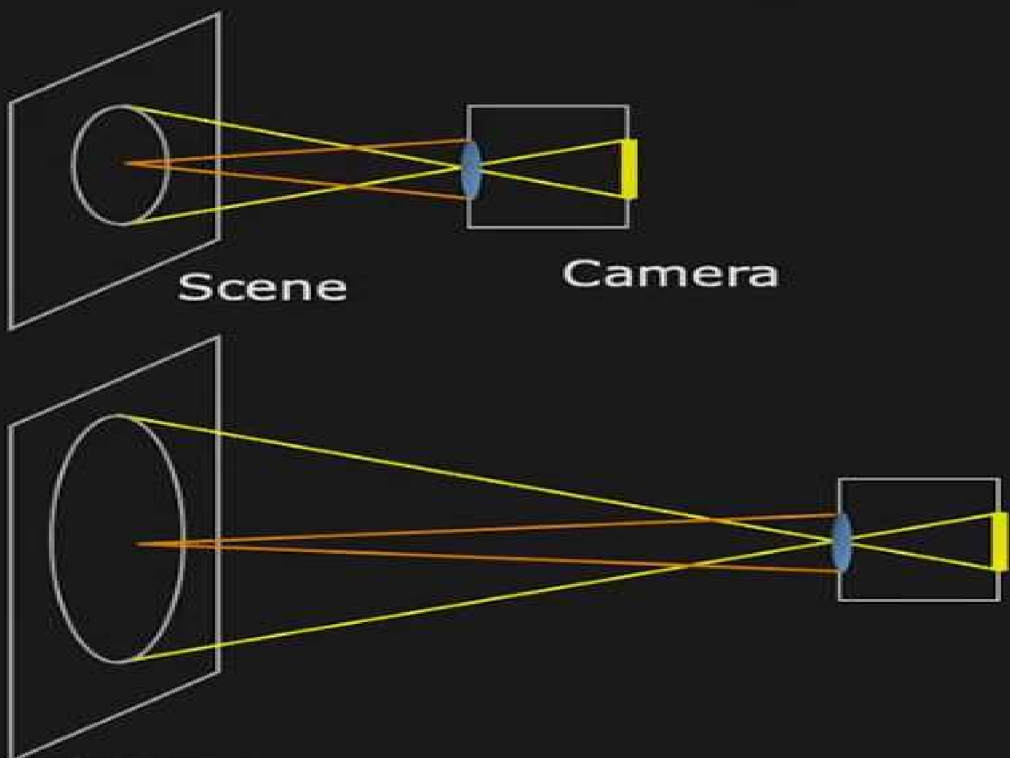
Scene Radiance

1/Effective F-number
(since f is *effective* focal length)

- Image Irradiance is proportional to Scene Radiance
- Image brightness falls off from image center as $\cos^4\alpha$
- For small fields of view, effects of $\cos^4\alpha$ are small

