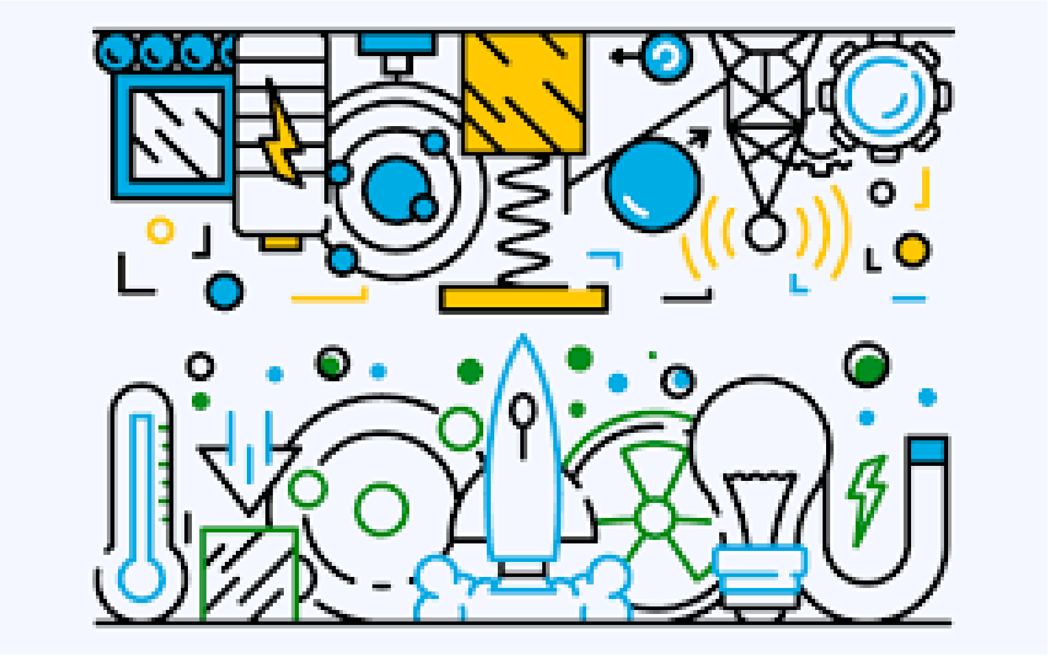
Engineering Physics

PHY1001

Dr Suchetana Sadhukhan

We will meet for 90 minutes, twice in a week!
Lets keep it interactive and enjoyable!!



Before starting, Lets know about some rules and regulations

1. Syllabus

2. Method of Assessment

3. Groups

4. Tutorials/ Assignment

Tutorial sheet has three sections

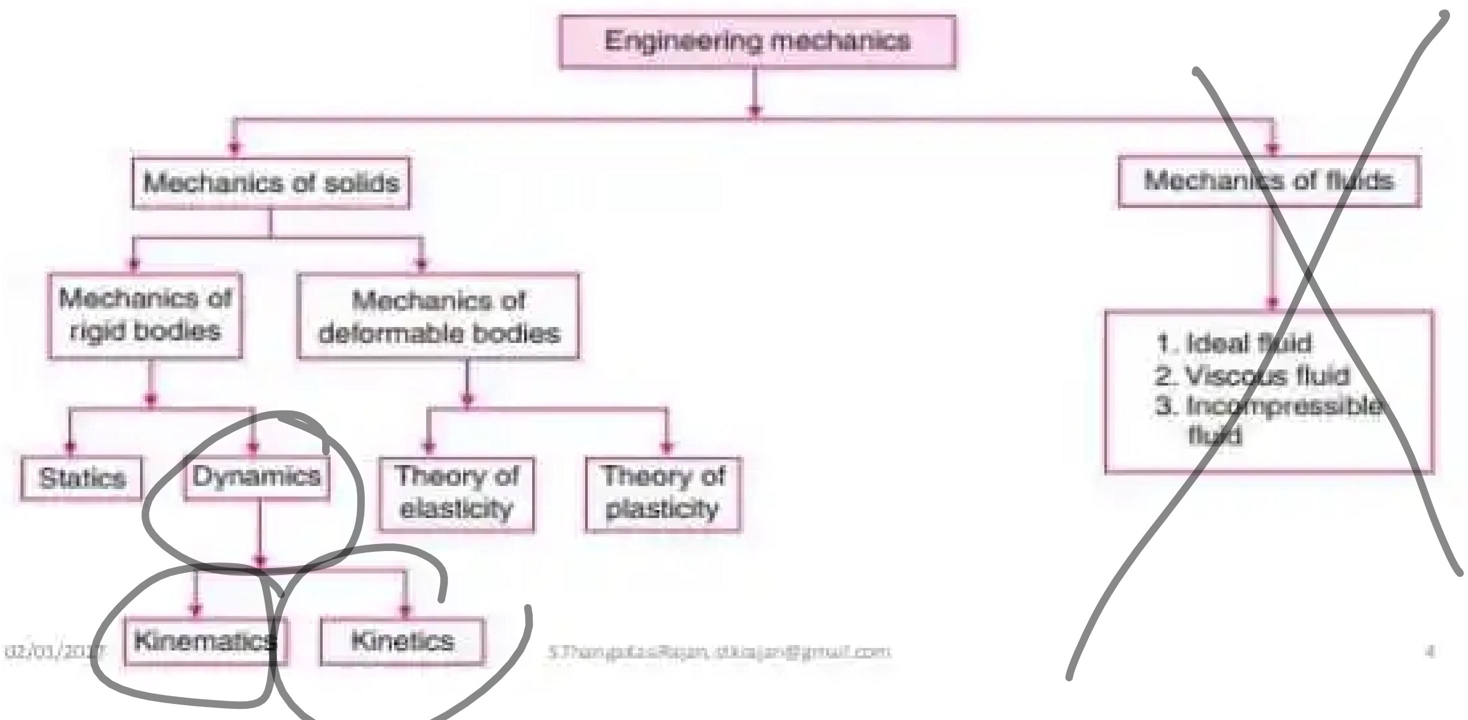
Section I: Discuss by the tutor
(2 questions)

Section II: Solve by the students in
the class (4 questions)

Section II: Solve by the students
As assignment
(4 questions)

Mechanics is a branch of the physical sciences that is concerned with the state of rest or motion of bodies subjected to the action of forces.

Branches of Engineering Mechanics



- Solid mechanics deals with the study of solids at rest or in motion.
- Fluid mechanics deals with the study of liquids and gases at rest or in motion.
- Deformable bodies change their shape or size when acted upon by forces.
- Rigid bodies do not deform acted upon by forces. The rigid bodies may change their orientation or position under the action of force, i.e. the relative position of Particles of rigid body remains unchanged.
- Statics deals with equilibrium of bodies under action of forces (bodies may be either at rest or move with a constant velocity).
- Dynamics deals with motion of bodies (accelerated motion).
- Kinematics is that branch of Dynamics, which deals with the bodies in motion, without any reference to the forces which are responsible for the motion.
- Kinetics is the branch of Dynamics, which deals with the bodies in motion due to the application of forces.

Engineering Mechanics

<u>Rigid-body Mechanics</u>

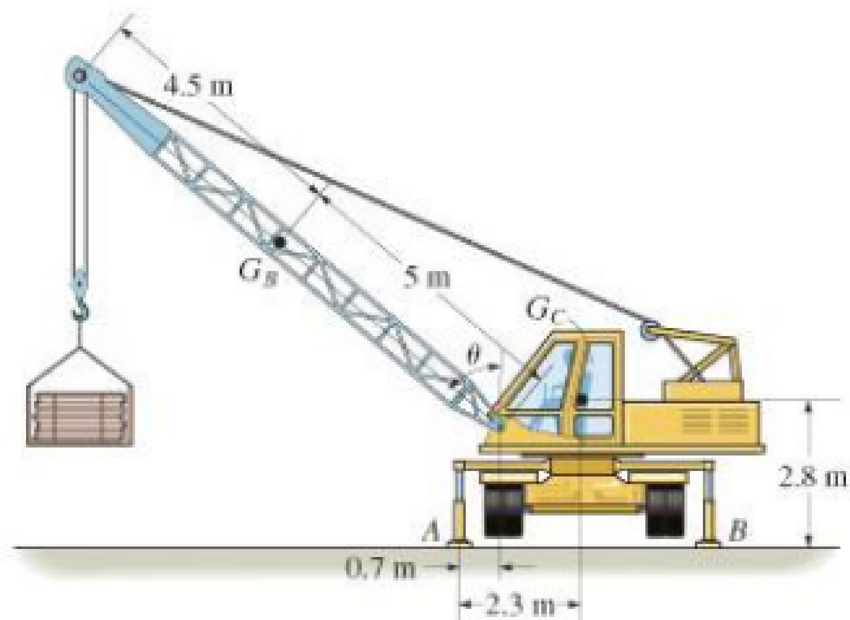
- a basic requirement for the study of the mechanics of deformable bodies and the mechanics of fluids (advanced courses).
- essential for the design and analysis of many types of structural members, mechanical components, electrical devices, etc, encountered in engineering.

