Construct the DFA equivalent to the NFA $M = (\{q_0, q_1\}, \{0,1\}, \delta, q_0, \{q_1\})$ and δ is defined as

States	Inputs 0	Inputs 1
q_0	$\{q_0, q_1\}$	$\{q_1\}$
q_1	ϕ	$\{q_0, q_1\}$

Solution:

DFA $M' = (Q', \{0,1\}, \delta', [q_0], F')$ accepting $L(M)$ as follows:

$$Q' = 2^Q = \{\phi, q_0, q_1, \{q_0, q_1\}\}$$

$\delta'(q_0, 0) = \{q_0, q_1\}$ —— New State 1

$\delta'(q_0, 1) = \{q_1\}$ —— New State 2

$\delta'((q_0, q_1), 0) = \{q_0, q_1\} \cup \{\phi\} = \{q_0, q_1\}$

$\delta'((q_0, q_1), 1) = \{q_1\} \cup \{q_0, q_1\} = \{q_0, q_1\}$

$\delta'(q_1, 0) = \{\phi\}$

$\delta'(q_1, 1) = \{q_0, q_1\}$

The DFA Transition Table is

States	Inputs	
	0	1
→ q_0	$\{q_0, q_1\}$	$\{q_1\}$
* q_1	ϕ	$\{q_0, q_1\}$
q_0, q_1	$\{q_0, q_1\}$	$\{q_0, q_1\}$

Replace the word.

$q_0 = A$, $q_1 = B$ $q_0, q_1 = C$ → Final
Starting final State.

States	Input	
	0	1
A	C	B
B	ϕ	C
C	C	C

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2. Construct a DFA equivalent to

$$M = (\{q_0, q_1\}, \{0,1\}, \delta, q_0, \{q_0\})$$

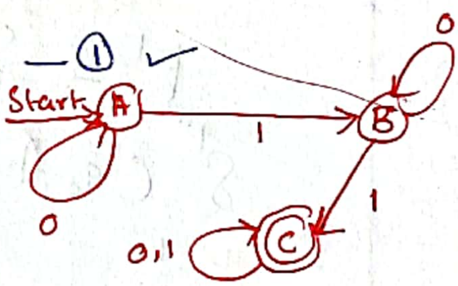
where δ is given as:-

States	Inputs	
	0	1
$\rightarrow * q_0$	q_0	q_1
q_1	q_1	$\{q_0, q_1\}$

Solution:-

$\delta'(q_0, 0) = q_0$

$\delta'(q_0, 1) = \{q_1\}$ New State — ①



$\delta'(q_1, 0) = q_1$

$\delta'(q_1, 1) = \{q_0, q_1\}$ — New State ②

$\delta'((q_0, q_1), 0) = \{q_0\} \cup \{q_1\} = \{q_0, q_1\}$

$\delta'((q_0, q_1), 1) = \{q_1\} \cup \{q_0, q_1\} = \{q_0, q_1\}$

— DFA Transition Table:

States	Inputs	
	0	1
$\rightarrow * q_0$	q_0	q_1
q_1	q_1	q_0, q_1
$* q_0, q_1$	q_0, q_1	q_0, q_1

③

$\Rightarrow$

States	Inputs	
	0	1
$\rightarrow *$ A	A	B
B	B	C
$*$ C	C	C

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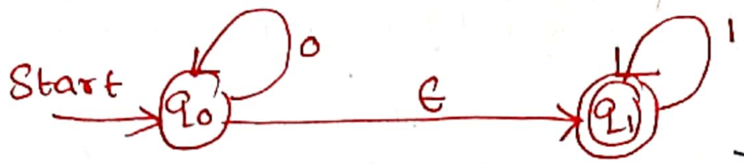
Construct a NFA without ϵ – moves from NFA with ϵ – moves.



States	Input 0	1	ϵ
$\rightarrow q_0$	q_0	ϕ	q_1
$* q_1$	ϕ	q_1	ϕ

Solution:

ϵ_ closure $(q_0) = \{q_0, q_1\}$

Let $M = (Q, \Sigma, q_0, \delta, F)$ be NFA with ϵ – transition.

Let $M' = (Q, \Sigma, q_0, \delta', F')$ be NFA without ϵ_ transition.

$\therefore F' = \{q_0, q_1\}$

$\delta'(q_0, \epsilon) = \epsilon$_ closure $(q_0) = \{q_0, q_1\}$ — ①

Step: 1

$\delta'(q_0, 0) = \hat{\delta}(q_0, 0)$

Move $(q_0, 0) = \{q_0\}$

ϵ_ closure (move $(q_0, 0)) = \{q_0, q_1\}$

Move $(q_0, 1) = \{\phi\}$

ϵ_ closure (move $(q_0, 1)) = \phi\ \{q_0\}$

$q_0 = \{q_0, q_1\}$

$q_1 = \{q_1\}$

Step 2:

ϵ – $\delta(q_0, 0) = \{q_0, q_1\}$

$= \text{cls}(q_0, 0) \cup q_1, 0)$

$= \{q_0\}$

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ϵ_closure $(q_0) = \{q_0, q_1\}$

ϵ_closure $(q_1) = \{q_1\}$

$\delta'(q_0, 0) = \cancel{\epsilon\text{-closure}}\ \delta(q_0, q_1\} = \{q_0, q_1\}$

δ' ϵ_closure $(q_0, 1) = \{q_0, q_1\} =$

Step 3:

$\delta'(q_1, 0) = \epsilon\text{-closure}(\text{move}(q_1, \epsilon), 0))$

$= \phi$

Step 4:

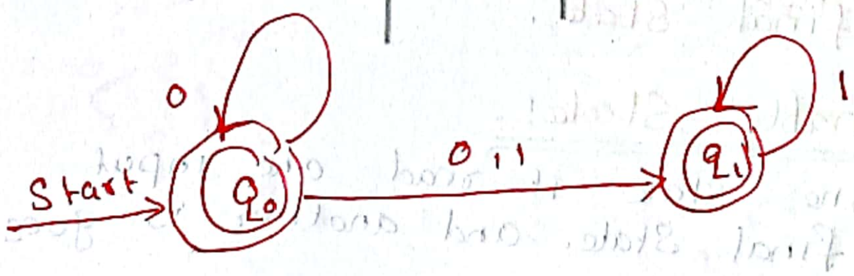
$\delta'(q_1, 1) = \hat{\delta}(q_1, 1)$

$= \epsilon\text{-closure}(\text{move}(\delta(\hat{\delta}(q_1, \epsilon), 1))$

$= \{q_1\}$

Transition Table:

States	Inputs 0	Inputs 1
$\rightarrow$ * q_0	$\{q_0, q_1\}$	$\{q_1\}$
* q_1	ϕ	$\{q_1\}$



DFA Minimization:

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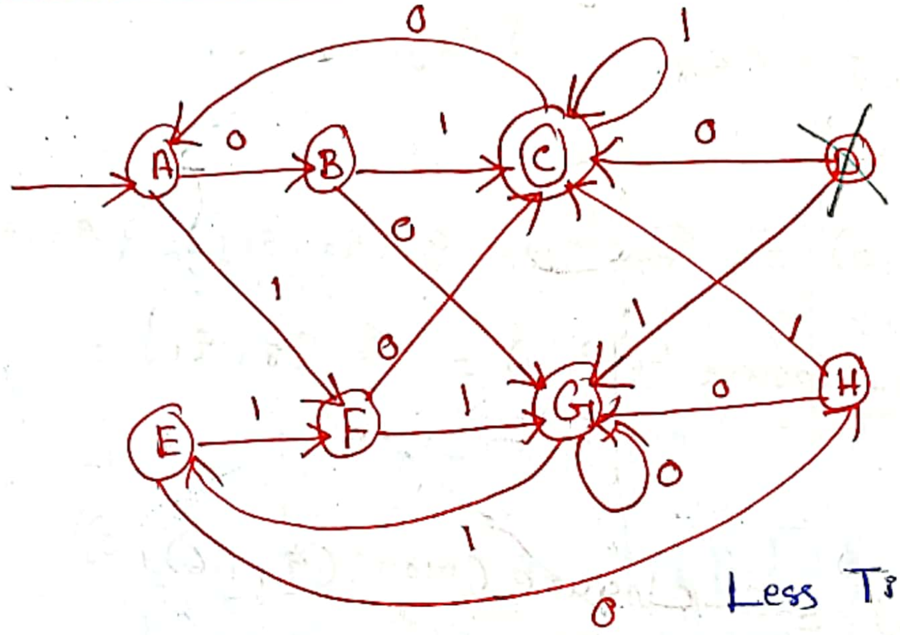

Less Time & Space

Less no. of State

Less no. of Transition

Not reachable state is 'D'.

Reachable States are A, B, C, E, F, G, H.

	A	B	C	E	F	G
B	X					
C	X	X				
E	E	X	X			
F	X	X	X	X		
G	X	X	X	X	X	
H	X	E	X	X	X	X

Equivalent State:

For all possible inputs if one States goes to final state then next state also should goes to final state.

Distinguishable State:

Take one state it read one input and move to final state. and another is goes to non final.

Final State is not equivalent to non final State because input is ε.

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States	Inputs 0	Inputs 1
→ A	B	F
B	G	C
C	A	C
E	H	F
F	C	G
G	G	E
H	G	C

B, H → Equivalent

1) Read Input '0' F only move to final State

∴ F ≠ any other State.

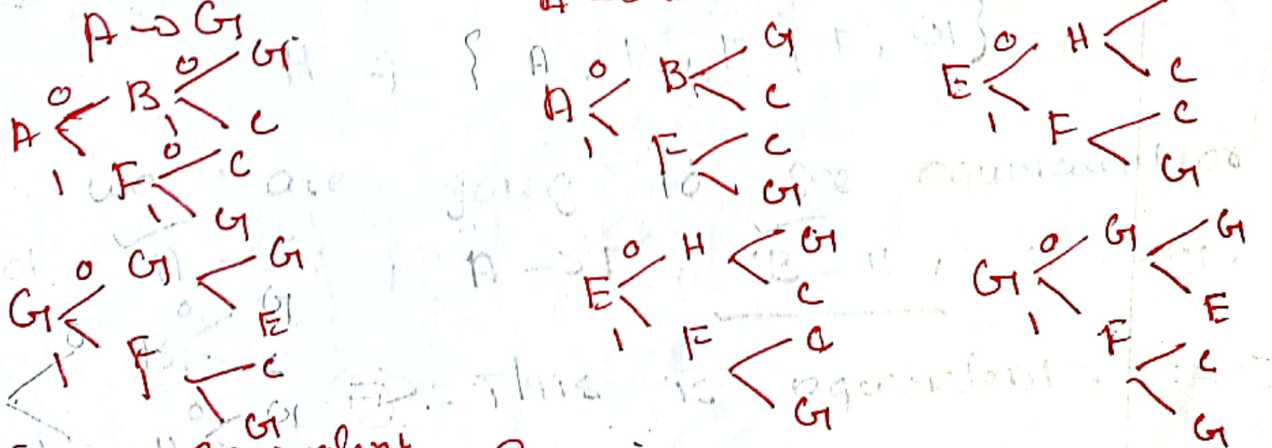
$\boxed{F \neq \{A, E, F, G\}}$ $F \neq \{A, B, C, E, G, H\}$

2) Read Input '1' B, C, H State only move to final State. other States are Distinguish State.

∴ B, C, H ≠ {A, E, F, G}

Now we take

A → G   A → E   E → G.



∴ Not equivalent   ∴ A → E is equivalent State.   Not

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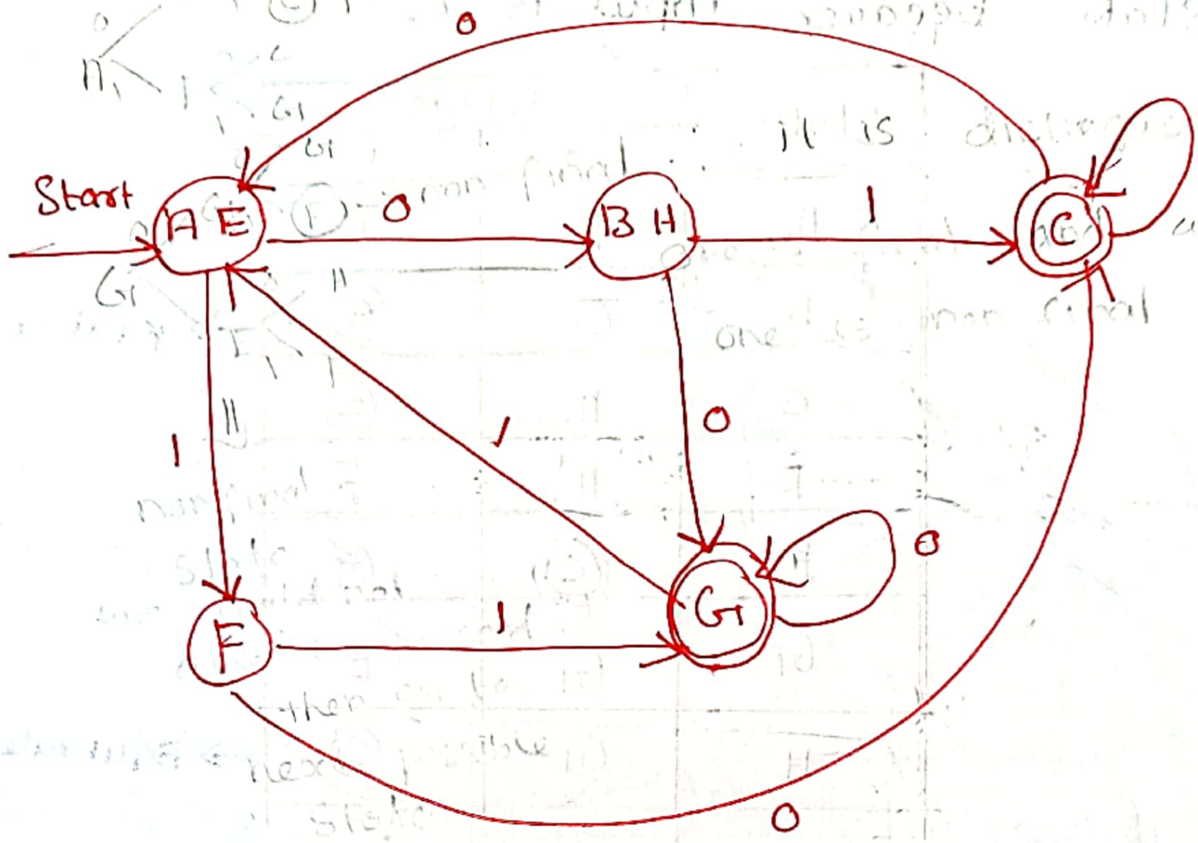

Demonstrate how to apply the minimization algorithm to reduce the number of given state in DFA.