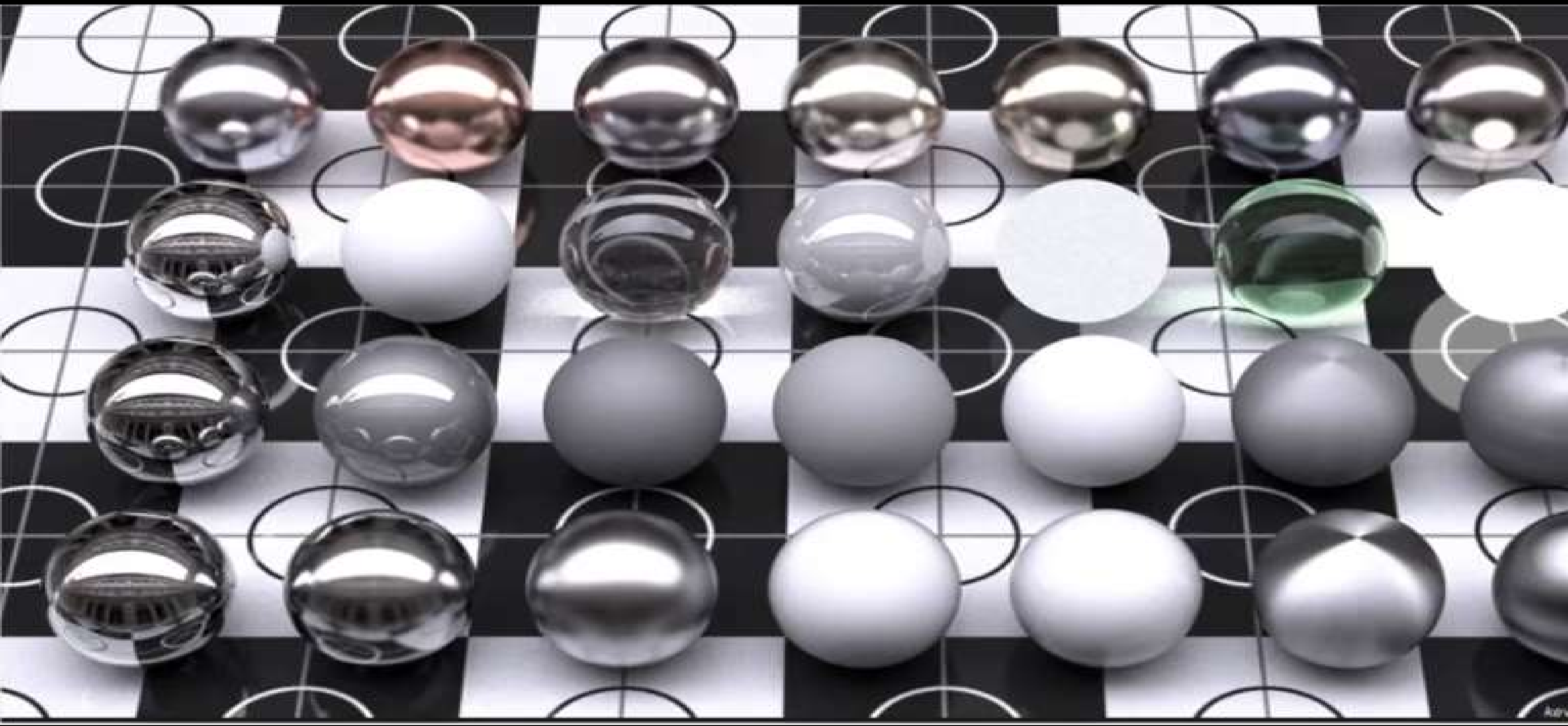
Scene Radiance and Image Irradiance

Does image brightness vary with scene depth? NO

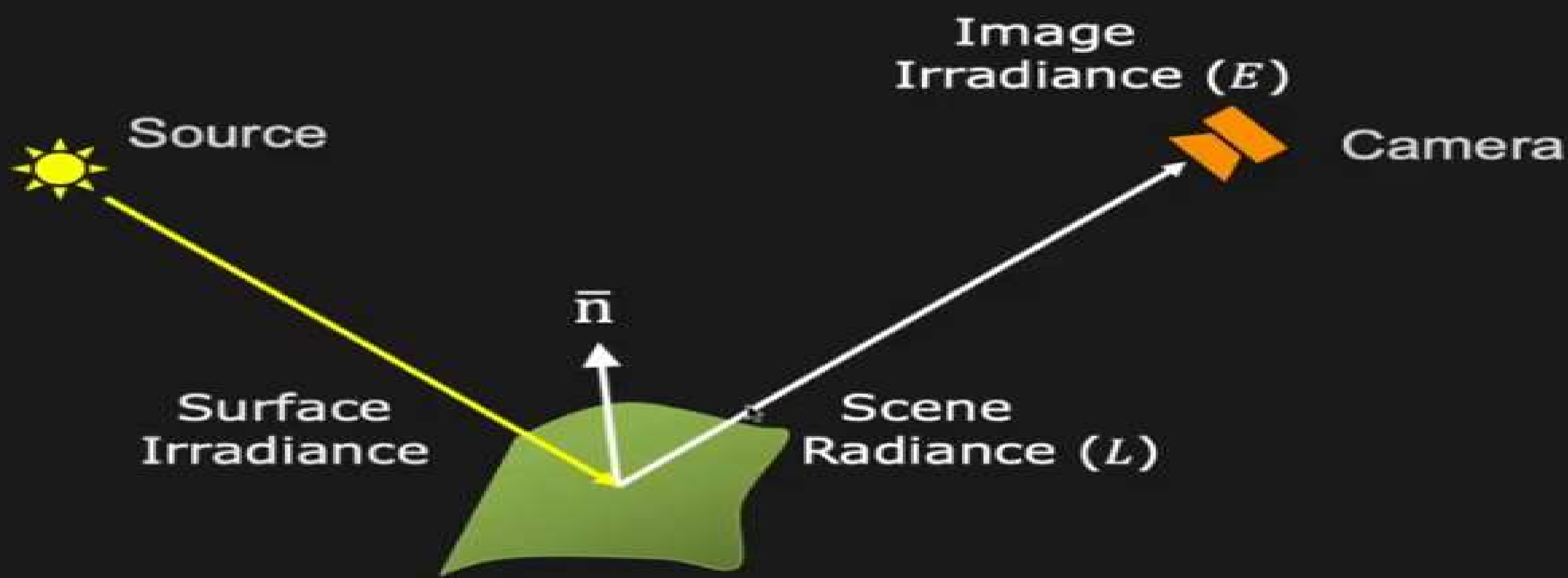


$$E = L\frac{\pi}{4}\left(\frac{d}{f}\right)^2 \cos^4\alpha$$

- Larger the scene depth, larger the area of light accumulation.
- Larger the scene depth, smaller the solid angle subtended by each point onto the lens, and hence less light from each point.



