⇛ Mathematical foundation for C.S. & DM

21 | 11 | 2012

→ Basic Counting.
→ Sets.
→ *Logic.
→ Relations.
→ Functions
→ *Graphs.
→ Groups.
→ Recurrence Relations.

⇛ Basic Counting:

i) Sum Rule:

ex:- (1) - CS courses.

Institute A → 5 course offered

B - 4.

C - 3.

How many ways you can choose one course.

→ 5 + 4 + 3 = 12.

If Event E_1 can happen in e_1 ways.
Event E_2 " " " e_2 ways.
Then " E_1 or E_2 " " " $e_1 + e_2$ ways.
→ **Occurring simultaneously and are disjoint i.e. nothing is common among them.
Assumptions.

ii) Product Rule:

ex: Hyd ⟶ Nagpur ⟶ Delhi

3 ways, 4 ways.

How many ways from Hyd to Delhi via Nag?

1 —— 4

3 ⟶ ?

3 × 4 = 12 ways.

If event E_1 can happen in e_1 ways.
event E_2 " " " e_2 ways.
Then E_1 followed by E_2 can happen in $e_1 \times e_2$ ways

Assumptions: happening one followed by other & are independent.

Ques: If two distinguishable dice are rolled. How many ways we get sum 4 or sum 8.

Sum 4	(or)	Sum 8
(1,3)		(2,6)
(2,2)		(3,5)
(3,1)		(4,4)
		(5,3)
		(6,2).
↓		↓
3 ways		5 ways

$\Rightarrow 3+5 = 8$ ways.

Ques: How many you get even sum = ?

Sum 2 or Sum 4 or Sum 6 or Sum 8 or Sum 10 or Sum 12.

↓ 1 way, ↓ 3 ways, ↓ 5 ways, ↓ 5 ways, ↓ 3 ways, ↓ 1 way.

$\Rightarrow 1+3+5+5+3+1 = 18$ ways.

Ques: How many ways you get odd sum = ?

Sum 3 or Sum 5 or Sum 7 or Sum 9 or Sum 11

$\downarrow$ 2 + $\downarrow$ 4 + $\downarrow$ 6 + $\downarrow$ 4 + $\downarrow$ 2 = 18

$\Rightarrow$ 18 ways

bigger as pivot

(*) → If two distinguishable dice are rolled. How many outcomes are possible?

D1 D2

$6 \times 6 = 36$ outcomes.

Indirect Counting

→ How many ways to get odd sum

$= \text{Total} - \#\text{of even outcomes}$

$= 36 - 18 = 18.$

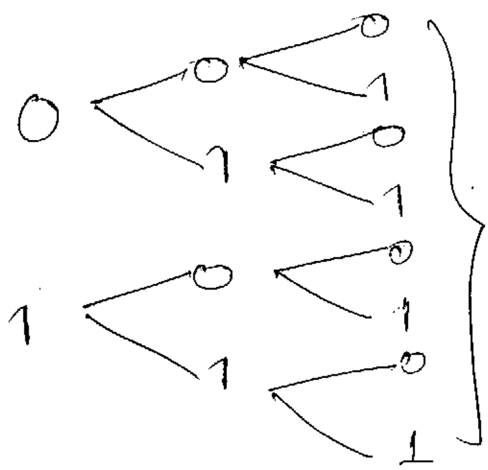
⇒ Binary Strings

(*) – How many binary string of length 3 are there?

Bit3 Bit2 Bit1

2 ways 2 ways 2 way $= 2 \times 2 \times 2 = 8$ ways

$= 2^3 = 8.$



8 possible Binary strings

(*) How many binary strings of length n are there?

1 2 3 ... n

$\underbrace{2 \;\; 2 \;\; 2 \; - - - - \; 2}_{n \text{ times}}$

$= 2^n$

(*) - How many binary strings of length 8 are those which start with "0".

1	2	3	4	5	6	7	8
0							
1	2	2	2	2	2	2	2

$= 2^7$ ways.

Ques:- IPv4 — 32bit addressing

Class A: $\underbrace{0 \ | \ }_{\text{Net id}} \ \underbrace{\ | \ | \ }_{\text{host ids}}$

Class B: $\underbrace{10 \ | \ }_{\text{Net ids}} \ \underbrace{\ | \ }_{\text{host ids}}$

Class C: $\underbrace{110. \ | \ | \ }_{\text{Net ids}} \ \underbrace{\ }_{\text{host ids.}}$

Not allotted to any one due to some technical reason.

	Net id	host ids
Class A	$2^7 - 2$	$2^{24} - 2$
Class B	2^{14}	$2^{16} - 2$
Class C	2^{21}	$2^8 - 2$

Ques:- How many Non-Negative integers less than 10^5 are there?

→ method 1 - Direct.

→ method 2 - Shortcut

⇒ using small instances we may solve large instances.

⇒ less than 3 - 3 Nos (0, 1, 2)

less than 5 - 5 ways (0, 1, 2, 3, 4).

So less than 10^5 — 10^5 Nos.

method - 3

10 10 10 10 10 = 10^5.

Ques: out of above, how many of them are even?

method 2 → half are even $= \frac{10^5}{2} = \underline{5*10^4}$.
as all Nos are consecutive

method 3

				0/2/4/6/8
10	10	10	10	5

$= \underline{5*10^4}$ //

Ques: How many are odd?

Indirect:- odd = Total - even
$= 10^5 - 5*10^4$
$10^4(10-5)$
$= 5*10^4$.

Ques: Palindrome:

Ques: How many palindrome of length 5 are there using english alphabet.

a) - 26^5, ~~b) - 26^3~~ c) - 26^2 d) - 26

26.	26.	26	1.	1.

choice choice

$\Rightarrow \underline{26^3}$.

Ques = for length 6 - Palindrome $= \underline{26}^3$.

(*)- How may palindromes of length k are there using English alphabet

$= 26^{\lceil k/2 \rceil}$.

<u>Que</u> How many 5 digits palindromes are there.

$\cancel{0}$

9 ways × 10 × 10 × 1 × 1 = 900.

<u>Que</u> How many are even?

$\cancel{0}$, 2, 4, 6, 8

1 10 10 1 4 = 400.

<u>Que</u> How many are odd?

900 − 400 = 500.

<u>Que</u> How many non-negative integers less than 10^5 contain digit 1

(contain 1) = Total − (do not contain 1).

(do not contain 1) $< 10^5$ =

$9 \times 9 \times 9 \times 9 \times 9 = 9^5$.

(contain 1) = $\underline{10^5 - 9^5}$ = 40,951

Ques (GATE): How many 4 digit even numbers are there without repetition of digits

even nos end with 0, 2, 4, 6, 8

end with 0

			~~0/2/4/6/8~~ 0
9	8	7	1

+

ending with 2, 4, 6, 8

~~0~~	0		2/4/6/8
8	8	7	4

$\Rightarrow 9*8*7*1 + 8*8*7*4$

Ques: 101 players participated in a knock-out Tennis tournament. Nos of matches played = ?

$\Rightarrow$ whenever, there is a match. There is a looser and vice versa.

ie. # of matches are in 1-1 correspondence with # of loosers.

So # of matches = # of loosers

= Total - [winner]

= 101 - 1

= 100 matches are conducted

11 players

	loosers
5 m + 1 B	- 5
3 + 0	- 3
1 + 1 B	- 1
1	- 1
10 matches	10

= Total - winner

11 - 1 = 10

So # of loosers = # of matches

Ques– How many 5 letter words can be formed using English alphabets with repetition but NOT consecutive repetition.

a) – 26×25^4
b) – $26^3 \times 25^2$
c) – $26^4 \times 25$
d) – None

26	25	25	25	25

$= \cancel{26 \times 26}$
$\Rightarrow 26 \times 25^4$

Ques– How many positive divisors are there for 300.

→ Prime factorization

$$\begin{array}{r|l} 2 & 300 \\ 2 & 150 \\ 3 & 75 \\ 5 & 25 \\ 5 & 5 \\ & 1 \end{array}$$

$\Rightarrow 2^2 \times 3^1 \times 5^2$

Nos of positive divisors

$\Rightarrow (2+1)(1+1)(2+1)$
$= 3 \times 2 \times 3 = 18.$

$2^a\, 3^b\, 5^c \mid 300$
$0 \le a \le 2,\ 0 \le b \le 1\ ;\ 0 \le c \le 2$

Ques:– # of positive divisors of $2^3 \times 7^2 \times 9^5$

$\Rightarrow 2^3 \times 7^2 \times (3^2)^5$
$\Rightarrow 2^3 \times 7^2 \times 3^{10}$
$\Rightarrow (3+1)(2+1)(10+1)$
$= 4 \times 3 \times 11 =$

⇒ Pigeonhole Principle – "If there are n-pigeonholes and (n+1) pigeons. Then some pigeonhole contain atleast 2 pigeons."

⇒ 3 pigeonhole
4 pigeons, atleast 2 pigeons.
5 " " 2 ".
6 " " 2 ".
7 " " 3 ".

If $2n+1$ pigeons, then some pigeonhole contain atleast 3 pigeons.

if $3n+1$ " " " " " atleast 4 pigeons.

$kn+1$ " " " " " atleast $k+1$ pigeons.

Ques There are 15 students in a class. Then atleast how many are born on the same day of a week.

a) - 2 b) - 3 c) - 4 d) - 5.

7 days —— 7 pigeonholes.

15 people —— 15 pigeonholes. i.e. $2n+1$.

$= \lceil 15/7 \rceil = \lceil 2.\text{some} \rceil = 3$ Ans

⇒ Formulae

> n — No of pigeonholes.
> k — No. of pigeons.
> $A = \frac{k}{n}$ = Avg # of pigeons per hole.
> Then some pigeonholes contains atleast $\lceil A \rceil$ pigeons

Pigeonhole principles provides the worst case analysis.

Ques Which of the following is min. No of students in class which assures atleast 5 members are born on same month of a year,

(a) → 37, (b) → 49 (c) → 61, (d) → 73

$\left\lceil \frac{37}{12} \right\rceil = 4$ $\left\lceil \frac{49}{12} \right\rceil = 5$ $\left\lceil \frac{61}{12} \right\rceil = 6$ $\left\lceil \frac{73}{12} \right\rceil = 7$

a) - 48 b) - 60 c) - 61 d) - 73.

$\lceil \frac{48}{12} \rceil = 4$ $\lceil \frac{60}{12} \rceil = 5$ $\lceil \frac{61}{12} \rceil = 6$ $\lceil \frac{73}{12} \rceil = 7$.

a) - 50 b) - 51 c) - 59 d) - 60.

$\lceil \frac{50}{12} \rceil = 5$ $\lceil \frac{51}{12} \rceil = 5$ $\lceil \frac{59}{12} \rceil = 5$ $\lceil \frac{60}{12} \rceil = 5$

(*) 401 letters distributed to 50 houses. Then some house receives atleast how many letters?

$$\lceil \frac{401}{50} \rceil = \underline{9}.$$

Que A box contains lot of red, green & blue socks. what is min # socks one one need to pick to be sure of getting

I - at least $\boxed{\text{one pair}}$ $\xrightarrow{2}$ of same colour?

a) → 3 b) → 4 c) → 7 d) → 10.

II - atleast $\boxed{\text{2 pairs}}$ $\xrightarrow{4}$ of same colors?

a) → 4 b) → 7 c) → 10 d) → 15.

$\lceil 4/3 \rceil = 2$ $\lceil 7/3 \rceil = 3$ $\lceil 10/3 \rceil = 4$ $\lceil 15/3 \rceil = 5$.

III → at least $\boxed{\text{3 pairs}}$ $\xrightarrow{6}$ of same color

a) - 13 b) → 16 c) → 19 d) → 22.

$\lceil \frac{13}{3} \rceil = 5$ $\lceil \frac{16}{3} \rceil = 6$ $\lceil \frac{19}{3} \rceil = 7$ $\lceil \frac{22}{3} \rceil = 8$.