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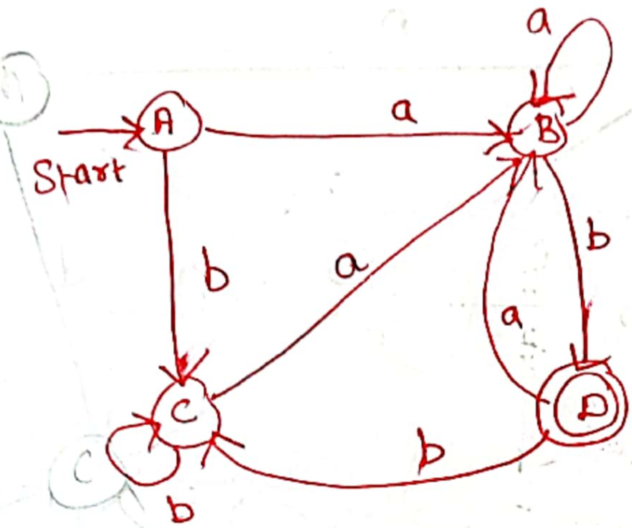

States	Inputs	
	a	b
A	B	C
B	B	D
C	B	C
D	B	C

AC is the equivalent state.

B	X		
C	E	X	
D	X	X	X
	A	B	C

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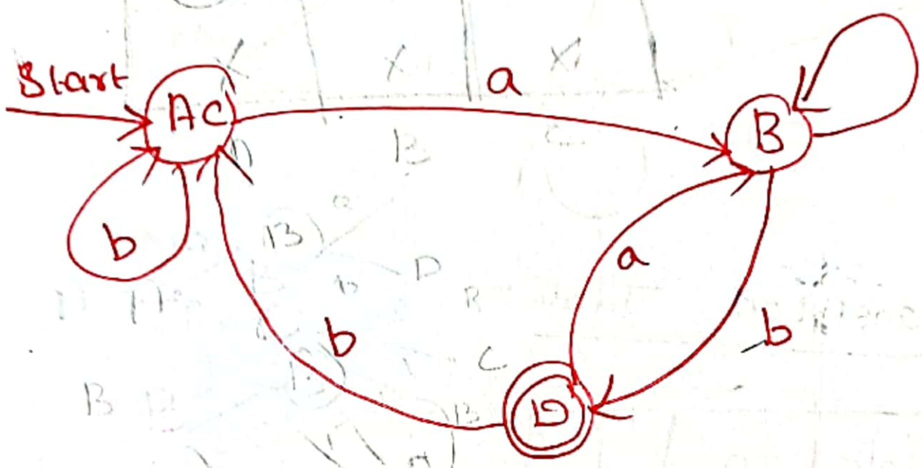
$A \to C$

$$A \begin{cases} \xrightarrow{a} B \begin{cases} B \\ D \end{cases} \\ \xrightarrow{b} C \begin{cases} B \\ C \end{cases} \end{cases}$$

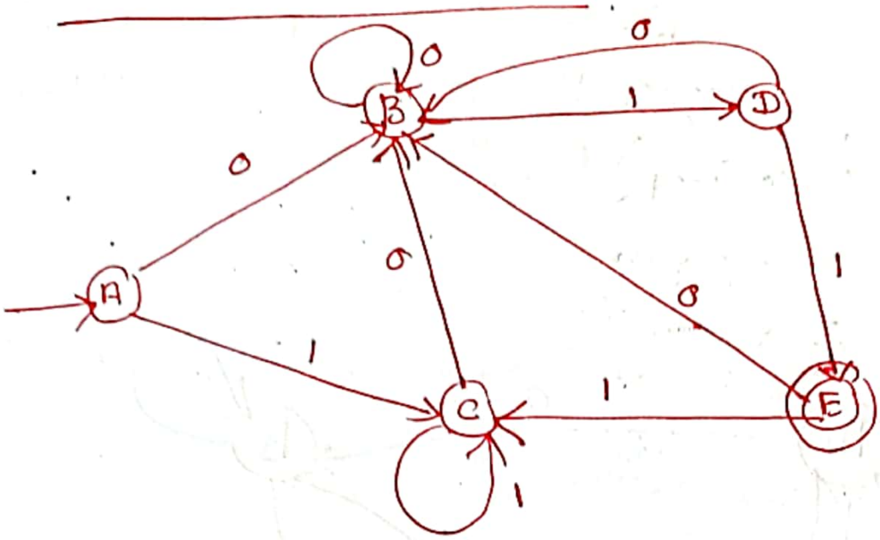
$$C \begin{cases} \xrightarrow{a} B \begin{cases} B \\ D \end{cases} \\ \xrightarrow{b} B \begin{cases} B \\ D \end{cases} \end{cases}$$

∴ AC is Equivalent State.

DFA:

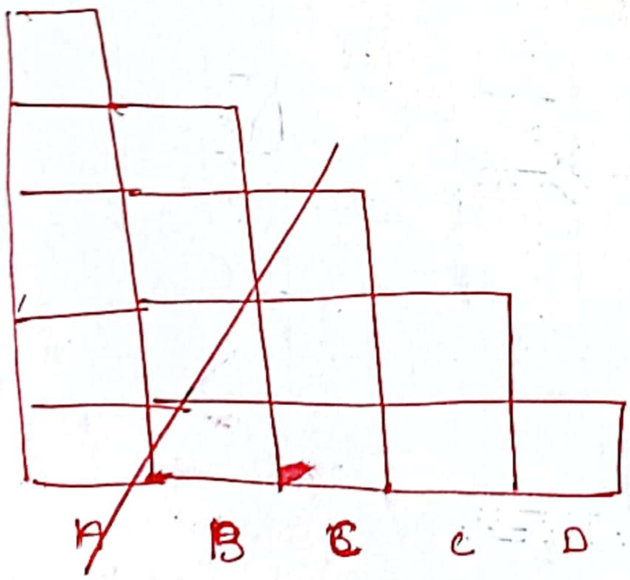


3) Minimization of DFA:



Transition Table:

States	0	1
→A	B	C
B	B	D
C	B	C
D	B	E
*E	B	E



	A	B	C	D
B	X			
C	E	X		
D	X	X	X	
E	X	X	X	X

11

A → B.

