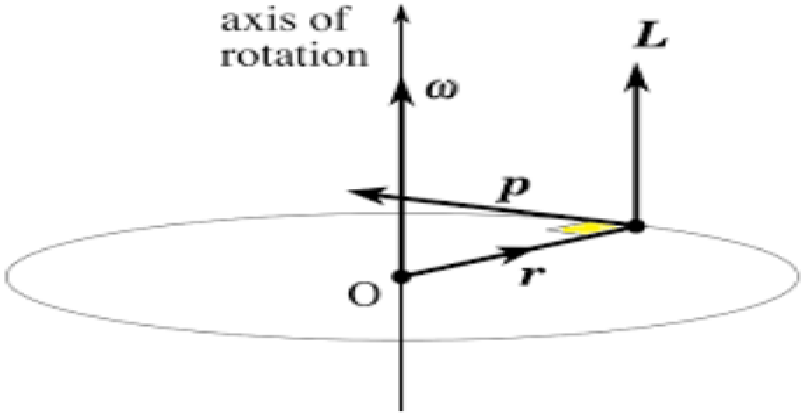
Angular Momentum



- Angular velocity: $\vec{\omega} = \frac{d\theta}{dt}\hat{n}$. Direction is defined along the perpendicular to the plane of rotation. By convention anti-clock wise motion correspond to the "positive" direction.
- Relation to linear velocity: $\vec{v} = \vec{\omega} \times \vec{r}$.
- By observation: "Difficulty" in rotating or to stop rotating a particle depends on its mass, linear velocity and its perpendicular distance from the axis of rotation.
- Mathematically, Angular Momentum $\vec{L} = \vec{r} \times m\vec{v} = \vec{r} \times \vec{p}$

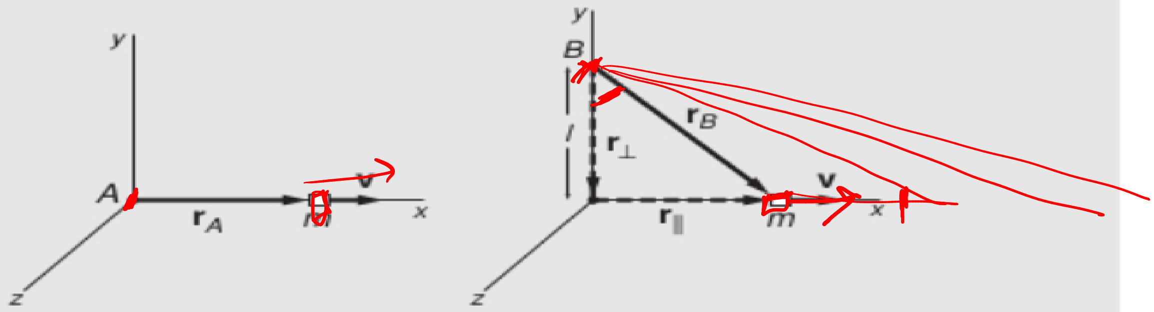
Angular Momentum: Ex-1

Example 7.1 Angular Momentum of a Sliding Block 1

A block of mass m and negligible dimensions slides freely in the x direction with velocity $\mathbf{v} = v\hat{\mathbf{i}}$, as shown in the sketch. What is its angular momentum $\mathbf{L}_A$ around origin A and its angular momentum $\mathbf{L}_B$ around origin B?

As shown in the drawing the vector from origin A to the block is $\mathbf{r}_A = x\hat{\mathbf{i}}$. Since $\mathbf{r}_A$ is parallel to $\mathbf{v}$, their cross product is zero:

$$\begin{aligned} \mathbf{L}_A &= m\mathbf{r}_A \times \mathbf{v} \\ &= 0. \end{aligned}$$



