Time dependent / independent

$$E\psi = \frac{p^2}{2m}\psi + V\psi$$

$$E = i\hbar \frac{\partial}{\partial t}$$

$$p = i\hbar \frac{\partial}{\partial x}$$

$$E\psi = -\frac{\hbar^2}{2m}\frac{\partial^2 \psi}{\partial x^2} + V\psi$$

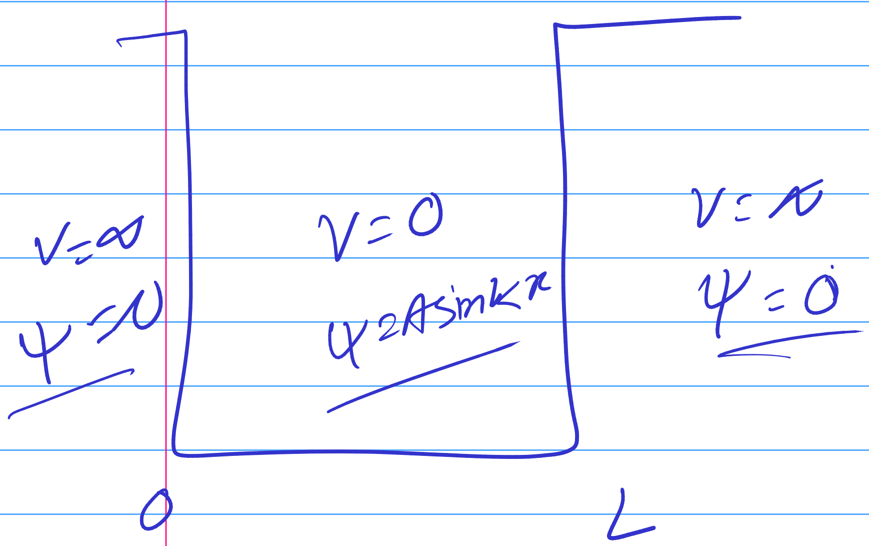
$$\int_{-\infty}^{+\infty} P(x)\, dx = 1$$

$$\psi = A \sin kx$$

$$\psi^* = \psi$$

$$\int_{-\infty}^{+\infty} \psi^* \psi \, dx = 1$$

$$\int_{-\infty}^{\infty} A^2 \sin^2 kx = 1 \Rightarrow \int_{-\infty}^{0} + \int_{0}^{L} + \int_{L}^{\infty}$$



$$\int_0^L \left(A^2 \sin^2 kx\right) dx = 1$$

$$A^2 \int_0^L \sin^2 kx \, dx = 1$$

$$A = \sqrt{\frac{2}{L}}$$

$$\psi_n = \sqrt{\frac{2}{L}} \sin kx = \sqrt{\frac{2}{L}} \sin \frac{n\pi x}{L}$$

