3

Diode Circuits and Rectifiers

IN THIS CHAPTER

- Diode Circuits with DC Source
- Freewheeling Diodes
- Diode and L Circuit
- Recovery of Trapped Energy
- Single-phase Diode Rectifiers
- Zener Diodes
- Performance Parameters
- Comparison of Single-phase Diode Rectifiers
- Three-phase Rectifiers
- Filters

A rectifier is a circuit that converts ac input voltage to dc output voltage. Semiconductor diodes are used extensively in power electronic circuits for the conversion of power from ac to dc. A rectifier employing diodes is called an *uncontrolled rectifier*, because its average output voltage is a fixed dc voltage.

In this chapter, first diode circuits involving different combinations of R, L and C are studied, and then diode rectifiers are described. For simplicity, the diodes are considered as ideal switches. An ideal diode has no forward voltage drop and reverse recovery time is negligible.

3.1 DIODE CIRCUITS WITH DC SOURCE

In this section, the effect of switching a dc source to a circuit consisting of diode and different circuit parameters is examined. The conclusions arrived at can then be applied to similar situations encountered later in power-electronic circuits.

3.1.1 Resistive Load

In the circuit of Fig. 3.1 (a), when switch S is closed, the current rises instantaneously to V_s/R as shown in Fig. 3.1 (b). Here V_s is the dc source voltage and R is the load resistance. When switch S is opened at t_1, the current at once falls to zero, Fig. 3.1 (b). Voltage v_D across diode is zero during the time diode conducts and is equal to $+V_s$ after diode stops conducting.

3.1.2 RC Load

A circuit with dc source, diode and RC load is shown in Fig. 3.2 (a). When switch S is closed at $t = 0$, KVL gives

$$Ri + \frac{1}{C}\int i\,dt = V_s$$

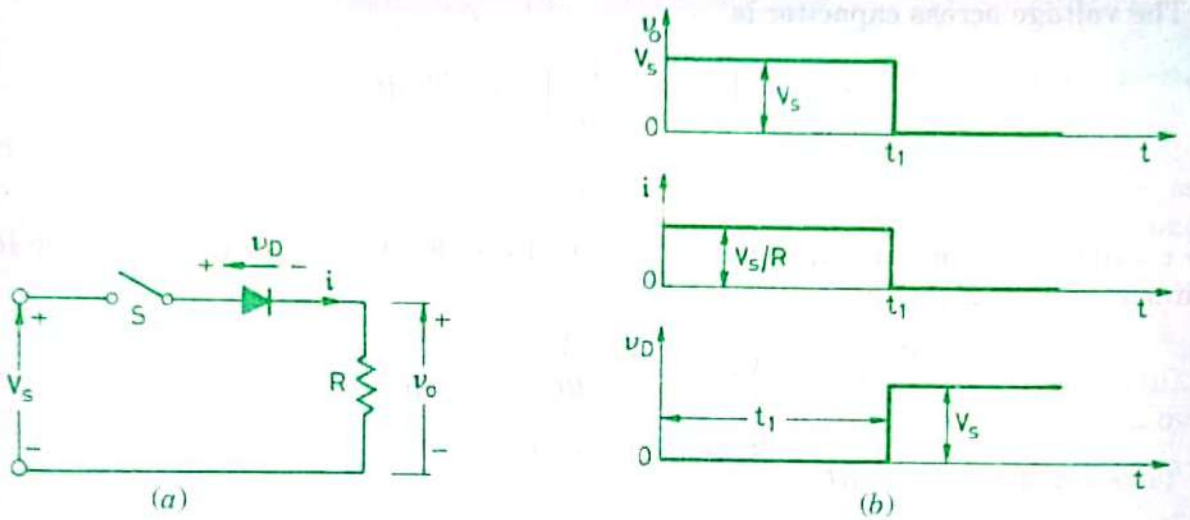


Fig. 3.1. Diode circuit with R load (a) circuit diagram and (b) waveforms.

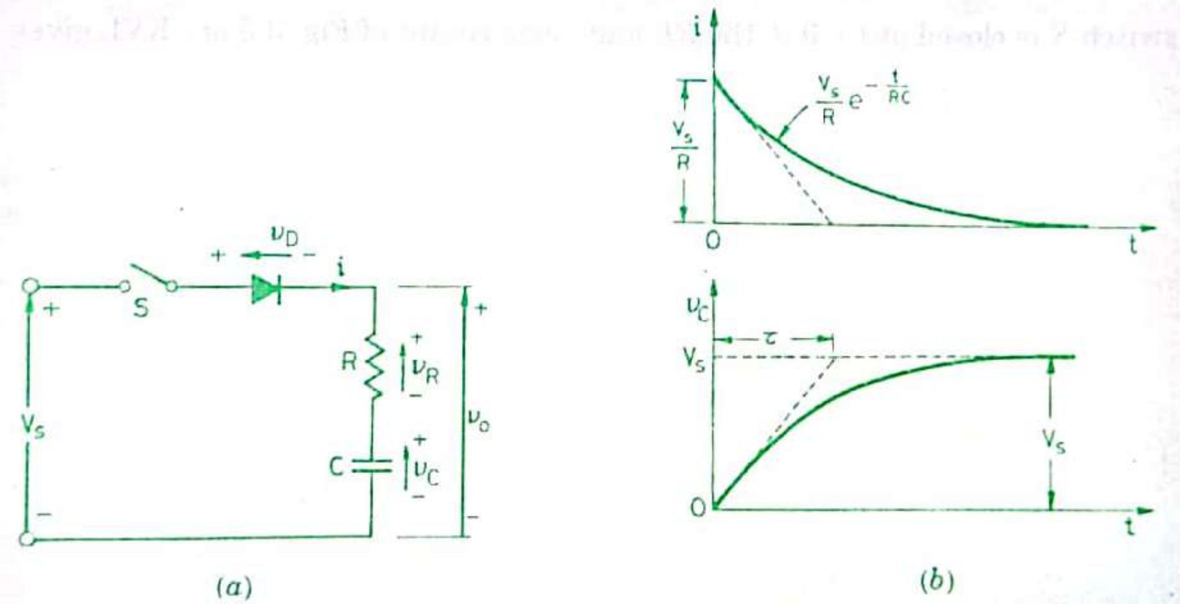


Fig. 3.2. Diode circuit with RC load (a) circuit diagram and (b) waveforms.

Its Laplace transform is $RI(s) + \frac{1}{C}\left[\frac{I(s)}{s} + \frac{q(o)}{s}\right] = \frac{V_s}{s}$ ...(3.1)

As the initial voltage across C is zero, $q\,(o) = 0$. With this, Eq. (3.1) becomes

$$I(s)\left[R + \frac{1}{Cs}\right] = \frac{V_s}{s}$$

or
$$I(s) = \frac{CV_s}{RC\left(s + \frac{1}{RC}\right)} = \frac{V_s}{R} \cdot \frac{1}{s + \frac{1}{RC}}$$

Its Laplace inverse is $i(t) = \frac{V_s}{R} \cdot e^{-t/RC}$ ...(3.2)

The voltage across capacitor is

$$v_c(t) = \frac{1}{C}\int_0^t i dt = \frac{V_s}{RC}\int_0^t e^{-t/RC} dt$$
$$= V_s(1 - e^{-t/RC}) \quad \text{...(3.3 a)}$$
$$= V_s(1 - e^{-t/\tau}) \quad \text{...(3.3 b)}$$

where $\tau = RC$ is the time constant for RC circuit. From Eq. (3.3 a), initial rate of change of capacitor voltage is given by

$$\left(\frac{dv_c}{dt}\right)_{t=0} = \left[V_s \cdot e^{-t/RC} \cdot \frac{1}{RC}\right]_{t=0} = \frac{V_s}{RC} \quad \text{...(3.4)}$$

Time constant, $RC = \frac{\text{Source voltage, } V_s}{(dv_c/dt)_{t=0}}$

In Fig. 3.2 (b), current through the circuit and voltage variation across C are shown.

3.1.3 RL Load

When switch S is closed at $t = 0$ in the RL and diode circuit of Fig. 3.3 (a), KVL gives

$$Ri + L\frac{di}{dt} = V_s \quad \text{...(3.5)}$$

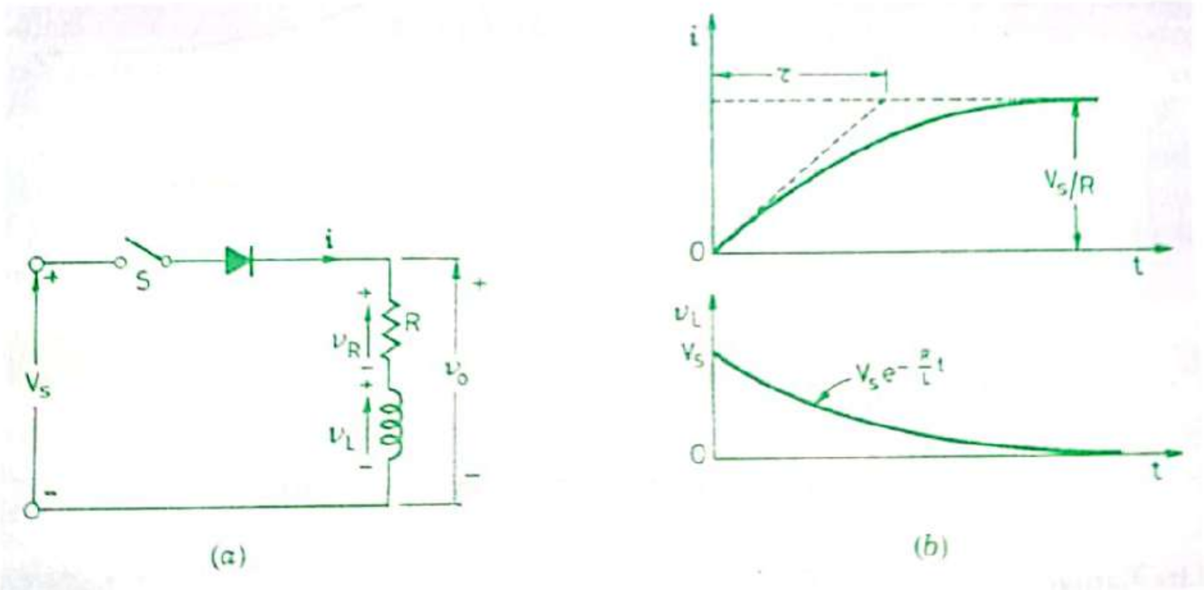


Fig. 3.3. Diode circuit with RL load (a) circuit diagram and (b) waveforms.

With initial current in the inductor as zero, the solution of Eq. (3.5) gives

$$i(t) = \frac{V_s}{R}\left(1 - e^{-\frac{R}{L}t}\right) \quad \text{...(3.6)}$$

Initial rate of rise of current is

$$\left.\frac{di}{dt}\right|_{t=0} = \left(\frac{V_s}{L}\cdot e^{-\frac{R}{L}t}\right)_{t=0} = \frac{V_s}{L} \qquad ...(3.7)$$

The voltage across L is $v_L(t) = L\frac{di}{dt} = V_s \cdot e^{-\frac{R}{L}t}$ ...(3.8)

For RL circuit, $\frac{L}{R} = \tau$ is the time constant. The waveforms of current through the circuit and voltage across inductance L are sketched in Fig. 3.3 (*b*).

It must be *noted* that the behaviour of circuits in Figs. 3.1 to 3.3 is not affected whether a diode is used or not. It is because, in these circuits, the current i does not have a tendency to reverse; *i.e.* current i remains unidirectional.

3.1.4 LC Load

A diode circuit with dc source voltage V_s, switch S and load LC is shown in Fig. 3.4 (*a*). When switch S is closed at $t = 0$, the voltage equation governing its performance is given by

$$L\frac{di}{dt} + \frac{1}{C}\int i\,dt = V_s$$

Its Laplace transform is $L[sI(s) - i(0)] + \frac{1}{C}\left[\frac{I(s)}{s} + \frac{q(0)}{s}\right] = \frac{V_s}{s}$

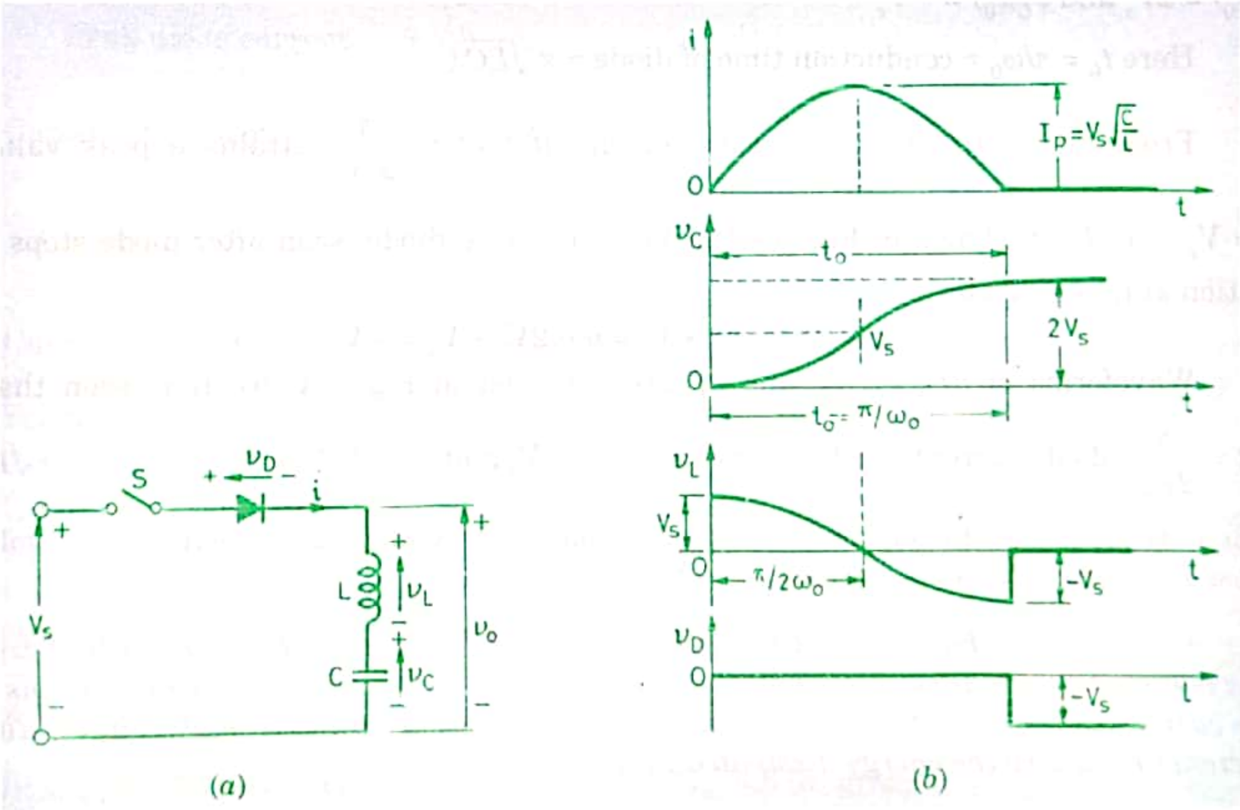


Fig. 3.4. Diode circuit with *LC* load (*a*) circuit diagram and (*b*) waveforms.

As the circuit is initially relaxed, $i(0) = 0$ and $v_C(0) = 0$ or $q(0) = C \cdot v_C(0) = 0$

$$\therefore \qquad I(s)\left[sL + \frac{1}{sC}\right] = \frac{V_s}{s}$$

or

$$I(s) = \frac{V_s}{L} \cdot \frac{1}{s^2 + \frac{1}{LC}}$$

Let $\omega_0 = \frac{1}{\sqrt{LC}}$. This gives $I(s) = \frac{V_s}{L \cdot \omega_0} \cdot \frac{\omega_0}{s^2 + \omega_0^2} = V_s \cdot \sqrt{\frac{C}{L}} \cdot \frac{\omega_0}{s^2 + \omega_0^2}$

Its Laplace inverse is $\quad i(t) = V_s \cdot \sqrt{\frac{C}{L}} \sin \omega_0 t \qquad$ ...(3.9)

Here $\omega_0 = \frac{1}{\sqrt{LC}}$ is called *resonant frequency* of the circuit. Capacitor voltage is given by

$$v_C(t) = \frac{1}{C}\int_0^t i(t) \cdot dt = \frac{1}{C}\int_0^t V_s \cdot \sqrt{\frac{C}{L}} \sin \omega_0 t \cdot dt$$

$$= V_s (1 - \cos \omega_0 t) \qquad \text{...(3.10a)}$$

Voltage across inductance is given by

$$v_L(t) = L\frac{di(t)}{dt} = V_s \cos \omega_0 t \qquad \text{...(3.10b)}$$

When $\omega_0 t_0 = \pi$ or when $t_0 = \pi/\omega_0$, from Eq. (3.9), $i(t_0) = 0$ and from Eq. (3.10*a*), $v_C(t_0) = 2V_s$ and $v_L(t_0) = -V_s$

Here $t_0 = \pi/\omega_0$ = conduction time of diode $= \pi\sqrt{LC}$

From Eq. (3.9), circuit or diode current at $t_0/2 = \frac{\pi}{2\omega_0}$ attains a peak value of $I_p = V_s \cdot \sqrt{C/L}$ as shown in Fig. 3.4 (*b*). Voltage across diode, soon after diode stops conduction at t_0 is given by

$$v_D = -v_L - v_C + V_s = 0 - 2V_s + V_s = -V_s.$$

Waveforms of $i(t)$, v_C, v_L and v_D are sketched in Fig. 3.4 (*b*). It is seen that at $t_0/2 = \frac{\pi}{2\omega_0}$, diode current reaches peak value, $v_C = V_s$ and $v_L = 0$. Also at $t_0 = \pi/\omega_0 = (\pi\sqrt{LC})$, diode current decays to zero and capacitor is charged to voltage $2V_s$. Soon after t_0, voltage across L is zero and diode voltage $v_D = -V_s$.

Example 3.1. *For the circuit shown in Fig. 3.5 (a), the capacitor is initially charged to a voltage V_0 with upper plate positive. Switch S is closed at $t = 0$. Derive expressions for the current in the circuit and voltage across capacitor C. What is the peak value of diode current? Find also the energy dissipated in the circuit.*

Solution. When switch S is closed, KVL gives

$$Ri + \frac{1}{C}\int i\,dt = 0$$

Its Laplace transform, including the initial voltage across capacitor, is

$$RI(s) + \frac{1}{C}\left[\frac{I(s)}{s} - \frac{CV_0}{s}\right] = 0$$

or

$$I(s)\left[R + \frac{1}{Cs}\right] = \frac{V_0}{s}$$

Its solution, as per Art. 3.1.2, is $i(t) = \frac{V_0}{R} e^{-t/RC}$

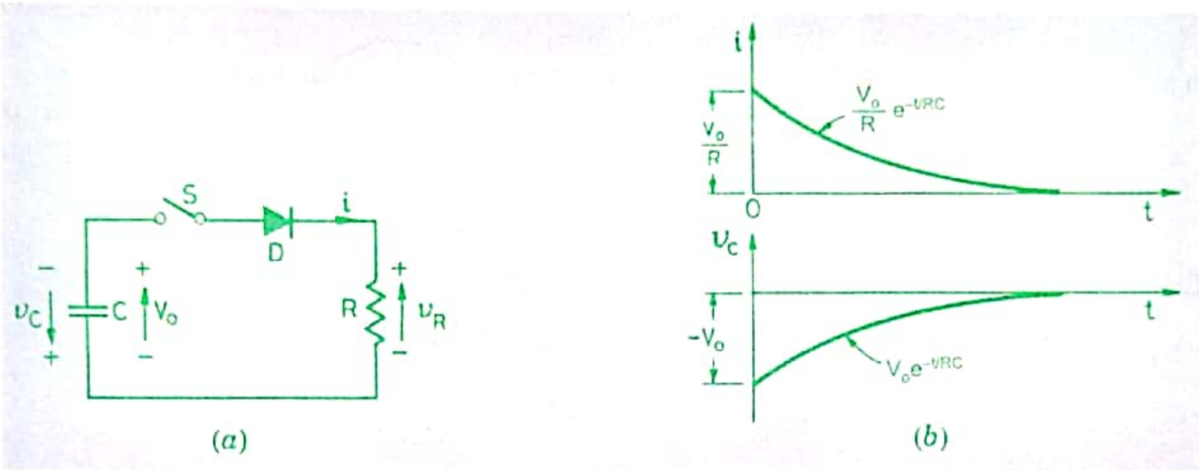


Fig. 3.5. Pertaining to Example 3.1 (*a*) circuit diagram and (*b*) waveforms.