Surface Reflection

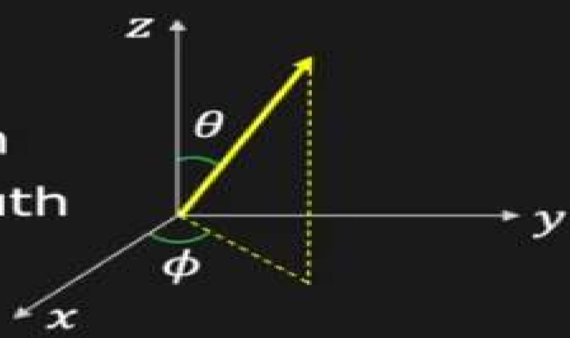


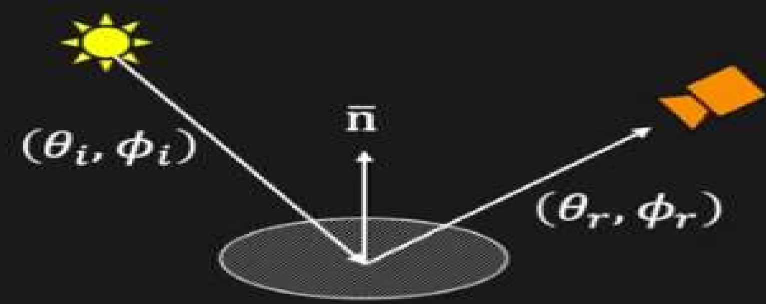
Surface reflection depends on both the viewing and illumination directions.

Bidirectional Reflectance Distribution Function

θ: zenith

ϕ: azimuth





$E(\theta_i, \phi_i)$: Irradiance due to source in direction (θ_i, ϕ_i)

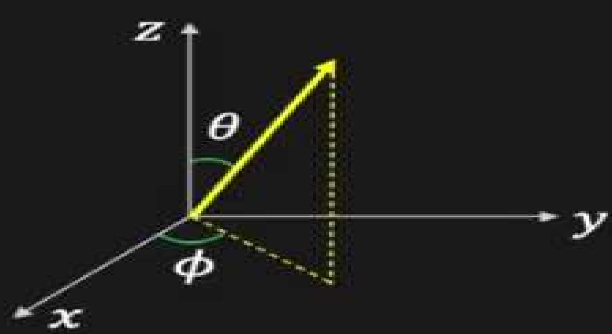
$L(\theta_r, \phi_r)$: Radiance of surface in direction (θ_r, ϕ_r)

