

Poynting Vector

$$\vec{\nabla} \times \vec{E} = - \frac{\partial \vec{B}}{\partial t} \quad (1)$$

$$\vec{\nabla} \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t} \quad (2)$$

Eqn (1)

$$\vec{H} \cdot (\vec{\nabla} \times \vec{E}) = - \vec{H} \cdot \frac{\partial \vec{B}}{\partial t} \quad (3)$$

Eqn (2)

$$\vec{E} \cdot (\vec{\nabla} \times \vec{H}) = \vec{E} \cdot \left(\vec{J} + \frac{\partial \vec{D}}{\partial t} \right) \quad (4)$$

Subtract (3) to (4)

$$\vec{H} \cdot (\vec{\nabla} \times \vec{E}) - \vec{E} \cdot (\vec{\nabla} \times \vec{H}) = - \vec{H} \cdot \frac{\partial \vec{B}}{\partial t} - \vec{E} \cdot \left(\vec{J} + \frac{\partial \vec{D}}{\partial t} \right)$$

$$\vec{\nabla} \cdot (\vec{A} \times \vec{B}) = \vec{B} \cdot (\vec{\nabla} \times \vec{A}) - \vec{A} \cdot (\vec{\nabla} \times \vec{B})$$

$$\vec{\nabla} \cdot (\vec{E} \times \vec{H}) = - \vec{H} \cdot \frac{\partial \vec{B}}{\partial t} - \vec{E} \cdot \left(\vec{J} + \frac{\partial \vec{D}}{\partial t} \right)$$
$$= - \vec{H} \cdot \mu \frac{\partial \vec{H}}{\partial t} - \vec{E} \cdot \vec{J} - \vec{E} \cdot \frac{\partial \vec{D}}{\partial t}$$

$$- \mu \vec{H} \cdot \frac{\partial \vec{H}}{\partial t} = - \frac{\mu}{2} \frac{\partial H^2}{\partial t}$$

$$\frac{\partial H^2}{\partial t} = \frac{\partial}{\partial t} (\vec{H} \cdot \vec{H}) = 2 \vec{H} \cdot \frac{\partial \vec{H}}{\partial t}$$

$$\vec{\nabla} \cdot (\vec{E} \times \vec{H}) = - \frac{1}{2} \frac{\partial}{\partial t} (\mu H^2 + \epsilon E^2) - \vec{E} \cdot \vec{J}$$

$$\vec{S} = \vec{E} \times \vec{H}$$

Electromagnetic wave Eqn

$$1) \quad \vec{\nabla} \cdot \vec{E} = 0 \quad \rho$$

$$2) \quad \vec{\nabla} \cdot \vec{B} = 0$$

$$3) \quad \vec{\nabla} \times \vec{E} = - \frac{\partial \vec{B}}{\partial t}$$

$$4) \quad \vec{\nabla} \times \vec{B} = \mu_0 \frac{\partial \vec{D}}{\partial t} = \mu_0 \epsilon_0 \frac{\partial \vec{E}}{\partial t}$$

$$\vec{\nabla} \times (\vec{\nabla} \times \vec{E}) = - \vec{\nabla} \times \frac{\partial \vec{B}}{\partial t} = - \frac{\partial}{\partial t} (\vec{\nabla} \times \vec{B})$$

$$= - \frac{\partial}{\partial t} \left(\mu_0 \epsilon_0 \frac{\partial \vec{E}}{\partial t} \right)$$

$$= - \mu_0 \epsilon_0 \frac{\partial^2 \vec{E}}{\partial t^2}$$

$$\vec{A} \times (\vec{B} \times \vec{C}) = \vec{A} (\vec{B} \cdot \vec{C}) - (\vec{A} \cdot \vec{B}) \vec{C}$$

$$\vec{\nabla} \times (\vec{\nabla} \times \vec{E}) = \vec{\nabla} (\vec{\nabla} \cdot \vec{E}) - \nabla^2 \vec{E}$$

$$= - \nabla^2 \vec{E}$$

$$\nabla^2 \vec{E} = \mu_0 \epsilon_0 \frac{\partial^2 \vec{E}}{\partial t^2}$$

$$\frac{\partial^2 \vec{E}}{\partial t^2} = \frac{1}{\mu_0 \epsilon_0} \nabla^2 \vec{E}$$

$$\frac{1}{\mu_0 \epsilon_0} = c^2$$

$$3 \times 10^8 \text{ m/s}$$

$c \Rightarrow$ light of speed