$A \xrightarrow{0} B$
$A \xrightarrow{1} C$

B → C

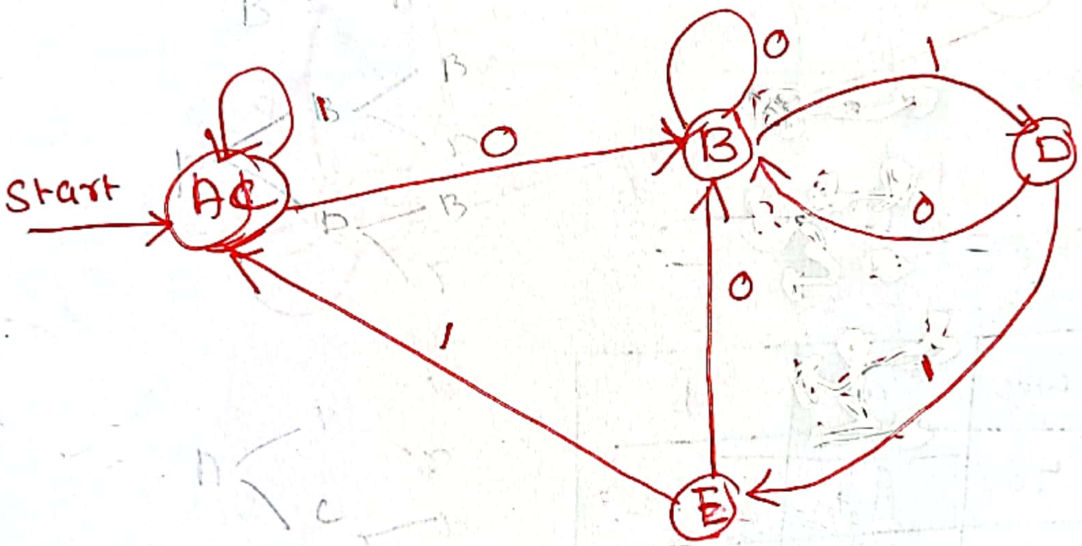
$B \xrightarrow{0} B$
$B \xrightarrow{1} D$

$B \xrightarrow{0} B$
$B \xrightarrow{1} D$

$C \xrightarrow{0} B$
$C \xrightarrow{1} C$

∴ A → B
Not equal

B → C is
Not equal



Home work

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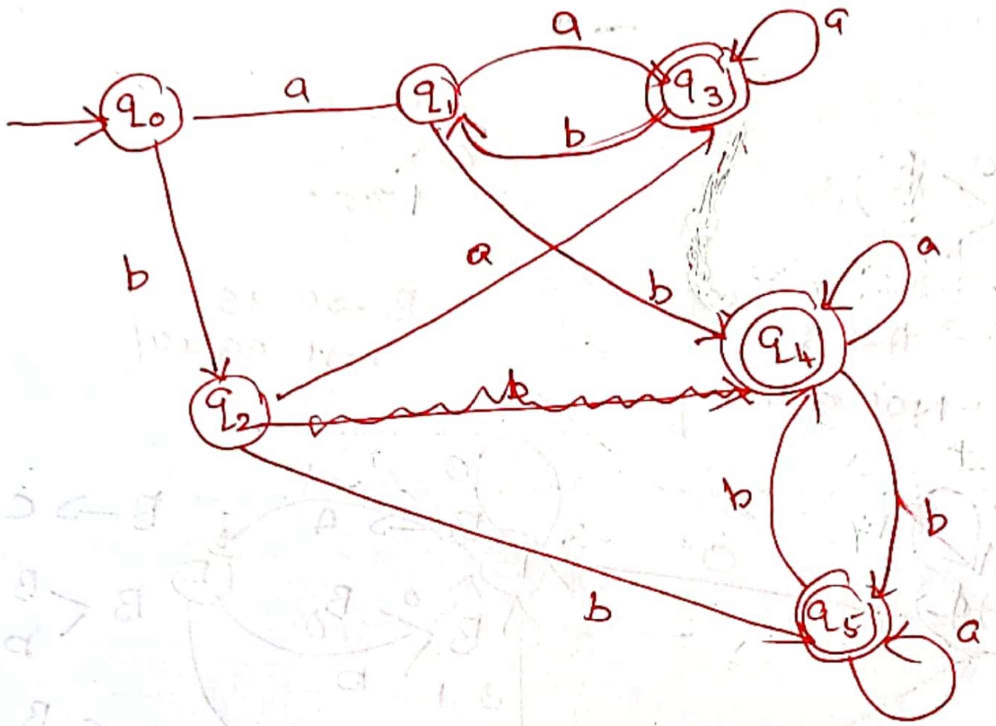
4) Minimization of DFA:



States	a	b
→ q_0	q_1	q_2
q_1	q_3	q_4
q_2	q_3	q_5
* q_3	q_3	q_1
* q_4	q_4	q_5
* q_5	q_5	q_4

Home work.

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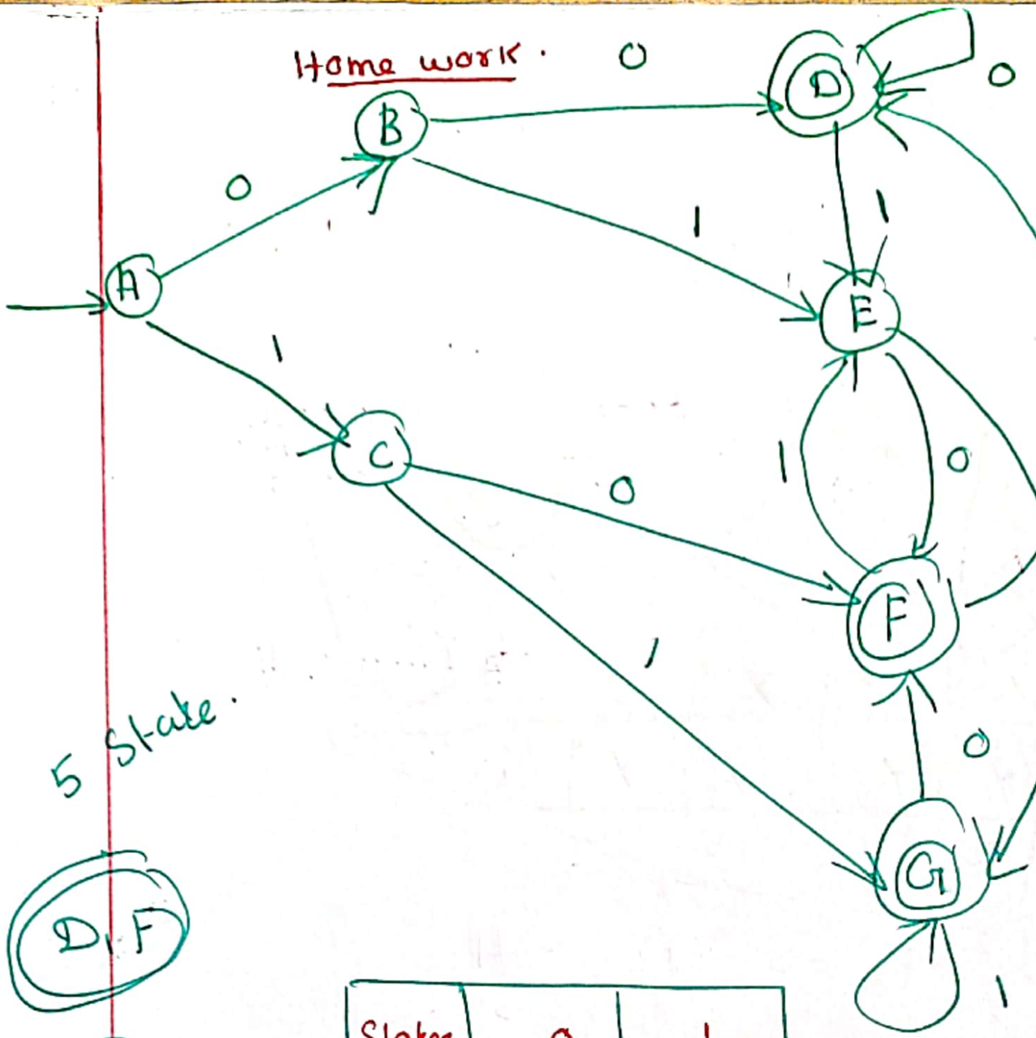

