RIGID BODY :

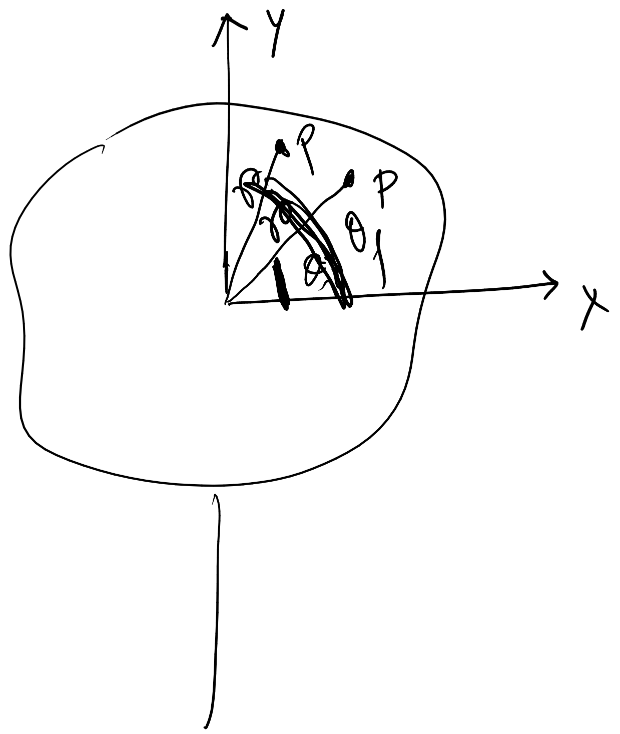
Rigid body is defined as a system of particles in which distance between each pair of particles remains constant (with respect to time). Remember, rigid body is a mathematical concept and any system which satisfies the above condition is said to be rigid as long as it satisfies it.

System behaves as a rigid body

- If a system is rigid, since there is no change in the distance between any pair of particles of the system, shape and size of system remains constant. Hence we intuitively feel that while a stone or cricket ball are rigid bodies, a balloon or elastic string is non rigid.
  But any of the above system is rigid as long as relative distance does not change, whether it is a cricket ball or a balloon. But at the moment when the bat hits the cricket ball or if the balloon is squeezed, relative distance changes and now the system behaves like a non-rigid system.

- For every pair of particles in a rigid body, there is no velocity of separation or approach between the particles. i.e. any relative motion of a point B on a rigid body with respect to another point A on the rigid body will be perpendicular to line joining A to B, hence with respect to any particle A of a rigid body the motion of any other particle B of that rigid body is circular motion.

Let velocities of A and B with respect ground be $\vec{V}_A$ and $\vec{V}_B$ respectively in the figure below.



$$\Delta\theta = \theta_f - \theta_i$$

$$\omega = \frac{\Delta\theta}{\Delta t}$$

$$\alpha = \frac{d\omega}{dt}$$

$$\int \alpha\, dt = \int d\omega$$

$$\int (\omega_f - \omega_i)\, dt = \int \alpha t\, dt$$

$$v - u = ft$$

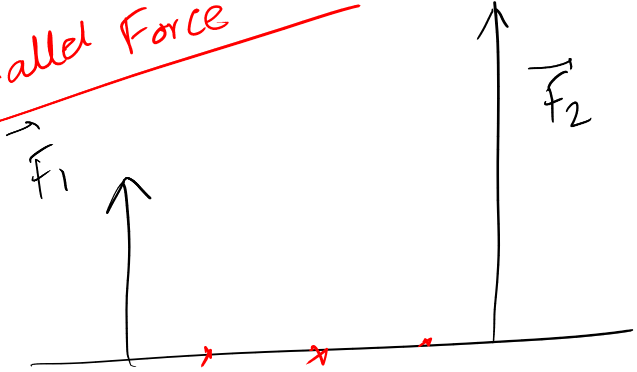
Rotational motion

Linear.