A rigid body does not deform under load!

Engineering Mechanics

Rigid-body Mechanics

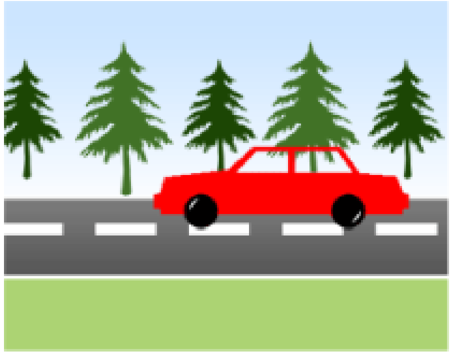
Statics: deals with equilibrium of bodies under action of forces (bodies may be either at rest or move with a constant velocity).

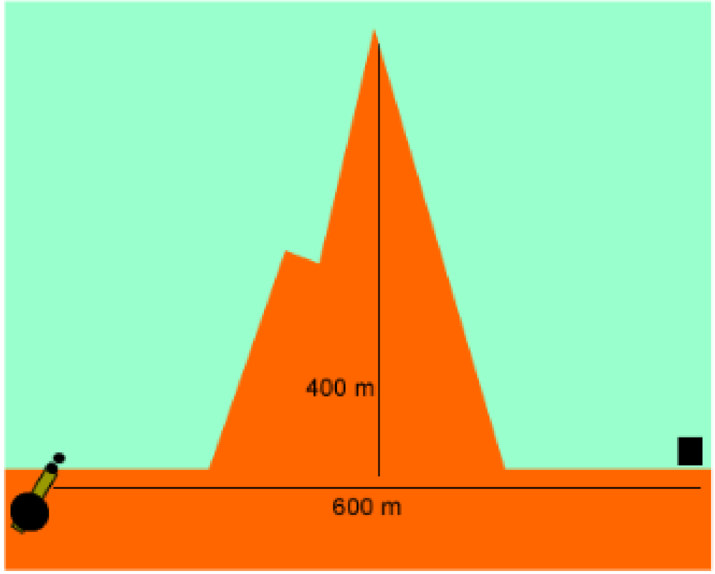


Engineering Mechanics

Rigid-body Mechanics

- Dynamics: deals with motion of bodies (accelerated motion)





- **Kinematics** deals with the geometry of motion of bodies without and application of external forces.
- **Kinetics** deals with the motion of bodies with the application of external forces.

Mechanics: Fundamental Concepts

Length (Space): needed to locate position of a point in space, & describe size of the physical system → Distances, Geometric Properties

Time: measure of succession of events → basic quantity in Dynamics

Mass: quantity of matter in a body → measure of inertia of a body (its resistance to change in velocity)

Force: represents the action of one body on another → characterized by its magnitude, direction of its action, and its point of application

→ Force is a Vector quantity.

Mechanics: Fundamental Concepts

Newtonian Mechanics

Length, Time, and Mass are absolute concepts
independent of each other

Force is a derived concept
not independent of the other fundamental concepts.
Force acting on a body is related to the mass of the body
and the variation of its velocity with time.

Force can also occur between bodies that are physically separated (Ex: gravitational, electrical, and magnetic forces)

Mechanics: Fundamental Concepts

Remember:

- Mass is a property of matter that does not change from one location to another.
- Weight refers to the gravitational attraction of the earth on a body or quantity of mass. Its magnitude depends upon the elevation at which the mass is located
- Weight of a body is the gravitational force acting on it.

Units:

Measurements are always made in comparison with certain standards. Example, ‘metre’ is the unit of length.

There are four systems of units used for the measurement of physical quantities. viz.

1.FPS (Foot – Pound – Second) system,

2.CGS (Centimetre – Gram – Second) system,

3.MKS (Meter-Kilogram–Second) system

4.SI (System international units– the French name)

The SI system of units is said to be an absolute system.

S.I Units (International System of Units)

The fundamental units of the system are

Metre (m) for length, Kilogram (kg) for mass and

Second (s) for time.

The unit for force is Newton (N).

One Newton is the amount of force required to induce an

acceleration of 1 m/sec2 on one kg mass.

Weight of a body (in N)

= Mass of the body (in kg) × Acceleration due to gravity (in /sec2).

The branch of mathematics dealing with dimensions of quantities is called dimensional analysis. There are two systems of dimensional analysis viz. absolute system and gravitational system.

Absolute system (MLT system)

A system of units defined on the basis of length, time and mass is referred to as an absolute system.

Gravitational system (FLT system)

A system of units defined on the basis of length, time and force is referred to as a gravitational system.

FLT system refers to the Force-Length-Time system.

MLT and FLT systems

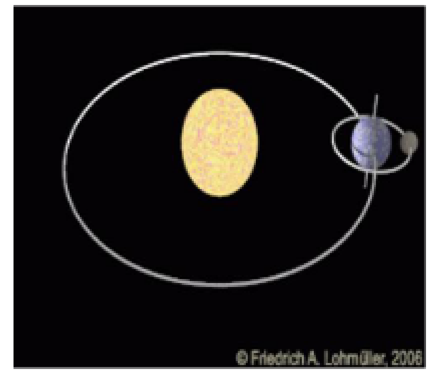
Quantity	MLT-System	FLT-system
Length	L	L
Mass	M	$FL^{-1}T^{2}$
Area	L^{2}	L^{2}
Volume	L^{3}	L^{3}
Velocity	LT^{-1}	LT^{-1}
Acceleration	LT^{-2}	LT^{-2}
Momentum	MLT^{-1}	FT
Stress	$ML^{-1}T^{-2}$	FL^{-2}
Weight	MLT^{-2}	F
Force	MLT^{-2}	F
Power	$ML^{2}T^{-3}$	FLT^{-1}
Density	ML^{-3}	$FL^{-4}T^{3}$

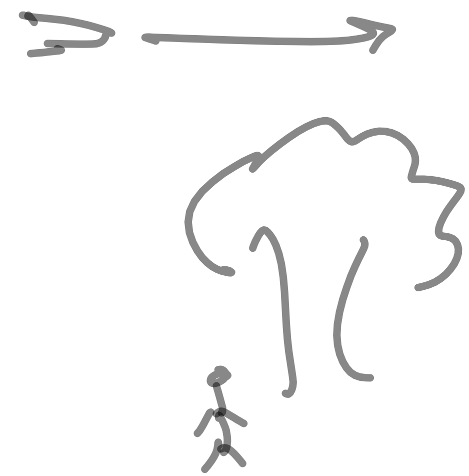
Mechanics: Idealizations

Idealizations are used in mechanics in order to simplify application of the theory.

1. **Particle**

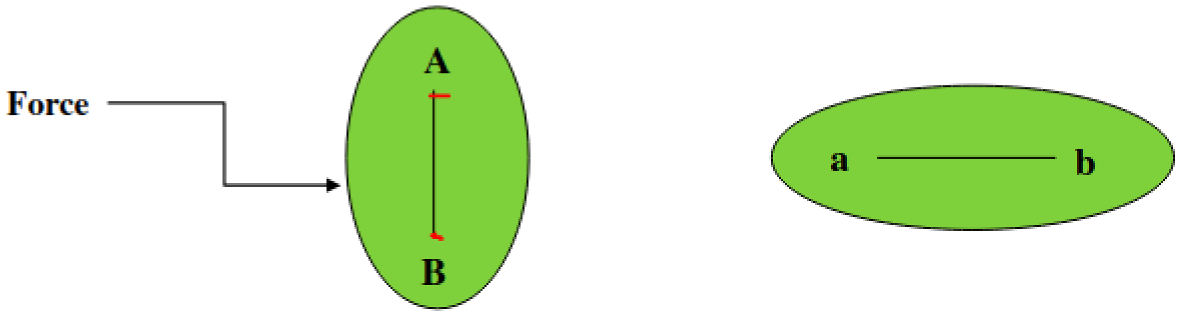
   - A particle has a mass, but a size that can be neglected, i.e. negligible dimensions.
   - For example, the size of the earth is insignificant compared to the size of its orbit, and therefore the earth can be modelled as a particle when studying its orbital motion.
   - Dimensions are considered to be near zero so that we may analyze it as a mass concentrated at a point
   - A body can be treated as a particle when its dimensions are irrelevant to the description of its position or the action of forces applied to it





Rigid body

- A rigid body can be considered as a combination of a large number of particles in which all the particles remain at a fixed distance from one another, both before and after applying a load.