5 state.

D, F

G

C, E

States	0	1
A	B	C
B	D	E
C	F	G
D	D	E
E	F	G
F	D	E
G	F	G

Minimization of DFA using Equivalent Theorem

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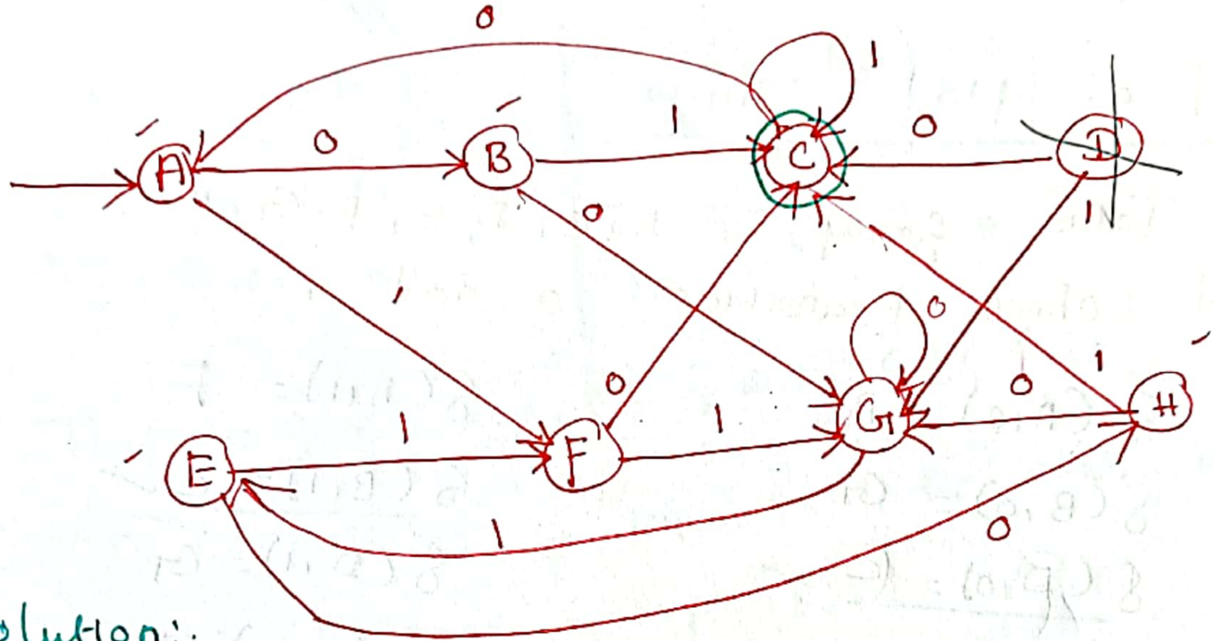
Home work.



Solution:

Step 1:

States	0	1
A	B	F
B	G	C
C	A	C
~~D~~	~~C~~	~~G~~
E	H	F
F	C	G
G	G	E
H	G	C

Step 2:

Initial State = A

Final State = C

The States are divided into two groups

Non final = { A, B, ~~D~~ E, F, G, H }

Final State = {c}

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Step 3: Find Π_{new}

Take group (A, B, D, E, F, G, H) and check transition 0 and 1

$\delta(A,0) = B$ $\quad\quad$ $\delta(A,1) = F$

$\delta(B,0) = G$ $\quad\quad$ $\delta(B,1) = C$ ✓

$\delta(D,0) = C$ ✓ $\quad\quad$ $\delta(D,1) = G$

$\delta(E,0) = A$ $\quad\quad$ $\delta(E,1) = F$

$\delta(F,0) = C$ ✓ $\quad\quad$ $\delta(F,1) = G$

$\delta(G,0) = G$ $\quad\quad$ $\delta(G,1) = E$

$\delta(H,0) = G$ $\quad\quad$ $\delta(H,1) = C$ ✓

→ On considering input 0, the states D & F goes to another group C.

→ On considering input 1, the states B, H goes to another group C.

$\Pi_{new} = \{A, B, E, G, H\} \{D, F\} \{C\}$

$\therefore \Pi_{new} = \{A, E, G\} \{B, H\} \{D, F\} \{C\}$

Step 4:

For group (A, E, G), check the input 0 & 1

$\delta(A,0) = B$
$\delta(E,0) = H$
$\delta(G,0) = G$

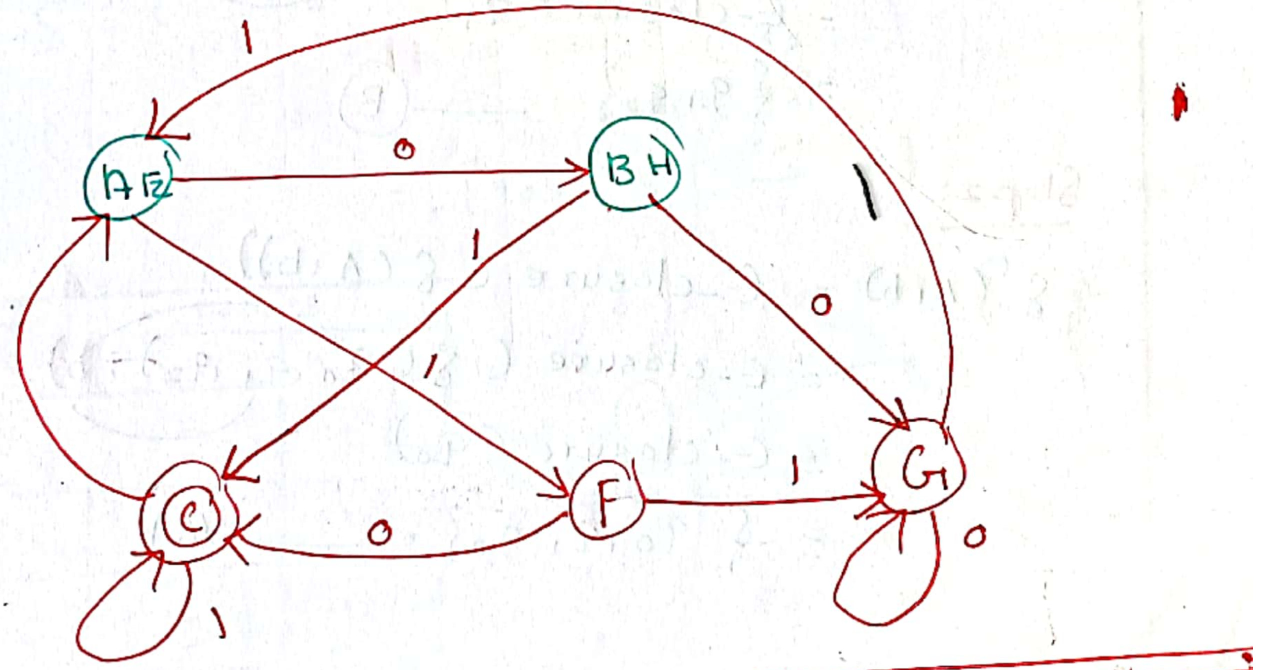
$\delta(A,1) = F$
$\delta(E,1) = F$
$\delta(G,1) = E$

