Thursday, July 21, 2022 10:09 AM

Regular Markov chain

Example

Applications of Regular Markov chain:

Equilibrium vector:

- $S = \{$ lower class, Middle class, Upper class $\}$.

$$P = \begin{array}{c} \\ \text{L.C} \\ \text{M.C} \\ \text{U.C} \end{array} \begin{array}{c} \begin{array}{ccc} \text{L.C} & \text{M.C} & \text{U.C} \end{array} \\ \begin{bmatrix} 0.65 & 0.28 & 0.07 \\ 0.15 & 0.67 & 0.18 \\ 0.12 & 0.36 & 0.52 \end{bmatrix} \end{array}.$$

This matrix is regular since all entries are positive.

Let P represents Transition Matrix and let V be the Prob. vector.

To find V, such that

$$\boxed{VP = V}$$

$$[v_1 \quad v_2 \quad v_3]\, P = [v_1 \quad v_2 \quad v_3]$$

Use matrix multiplication on the left.

$$[0.65v_1 + 0.15v_2 + 0.12v_3,\ 0.28v_1 + 0.67v_2 + 0.36v_3,$$

$$0.07v_1 + 0.18v_2 + 0.52v_3] = [v_1\ \ v_2\ \ v_3].$$

Form set of eq.

$$0.65v_1 + 0.15v_2 + 0.12v_3 = v_1 \quad -(1)$$
$$0.28v_1 + 0.67v_2 + 0.36v_3 = v_2 \quad -(2)$$
$$0.07v_1 + 0.18v_2 + 0.52v_3 = v_3 \quad -(3)$$

After simplication.

$$-0.35v_1 + 0.15v_2 + 0.12v_3 = 0. \quad -(4)$$
$$0.28v_1 - 0.33v_2 + 0.36v_3 = 0 \quad -(5)$$
$$0.07v_1 + 0.18v_2 - 0.48v_3 = 0 \quad -(6)$$

We know that

$$v_1 + v_2 + v_3 = 1 \quad -(7)$$

Solve 4, 5, 6, 7

Find the value of v_1, v_2 and v_3.

v_1 ↓ 0.2865 , v_2 ↓ 0.4885 , v_3 ↓ 0.2250

↓

Equilibrium vector.

or

Steady state of Markov chain

or

Stationary dist. (limiting form of Markov chain)

As the time Progressed, the markov chain forgets about initial dist.

$$P = \begin{array}{c} \\ L.C \\ M.C \\ U.C \end{array} \begin{array}{c} \begin{array}{ccc} L.C & M.C & U.C \end{array} \\ \begin{bmatrix} 0.65 & 0.28 & 0.07 \\ 0.15 & 0.67 & 0.18 \\ 0.12 & 0.36 & 0.52 \end{bmatrix} \end{array}.$$

$$P^{10} = \begin{bmatrix} 0.2865 & 0.4885 & 0.2250 \\ 0.2865 & 0.4885 & 0.2250 \\ 0.2865 & 0.4885 & 0.2250 \end{bmatrix}$$

$$P^{50} = \begin{bmatrix} 0.2865 & 0.4885 & 0.2250 \\ 0.2865 & 0.4885 & 0.2250 \\ 0.2865 & 0.4885 & 0.2250 \end{bmatrix}$$

Higher and higher powers of the transition matrix P approches a matrix having all rows

identical. These identical rows have an entries of the equilibrium vector v.

Basic Property of Regular Markov chain

The limiting dist. is independent of the initial dist.

- x -