1. $V = u + ft$

2. $v^2 = u^2 + 2fS$

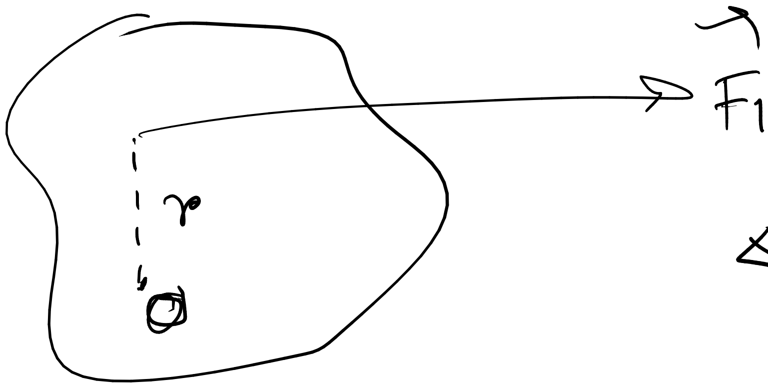
Parallel Force



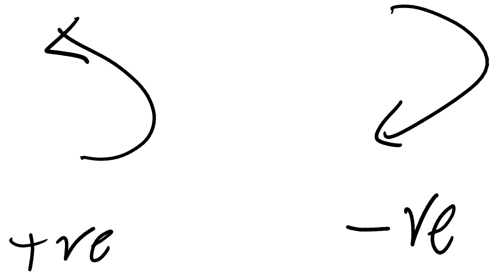
$$|\vec{R}| = |\vec{F_1}| + |\vec{F_2}|$$

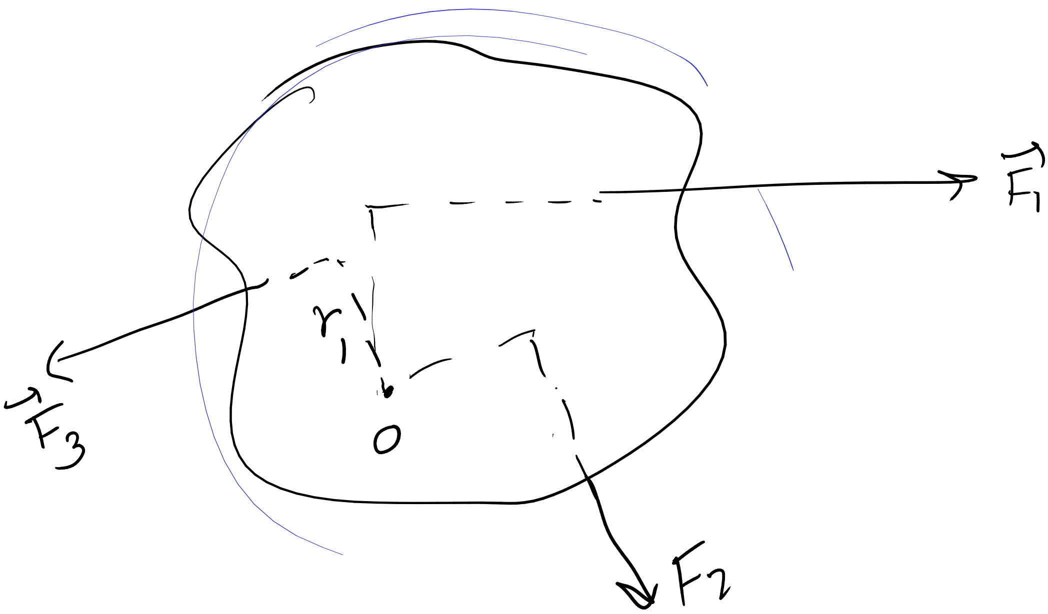
Moment of a force

$$\vec{M} = \vec{F} \times \vec{r}$$



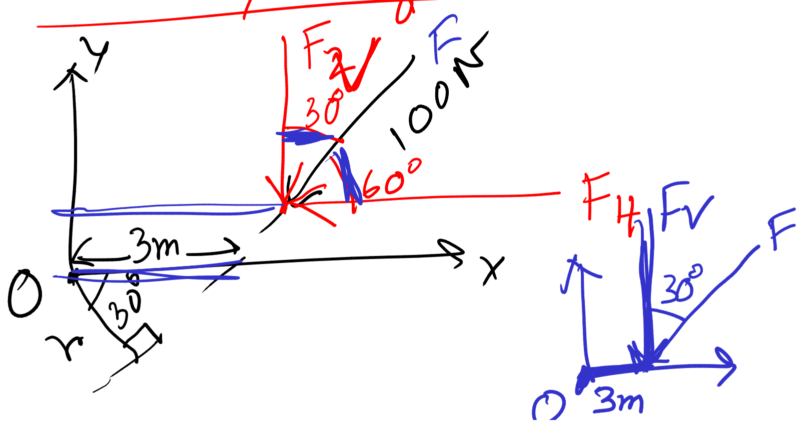
$$|\vec{M}| = Fr$$





$$M_{/O} = -\vec{F_1} \times \vec{r_1} - \vec{F_2} \times \vec{r_2} + \vec{F_3} \times \vec{r_3}$$