$\therefore$ Peak diode current $= \frac{V_0}{R}$

Capacitor voltage, $v_C(t) = \frac{1}{C}\int_0^t i\,dt - V_0$

$$= \frac{1}{C}\int_0^t \frac{V_0}{R} \cdot e^{-t/RC} \cdot dt - V_0$$

$$= -V_0 e^{-t/RC}$$

Current $i(t)$ and voltage $v_C(t)$ are sketched in Fig. 3.5 (*b*).

Energy dissipated in the circuit $= \frac{1}{2} C V_0^2$ Joules.

Example 3.2. *In the diode and LC network shown in Fig. 3.6 (a), the capacitor is initially charged to voltage V_0 with upper plate positive. Switch S is closed at $t = 0$. Derive expressions for current through and voltage across C.*

Find the conduction time of diode, peak current through the diode and final steady-state voltage across C in case $V_s = 400$ V, $V_0 = 100$ V, $L = 100$ μH and $C = 30$ μF. Determine also the voltage across diode after it stops conduction.

Solution. When switch S is closed, KVL for Fig. 3.6 (*a*) gives

$$L\frac{di}{dt} + \frac{1}{C}\int i\,dt = V_s$$

Its Laplace transform gives

$$L[sI(s) - i(0)] + \frac{1}{C}\left[\frac{I(s)}{s} + \frac{C \cdot V_0}{s}\right] = \frac{V_s}{s}$$

As initially $i(0) = 0$, the above equation becomes

$$I(s)\left[sL + \frac{1}{sC}\right] = \frac{V_s - V_0}{s}$$

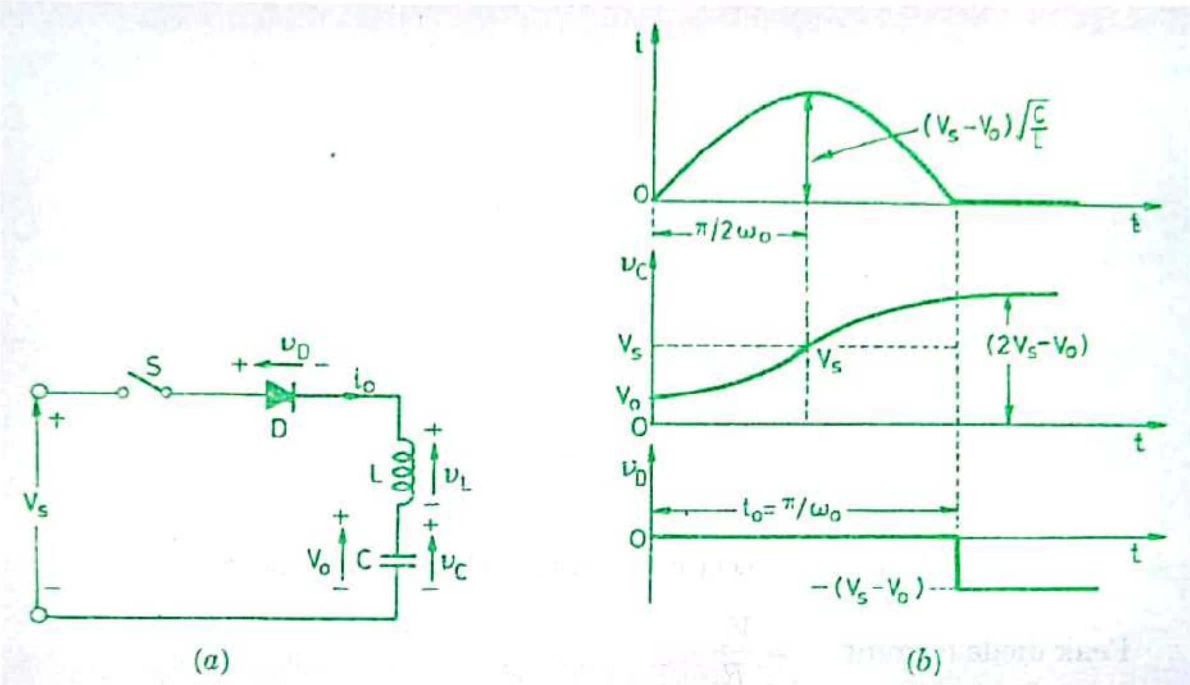


Fig. 3.6. Pertaining to Example 3.2 (*a*) circuit diagram and (*b*) waveforms.

This equation in s-domain can be solved as in Art 3.1.4. Its solution is

$$i(t) = (V_s - V_0) \cdot \sqrt{\frac{C}{L}} \sin \omega_0 t$$

$$v_C(t) = \frac{1}{C}(V_s - V_0)\sqrt{\frac{C}{L}} \int_0^t \sin \omega_0 t \, dt + V_0$$

$$= (V_s - V_0)(1 - \cos \omega_0 t) + V_0$$

At $\omega_0 t = 0$, $\quad v_C(t) = V_0$

At $\omega_0 t = \pi/2$, $\quad v_C(t) = V_s$ and at $\omega_0 t = \pi$, $v_C(t) = 2(V_s - V_0) + V_0 = 2V_s - V_0$

Diode conduction time, $t_0 = \dfrac{\pi}{\omega_0} = \pi\sqrt{LC} = \pi\sqrt{30 \times 100} \times 10^{-6} = 54.77 \ \mu s$

Peak current through diode, $I_p = (V_s - V_0)\sqrt{\dfrac{C}{L}}$

$$= 300\sqrt{\frac{30}{100}} = 164.32 \text{ A}$$

Steady state voltage across C occurs when $\omega_0 t_0 = \pi$

$$\therefore \qquad V_C = 2(V_s - V_0) + V_0 = 2V_s - V_0 = 2 \times 400 - 100 = 700 \text{ V}$$

Voltage across diode, after it stops conducting, is given by

$$v_D = -v_L - v_C + V_s = 0 - (2V_s - V_0) + V_s = -V_s + V_0 = -400 + 100$$
$$= -300 \text{ V}.$$

Example 3.3. *In the circuit shown in Fig. 3.7 (a), the capacitor has initial voltage V_0 with upper plate positive. The circuit is switched at $t = 0$. Derive expressions for current and voltage across capacitor. Find the conduction time for diode and steady-state capacitor voltage.*

Solution. The voltage equation for the circuit of Fig. 3.7 (a), after switch S is closed at $t = 0$, is

$$L\frac{di}{dt} + \frac{1}{C}\int i\,dt = 0$$

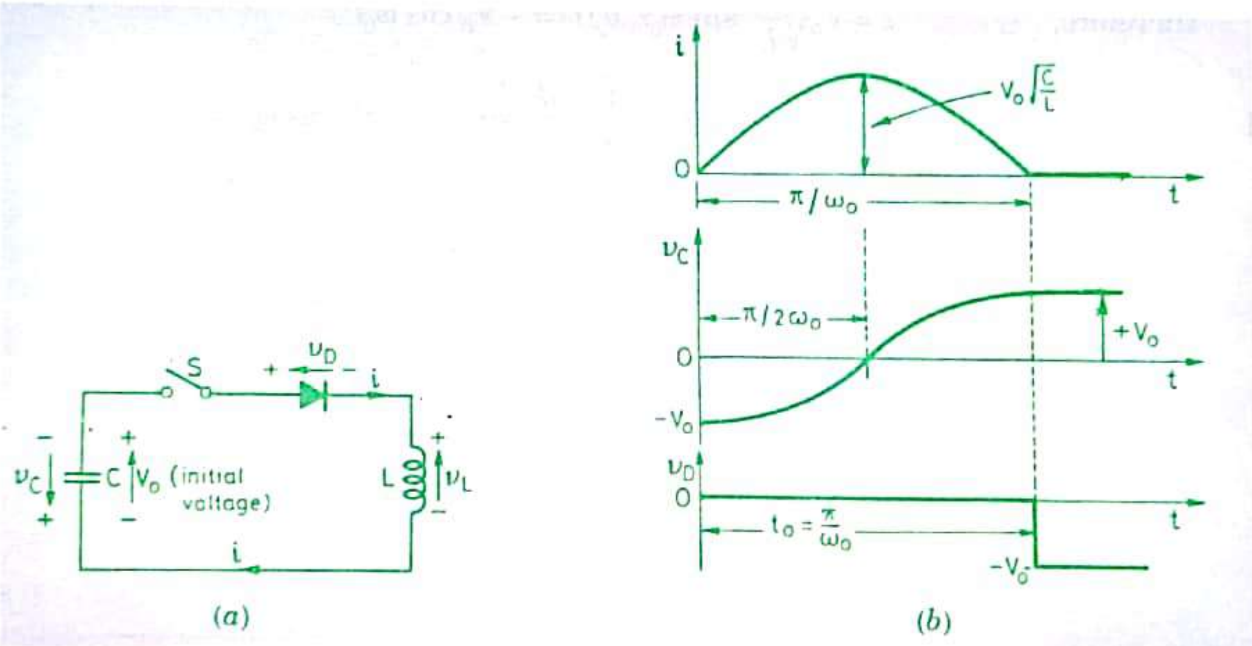


Fig. 3.7. Pertaining to Example 3.3 (a) circuit diagram and (b) waveforms.

Its Laplace transform, including initial voltage across capacitor, is

$$I(s)\cdot sL + \frac{1}{C}\left[\frac{I(s)}{s} - \frac{CV_0}{s}\right] = 0$$

$$I(s)\left[sL + \frac{1}{sC}\right] = \frac{V_0}{s}$$

Here minus sign is put before V_0, because for the direction of positive current flow, polarity of V_0 is opposite.

Solution of above s-domain equation, from Art. 3.1.4, is

$$i(t) = V_0\sqrt{\frac{C}{L}}\sin\omega_0 t$$

Voltage across C is $$v_C(t) = \frac{1}{C} \cdot V_0\sqrt{\frac{C}{L}} \int_0^t \sin \omega_0 t\, dt - V_0 = -V_0 \cos \omega_0 t$$

Diode conduction time, $$t_0 = \frac{\pi}{\omega_0} = \pi\sqrt{LC}$$

Steady-state capacitor voltage $= -V_0 \cos \pi = +V_0$

Voltage across diode, $v_D = -V_0$.

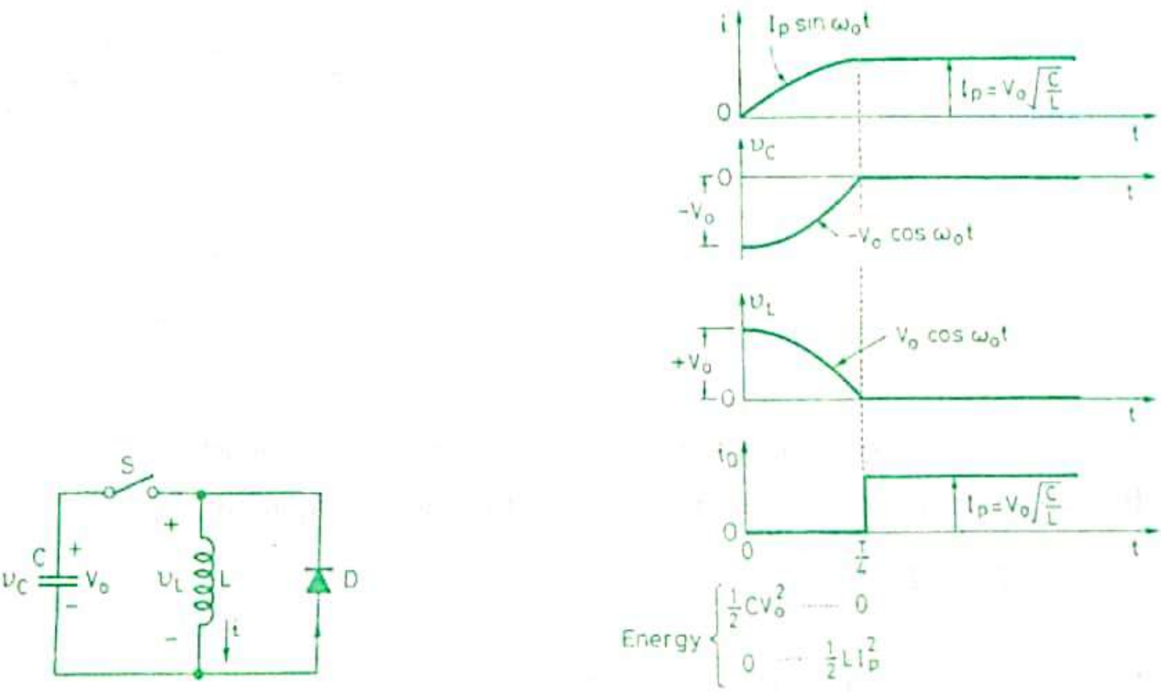
Waveforms for i, V_C and v_D are sketched in Fig. 3.7 (*b*).

Example 3.4. *In the circuit shown in Fig. 3.8 (a), capacitor is initially charged to voltage V_o with upper plate positive. Sketch waveforms of i, v_C, v_L and i_D after the switch S is closed.*

Solution. When switch S is closed, capacitor C begins to discharge through L and C. For obtaining i, v_C expressions, refer to Example 3.3.

Therefore, $$i = V_0\sqrt{\frac{C}{L}} \sin \omega_0 t,\; v_c(t) = -V_0 \cos \omega_0 t$$

and $$v_L = L\frac{di}{dt} = L \cdot V_0 \cdot \sqrt{\frac{C}{L}} \cdot \frac{d}{dt} \sin \omega_0 t = V_0 \cos \omega_0 t = -v_c.$$



(*a*) (*b*)

Fig. 3.8. Pertaining to Example 3.4 (*a*) circuit diagram and (*b*) waveforms.

The waveforms for i, v_C, v_L and i_D are shown in Fig. 3.8 (*b*). At $t = 0$, energy stored in C is $\frac{1}{2}CV_0^2$ and $i = 0$. At $t = T/4$, current i reaches peak value $V_0\sqrt{\frac{C}{L}} = I_p$, $v_C = v_L = 0$ and energy stored in C gets transferred to L as $\frac{1}{2}LI_p^2$. Soon after $T/4$ (ω_0 $T/4 = \pi/2$), as v_L tends to reverse, diode D gets forward biased. Current I_p now begins to flow as i_D through D and as i through L. If there were no resistance in this closed path, current I_p would continue flowing unabated. In practice, inherent resistance in the closed path would cause this current to decay exponentially.

Example 3.5. *In the circuit of Fig. 3.9 (a), current in inductor L is I_0 before $t = 0$. Sketch the waveforms of i, v_c and v_L after switch S is opened at $t = 0$.*

Solution. When switch S is opened at $t = 0$, current I_0 begins to flow in the path D, C and L. KVL for this path is

$$L\frac{di}{dt} + \frac{1}{C}\int i\,dt = 0$$

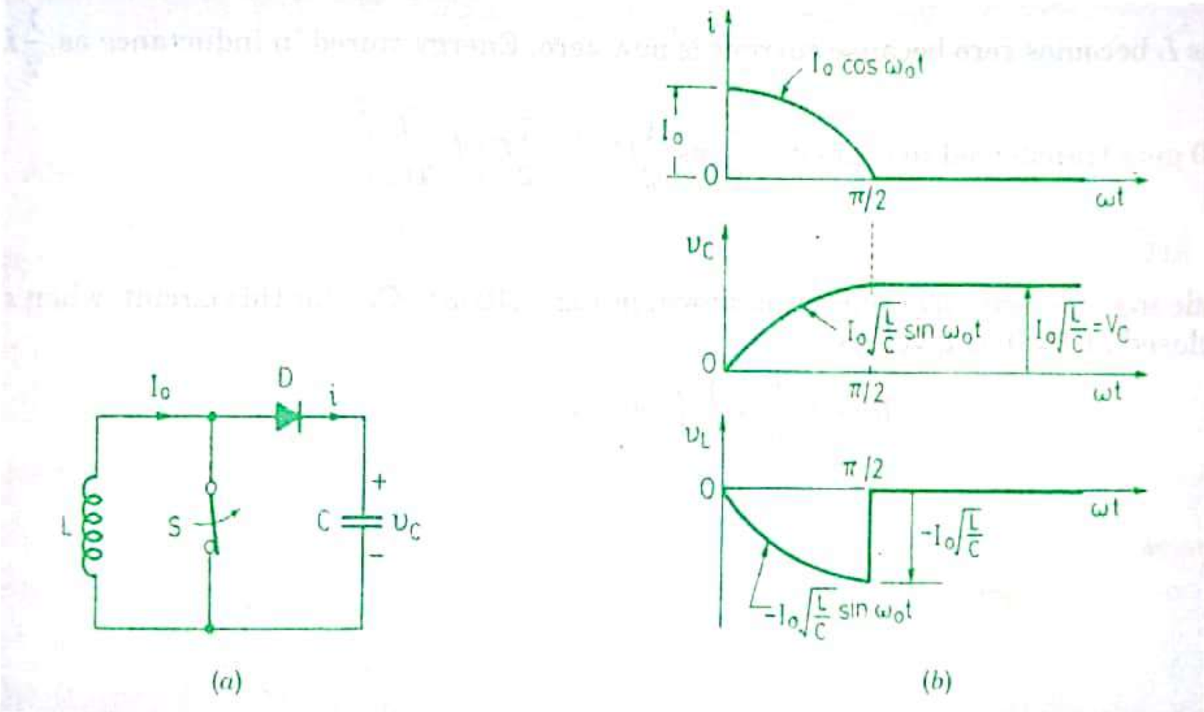


Fig. 3.9. Pertaining to Example 3.5. (*a*) Circuit diagram and (*b*) waveforms.

Its Laplace transform is $L[sI(s) - i(0)] + \frac{1}{C}\left[\frac{I(s)}{s} - \frac{CV_0}{s}\right] = 0$

It is given that initially $i_0 = I_0$ and $V_0 = 0$. Therefore, we get

$$I(s)\left[sL + \frac{1}{sC}\right] = LI_0$$

or

$$I(s) = I_0\frac{s}{s^2 + \omega_o^2}, \text{ where } \omega_0 = \frac{1}{\sqrt{LC}} \text{ as before.}$$

Its Laplace inverse is $\quad i(t) = I_0 \cos \omega_0 t$

Also $$v_c = \frac{1}{C}\int i\,dt = \frac{1}{C}\int_0^t I_0 \cos \omega_0 t \,.\, dt = I_0\sqrt{\frac{L}{C}} \sin \omega_0 t$$

and $$v_L = L\frac{di}{dt} = L\frac{d}{dt}(I_0 \cos \omega_0 t) = -\, I_0\sqrt{\frac{L}{C}} \sin \omega_0 t$$

At $\omega t = \frac{\pi}{2}$, current i tends to reverse, but diode D blocks this current reversal. Also, at $\omega t = \frac{\pi}{2}$, capacitor is charged to $I_0\sqrt{\frac{L}{C}} \sin\frac{\pi}{2} = I_0\sqrt{\frac{L}{C}} = V_C$ and voltage across inductance is $V_L = -\, I_0\sqrt{\frac{L}{C}}$.

Thus, after $\omega t = \frac{\pi}{2}$, capacitor voltage remains constant at $I_0\sqrt{\frac{L}{C}}$ whereas voltage across L becomes zero because current is now zero. Energy stored in inductance as $\frac{1}{2}LI_0^2$ at $\omega t = 0$ gets transferred to C at $\omega t = \frac{\pi}{2}$ as $\frac{1}{2}CV_c^2 = \frac{1}{2}C\left(I_0\sqrt{\frac{L}{C}}\right)^2$.