Before applying force **After applying force**

- From figure, body will be called rigid, if AB=ab, i.e. the distance between the particles A & B will be same before and after the application of force.
- Statics deals primarily with the calculation of external forces which act on rigid bodies in equilibrium
- The actual deformations occurring in structures, machines, mechanisms, and the like are relatively small, and the rigid-body assumption is suitable for analysis

Concentrated Force

- A concentrated force represents the effect of a loading which is assumed to act at a point on a body.
- A load can be represented by a concentrated force, provided the area over which the load is applied is very small compared to the overall size of the body.
- For example, contact force between a wheel and the ground.

- **Scalar Quantity:** A quantity is said to be scalar if it is completely defined by its ***magnitude alone***. Examples of scalar quantities are: ***Area, length, Mass, Moment of inertia, Energy, Power, Volume And Work etc.***
- **Vector Quantity:** A quantity is said to be vector if it is completely defined only when its ***magnitude as well as direction are specified***. Examples of vector quantities include: ***Force, Moment, Momentum, Displacement, Velocity and Acceleration.***

$\vec{A} = 10\,\hat{i}$ $\qquad\qquad |\vec{A}|$

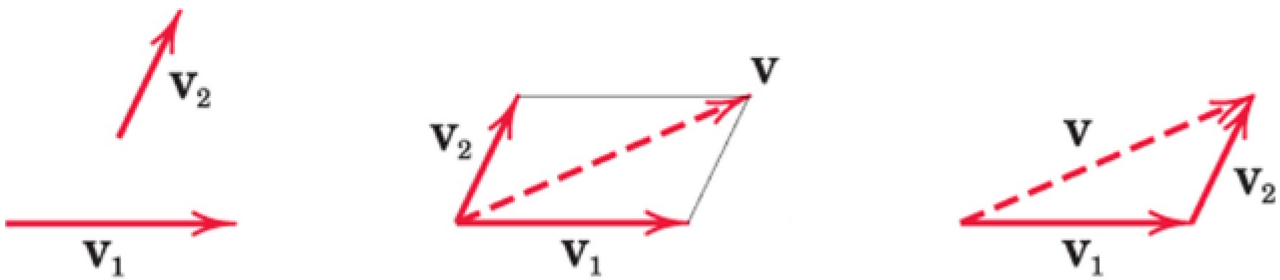
Scalars and Vectors

Scalars: only magnitude is associated.
Ex: time, volume, density, speed, energy, mass

Vectors: possess direction as well as magnitude, and must obey the parallelogram law of addition (and the triangle law).

Ex: displacement, velocity, acceleration,
force, moment, momentum

Equivalent Vector: $\mathbf{V} = \mathbf{V_1} + \mathbf{V_2}$ (Vector Sum)



Speed is the magnitude of velocity.

Vectors

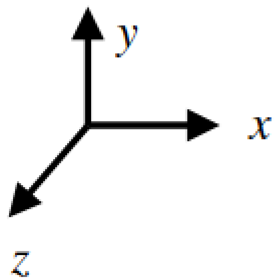
A Vector **V** can be written as: **V** = *V***n**

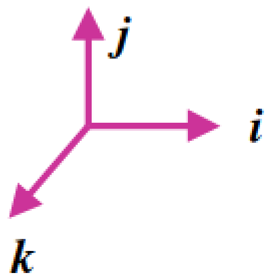
V = magnitude of **V**
n = unit vector whose magnitude is one and whose direction coincides with that of **V**

Unit vector can be formed by dividing any vector, such as the geometric position vector, by its length or magnitude

Vectors represented by Bold and Non-Italic letters (**V**)

Magnitude of vectors represented by Non-Bold, Italic letters (*V*)





i, ***j***, ***k*** – unit vectors

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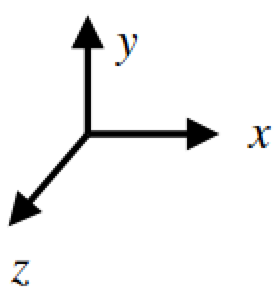
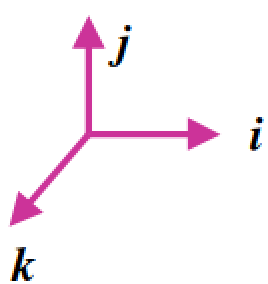
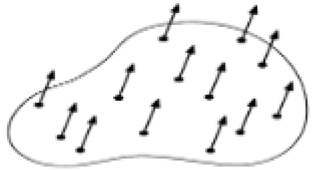




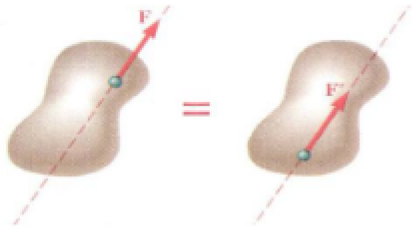
i, ***j***, ***k*** – unit vectors

- Vectors representing physical quantities can be classified as free, sliding, or fixed vector.

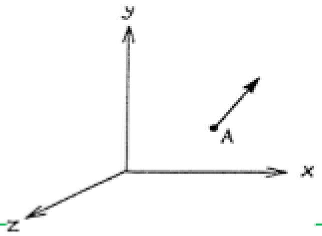
   - A free vector is one whose action is not confined to or associated with a unique line in space. For example, if a body moves without rotation, then the movement or displacement of any point in body may be a free vector.



   - A sliding vector has a unique line of action in space but not a unique point of application. For example, when an external force acts on a rigid body, the force can be applied at any point along its line of action without changing its effect on the body as a whole (Law of transmissibility of forces).

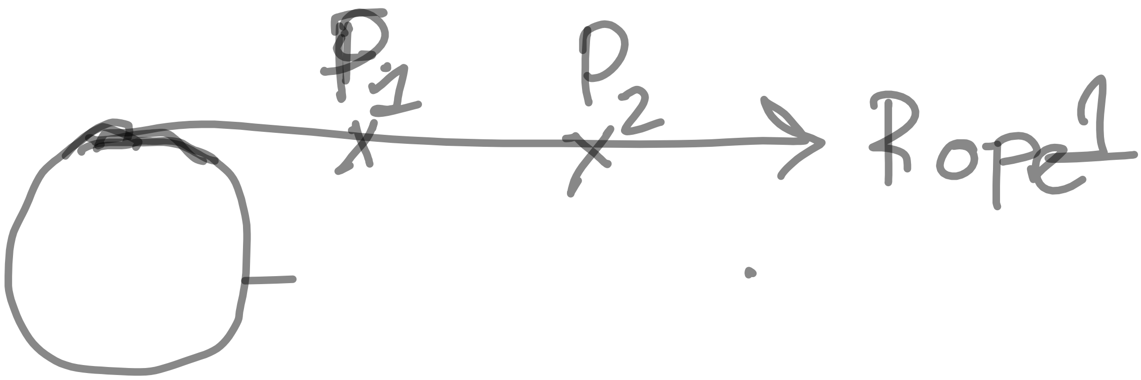


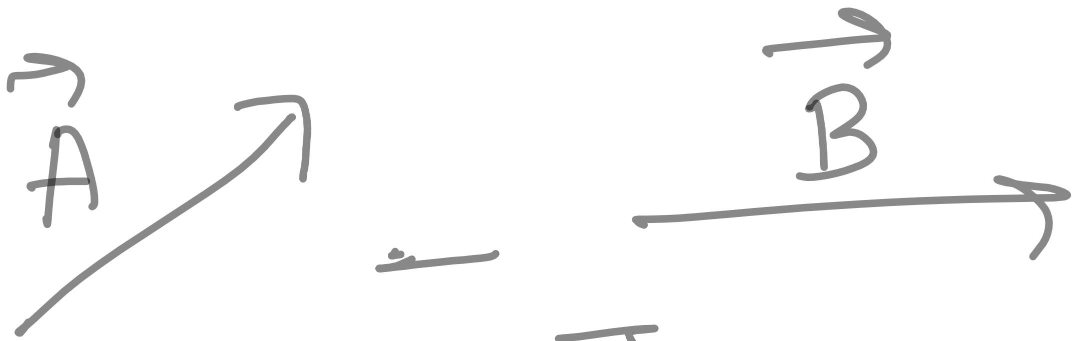
   - A fixed vector is one for which a unique point of application is specified. The action of a force on a deformable or nonrigid body must be specified by a fixed vector at the point of application of the force.