$$P_n = \hbar k = \frac{n\pi\hbar}{L}$$

$$E_n = \frac{p_n^2}{2m} = \frac{n^2\pi^2\hbar^2}{2mL^2}$$

$$n = \cancel{0}, 1, 2, \ldots$$

$n=1$ $$E_1 = \frac{\pi^2\hbar^2}{2mL^2} \Rightarrow$$ Ground State

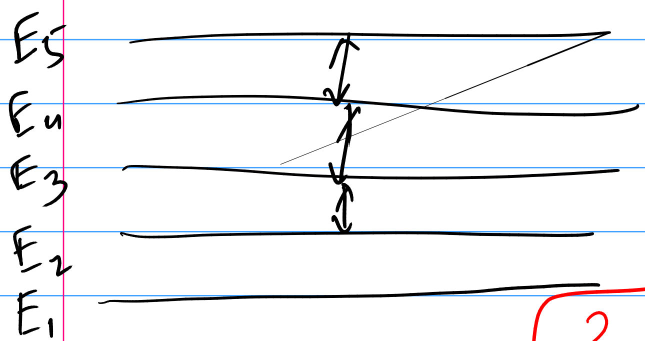
zero point energy

$n=2$ $$E_2 = \frac{4\pi^2\hbar^2}{2mL^2}$$

$$\boxed{E_{n+1} - E_n = \frac{(2n+1)\,\pi^2\hbar^2}{2mL^2}}$$

$$E_{n+1} - E_n \propto n$$

$n \uparrow$ $\uparrow$



$$\psi_1 = \sqrt{\frac{2}{L}} \sin \frac{\pi x}{L}$$

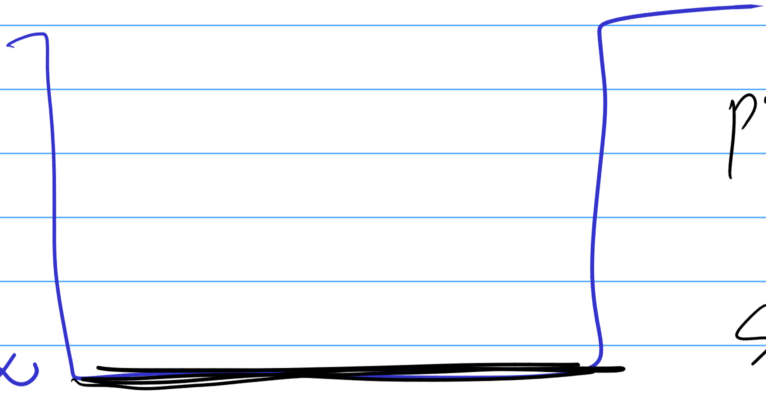
$$\psi_2 = \sqrt{\frac{2}{L}} \sin \frac{2\pi x}{L}$$

$$\psi_3 = \sqrt{\frac{2}{L}} \sin \frac{3\pi x}{L}$$

$$x = 0, L/6, L/3, L/2, 2L/3$$

$n \uparrow$ nodes $\uparrow$

$n = \infty$ # nodes $= \infty$

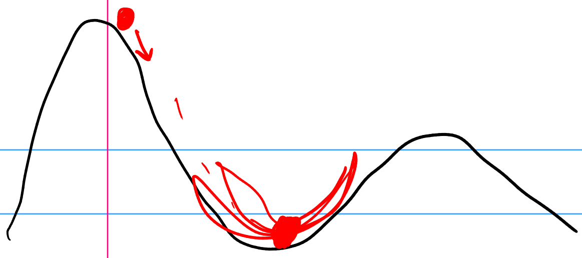


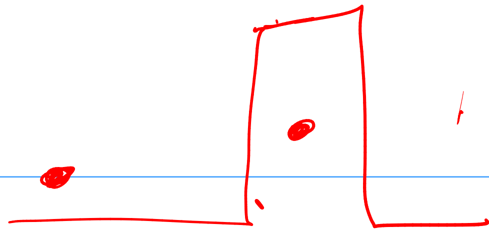
$n = \infty$

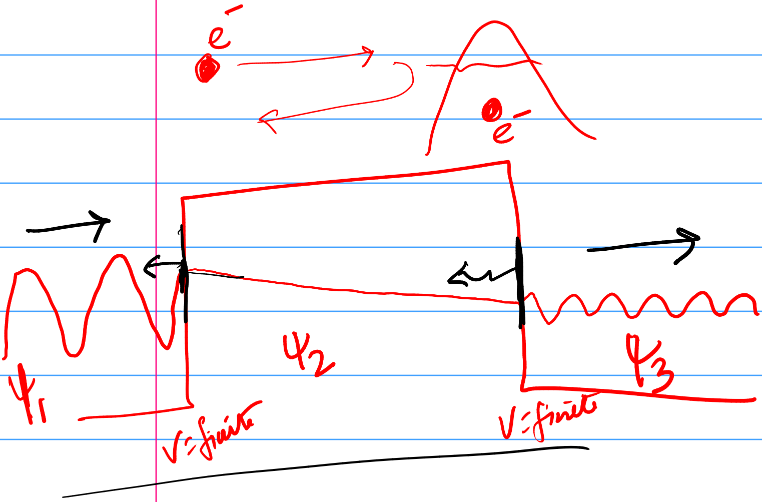
prob. = zero at $n = \infty$
so electron can't have energy
$V = \infty$