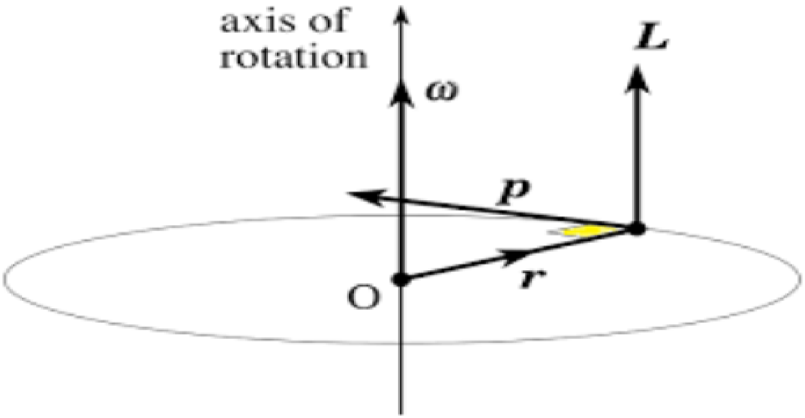
Taking origin B, we can resolve $\mathbf{r}_B$ into a component $\mathbf{r}_\parallel$ parallel to $\mathbf{v}$ and a component $\mathbf{r}_\perp$ perpendicular to $\mathbf{v}$. Then

$$\mathbf{L}_B = m\mathbf{r}_B \times \mathbf{v} = m(\mathbf{r}_\parallel + \mathbf{r}_\perp) \times \mathbf{v}.$$

With $\mathbf{r}_\parallel \times \mathbf{v} = 0$ and $\boxed{|\mathbf{r}_\perp \times \mathbf{v}| = lv\hat{\mathbf{k}}}$ we have

$$\boxed{\mathbf{L}_B = mlv\hat{\mathbf{k}}.}$$

Moment of Inertia



$\vec{L} = \vec{r} \times \vec{P}$

$\frac{d\vec{L}}{dt}$

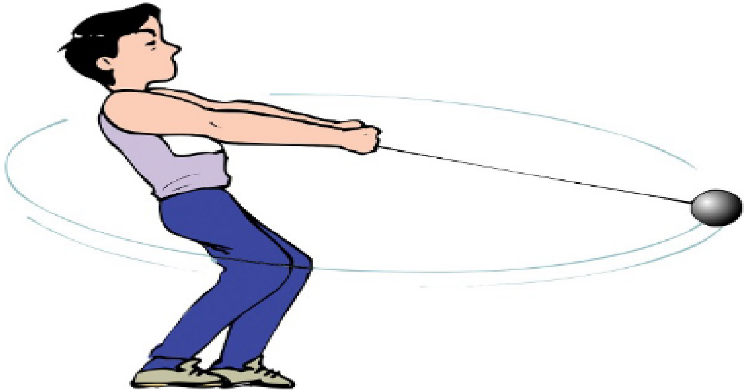
- Note that Angular Momentum $\vec{L} = \vec{r} \times m\vec{v} = \vec{r} \times \vec{p}$ is a Moment of Liner Momentum!
- Now $\vec{L} = \vec{r} \times m\vec{v} = -m(\vec{r} \times \vec{r} \times \vec{\omega}) = mr^2\vec{\omega} = I\vec{\omega}$ for a point particle.
- Here $m \Rightarrow$ Inertia and $I = mr^2$ is Moment of Inertia.
- I depends on the choice of the axis of rotation and defines inertia for rotational motion.

Torque

- Given I and $\vec{\omega}$, we can determine $\vec{L}$.
- Now consider time variation of $\vec{L}$: $\frac{d\vec{L}}{dt} = \frac{d(\vec{r}\times\vec{p})}{dt} = \vec{r} \times \frac{d\vec{p}}{dt} = \vec{r} \times \vec{F}$
- Rate of change of Angular Momentum depends on the moment of applied Force which is called Torque: $\vec{\tau} = \vec{r} \times \vec{F}$
- Note that it has a dimension of energy!

Conservation of Angular Momentum

- "Newton's Law" for angular motion: $\vec{\tau} = \frac{d\vec{L}}{dt}$, $\vec{\tau} = \vec{r} \times \vec{F}$
- If $\tau = 0$, $\vec{L}$ is conserved.
- $\vec{F} = 0$, NO motion, $\vec{L}$ is trivially conserved.
- However, $\tau = 0$ does not imply $F = 0$. It becomes zero if $\vec{r} || \vec{F}$ also.
- Example: Central force, Keppler's law., to be discussed in the next class.



- Torque $\vec{\tau} = \vec{r} \times \vec{F}$ is responsible for changing rotational state of motion.
- Apart from the applied Force, it depends on the choice of origin and axis of rotation.
- If $\vec{\tau} = 0$ the angular momentum is conserved.