3.1.5 RLC Load

A diode in series with RLC circuit is shown in Fig. 3.10 (a). KVL for this circuit, when switch S is closed at $t = 0$, is given by

$$Ri + L\frac{di}{dt} + \frac{1}{C}\int i\,dt = V_s$$

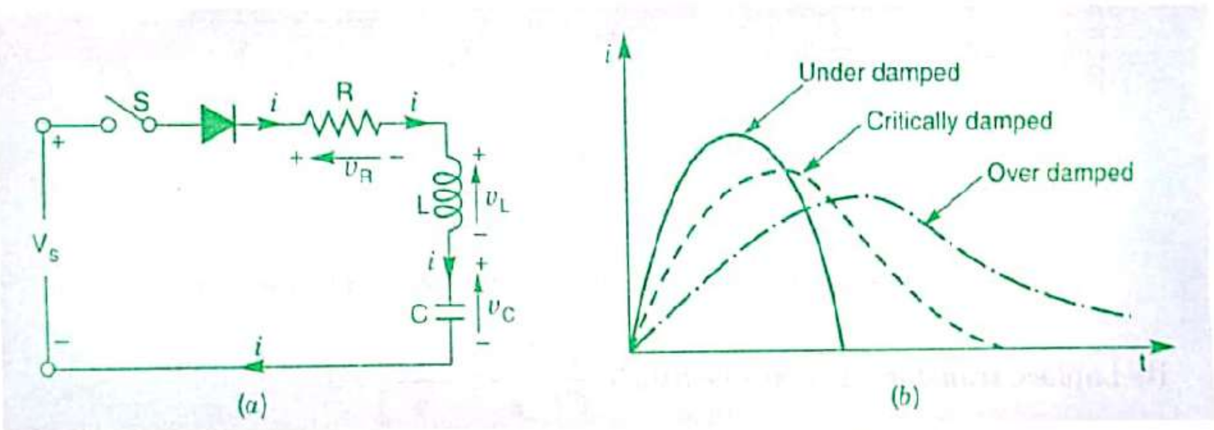


Fig. 3.10. Diode circuit with RLC load (a) circuit diagram and (b) waveforms.

With zero initial conditions, the Laplace transform of above equation is

$$I(s)\left[R + sL + \frac{1}{sC}\right] = \frac{V_s}{s}$$

or $$I(s) = \frac{V_s}{L}\cdot\frac{1}{s^2 + \frac{R}{L}s + \frac{1}{LC}}$$

Here $s^2 + \frac{R}{L}s + \frac{1}{LC} = 0$ is the characteristic equation in s-domain. The roots of this equation are

$$s = -\frac{R}{2L} \pm \sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}}$$

or $$s = -\xi \pm \sqrt{\xi^2 - \omega_0^2} \quad \text{...(3.11)}$$

where $$\xi = \frac{R}{2L} \quad \text{...(3.12)}$$

is called the *damping factor* $$\omega_0 = \frac{1}{\sqrt{LC}} \quad \text{...(3.13)}$$

is called resonant frequency in rad/sec

and $$\omega_r = \sqrt{\frac{1}{LC} - \left(\frac{R}{2L}\right)^2} \quad \text{...(3.14)}$$

$$= \sqrt{\omega_0^2 - \xi^2} = \text{ringing frequency in rad/sec.}$$

Also $$\omega_0 = \sqrt{\omega_r^2 + \xi^2}$$

Depending upon the values of ξ and ω_0, the solution for the current can have three possible solutions.

Case 1. In case $\xi < \omega_0$, it is seen from Eq. (3.11) that the roots are complex and the circuit is said to be *underdamped*. The two roots are

$$s_1 = -\xi + j\omega_r \quad \text{and} \quad s_2 = -\xi - j\omega_r$$

and the current is given by $$i(t) = \frac{V_s}{\omega_r L} . e^{-\xi t} \sin \omega_r t \quad \text{...(3.15)}$$

Case 2. If $\xi > \omega_0$, the two roots are real and the circuit is said to be *overdamped*. The two roots are

$$s_1 = -\xi + \sqrt{\xi^2 - \omega_0^2} \quad \text{and} \quad s_2 = -\xi - \sqrt{\xi^2 - \omega_0^2}$$

and the solution for current is

$$i(t) = \frac{V_s}{L\sqrt{\xi^2 - \omega_0^2}} . \sinh \sqrt{(\xi^2 - \omega_0^2)} . t \quad \text{...(3.16)}$$

Case 3. In case $\xi = \omega_0$, the roots are equal and the circuit is said to be *critically damped*. The roots are $s_1 = s_2 = -\xi$ and the solution for the current is

$$i(t) = \frac{V_s}{L} . t . e^{-\xi t} \quad \text{...(3.17)}$$

Waveforms of current for the three different levels of damping are sketched in Fig. 3.10 (*b*).

Example 3.6. *For the circuit of Fig. 3.10 (a), the data is as under :*

$$R = 10\ \Omega,\ L = 1\ mH,\ C = 5\ \mu F,\ V_s = 230\ V$$

The circuit is initially relaxed. With switch closed at $t = 0$, determine (a) current $i(t)$ (b) conduction time of diode (c) rate of change of current at $t = 0$.

Solution. (*a*) From Eq. (3.12),

$$\xi = \frac{10 \times 1000}{2 \times 1} = 5000$$

From Eq. (3.13),
$$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\left[1 \times 10^{-3} \times 5 \times 10^{-6}\right]^{1/2}} = \frac{10^5}{\sqrt{50}} = 14142.136 \text{ rad/s}$$

From Eq. (3.14),
$$\omega_r = \left[\frac{10^{10}}{50} - (5000)^2\right]^{1/2} = 13228.76 \text{ rad/s}$$

Here as $\xi < \omega_0$, the circuit is underdamped. The current is, therefore, given by Eq. (3.15).

$$i(t) = \frac{230 \times 1000}{13228.76 \times 1} . e^{-5000t} . \sin(13228.76)t$$
$$= 17.3864 . e^{-5000t} . \sin(13228.76t)$$

(*b*) Diode stops conducting when $\omega_r t_1 = \pi$

∴ Conduction time of diode,

$$t_1 = \frac{\pi}{\omega_r} = \frac{\pi}{13228.76} = 237.482 \ \mu\text{s}$$

(*c*) From Eq. (3.15),
$$\frac{di}{dt} = \frac{V_s}{\omega_r L}\left[e^{-\xi t} . \omega_r \cos \omega_r t - \sin \omega_r t . (-\xi)\, e^{-\xi t}\right]$$

$$\left.\frac{di}{dt}\right|_{t=0} = \frac{V_s}{L} = \frac{230 \times 1000}{1} = 230{,}000 \text{ A/s.}$$

3.2 FREEWHEELING DIODES

In Fig. 3.3 (*a*), steady state current, after switch S is closed, is equal to V_s/R. Energy stored in inductance L is $\frac{1}{2}$. $L(V_s/R)^2$. If the switch S is now opened, current V_s/R would eventually decay to zero. As the current V_s/R tends to decay with the opening of switch S, a high reverse voltage appears across switch as well as the diode. High voltage across switch leads to spark across the switch contacts, thus dissipating the stored energy. In the process, the diode, subjected to high reverse voltage, may get damaged. In order to avoid such an occurrence, a diode FD, called *freewheeling, or flywheel,* diode, is connected across load RL as shown in Fig. 3.11 (*a*). For understanding how FD comes into play, the working of circuit of Fig. 3.11 (*a*) is divided into two modes.

Mode I : When switch S is closed in Fig. 3.11 (*a*) at $t = 0$, current flows through V_s, S, D, R and L as shown in Fig. 3.11 (*b*). In this circuit, current i is given by

$$i = \frac{V_s}{R}\left(1 - e^{-\frac{R}{L}t}\right) \qquad ...(3.18)$$

Final value of current, $I = \dfrac{V_s}{R}$

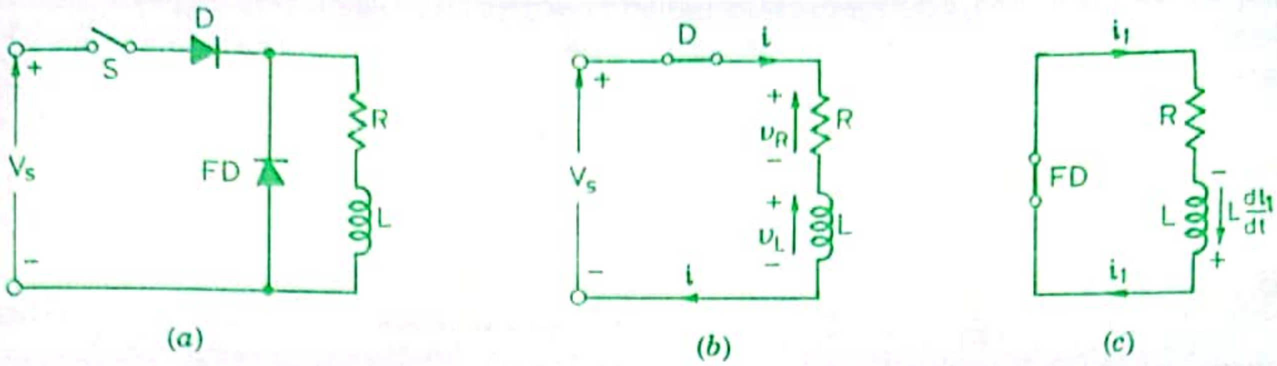


Fig. 3.11. Circuit of Fig. 3.3 with freewheeling diode.

Mode II : When switch S is opened at $t = 0$, current in the circuit tends to decay and so a voltage $L\dfrac{di}{dt}$ is induced in L which forward biases freewheeling diode. The current is, therefore, transferred to the circuit consisting of FD, R and L as shown in Fig. 3.11 (c). In this circuit, current is given by

$$i_1 = \frac{V_s}{R} \cdot e^{-\frac{R}{L}t} \qquad ...(3.19)$$

The current i_1 will eventually decay to zero exponentially in mode II of Fig. 3.11 (c). The current build up during mode I and current decay during mode II are shown in Fig. 3.11 (d).

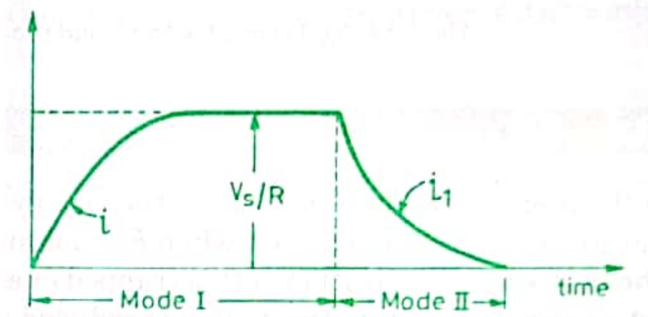


Fig. 3.11. (d) Current variation in the circuit of Fig. 3.11.

3.3 DIODE AND L CIRCUIT

Consider the circuit of Fig. 3.12 (a) where dc source feeds L through diode D. A freewheeling diode FD is connected across L. When switch S is closed at $t = 0$, KVL gives

$$V_s = L\frac{di}{dt}$$

or

$$i = \frac{V_s}{L}t \qquad ...(3.20)$$

This shows that current i rises linearly with time t. In case switch S is opened at t_1, load current $\dfrac{V_s}{L}t_1$ begins to flow through FD. As there is no resistance in the circuit formed by L and FD, current continues to flow at its constant value of $\dfrac{V_s}{L}t_1$. Energy stored in the inductance is $\dfrac{1}{2}\left(\dfrac{V_s}{L}t_1\right)^2 . L = \dfrac{1}{2} \cdot \dfrac{V_s^2}{L}t_1$ joules. Current waveforms are shown in Fig. 3.12 (b).

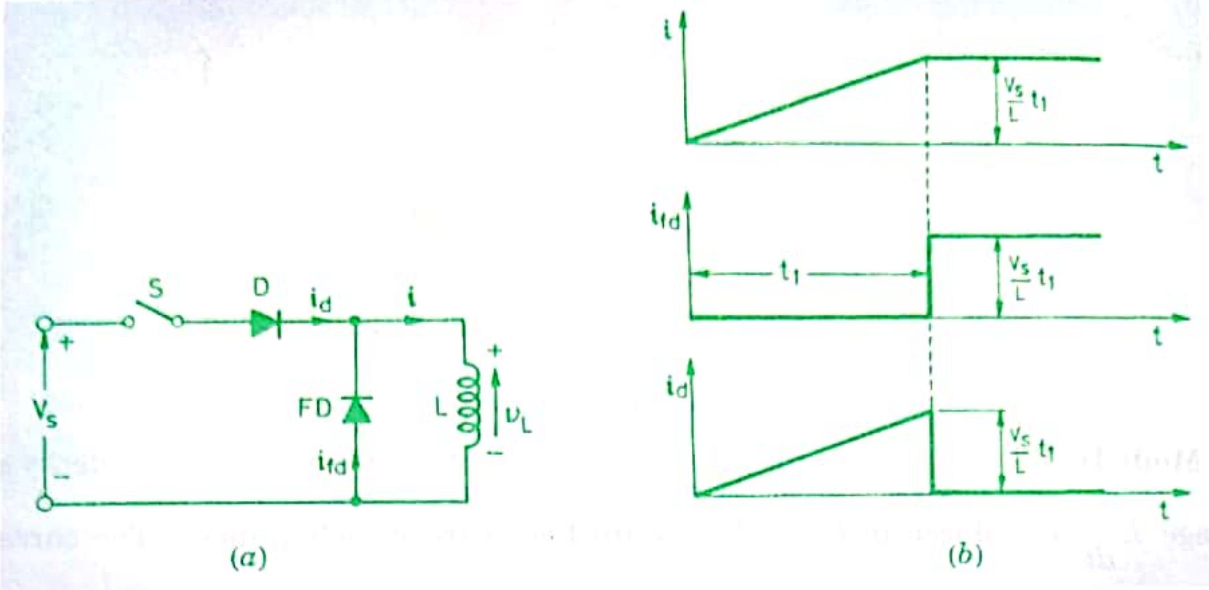


Fig. 3.12. Diode circuit with *FD* and *L* load (*a*) circuit diagram and (*b*) waveforms.

3.4 RECOVERY OF TRAPPED ENERGY

In the ideal circuit of Fig. 3.12 (*a*), the energy stored in the inductor is trapped. This trapped energy is not dissipated even when *FD* conducts because circuit does not contain resistance. The best way of utilization of this trapped energy is to return it to the source. In this manner, net energy taken from the source is reduced and the system efficiency improves.

One way of returning this trapped energy back to the source is to add a second winding closely coupled with the inductor winding as shown in Fig. 3.13. A diode *D* is also placed in series with the second winding. The inductor now behaves like a transformer. The two windings are so arranged that their polarity markings are opposite to each other.

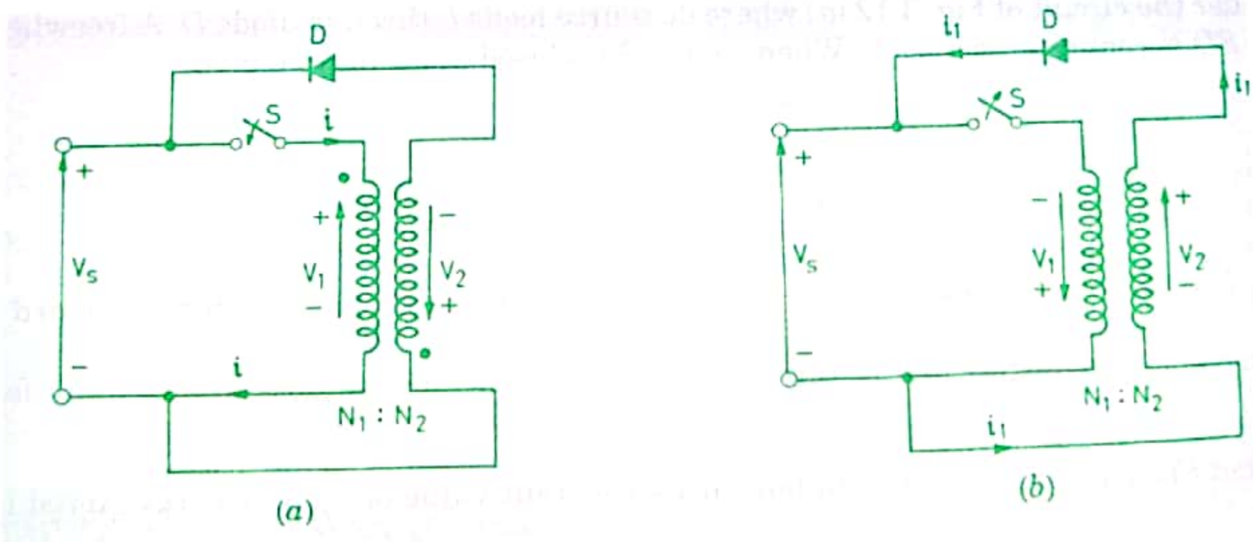


Fig. 3.13. Energy-recovery circuit (*a*) switch *S* closed and (*b*) switch *S* opened.

When switch S is closed, current i begins to flow and energy is stored in the inductance of primary winding with N_1 turns. The polarity of the secondary winding voltage V_2 is as shown. The diode D is reverse biased by voltage $(V_2 + V_s)$.

When switch S is opened, polarities of voltages V_1 and V_2 get reversed, the diode is now forward biased by voltage $(V_2 - V_s)$. As a result, diode begins to conduct a current i_1 into the positive terminal of source voltage V_s and so the trapped energy is fed back to the source.

Energy fedback to dc source = V_s × current i_1 dependent upon $(V_2 - V_s)$.

The energy stored in L of N_1 turns is transferred to secondary winding of N_2 turns from where it is fed back into the dc source.

3.5 SINGLE-PHASE DIODE RECTIFIERS

Rectification is the process of conversion of alternating input voltage to direct output voltage. As stated before, a rectifier converts ac power to dc power. In diode-based rectifiers, the output voltage cannot be controlled.

In this section, uncontrolled single-phase rectifiers are studied. The diode is assumed ideal as before.

A rectifier may be half-wave type or full-wave type. A half-wave rectifier is one in which current in any one line, connected to ac source, is *unidirectional*. However, a full-wave rectifier has *bidirectional* current in any one line connected to ac surface.

A rectifier may be one-pulse, two-pulse, three-pulse or n-pulse type. The number of pulses in any rectifier-configuration is obtained as under :

pulse number = number of load current (or voltage) pulses during one cycle of ac source voltage.

3.5.1 Single-Phase Half-wave Rectifier

This is the simplest type of uncontrolled rectifier. It is never used in industrial applications because of its poor performance. Its study is, however, useful in understanding the principle of rectifier operation.