$V =$ finite $V =$ finite

Quantum tunnelling



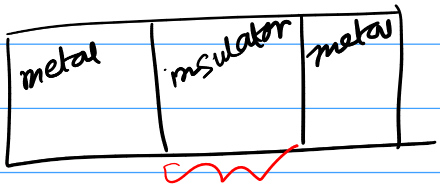


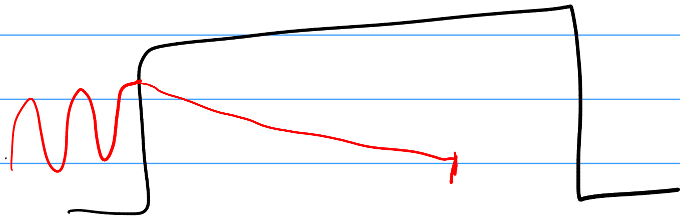


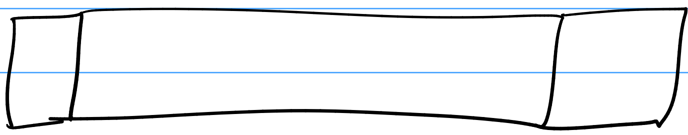
$$\psi_1 = Ae^{k_1x} + Be^{-k_1x}$$

$$\psi_2 = Ce^{k_2x} + De^{-k_2x}$$

$$\psi_3 = Ee^{k_p x}$$







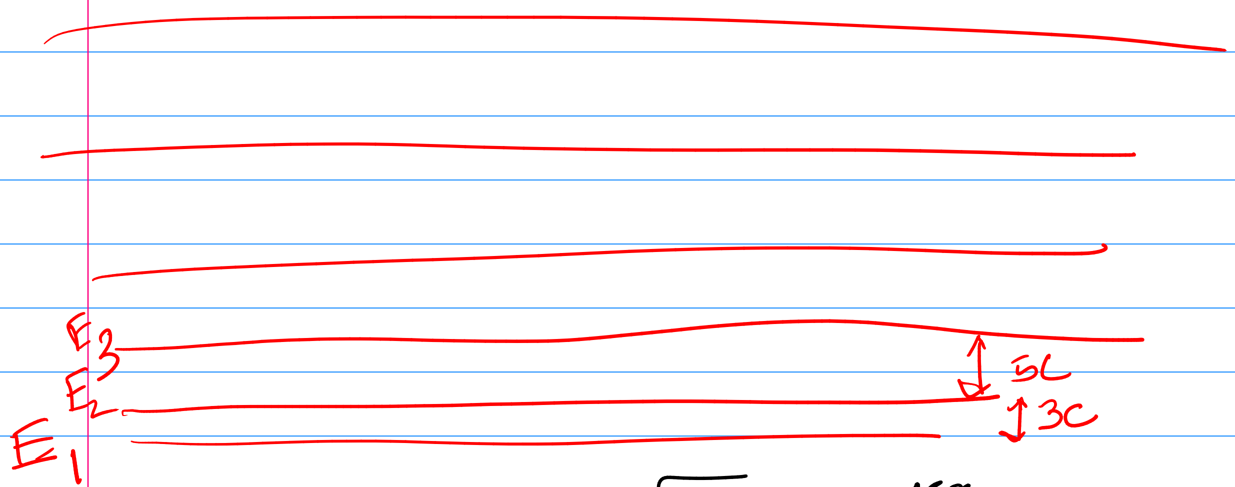
$$E_{n+1} - E_n = (2n+1)\,c$$

$$c = \frac{\pi^2 \hbar^2}{2mL^2}$$

$n=1 \quad E_2 - E_1 = 3c$

$n=2 \quad E_3 - E_2 = 5c$

$n=3 \quad E_4 - E_3 = 7c$



$$\psi_n = \sqrt{\frac{2}{L}}\ \sin \frac{kx}{L}$$

$$\psi_n^2 = \frac{2}{L}\ \sin^2 \frac{kx}{L}$$