Que what is the min. # of cards one need to pick from a deck of 52 cards to be sure of getting 3 cards of same suit?

a) - 3 b) - 9 c) - 13 d) - 17

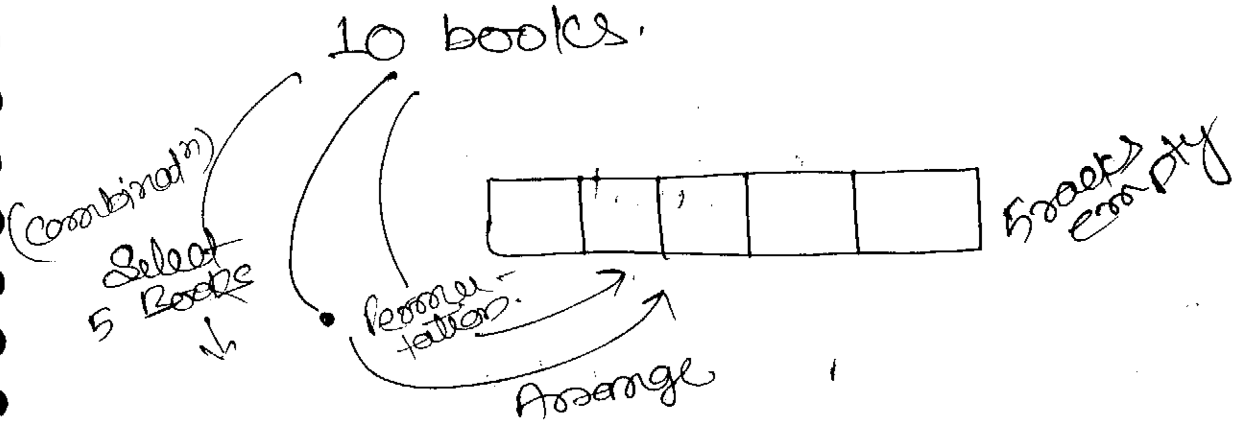
52 cards → 4 suits of 13 cards.

$\lceil \frac{3}{4} \rceil = 1$ $\lceil \frac{9}{4} \rceil = 3$

♠ ♣ ♦ ♥ – 4 suit

A – Ace
2
|
10
J – Jack
Q – queen
K – King

⇒ Permutation ~~and~~ Combinations



⇒ Permutations without Repetitions

$${}^nP_r = P(n,r) = r\text{- permutation of } n\text{-objects without repetitions}$$

n objects

1	2	3	- - - - - -	r
n	$n-1$	$n-2$		$(n-(r-1))$

$${}^nP_r = n \quad n-1 \quad n-2 \quad \cdots \quad (n-(r-1))$$

$$= \frac{n(n-1)(n-2) \cdots (n-r+1)(n-r)!}{(n-r)!}$$

$$= \boxed{{}^nP_r = \frac{n!}{(n-r)!}}$$

* $\boxed{{}^nP_r = \frac{n!}{(n-r)!}}$

$\boxed{{}^nP_0 = 1.}$

$\boxed{{}^nP_n = n!} \equiv$ n-permutations of n-objects.

Ques How many ways 10 people can be arranged in a series ?. ${}^{10}P_{10} = 10!$.

a) So that a certain pair always together?

$\boxed{AB} + 8$ ~~units~~ people.

1 unit + 8 units = 9 units

~~For~~ ⇒ 9 units in arrow = 9! ways

For 1 arrangement — A-B can be arranged in 2! way

for 9! arrangements ——— ?

$\boxed{9! \times 2}$.

$\boxed{9! \times 2}$. $\left\{ \begin{array}{l} \boxed{AB}\ m_3 m_4 \cdots m_{10} \\ \downarrow \\ \boxed{BA}\ m_3 m_4 \cdots m_{10} \end{array} \right.$

b) → So that certain pair are never together.

$\boxed{\cancel{AB}}$ NOT together. = Total − $\boxed{AB}$ always together.

AB Never together

$= 10! - 9! \times 2$

$= 9!\,(10-2) =$

$9! \times 8 = 8 \times 9!$

Ques- How many ways 7W and 3M can be arranged in a row?

$\Rightarrow (7+3)! \Rightarrow 10!$.

a) → Such that all 3 men are together.

$\boxed{3\text{ men}} + 7W$

$\Downarrow \qquad\qquad \Downarrow$

1 unit + 7 units = 8 units.

8 units in a row ⟶ 8! ways.

In 1 such arrangement - 3 men can be arranged in ⟶ 3! ways.

So, for 8! " " " " " $\boxed{3! \times 8!}$ ways

b) → Such that No 2 men are together?

7W & 3M

7W can be arranged in — 7! ways

$\times W_1 \times W_2 \times W_3 \times W_4 \times W_5 \times W_6 \times W_7 \times$

⟶ 3 men can be arranged in 8 gaps in $^8P_3$ ways

1 arrangement of W. ⟶ $^8P_3$ arrangements

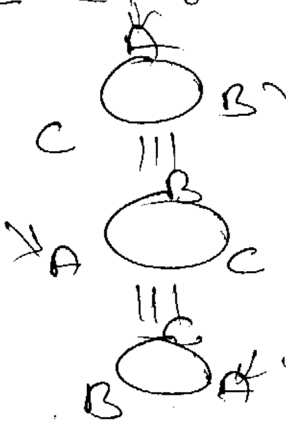
7! arrangements of W ⟶ $7! * {^8P_3}$ arrangements.

⇒ Arrangement in a row → linear permutation
,, ,, a circle → circular ,, .

⇒ Circular permutations

No. of circular permutations of n objects
$\equiv (n-1)! \equiv$ [fix 1 object and arrange remaining n-1 in linear manner]

~~ABC~~
A CB
B AC
~~BCA~~
~~CAB~~
C B A



are similar as we change our point of view everytime.

Ques No. of ways 10 people can be arranged in a circle? $= (10-1)! = 9!$.

a)→ So that certain pair is always together.

$\boxed{AB}$ + 8 people
1 unit + 8 units

$\underbrace{\hspace{2em}}$
||| = 9 units in a circle ≡ 8! ways

for 1 arrangement — AB can be arranged in 2! ways

So, for 8! ,, — $\boxed{8! \times 2!}$.

b)→ So that certain pair are never together.

$\boxed{\cancel{AB}}$ = Total − $\boxed{AB}$
$= 9! - 2! \times 8!$
$= 8!\,(9-2) = 8! \times 7$

Ques:- How many ways 7w & 3m can be arranged in a circle?

$9!$

(a) → So that 3m are always together.

$$\boxed{3m} + 7w$$

$1+7 = 8$ units

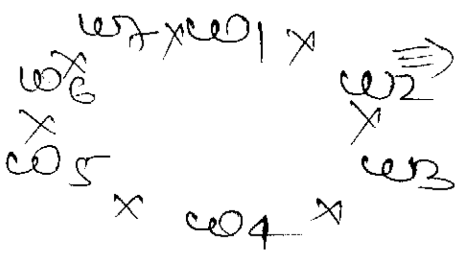
8 units in circle ⟶ $7!$ ways.

for 1 arrangement 3m can be arranged in ⟶ $3!$ ways

for $7!$ " " " ⟶ $7! \times 3!$ ways

(b) → So that no two men are ~~together~~ together?

7 w in circle ≡ ⟵ $6!$ ways



⟹ 3 men can be arranged in 7 gaps in $^7P_3$ ways

1 ⟶ $^7P_3$ ways.

$6!$ ⟶ $6! \times {}^7P_3$ ways.

⟹ Combinations without repetitions:-

$$^nC_r \equiv C(n,r) \equiv \binom{n}{r}$$

≡ r-combinations of n-objects without repetitions

$$\boxed{^nP_r = {}^nC_r \times r!} \equiv$$

^nC_r → selecting r-objects

$r!$ → arranging r objects in r-places

$$\Rightarrow \boxed{{}^{n}C_{r} = \frac{{}^{n}P_{r}}{r!} = \frac{n!}{r!\,(n-r)!}}$$

$$\Rightarrow \boxed{{}^{n}C_{0} = 1}$$

$$\Rightarrow \boxed{{}^{n}C_{n} = 1}$$

$$\Rightarrow \boxed{{}^{n}C_{1} = n}$$

$$ {}^{n}C_{2} = \frac{n!}{2!(n-2)!} = \frac{n\cdot(n-1)\,\cancel{(n-2)!}}{2\ \cancel{(n-2)!}} = \boxed{\frac{n\cdot(n-1)}{2}}$$

$$\Rightarrow \boxed{{}^{n}C_{r} = {}^{n}C_{n-r}}$$

i.e. out of n objects

selected r objects $\equiv$ selected for rejection $(n-r)$ objects.

Ques → No. of 5 combinations of 10 objects.
$= {}^{10}C_{5}$

→ How many ways 5 members can be selected from 10 members $= {}^{10}C_{5}$

→ 5 members can be selected from 7 W & 6 M
$= {}^{13}C_{5} = {}^{(7+6)}C_{5}$.

→ How many ways 3 men & 2 women can be selected from 7 men & 5 women
$= \boxed{{}^{7}C_{3} \times {}^{5}C_{2}}$

(*) - How many ways a 5 member committ can be selected from 6M & 7W. with atleast 2W in a committee

M=6	W=7		
5	0		$\to {}^6C_5 \times {}^7C_0$
4	1		$\to {}^6C_4 \times {}^7C_1$
3	2	atleast	$\to {}^6C_3 \times {}^7C_2$
2	3	2W.	$\to {}^6C_2 \times {}^7C_3$
1	4		$\to {}^6C_1 \times {}^7C_4$
0	5.		$\to {}^6C_0 \times {}^7C_5$

$\Rightarrow \quad {}^6C_3 \times {}^7C_2 + {}^6C_2 \times {}^7C_3 + {}^6C_1 \times {}^7C_4 + {}^6C_0 \times {}^7C_5.$

$\Rightarrow$ (atleast 2W) = T - (atmost 1W in committee).

$= {}^{13}C_5 - \{ {}^6C_5 \times {}^7C_0 + {}^6C_4 \times {}^7C_1 \}.$

* atleast 1 men in committee

= (Atleast 1 M) = Total - (No men)

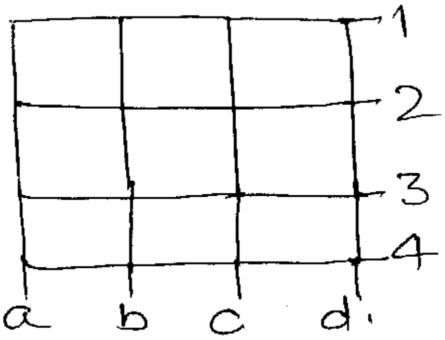
$\Rightarrow {}^{13}C_5 - \{ {}^6C_0 \times {}^7C_5 \}$

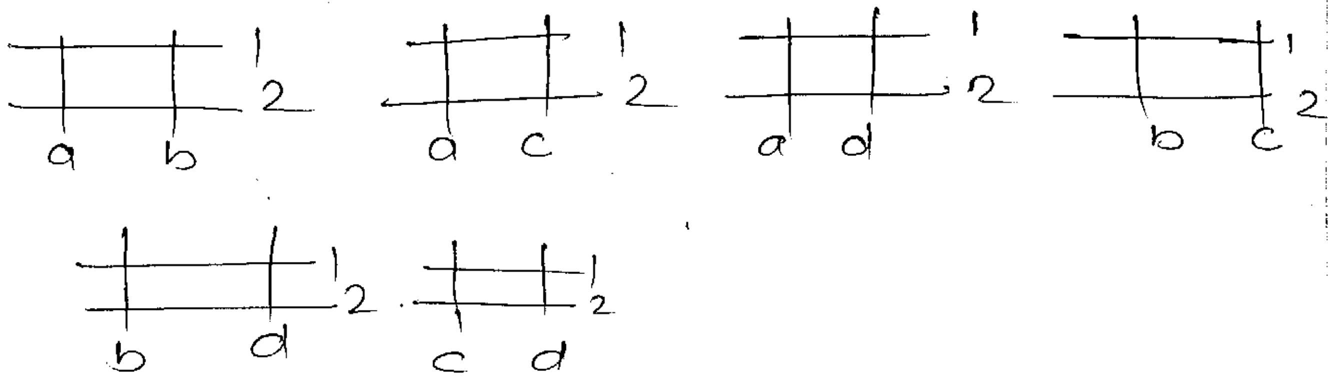
⇒ Product Rule ⇒ Pg/21.
chapter 4
→ 3, 8, 9, 26, 29
⇒ Pigeon hole principle → 28.
⇒ Permutation → 31, 15.
⇒ Circular perm → 1, 10.
⇒ combination → 2, 7, 13, 23, 33, 41.

Ques How many rectangles are there in a 8×8 chess board ?

22/10/2012

eg:-





2 vertical lines & followed by 2 horizontal line combination will give a rectangle

⇒ in 8X8 chessboard.

NO. of horizontal line = 9
NO of 2 combinations of horizontal lines = $9C_2$.
NO of vertical lines = 9.
NO of 2 combn of vertical lines = $9C_2$.