In a single-phase half-wave rectifier, for one cycle of supply voltage, there is one half-cycle of output, or load, voltage. As such, it is also called *single-phase one-pulse rectifier.*

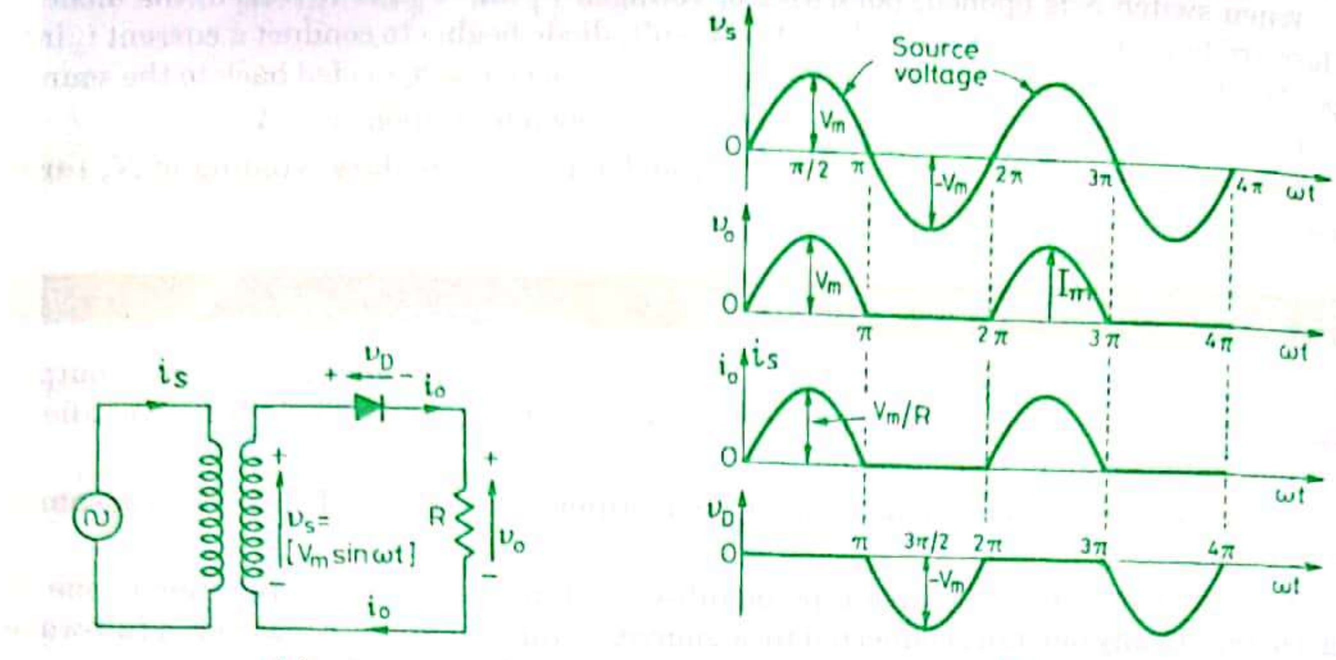
The load on the output side of rectifier may be R, RL or RL with a flywheel diode. These are now discussed briefly.

(*a*) **R load :** The circuit diagram of a single-phase half-wave rectifier is shown in Fig. 3.14 (*a*). During the positive half cycle, diode is forward biased, it therefore conducts from $\omega t = 0°$ to $\omega t = \pi$. During the positive half cycle, output voltage v_0 = source voltage v_s and load current $i_0 = v_0/R$. At $\omega t = \pi$, $v_0 = 0$ and for R load, i_0 is also zero. As soon as v_s tends to become negative after $\omega t = \pi$, diode D is reverse biased, it is therefore turned off and goes into blocking state. Output voltage, as well as output current, are zero from $\omega t = \pi$ to $\omega t = 2\pi$. After $\omega t = 2\pi$, diode is again forward biased, conduction begins and the cycle repeats.

For a resistive load, output current i_0 has the same waveform as that of the output voltage v_0. Diode voltage v_D is zero when diode conducts. Diode is reverse biased from $\omega t = \pi$ to $\omega t = 2\pi$ as shown. The waveforms of v_s, v_0, i_0, and v_D are sketched in Fig. 3.14 (*b*). Here source voltage is sinusoidal *i.e.* $v_s = V_m \sin \omega t$. KVL for the circuit of Fig. 3.14 (*a*) gives $v_s = v_0 + v_D$.

Average value of output (or load) voltage,

$$V_0 = \frac{1}{2\pi}\left[\int_0^{\pi} V_m \sin \omega t \, d(\omega t)\right]$$



(a) (b)

Fig. 3.14. Single-phase half-wave diode rectifier with *R* load (*a*) circuit diagram and (*b*) waveforms.

$$= \frac{V_m}{2\pi} \left| -\cos \omega t \right|_0^{\pi} = \frac{V_m}{\pi} \qquad \text{...(3.21)}$$

Rms value of output voltage,
$$V_{or} = \left[\frac{1}{2\pi} \int_0^{\pi} V_m^2 \sin^2 \omega t \, . \, d(\omega t) \right]^{1/2}$$
$$= \frac{V_m}{\sqrt{2\pi}} \left[\int_0^{\pi} \frac{1 - \cos 2\omega t}{2} \, . \, d(\omega t) \right]^{1/2}$$
$$= \frac{V_m}{2} \qquad \text{...(3.22)}$$

Here the subscript '*r*' is used to denote rms value.

Average value of load current,

$$I_0 = \frac{V_0}{R} = \frac{V_m}{\pi R} \qquad \text{...(3.23)}$$

Rms value of load current, $$I_{or} = \frac{V_{or}}{R} = \frac{V_m}{2R} \qquad \text{...(3.24)}$$

Peak value of load, or diode, current

$$= \frac{V_m}{R} \qquad \text{...(3.25)}$$

Peak inverse voltage, PIV, is an important parameter in the design of rectifier circuits. PIV is the maximum voltage that appears across the device (here diode) during its blocking state. In Fig. 3.14, PIV = $V_m = \sqrt{2} \, . \, V_s = \sqrt{2}$ (rms value of transformer secondary voltage). It is seen from the waveform of source current i_s (or i_o) that the transformer has to handle dc component of i_s. It leads to magnetic saturation of the transformer core, therefore more iron losses, more transformer heating and reduced efficiency.

Power delivered to resistive load = (rms load voltage) (rms load current)

$$= V_{or} I_{or} = \frac{V_m}{2} \cdot \frac{V_m}{2R} = \frac{V_m^2}{4R} = \frac{V_s^2}{2R} = I_{or}^2 R \qquad ...(3.26)$$

Input power factor $= \dfrac{\text{Power delivered to load}}{\text{Input } VA}$

$$= \frac{V_{or} \cdot I_{or}}{V_s \cdot I_{or}} = \frac{V_{or}}{V_s} = \frac{\sqrt{2} V_s}{2 V_s} = 0.707 \text{ lag.}$$

(*b*) **L load :** Single-phase half-wave diode rectifier with L load is shown in Fig. 3.15 (*a*). When switch S is closed at $\omega t = 0$, diode starts conducting. KVL for this circuit gives

$$v_s = v_0 = L \frac{di_0}{dt} = V_m \sin \omega t$$

or

$$i_0 = \frac{V_m}{L} \int \sin \omega t \, . \, dt$$

$$= -\frac{V_m}{\omega L} \cos \omega t + A \qquad ...(3.27a)$$

At $\omega t = 0, i_0 = 0,\quad \therefore\quad 0 = -\dfrac{V_m}{\omega L} + A$

or

$$A = V_m / \omega L$$

Substitution of the value of A in Eq. (3.27*a*) gives

$$i_0 = \frac{V_m}{\omega L} (1 - \cos \omega t) \qquad ...(3.27b)$$

Output voltage, $\quad v_0 = L \dfrac{di_o}{dt} = L \dfrac{V_m}{\omega L} [\sin \omega t] \, \omega = V_m \sin \omega t = v_s$

Source voltage v_s and both output voltage v_0 and output current i_0 are plotted in Fig. 3.15 (*b*).

Average value of output voltage, $V_0 = 0$

The output of current i_0 consists of dc component and fundamental frequency component of frequency ω.

Peak value of current I_{max} occurs at $\omega t = \pi$

$$\therefore \qquad I_{max} = \frac{V_m}{\omega L} (1 + 1) = \frac{2 V_m}{\omega L} \qquad ...(3.28)$$

Average value of current, $I_0 = \dfrac{1}{2\pi} \displaystyle\int_0^{2\pi} \frac{V_m}{\omega L} (1 - \cos \omega t) d(\omega t)$

$$= \frac{V_m}{\omega L} = \frac{1}{2} I_{max} \qquad ...(3.29)$$

Rms value of fundamental current, I_{1r} is given by

$$I_{1r} = \left[\frac{1}{2\pi} \left(\frac{V_m}{\omega L} \right)^2 \int_0^{2\pi} (\cos \omega t)^2 \, d(\omega t) \right]^{1/2}$$

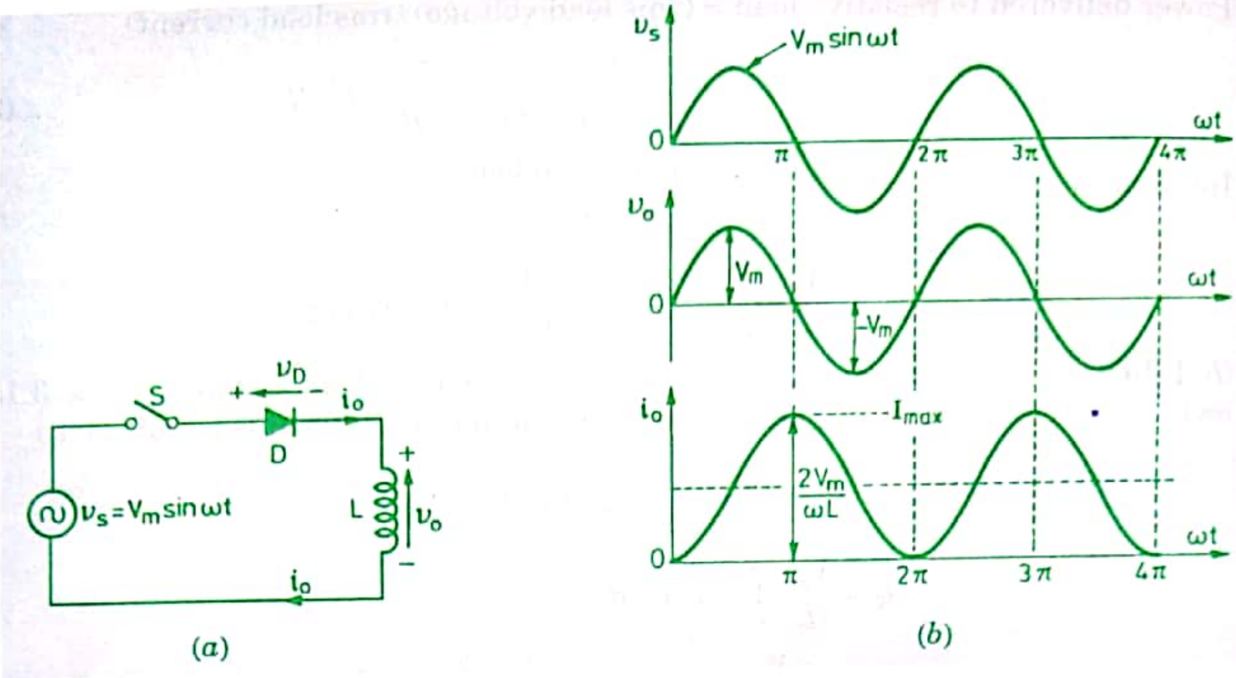


Fig. 3.15. Single-phase one-pulse rectifier with *L* load (*a*) circuit diagram and (*b*) waveforms.

$$= \frac{V_m}{\sqrt{2}\,.\,\omega L} = \frac{V_s}{\omega L} = \frac{I_0}{\sqrt{2}} \quad ...(3.30)$$

Rms value of rectified current $= \left[I_0^2 + I_{1r}^2 \right]^{1/2}$

$$= \left[I_0^2 + \frac{I_0^2}{2} \right]^{1/2} = 1.225\, I_0 \quad ...(3.31)$$

Voltage across diode, $v_D = 0$.

(*c*) ***C* load :** In Fig. 3.16 (*a*), when switch *S* is closed at $\omega t = 0$, the equation governing the behaviour of the circuit is

$$i_0 = C \frac{dv_s}{dt} = C \frac{d}{dt} (V_s \sin \omega t)$$
$$= \omega C\, V_m \cos \omega t \quad ...(3.32)$$

Output voltage, $v_0 = \frac{1}{C} \int i dt = V_m \sin \omega t = v_s = v_C$

Capacitor is charged to voltage V_m at $\omega t = \frac{\pi}{2}$ and subsequently this voltage remains constant at V_m. This is shown as $v_0 = v_c$ in Fig. 3.16 (*b*).

Capacitor current or load current is maximum at $\omega t = 0$. Its value at $\omega t = 0$ is $\omega C V_m$ as shown.

The diode conducts for $\frac{\pi}{2\omega}$ seconds only from $\omega t = 0$ to $\omega t = \frac{\pi}{2}$. During this interval, diode voltage is, therefore, zero. After $\omega t = \pi/2$, diode voltage v_D is given by

$$v_D = -v_0 + v_s = -V_m + V_m \sin \omega t$$
$$= V_m(\sin \omega t - 1) \quad ...(3.33)$$

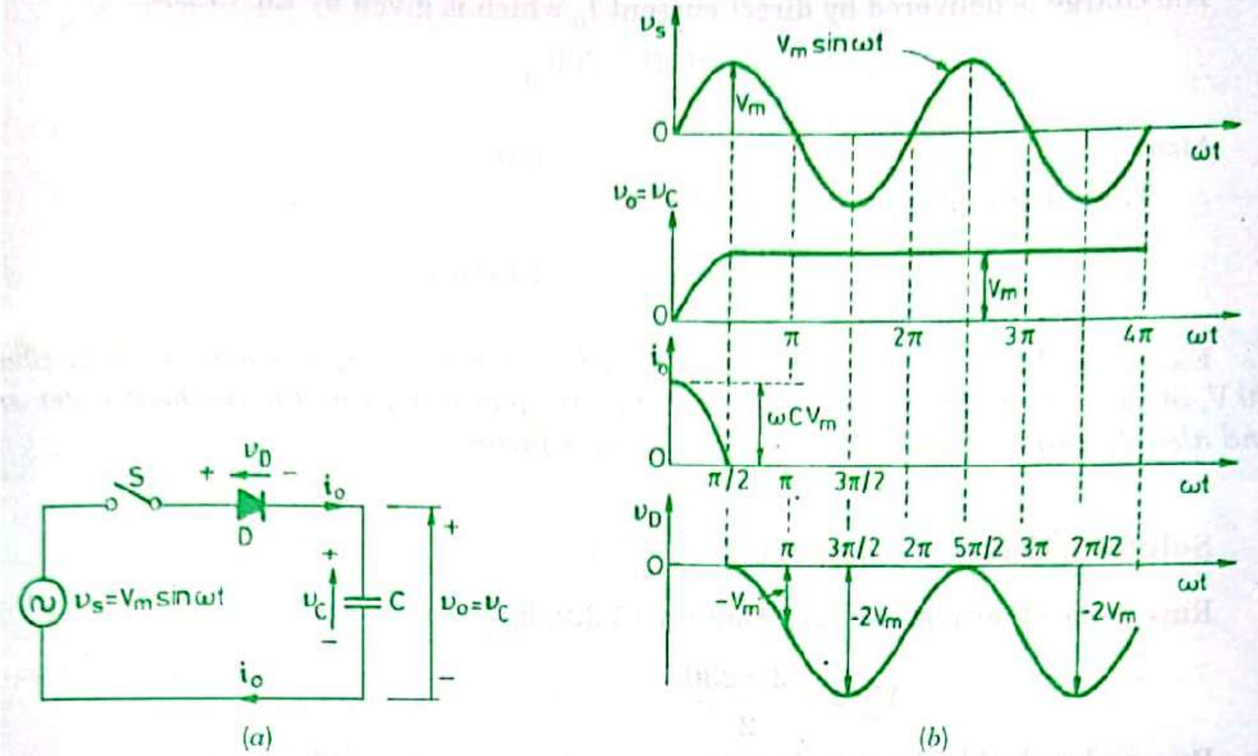


Fig. 3.16. Single-phase half-wave diode rectifier with *C* load (*a*) circuit diagram and (*b*) waveforms.

For Eq. (3.33), the time origin is redefined at $\omega t = \pi/2$.

After $\omega t = \pi/2$, diode voltage is plotted as shown in Fig. 3.16 (*b*). At $\omega t = \dfrac{3\pi}{2}$, $v_D = -2V_m$.

Average value of voltage across diode,

$$V_D = \frac{1}{2\pi}\int_0^{2\pi} V_m (\sin \omega t - 1) d(\omega t)$$

$$= V_m = \sqrt{2}\, V_s \qquad \text{...(3.34a)}$$

Rms value of fundamental component of voltage across diode,

$$V_{1r} = \left[\frac{1}{2\pi}\int_0^{2\pi} V_m^2 \sin^2 \omega t\, d(\omega t)\right]^{1/2} = \frac{V_m}{\sqrt{2}} \qquad \text{...(3.34b)}$$

Rms value of voltage across diode

$$= \sqrt{V_D^2 + V_{1r}^2} = 1.225\, V_m. \qquad \text{...(3.35)}$$

Example 3.7. *Find the time required to deliver a charge of 200 Ah through a single-phase half-wave diode rectifier with an output current of 100 A rms and with sinusoidal input voltage. Assume diode conduction over a half-cycle.*

Solution. For 1-phase half-wave diode rectifier, rms value of output current, from Eq. (3.24), is

$$I_{or} = \frac{V_m}{2R} = 100 \text{ A or } V_m = 200 \text{ R}$$

The charge is delivered by direct current I_0 which is given by Eq. (3.23).

$$\therefore \quad I_0 = \frac{V_m}{\pi R} = \frac{200\,R}{\pi R} = \frac{200}{\pi}\,\text{A}$$

Also $\quad I_0 \times \text{time in hours} = 200$ Ah

$\therefore$ Time required to deliver this charge

$$= \frac{200 \times \pi}{200}\ \text{hrs} = \pi = 3.1416\ \text{hrs.}$$

Example 3.8. *A single-phase 230 V, 1 kW heater is connected across single-phase 230 V, 50 Hz supply through a diode. Calculate the power delivered to the heater element. Find also the peak diode current and input power factor.*

Solution. Heater resistance, $R = \dfrac{230^2}{1000}\ \Omega$

Rms value of output voltage, from Eq. (3.22), is

$$V_{or} = \frac{\sqrt{2} \times 230}{2}$$

Power absorbed by heater element

$$= \frac{V_{or}^2}{R} = \frac{2 \times 230^2}{4} \times \frac{1000}{230^2} = 500\ \text{W}$$

Peak value of diode current, from Eq. (3.25), is given by

$$\frac{\sqrt{2} \times 230}{230^2} \times 1000 = 6.1478\ \text{A}$$

Input power factor $\quad = \dfrac{V_{or}}{V_s} = \dfrac{\sqrt{2} \times 230}{2} \times \dfrac{1}{230} = 0.707$ lag.