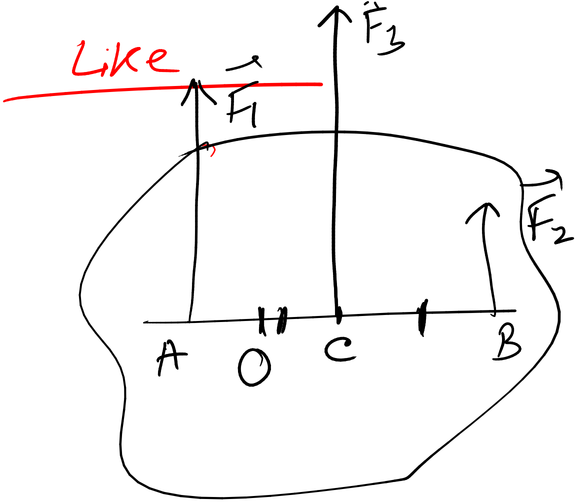
Principle of moments



$$M = 3\cos 30^\circ \times 100 \text{ Nm}$$
$$= \frac{3\sqrt{3}}{2} \times 100 \text{ Nm}$$
$$= 150\sqrt{3} \text{ Nm}$$

$$M_H = 0$$

$$M_V = 100\cos 30^\circ \times 3 \text{ Nm}$$

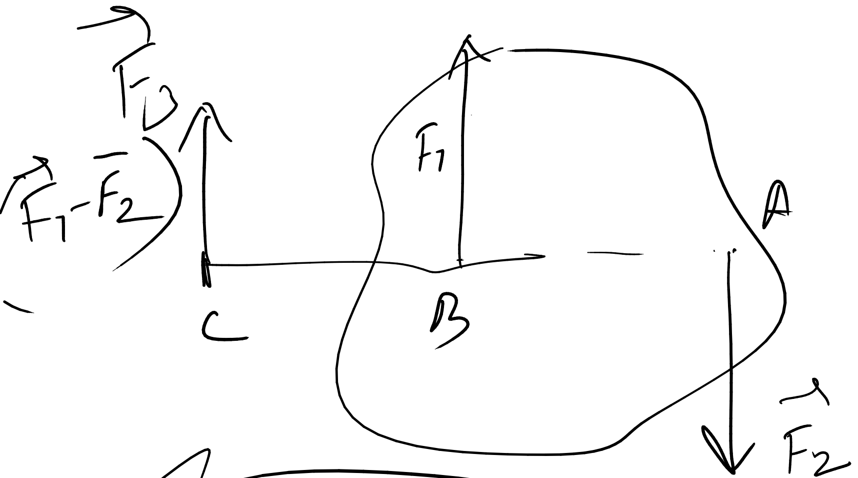


$$F_2 \times BO - F_1 \times AO$$
$$= F_3 \times CO$$
$$= (F_1 + F_2) \times CO$$

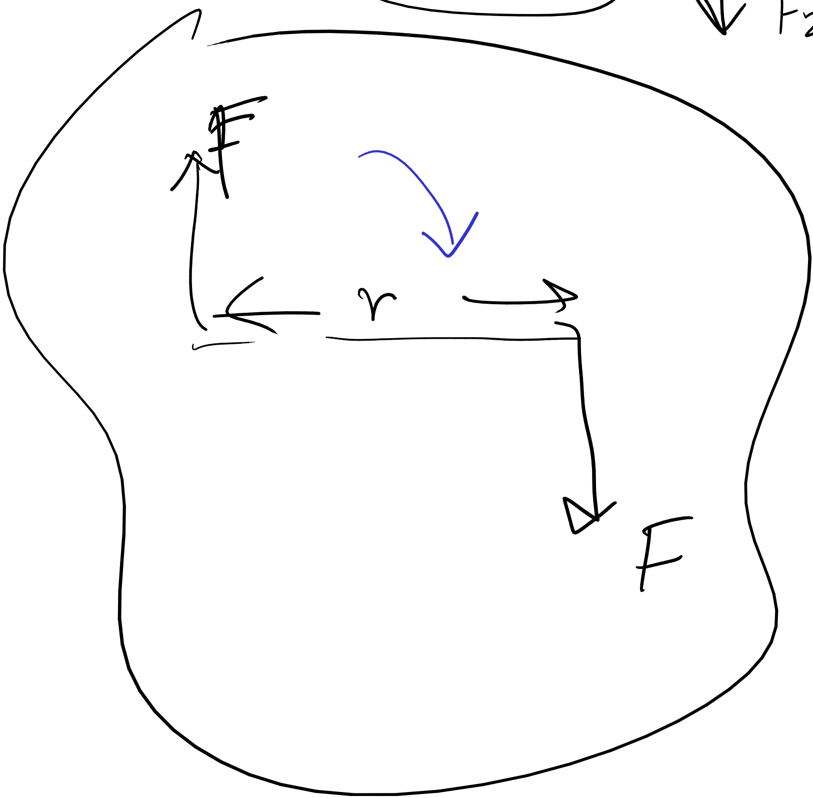
$$F_2 \times BO - F_2 \times CO$$
$$= +F_1 \times AO + F_1 \times CO$$