1 horizontal 2 combn —— $9C_2$ rectangles

$9C_2$ " " " —— ?

$$9C_2 \times 9C_2 = 36 \times 36 = 1296.$$

⇒ HAND-SHAKING LEMMA in G.T ⇐

case 10 people meet in a party, everyone shakes hand with other. How many hand shakes?

Every 2 combn of people ⟶ one hand shake

∴ 2 combn of 10 people → $10C_2$ handshakes.

Degrees as pivot

Ques How many binary string of length 6 are there with exactly four 1s.

4 1's and 2 0's

${}^{6}C_4$ ways to select 4 one's at 6 places

∴ 1 arrangement

1	1		1		1

all ones can they can be arranged one way only

Now 0 can be arranged in 1 way i.e. 2C2 ∴ No po ... possible.

$\equiv {}^{6}C_4$ strings

⊛ 8 letter words formed using 5 consonants & 3 vowels without repetition of letter.

a) How many such words can be formed?

a) → ${}^{21}C_5 \times {}^{5}C_3 \times 8!$ ≡ Bcoz first selection & then arrangement as letters are taken from different category

b) → ${}^{21}C_5 \times {}^{8}C_3$

c) → ${}^{21}P_5 \times {}^{5}P_3$ ≡ This does shuffling for consonents & vowels separa...

d) → None

⊛ How many such words contain "a"

a) ${}^{21}C_5 \times {}^{4}C_2 \times 7!$ $("a" + {}^{21}C_5 * {}^{4}C_2) \times 8!$

b) ${}^{21}C_5 \times {}^{4}C_2 \times 8!$

c) ${}^{21}C_5 \times {}^{5}C_3 \times 7!$

d) None

⊛ How many such words start with "a".

[a] $\equiv {}^{21}C_5 \times {}^{4}C_2 \times 7!$

"a" ____________

⇒ Binomial Expansion

$(x+y)^n = {}^nC_0 x^{n-0} y^0 + {}^nC_1 x^{n-1} y^1 + {}^nC_2 (x)^{n-2} y^2 + \cdots$
n is positive
$+ {}^nC_r x^{n-r} y^r + \cdots + {}^nC_n x^0 y^n$

$$\Rightarrow \boxed{(x+y)^n = \sum_{r=0}^{n} {}^nC_r (x)^{n-r} y^r}$$

$(1+1)^n = {}^nC_0 + {}^nC_1 + {}^nC_2 + \cdots + {}^nC_n$

$$\Rightarrow \boxed{{}^nC_0 + {}^nC_1 + {}^nC_2 + \cdots + {}^nC_n = 2^n} \quad \text{—— } (1)$$

$\Rightarrow 0 = (1-1)^n = {}^nC_0 - {}^nC_1 + {}^nC_2 - {}^nC_3 + \cdots$

$\Rightarrow 0 = {}^nC_0 - {}^nC_1 + {}^nC_2 - {}^nC_3$

$\Rightarrow$ ** $\boxed{{}^nC_0 + {}^nC_2 + {}^nC_4 + \cdots = {}^nC_1 + {}^nC_3 + \cdots = K}$.

from (1) –

$({}^nC_0 + {}^nC_2 + {}^nC_4 + \cdots) + ({}^nC_1 + {}^nC_3 + \cdots) = 2^n$

$K + K = 2^n$

$2K = 2^n$.

$K = \frac{2^n}{2}$

$\boxed{K = 2^{n-1}}$.

$$\Rightarrow \boxed{{}^nC_0 + {}^nC_2 + {}^nC_4 + \cdots = {}^nC_1 + {}^nC_3 + \cdots = 2^{n-1}}$$

⇒ SETs:- Unordered collection of well-defined objects.

$A = \{1, 2, 3\}$ or $\{3, 1, 2\}$ ≡ Unordered.

* $\in$ (Belongs to (or) member of)

$1 \in A$
$5 \notin A$ } ≡ well-defined.

→ Subset:- ($\subseteq$)

$A \subseteq B$ iff every element of A is an element of B
i.e. $x \in A$ then $x \in B$.

eg: $A = \{1, 2\}$ $B = \{1, 2\}$ $C = \{1, 2, 3\}$.

$A \subseteq B$, $B \subseteq A$
$B \subseteq C$, $C \nsubseteq B$.

→ Equal:⇒

$A = B$ iff $A \subseteq B$ and $B \subseteq A$.

→ Proper Subset:-

$A \subset B$ if $A \subseteq B$ and $A \neq B$.

→ U = Set containing all elements under consideration.

$\phi = \{\ \}$ = Set with no elements.

Ques A

which is always true

(I) $\phi \in A$

(II) $\phi \subseteq A$

a) - only I true
b) - only II true
c) - Both True
d) - None True

→ Properties

1.) $A \subseteq A$.

2.) $\phi \subseteq A$.

3.) $A \subseteq U$.

→ Cardinality $|A|$ = No. of elements in A.

$A = \{1, 2, \{3,4\}\}$

$|A| = 3$.

→ Ques – Find all subsets of $A = \{1,2\}$

Subsets

$2C_0 \leftarrow \phi$

$2C_1 \leftarrow \{1\}, \{2\}$

$2C_2 \leftarrow \{1,2\}$

Binary string of length

1	2
0	0
1	0
0	1
1	1

→ So, No. of subsets of a set with n-elements
= No. of binary strings of length n
$= 2^n$.

→ No. of subsets of a set with n elements.

$= nC_0 + nC_1 + \cdots\cdots + nC_n = 2^n$.

nC_0 ↓ No. of 0 elements subsets

nC_1 ↓ No. of 1 element subsets.

nC_n ↓ No. of n element subsets

No. of subsets of a sets with n element having odd cardinality

⇒ 1 length + 3 length + ——

$\Rightarrow nC_1 + nC_3 + nC_5 + \cdots$

$\Rightarrow 2^{n-1}$

(*) → Find all subsets of $A = \{1,2,3\}$.

Binary strings of length $3 = 2^3 = 8$

$000 \rightarrow \phi$

$001 \rightarrow \{3\}$

$010 \rightarrow \{2\}$

$011 \rightarrow \{2,3\}$

$100 \rightarrow \{1\}$

$101 \rightarrow \{1,3\}$

$110 \rightarrow \{1,2\}$

$111 \rightarrow \{1,2,3\}$.

$P(A) = \{ \phi, \{1\}, \{2\}, \{3\}, \{1,2\}, \{1,3\}, \{2,3\}, \{1,2,3\} \}$

$\Rightarrow P(A) =$ Power set
(set of all subsets of A).

(*) - $A = \{1, 2, \{3,4\}\}$

$|P(A)| =$ (a) - 4
~~(b)~~ - 8
(c) - 16
(d) - 32

\# $|A| = n$.

$\therefore |P(A)| = 2^n$.

⇒

Algebra.	Set Algebra	Prop. Algebra
objects	Sets.	Propositions
Operations	$\cap, \cup, ^c$	$\neg, \wedge, \vee$

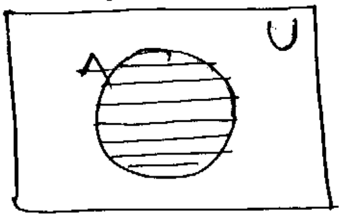
A.D.T.

→ Boolean Algebra. ←

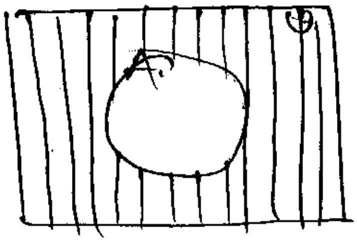
⇒ Set operations:

(I): $A^c = \{x \mid x \notin A \text{ and } x \in U\}$.

Venn Diagram

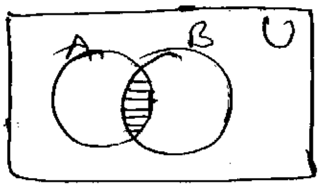


A

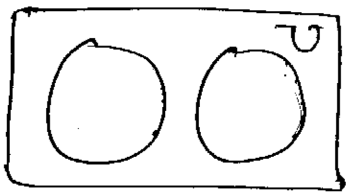


A^c

(II): $A \cap B = \{x \mid x \in A \text{ and } x \in B\}$.

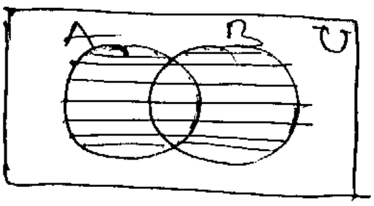


$A \cap B$



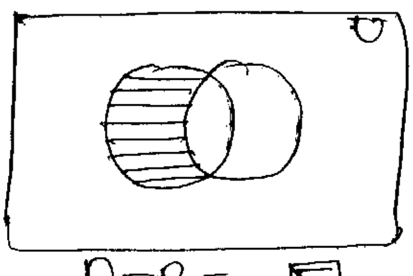
$A \cap B = \phi$.
Disjoint sets.

(III): $A \cup B = \{x \mid x \in A \text{ or } x \in B \text{ or both}\}$
i.e. $x \in$ at least one set.



$A \cup B =$

(IV): $A - B = \{x \mid x \in A \text{ and } x \notin B\}$

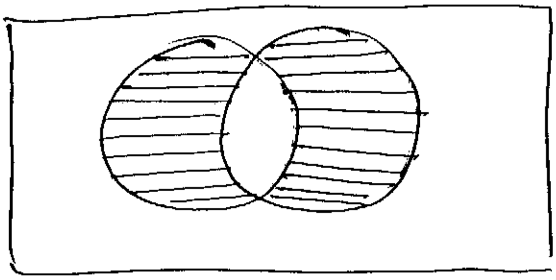


$A - B = A \cap B^c$.

$A - B =$

(V) - Symmetric difference or Boolean Sum

$A \Delta B = A \oplus B = \{x \mid x \in A$ or $x \in B$ but NOT bo



$$A \Delta B = A \oplus B = (A-B) \cup (B-A).$$
$$= (A \cup B) - (A \cap B)$$

Some Properties

(i) → $A \Delta A = (A \cup A) - (A \cap A) = A - A = \phi.$

(ii) → $A \Delta A^C = (A \cup A^c) - (A \cap A^c) = U - \phi = U.$

iii) → $A \Delta \phi = A$

iv) → $A \Delta U = (A \cup U) - (A \cap U) = U - A = A^c.$

⇛ Set Identities

(I) → Idempotent -

$A \cup A = A.$

$A \cap A = A.$

(II) → Identity -

$A \cup \phi = A$

$A \cap U = A.$

(III) → Domination :-

$A \cup U = U$

$A \cap \phi = \phi.$

(IV) → Complementation

$A \cup A^c = U$

$A \cap A^c = \phi.$

(V) → Double Complement

$(A^c)^c = A.$

(VI) → Commutative

$A \cup B = B \cup A$

$A \cap B = B \cap A$.

(VII) → Associative

$A \cup (B \cup C) = (A \cup B) \cup C$.

$A \cap (B \cap C) = (A \cap B) \cap C$

(VIII) → Absorption

$A \cup (A \cap B) = A$.

$A \cap (A \cup B) = A$.

(IX) → De-Morgan's

$(A \cup B)^C = A^C \cap B^C$

$(A \cap B)^C = A^C \cup B^C$.

(X) → Distributive

$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$.

$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$.

→ Prob:

1.)- $(\underline{X} \cup Y) \cap (\underline{X} \cup Z) = X \cup \{Y \cap Z\}$.

11.)- $(P \cup Q) \cap (P^C \cap Q^C)$

$\Rightarrow \underline{(P \cup Q)} \cap \underline{(P \cup Q)^C}$

$X \cap X^C$

$= \phi$.