(*d*) **RE Load :** Single-phase half-wave diode rectifier with load resistance R and load counter emf E is shown in Fig. 3.17 (*a*). If the switch S is closed at $\omega t = 0°$ or when $v_s = 0$, then diode would not conduct at $\omega t = 0$ because diode is reverse biased until source voltage v_s equals E. When $V_m \sin \theta_1 = E$, diode D starts conducting and the turn-on angle θ_1 is given by

$$\theta_1 = \sin^{-1}\left(\frac{E}{V_m}\right) \qquad \text{...(3.36)}$$

The diode now conducts from $\omega t = \theta_1$ to $\omega t = (\pi - \theta_1)$, *i.e.* conduction angle for diode is $(\pi - 2\theta_1)$ as shown in Fig. 3.17 (*b*). During the conduction period of diode, the voltage equation for the circuit is

$$V_m \sin \omega t = E + i_0 R$$

or

$$i_0 = \frac{V_m \sin \omega t - E}{R} \qquad \text{...(3.37)}$$

Average value of this current is given by

$$I_0 = \frac{1}{2\pi R}\left[\int_{\theta_1}^{\pi - \theta_1} (V_m \sin \omega t - E)\, d(\omega t)\right]$$

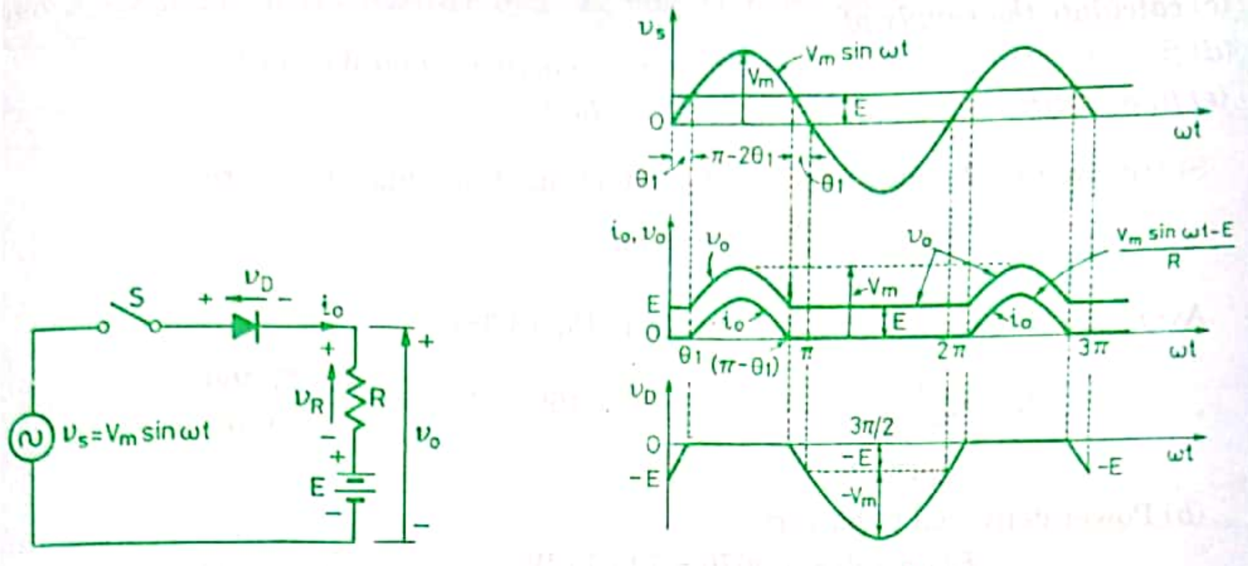


Fig. 3.17. Single-phase half-wave diode rectifier with *RE* load (*a*) circuit diagram and (*b*) waveforms.

$$= \frac{1}{2\pi R}\left[2V_m \cos\theta_1 - E(\pi - 2\theta_1)\right] \qquad \text{...(3.38)}$$

Rms value of the load current of Eq. (3.37) is

$$I_{or} = \left[\frac{1}{2\pi}\int_{\theta_1}^{\pi-\theta_1}\left(\frac{V_m \sin\omega t - E}{R}\right)^2 . d(\omega t)\right]^{1/2}$$

$$= \left[\frac{1}{2\pi R^2}\int_{\theta_1}^{\pi-\theta_1}(V_m^2 \qquad\qquad - 2V_m E \sin\omega t)\, d(\omega t)\right]^{1/2}$$

$$= \left[\frac{1}{2\pi R^2}\left\{(V_s^2 + E^2)(\pi - 2\theta_1) + V_s^2 \sin 2\theta_1 - 4V_m E\cos\theta_1\right\}\right]^{1/2} \qquad \text{...(3.39)}$$

Power delivered to load,

$$P = EI_0 + I_{or}^2 R \text{ watts} \qquad \text{...(3.40)}$$

$$\text{Supply pf} = \frac{\text{Power delivered to load}}{\text{(Source voltage)(rms value of source current)}}$$

$$= \frac{EI_0 + I_{or}^2 R}{V_s \,.\, I_{or}} \qquad \text{...(3.41)}$$

It is seen from Fig. 3.17 (*a*) that at $\omega t = 0°$, $v_D = -E$ and at $\omega t = \theta_1$, $v_D = 0$. During the period diode conducts, $v_D = 0$. When $\omega t = 3\pi/2$, $v_s = -V_m$ and $v_D = -(V_m + E)$. Thus PIV for diode is $(V_m + E)$.

Example 3.9. *A dc battery of constant emf E is being charged through a resistor as shown in Fig. 3.17 (a). For source voltage of 230 V, 50 Hz and for R = 8 Ω, E = 150 V*

(a) find the value of average charging current,

(b) find the power supplied to battery and that dissipated in the resistor,

(c) calculate the supply pf,
(d) find the charging time in case battery capacity is 1000 Wh, and
(e) find rectifier efficiency and PIV of the diode.

Solution. (*a*) The diode will start conducting at an angle θ_1, where

$$\theta_1 = \sin^{-1} \frac{150}{\sqrt{2} \times 230} = 27.466°$$

Average value of charging current, from Eq. (3.38), is

$$I_0 = \frac{1}{2\pi \times 8}\left[2 . \sqrt{2} \times 230 \cos 27.466° - 150\left(\pi - \frac{2 \times 27.466 \times \pi}{180}\right)\right]$$
$$= 4.9676 \text{ A}$$

(*b*) Power delivered to battery

$$= EI_0 = 150 \times 4.9676 = 745.14 \text{ W}$$

Rms value of charging current, from Eq. (3.39), is

$$I_{or} = \left[\frac{1}{2\pi \times 64}\left\{(230^2 + 150^2)\left(\pi - 2 \times 27.466 \times \frac{\pi}{180}\right) + 230^2 \sin 2 \times 27.466° - 4\sqrt{2} \times 230 \times 150 \cos 27.466°\right\}\right]^{1/2} = 9.2955 \text{ A}$$

Power dissipated in resistor

$$= I_{or}^2 R = (9.2955)^2 \times 8 = 691.25 \text{ W}$$

(*c*) From Eq. (3.41), the supply

$$pf = \frac{745.14 + 691.25}{230 \times 9.2955} = 0.672 \text{ lag}$$

(*d*) (Power delivered to battery) (charging time in hours)
= Battery capacity in Wh

$$\therefore \quad \text{Charging time} = \frac{1000}{745.14} = 1.342 \text{ h}$$

(*e*) Rectifier efficiency $= \dfrac{\text{Power delivered to battery}}{\text{Total input power}}$

$$= \frac{745.14}{745.14 + 691.25} \times 100 = 51.876\%$$

(*f*) PIV of diode $= V_m + E = \sqrt{2} \times 230 + 150 = 475.22$ V.

(*e*) **RL load :** A single-phase one-pulse diode rectifier feeding *RL* load is shown in Fig. 3.18 (*a*). Current i_0 continues to flow-even after source voltage v_s has become negative; this is because of the presence of inductance *L* in the load circuit. After positive half cycle of source voltage, diode remains on, so the negative half cycle of source voltage appears across load until load current i_0 decays to zero at $\omega t = \beta$. Voltage $v_R = i_0 R$ has the same waveshape as that of i_0. Inductor voltage $v_L = v_s - v_R$ is also shown. The current i_0 flows till the two areas *A* and *B* are equal. Area *A* (where $v_s > v_R$) represents the energy stored by *L* and area *B* (where $v_s < v_R$) the energy released by L. It must be noted that average value of voltage v_L across inductor L is zero.

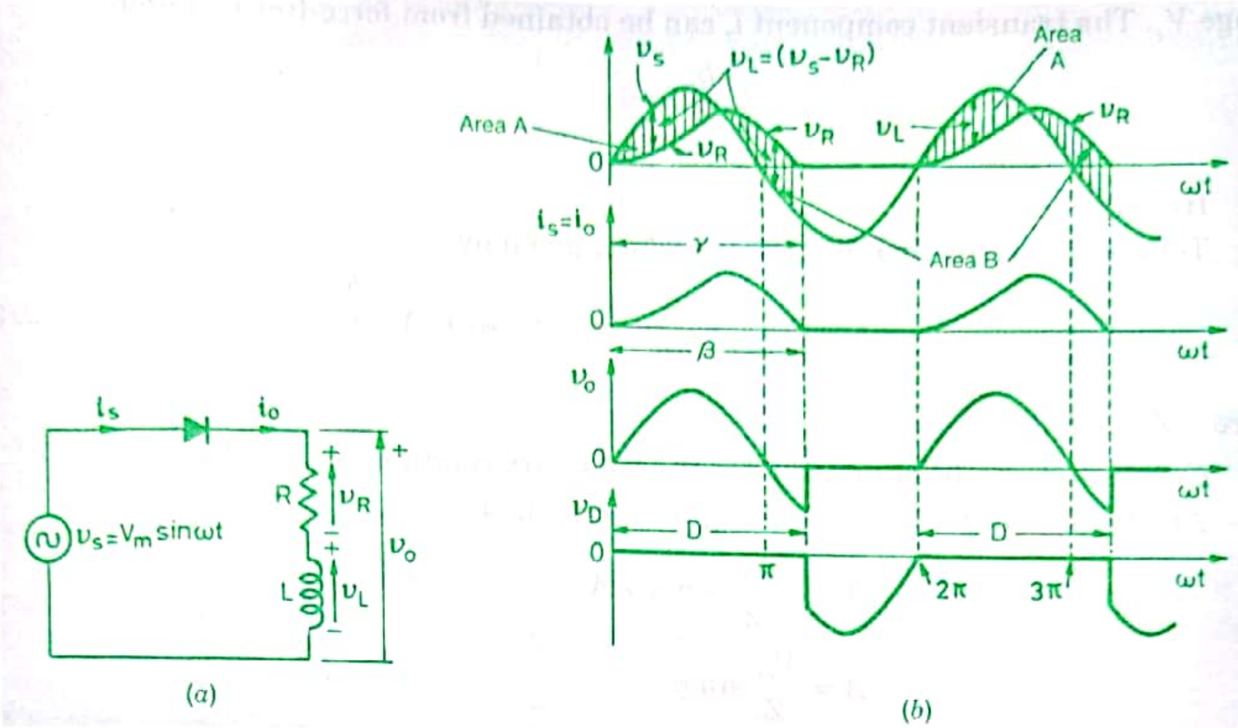


Fig. 3.18. Single-phase half-wave diode rectifier with *RL* load (*a*) circuit diagram and (*b*) waveforms.

When $i_0 = 0$ at $\omega t = \beta$; $v_L = 0$, $v_R = 0$ and voltage v_s appears as reverse bias across diode *D* as shown. At β, voltage v_D across diode jumps from zero to $V_m \sin\beta$ where $\beta > \pi$. Here $\beta = \gamma$ is also the conduction angle of the diode.

Average value of output voltage,

$$V_0 = \frac{1}{2\pi}\int_0^{\beta} V_m \sin\omega t \,.\, d(\omega t)$$

$$= \frac{V_m}{2\pi}(1 - \cos\beta) \quad \text{...(3.42)}$$

Average value of load or output current

$$I_0 = \frac{V_0}{R} = \frac{V_m}{2\pi R}(1 - \cos\beta) \quad \text{...(3.43)}$$

A general expression for output current i_0 for $0 < \omega t < \beta$ can be obtained as under :

When diode is conducting, KVL for the circuit of Fig. 3.18 (*a*) gives

$$Ri_0 + L\frac{di_0}{dt} = V_m \sin\omega t$$

The load, or output, current i_0 consists of two components, one steady state component i_s and the other transient component i_t. Here i_s is given by

$$i_s = \frac{V_m}{\sqrt{R^2 + X^2}} \sin(\omega t - \phi)$$

where $\phi = \tan^{-1} \dfrac{X}{R}$ and $X = \omega L$. Here ϕ is the angle by which rms current I_s lags source voltage V_s. The transient component i_t can be obtained from force-free equation

$$Ri_t + L\frac{di_t}{dt} = 0$$

Its solution gives $\qquad i_t = Ae^{-\frac{R}{L}t}$

Total solution for current i_0 is, therefore, given by

$$i_0 = i_s + i_t = \frac{V_m}{Z}\sin(\omega t - \phi) + Ae^{-\frac{R}{L}t} \qquad ...(3.44)$$

where $\quad Z = \sqrt{R^2 + X^2}$

Constant A can be obtained from the boundary condition at $\omega t = 0$.

At $\omega t = 0$, or at $t = 0$, $i_0 = 0$. Thus, from Eq. (3.44)

$$0 = -\frac{V_m}{Z}\sin\phi + A$$

$$\therefore \qquad A = \frac{V_m}{Z}\sin\phi$$

Substitution of A in Eq. (3.44) gives

$$i_0 = \frac{V_m}{Z}\left[\sin(\omega t - \phi) + \sin\phi \,.\, e^{-\frac{R}{L}t}\right] \qquad ...(3.45)$$

$$\text{for } 0 \le \omega t \le \beta$$

It is also seen from the waveform of i_0 in Fig. 3.18 (b) that when $\omega t = \beta$, $i_0 = 0$. With this condition, Eq. (3.45) gives

$$\sin(\beta - \phi) + \sin\phi \,.\, \exp\left[-\frac{R}{\omega L}\beta\right] = 0$$

The solution of this transcendental equation can give the value of extinction angle β.

(f) **RL load with freewheeling diode***: Performance of single-phase one-pulse diode rectifier with RL load can be improved by connecting a freewheeling diode across the load as shown in Fig. 3.19 (a). Output voltage is $v_0 = v_s$ for $0 \le \omega t \le \pi$. At $\omega t = \pi$, source voltage v_s is zero, but output current i_0 is not zero because of L in the load circuit. Just after $\omega t = \pi$, as v_s tends to reverse, negative polarity of v_s reaches cathode of FD through conducting diode D, whereas positive polarity of v_s reaches anode of FD direct. Freewheeling (or flywheel) diode FD, therefore, gets forward-biased. As a result, load current i_0 is immediately transferred from D to FD as v_s tends to reverse. After $\omega t = \pi$, diode, or source, current $i_s = 0$ and diode D is subjected to reverse voltage with PIV equal to V_m at $\omega t = \dfrac{3\pi}{2}, \dfrac{7\pi}{2}$ etc.

After $\omega t = \pi$, current freewheels through circuit R, L and FD. The energy stored in L is now dissipated in R. When energy stored in L = energy dissipated in R, current falls to zero

*Freewheeling diode is also called bypass diode or commutating diode.

at $\omega t = \beta < 2\pi$. Depending upon the value of R and L, the current may not fall to zero even when $\omega t = 2\pi$, this is called continuous conduction. But in Fig. 3.19 (*b*), load current decays to zero before $\omega t = 2\pi$; load current is therefore discontinuous.

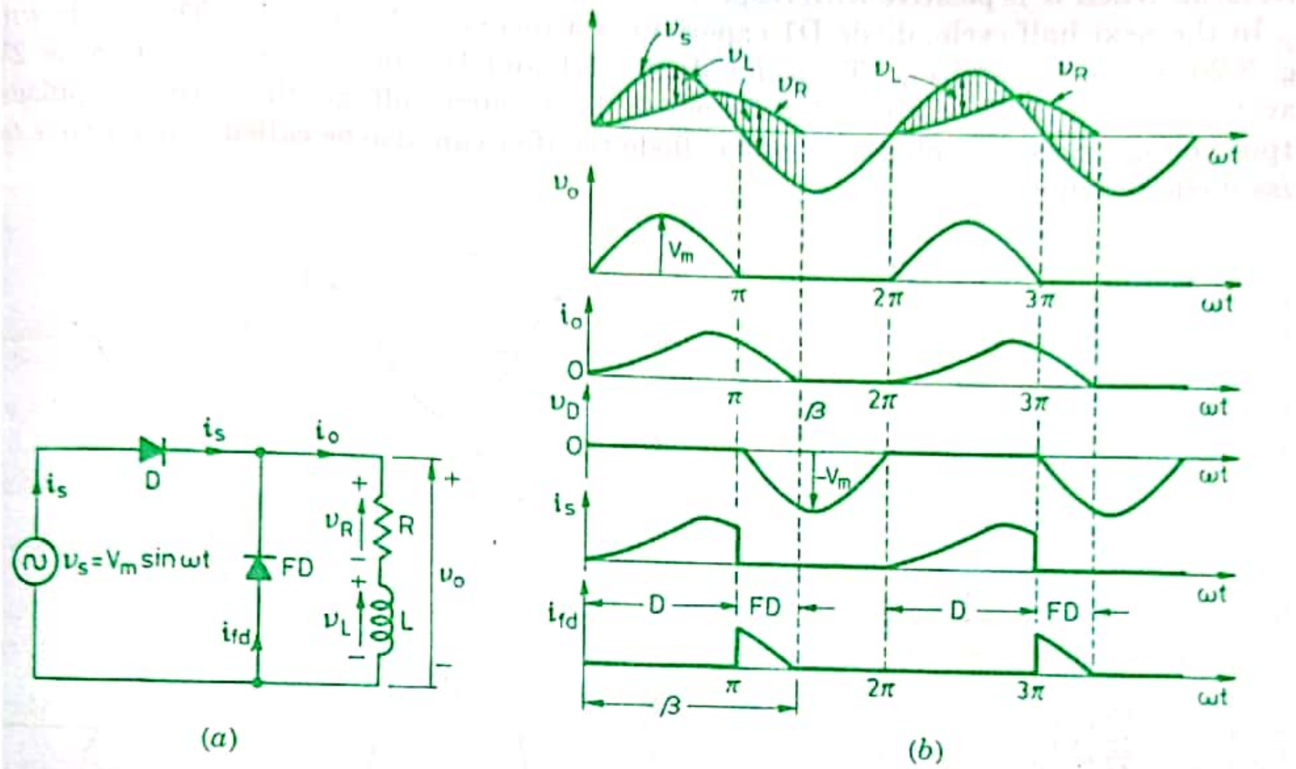


Fig. 3.19. Single-phase one-pulse diode rectifier with *RL* load and freewheeling diode (*a*) circuit diagram and (*b*) waveforms.

The effects of using freewheeling diode are as under :

(*i*) It prevents the output (or load) voltage from becoming negative.

(*ii*) As the energy stored in L is transferred to load R through FD, the system efficiency is improved.

(*iii*) The load current waveform is more smooth, the load performance, therefore, gets better.

The waveforms for v_s, v_0, i_0, v_D, i_s and i_{fd} are drawn in Fig. 3.19 (*b*).

The expression for the load current i_0 can be obtained from Art. 6.1.2 if required. It is seen from Fig. 3.19 (*b*) that

average output voltage,
$$V_0 = \frac{1}{2\pi}\int_0^{\pi} V_m \sin \omega t \, d(\omega t) = \frac{V_m}{\pi} \quad ...(3.46)$$

and average load current,
$$I_0 = \frac{V_m}{\pi R} \quad ...(3.47)$$

(*g*) **Single-phase full-wave diode rectifier :** There are two types of full-wave diode rectifiers, one is centre-tapped (or mid-point) full-wave diode rectifier and the other is full-wave diode bridge rectifier. These are now described briefly :

(*i*) **Single-phase full-wave mid-point diode rectifier :** Fig. 3.20 (*a*) illustrates a single-phase full-wave mid-point rectifier using diodes. The turns ratio from each secondary to primary is taken as unity for simplicity. When '*a*' is positive with respect to '*b*', or mid-point O, diode D1 conducts for π radians. In the next half cycle, 'b' is positive with respect to 'a', or mid-point O, and therefore diode D2 conducts. The output voltage is shown as v_0 in Fig. 3.20 (b). The waveform for output current i_0 (not shown in the figure) is similar to v_0 waveform. When 'a' is positive with respect to 'b', diode D2 is subjected to a reverse voltage of $2v_s$. In the next half cycle, diode D1 experiences a reverse voltage of $2v_s$. This is shown in Fig. 3.20 (b) as v_{D1} and v_{D2}. Thus, for diodes D1 and D2, peak inverse voltage is $2V_m$. Waveforms of Fig. 3.20 (b) show that for one cycle of source voltage, there are two pulses of output voltage. So single-phase full-wave diode rectifier can also be called *single-phase two-pulse* diode rectifier.