$\Pi_{new} = \{AE\} \{BH\} \{D,F\} \{C\} \{G\}$

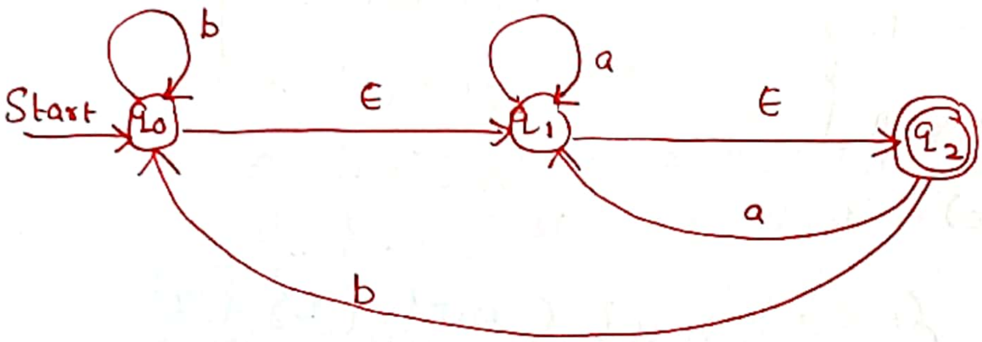
On considering input 0, the states A & E goes to another group {B, H}

On considering input 1, the states cannot be splitted.

$\Pi_{final} = \Pi_{new}$ ~~(G)~~



1) Convert the following NFA with ϵ to equivalent DFA ?



Solution:

ϵ-closure $\{q_0\} = \{q_0, q_1, q_2\}$ — A

ϵ-closure $\{q_1\} = \{q_1, q_2\}$ — B

ϵ-closure $(q_2) = \{q_2\}$ — C

Step 1:

$\delta'(A, a) = \epsilon\text{-closure}(\delta(A, a))$

$\delta'(A, a) = \epsilon\text{-closure}(\delta(q_0, q_1, q_2), a)$

$= \epsilon\text{-closure}(q_1)$

$= \{q_1, q_2\}$ — B

Step 2:

$\delta'(A, b) = \epsilon\text{-closure}(\delta(A, b))$

$= \epsilon\text{-closure}(\delta(q_0, q_1, q_2), b)$

$= \epsilon\text{-closure}(q_0)$

$= \{q_0, q_1, q_2\}$ — A

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Step 3:

$$\delta'(B, a) = \epsilon\text{-closure}(\delta(B, a))$$
$$= \epsilon\text{-closure}(\delta(q_1, q_2), a)$$
$$= \epsilon\text{-closure}(q_1)$$
$$= \{q_1, q_2\} \text{ ——— } B$$

Step 4:

$$\delta'(B, b) = \epsilon\text{-closure}(\delta(B, b))$$
$$= \epsilon\text{-closure}((q_1, q_2), b)$$
$$= \epsilon\text{-closure}(q_0)$$
$$= \{q_0, q_1, q_2\} \text{ ——— } A$$

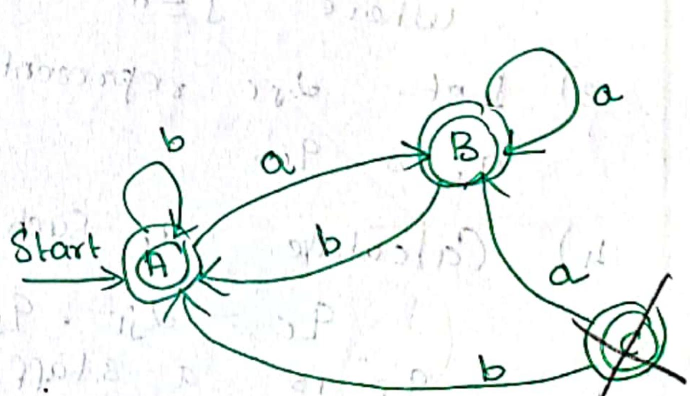
Step 5:

$$\delta'(C, a) = \epsilon\text{-closure}(\delta(C, a))$$
$$= \epsilon\text{-closure}(q_2), a)$$
$$= \epsilon\text{-closure}(q_1)$$
$$= \{q_1, q_2\} \text{ ——— } B$$

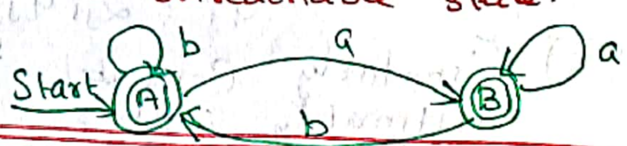
Step 6:

$$\delta'(C, b) = \epsilon\text{-closure}(\delta(C, b))$$
$$= \epsilon\text{-closure}(q_2, b)$$
$$= \epsilon\text{-closure}(q_0)$$
$$= \{q_0, q_1, q_2\} \text{ — } A$$

States	Inputs a	Inputs b
→ * A	B	A
* B	B	A
* C	B	A



∴ C unreachable state:



Conversion of FA from Regular Expression

Dr. I Jasmine

Arden's Theorem:-

Let p and q be two regular expressions over Σ. If P does not contain ϵ, then the equation in

$$\boxed{R = Q + RP}$$

has a solution

$$\boxed{R = QP^*}$$

using this theorem, its easy to find the regular expression.

The condition to apply this theorem are

i) Finite Automata does not have ϵ-moves.

(ii) It has only 1 start state.

(iii) The following algorithm is used to build the regular expression from given DFA.

1) Let q_1 be the initial state.

2) Then are $q_2, q_3, \ldots q_n$ number of states.

The final state may be same q_j where $j \leq n$

3) Let α_{ji} represents the transition from q_j to q_i.

4) Calculate q_i such that

$$q_i = \alpha_{ji} \cdot q_j$$

q_i is a start state

$$q_i = \alpha_{ji} q_j + \epsilon.$$

5) Similarly compute the final state which ultimately gives the regular expression r.