$$F_2 \times CB = F_1 \times AC$$

$$\boxed{\frac{CB}{AC} = \frac{F_1}{F_2}}$$



$$\frac{F_1}{F_2} = \frac{BC}{AC}$$



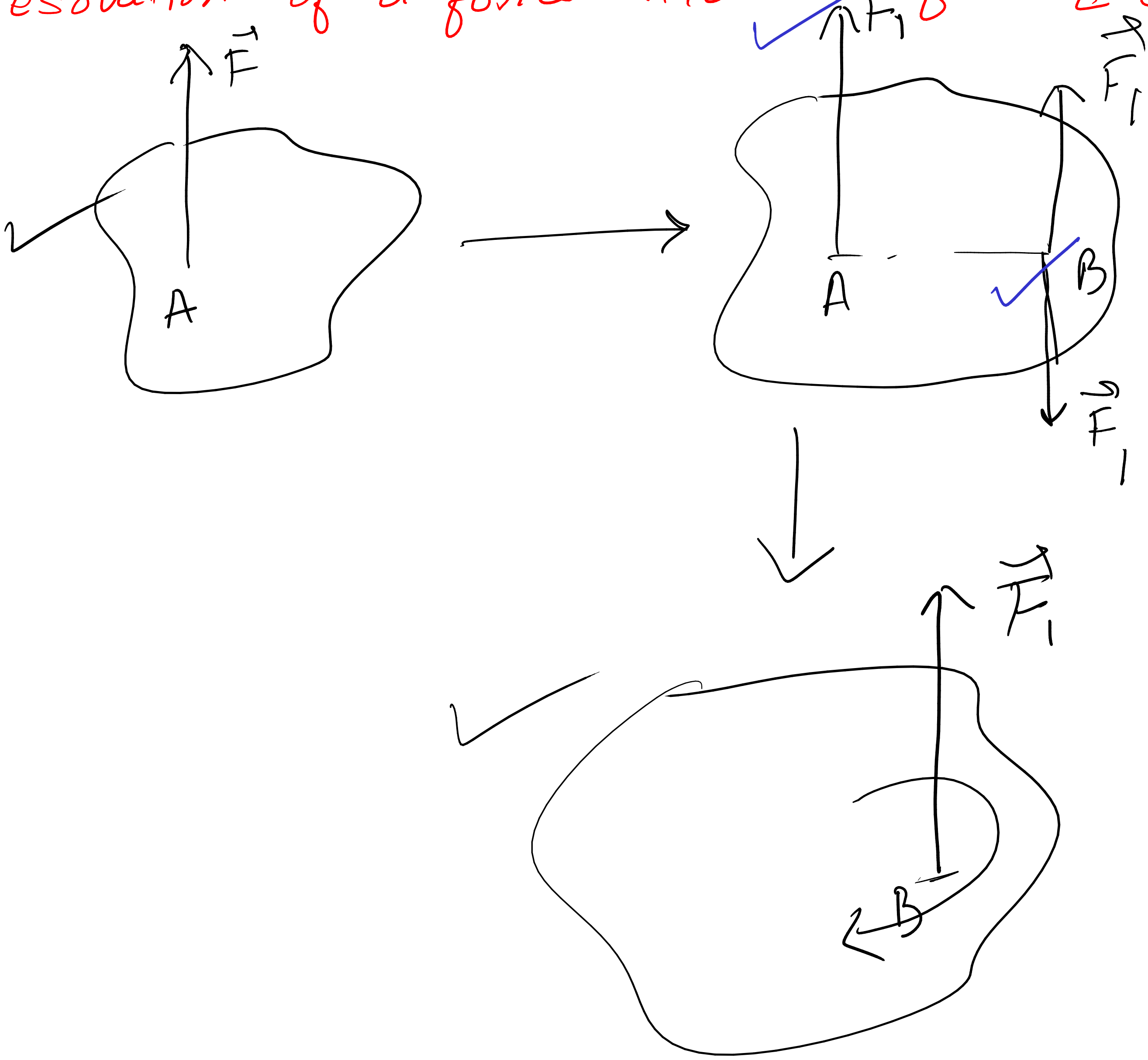
Unlike + Equal force

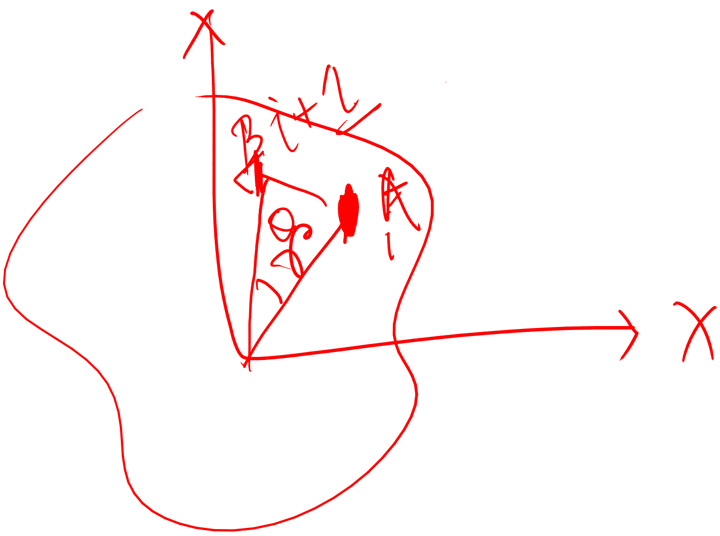
forms couple

Clockwise dirn

$$\vec{M} = \vec{F} \times \vec{r}$$

Resolution of a force into a force & couple





$$\Delta\theta = \theta_f - \theta_i$$

$$\omega = \frac{\Delta\theta}{\Delta t}$$

$$\alpha = \frac{d\omega}{dt}$$

Types of Motion of rigid body

- Pure Translational Motion
- Pure Rotational Motion
- Combined Translational and Rotational Motion

