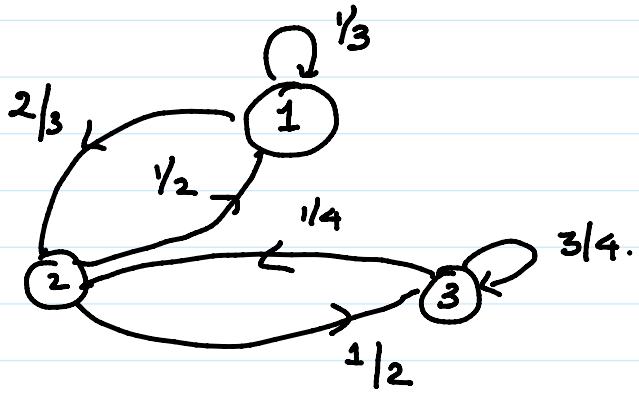
# Accessibility

Example-1.

$$P = \begin{array}{c} \\ 1 \\ 2 \\ 3 \end{array} \begin{array}{c} \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ \begin{bmatrix} 1/3 & 2/3 & 0 \\ 1/2 & 0 & 1/2 \\ 0 & 1/4 & 3/4 \end{bmatrix} \end{array}$$




$P_{11} = \frac{1}{3} > 0$

- $1 \leftarrow 1$ [1 is accessible from 1]

Every state is accessible from itself.

$P_{12} = \frac{2}{3} > 0$

- $2 \leftarrow 1$ [2 is accessible from 1]

$P_{13} = 0$

- 3 is not accessible from 1.

$P_{21} = \frac{1}{2} > 0$

$1 \leftarrow 2$.

$P_{22} = 0$

· 2 is not accessible from 2.

$P_{23} = \frac{1}{2} > 0$

$3 \leftarrow 2$.

$P_{31} = 0$

· 1 is not accessible from 3.

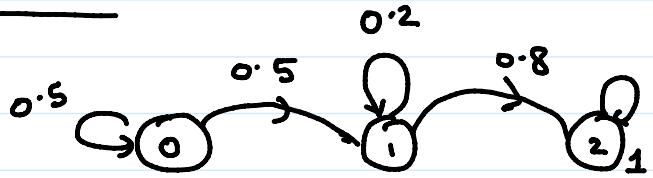
$P_{32} = \frac{1}{4} > 0$.

$2 \leftarrow 3$.

$P_{33} = \frac{3}{4}$

$3 \leftarrow 3$.

Example-2




(i) Is state 2 accessible from 0?

Ans $2 \leftarrow 0$.

$P_{02} = 0$ [ 2 is not accessible from 0].

(ii) Is state 0 accessible from 2?

Ans $0 \leftarrow 2$

$P_{20} = 0$ [Not accessible].

(iii) Is state 1 accessible from state 0?

Ans $1 \leftarrow 0$.

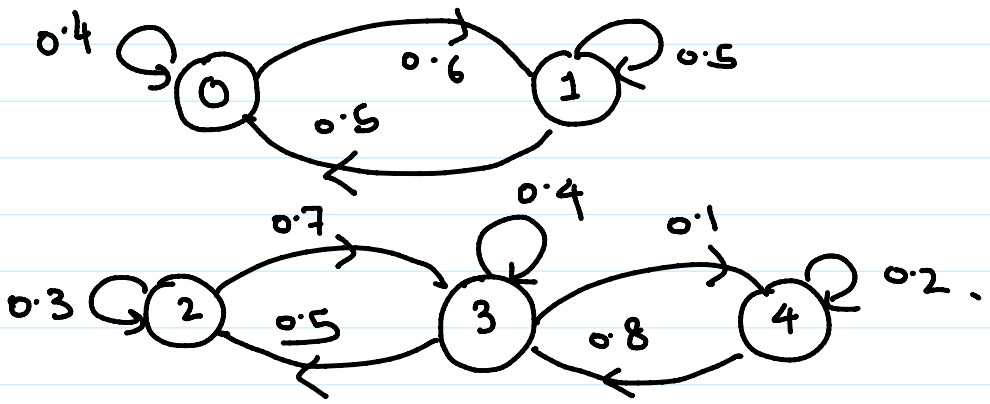
$P_{01} = 0.5$ [Accessible]

(iv) Is state 0 accessible from 1?