111.)-

I) → $A - (B \cup C) = (A - B) \cup (A - C)$ ✗

II) → $A - (B \cap C) = (A - B) \cap (A - C)$.

a) only I true

b)- only II true

c)- both true

d)- None true

$\Rightarrow A-(B\cup C) = A\cap (B\cup C)^c$

$= A\cap (B^c\cap C^c)$

$= (A\cap B^c)\cap (A\cap C^c).$

$\boxed{A-(B\cup C) \Rightarrow (A-B)\cap (A-C)}$

$\Rightarrow A-(B\cap C) = A\cap (B\cap C)^c$

$= A\cap (B^c\cup C^c)$

$= (A\cap B^c)\cup (A\cap C^c)$

$\boxed{A-(B\cap C) = (A-B)\cup (A-C)}$

(*) – $(P^c \cap Q\cap R)\cup (P\cap Q\cap R)\cup Q^c\cup R^c.$

(a) → $Q^c\cup R^c$

(b) → $P\cup Q^c\cup R^c$

(c) → $P^c\cup Q^c\cup R^c$

✓(d) → U

$\Rightarrow (P^c \cap Q\cap R)\cup (P\cap Q\cap R)\cup Q^c\cup R^c$

$= [(P^c\cup P)\cap Q\cap R]\cup (Q\cap R)^c$

$= [U\cap (Q\cap R)]\cup (Q\cap R)^c$

$= (Q\cap R)\cup (Q\cap R)^c$

$= U$

ans: $X = (A-B)-C$

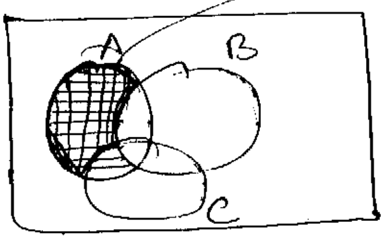
$Y = (A-C)-(B-C)$

✓a) – $X=Y$

b) – $X\subset Y$

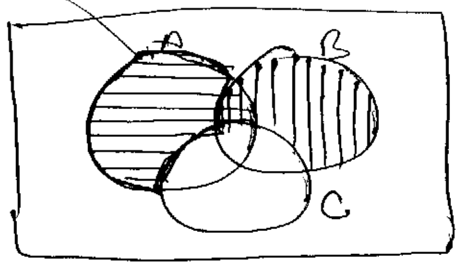
c) – $Y\subset X$

d) – None



X

So, $X = Y$



Y

AB

$\rightarrow (A-B)-C$.

$A-C =$

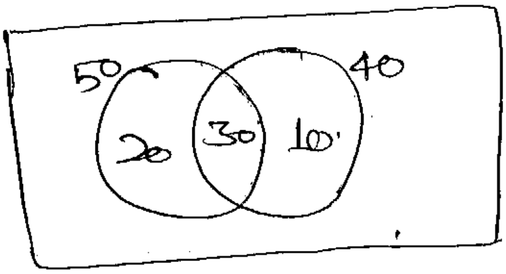
$B-C =$

⇒ Principle of Inclusion - exclusion

$$\boxed{|A\cup B| = |A| + |B| - |A\cap B|}$$

exe $|A| = 50$,
$|B| = 40$,
$|A\cap B| = 30$

$|A\cup B| = 50 + 40 - 30$
$= 50 + 10 = 60$.



$|\text{only } A| = 20$.
$|\text{only } B| = 10$.
$|\text{Exactly one}| = 20 + 10 = 30$.

(*) How many positive integers not exceeding 1000 are divisible by 7 or 11.

Divisible by 7 = A,
Divisible by 11 = B

$|A\cup B| = |A| + |B| - |A\cap B|$.

$|A| = \left\lfloor \frac{1000}{7} \right\rfloor = 142$

$|B| = \left\lfloor \frac{1000}{11} \right\rfloor = 90$

$|A\cap B| = \left\lfloor \frac{1000}{77} \right\rfloor = 12$.

$|A\cup B| = |A| + |B| + |A\cap B|$
$= 142 + 90 - 12 = \underline{220}$

Ques How many binary strings of length 8 start with "0" or end with "11".

Start with "0" $|A| =$ 0 _ _ _ _ _ _ _ $= 2^7$.

end with "11" $|B| =$ _ _ _ _ _ _ 11 $= 2^6$

$|A \cap B| =$ 0 _ _ _ _ _ 11 $= 2^5$

$|A \cup B| = |A| + |B| - |A \cap B|$

$= 2^7 + 2^6 - 2^5$

$= 2^5(2^2 + 2^1 - 2^0)$

$= 2^5(4+2-1) = 2^5 * 5$.

$= 32 \times 5 = 160$.

(*) for 3 sets

$|A \cup B \cup C| = |A| + |B| + |C| - |A \cap B| - |B \cap C| - |A \cap C|$
$+ |A \cap B \cap C|$

(*) How many positive integers not exceeding 123 are divisible by 2, 3, or 5.

$|A \cup B \cup C| = |A| + |B| + |C| - |A \cap B| - |B \cap C| - |C \cap A| + |A \cap B \cap C|$

Divisible by 2 $|A| = \left\lfloor \frac{123}{2} \right\rfloor = 61$

" 3 $|B| = \left\lfloor \frac{123}{3} \right\rfloor = 41$.

" 5 $|C| = \left\lfloor \frac{123}{5} \right\rfloor = 24$.

" 2&3 $|A \cap B| = \left\lfloor \frac{123}{6} \right\rfloor = 20$

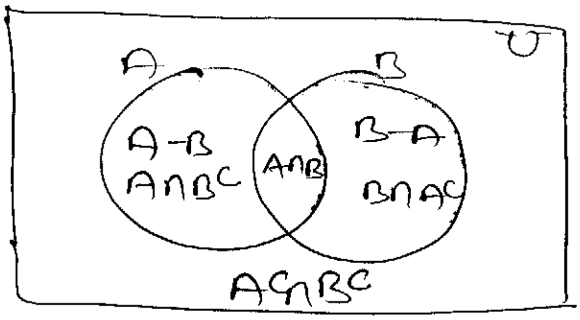
" 3&5 $|B \cap C| = \left\lfloor \frac{123}{15} \right\rfloor = 8$.

" 2&5 $|A \cap C| = \left\lfloor \frac{123}{10} \right\rfloor = 12$

" 2&3&5 $|A \cap B \cap C| = \left\lfloor \frac{123}{30} \right\rfloor = 4$

$|A \cup B \cup C| = 61 + 41 + 24 - 20 - 12 - 8 + 4$

$= 130 - 40 = 90$



$\Rightarrow |A^c \cap B^c| = |(A\cup B)^c|$

$= |U| - |A\cup B|$

Que: How many positive integers not exceeding 1000 are NOT divisible by 7 or 11

Divisible by 7 = A
Divisible by 11 = B

NOT divisible by 7 or 11 $|A^c\cap B^c|$

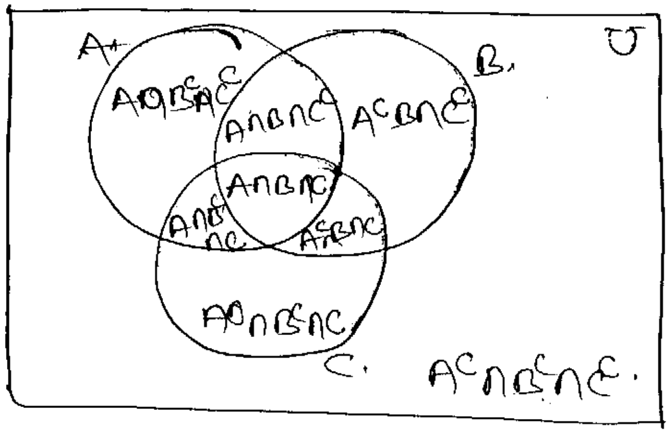
$= |(A\cup B)^c|$

$= U - |A\cup B|$

$= 1000 - 220$

$= 780.$

*:



$|A^c \cap B^c \cap C^c| = |U| - |A\cup B\cup C|.$

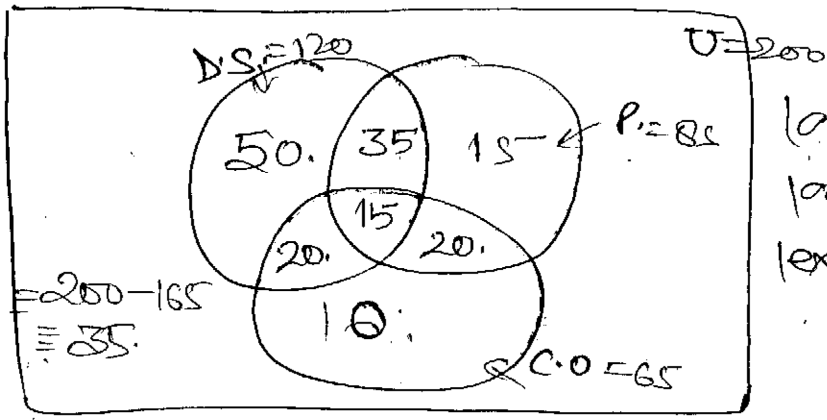
Ques In a class, there are 200 students

120 - D.S.
85 . Prog.
65 . - CO.
50 - D.S. & Prog.
35 - P & CO.
35 - DS & CO.
15 - All 3 courses

How many taken none of the courses.

$|A \cup B \cup C| = 120 + 85 + 65 - 50 - 35 - 35 + 15$
$= 285 - 120$
$= 165$

$|A^c \cap B^c \cap C^c| = 200 - 165 = 35.$



|only DS| = 50
|only DS&P| = 3
|exactly 1| = 75

⇒ Ques- what is the cardinality of the following Set.

$X = \{n \mid 1 \le n \le 123,\ n \in Z^+ \text{ and } n \text{ is NOT divisible by } 2, 3, \text{or } 5\}$.

→ How many positive integers not exceeding 123
⇒ are NOT divisible by 2, 3 or 5.

$|A^c \cap B^c \cap C^c| = |U| - |A \cup B \cup C|$
$= 123 - 90$
$= \underline{\underline{33}}.$

⇒ Derangements :- The arrangement of n objects in such a way that no objects to its natural position

eg :- $(1,2) \rightarrow (2,1) = 1$

$(1,2,3) \rightarrow (3,1,2), (2,3,1) = 2$

* D_n = No of Derangements of n objects.

$$D_n = n! \left[\frac{1}{2!} - \frac{1}{3!} + \cdots + \frac{(-1)^n}{n!}\right]; (n \geq 2)$$

$D_1 = 0$

$D_2 = 1.$

$$D_3 = 3! \left[\frac{1}{2!} - \frac{1}{3!}\right] = \cancel{3!}\left[\frac{3-1}{\cancel{3!}}\right] = 2$$

$$D_4 = 4! \left[\frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!}\right] = \cancel{4!}\left[\frac{4\times3-4+1}{\cancel{4!}}\right]$$

$= 12-4+1 = 9.$

$$D_5 = 5! \left[\frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!} - \frac{1}{5!}\right] = \cancel{5!}\left[\frac{5\times4\times3 - 5\times4 + 5 - 1}{\cancel{5!}}\right]$$

$= 60 - 20 + 5 - 1$

$= 44.$

Ques :- five balls b_1, b_2, b_3, b_4, b_5 are to be kept in 5 cells - C_1, C_2, C_3, C_4, C_5 (each cell can take one ball). How many ways. This can be done so that ball b_i is not in cell C_i ($i = 1,2,3,4,$

⇒ Derangement $= D_5 = 44$ ways

(9) ⇒

4 4 4

$4 \times 4 \times 4 = 64$.

Unsuccesful events = Total - Successful

$= 64 - 1 = 63$

(24) → 10 Roses | 15 sun flowers | 14 dafodils

10 Roses		15 sun flowers	14 dafodils
0	10		
1	9		
2	8		
⋮			
10	0		

↓ 11 ways ↓ 16 ways ↓ 15 way

$= 11 \times 16 \times 15 = \boxed{2640}$

(31) → $^{25}P_{15} = \dfrac{25!}{10!}$

(13) → If every team plays a match with every other team.

Then # of matches $= {}^{n}C_2 = \dfrac{n(n-1)}{2} = 153$

$\Rightarrow n(n-1) = 306$

$n = 18$.