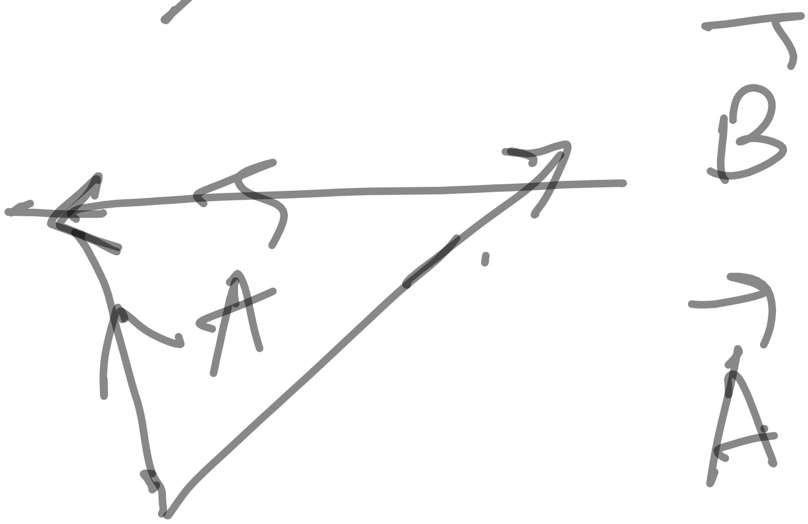


300 m → North

400 m → East



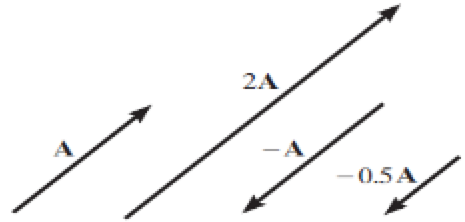




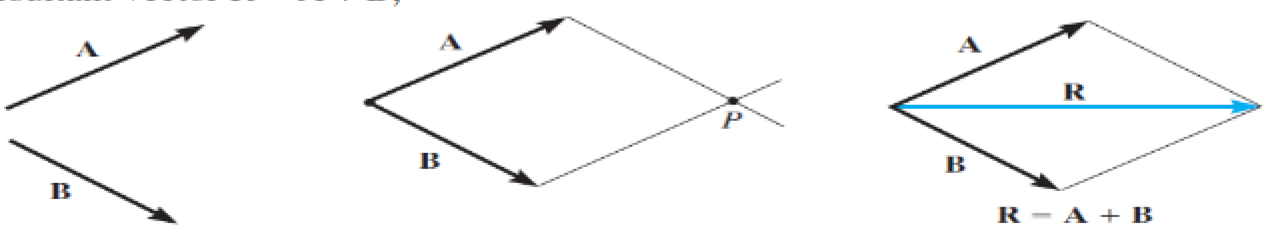
$$\vec{A} + \vec{B} = \vec{B} + \vec{A}$$

Vector Operations

- **Multiplication and Division of a Vector by a Scalar:** If a vector is multiplied by a positive scalar, its magnitude is increased by that amount. When multiplied by a negative scalar it will also change the directional sense of the vector.

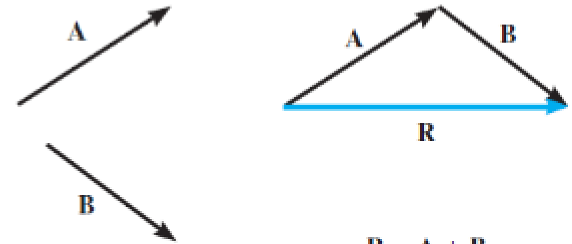


- **Vector Addition:** All vector quantities obey the parallelogram law of addition. Two "component" vectors A and B are added to form a "resultant" vector R =A+B using the following procedure:
  - First join the tails of the components at a point so that it makes them concurrent,
  - From the head of B, draw a line parallel to A. Draw another line
  - From the head of A that is parallel to B. These two lines intersect at point P to form the adjacent sides of a parallelogram.
  - The diagonal of this parallelogram that extends to P forms R, which then represents the resultant vector R = A + B,



Parallelogram law

We can also add B to A using the triangle rule, which is a special case of the parallelogram law, whereby vector B is added to vector A in a "head-to-tail" fashion, i.e., by connecting the head of A to the tail of B. The resultant R extends from the tail of A to the head of B.

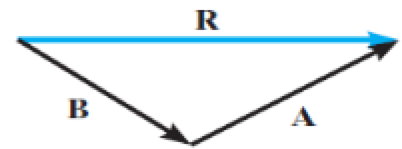


$\mathbf{R} = \mathbf{A} + \mathbf{B}$

Triangle rule

In a similar manner, R can also be obtained by adding A to B. By comparison, it is seen that vector addition is commutative; in other words, the vectors can be added in either order, i.e.,

$$\mathbf{R} = \mathbf{A} + \mathbf{B} = \mathbf{B} + \mathbf{A}$$



$\mathbf{R} = \mathbf{B} + \mathbf{A}$

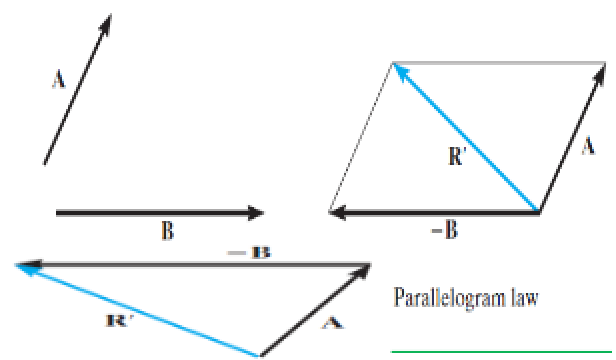
Triangle rule

As a special case, if the two vectors A and B are collinear, i.e., both have the same line of action, the parallelogram law reduces to an algebraic or scalar addition R = A + B

$R = A + B$

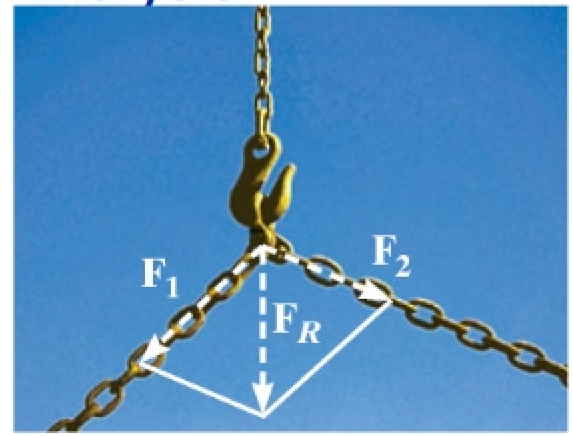
Vector Subtraction: The resultant of the difference between two vectors A and B of the same type may be expressed as, $\mathbf{R'} = \mathbf{A} - \mathbf{B} = \mathbf{A} + (-\mathbf{B})$

Subtraction is therefore defined as a special case of addition, so the rules of vector addition also apply to vector subtraction



Parallelogram law

Vector Addition: Procedure for Analysis



Parallelogram Law (Graphical)

Resultant Force (diagonal)

Components (sides of parallelogram)

Algebraic Solution

Using the coordinate system

Cosine law:

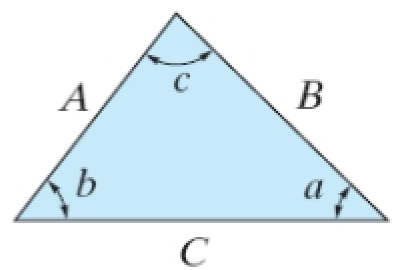
$$C = \sqrt{A^2 + B^2 - 2AB\cos c}$$

Sine law:

$$\frac{A}{\sin a} = \frac{B}{\sin b} = \frac{C}{\sin c}$$

Trigonometry (Geometry)

Resultant Force and Components from Law of Cosines and Law of Sines



a) Parallelogram law of forces

It states that "if two forces acting at a point are represented in magnitude and direction by the two adjacent sides of a parallelogram, then their resultant is represented in magnitude and direction by the diagonal of the parallelogram passing through the point".