Ans $0 \leftarrow 1$.

$P_{10} = 0$ [0 is not accessible from 1].

Example 3




(i) Which states are accessible from state 0?

Ans $0 \leftarrow 0$, $1 \leftarrow 0$. } States 0 and 1 are accessible from state 0

(ii) Which states are accessible from state 3?

Ans:- States 2, 3 and 4 are accessible from state 3.

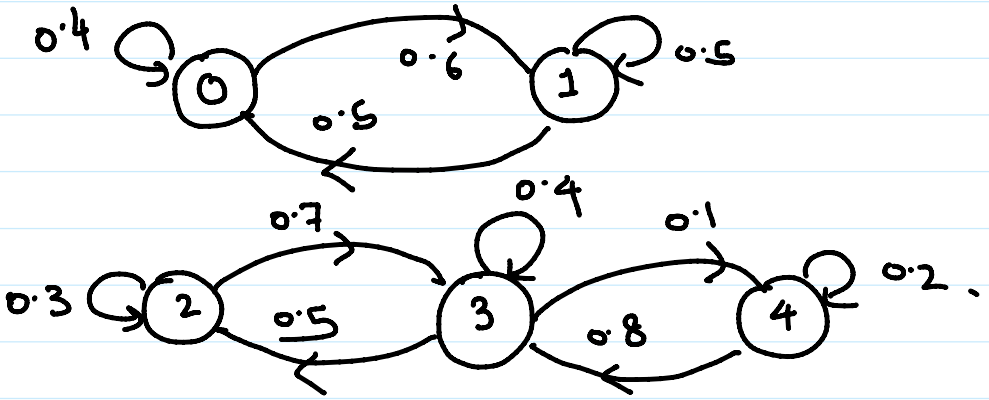
$3 \leftarrow 3$, $2 \leftarrow 3$, $4 \leftarrow 3$.

(iii) Is state 0 accessible from state 4?

Ans Not accessible

## Communicability

Example 1:-

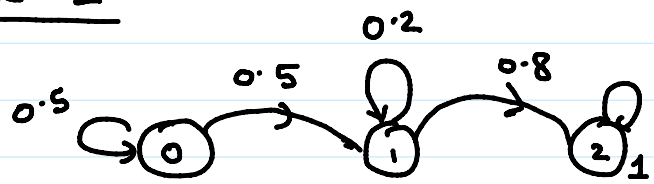



Q1 Which states communicate with each other?

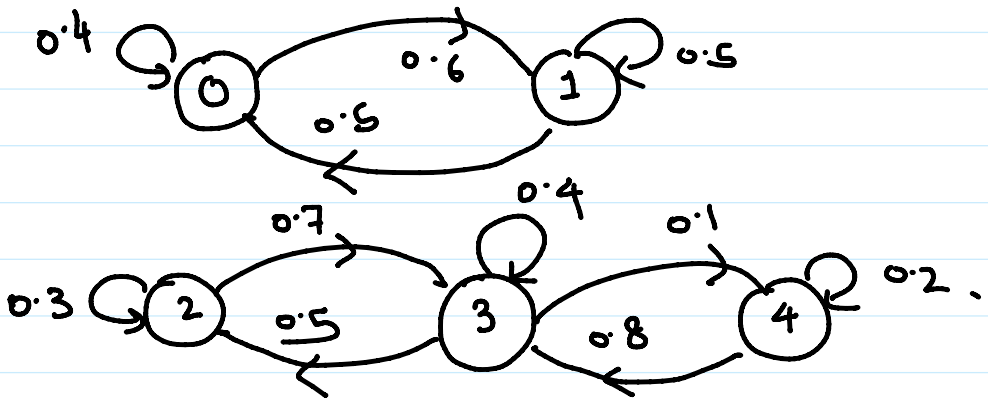
Ans (1) $1 \leftrightarrow 1$, $2 \leftrightarrow 2$, $3 \leftrightarrow 3$, $4 \leftrightarrow 4$, $0 \leftrightarrow 0$

$0 \leftrightarrow 1$, $2 \leftrightarrow 3$, $3 \leftrightarrow 4$ $2 \leftrightarrow 3 \leftrightarrow 4$

## Example-2




$0 \leftrightarrow 0$, $1 \leftrightarrow 1$, $2 \leftrightarrow 2$.