(23) → length of Binary string - $\underline{n+k}$

n-zero can be arranged in 1 way

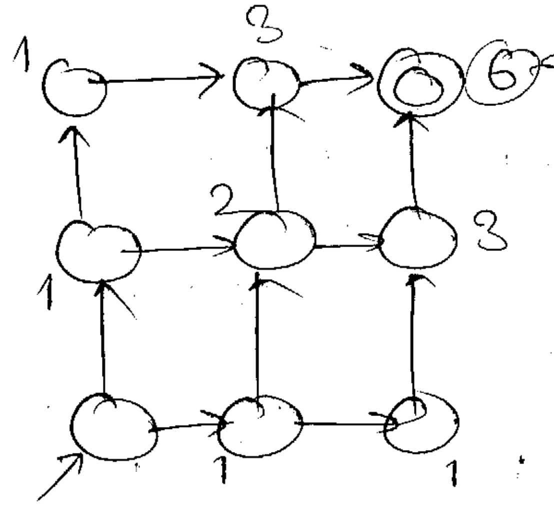
$x\ 0_1\ x\ 0_2\ x\ 0_3\ x - - - - - x\ 0_n\ x$ - n zero

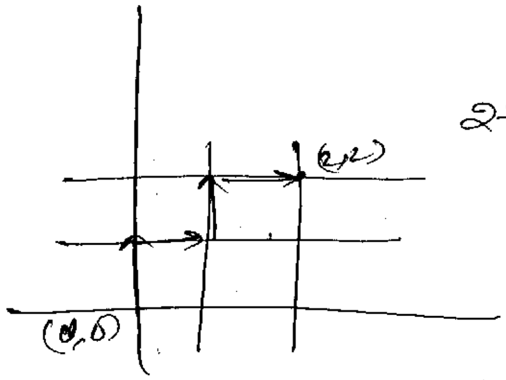
n+1 Gaps

$\Rightarrow 1 * \underline{^{n+1}C_k}$

6! =

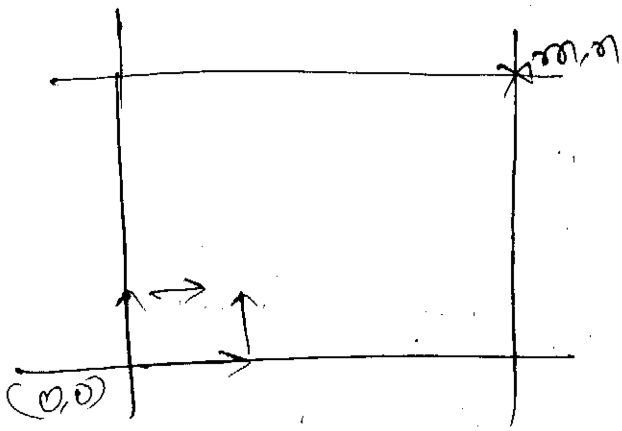
⇒ (33) ≡





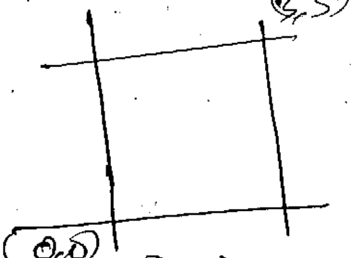
$2+2 \equiv 4$ steps
$2 \uparrow$ steps
$2 \rightarrow$ steps
$\Rightarrow {}^4C_2 = 6$

⇒



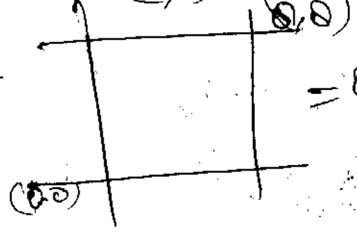
No. of ways to reach from (0,0) to (m,n) is ${}^{m+n}C_m$
$= {}^{m+n}C_n$

⇒ 33) ≡



$\Rightarrow \quad {}^6C_3 = \underline{\underline{20}}$

41) ≡



$= {}^{8+8}C_8 = {}^{16}C_8 =$

9 cellse

⇒ Binomial - 35/23

Set ≡ 4, 7, 8, 27, 25 / 3rd unit

21, 10, 43, 44 / 4th unit

⇒ 23/11/2012

LOGIC

Provides Rules to verify validity of arguments

→ Propositions – Declarative sentence which is either ~~true~~ true, or false, but NOT both.

eg:- 2+2=5 [False].

Delhi is a city [True].

~~This sentence is false~~ [T, F]

*)- P is a proposition variable.

P
T
F

It can take one of the two possible truth values.

$2 \times 2 = 4$.

p	q
T	T
T	F
F	T
F	F

} Truth combn

$2 \times 2 \times 2 = 8$

p	q	r
T	T	T
T	T	F
T	F	T
T	F	F
F	T	T
F	T	F
F	F	T
F	F	F

} Truth combn

⇒ If there are n propositional variables. The no. of truth combinations = 2^n.

$$\underbrace{\begin{matrix} p_1 & p_2 & \cdots & p_n \\ \downarrow & \downarrow & & \downarrow \\ 2 & 2 & & 2 \end{matrix}}_{n \text{ times}} = 2^n$$

→ Propositions can be combined using connectives (logical operators) to form compound propositions.

→ Five fundamental connectives

Negation $\sim, \neg$
Conjunction $\wedge$
Disjunction $\vee$
Implication $\rightarrow, \Rightarrow$
Biimplication $\leftrightarrow$

(I) → Negation: "not p"

p	$\neg p$
T	F
F	T.

Truth Table – Table containing truth value for all possible truth combinations.

(II) → Conjunction: "AND"
"$p \wedge q$" is true when both p and q are true.

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

*) – "but" can be used instead of "AND" in logic somewhere

→ { Sunshining, but it is raining.
{ Sun shining and it is raining
$\underbrace{\text{Sun shining}}_{p}$ and $\underbrace{\text{it is raining}}_{q}$

$p \wedge q$.

→ "Jack and Jill went up the hill" → Jack went up the hill, Jill went up the hill
$p \wedge q$.

→ Jack and Jill are cousins.
↓ × Not connective.

*) Ques – How many lines the following proposition is true

$\neg P_1 \wedge P_2 \wedge P_3 \wedge P_4 \wedge P_5$

✓ a) → 1 b) – 2 c) – 31 d) – 32.

P_1	P_2	P_3	P_4	P_5	$\neg P_1 \wedge P_2 \wedge P_3 \wedge P_4 \wedge P_5$
F	T	T	T	T	T.

(III) - Disjunction - "OR"

"$p \vee q$ is false when both p and q are false"

p	q	p∨q
T	T	T
T	F	T
F	T	T
F	F	F

ex: $p \vee (q \wedge r)$.

p	q	r	$(q \wedge r)$	$p \vee (q \wedge r)$
T	T	T	T	T
T	T	F	F	T
T	F	T	F	T
T	F	F	F	T
F	T	T	T	T
F	T	F	F	F
F	F	T	F	F
F	F	F	F	F

Is $p \vee (q \wedge r)$ always True = (NO) -

p	q	r
F	F	F
F	F	T
F	T	F

(IV) → Implication (Conditional) "→ implies"

"$p \to q$" A true statement cannot imply false" -

p	q	p→q
T	T	T
T	F	F
F	T	T
F	F	T

⇒ $p \to q$
when it is false?

check T → F Take it false

not check possible cannot any put it always true

①

p	q	r	$(p \vee q) \to r$
T	T	F	F
T	F	F	F
F	T	F	F

NO

② $(p \vee q) \to p$ is it always true? No

p	q	$(p \vee q) \to p$
F	T	F

③ $(p \wedge q) \to p$ is it always True?

p	q	$(p \wedge q) \to p$
F		

But $(p \wedge q)$ can't be T for any p to F
So it is always True

→ Different forms of an implication $p \to q$.

→ p implies q
→ if p then q.
→ p only if q. (≡) If not q then NOT p.
→ q if p.
→ q follows from p.
→ q unless $\neg p$. ≡ q if NOT ($\sim p$)
   (unless = if NOT)
   if $\sim(\neg p)$ then q
   if p, then q.

(V) → Bi-implication (Double implication)
Double conditional "$\leftrightarrow$"

$p \leftrightarrow q$: p iff q

"$p \leftrightarrow q$ is true when both p and q have the same truth value".

p	q	$p \leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

p	q	$(p \wedge q)$	$(p \wedge q) \to p$
T	T	T	T
T	F	F	T
F	T	F	T
F	F	F	T.

⇒ Tautology ∈ "T" A proposition which is always true. Denoted by "T"

eg:- $p \vee \neg p$

p	$p \vee \neg p$	$\neg p$
T	T	F
F	T	T

Tautology

⇒ Contradiction (absurdity) ∈ A proposition which is always false.

eg:- $P \wedge \neg P$

Reasoning: one of P and ¬P are always f.

⇒ Satisfiable ∈ A proposition which is true for at least one combination of truth values.

→ ** Every Taut. is satisfiable.
But every satisfiable need not be Tautology.

eg. $(P \vee q) \rightarrow P$

(a) - Tautology
(b) - Contradiction
(c) - Satisfiable
(d) - None

P	q	$(P\vee q) \rightarrow P$
F	T	F So, NOT Tautology
T	—	T. So, NOT contradi... But satisfiable

* Alternative Names

*** Tautology (T) → valid.
Contradiction (F) → Unsatisfiable.
Neither T or F → Satisfiable but NOT valid (Contingency)

ex:-

P	q	$\neg(P \vee q)$ x	$\neg P \wedge \neg q$ y	$x \leftrightarrow y$
T	T	F	F.	T
T	F	F	F	T
F	T	F	F	T
F	F	T	T	T.

Same Truth values. So equivalent.

→ Equivalent: Two propositions P and q are equivalent if they have same truth table. Denoted by $P \equiv q$.

Result:

$\boxed{P \equiv q \text{ iff } P \leftrightarrow q \text{ is tautology.}}$

→ Dual: Dual of a compound proposition having connectives $\sim, \wedge, \vee$ is obtained by replacing

each $\wedge$ with $\vee$,
each $\vee$ with $\wedge$,
each T with F,
each F with T.

ex: $P \vee q$
Dual $= P \wedge q$.

ex: $P \vee (q \wedge r)$
$P \wedge (q \vee r)$.

→ Result

i) → S is a compound proposition and S^* is dual of S.
Then $(S^*)^* = S$.

ii) → $A \equiv B$.
$A^* \equiv B^*$.

*) Forms of Equivalence

⇒ I → Equivalence with $\sim, \wedge, \vee$.

1) → Idempotent:
$P \vee P \equiv P$.
$P \wedge P \equiv P$.

2) → Identity: $P \vee F \equiv P$
$P \wedge T \equiv P$.

3) Domination:

$$p \vee T \equiv T$$
$$p \wedge F \equiv F$$

4) Negation:

$$p \vee \neg p \equiv T$$
$$p \wedge \neg p \equiv F$$

5) Double Negation:

$$\neg(\neg p) \equiv p.$$

6) Commutative:

$$p \vee q \equiv q \vee p$$
$$p \wedge q \equiv q \wedge p.$$

7) Associative:

$$p \vee (q \vee r) \equiv (p \vee q) \vee r$$
$$p \wedge (q \wedge r) \equiv (p \wedge q) \wedge r.$$

8) Absorption:

$$p \vee (p \wedge q) \equiv p.$$
$$p \wedge (p \vee q) \equiv p.$$

9) De-morgan's:

$$\neg (p \vee q) \equiv \neg p \wedge \neg q.$$
$$\neg (p \wedge q) \equiv \neg p \vee \neg q.$$

10) Distributive:

$$p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r).$$
$$p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r).$$

Ans → $(p \vee q) \wedge (\neg p \wedge \neg q)$

$(p \vee q) \wedge \neg (p \vee q)$ using De-morgan's

$x \wedge \neg x \equiv F.$ using Negation.

→ Well formed formula :-

1.) Every propositional variable P is WFF.

2.) If P is WFF, then $(\neg(P))$

3.) If P and q are WFF, then $(P \wedge q)$, $(P \vee q)$, $(P \rightarrow q)$, $(P \leftrightarrow q)$ are also WFF.

4.) only formulas formed using steps 1, 2, 3 are WFF.

→ Operator Precedence

Highest ↑
1. $\neg$
2. $\wedge$
3. $\vee$
4. $\rightarrow$
5. $\leftrightarrow$

Ques :- $P \rightarrow q \wedge r$.

a) :- $P \rightarrow (q \wedge r)$

b) :- $(P \rightarrow q) \wedge r$.

c) :- Both

d) :- None

→ II :- Equivalence involving implications :-

1.) → Law of implication :-

$$\boxed{P \rightarrow q \equiv \neg P \vee q}$$

verify

P	q	$P \rightarrow q$	$\neg P \vee q$
T	F	F	F.

So equivalent.

$\underset{\text{Negate}}{\ }$ $\underset{\to \text{ to } \vee}{\ }$

1.> $p \to q \equiv \neg p \vee q$. leave it as it is.

2.> $\neg p \to q \equiv p \vee q$.

3.> $p \to \neg q \equiv \neg p \vee \neg q$

4.> $\neg p \to \neg q \equiv p \vee \neg q$

5.> $p \vee q \equiv \neg p \to q$

6.> $\neg p \vee q \equiv p \to q$

7.> $p \vee \neg q \equiv \neg p \to \neg q$

8.> $\neg p \vee \neg q \equiv p \to \neg q$.