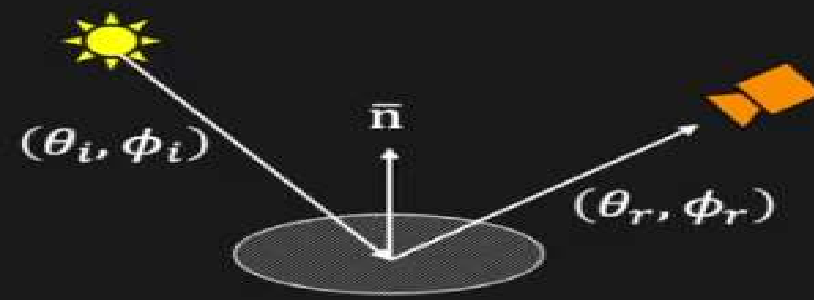
$$\text{BRDF: } f(\theta_i, \phi_i, \theta_r, \phi_r) = \frac{L(\theta_r, \phi_r)}{E(\theta_i, \phi_i)}$$

Unit: 1/sr

[Nicodemus 1977]

Properties of BRDF





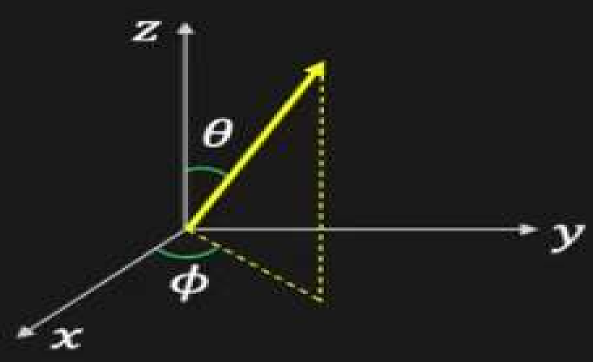
Non-Negative:

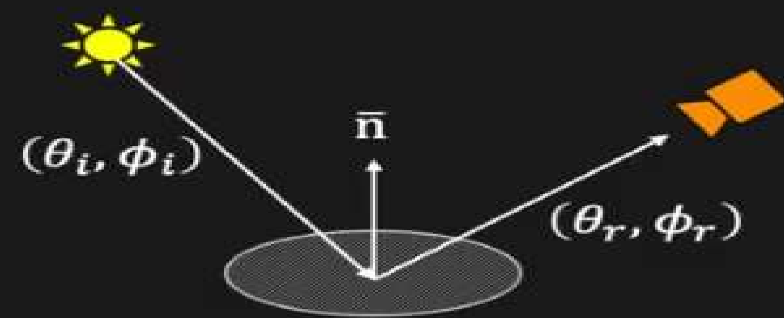
$$f(\theta_i, \phi_i, \theta_r, \phi_r) > 0$$

Helmholtz Reciprocity:

$$f(\theta_i, \phi_i, \theta_r, \phi_r) = f(\theta_r, \phi_r, \theta_i, \phi_i)$$

BRDF of Isotropic Surfaces





In general, BRDF is a 4-D function: $f(\theta_i, \phi_i, \theta_r, \phi_r)$

For rotationally symmetric reflectance (Isotropic Surfaces), BRDF is a 3-D function:

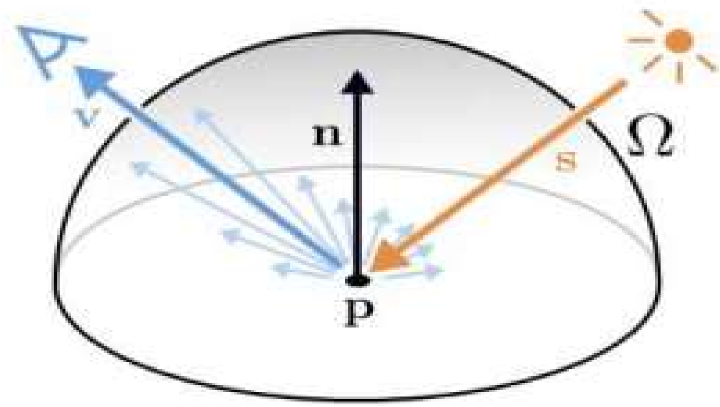
$$f(\theta_i, \theta_r, \phi_i - \phi_r)$$