Identity Rules:

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1) $\epsilon R = R\epsilon = R$

2) $\epsilon^* = \epsilon$

3) $(\phi)^* = \epsilon$

4. $\phi R = R\phi = \phi$

5) $\phi + R = R$

6) $R + R = R$

7) $RR^* = R^*R = R^+$

8 $(R^*)^* = R^*$

9 $\epsilon + RR^* = R^*$

10 $(P + Q)R = PR + QR$

11) $(P + Q)^* = (P^*Q^*) = (P^* + Q^*)^*$

12) $R^*(* + R) = (* + R)R^* = R^*$

13) $(R + *)^* = R^*$

14. $\epsilon + R^* = R^*$

15) $(PQ)^*P = P(QP)^*$

16 $R^*R + R = R^*R$

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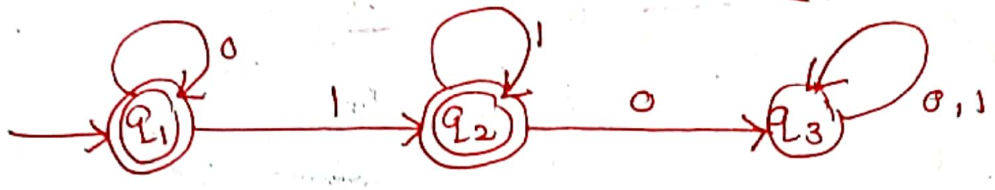
1) Find the regular Expression for the following DFA. Using Arden's Theorem.



Solution:

Equations for all the States:

$q_1 = \epsilon + q_1 0$ ——— (1)

$q_2 = q_1 1 + q_2 1$ ——— (2) $RE = q_1 + q_2$

$q_3 = q_2 0 + q_3 0 + q_3 1$ ——— (3)

Equation for final State q_1

Arden's Theorem

$$\boxed{\begin{array}{l} R = Q + RP \\ R = QP^* \end{array}}$$

$\underset{R}{q_1} = \underset{Q}{\epsilon} + \underset{R}{q_1}\underset{P}{0}$

$q_1 = \epsilon 0^*$ $\epsilon R = R$, by identity

$\boxed{q_1 = 0^*}$ ——— (4)

Equation for final State q_2

$q_2 = q_1 1 + q_2 1$

$\underset{R}{q_2} = \underset{Q}{\underbrace{0^* 1}} + \underset{RP}{\underbrace{q_2 1}}$

$$\boxed{\begin{array}{l} R = Q + RP \\ R = QP^* \end{array}}$$

$q_2 = 0^* 1 1^*$ ——— (5)

∴ Regular expression: Union of Two RE

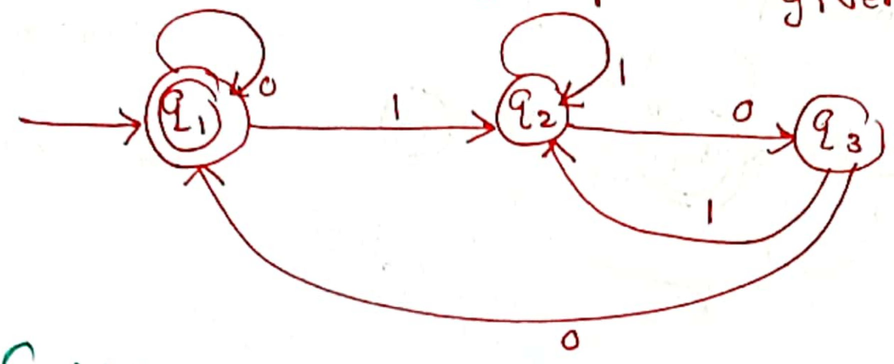
$RE = 0^* + 0^* 1 1^*$

$= 0^* (\epsilon + 1 1^*)$

$\boxed{\epsilon + RR^* = R^*}$

$\boxed{RE = 0^* 1^*}$

2) Find out the RE from given DFA? using Arden's Theorem.



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Solution:.

The equation for all States

$$q_1 = \epsilon + q_1 0 + q_3 0 \quad \text{——— (1)}$$

$$q_2 = q_2 1 + q_3 1 + q_1 1 \quad \text{——— (2)}$$

$$\boxed{q_3 = q_2 0} \quad \text{——— (3)}$$

Substitute q_3 in equation 2.

$$q_2 = q_2 1 + q_2 01 + q_1 1$$

$$q_2 = q_1 1 + q_2 (1+01)$$

$$R = Q + R P$$

$R = Q + RP$

$R = QP^*$

$$q_2 = q_1 1 \cdot (1+01)^* \quad \text{——— (4)}$$

∴ Equation for the final State:

$$q_1 = \epsilon + q_1 0 + q_3 0$$

$$q_1 = \epsilon + q_1 0 + q_2 00 \qquad \text{Sub} \Rightarrow q_2$$

$$q_1 = \epsilon + q_1 0 + q_1 1 \cdot (1+01)^* 00$$

$$q_1 = \epsilon + q_1 (0 + 1 \cdot (1+01)^* 00$$

$$R = Q + R P^*$$

$R = QP^*$

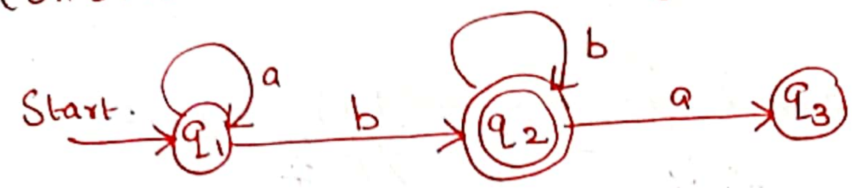
$\epsilon R = R$.

$$q_1 = \epsilon (0+1 \cdot (1+01)^* 00)^*$$

∴ $q_1 = (0+1(1+01)^* 00)^* \Rightarrow$ & q_1 is the final State∴

∴ $\boxed{RE = (0+1(1+01)^* 00)^*}$

3) Construct RE from given DFA Using Arden's Theorem.



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Solution:

$q_1 = \epsilon + q_1 a$ ——— ①

$q_2 = q_1 b + q_2 b$ ——— ②

$q_3 = q_2 a$ ——— ③

Substitute

$q_1 = \epsilon + q_1 a$

$\Downarrow$ R, $\Downarrow$ Q, $\Downarrow$ R, $\Downarrow$ P

$\boxed{q_1 = \epsilon a^*}$

$\boxed{\begin{array}{l} R = Q \cancel{+} RP \\ R = QP^* \end{array}}$

Sub q_1 in Equ ②

$q_2 = \epsilon a^* b + q_2 b$

$q_2 = a^* b + q_2 b$

$R = Q + R P$

$q_2 = (a^* b) b^*$

$\boxed{RR^* = R^+}$

$q_2 = a^* b^+$

$\therefore$ q_2 is the final state.