<u>Ques</u> $p \to q \equiv ?$ $= \boxed{\neg(\neg(\neg p \vee q)) \equiv \neg(p \wedge \neg q)}$

a)→ $\neg(\neg p \wedge q)$

b)→ $\neg(p \wedge q)$

c)→ $\neg(\neg p \wedge \neg q)$

d)→ $\boxed{\neg(p \wedge \neg q)}$

9). $p \to q \equiv \neg(p \wedge \neg q)$ 1st Negate whole, write p as it is, change → to ∧, make q to ¬q i.e. Negate

10). $p \to \neg q \equiv \neg(p \wedge q)$.

11). $\neg p \to q \equiv \neg(\neg p \wedge \neg q)$

12). $\neg p \to \neg q \equiv \neg(\neg p \wedge q)$

13)→ $p \wedge q \equiv \neg(p \to \neg q)$

14)→ $\neg p \wedge q \equiv \neg(\neg p \to \neg q)$

15)→ $p \wedge \neg q \equiv \neg(p \to q)$

16)→ $\neg p \wedge \neg q \equiv \neg(\neg p \to q)$

2)→ <u>law of contrapositives</u> $\boxed{p \to q \equiv \neg q \to \neg p}$

Implication : $p \to q$.

Converse : $q \to p$

Inverse : $\neg p \to \neg q$.

Contrapositive : $\neg q \to \neg p$.

an implication is always equivalent to its contrapositive

Exe ~~Ex~~

Implication: If $\underset{p}{\text{it is cold}}$, then $\underset{q}{\text{I stay at home}}$

p:→ it is cold
q:→ I stay at home
$\therefore p \to q$

Converse:- $q \to p$;
If I stay at home, then it is cold.

Inverse:- $\neg p \to \neg q$;
If it is NOT cold, then I donot stay at hom

Contrapositive:- $\neg q \to \neg p$;
If I don't stay at home, then it is NOT cold.

→ verification

p	q	$p \to q \equiv$	$\neg q \to \neg p$
T	F	F	F

So equivalent

$$\boxed{q \to p \equiv \neg p \to \neg q}$$

Converse is equivalent to its inverse.

3) → Exportation Laws

$$\boxed{p \to (q \to r) \equiv (p \wedge q) \to r.}$$

Proof: $p \to (q \to r)$ Given

$\equiv \neg p \vee (q \to r)$ Law of implication.

$\equiv \neg p \vee (\neg q \vee r)$ implication.

$\equiv (\neg p \vee \neg q) \vee r$ — associative.

$\equiv \neg (p \wedge q) \vee r$ De morgan's

$(p \wedge q) \to r$ implication.

4) →

I :- $(p \to q) \wedge (p \to r) \equiv p \to (q \vee r)$.

II :- $(p \to q) \vee (p \to r) \equiv p \to (q \wedge r)$.

a) only I true

b) only II true

c) Both True

d) Both false

I) - $(p \to q) \wedge (p \to r)$

$\equiv (\neg p \vee q) \wedge (\neg p \vee r)$ law of implication

$\equiv \neg p \vee (q \wedge r)$ Distributive.

$\equiv p \to (q \wedge r)$ implication

(II) - $(p \to q) \vee (p \to r)$ ~~law~~

$\equiv (\neg p \vee q) \vee (\neg p \vee r)$. law of implication

$\equiv \neg p \vee (q \vee r)$. Distributive

$\equiv p \to (q \vee r)$ implication.

**

$$\boxed{\begin{array}{l} \text{I}- (p \to q) \wedge (p \to r) \equiv p \to (q \wedge r). \\ \text{II}- (p \to q) \vee (p \to r) \equiv p \to (q \vee r). \end{array}}$$

(5) :-
$$\boxed{\begin{array}{l} \text{I}- (p \to r) \wedge (q \to r) \equiv (p \vee q) \to r \\ \text{II}- (p \to r) \vee (q \to r) \equiv (p \wedge q) \to r \end{array}}$$

Soln :- (I) :- verification

$(\neg p \vee r) \wedge (\neg q \vee r) \equiv (\neg p \wedge \neg q) \vee r$ Distri.

$\equiv \neg (p \vee q) \vee r$ Demorgans

$\equiv (p \vee q) \to r$. implication.

→ (III) Equivalence ~~Implications~~ involving ~~bi-implications~~

1.) *** $\boxed{p \leftrightarrow q \equiv (p \to q) \wedge (q \to p)}$

$p \leftrightarrow q$ means
"p is necessary & sufficient for q"

2.) ** $$\boxed{\begin{aligned} p \leftrightarrow q &\equiv (p \to q) \wedge (q \to p) \\ &\equiv (\neg p \vee q) \wedge (\neg q \vee p) \end{aligned}}$$

3.) ** $\boxed{p \leftrightarrow q \equiv \neg p \leftrightarrow \neg q}$

* → Find $\neg(p \leftrightarrow q)$. (Express in $\vee, \wedge, \neg$ only).

$$\begin{aligned} &\neg(p \leftrightarrow q) \\ \equiv\ &\neg((p \to q) \wedge (q \to p)) \\ \equiv\ &\neg((\neg p \vee q) \wedge (\neg q \vee p)) \\ \equiv\ &\neg(\neg p \vee q) \vee \neg(\neg q \vee p) \\ \equiv\ &(p \wedge \neg q) \vee (q \wedge \neg p). \end{aligned}$$

⇛ Functionally Complete Sets

A set of connectives is said to be functionally complete if every compound proposition can be expressed in terms of these connectives. $\equiv \{\neg, \wedge, \vee\}$. $\equiv$

Functionally complete set:
- $\equiv \{\neg, \wedge\}$
- $\equiv \{\neg, \vee\}$.

→ minimal functionally complete set
$\equiv$ No proper subset of this set is functionally complete

NOT functionally complete set $\equiv \{\vee, \wedge\}$

⇛ Other Connectives:

→ Exclusive "OR", $\bar{\vee}$, $\oplus$

"$p \bar{\vee} q$ is true when any one of them is true, but Not both."

p	q	$p \bar{\vee} q$
T	T	F
T	F	T
F	T	T
F	F	F

***

$$\boxed{\neg(p \leftrightarrow q) \equiv p \bar{\vee} q}$$

→ "NAND" - "NOT and" ($\uparrow$) or ($|$) Sheffer Stroke.

p	q	$p \uparrow q$
T	T	F
T	F	T
F	T	T
F	F	T

$$\boxed{(p \uparrow q) \equiv \neg(p \wedge q)}$$

Results:

(i) → $(p \uparrow p) = \neg(p \wedge p)$

$= \neg p$

(ii) → $(p \uparrow q) \uparrow (p \uparrow q) \equiv \neg(\neg(p \wedge q) \wedge \neg(p \wedge q))$

$\equiv \neg(\neg(p \wedge q)) \equiv (p \wedge q)$

(iii) → $(p \uparrow p) \uparrow (q \uparrow q) \equiv \neg(\neg(p \wedge p) \wedge \neg(q \wedge q))$

$\equiv \neg(\neg p \wedge \neg q)$

$\equiv (p \vee q)$

($\uparrow$) is functionally complete as ($\neg, \wedge, \vee$) can be derived from $\uparrow$.

→ NOR (Not OR) "↓" pierce arrow.

P	q	P↓q
T	T	F
T	F	F
F	T	F
F	F	T

$(P \downarrow q) \equiv \neg(P \vee q)$

⇒ $(P \downarrow P) \equiv \neg P$.

$(P \downarrow q) \downarrow (P \downarrow q) \equiv P \vee q$.

$(P \downarrow P) \downarrow (q \downarrow q) \equiv P \wedge q$.

as $\neg, \vee, \wedge$ can be derived from ↓.
So, (↓) is also functionally complete.

⇒ Smallest minimal functionally complete set.
{↑}
{↓}.

26/11/2012

⇒ Logical Implication - An implication which is tautology.

Ex:- $P \rightarrow (P \vee q)$

P	q	$P \rightarrow (P \vee q)$
F	F.	P

$P \vee q \equiv F$
So, $P \equiv F$
$q \equiv F$

Now, P can't be True

So T→F never possible
So, tautology.

To prove tautology
$\boxed{P \rightarrow Q}$
Take Q as false
P can't be true
⇒ Never get T→F situation
So, tautology

⇒ General form of argument: (Inference):
$p \rightarrow q$

antecedent — consequent.
hypothesis — conclusion.
premise — "

(conjunction of premises) $\longrightarrow$ conclusion

$$\boxed{(P_1 \wedge P_2 \wedge \cdots \wedge P_n) \longrightarrow q}$$

→ An inference which is tautology, is called valid inference.
otherwise it is invalid inference.

Any valid inference is called rule of inference.

→ Some important Rules of Inference:

Name / Form	Tautological form / Proof
1) Conjunction: p q } premises $\therefore p \wedge q$ } conclusion $p, q \vdash p \wedge q$.	$p \wedge q \longrightarrow (p \wedge q)$.
2) Simplification: $p \wedge q$ $\therefore p$ $p \wedge q \vdash p$,	$(p \wedge q) \rightarrow p$.
3) Addition: p $\therefore p \vee q$	$p \rightarrow (p \vee q)$

4) - Modus Ponens
(Rule of Detachment)

$p \to q$
p detachment

$\therefore q$

$p \to q, p \vdash q$

Try to make T.

$[\overset{T}{(p \to q)} \wedge \overset{T}{p}] \to q$

can't be true

So tautology

Take F
$q \equiv F$

5) - Hypothetical Syllogism
(Transitive Rule)

$p \to q$
$q \to r$

$\therefore p \to r$

Try to make it T

$[\overset{T}{(p \to q)} \wedge \overset{T}{(q \to r)}] \to p \to r$

So it can't be true.
So, This combn NOT possible
So, tautology.

Take F
$p \equiv T$
$r \equiv F$

6) - Modus Tollens

$p \to q$
$\sim q$

$\therefore \sim p$

Proof

1) $p \to q$ Given
2) $\sim q \to \sim p$ contrapositive
3) $\sim q$

$\therefore \sim p$ using modus Ponens on 2, Proved

$[(p \to q) \wedge \sim q] \to \sim p$

ex:

I_1: If it rains, then the cricket match will not be played
The cricket match was played

There was no rain

I_2: If it rain, then cricket match will not be played
It didn't rain

$\therefore$ cricket match was played.

a) → only I_1 True
b) → only I_2 true
c) → both true
d) - None true

$\therefore p$: it rains
q : cricket match was played.

D_1: $p \to \sim q$ Method 1

$$\begin{array}{l} p \to \sim q \\ q \\ \hline \therefore \sim p \end{array}$$

D_1: $[(p \to \neg q) \wedge q] \to \sim p$ — make T, make T, False, $p \equiv T$.

$T \to F = F$, $F \wedge T = F$

can't be true
So tautology

D_2: $p \to \sim q$

$$\begin{array}{l} p \to \sim q \\ \sim p \\ \hline \therefore q \end{array}$$

D_2: $((p \to \neg q) \wedge \neg p) \to q$ — make T, make T, F

p-F, $T \wedge T$, $\to F$, $T \to F$

So, it can be true. So NOT tautology.

Method 2 :-

D_1 :-
$$\left.\begin{array}{l} p \to \neg q \\ q \\ \hline \therefore \neg p \end{array}\right]$$ modus Tollens $\equiv$ true

D_2 -
$$\left.\begin{array}{l} p \to \sim q \\ \sim p \\ \hline \therefore q \end{array}\right]$$ Fallacy $\equiv$ invalid inference which resembles ~~...~~ valid inference

~~It resembles~~ ~~...~~ modus Tollens. But it is NOT so, it is invalid.

Fallacy :- An invalid inference which resembles a valid inference, is called fallacy.

→ Fallacy of affirming consequent

$$\left.\begin{array}{l} p \to q \\ q \\ \hline \therefore p \end{array}\right\}$$ INVALID.

→ Fallacy of ~~...~~ denying antecedent

$$\left.\begin{array}{l} p \to q \\ \sim p \\ \hline \therefore \sim q \end{array}\right\}$$ INVALID.