Rendering Equation

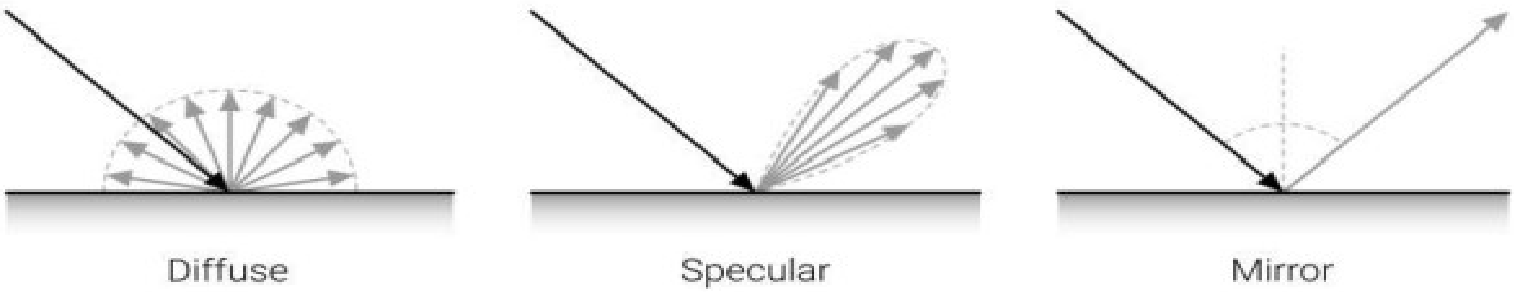
Let $\mathbf{p} \in \mathbb{R}^3$ denote a 3D surface point, $\mathbf{v} \in \mathbb{R}^3$ the viewing direction and $\mathbf{s} \in \mathbb{R}^3$ the incoming light direction. The **rendering equation** describes how much of the light L_{in} with wavelength λ arriving at $\mathbf{p}$ is reflected into the viewing direction $\mathbf{v}$:

$$L_{\text{out}}(\mathbf{p}, \mathbf{v}, \lambda) = L_{\text{emit}}(\mathbf{p}, \mathbf{v}, \lambda) + \int_{\Omega} \text{BRDF}(\mathbf{p}, \mathbf{s}, \mathbf{v}, \lambda) \cdot L_{\text{in}}(\mathbf{p}, \mathbf{s}, \lambda) \cdot (-\mathbf{n}^\top \mathbf{s})\, d\mathbf{s}$$

- Ω is the unit hemisphere at normal $\mathbf{n}$
- The bidirectional reflectance distribution function $\text{BRDF}(\mathbf{p}, \mathbf{s}, \mathbf{v}, \lambda)$ defines how light is reflected at an opaque surface.
- $L_{\text{emit}} > 0$ only for light emitting surfaces