$\therefore$ $\boxed{RE = a^* b^+}$

4. Construct the regular Expression for the following DFA?

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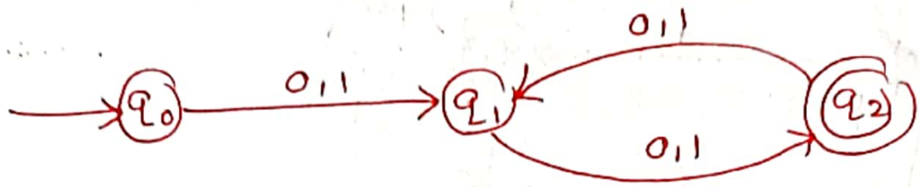


Solution::

$q_0 = \epsilon$ —— ①

$q_1 = q_0 0 + q_0 1 + q_2 0 + q_2 1$ —— ②

$q_2 = q_1 0 + q_1 1$ —— ③

Subtitute ① in ②

$q_1 = \epsilon 0 + \epsilon 1 + q_2(0+1)$

$= \epsilon(0+1) + q_2(0+1)$ $\qquad \epsilon R = R.$

$\boxed{q_1 = (0+1) + q_2(0+1).}$

$\cancel{q_1 = 0+1[1+q_2]}$

q_2 is the final state.

$\therefore \quad q_2 = q_1 0 + q_1 1$ $\quad$ sub. q_1 in

$q_2 = q_1(0+1)$ $\quad$ sub q_1 in

$\cancel{q_2 = (0+1).(0+1)(1+q_2)}$

$\cancel{q_2 = (0+1)(0+1)(1+q_2)}$

$q_2 = [(0+1) + q_2(0+1)]\, 0+1$

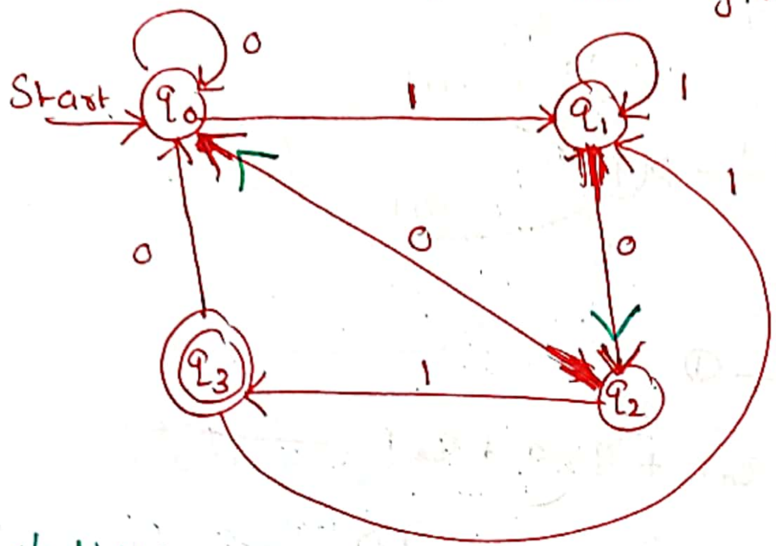
$q_2 = \underbrace{(0+1).(0+1)}_{Q} + \underbrace{q_2}_{R}\underbrace{(0+1).(0+1)}_{P}$

$R = Q + RP$

$\boxed{q_2 = (0+1).(0+1)[(0+1).(0+1)]^*}$

$\boxed{\begin{aligned} &\therefore R = Q + RP \\ &R = QP^* \end{aligned}}$

5. Construct RE for the given DFA: Dr. I Jasmine



Solution:

$$q_0 = \epsilon + q_0 0 + q_2 0 + q_3 0 \quad \text{—— (1)}$$

$$q_1 = q_0 1 + q_1 1 + \cancel{q_2 0} + q_3 1 \quad \text{—— (2)}$$

$$q_2 = q_1 0 \quad \text{—— (3)}$$

$$\boxed{q_3 = q_2 1} \quad \text{—— (4)}$$

Sub. q_2 in Equ (4)

$$q_3 = q_2 1$$

$$\boxed{q_3 = q_1 0 1} \quad \text{— Sub } q_1 \text{ Eq. (2)}$$

$$= \cancel{q_0 1 + q_1 1 + q_3 1 \ (01)}$$

Equ q_1

$$q_1 = q_0 1 + q_1 1 + q_1 011$$

$$\underset{R}{q_1} = \underset{Q}{q_0 1} + \underset{R}{q_1} \underbrace{(1 + 011)}_{P}$$

$$\boxed{q_1 = q_0 1 (1 + 011)^*}$$

$$\boxed{\begin{array}{l} R = Q RP \\ R = QP^* \end{array}}$$

Equ ①

$$q_0 = \epsilon + q_0 0 + q_2 0 + q_3 0$$

$$= \epsilon + q_0 0 + q_1 00 + q_1 010$$

$R = Q + RP$

$$q_0 = \epsilon + q_0 0 + q_1 (00 + 010)$$

$R = QP^*$

$$= \epsilon + q_0 0 + q_0 1 (1 + 011)^* (00 + 010)$$

$$q_0 = \epsilon + q_0 [0 + 1 (1 + 011)^* (00 + 010)$$

R Q R P

$$q_0 = \epsilon [(0 + 1 (1 + 011)^* (00 + 010)]^*$$

$$\therefore q_3 = q_1 01$$

$$= q_0 1 (1 + 011)^* 01$$

Sub q_0.

$$q_3 = \{ 0 + 1 [(1 + 011)^* (00 + 010] 1 (1 + 011)^* 01 \}$$

Dr. I. Jasmin

6. Convert the following NFA into RE

Solution:

Equ for all States.

$A = \epsilon + A0 + A1$ —— (1)

$B = A1$ —— (2)

$\boxed{\begin{array}{l} C = B0 + B1 \text{ —— (3)} \\ D = \cancel{B}\; C0 + C1 \text{ —— (4)} \end{array}}$ Final State.

Consider the State C

$\therefore C = B0 + B1$ Sub (2) in (3)

$\cancel{C = A10 + A11}$

Consider Equ (1)

$A = \epsilon + A0 + A1$

$\underset{R}{A} = \underset{Q}{\epsilon} + \underset{R}{A}\underset{P}{(0+1)}$

$\boxed{\begin{array}{l} R = Q + RP \\ R = QP^* \end{array}}$

$\boxed{\epsilon R^* = R}$

$\therefore A = \epsilon (0+1)^*$

$\therefore \boxed{A = (0+1)^*}$ —— (5)

$\therefore$ Sub (5) in (2)

$\boxed{B = (0+1)^* 1}$ —— (6)

$\therefore C = (0+1)^* 10 + (0+1)^* 11$

$\boxed{C = (0+1)^* 1 [0+1)}$ —— (7)

Consider the State D. Dr. I. Jasmine.

$$D = C0 + C1 \quad \text{—— (4)}$$

Sub Equ (1) in 4

$$D = (0+1)^* 1 (0+1) D + (0+1)^* 1 (0+1) 1$$

$$D = (0+1)^* 1 (0+1)(0+1)$$

$$\therefore \quad RE = C + D$$

$$\boxed{RE = (0+1)^* 1 (0+1) + (0+1)^* 1 (0+1)(0+1)}$$

$T_{new} = \{q_4\} \{q_0, q_1\} \{q_3 q_5\} \{q_2\}$

$\{q_3 q_5\} \{q_4\} \{q_0, q_1\} \{q_2\}$