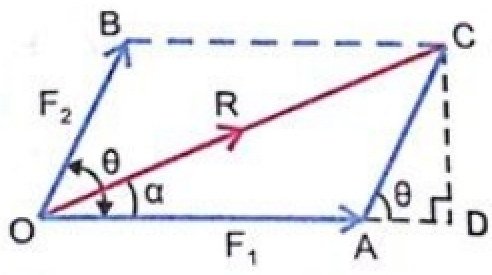


Fig. 2.7

Mathematically Resultant force, $R = \sqrt{F_1^2 + F_2^2 + 2F_1F_2\cos\theta}$

and Direction of resultant, $\tan\alpha = \dfrac{F_2\sin\theta}{F_1 + F_2\cos\theta}$

Where F_1 and F_2 = Forces whose resultant is required to be found out,

θ = Angle between the forces F_1 and F_2, and

α = Angle which the resultant force makes with one of the forces (say F_1)

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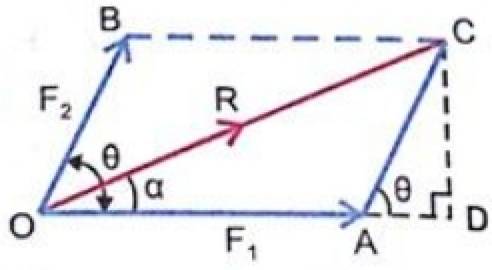

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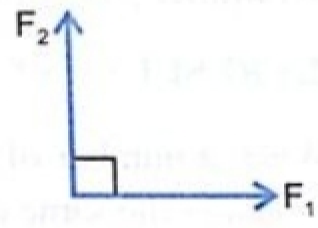
Special cases

Case i) If F_1 and F_2 are at right angles, then $\theta = 90°$

$$\text{Resultant } R = \sqrt{F_1^2 + F_2^2}$$



$$\text{Direction of resultant, } \tan\alpha = \frac{F_2}{F_1}$$

Case ii) If F_1 and F_2 are collinear and are in the same direction, then $\theta = 0$

$$\text{Resultant } R = F_1 + F_2$$

$$\text{Direction of resultant, } \tan\alpha = 0$$

$$\therefore\ \alpha = 0$$

Case iii) If F_1 and F_2 are collinear and are in opposite direction, then $\theta = 180°$

$$\text{Resultant } R = F_1 - F_2$$

$$\text{Direction of resultant, } \tan\alpha = 0$$

$$\therefore\ \alpha = 0$$

b) Triangle law of forces

It states that "if two forces (F_1 & F_2) acting simultaneously on a particle can be represented by the two sides of a triangle (in magnitude and direction) taken in order, then the third side (closing side) represents the resultant in the opposite order".

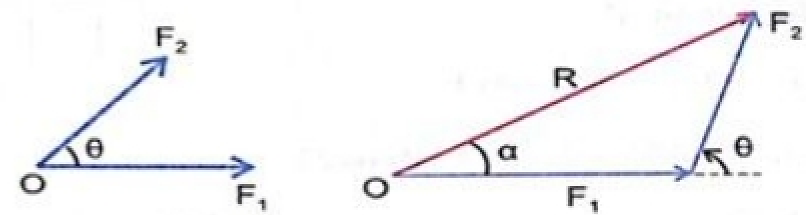


Fig. 2.8

Thus all the trigonometric relations can be applied

From Fig.2.9, Sine law is written as

$$\frac{A}{\sin\alpha}=\frac{B}{\sin\beta}=\frac{C}{\sin\gamma}$$

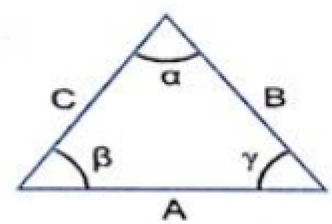


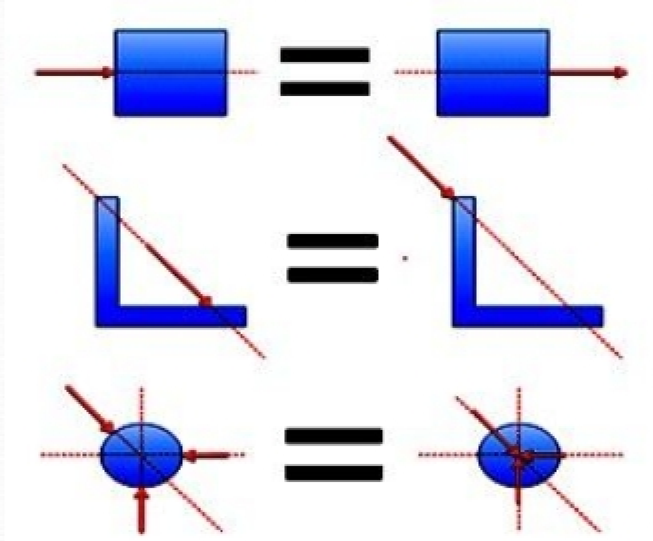
Fig. 2.9

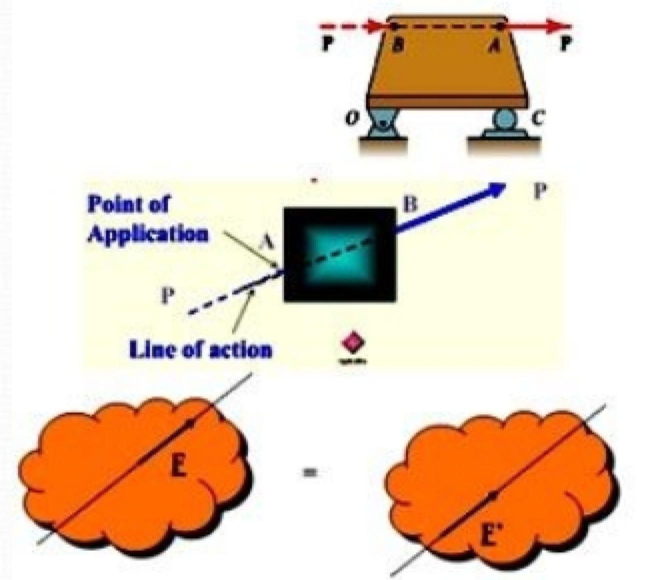
Cosine law is written as

$$A^2 = B^2 + C^2 - 2BC\cos\alpha$$
$$B^2 = A^2 + C^2 - 2AC\cos\beta$$
$$C^2 = A^2 + B^2 - 2AB\cos\gamma$$

Law of Transmissibility of Forces

- The Law of the Transmissibility of the Forces is states that When change the point of application of the force acting on a body to any other point which lies on the line of action of the force without any change in the direction of the force then there is no change in the equilibrium state of the body.



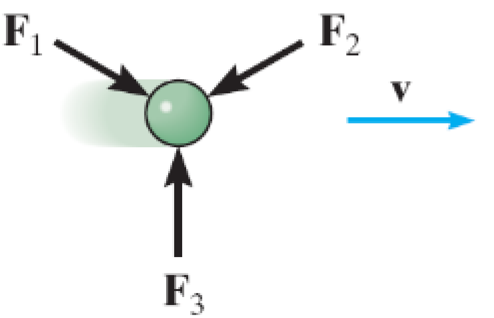


Mechanics: Newton's Three Laws of Motion

Basis of formulation of rigid body mechanics.

First Law: A particle originally at rest, or moving in a straight line with constant velocity, tends to remain in this state provided the particle is not subjected to an unbalanced force.

First law contains the principle of the equilibrium of forces → main topic of concern in Statics



Equilibrium

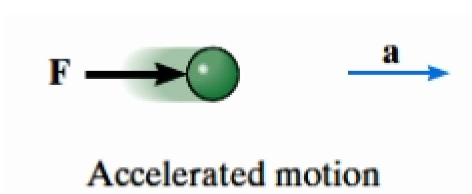
Mechanics: Newton’s Three Laws of Motion

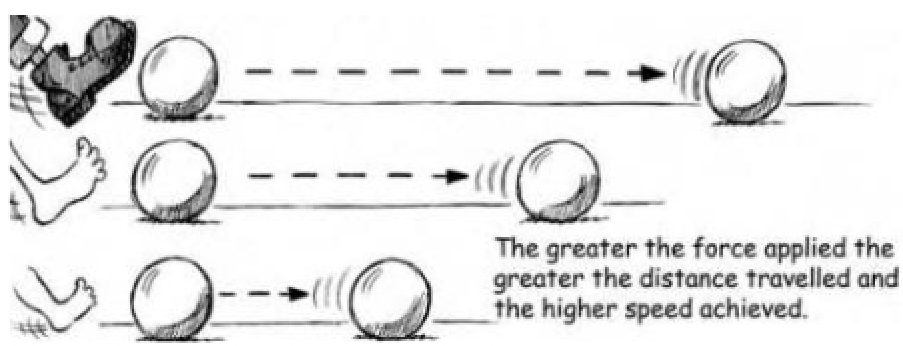
Second Law: A particle of mass “m” acted upon by an unbalanced force “F” experiences an acceleration “a” that has the same direction as the force and a magnitude that is directly proportional to the force.

F = ma

Accelerated motion

Second Law forms the basis for most of the analysis in Dynamics





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F = ma

Accelerated motion

Second Law forms the basis for most of the analysis in Dynamics

Force = rate of change of momentum.

But momentum = mass × velocity

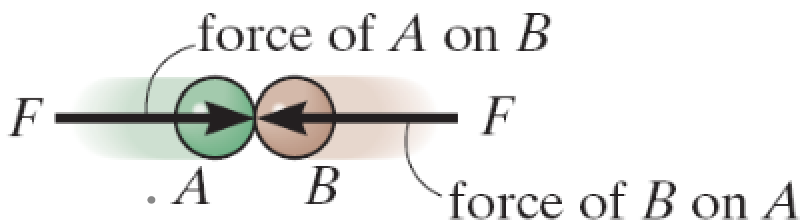
As mass do not change,

Force = mass × rate of change of velocity

i.e. Force = mass × acceleration $F = m \times a$

Mechanics: Newton's Three Laws of Motion

Third Law: The mutual forces of action and reaction between two particles are equal, opposite, and collinear.