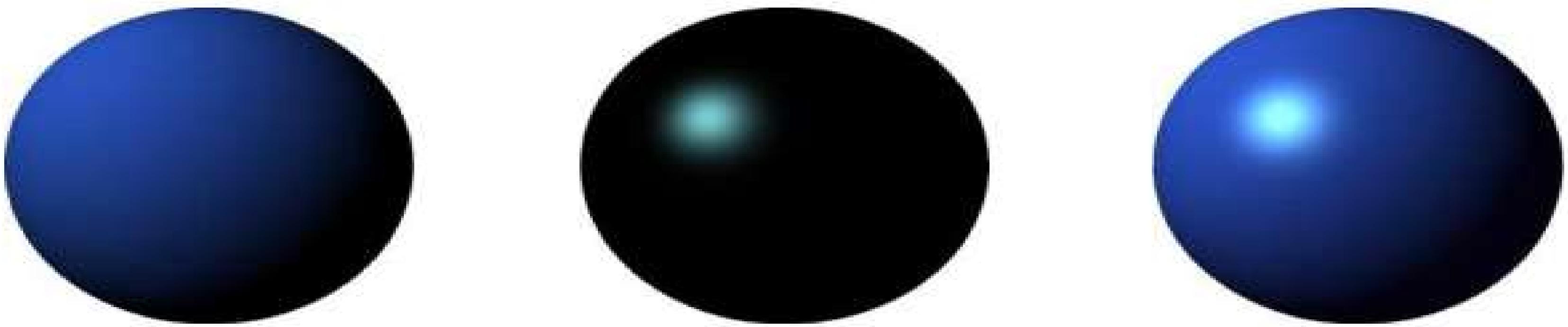


Diffuse and Specular Reflection



- Typical BRDFs have a **diffuse** and a **specular** component
- The diffuse (=constant) component scatters light uniformly in all directions
- This leads to shading, i.e., smooth variation of intensity wrt. surface normal
- The specular component depends strongly on the outgoing light direction

Diffuse and Specular Reflection



Diffuse   Specular   Combined

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Diffuse and Specular Reflection

Clay Pot

Cloud Gate (Kapoor, 2006)

Simplifying the Rendering Equation

Dropping the dependency on λ and $\mathbf{p}$ for notational simplicity, and considering a **single point light source** located in direction $\mathbf{s}$, the rendering equation

$$L_{\text{out}}(\mathbf{p},\mathbf{v},\lambda) = L_{\text{emit}}(\mathbf{p},\mathbf{v},\lambda) + \int_{\Omega} \text{BRDF}(\mathbf{p},\mathbf{s},\mathbf{v},\lambda)\cdot L_{\text{in}}(\mathbf{p},\mathbf{s},\lambda)\cdot(-\mathbf{n}^\top\mathbf{s})\, d\mathbf{s}$$

simplifies as follows:

$$L_{\text{out}}(\mathbf{v}) = \text{BRDF}(\mathbf{s},\mathbf{v})\cdot L_{\text{in}}\cdot(-\mathbf{n}^\top\mathbf{s})$$

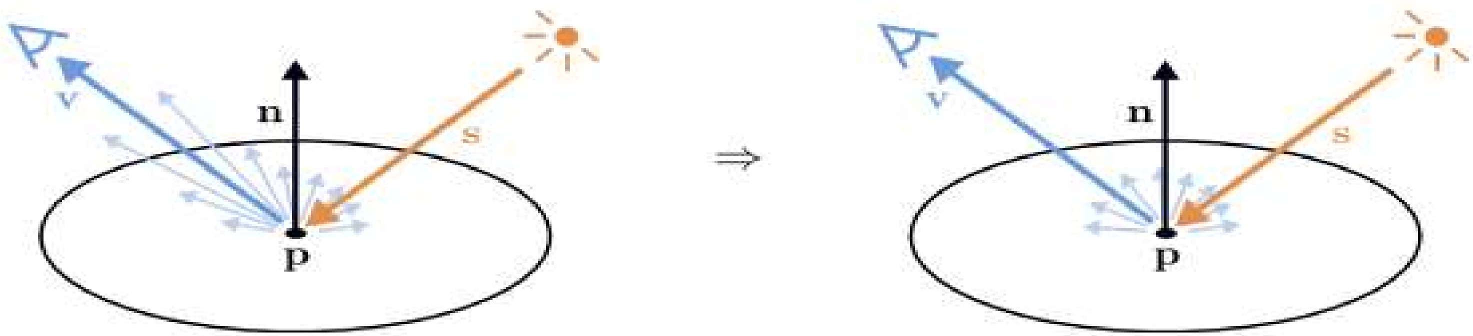
Simplifying the Rendering Equation

Assuming a purely **diffuse material** with albedo (=diffuse reflectance) $\text{BRDF}(\mathbf{s}, \mathbf{v}) = \rho$

$$L_{\text{out}}(\mathbf{v}) = \text{BRDF}(\mathbf{s}, \mathbf{v}) \cdot L_{\text{in}} \cdot (-\mathbf{n}^\top \mathbf{s})$$

further simplifies to the following equation (L_{out} becomes independent of $\mathbf{v}$):

$$L_{\text{out}} = \rho \cdot L_{\text{in}} \cdot (-\mathbf{n}^\top \mathbf{s})$$