7.) Disjunctive Syllogism $[(p \vee q) \wedge \neg p] \rightarrow q$

$$\begin{array}{l} p \vee q \\ \neg p \\ \hline \therefore q \end{array}$$

Proof:-

1.) $p \vee q$ Given

2.) $\neg p \rightarrow q$ law of implication

3.) $\neg p$ } modus ponens

$\therefore q$ proved

8.) Conditional Syllogism

$$\begin{array}{l} p \rightarrow (q \rightarrow r) \\ p \wedge q \\ \hline \therefore r \end{array}$$

1.) $p \rightarrow (q \rightarrow r)$ Given

2.) $(p \wedge q) \rightarrow r$ Exportation law

3.) $(p \wedge q)$ Given

4.) r modus ponens using 2, 3.

Method:-

1.) $p \wedge q$ Given

2. $\therefore p$ simplification of 1

3.) $\therefore q$ " 1

4.) $p \rightarrow (q \rightarrow r)$ modus ponens 2, 4

5.) $q \rightarrow r$ modus ponens 3, 5.

6.) r

9.) Resolutions

$$\begin{array}{l} p \vee q \\ \neg p \vee r \\ \hline \therefore q \vee r \end{array}$$

Proof make true

$[(p \vee q) \wedge (\neg p \vee r)] \rightarrow (q \vee r)$

make T make T

$(T \vee F)$ $(F \vee F)$

$T \wedge F$

F

Take F $q \equiv F$ $r \equiv F$

Can't be True, So, tautology.

method 2 1.) $p \vee q$ Given

2.) $q \vee p$ Commutative

3.) $\neg q \rightarrow p$ implication on 2

4.) $\neg p \vee r$ Given

5.) $p \rightarrow r$ implication on 4

6.) $\neg q \rightarrow r$ using hypothetical syllogism on 3, 5

7.) $q \vee r \equiv$ implication

Proved

10) → Disjunctive Elimination:

$p \vee q$
$p \rightarrow r$
$q \rightarrow r$
$\therefore r$

reading DM or DS.
If read DM., then I get 10% marks
If read DS., then I get 10% marks
∴ I get 10% marks.

Proof
1) $p \rightarrow r$ Given
2) $q \rightarrow r$ Given
3) $(p \rightarrow r) \wedge (q \rightarrow r)$ conjunction
4) $(p \vee q) \rightarrow r$
5) $(p \vee q)$ Given
6) $\therefore r$ modus ponens on 4, 5.

11) → Constructive Dilemma:

$(p \rightarrow q) \wedge (r \rightarrow s)$
$p \vee r$
$\therefore q \vee s$.

(cold then go to movie)
∧ (raining then go to class)
It is cold or It is raining
I go to movie or go to class.

Proof:
1) $(p \rightarrow q) \wedge (r \rightarrow s)$
2) $(p \rightarrow q)$ simplification
3) $(r \rightarrow s)$ "
4) $\sim q \rightarrow \sim p$ contrapositive
5) $p \vee r$ Given
6) $\sim p \rightarrow r$ implication
7) $\sim q \rightarrow r$ Transitive 4, 6
8) $r \rightarrow s$ 3.
9) $\sim q \rightarrow s$ Transitive rule 7, 8
10) $q \vee s$

12) → Destructive Dilemma:

$(p \rightarrow q) \wedge (r \rightarrow s)$
$\sim q \vee \sim s$
$\therefore \sim p \vee \sim r$.

Proof
1) $p \rightarrow q$ Given
2) $q \rightarrow \neg s$ implication of given
3) $\neg s \rightarrow \neg r$ contrapositive of given
$p \rightarrow \neg r$
$\sim p \vee \sim r$ Proved

Ques: which of the following is NOT a tautology

Proof is easy by Rule & Disproof is easier by Truth table

1.) $(p \wedge q) \rightarrow p \vee q$

2.) $\neg p \rightarrow (p \rightarrow q)$

3.) $p \rightarrow (p \wedge q)$.

4.) $q \rightarrow (p \vee q)$

5.) $q \rightarrow (p \rightarrow q)$

6.) $p \vee (q \rightarrow p)$.

1)→ $p \wedge q \rightarrow p$ Simplification

$\underline{p \rightarrow p \vee q}$ Disjunction (Addition).

$\therefore (p \wedge q) \rightarrow (p \vee q)$ using transitive Tautology

2)→ $\neg p \rightarrow (p \rightarrow q)$

$\neg p \rightarrow (\neg p \vee q) \Rightarrow$ Addition ~~Disjunction~~

$(p \rightarrow q)$ Implication

So tautology

3)→ $p \rightarrow (p \wedge q)$ $\boxed{(p \wedge q) \rightarrow p}$ Simplification Rules of Inference are one-sided.

So it is false

So NOT tautology

$p \rightarrow (p \wedge q)$ Make false

$p \rightarrow T$ $p - T$

$q \rightarrow F$

$T \rightarrow F$ So getting a false situation

So NOT tautology

(4)→ $q \rightarrow (p \vee q)$ Addition property

(5)→ $q \rightarrow (p \rightarrow q)$

$q \rightarrow (\neg p \vee q) \equiv (q \wedge \neg p) \equiv$ Addition

So tautology

6) → $p \vee (q \to p)$ make F.

p F F

$p \vee (q \to p)$ F

$p \equiv F$ T F

F $\vee$ F

F So NOT tautology

x

__WB__

2) → This sentence is true $\begin{pmatrix}T\\T\end{pmatrix}, \begin{pmatrix}F\\F\end{pmatrix}$ } unique value so proposition

This sentence is false $\begin{pmatrix}T\\F\end{pmatrix}, \begin{pmatrix}F\\T\end{pmatrix}$ = NOT unique. So NOT proposition

⇒ Predicate (open proposition) :- A predicate is a proposition except for the fact that it contains variables whose values are to be taken from some universe of discourse.

eg:-

		Notation
	$x+2=5$.	: $E(x)$
	$x+y \leq 7$.	: $L(x,y)$
	x loves y.	: loves(x,y).

D(x) : x is a dog

P(x) : —— 1 place predicate.
P(x,y) : —— 2 " ".
P(x,y,z) : —— 3 " ".
⋮
$P(x_1, x_2 \ldots x_n)$ —— n " ".

⇒ Predicate to proposition :-

1) - Substitution.
2) - Quantification.

1.) → Substitution :-

Ex:- $U = \{1,2,3,4,5,6,7\}$

$P(x)$: $x+2 \leq 7$
$P(1)$: $1+2 \leq 7$ [T]
$P(3)$: $3+2 \leq 7$ [T]
$P(6)$: $6+2 \leq 7$ [F]

2.) - Quantification :-

Some ex:- For every x, P(x) is true [F].
For some x, P(x) is true [T].

eg:- $U = \{1, 2, 3\}$
$P(x): x+3 \le 6$
for every x $P(x)$ is true [T]

$U = \{1, 2, 3, 4\}$
$P(x): x+3 \le 6$
for every x $P(x)$ is true [F].

So, truth values vary with variation in universe discourse as well as in problem.

⇒ Universal quantifier → "for all" $\forall$
Existential quantifier "there exists", "for some" $\exists$.

For every x $P(x)$ is true $\forall x\, P(x)$.
For some x $P(x)$ is true $\exists x\, P(x)$.

⇒

Form	Meaning
1) → $\forall x\, P(x)$	all true.
2) → $\exists x\, P(x)$	Some true. (or) at least one true.
3) → $\neg \forall x\, P(x)$	Not all true.
4) → $\neg \exists x\, P(x)$ $\forall x \neg P(x)$	None True.
5) → $\forall x \neg P(x)$	All false.
6) → $\exists x \neg P(x)$	Some false (or) atleast one false.
7) → $\neg \forall x \neg P(x)$	Not all false.
8) → $\neg \exists x \neg P(x)$	None false

① ≡ ⑧
② ≡ ⑦
③ ≡ 6
④ ≡ ⑤

equivalent forms

① $\forall x\, P(x) \equiv \sim \exists x \sim P(x)$.
② $\exists x\, P(x) \equiv \sim \forall x \sim P(x)$.
③ $\neg \forall x\, P(x) \equiv \exists x \sim P(x)$.
④ $\neg \exists x\, P(x) \equiv \forall x \sim P(x)$

Negating quantifiers.

$\sim \forall x \equiv \exists x \sim$
$\sim \exists x \equiv \forall x \sim$.

⇒ Negate the following quantified predicate
$\forall x [P(x) \rightarrow Q(x)]$.

Q1 ⇒ $\sim \forall x \, [P(x) \rightarrow Q(x)]$

$\exists x \sim [P(x) \rightarrow Q(x)]$

$\exists x \sim [\sim P(x) \vee Q(x)]$ (implication)

$\exists x \, [P(x) \wedge \sim Q(x)]$ (De-morgan's)

Q2 ⇒ $\sim \exists x \, [P(x) \wedge Q(x)]$.

$\forall x \sim [P(x) \wedge Q(x)]$

$\forall x \, [\sim P(x) \vee \sim Q(x)]$. (De-morgan's).

$\forall x \, [P(x) \rightarrow \sim Q(x)]$.

... into symbolic form

⇒ Negate the following quantified predicate

$$\forall x [P(x) \rightarrow Q(x)].$$

<u>Q1</u> ⇒ $\sim \forall x [P(x) \rightarrow Q(x)]$

$\exists x \sim [P(x) \rightarrow Q(x)]$

$\exists x \sim [\sim P(x) \vee Q(x)]$ (implication)

$\exists x [P(x) \wedge \sim Q(x)]$ (De-morgan's)

<u>Q2</u> ⇒ $\sim \exists x [P(x) \wedge Q(x)]$.

$\forall x \sim [P(x) \wedge Q(x)]$

$\forall x [\sim P(x) \vee \sim Q(x)]$. (De-morgan's).

$\forall x [P(x) \rightarrow \sim Q(x)]$.

... symbolic form

⇒ Negate the following quantified predicate

$$\forall x [P(x) \longrightarrow Q(x)].$$

Q1 ⇒ $\sim \forall x \; [P(x) \longrightarrow Q(x)]$

$\exists x \sim [P(x) \rightarrow Q(x)]$

$\exists x \sim [\sim P(x) \vee Q(x)]$ (implication)

$\exists x \; [P(x) \wedge \sim Q(x)]$ (De-morgan's)

Q2 ⇒ $\sim \exists x \; [P(x) \wedge Q(x)]$.

$\forall x \sim [P(x) \wedge Q(x)]$

$\forall x \; [\sim P(x) \vee \sim Q(x)]$. (De-morgan's).

$\forall x \; [P(x) \longrightarrow \sim Q(x)]$.

symbolic form

eg:- Some students in this class visited Hyderabad.
→ V(x) : x visited hyderabad
U ≡ {~~set of~~ students}

$\boxed{\exists x\, V(x)}$.

→ U ≡ {set of all people}
S(x) :- x is a student
V(x) :- x is visited hyderabad.

for some x, x is student and x visited hyderabad.

$\boxed{\exists x [S(x) \wedge V(x)]}$. $\cancel{\exists x [S(x) \rightarrow V(x)]}$

as it may be true when x (F) the result V(x) not be true

So, in general,
∀ is followed by "→".
∃ is followed by "∧".

→ Aristotle forms
1) → All P's are Q's
$\forall x [P(x) \rightarrow Q(x)]$

2) → Some P's are Q's
$\exists x [P(x) \wedge Q(x)]$.

3. → Not all P's are Q's
$\cancel{\exists x [P(x) \wedge \sim Q(x)]}$
$\sim \forall x [P(x) \rightarrow Q(x)]$
$\exists x [P(x) \wedge \sim Q(x)]$.
i.e. Some P's are NOT Q's

1→ No P's are Q's.

$$\sim \exists x \, [P(x) \wedge Q(x)]$$

$$\equiv \forall x \, [P(x) \rightarrow \sim Q(x)].$$

(P's are NOT Q's)

Ques Gold and silver ornaments are precious.

$G(x)$: x is gold ornament

$S(x)$: x is silver "

$P(x)$: x is precious.

a)→ $\forall x \, [P(x) \rightarrow [G(x) \wedge S(x)]]$.

b)→ $\forall x \, [[G(x) \wedge S(x)] \rightarrow P(x)]$.

c)→ $\exists x \, [[G(x) \wedge S(x)] \rightarrow P(x)]$.

d)→ $\forall x \, [[G(x) \vee S(x)] \rightarrow P(x)]$.

→ Gold ornaments are precious and silver ornaments are precious.

$$\forall x \, [[G(x) \rightarrow P(x)] \wedge [S(x) \rightarrow P(x)]]$$

$$\forall x \, [[G(x) \vee S(x)] \rightarrow P(x)]$$

Ques WB/53/7.