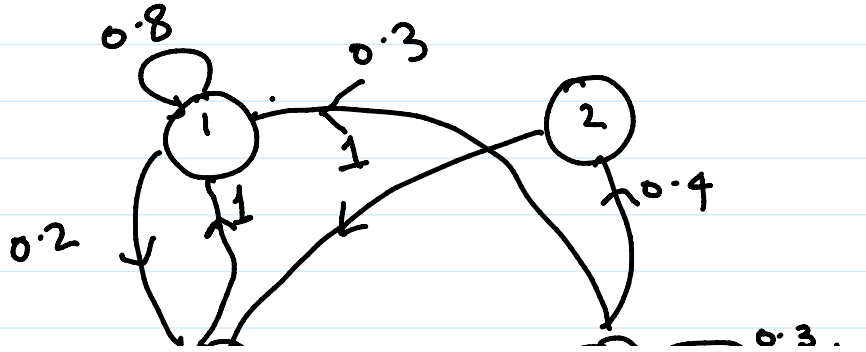


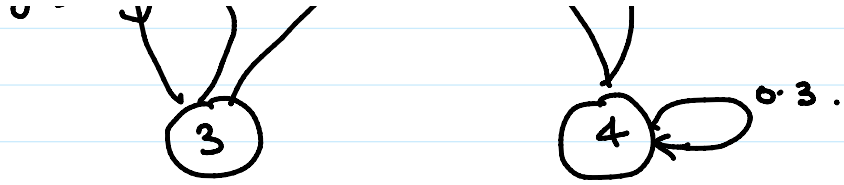

State classes:-

$\{0,1\}$, $\{2,3,4\}$ [2 classes]

## Transient vs. Recurrent.

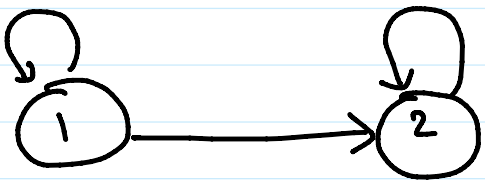
$$P = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array} \begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{bmatrix} 0.8 & 0 & 0.2 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0.3 & 0.4 & 0 & 0.3 \end{bmatrix} \end{array}.$$






| State | Transient | Recurrent |
|---|---|---|
| | 2. | 1 |
| | 4 | 3 |

Example




$P_{ii} = 1$ [Recurrent]

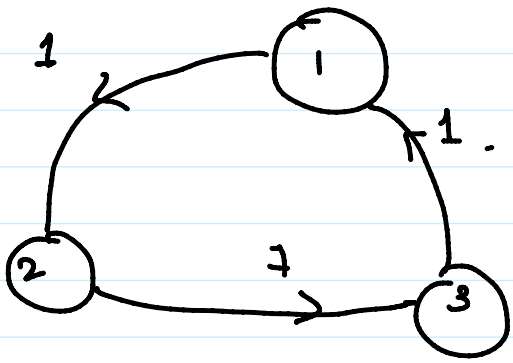
$P_{ii} < 1$ [Transient]

State 1 — Transient.

2 → Recurrent [might be]

Irreducible vs. Reducible

$$P = \begin{array}{c} \\ 1 \\ 2 \\ 3 \end{array} \begin{array}{c} \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix} \end{array}$$




[irreducible Markov chain]

# Transient and Recurrent State

Define.

$f_{ii}^{(n)}$ = Prob. of first recurrence to i is at the $n^{th}$ step.

$$f_i = f_{ii} = \sum_{n=1}^{\infty} f_{ii}^{(n)} = \text{Prob. of recurrence to i}$$

A State i is recurrent if $f_i = 1$.

A State i is Transient if $f_i < 1$