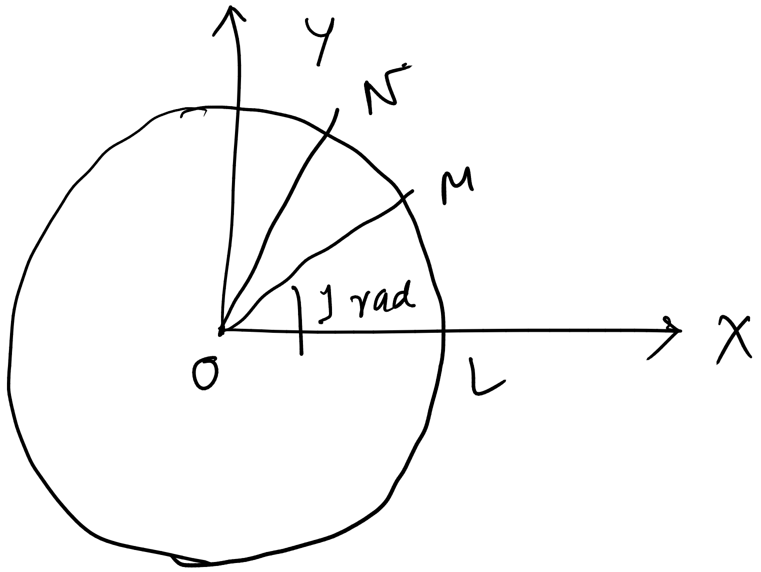
$$\alpha = \frac{d\omega}{dt}$$

$$\int d\omega = \int \alpha \, dt$$

$$\omega_f - \omega_i = \alpha t$$

$$v - u = ft$$

Linear	Rotational
1. $v = u + ft$	1. $\omega_f = \omega_i + \alpha t$
2. $x_f = x_i + \frac{1}{2} f t^2 + vt$	2. $\theta_f = \theta_i + \frac{1}{2}\alpha t^2 + \omega t$
3. $v_f^2 = v_i^2 + 2f(x_f - x_i)$	3. $\omega_f^2 = \omega_i^2 + 2\alpha(\theta_f - \theta_i)$



$\angle LOM = 1\ \text{rad}$

$\angle LON = \theta$

$\text{arc } LM = r$

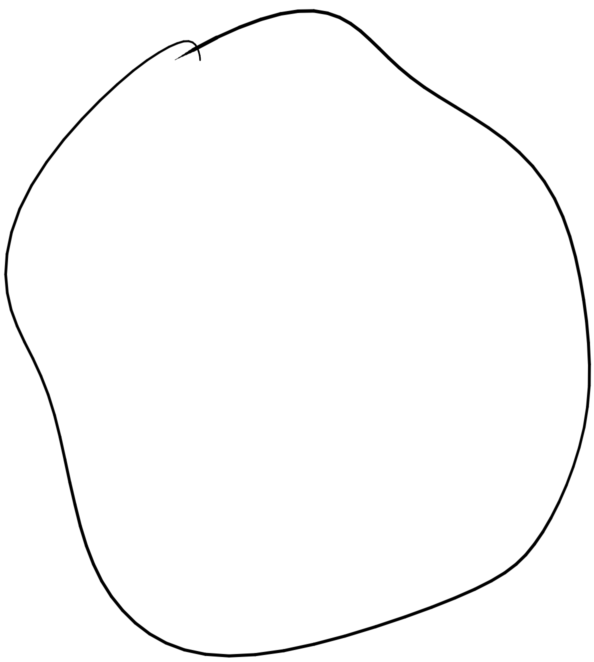
$$\frac{\angle LOM}{\angle LON} = \frac{LM}{LN}$$