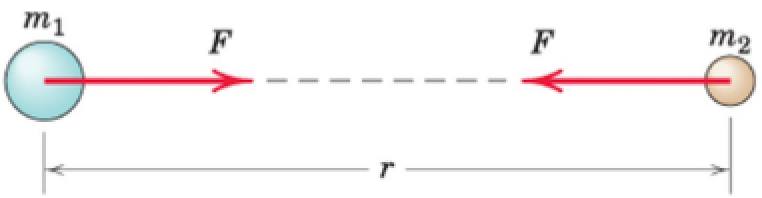
Action – reaction

Third law is basic to our understanding of Force → Forces always occur in pairs of equal and opposite forces.

Mechanics: Newton's Law of Gravitational Attraction

Weight of a body (gravitational force acting on a body) is required to be computed in Statics as well as Dynamics.
This law governs the gravitational attraction between any two particles.

$$F = G\frac{m_1 m_2}{r^2}$$



F = mutual force of attraction between two particles

G = universal constant of gravitation

Experiments → G = 6.673x10^{-11} m^3/(kg.s^2)

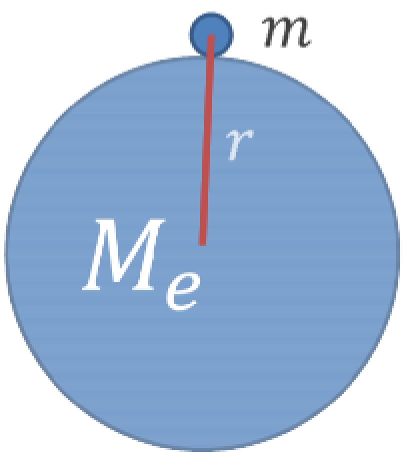
Rotation of Earth is not taken into account

$\boldsymbol{m_1}$, $\boldsymbol{m_2}$ = masses of two particles

r = distance between two particles

Gravitational Attraction of the Earth

Weight of a Body: If a particle is located at or near the surface of the earth, the only significant gravitational force is that between the earth and the particle



Weight of a particle having mass $m_1 = m$:

Assuming earth to be a non-rotating sphere of constant density and having mass $m_2 = M_e$

$$W = G\frac{mM_e}{r^2}$$

r = distance between the earth's center and the particle

$$W = mg$$

Let $g = G\,M_e/r^2$ = acceleration due to gravity (9.81m/s²)

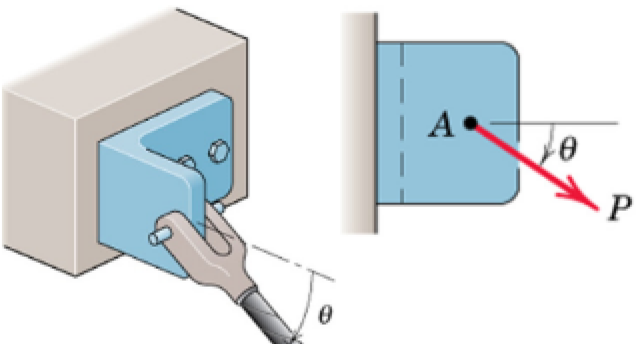
Force Systems

Force: Magnitude (*P*), direction (arrow) and point of application (point A) is important

Change in any of the three specifications will alter the effect on the bracket.

Force is a Fixed Vector

In case of rigid bodies, line of action of force is important (not its point of application if we are interested in only the resultant external effects of the force), we will treat most forces as



Cable Tension *P*

External effect: Forces applied (applied force); Forces exerted by bracket, bolts, Foundation (reactive force)

Internal effect: Deformation, strain pattern – permanent strain; depends on material properties of bracket, bolts, etc.

Force Systems

Force: Magnitude (P), direction (arrow) and point of application (point A) is important

Change in any of the three specifications will alter the effect on the bracket.

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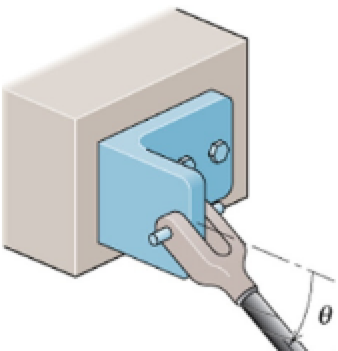
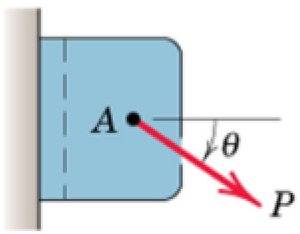


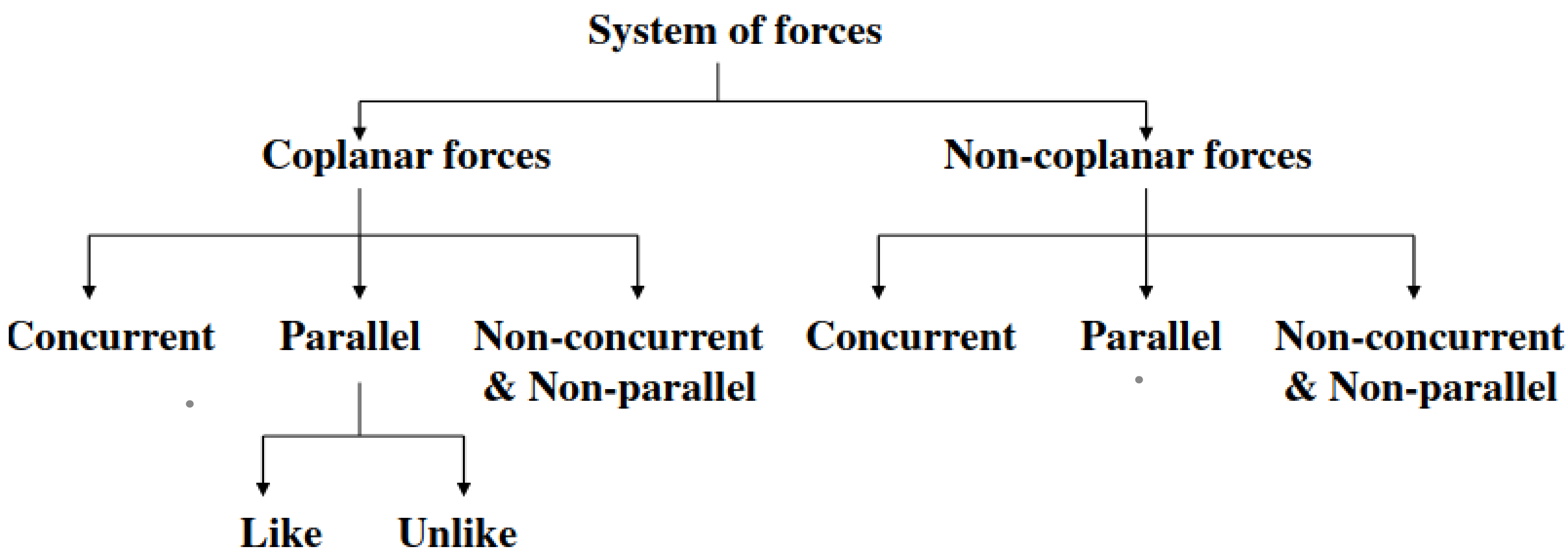
Cable Tension P



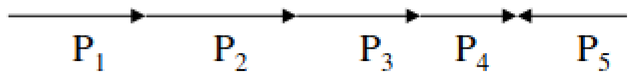
External effect: Forces applied (applied force); Forces exerted by bracket, bolts, Foundation (reactive force)

Internal effect: Deformation, strain pattern – permanent strain; depends on material properties of bracket, bolts, etc.

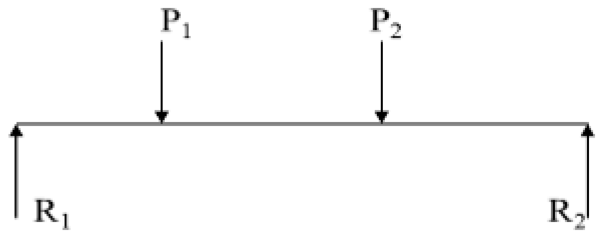
- Considering the plane in which force is applied and depending upon the position of line of action, forces may be classified as,



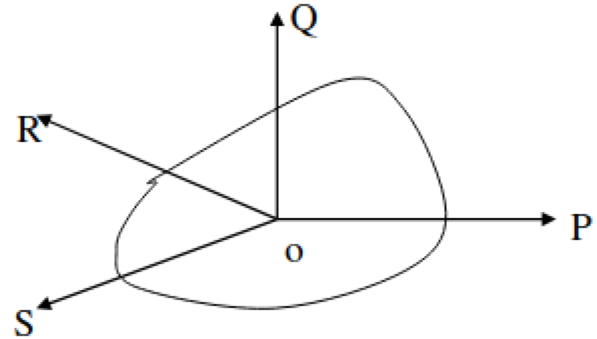
- **Collinear forces**: Line of action of all forces lie in same straight line. Example: Forces on a rope in tug of war.