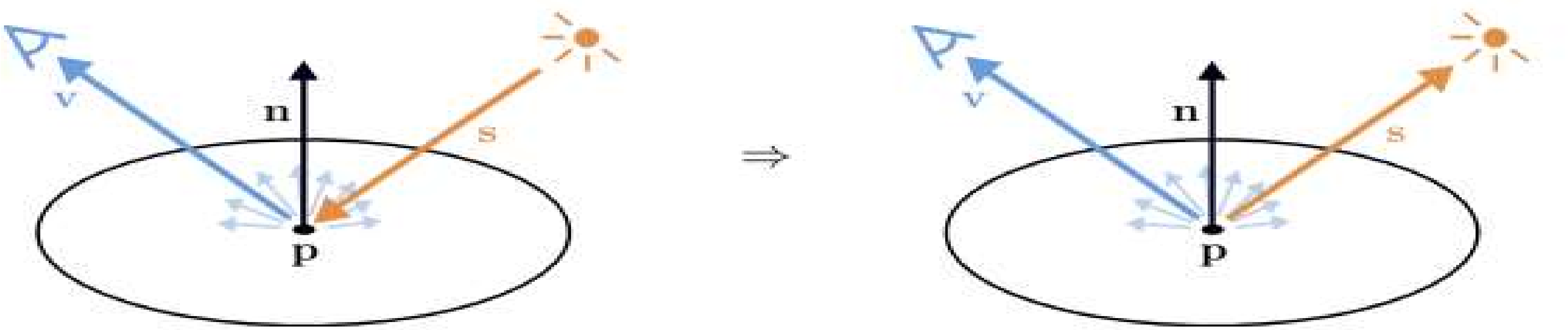
Simplifying the Rendering Equation

For simplicity, we further eliminate the **minus sign** in

$$L_{\text{out}} = \rho \cdot L_{\text{in}} \cdot (-\mathbf{n}^\top \mathbf{s})$$

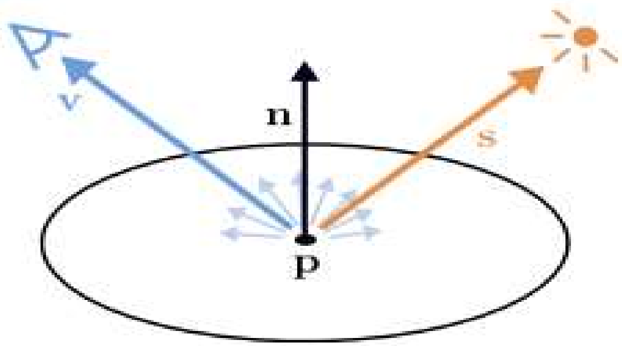
by reversing the orientation (definition) of the light ray $\mathbf{s}$ and obtain:

$$L_{\text{out}} = \rho \cdot L_{\text{in}} \cdot \mathbf{n}^\top \mathbf{s}$$



Simplifying the Rendering Equation

$$L_{\text{out}} = \rho \cdot L_{\text{in}} \cdot \mathbf{n}^\top \mathbf{s} = R(\mathbf{n})$$



- For a fixed material and light source, the reflected light L_{out} is a function of $\mathbf{n}$
- This function $R(\mathbf{n})$ is called **reflectance map** (we will see examples)
- If we would know $\mathbf{n}$ at each surface point, we can integrate the geometry
- Can we determine $\mathbf{n}$ from the observation L_{out} for every pixel in an image?
- This problem is called **Shape-from-Shading** (Berthold Horn, 1970)