$$\theta = \frac{LN}{LM} \times 1\ \text{rad}$$

$$\boxed{r\theta = s}$$

$$v = r\frac{d\theta}{dt} = r\omega$$

$$K.E = \frac{1}{2}mv^2$$



$$= \sum_i \frac{1}{2} m_i v_i^2$$

$$= \sum_i \frac{1}{2} m_i r_i^2 \omega^2$$

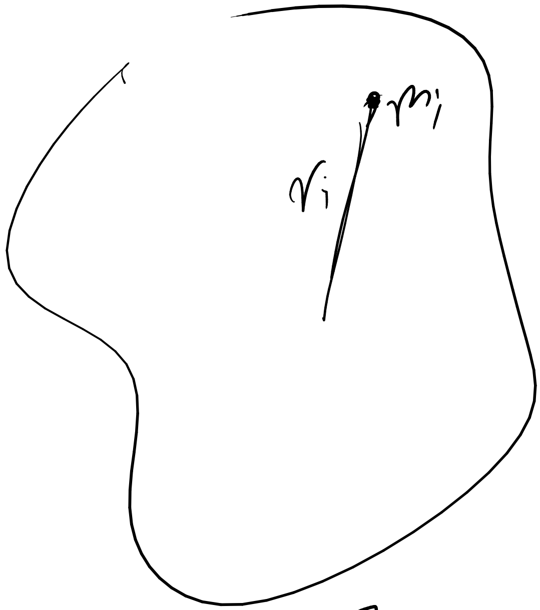
$$= \frac{1}{2} I \omega^2$$

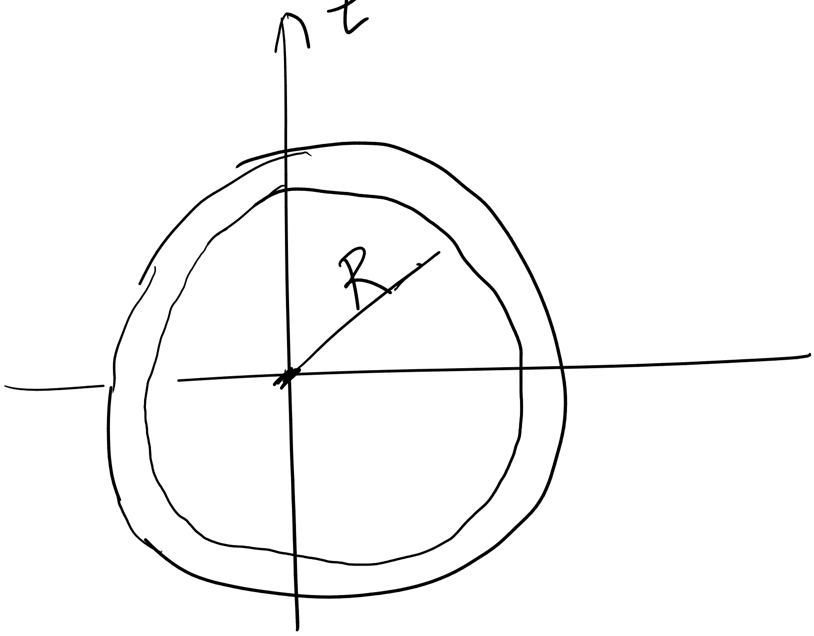
$$\boxed{I = \sum_i m_i r_i^2}$$

$$I = \sum_i m_i r_i^2$$

$$= \sum_i r_i^2 \Delta m_i$$

$$= \int r^2 dm = \int r^2 \rho \, dV$$





$$I_z = \int r^2 dm$$

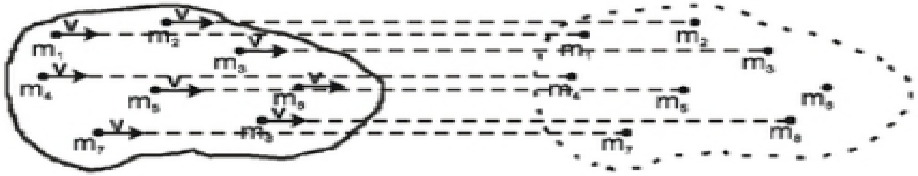
$$= r^2 \int dm$$

$$= r^2 M$$

I. Pure Translational Motion :

A body is said to be in pure translational motion, if the displacement of each particle of the system is same during any time interval. During such a motion, all the particles have same displacement $(\vec{s})$, velocity $(\vec{v})$ and acceleration $(\vec{a})$ at an instant.