All purple mushrooms are poisonous.

purp(x) mush(x) pois(x)

$$\forall x \, [[\mathrm{Purp}(x) \wedge \mathrm{mush}(x)] \rightarrow P(x)]$$

Ques: Every connected and acyclic graph is tree.

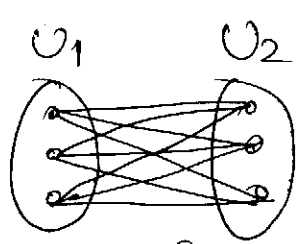
$G(x)$: x is connected
$S(x)$: x is acyclic
$P(x)$: x is tree

$\forall x\,[\,[G(x) \land S(x)] \rightarrow P(x)\,]$.

2-place predicates

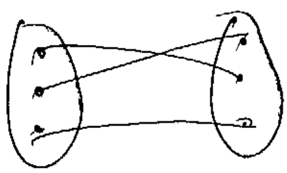
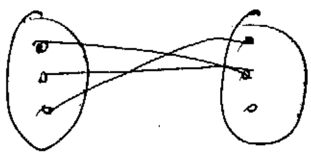
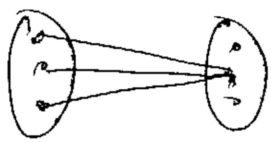
eg:- $L(x,y)$: x loves y.

I → $\forall x\,\forall y\,L(x,y)$:
Every1 loves every body.

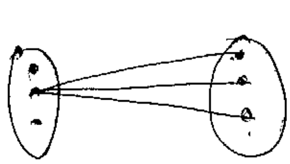


$\forall x\,\forall y$

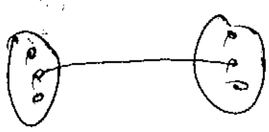
II → $\forall x\,\exists y\,L(x,y)$: Everyone loves somebody.

 (or)  or 

III → $\exists x\,\forall y\,L(x,y)$: Someone loves everybody



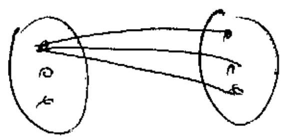
IV → $\exists x\,\exists y\,L(x,y)$: Someone loves somebody

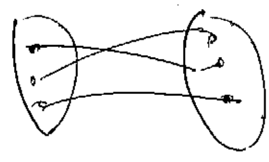


$\boxed{L(x,y) \neq L(y,x)}$ i.e. Both statements are different

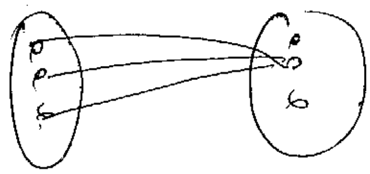
5)- $\forall y \forall x\ L(x,y) \equiv$ Everybody loved by everyone

6)- $\forall y \exists x\ L(x,y) \equiv$ Everybody loved someone.





7)- $\exists y \forall x\ L(x,y) \equiv$ Somebody loved by everyone

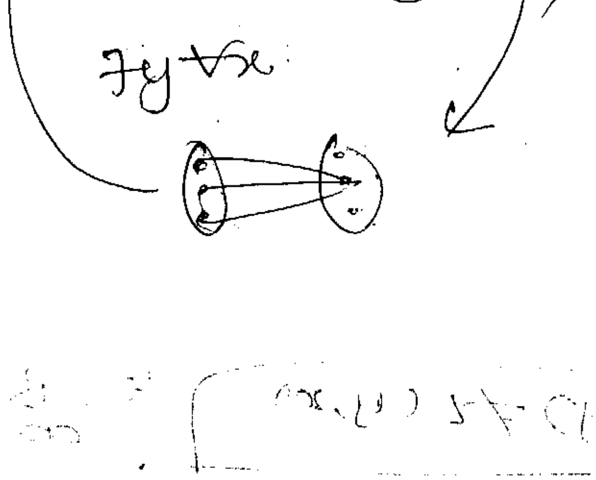


8)- $\exists y \exists x\ L(x,y) \equiv$ Somebody loved by someone.

27/11/2012

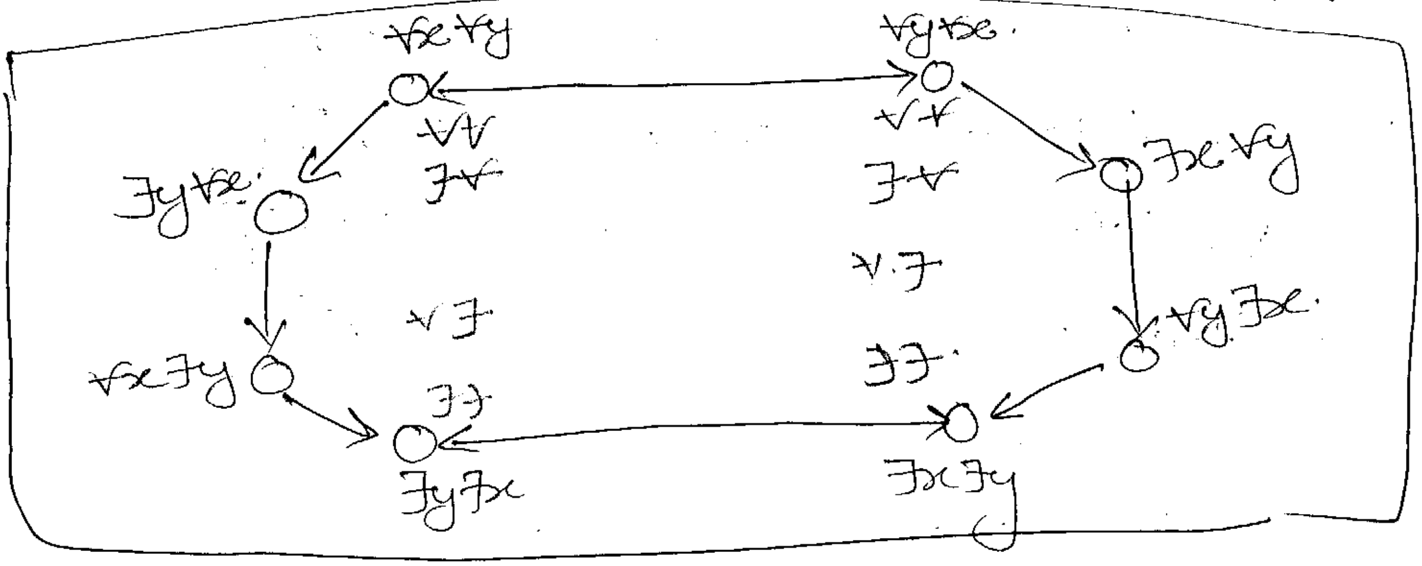
⇒ Relations b/w quantified two place predicates

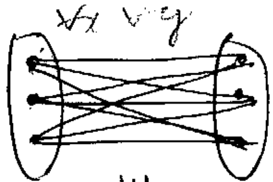
$$\left.\begin{array}{l} \forall x \forall y \equiv \forall y \forall x \\ \exists x \exists y \equiv \exists y \exists x \end{array}\right\}$$ i.e. when both quantifiers are of same type, their order doesn't matter



$\longleftrightarrow$ equivalence

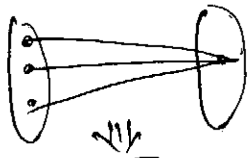
$\longrightarrow$ logical implication





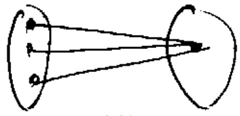
$\Downarrow$

$\exists y \forall x$

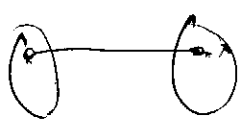


$\Downarrow$

$\forall x \exists y$



$\Downarrow$



$\exists y \exists x$

Ques: which of the following is NOT valid?

1) $\forall x \forall y\, P(x,y) \longrightarrow \exists x \forall y\, P(x,y)$.
2) $\exists x \forall y\, P(x,y) \longrightarrow \exists y \exists x\, P(x,y)$.
3) $\exists y \forall x\, P(x,y) \longrightarrow \forall y \exists x\, P(x,y)$. Invalid from above diagram
4) $\exists y \forall x\, P(x,y) \longrightarrow \forall x \exists y\, P(x,y)$.

⇒ Every teachers liked By some students.

a) ⇒ $\forall x\, [T(x) \rightarrow \exists y [S(y) \rightarrow likes(y,x)]]$ must not come

b) ⇒ $\boxed{\forall x\, [T(x) \rightarrow \exists y [S(y) \wedge likes(y,x)]]}$

c) ⇒ $\forall x\, [T(x) \wedge \exists y\, [S(y) \rightarrow likes(y,x)]]$

NO world idle

d) ⇒ $\exists y \forall x\, [T(x) \rightarrow [S(y) \wedge like(y,x)]]$

For sentence is in passive voice. So as $\forall x$ should come...

Some likes(y,x)

Put quantifiers of teacher first

$\forall x \exists y$ likes(y,x)

T(x) :- x is a teacher.

S(y) :- y is a student.

likes(y,x) :- y likes x.

⇒ $\forall x\, [T(x) \rightarrow \exists(y) [S(y) \wedge likes(y,x)]]$

⇒ Relations b/w quantifiers & ~~forms~~ connective

$U = \{1, 2\}$

→ $\forall x\, P(x) = P(1) \wedge P(2)$.

→ $\exists x\, P(x) = P(1) \vee P(2)$.

Results:

1). $\forall x\, [P(x) \wedge Q(x)] \equiv \forall x P(x) \wedge \forall x\, Q(x)$.

2). $\exists x\, [P(x) \vee Q(x)] \equiv \exists x\, P(x) \vee \exists x\, Q(x)$.

eg:- $P(1)$ T $Q(1) = F$

$P(2)$ F $Q(2) = T$

→ $\forall x\, [P(x) \vee Q(x)] \equiv [T \vee F] \wedge [F \vee T] \equiv T$

$\equiv [P(1) \vee Q(1)] \wedge [P(2) \vee Q(2)]$

$\forall x\, P(x) \vee \forall x\, Q(x) \equiv (T \wedge F) \vee (F \wedge T)$ ✗

$= [P(1) \wedge P(2)] \vee [Q(1) \wedge Q(2)]$

$\equiv F \vee F \equiv F$

Logical implications

3) $\forall x P(x) \vee \forall x Q(x) \rightarrow \forall x [P(x) \vee Q(x)]$

4) $\exists x [P(x) \wedge Q(x)] \rightarrow \exists x P(x) \wedge \exists x Q(x)$

5) $\forall x [P \wedge Q(x)] \equiv P \wedge \forall x Q(x)$

6) $\exists x [P \vee Q(x)] \equiv P \vee \exists x Q(x)$

7) $\forall x [P \vee Q(x)] \equiv P \vee \forall Q(x)$.

8) $\exists x [P \wedge Q(x)] \equiv P \wedge \exists x Q(x)$.

eg: $\forall x [P \rightarrow Q(x)]$

$\equiv \forall x [\neg P \vee Q(x)]$

$\equiv \neg P \vee (\forall x Q(x))$

$\equiv P \rightarrow \forall x Q(x)$.

9) $\forall x [P \rightarrow Q(x)] \equiv P \rightarrow \forall x Q(x)$.

10) $\exists x [P \rightarrow Q(x)] \equiv P \rightarrow \exists x Q(x)$.

eg: $\forall x [P(x) \rightarrow Q]$.

$\forall x [\neg P(x) \vee Q]$.

$\forall x (\neg P(x)) \vee Q$.

$\neg (\exists x P(x)) \vee Q$

$\exists x P(x) \rightarrow Q$.

11) $\forall x [P(x) \rightarrow Q] \equiv \exists x P(x) \rightarrow Q$.

12) $\exists x [P(x) \rightarrow Q] \equiv \forall x P(x) \rightarrow Q$.

Ques: $F(x, y, t)$: A person x can fool y at time t.

$\forall x \exists y \exists t \quad \neg F(x, y, t)$.

a) Everyone can fool some person at some time.

b) No one can fool everyone at all time

c) Everyone cannot fool some person all the time

d) No one can fool some person at some time

$\forall x \; \exists y \; \exists t \; \neg f(x, y, t)$.

Everyone — cannot fool — Some person — Sometime

Take Negation of it.

$\neg \exists x \forall y \forall t f(x, y, t)$

None — can fool — everyone at all the time

write symbolic form for remaining.

Option a) $\forall x \; \exists y \; \exists t \; f(x, y, t)$.

c) $\forall x \; \exists y \; \forall t \; \neg f(x, y, t)$.

d) $\neg \exists x \; \exists y \; \exists t \; f(x, y, t)$.