	Reg. No.:	
	Name :	

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TERM END EXAMINATIONS (TEE) – January 2023

Programme	:	**B.Tech.**	**Semester**	:	**Interim 2022-23**
Course Title	:	**Statistical Inferences and Series of Function**	**Course Code**	:	**MAT3017**
Time	:	**1½ hours**	**Max. Marks**	:	**50**

Answer ALL the Questions

Q. No.		Question Description	Marks
		PART - A (30 Marks)	
1	(a)	In random sampling from the probability distribution function $f(x;\theta) = \left(1/\theta^2\right) xe^{-x/\theta}$, $0 < x < \infty, 0 < \theta < \infty$ show that asymptotic maximum likelihood estimator attains minimum variance bound (Rao-Cramer lower bound)	10
		OR	
	(b)	Let the pdf of X be defined by $f(x) = \begin{cases} \left(4/\theta^2\right)x & 0 < x < \theta/2 \\ -\left(4/\theta^2\right)x + 4/\theta & \theta/2 < \theta \\ 0 & elsewhere \end{cases}$ where $0 < \theta \leq 2$. (a) Sketch the graph of this pdf when $\theta = 1/2$, $\theta = 1$ and $\theta = 2$. (b) Find the estimator of θ by the methods of moments. (c) For the following observations of X, give a point estimate of θ: 0.3206, 0.2408, 0.2577, 0.3557, 0.4188, 0.5601, 0.0240, 0.5422, 0.4532, 0.5592	10
2	(a)	Let $X \sim N(\mu, 4)$, μ is unknown. To test hypothesis H_0; $\mu = -1$ against H_1; $\mu = 1$, based on the sample of size 5 from this population, we use the critical region $0 \leq X_1 + 2X_2 + 3X_3 + 4X_4 + 5X_5 + 6X_6$. Find type I and type II error.	10
		OR	
	(b)	Students looked at the effect of a certain fertilizer on plant growth. The students tested this fertilizer on one group of plants (Group A) and did not give fertilizer to a second group (Group B). The growths of the plants, in mm, over six weeks were as follows: Group A: 55, 61, 33, 57, 17, 46, 50, 42, 71, 51, 63 Group B: 31, 27, 12, 44, 9, 25, 34, 53, 33, 21, 32 Test the null hypothesis (at 5% level of significance) that the mean growths are equal against the alternative that the fertilizer enhanced growth. Assume that the variances are equal.	10
3	(a)	Examine the convergence of series: $\sum \sqrt[3]{n^3 - 1} - n$.	10