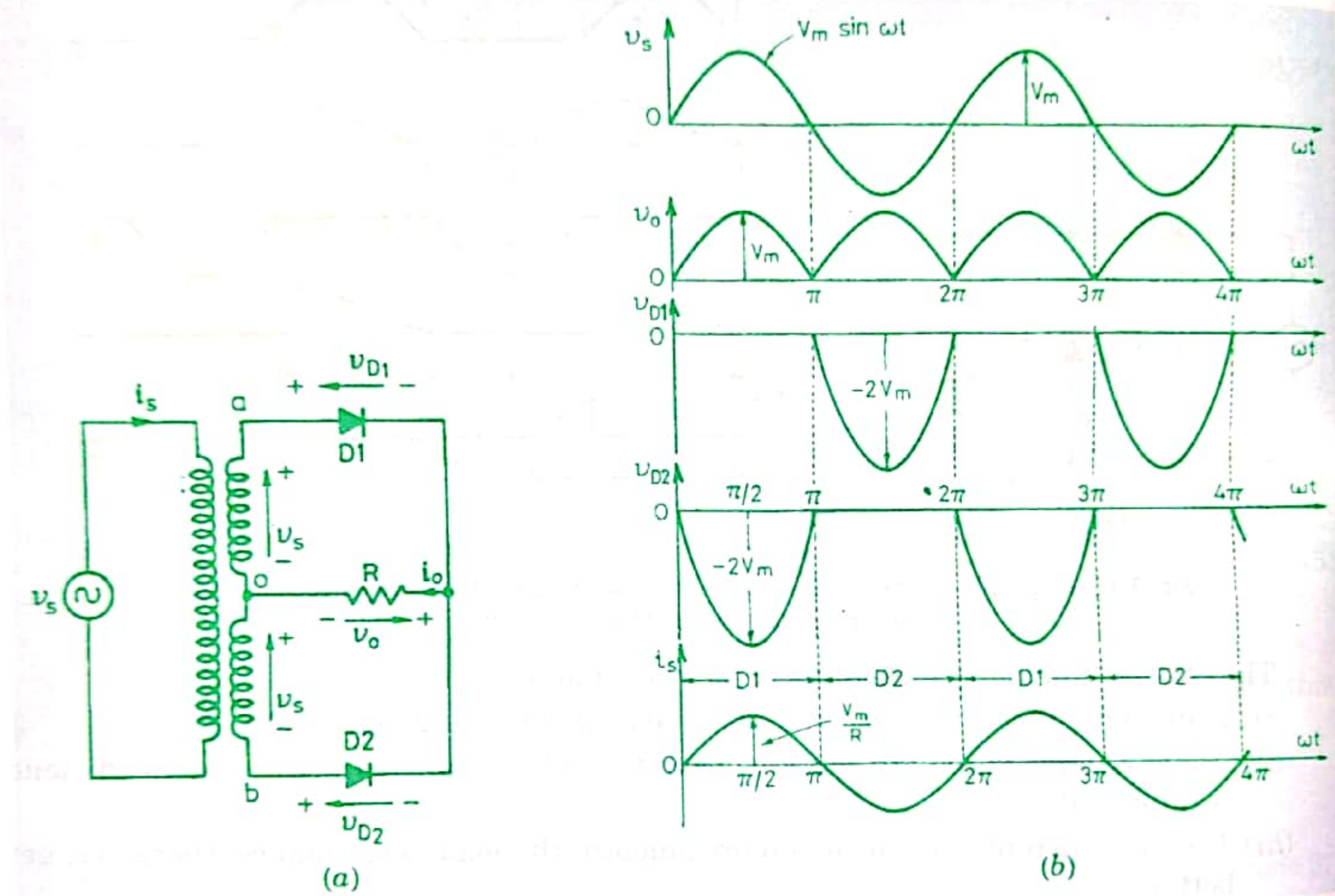


Fig. 3.20. Single-phase full-wave mid-point diode rectifier (a) circuit diagram and (b) waveforms.

Source current waveform i_s is also shown in Fig. 3.20 (b). When D1 conducts, current in secondary flows upward from o to a and therefore, primary current i_s must flow *downward* to balance the secondary mmf from 0 to π rad. When D2 conducts, secondary current flows downward from o to b, therefore, primary current i_s must flow *upward* to balance the secondary mmf from π to 2π and so on.

Average output voltage, $$V_0 = \frac{1}{\pi}\int_0^{\pi} V_m \sin \omega t \, d(\omega t) = \frac{2V_m}{\pi} \quad ...(3.48a)$$

Average output current, $$I_0 = \frac{V_0}{R}$$

Rms value of output voltage, $$V_{or} = \left[\frac{1}{\pi}\int_0^{\pi} V_m^2 \sin^2 \omega t \, d(\omega t)\right]^{1/2}$$

$$= \frac{V_m}{\sqrt{2}} = V_s \qquad ...(3.48b)$$

Rms value of load current, $I_{or} = \dfrac{V_s}{R}$

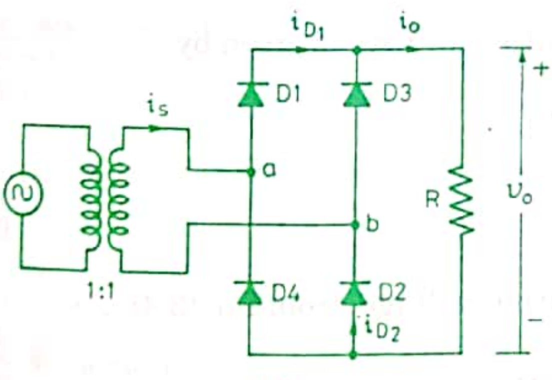
Power delivered to load $= V_{or} \cdot I_{or} = I_{or}^2 \cdot R$

Input volt-amperes $= V_s \cdot I_{or}$

$\therefore$ Input $pf = \dfrac{V_{or} \cdot I_{or}}{V_s \cdot I_{or}} = 1$

(*ii*) **Single-phase full-wave diode bridge rectifier :** A single-phase full-wave bridge rectifier employing diodes is shown in Fig. 3.21 (*a*). When '*a*' is positive with respect to '*b*', diodes D1, D2 conduct together so that output voltage is v_{ab}. Each of the diodes D3 and D4 is subjected to a reverse voltage of v_s as shown in Fig. 3.21 (*b*). When '*b*' is positive with respect to '*a*', diodes D3, D4 conduct together and output voltage is v_{ba}. Each of the two diodes D1 and D2 experience a reverse voltage of v_s as shown.

A comparison of Figs. 3.20 (*b*) and 3.21 (*b*) reveals that a diode in mid-point full-wave rectifier is subjected to PIV of $2V_m$ whereas a diode in full-wave bridge rectifier has PIV of V_m only. However, average and rms values of output voltage are the same for both rectifier configurations.



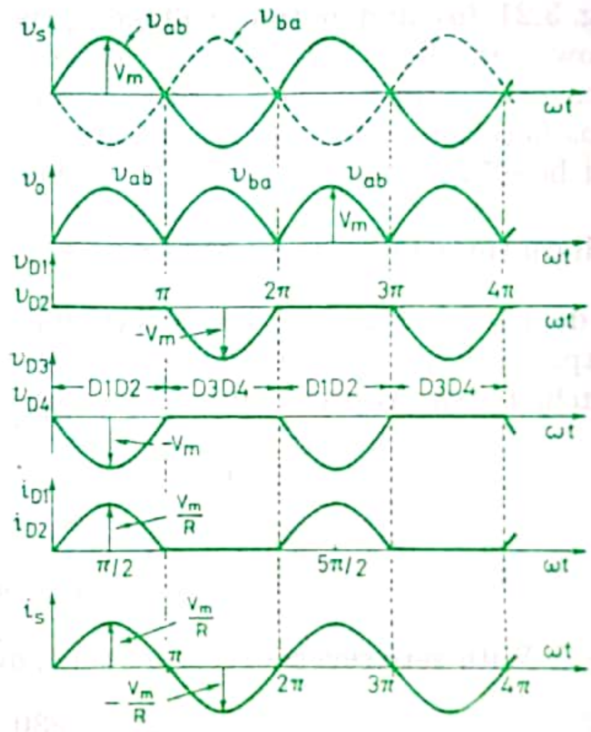


Fig. 3.21. Single-phase full-wave diode bridge rectifier (*a*) circuit diagram and (*b*) waveforms.

For the waveforms of diode current i_{D1} or i_{D2} in Fig. 3.21 (*b*) and also for i_{D3}, i_{D4} for the circuit of Fig. 3.20 (*a*) (not shown in Fig. 3.20 (*b*)), the average and rms values for diode current are obtained as under :

Average value of diode current, $I_{DA} = \dfrac{1}{2\pi}\displaystyle\int_0^{\pi} I_m \sin \omega t \, . \, d(\omega t) = \dfrac{I_m}{\pi}$ ...(3.49*a*)

Rms value of diode current, $I_{Dr} = \left[\frac{1}{2\pi}\int_0^{\pi} I_m^2 \sin^2 \omega t\, d(\omega t)\right]^{1/2} = \frac{I_m}{2}$ ...(3.49b)

Peak repetitive diode current, $I_m = \frac{V_m}{R}$ ...(3.49c)

It can similarly be shown that average value of voltage across each diode in Fig. 3.20 (b) is $\frac{2V_m}{\pi}$ and that in Fig. 3.21 (b) is $\frac{V_m}{\pi}$. The corresponding rms values of voltage across each diode is $V_m = \sqrt{2}\, V_s$ in Fig. 3.20 (b) and $\frac{V_m}{\sqrt{2}} = V_s$ in Fig. 3.21 (b).

Three-phase rectifiers using diodes are discussed in Art. 3.9. Example 3.10 is formulated to illustrate the effect of reverse recovery time on the average output voltage.

Example 3.10. *In a single-phase full-wave diode bridge rectifier, the diodes have a reverse recovery time of 40 μs. For an ac input voltage of 230 V, determine the effect of reverse recovery time on the average output voltage for a supply frequency of (a) 50 Hz and (b) 2.5 kHz.*

Solution. Single-phase full-wave diode bridge rectifier is shown in Fig. 3.21 (a) and output voltage v_0 is shown in Fig. 3.21 (b). If reverse recovery time is taken into consideration, the diodes D1 and D2 will not be off at $\omega t = \pi$ in Fig. 3.21 (b), but will continue to conduct until $t = \frac{\pi}{\omega} + t_{rr}$ as depicted in Fig. 3.22. The reduction in output voltage is given by the cross-hatched area. Average value of this reduction in output voltage is given by

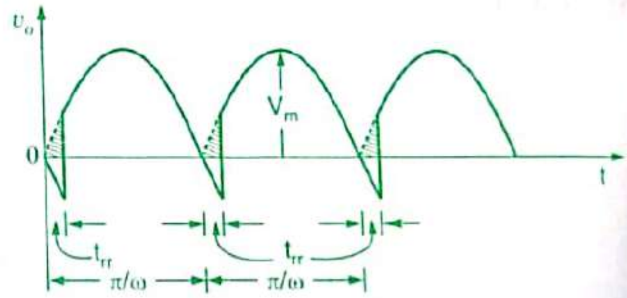


Fig. 3.22. Effect of reverse recovery time on output voltage.

$$V_r = \frac{1}{\pi}\int_0^{t_{rr}} V_m \sin \omega t\, d(\omega t)$$

$$= \frac{V_m}{\pi}(1 - \cos \omega t_{rr}) \qquad ...(3.50)$$

With zero reverse recovery time, average output voltage, from Eq. (3.48), is

$$V_0 = \frac{2\sqrt{2} \times 230}{\pi} = 207.40 \text{ V}$$

(a) For $f = 50$ Hz and $t_{rr} = 40$ μs, the reduction in the average output voltage, from Eq. (3.50), is

$$V_r = \frac{V_m}{\pi}(1 - \cos 2\pi f\, t_{rr})$$

$$= \frac{\sqrt{2} \times 230}{\pi}\left(1 - \cos 2\pi \times 50 \times 40 \times 10^{-6} \times \frac{180}{\pi}\right)$$

$$= 8.174 \text{ mV}$$

Percentage reduction in average output voltage

$$= \frac{8.174 \times 10^{-3}}{207.04} \times 100 = 3.948 \times 10^{-3}\%$$

(*b*) For $f = 2500$ Hz, the reduction in the average output voltage, from Eq. (3.50), is

$$V_r = \frac{\sqrt{2} \times 230}{\pi}\left(1 - \cos 2\pi \times 2500 \times 40 \times 10^{-6} \times \frac{180}{\pi}\right)$$

$$= 19.77 \text{ V}$$

Percentage reduction in average output voltage $= \dfrac{19.77}{207.04} \times 100 = 9.594\%$

It is seen from above that the effect of reverse recovery time is negligible for diode operation at 50 Hz, but for high-frequency operation of diodes, the effect is noticeable.

Example 3.11. *A single-phase full bridge diode rectifier is supplied from 230 V, 50 Hz source. The load consists of R = 10 Ω and a large inductance so as to render the load current constant. Determine :*

(a) average values of output voltage and output current,

(b) average and rms values of diode currents,

(c) rms values of output and input currents, and supply pf.

Solution. The circuit diagram and relevant waveforms for this uncontrolled rectifier are shown in Fig. 3.23.

(*a*) Average value of output voltage,

$$V_0 = \frac{2V_m}{\pi} = \frac{2\sqrt{2} \times 230}{\pi} = 207.04 \text{ V}$$

Average value of output current,

$$I_0 = \frac{V_0}{R} = \frac{207.04}{10} = 20.704 \text{ A}$$

(*b*) Average value of diode current,

$$I_{DA} = \frac{I_0 . \pi}{2\pi} = \frac{I_0}{2} = \frac{20.704}{2} = 10.352 \text{ A}$$

Rms value of diode current, $I_{Dr} = \sqrt{\dfrac{I_0^2 \pi}{2\pi}} = \dfrac{I_0}{\sqrt{2}} = \dfrac{20.704}{\sqrt{2}} = 14.642$ A

(*c*) As load, or output, current is ripple free, rms value of output current

$= \text{average value of output current} = I_0 = 20.704$ A

Rms value of source current, $I_s = \sqrt{\dfrac{I_0^2 \pi}{\pi}} = I_0 = 20.704$ A

Load power $= V_0 I_0 = 207.04 \times 20.704$ W

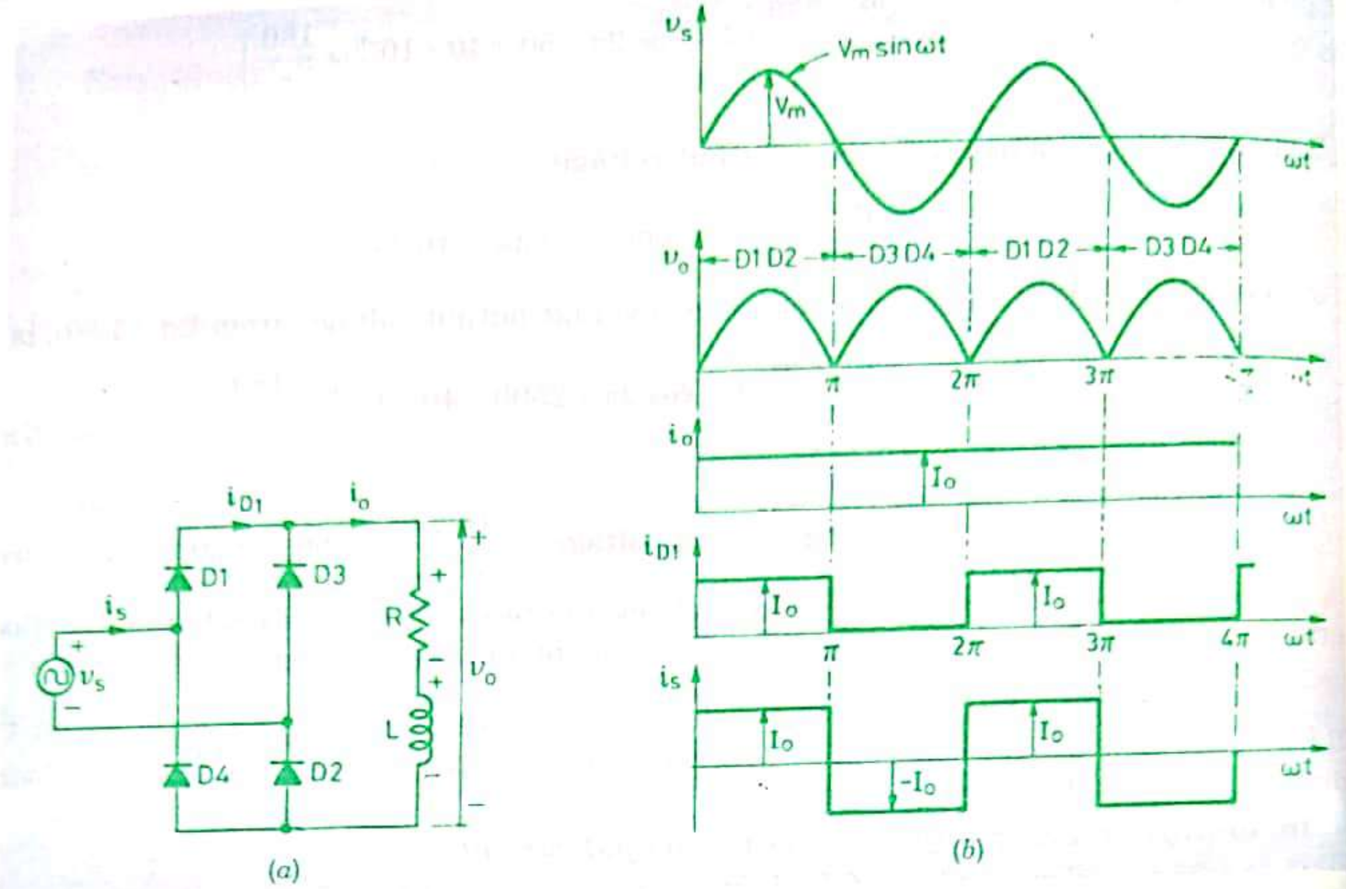


Fig. 3.23. Pertaining to Example 3.11 (*a*) circuit diagram and (*b*) waveforms.

Input power $= V_s I_s \cos\phi$

$\therefore$ $230 \times 20.704 \times \cos\phi = 207.04 \times 20.704$

$\therefore$ Supply *pf* $= \cos\phi = \dfrac{207.04}{230} = 0.90$ lagging.

Example 3.12. *A diode whose internal resistance is 20 Ω is to supply power to a 1000 Ω load from a 230 V (rms) source of supply. Calculate (a) the peak load current (b) the dc load current (c) the dc diode voltage (d) the percentage regulation from no load to given load.* *(I.A.S., 1983)*

Solution. A voltage of 230 V supplying power to 1000 Ω, through a single diode, is shown in Fig. 3.24 (*a*). Waveforms for the source voltage, load current i_0 and diode voltage v_D are shown in Fig. 3.24 (*b*).

(*a*) It is seen from the waveform of i_0 that peak load current I_{om} is given by

$$I_{om} = \frac{V_m}{R + R_D} = \frac{\sqrt{2} \times 230}{1020} = 0.3189 \text{ A}$$

Here R = load resistance and R_D = internal resistance of diode

(*b*) DC load current, $I_0 = \dfrac{1}{2\pi} \displaystyle\int_0^{\pi} I_{om} \sin \omega t \, d(\omega t)$

$$= \frac{I_{om}}{\pi} = 0.10151 \text{ A}$$

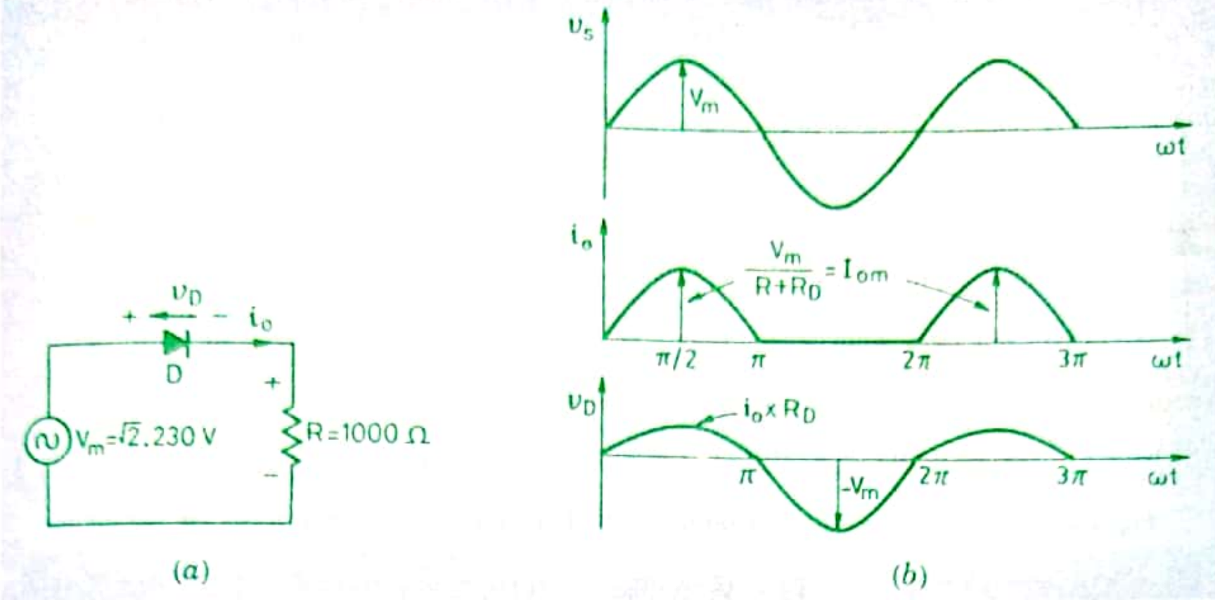


Fig. 3.24. Pertaining to Example 3.12 (*a*) circuit diagram and (*b*) waveforms.

