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Module 1.

1) The random sequence $(X_n, n \in N)$ is a Markow Chain for all $x_0, x_1 \dots x_n \in I$:
$P(x_n = j_n | X_0 = j_0, X_1 = j_1 \dots x_{n-1} = j_{n-1}) =$
$P(x_n = j_n | X_{n-1} = j_{n-1})$ provided this probability has meaning. i.e A markow chain $(J_n, n \geq 0)$ is homogeneous if the probabilities do not depend on n and for this case.

$$P(x_n = j | x_{n-1} = i) = P_{ij}$$

$$\text{Matrix } P = |P_{ij}|$$

2) It provides a method for computing the n-step transition probabilities.

$$P_{ij}^{(n)} = \sum_{k=0}^{M} P_{ik}^{(m)} P_{ij}^{(n-m)}$$ for all i, j and any $m = 1, 2 \dots n-1,$ $n = m+1, m+2 \dots$

These eq^n point out that in going from state i to state j in n steps, the process will be some state k after exactly m steps.

$$P(x_{t+2} = j | X_t = i) = \sum P(X_{t+2} = j | X_{t+1} = k) P(X_{t+1} = k | X_t = i)$$

i.e

$$Q(x(t_f) | x(t_i)) = \int dx(t') Q(x(t_f) | x(t')) Q(x(t') | x(t_i))$$

Tutorial

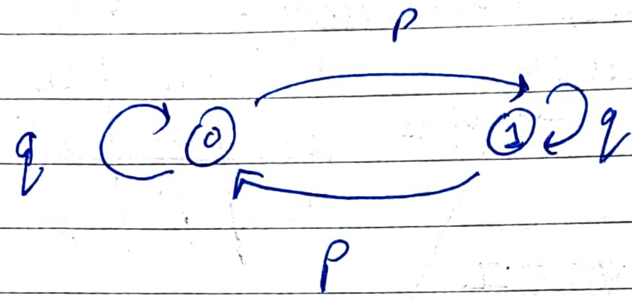
Soln) Given :- Stages → 0 and 1

- $P_{01} = P_{10} = p$
- $P_{00} = P_{11} = q$
- $q = 1 - p$

Taking as states the digits 0 and 1, we can identify Markov chain by specifying and transition probability matrix.

$$
TPM = \begin{array}{c c} & \begin{array}{cc} 0 & 1 \end{array} \\ \begin{array}{c} 0 \\ 1 \end{array} & \begin{bmatrix} q & p \\ p & q \end{bmatrix} \end{array}
$$

Tree $\Rightarrow$



Now, prob of starting from 0 and after two stages & producing the digit '0'

can be :- $\underline{\underline{P_{00}^{(2)}}}$

$$
P^{(2)} = \begin{pmatrix} q & p \\ p & q \end{pmatrix} \cdot \begin{pmatrix} q & p \\ p & q \end{pmatrix} = \begin{pmatrix} q^2+p^2 & 2pq \\ 2pq & q^2+p^2 \end{pmatrix}
$$

Hence required prob. $= \underline{\underline{p^2+q^2}}$

It can also be rewritten as

$$
p^2 + (1-p)^2
$$

$$
p^2 + 1 - 2p + p^2
$$

$$
= \boxed{2p^2 - 2p + 1}
$$

Assignment.

1) Markov chain containing values in set $S = \{ i : i = 0, 1, 2, 3, 4 \}$ where i = no. of umbrella. If $i=1$ and it rains then I take umbrella, move to other place where there are already 3 umbrellas and including I bring, now total = 4.