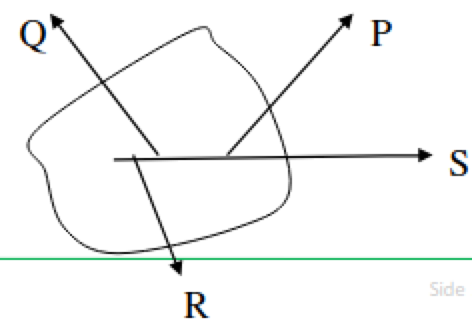
- **Coplanar parallel forces**: Line of action of all forces are parallel and lie on same plane. Example: Vertical loads acting on a beam.



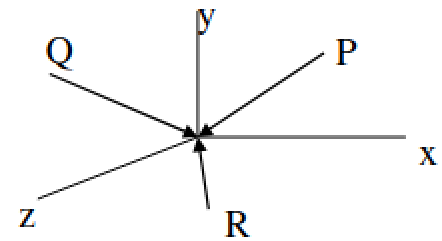
- **Coplanar concurrent forces**: Line of action of all forces, acting on same plane, pass through one point and directions will be different. Example: Forces on a rod resting against a wall.



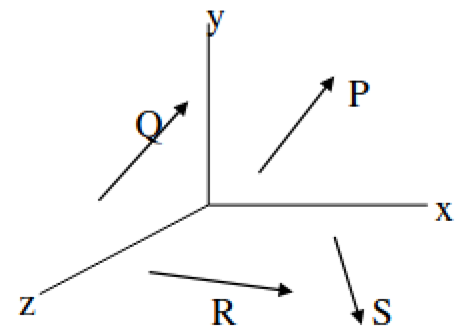
- **Coplanar non-concurrent forces:** Line of action of all forces, acting on same plane, do not pass through one point. Example: Forces on a ladder against a wall and a man standing on a rung which is not its centre of gravity.



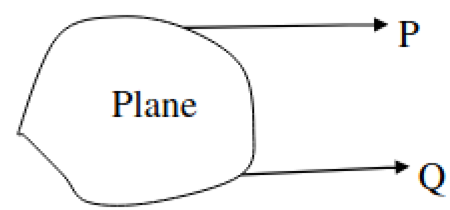
- **Non-coplanar concurrent forces:** All forces do not lie on same plane but their line of action pass through a single point. Example: Forces on a tripod carrying a camera.



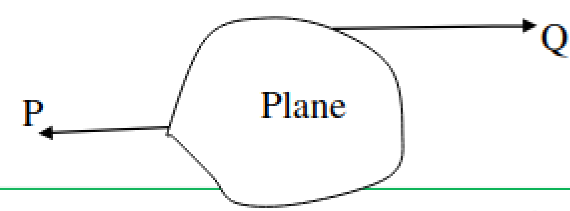
- **Non-coplanar non-concurrent forces:** All forces do not lie on same plane and also their line of action do not pass through a single point. Example: Forces on a moving bus.



- **Coplanar like parallel forces**: Line of action of all forces are parallel, lie on same plane and in the same direction.



- **Coplanar unlike parallel forces**: Line of action of all forces are parallel, lie on same plane and in the opposite direction.

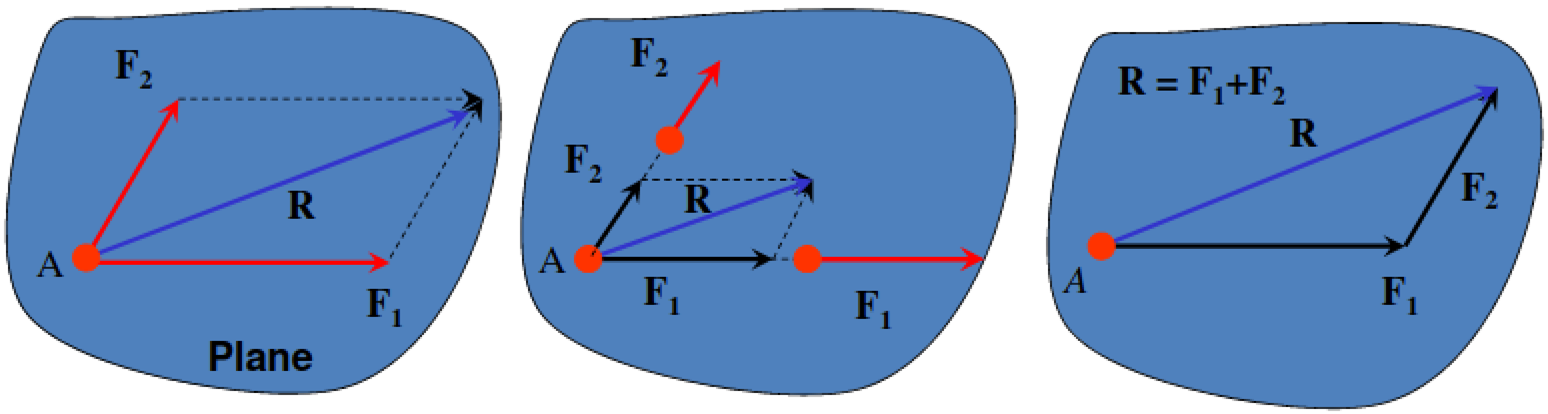


Force Systems

Concurrent force:

Forces are said to be concurrent at a point if their lines of action intersect at that point

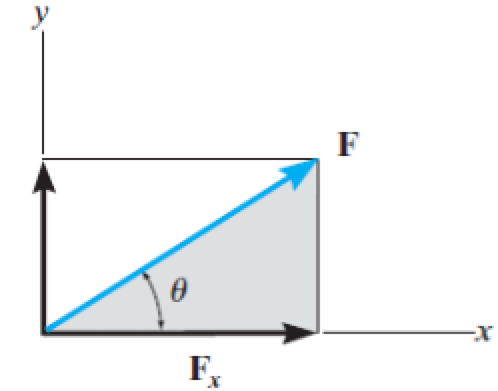
F_1, F_2 are concurrent forces; **R** will be on same plane; **R = F_1+F_2**



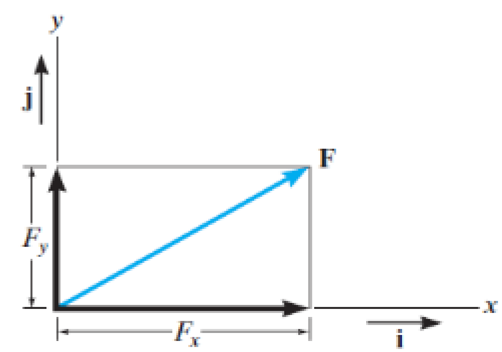
Forces act at same point Forces act at different point Triangle Law

(Apply Principle of Transmissibility)

Addition of a System of Coplanar Forces



- When a force is resolved into two components along the x and y axes, the components are then called rectangular components.
- One can represent these components in one of two ways, using either scalar notation or Cartesian vector notation
- **Scalar notation**: Rectangular components of force F are found using the parallelogram law, so that $F = F_x + F_y$
  - ➢ magnitudes can be determined by $F_x = F\cos\theta$
    $F_y = F\sin\theta$
- **Cartesian Vector Notation**: It is also possible to represent the *x* and *y* components of a force in terms of Cartesian unit vectors **i** and **j**.
  - ➢ Since the magnitude of each component of F is always a positive quantity, which is represented by the (positive) scalars Fx and Fy, then we can express F as a Cartesian vector, $$F = F_x i + F_y j$$



Coplanar Force Resultants

- Each force is first resolved into its x and y components, and then respective components are added using scalar algebra since they are collinear.
- The resultant force is then formed by adding the resultant components using the parallelogram law.
- We can represent the components of the resultant force of any number of coplanar forces symbolically by the algebraic sum of the x and y components of all the forces, i.e. $F_{Rx} = \sum F_x$ and $F_{Ry} = \sum F_y$
- Magnitude of resultant, $F_R = \sqrt{F_{Rx}^2 + F_{Ry}^2}$
- Direction of resultant, $\theta = \tan^{-1}(\frac{F_{Ry}}{F_{Rx}})$
- From given problem, $F_1 = F_{1x}i + F_{1y}j$, $F_2 = -F_{2x}i + F_{2y}j$