Consider a system of n particle of mass $m_1, m_2, m_3, \ldots\ldots m_n$ under going pure translation. then from above definition of translational motion



$$\vec{a}_1 \quad \vec{a}_2 \quad \vec{a}_3 \quad \ldots\ldots \vec{a}_n = \vec{a} \text{ (say)}$$

and $\quad \vec{v}_1 \quad \vec{v}_2 \quad \vec{v}_3 \quad \ldots\ldots \vec{v}_n = \vec{v}$ (say)

From Newton's laws for a system.

$$F_{ext} = m_1a_1 + m_2a_2 + m_3a_3 + \ldots\ldots\ldots\ldots\ldots\ldots$$

$$F_{ext} = M\ a$$

Where $\quad M$ = Total mass of the body

$$\vec{P} = m_1v_1 + m_2v_2 + m_3v_3 + \ldots\ldots\ldots\ldots\ldots\ldots$$

$$\vec{P} = Mv$$

Total Kinetic Energy of body $= \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 + \ldots\ldots\ldots\ldots\ldots = \frac{1}{2}Mv^2$

Pure Rotational Motion :

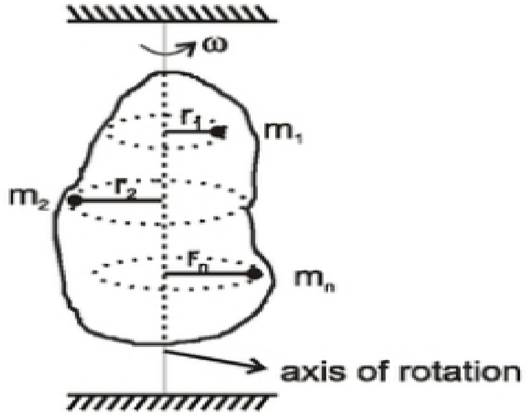


Figure shows a rigid body of arbitrary shape in rotation about a fixed axis, called the axis of rotation. Every point of the body moves in a circle whose center lies on the axis of rotation, and every point moves through the same angle during a particular time interval. Such a motion is called pure rotation.

We know that each particle has same angular velocity (since the body is rigid.)

so, $v_1 = \ \ r_1$, $v_2 = \ \ r_2$, $v_3 = \ \ r_3$ ...... $v_n = \ \ r_n$

$$\text{Total Kinetic Energy} = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 + \ldots\ldots\ldots\ldots\ldots\ldots\ldots$$

$$= \frac{1}{2} [m_1 r_1^2 + m_2 r_2^2 + \ldots\ldots\ldots\ldots\ldots\ldots\ldots] \quad ^2$$

$$= \frac{1}{2} \quad ^2 \text{ Where } \quad = m_1 r_1^2 + m_2 r_2^2 + \ldots\ldots\ldots \text{ (is called moment of inertia)}$$

= angular speed of body.

Combined Translational and Rotational Motion :

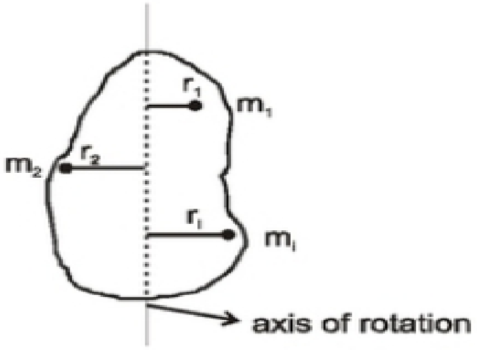
A body is said to be in combined translation and rotational motion if all point in the body rotates about an axis of rotation and the axis of rotation moves with respect to the ground. Any general motion of a rigid body can be viewed as a combined translational and rotational motion.

MOMENT OF INERTIA (I) ABOUT AN AXIS :

(i) Moment of inertia of a system of n particles about an axis is defined as

$m_1 r_1^2 + m_2 r_2^2 + \ldots\ldots\ldots\ldots\ldots\ldots + m_n r_n^2$

i.e. $= \sum_{i=1}^{n} m_i r_i^2$



where, r_i = It is perpendicular distance of mass m_i from axis of rotation

S units of Moment of nertia is Kgm^2.

Moment of inertia is a scalar positive quantity.

(ii) For a continuous system :

$= \int r^2 (dm)$

where dm = mass of a small element

r = perpendicular distance of the mass element dm from the axis

Moment of Inertia depends on :

(i) density of the material of body

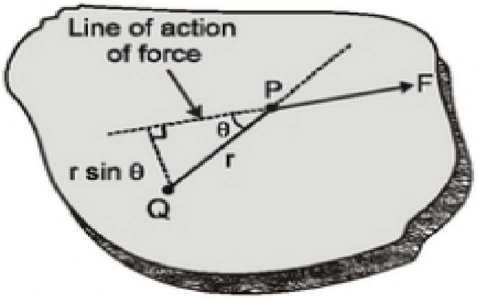
(ii) shape & size of body

(iii) axis of rotation

In totality we can say that it depends upon distribution of mass relative to axis of rotation.