(*c*) DC diode voltage, $$V_D = I_0 R_D - \frac{1}{2\pi}\int_0^{\pi} 230\sqrt{2}\sin\omega t\, d(\omega t)$$

$$= I_0 R_D - \frac{V_m}{\pi} = 0.10151 \times 20 - \frac{230\sqrt{2}}{\pi} = -101.5 \text{ V}$$

(*d*) At no load, load voltage, $$V_{on} = \frac{V_m}{\pi} = \frac{\sqrt{2}\times 230}{\pi} = 103.521 \text{ V}$$

At given load, load voltage, $$V_{01} = \frac{230\sqrt{2}}{\pi}\times\frac{1000}{1020} = 101.491 \text{ V}$$

∴ Voltage regulation $$= \frac{V_{on} - V_{01}}{V_{on}}\times 100 = \frac{103.521 - 101.491}{103.521} = 1.961\%.$$

3.6 ZENER DIODES

Zener diodes are specially constructed to have accurate and stable reverse breakdown voltage.

Circuit symbol for Zener diode is shown in Fig. 3.25 (*a*). When it is forward biased, it behaves as a normal diode. When reverse biased, a small leakage current flows. If the reverse voltage across Zener diode is increased, a value of voltage is reached at which reverse breakdown occurs. This is indicated by a sudden increase of Zener current, Fig. 3.25 (*b*). The voltage after reverse breakdown remains practically constant over a wide range of Zener current. This makes it suitable for use as a voltage regulator to furnish constant voltage from a source whose voltage may vary noticeably.

For the operation of Zener diode as a voltage regulator, (*i*) it must be reverse biased with a voltage greater than its breakdown, or Zener, voltage and (*ii*) a series resistor R_s, Fig. 3.25 (*c*) is necessary to limit the reverse current through the diode below its rated value.

If V_z = voltage across Zener diode, then it is seen from Fig. 3.25 (*c*) that source current I_s is

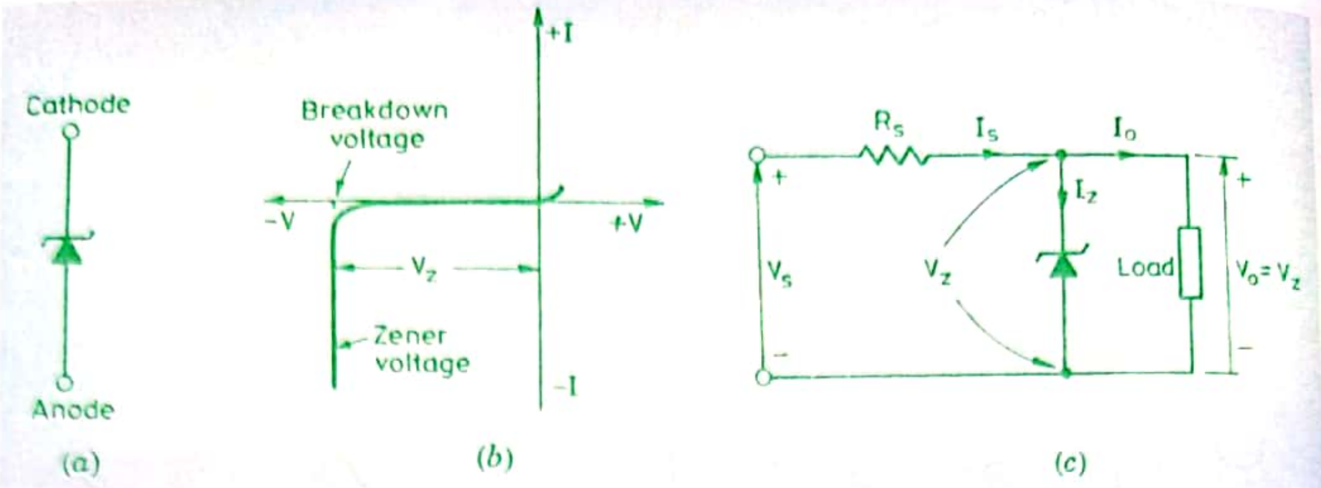


Fig. 3.25. Zener diode (*a*) circuit symbol (*b*) I-V characteristics (*c*) use as a voltage regulator.

$$I_s = \frac{V_s - V_z}{R_s}$$

Load, or output, current, $I_0 = \dfrac{V_z}{R}$ where R = load resistance. Current through Zener diode,

$$I_z = I_s - I_0$$

Power rating of a Zener diode is $V_Z \cdot I_Z$. These are available in a voltage range from few volts to about 280 V.

Example 3.13. *Design a Zener voltage regulator, shown in Fig. 3.26, to meet the following specifications :*

Load voltage = 6.8 V, Source voltage V_s is 20 V ± 20% and load current is 30 mA ± 50%.

The Zener requires a minimum current of 1 mA to breakdown. The diode D has a forward voltage drop of 0.6 V.

Solution. When source voltage is maximum and load current is minimum, then source resistance should be maximum.

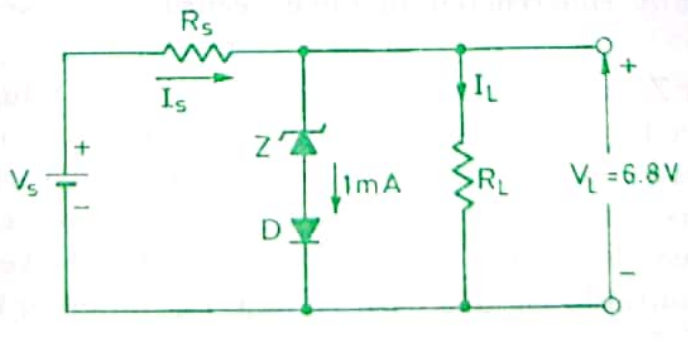


Fig. 3.26. Pertaining to Example 3.13.

$$\therefore \qquad V_{s\,.\,max} = V_L + (I_{L.min} + I_z)\,R_{s\,.\,max}$$

$$\therefore \qquad R_{s\,.\,max} = \frac{(20 \times 1.2) - 6.8}{[30 \times 0.5 + 1] \times 10^{-3}} = 1075\ \Omega$$

Similarly,

$$V_{s\,.\,min} = V_L + (I_{L\,.\,max} + I_z)R_{s\,.\,min}$$

$$\therefore \quad R_{s\,.\,min} = \frac{(20 \times 1.2) - 6.8}{[30 \times 0.5 + 1] \times 10^{-3}} = 200\ \Omega$$

Maximum load resistance,

$$R_{L\,.\,max} = \frac{V_L}{I_{L\,.\,min}} = \frac{6.8}{30 \times 0.5 \times 10^{-3}} = 453.3\ \Omega$$

Minimum load resistance, $R_{L\,.\,min} = \dfrac{V_L}{I_{L\,.\,max}} = \dfrac{6.8}{30 \times 1.5 \times 10^{-3}} = 151.5\ \Omega$

The voltage rating of the Zener diode is

$$6.8 - 0.6 = 6.2\ \text{V}.$$

Example 3.14. *The complete circuit shown in Fig. 3.27 (a) represents a 25 V dc voltmeter where G is a PMMC galvanometer having full-scale deflection current I_{fsd} = 200 micro-A and resistance R_G = 500 ohms, and D is a 20-V Zener diode. Find R_1 and R_2. What is the function of the diode D in this circuit ?* *(GATE, 1990)*

Solution. Current through galvanometer,

$$I_{fsd} = I_2 = \frac{\text{Zener voltage}}{R_2 + R_G}$$

or
$$\frac{20}{R_2 + 500} = 200 \times 10^{-6}$$

or
$$R_2 = \frac{20 \times 10^6}{200} - 500 = 99.5\ \text{k}\Omega$$

As Zener diode current is not specified, let it be assumed zero. Therefore, from Fig. 3.27 (*b*),

$$I_1 - I_2 = I_z = 0 \text{ or } I_1 = I_2 = 200\ \mu\text{A}$$

Also
$$I_1 = \frac{25 - 20}{R_1} = 200 \times 10^{-6}$$

or
$$R_1 = \frac{5 \times 10^6}{200} = 25\ \text{k}\Omega$$

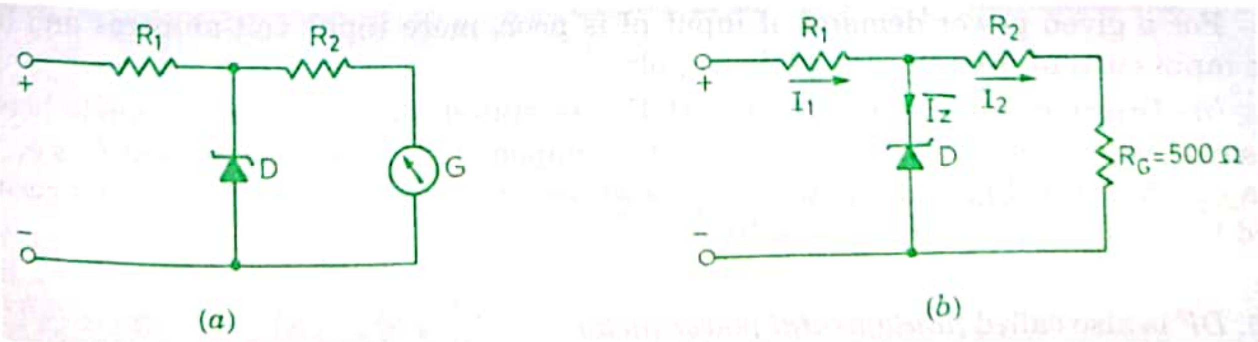


Fig. 3.27. Pertaining to Example 3.14.

Function of Zener diode is to provide a constant voltage to the galvanometer circuit. Whenever voltage across this diode exceeds 20 V, it conducts and the excess current is shunted away from galvanometer G. So here diode D prevents overloading of the PMMC galvanometer.

3.7 PERFORMANCE PARAMETERS

The input voltage to rectifiers is usually sinusoidal. It is desired that the output voltage from a rectifier should be constant with no ripples in it. This, however, is not the case. This shows that the rectified output voltage is made up of constant dc voltage plus harmonic components. The waveform of input and output currents depend on the nature of load and the rectifier configuration. In order to evaluate the overall performance of rectifier-load combinations, certain performance parameters relating to their input and output must be known. The object of this article is to define the various performance parameters (or indices) relating to input as well as output voltages and currents.

3.7.1 Input Performance Parameters

The various parameters relating to the source (or input) side of the converter-load combination are defined below :

(*i*) **Input power-factor.** Input voltage taken from power-supply undertaking is generally sinusoidal. However, ac input current is usually non-sinusoidal. Under such a condition, only the fundamental component of input current takes part in extracting mean ac input power from the source.

The input power factor PF is defined as the ratio of mean input power (real power) to the total rms input voltamperes (apparent power) given to the converter (or rectifier) system.

If V_s = rms value of supply phase voltage.

I_s = rms value of supply phase current including fundamental and harmonics

I_{s1} = rms value of fundamental component of supply current I_s and

ϕ_1 = phase angle between supply voltage V_s and fundamental component I_{s1} of supply current I_s ; see Fig. 3.28 ;

then, the input power factor, as per the definition, is given by

$$PF = \frac{\text{Mean ac input power}}{\text{Total rms input voltamperes}} = \frac{\text{Real powers}, V_s \,.\, I_{s1} \,.\, \cos\phi_1}{\text{Apparent power}, V_s \,.\, I_s}$$

$$= \frac{I_{s1}}{I_s} \,.\, \cos\phi_1 \qquad \text{...(3.51)}$$

For a given power demand, if input pf is poor, more input volt-amperes and hence more input current are taken from the supply.

(*ii*) **Input displacement factor (DF).** As stated above, the phase angle between sinusoidal supply voltage V_s and fundamental component I_{s1} of supply current I_s is ϕ_1. This angle ϕ_1, shown in Fig. 3.28, is usually known as *input displacement angle*. Its cosine is called the input displacement factor DF.

$$\therefore \quad DF = \cos\phi_1 \qquad \text{...(3.52)}$$

DF is also called *fundamental power factor*.

(*iii*) **Input current distortion factor (CDF).** It is defined as the ratio of the rms value of fundamental component I_{s1} of the input current to the rms value of input, or supply, current I_s.

$$\therefore \qquad CDF = \frac{I_{s1}}{I_s} \qquad ...(3.53)$$

It is seen from Eqs. (3.51) to (3.53) that $PF = (CDF) \times (DF)$

or input power factor = (input current distortion factor) × (input displacement factor)

(*iv*) **Input current harmonic factor (HF).** Non-sinusoidal input, or supply, current is made up of fundamental current plus current components of higher frequencies. The harmonic factor (*HF*) is equal to the rms value of all the harmonics divided by the rms value of fundamental component of the input current.

If I_h = rms value of all the harmonic-components combined

$$= \sqrt{I_s^2 - I_{s1}^2}$$

then, as per the definition,
$$HF = \frac{I_h}{I_{s1}} = \frac{\sqrt{I_s^2 - I_{s1}^2}}{I_{s1}} = \frac{\left[\sum_{n=2}^{\infty} I_{sn}\right]}{I_{s1}} \qquad ...(3.54)$$

where I_{sn} = rms value of nth harmonic content.

Harmonic factor is a measure of the harmonic content in the input supply current. *HF* is also known as total harmonic distortion (*THD*). Greater the value of *HF* (or *THD*), greater is the harmonic content and hence greater is the distortion of input supply current.

Also,
$$HF = \sqrt{\left(\frac{I_s}{I_{s1}}\right)^2 - 1} = \sqrt{\frac{1}{CDF^2} - 1} \qquad ...(3.55)$$

Higher value of input distortion factor *CDF* indicates lower magnitude of harmonic content in the source current.

Non-sinusoidal input current can be resolved into Fourier series as under :

$$i = \frac{a_0}{2} + \sum_{n=1,2,3,...}^{\infty} (a_n \cos n\omega t + b_n \sin n\omega t) \qquad ...(3.56)$$

$$= \frac{a_0}{2} + \sum_{n=1,2,3,...}^{\infty} C_n \sin (n\omega t + \phi_n) \qquad ...(3.57)$$

where $a_0 = \frac{2}{T}\int_0^T i(t)\,.\,dt$, $a_n = \frac{2}{T}\int_0^T i(t)\,.\cos n\omega t\, dt$ and $b_n = \frac{2}{T}\int_0^T i(t)\,.\sin n\omega t.dt$

$$C_n = \left[\frac{a_n^2 + b_n^2}{2}\right]^{1/2} \text{ and } \phi_n = \tan^{-1}\left(\frac{a_n}{b_n}\right) \qquad ...(3.58)$$

(*v*) **Crest Factor (CF).** Crest factor for input current is defined as the ratio of peak input current I_{sp} to its rms value I_s

$$\therefore \qquad CF = \frac{I_{sp}}{I_s} \qquad ...(3.59)$$

CF is used for specifying the current ratings of power semiconductor devices and other components.

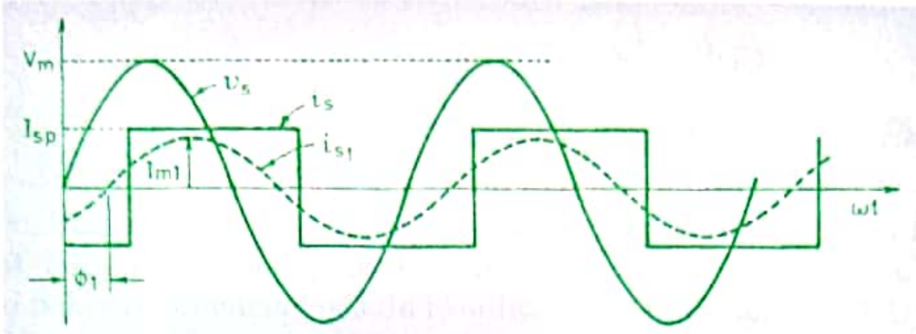


Fig. 3.28. Waveforms for source voltage v_s, source current i_s, fundamental component i_{s1} of source current and ϕ_1 = phase angle between v_s and i_{s1}.

3.7.2 Output Performance Parameters

The load, or output, voltage and the load (or output) current at the output terminals of ac to dc converters are unidirectional but pulsating in nature. Fourier series is used to express these output quantities in terms of its two components, namely (*i*) average (or dc) value and (*ii*) ac component superimposed on dc value as under :

In general, average value of output quantity y is, $Y_o = Y_{dc} = \dfrac{1}{T}\displaystyle\int_{t_1}^{t_1+T} y \,.\, dt$

and its rms value is, $$Y_{or} = \left[\frac{1}{T}\int_{t_1}^{t_1+T} y^2 dt\right]^{1/2}$$

where y = instantaneous value of the function in terms of t

and T = time period for one cycle of y variation.

$\therefore$ Output dc power, P_{dc} = (average output voltage, V_o) × (average output current, I_o)

$$= V_o I_o \qquad ...(3.60)$$

where subscript "*o*" denotes output dc values.

Output ac power $P_{ac} = V_{or} \,.\, I_{or}$

where subscript "*or*" denotes rms value of output quantities.

The various output parameters are now defined below.

(*i*) **Rectification ratio η.** Rectification ratio, also called efficiency of a converter, is defined as the ratio of dc output power P_{dc} to ac output power P_{ac}.

$$\therefore \qquad \eta = \frac{P_{dc}}{P_{ac}} \qquad ...(3.61)$$

Rectifier ratio is also known as *rectifier efficiency* or *figure of merit*. In case R_d = forward rectifier resistance, then

$$\eta = \frac{P_{dc}}{P_{ac} + I_{or}^2 R_d} \qquad ...(3.62)$$

(*ii*) Effective, or ripple, value of the ac component of output voltage is given by

$$V_r = \sqrt{V_{or}^2 - V_o^2} \qquad ...(3.63)$$

where V_r is called ripple voltage, or effective value of ac component of output voltage.

(*iii*) **Form factor (FF).** It is defined as the ratio of rms value V_{or} of output voltage to the dc value V_o of output voltage.

$$\therefore \qquad FF = \frac{V_{or}}{V_o} \qquad ...(3.64)$$

FF is a measure of the shape of the output voltage. The closer *FF* is to unity, the better is the dc output voltage waveform. For constant dc output voltage, rms value of output voltage, V_{or} = average value of output voltage, V_o.

(*iv*) **Voltage ripple factor (VRF).** It is defined as the ratio of ripple voltage V_r to the average output voltage V_o.

$$\therefore \qquad VRF = \frac{V_r}{V_o} \qquad ...(3.65)$$

Substituting the value of V_r from Eq. (3.63) in Eq. (3.65) gives

$$VRF = \left[\left(\frac{V_{or}}{V_o}\right)^2 - 1\right]^{1/2} = \sqrt{FF^2 - 1} \qquad ...(3.66\ a)$$

or

$$FF = \sqrt{VRF^2 + 1} \qquad ...(3.66\ b)$$

(*v*) **Per-unit average output voltage.** It is defined as the ratio of the average output voltage V_o for any value of triggering angle to the average output voltage V_{om} for zero-degree firing angle.

$$\therefore \qquad V_{o\,.\,pu} = \frac{V_o}{V_{om}} \qquad ...(3.67)$$

(*vi*) **Current ripple factor (CRF).** It is defined as the ratio of rms value of all harmonic components of output current to the dc component I_o of the output current.

$$\therefore \qquad CRF = \frac{I_r}{I_o} = \frac{\sqrt{I_{or}^2 - I_o^2}}{I_o} = \left[\left(\frac{I_{or}}{I_o}\right)^2 - 1\right]^{1/2} \qquad ...(3.68)$$

Here I_{or} = rms value of output current including dc and harmonics,

I_r = rms value of all harmonic components of output current

I_o = dc component of output current.