		OR	
	(b)	Find the Taylor series for (a) $f(x) = \frac{1}{1+4x}$ and (b) $f(x) = \frac{1}{2-x^2}$.	10
PART - B (20 Marks)			
4		A random sample X_1, X_2, X_3, X_4, X_5 is taken from a Poisson distribution ($X \sim P(\theta)$) to test $H_0: \theta = 1$ against $H_1: \theta = 2$. It is decided to reject H_0 if $Y = \sum_{i=1}^{5} X_i \geq 8$. Find the level of significance and power of test.	10
5		A new machine is purchased for \$150,000. Each year the machine loses 25% of its value. Find its value at the end of the sixth year.	10
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One-sided	75%	80%	85%	90%	95%	97.5%	99%	99.5%	99.75%	99.9%	99.95%
Two-sided	50%	60%	70%	80%	90%	95%	98%	99%	99.5%	99.8%	99.9%
1	1.000	1.376	1.963	3.078	6.314	12.706	31.821	63.657	127.321	318.309	636.619
2	0.816	1.080	1.386	1.886	2.920	4.303	6.965	9.925	14.089	22.327	31.599
3	0.765	0.978	1.250	1.638	2.353	3.182	4.541	5.841	7.453	10.215	12.924
4	0.741	0.941	1.190	1.533	2.132	2.776	3.747	4.604	5.598	7.173	8.610
5	0.727	0.920	1.156	1.476	2.015	2.571	3.365	4.032	4.773	5.893	6.869
6	0.718	0.906	1.134	1.440	1.943	2.447	3.143	3.707	4.317	5.208	5.959
7	0.711	0.896	1.119	1.415	1.895	2.365	2.998	3.499	4.029	4.785	5.408
8	0.706	0.889	1.108	1.397	1.860	2.306	2.896	3.355	3.833	4.501	5.041
9	0.703	0.883	1.100	1.383	1.833	2.262	2.821	3.250	3.690	4.297	4.781
10	0.700	0.879	1.093	1.372	1.812	2.228	2.764	3.169	3.581	4.144	4.587
11	0.697	0.876	1.088	1.363	1.796	2.201	2.718	3.106	3.497	4.025	4.437
12	0.695	0.873	1.083	1.356	1.782	2.179	2.681	3.055	3.428	3.930	4.318
13	0.694	0.870	1.079	1.350	1.771	2.160	2.650	3.012	3.372	3.852	4.221
14	0.692	0.868	1.076	1.345	1.761	2.145	2.624	2.977	3.326	3.787	4.140
15	0.691	0.866	1.074	1.341	1.753	2.131	2.602	2.947	3.286	3.733	4.073
16	0.690	0.865	1.071	1.337	1.746	2.120	2.583	2.921	3.252	3.686	4.015
17	0.689	0.863	1.069	1.333	1.740	2.110	2.567	2.898	3.222	3.646	3.965
18	0.688	0.862	1.067	1.330	1.734	2.101	2.552	2.878	3.197	3.610	3.922
19	0.688	0.861	1.066	1.328	1.729	2.093	2.539	2.861	3.174	3.579	3.883
20	0.687	0.860	1.064	1.325	1.725	2.086	2.528	2.845	3.153	3.552	3.850
21	0.686	0.859	1.063	1.323	1.721	2.080	2.518	2.831	3.135	3.527	3.819
22	0.686	0.858	1.061	1.321	1.717	2.074	2.508	2.819	3.119	3.505	3.792
23	0.685	0.858	1.060	1.319	1.714	2.069	2.500	2.807	3.104	3.485	3.767
24	0.685	0.857	1.059	1.318	1.711	2.064	2.492	2.797	3.091	3.467	3.745
25	0.684	0.856	1.058	1.316	1.708	2.060	2.485	2.787	3.078	3.450	3.725
26	0.684	0.856	1.058	1.315	1.706	2.056	2.479	2.779	3.067	3.435	3.707
27	0.684	0.855	1.057	1.314	1.703	2.052	2.473	2.771	3.057	3.421	3.690
28	0.683	0.855	1.056	1.313	1.701	2.048	2.467	2.763	3.047	3.408	3.674
29	0.683	0.854	1.055	1.311	1.699	2.045	2.462	2.756	3.038	3.396	3.659
30	0.683	0.854	1.055	1.310	1.697	2.042	2.457	2.750	3.030	3.385	3.646
40	0.681	0.851	1.050	1.303	1.684	2.021	2.423	2.704	2.971	3.307	3.551
50	0.679	0.849	1.047	1.299	1.676	2.009	2.403	2.678	2.937	3.261	3.496
60	0.679	0.848	1.045	1.296	1.671	2.000	2.390	2.660	2.915	3.232	3.460
80	0.678	0.846	1.043	1.292	1.664	1.990	2.374	2.639	2.887	3.195	3.416

100	0.677	0.845	1.042	1.290	1.660	1.984	2.364	2.626	2.871	3.174	3.390
120	0.677	0.845	1.041	1.289	1.658	1.980	2.358	2.617	2.860	3.160	3.373
∞	0.674	0.842	1.036	1.282	1.645	1.960	2.326	2.576	2.807	3.090	3.291
One-sided	**75%**	**80%**	**85%**	**90%**	**95%**	**97.5%**	**99%**	**99.5%**	**99.75%**	**99.9%**	**99.95%**
Two-sided	**50%**	**60%**	**70%**	**80%**	**90%**	**95%**	**98%**	**99%**	**99.5%**	**99.8%**	**99.9%**