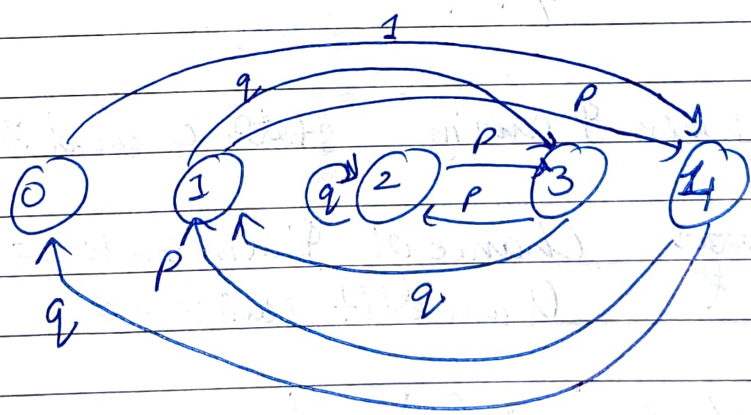
$$\boxed{P_{14} = p}$$

p = prob of rain

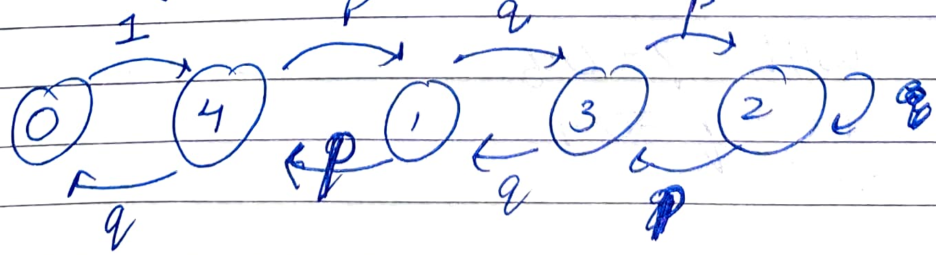
If $i = 1$ but does not rain then I don't take umbrella. i.e other place has 3 umbrellas.

$$\boxed{P_{13} = 1 - p} \equiv q.$$

Tree:-



Rearranged.



Stationary distribution
$$\pi(2) = \pi(3) = \pi(1) = \pi(4)$$

$$\pi(0) = \pi(4)\, q$$

So $\sum_{i=0}^{4} \pi(i) = 1$

↓

When $i=1$, it rain, I take umbrella & more

Expressing all prob. interms of $\pi(4)$ ∴

$$\pi(4)q + 4\pi(4) = 1$$

$$\Rightarrow \pi(4) = \frac{1}{q+4} = \pi(1) = \pi(2) = \pi(3)$$

$$\pi(0) = \frac{q}{q+4}$$

When I am in state 0 and it rains I get wet

~~change~~ Chance of I'm in state 0 = $\pi(0)$
& Chance it rains = p

$$P(\text{wet}) = \pi(0) \cdot p = \frac{qp}{q+4}$$

with $p = 0.6$
i.e $q = 0.4$

$$P(\text{wet}) \simeq \frac{0.4 \times 0.6}{0.4 + 4}$$

$$\boxed{P(\text{wet}) \simeq 0.0545}$$

ii) P(wet) less than 6% currently
If I want chance less than 1% I need to more umbrellas.

$\therefore$ N = no. of umbrella.

$$\pi(N) = \pi(N-1) = \cdots = \pi(1)$$

Probability =

$$\pi(0) = \pi(N)\, q$$

Insert $\sum_{i=0}^{N} \pi(i)$

$$\pi(N) = \frac{1}{q+N} = \pi(N-1) = \cdots = \pi(1)$$

$$\pi(0) = \frac{q}{q+N}$$

So, $P(\text{wet}) = \dfrac{pq}{q+N}$

we want $P(\text{wet}) = \dfrac{1}{100}$ or $q + N > 100pq$

or $N > 100pq - q$

$= 100 \times 0.4 \times 0.6 - 0.4 = 23.6 \approx 24$

So we need to reduce apx 24 umbrellas. to have less than 1%.

2)
```
def TPM_matrix (A, s) :
    n, u, accum = 0 , np.random.uniform (0,1),
                                        A [s] [0]
while u > accum :
    n = n+1
    accum = accum +A [s] [n]
if
    n >= 10
  def transition (A, s, n, e) :
    for i in range (n) :
      s = TPM_matrix (A, s)
return s == e
Else
  return n .
```