Note that $I_{or}^2 = I_o^2 + I_r^2$.

(*vii*) **Transformer utilization factor (TUF).** If $V_2 (= V_s)$ and $I_2 (= I_s)$ are respectively the rms voltage and rms current ratings of the secondary winding of a transformer, then *TUF* is defined as

$$TUF = \frac{P_{dc}}{V_2 I_2} = \frac{P_{dc}}{V_s I_s} \qquad ...(3.69)$$

$$\therefore \quad \text{Transformer } VA \text{ rating} = \frac{P_{dc}}{TUF} \qquad ...(3.70)$$

Lower the *TUF*, higher is the transformer *VA* rating required.

It is desirable that a rectifier produces a perfect dc output voltage so that (*i*) rms value = dc value (*ii*) *FF* = 1.0 (*iii*) ac component of output voltage = 0 (*iv*) *HF* = 0 (*v*) *PF* = 1.0 and *TUF* = 1.

3.8 COMPARISON OF SINGLE-PHASE DIODE RECTIFIERS

In this article, the performance parameters of single-phase diode rectifiers feeding resistive loads are evaluated. The rectifier types discussed are 1-phase half-wave rectifier and 1-phase full-wave mid-point and bridge types. The performance parameters are then collated in tabular form.

3.8.1 Single-phase Half-wave Rectifier

This rectifier, when feeding a resistive load, Fig. 3.14 (*a*), has waveforms for source voltage v_s, output voltage v_o and output current i_o in Fig. 3.14 (*b*). Its various performance parameters are obtained as under :

From Eq. (3.21), dc output voltage, $V_o = \dfrac{V_m}{\pi}$

and dc output current, $I_o = \dfrac{V_m}{\pi . R} = \dfrac{I_m}{\pi}$

where $I_m = \dfrac{V_m}{R}$ = maximum values of dc current as shown in Fig. 3.14 (*b*).

Output dc power, $P_{dc} = V_o I_o = \dfrac{V_m . I_m}{\pi^2}$ ...(*i*)

From Eq. (3.22), rms output voltage, $V_{or} = \dfrac{V_m}{2}$

and rms output current, $I_{or} = \dfrac{V_m}{2.R} = \dfrac{I_m}{2}$

Output power, $P_{ac} = V_{or} I_{or} = \dfrac{V_m I_m}{4}$ ...(*ii*)

Rectifier efficiency, $\eta = \dfrac{P_{dc}}{P_{ac}} = \dfrac{V_m I_m}{\pi^2} \cdot \dfrac{4}{V_m I_m} = \dfrac{4}{\pi^2} = 0.4053$ or 40.53%

Form factor, $FF = \dfrac{V_{or}}{V_0} = \dfrac{V_m}{2} \cdot \dfrac{\pi}{V_m} = \dfrac{\pi}{2} = 1.5708$

Ripple voltage, $V_r = \sqrt{V_{or}^2 - V_o^2} = \sqrt{\left(\dfrac{V_m}{2}\right)^2 - \left(\dfrac{V_m}{\pi}\right)^2} = 0.3856\ V_m$

Voltage ripple factor, $VRF = \dfrac{V_r}{V_0} = \dfrac{0.3856\ V_m . \pi}{V_m} = 1.211$

Also, $VRF = \sqrt{FF^2 - 1} = \sqrt{1.5708^2 - 1} = 1.211$

Since source voltage v_s is a sine wave, its rms value, $V_s = \dfrac{V_m}{\sqrt{2}}$

Load current i_o waveform is the same as that of source current i_s.

$\therefore$ Rms value of source current, I_s = rms value of output current, $I_{or} = \dfrac{I_m}{2}$.

VA rating of transformer $= V_s I_s = \frac{V_m}{\sqrt{2}} \times \frac{I_m}{2} = \frac{V_m I_m}{2\sqrt{2}}$

$\therefore$ Transformer utilization factor, $TUF = \frac{P_{dc}}{V_s I_s} = \frac{V_m I_m}{\pi^2} \times \frac{2\sqrt{2}}{V_m I_m} = 0.2865$

A $TUF = 0.2865$ means that VA rating of transformer is $\frac{1}{TUF}$ times the dc power output. For a load of 100 watt, a transformer having a rating of $\frac{100}{0.2865} = 349.6$ VA would be required.

$$PIV = \sqrt{2}\, V_s = V_m$$

Peak value of source current, $I_{sp} = I_m$

Rms value of source current, $I_s = I_{or} = \frac{I_m}{2}$

Crest factor, $CF = \frac{I_{sp}}{I_s} = \frac{I_m}{I_m} \times 2 = 2$

3.8.2 Single-phase Full-wave Mid-point Rectifier

Its circuit diagram and various waveforms are shown in Fig. 3.20. Its different performance parameters are obtained as under :

From Eq. (3.48), dc output voltage, $V_o = \frac{2V_m}{\pi}$

and dc output current, $I_o = \frac{2V_m}{\pi R} = \frac{2}{\pi} I_m$

where I_m = maximum value of load current $= \frac{V_m}{R}$

Output dc power, $P_{dc} = V_o I_o = \frac{2V_m}{\pi} . \frac{2}{\pi} I_m = \left(\frac{2}{\pi}\right)^2 . V_m I_m$

Rms output voltage, $V_{or} = \frac{V_m}{\sqrt{2}} = V_s$

Rms output current, $I_{or} = \frac{V_m}{\sqrt{2}} \times \frac{1}{R} = \frac{1}{\sqrt{2}}\left(\frac{V_m}{R}\right) = \frac{1}{\sqrt{2}} I_m = I_s$

Output ac power, $P_{ac} = V_{or} . I_{or} = \frac{V_m}{\sqrt{2}} . \frac{I_m}{\sqrt{2}} = \frac{V_m I_m}{2}$

Rectifier efficiency, $\eta = \frac{P_{dc}}{P_{ac}} = \frac{4}{\pi^2} V_m I_m . \frac{2}{V_m I_m} = \frac{8}{\pi^2} = 0.8106$

Form factor, $FF = \frac{V_{or}}{V_o} = \frac{V_m}{\sqrt{2}} . \frac{\pi}{2V_m} = \frac{\pi}{2\sqrt{2}} = 1.11$

Ripple voltage, $V_r = \sqrt{V_{or}^2 - V_o^2} = \left[\left(\frac{V_m}{\sqrt{2}}\right)^2 - \left(\frac{2V_m}{\pi}\right)^2\right]^{1/2} = 0.3077V_m$

Voltage ripple factor, $VRF = \frac{V_r}{V_o} = 0.3077\ V_m \times \frac{\pi}{2V_m} = 0.483$

Also, $VRF = \sqrt{FF^2 - 1} = \sqrt{1.11^2 - 1} = 0.482$

TUF can be obtained as under :

Rms value of voltage for each secondary winding $= \frac{V_m}{\sqrt{2}}$

Note that current in each secondary winding flows for half cycle only.

$\therefore$ Rms value of current in each secondary winding $= \frac{I_m}{2}$

VA rating of secondary winding = 2[voltage rating of each secondary winding] × [current rating of each secondary winding]

$$= 2 \times \frac{V_m}{\sqrt{2}} \times \frac{I_m}{2} = \frac{V_m I_m}{\sqrt{2}} = 0.707\ V_m I_m$$

Primary winding current is, however, made up of both positive and negative half cycles.

$\therefore$ Primary rms current $= \frac{I_m}{\sqrt{2}}$

Primary rms voltage $= \frac{V_m}{\sqrt{2}}$

Primary *VA* rating $= \frac{V_m I_m}{2} = 0.5\ V_m I_m$

$\therefore$ Average *VA* rating of transformer $= \frac{0.5 + 0.707}{2} . V_m I_m = 0.6035\ V_m I_m$

$$TUF = \frac{P_{dc}}{\text{Average } VA \text{ rating of transformer}} = \frac{4}{\pi^2} . V_m I_m \times \frac{1}{0.6035\ V_m I_m} = 0.672$$

PIV for each diode $= 2V_m$

Peak value of source current, $I_{sp} = I_m$

Rms value of source current, $I_s = \frac{I_m}{\sqrt{2}}$

$\therefore$ CF of input current $= \frac{I_{sp}}{I_s} = \frac{I_m}{I_m}\sqrt{2} = \sqrt{2} = 1.414.$

3.8.3 Single-phase full-wave Bridge Rectifier

Its circuit diagram is given in Fig. 3.21 (*a*). It is seen from Fig. 3.20 (*b*) and 3.21 (*b*) that waveform of output (or load) voltage v_o and output current i_o are identical in both $M-2$ and $B-2$ types of diode rectifiers. Therefore, in single-phase $B-2$ diode rectifier also,

$$V_o = \frac{2V_m}{\pi},\ I_o = \frac{2I_m}{\pi},\ V_{or} = \frac{V_m}{\sqrt{2}} \text{ and } I_{or} = \frac{I_m}{\sqrt{2}}$$

This shows that the rectifier efficiency, FF, ripple voltage V_r, VRF are the same for both types of diode rectifiers. However, PIV of diode in single-phase $B-2$ rectifier is V_m whereas it is $2V_m$ in 1-phase $M-2$ rectifier.

TUF : Rms value of source voltage $V_s = \frac{V_m}{\sqrt{2}}$

Rms value of source current, $I_s = \frac{I_m}{\sqrt{2}}$

VA rating of transformer $= V_s I_s = \frac{V_m I_m}{2}$

$$P_{dc} = V_o I_o = \frac{2V_m}{\pi} \cdot \frac{2I_m}{\pi} = \left(\frac{2}{\pi}\right)^2 \cdot V_m I_m$$

$$\therefore \quad TUF = \frac{P_{dc}}{VA \text{ rating of transformer}} = \frac{4}{\pi^2} \cdot V_m I_m \times \frac{2}{V_m I_m} = \frac{8}{\pi^2} = 0.8106$$

Source current waveforms for both types are identical, therefore, $CF = \sqrt{2}$.

A comparison of three types of 1-phase diode rectifiers discussed above is given in the table below where $V_m = \sqrt{2}\, V_s$. Here V_s = rms value of sinusoidal source voltage and f = source frequency in Hz.

S. No.	Parameters	Half-wave (or one-pulse)	Full-wave (or Two pulse)	
			Centre-tap $(M-2)$	Bridge $(B-2)$
1.	DC output voltage, V_o	$\frac{V_m}{\pi}$	$\frac{2V_m}{\pi}$	$\frac{2V_m}{\pi}$
2.	Rms value of output voltage, V_{or}	$\frac{V_m}{2}$	$\frac{V_m}{\sqrt{2}}$	$\frac{V_m}{\sqrt{2}}$
3.	Ripple voltage, V_r	$0.3856\, V_m$	$0.3077\, V_m$	$0.3077\, V_m$
4.	Voltage ripple factor, VRF	1.211	0.482	0.482
5.	Rectification efficiency, η	40.53%	81.06%	81.06%
6.	Transformer utilization factor, TUF	0.2865	0.672	0.8106
7.	Peak inverse voltage, PIV	V_m	$2V_m$	V_m
8.	Crest factor, CF	2	$\sqrt{2}$	$\sqrt{2}$
9.	Number of diodes	1	2	4
10.	Ripple frequency	f	$2f$	$2f$

It is seen from the above table that both full-wave diode rectifiers :

(*i*) are better than the half-wave rectifier in so far as voltage ripple factor, rectification efficiency, *TUF* and crest factor are concerned,

(*ii*) have average output voltage double of that of the half-wave rectifier (for the same input voltage),

(*iii*) have ripple frequency double of that of half-wave rectifier.

For both the full-wave rectifiers, the following is observed from the table :

(*i*) *TUF* of $B - 2$ rectifier is superior than the $M - 2$ type. Therefore, transformer required in $M - 2$ configuration is bulky and weighty.

(*ii*) PIV of diodes in $B - 2$ rectifier is half of that of the diodes used in $M - 2$ rectifier.

(*iii*) $B - 2$ rectifier requires four diodes whereas $M - 2$ requires only two diodes

(*iv*) Overall, a bridge rectifier using four diodes is more economical.

Example 3.15. *A load of $R = 60\ \Omega$ is fed from 1-phase, 230 V, 50 Hz supply through a step-up transformer and then one diode. The transformer turns ratio is two. Find the VA rating of transformer.*

Solution. The half-wave diode rectifier uses a step-up transformer therefore, ac voltage applied to rectifier = 230 × 2 = 460 V = V_s

Average value of load voltage, $V_o = \dfrac{V_m}{\pi} = \dfrac{\sqrt{2} \times 460}{\pi} = 207.04$ V

Output dc power, $P_{dc} = \dfrac{V_o^2}{R} = \dfrac{207.04^2}{60} = 714.43$ W

It is seen from the table that *TUF* for 1-phase half-wave diode rectifier is 0.2865.

$\therefore$ *VA* rating of transformer $= \dfrac{P_{dc}}{TUF} = \dfrac{714.43}{0.2865} = 2493.65$ VA

So choose a transformer with 2.5 kVA (next round figure) rating.

Example 3.16. *A 230 V, 50 Hz supply is connected to a 1-phase transformer which feeds a diode bridge as shown in Fig. 3.23 (a). Primary to secondary turns ratio for transformer is 0.5 and load RL has a ripple free current $I_o = 10$ A. Determine (i) average value of output voltage (ii) input current distortion factor (iii) input displacement factor DF (iv) input power factor (v) input current harmonic factor HF (or THD) and (vi) crest factor.*

Solution. Waveforms for supply voltage v_s, constant load current $i_o = I_o = 10$ A and source current i_s are shown in Fig. 3.23 (*b*).

Rms value of input voltage to bridge rectifier, $\dfrac{N_1}{N_2} = \dfrac{230}{V_s} = 0.5$

$$\therefore \qquad V_s = \frac{230}{0.5} = 460 \text{ V}$$

The source current, or input current, i_s can be expressed in Fourier series as under :

$$i_s = I_{dc} + \sum_{n=1,3,5}^{\infty} (a_n \cos n\omega t + b_n \sin n\omega t) \qquad ...(3.56)$$

Here I_{dc} = dc value of source current

$$= \frac{1}{2\pi}\int_0^{2\pi} i_s \, . \, d(\omega t) = \frac{1}{2\pi}\left[\int_0^{\pi} I_o \, . \, d(\omega t) - \int_{\pi}^{2\pi} I_o \, d(\omega t)\right] = 0$$

It can also be stated from the waveform of i_s that as the area of positive and negative half cycle are equal, average value of i_s *i.e.* $I_{dc} = 0$.

$$a_n = \frac{1}{\pi}\int_o^{2\pi} i_o(t) \, . \cos n\omega t \, . \cos n\omega t \, . \, d(\omega t)$$

$$= \frac{2}{\pi}\int_o^{\pi} I_o \cos n\omega t \, . \, d(\omega t) = \frac{2I_o}{n\pi}\left| \sin n\omega t \right|_o^{\pi} = 0 \text{ for all } n$$

$$b_n = \frac{1}{\pi}\int_o^{2\pi} i_o(t) \, . \sin n\omega t \; d(\omega t) = \frac{2}{\pi}\int_o^{\pi} I_o \sin n\omega t \, . \, d(\omega t)$$

$$= \frac{2I_o}{n\pi}\left[-\cos n\omega t\right]_o^{\pi} = \frac{2I_o}{n \, . \, \pi}\,[1 - \cos n\pi]$$

$$= \frac{4I_o}{n\pi} \text{ for } n = 1, 3, 5 \, ... \text{ (for odd values of } n)$$

and $$b_n = 0 \text{ for } n = 2, 4, 6 \, ... \text{ (for even values of } n)$$

Substituting the values of I_{dc}, a_n and b_n in Eq. (3.56), we get

$$i_s = \frac{4I_o}{n\pi} \sin n\omega t \quad \text{and} \quad \phi_n = \tan^{-1}\left[\frac{0}{b_n}\right] = 0$$

$$\therefore \quad i_s = \frac{4I_o}{\pi}\left[\sin \omega t + \frac{1}{3}\sin 3\omega t + \frac{1}{5}\sin 5\omega t + \frac{1}{7}\sin 7\omega t + ...\right]$$

(*i*) Average value of output voltage, $V_o = \frac{2V_m}{\pi} = \frac{2\sqrt{2} \times 460}{\pi} = 414.08 \text{ V}$

(*ii*) Since fundamental component of input source current $\frac{4I_o}{\pi}\sin \omega t$ is in phase with source voltage $V_m \sin \omega t$, the displacement angle = 0. Also, from above, $\phi_1 = 0$.

$\therefore$ Input displacement factor, $DF = \cos \phi_1 = \cos 0° = 1$.

(*iii*) Rms value of fundamental component of source current, $I_{s1} = \frac{4I_o}{\pi} \times \frac{1}{\sqrt{2}}$ A. Rms value of source current, $I_s = \left[\frac{I_o^2 \times \pi}{\pi}\right]^{1/2} = I_o = 10 \text{ A}$

Input current distortional factor, $CDF = \frac{I_{s1}}{I_s} = \frac{4I_o}{\pi.\sqrt{2}} \times \frac{1}{I_o} = \frac{\sqrt{2} \times 2}{\pi} = 0.9$

(*iv*) Input $pf = CDF \times DF = 0.9 \times 1 = 0.90$ (lagging)

(*v*) $$HF = THD = \left[\left(\frac{I_s}{I_{s1}}\right)^2 - 1\right]^{1/2} = \left[\left(\frac{1}{0.9}\right)^2 - 1\right]^{1/2} = 0.4843 \text{ or } 48.43\%$$

(*vi*) *Crest factor*. Here $I_{sp} = I_o = 10$ A and $I_s = 10$ A.

$$\therefore \quad CF = \frac{10}{10} = 1.00.$$

Example 3.17. *A single-phase B-2 diode rectifier is required to supply a dc output voltage of 230 V to a load of R = 10 Ω. Determine the diode ratings and transformer rating required for this configuration.*

Solution. Average, or dc output voltage, $V_o = \frac{2V_m}{\pi} = \frac{2\sqrt{2}\,.\,V_s}{\pi} = 230$ V

∴ Rms value of input voltage to rectifier = transformer secondary voltage,

$$V_s = \frac{230 \times \pi}{2\sqrt{2}} = 255.5 \text{ V}$$

Average load current, $I_o = \frac{V_o}{R} = \frac{230}{10} = 23$ A

Maximum value of diode current, $I_m = \frac{V_m}{R} = \frac{\sqrt{2} \times 255.5}{10} = 36.13$ A

It is seen from the waveform of diode current i_{D1} from Fig. 3.23 (*b*) that average value of diode current is

$$I_{DAV} = \frac{1}{2\pi}\int_0^{\pi} I_m \sin \omega t \,.\, d(\omega t) = \frac{I_m}{\pi} = \frac{36.13}{\pi} = 11.50 \text{ A}$$

and rms value of diode current, $I_{Dr} = \left[\frac{1}{2\pi}\int_0^{\pi} I_m^2 \sin^2 \omega t \,.\, d(\omega t)\right]^{1/2} = \frac{I_m}{2} = \frac{36.13}{2} = 18.07$ A

$$PIV = \sqrt{2}\, V_s = \sqrt{2} \times 255.5 = 361.3 \text{ V}$$

Transformer secondary current $= \frac{I_m}{\sqrt{2}} = \frac{36.13}{\sqrt{2}} = 25.55 \text{ A} = I_s$

Transformer rating $= V_s I_s = 255.5 \times 25.55 = 6528$ VA $= 6.528$ (kVA)

[Check : $P_{dc} = V_o I_o = 230 \times 23 = 5290$ W

∴ Transformer rating $= \frac{P_{dc}}{TUF} = \frac{5290}{0.81} = 6530$ VA $= 6.53$ kVA]

Thus, diode ratings are : $I_{DAV} = 11.50$ A, $I_{Dr} = 18.07$ A

Peak diode current, $I_m = 36.13$ A and $PIV = 361.3$ V and transformer rating = 6.528 kVA