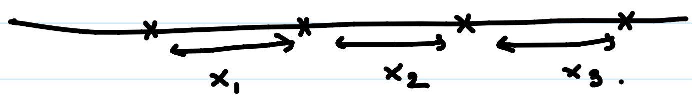
Renewal Process

It is a generalization of Poisson Process.

$N(t) > 0$



(x_i = i.i.d).

- Renewal Process is a counting process $N(t)$.
- Renewal Process is characterized by certain inter-arrival times.

<u>Definition</u> :- A renewal process is a counting process $\{N(t);\, t \geq 0\}$ in which arrival time are i.i.d with $P[x>0]=1$, $P[x_0]=0$.

Renewal Function

The function $M(t) = E[N(t)]$ is called renewal function of the process with dist. F. It is

$$\{N(t) \geq n\} \iff \{S_n \leq t\}$$

[$\because S_n$ = time of n^{th} renewal]

or

Imp

$$\{N(t) \geq n\} \text{ iff } \{S_n \leq t\}.$$

Theorem - 1

The dist. of $N(t)$ is given by

$$P_n(t) = P[N(t)=n] = F_n(t) - F_{n+1}(t).$$

and The expected no. of renewals by

$$M(t) = \sum_{n=1}^{\infty} F_n(t)$$

<u>Proof</u> :- we have.

$$P[N(t)=n] = P[N(t) \geq n] - P[N(t) \geq n+1].$$

$$= P[S_n \leq t] - P[S_{n+1} \leq t].$$

$$= F_n(t) - F_{n+1}(t).$$

More over

$$M(t) = E[N(t)]$$

$$M(t) = \sum_{n=0}^{\infty} n \cdot P_n(t) \qquad [n = 0, 1, 2, ---].$$

$$= 0 P_0(t) + 1 \cdot P_1(t) + 2 \cdot P_2(t) + 3 \cdot P_3(t) + ---$$

$$= 0 + [F_1(t) - F_2(t)] + 2[F_2(t) - F_1(t)] + 3[F_3(t) - F_2(t)] + ----$$

$$= F_1(t) + F_2(t) + F_3(t) + ---$$

$$\boxed{M(t) = \sum_{n=1}^{\infty} F_n(t)} \qquad \text{Hence Proved.}$$

Home work.

Find out 5 examples of renewal Process.