TORQUE :

Torque represents the capability of a force to produce change in the rotational motion of the body.



6.1 Torque about a point :

Torque of force $\vec{F}$ about a point r F

Where $\vec{F}$ = force applied

P = point of application of force

Q = Point about which we want to calculate the torque.

$\vec{r}$ = position vector of the point of application of force w.r.t. the point about which we want to determine the torque.

$|\ ^{-}\ |$ = r F sin = r F = rF

Where = angle between the direction of force and the position vector of P wrt. Q.

r = r sin = perpendicular distance of line of action of force from point Q ,it is also called force arm.

F = F sin = component of F perpendicular to $\vec{r}$

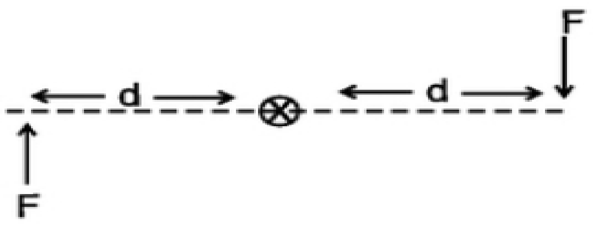
S unit of torque is Nm

☛ Torque is a vector quantity and its direction is determined using right hand thumb rule and its always perpendicular to the plane of rotation of the body.

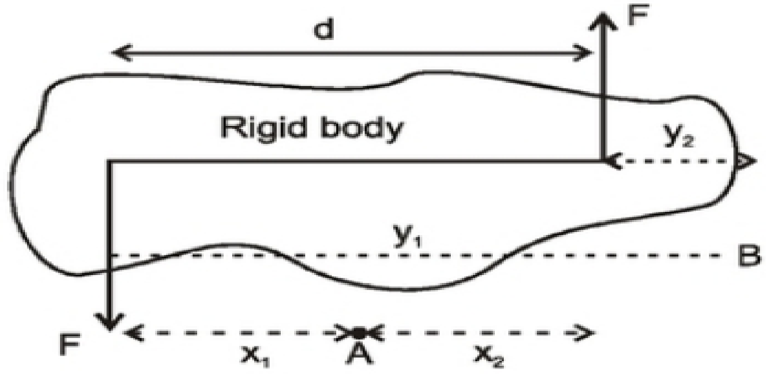
Force Couple :

A pair of forces each of same magnitude and acting in opposite direction is called a force couple. Torque due to couple = Magnitude of one force × distance between their lines of action.

Magnitude of torque = = F (2d)



- A couple does not exert a net force on an object even though it exerts a torque.
- Net torque due to a force couple is same about any point.



$$\text{Torque about A} = x_1F + x_2F$$
$$= F(x_1 + x_2) = Fd$$
$$\text{Torque about B} = y_1F - y_2F$$
$$= F(y_1 - y_2) = Fd$$

- If net force acting on a system is zero, torque is same about any point.
- A consequence is that, if $F_{net} = 0$ and $_{net} = 0$ about one point, then $_{net} = 0$ about any point.

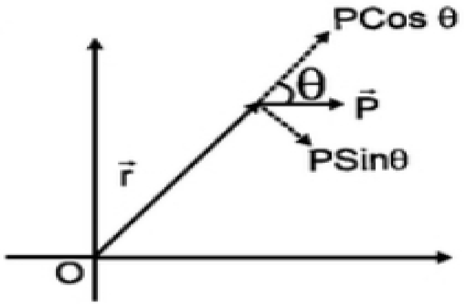
ANGULAR MOMENTUM ($\vec{L}$)

Angular momentum of a particle about a point.

$\vec{L} = \vec{r} \quad \vec{P}$ $\qquad$ L = rpsin

or $|\vec{L}| = r \quad \times P$

or $|\vec{L}| = P \quad \times r$



Where $\vec{P}$ = momentum of particle

$\vec{r}$ = position of vector of particle with respect to point O about which angular momentum is to be calculated .

= angle between vectors $\vec{r}$ & $\vec{P}$

r = perpendicular distance of line of motion of particle from point O.

P = component of momentum perpendicular to $\vec{r}$.

SI unit of angular momentum is kgm^2/sec.

Conservation of Angular Momentum

Newton's 2nd law in rotation : $\frac{d\vec{L}}{dt}$

where and $\vec{L}$ are about the same axis.

☞ Angular momentum of a particle or a system remains constant if $_{ext}$ = 0 about the axis of rotation.

Even if net angular momentum is not constant, one of its component of an angular momentum about an axis remains constant if component of torque about that axis is zero

Impulse of Torque : dt J

J Charge in angular momentum.