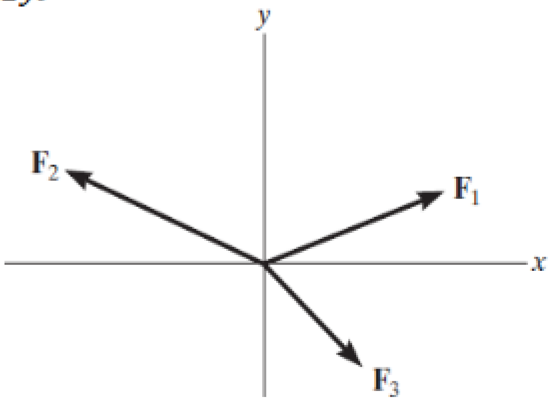
$$F_3 = F_{3x}i - F_{3y}j$$

Resultant, $F_R = F_1 + F_2 + F_3$

$$F_R = F_{1x}i + F_{1y}j - F_{2x}\,i + F_{2y}j + F_{3x}i - F_{3y}j$$

$$F_R = (F_{1x} - F_{2x} + F_{3x})i + (F_{1y} + F_{2y} - F_{3y})j$$

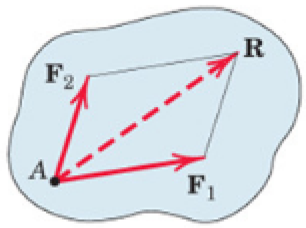
$$F_R = F_{Rx}i + F_{Ry}j$$



Components and Projections of Force

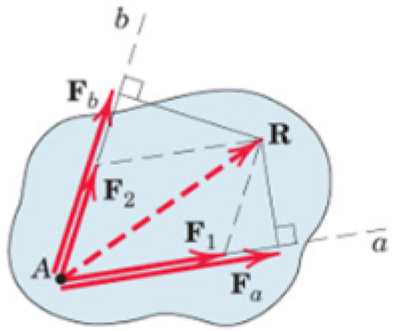
Components of a Force are not necessarily equal to the Projections of the Force unless the axes on which the forces are projected are orthogonal (perpendicular to each other).

F_1 and **F_2** are components of **R**. **R = F_1 + F_2**



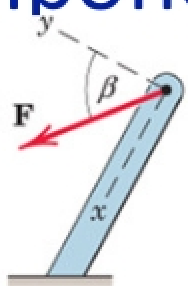
F_a and **F_b** are perpendicular projections on axes a and b, respectively.

R ≠ F_a + F_b unless a and b are perpendicular to each other

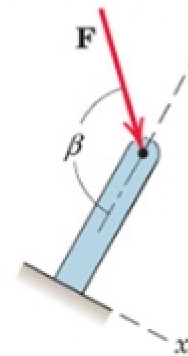


Components of Force

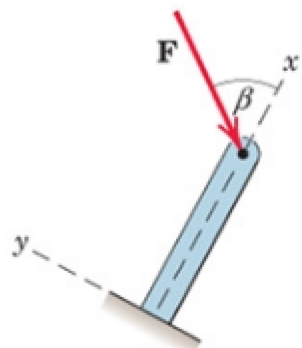
Examples



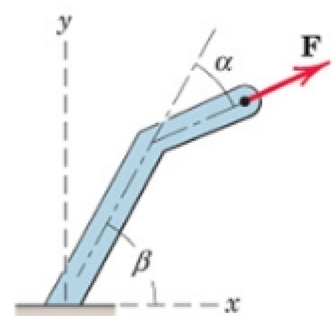
$F_x = F \sin \beta$
$F_y = F \cos \beta$



$F_x = F \sin(\pi - \beta)$
$F_y = -F \cos(\pi - \beta)$

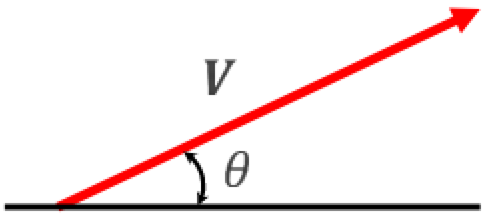


$F_x = -F \cos \beta$
$F_y = -F \sin \beta$

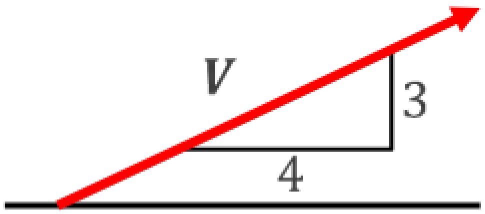


$F_x = F \cos(\beta - \alpha)$
$F_y = F \sin(\beta - \alpha)$

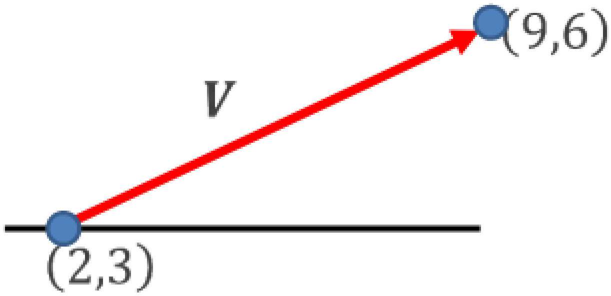
Vector



$$\boldsymbol{V} = V(cos\theta\boldsymbol{i} + sin\theta\boldsymbol{j})$$



$$\boldsymbol{V} = V\left(\frac{4\boldsymbol{i} + 3\boldsymbol{j}}{\sqrt{4^2 + 3^2}}\right)$$

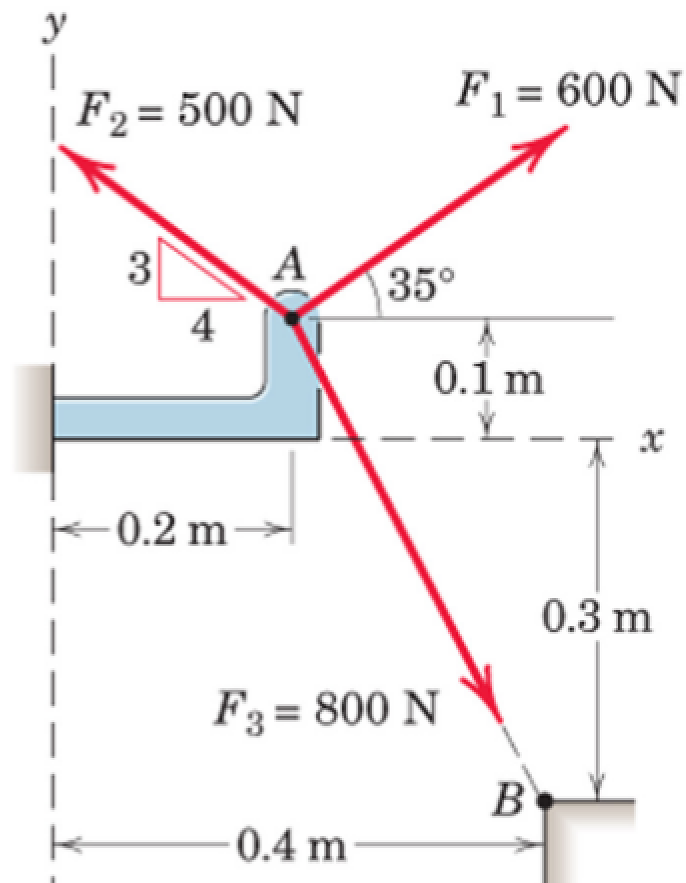


$$\boldsymbol{V} = V\left(\frac{(9-2)\boldsymbol{i} + (6-3)\boldsymbol{j}}{\sqrt{(9-2)^2 + (6-3)^2}}\right)$$

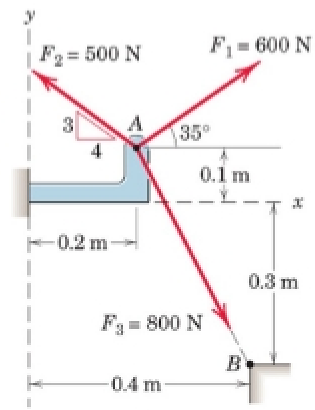
Components of Force

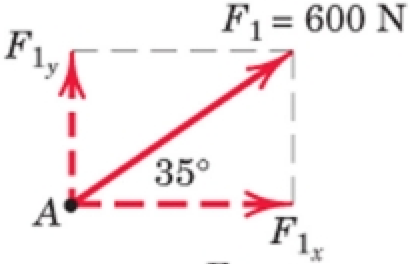
Example 1:

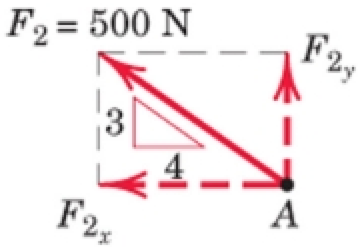
Determine the x and y scalar components of F_1, F_2, and F_3 acting at point A of the bracket

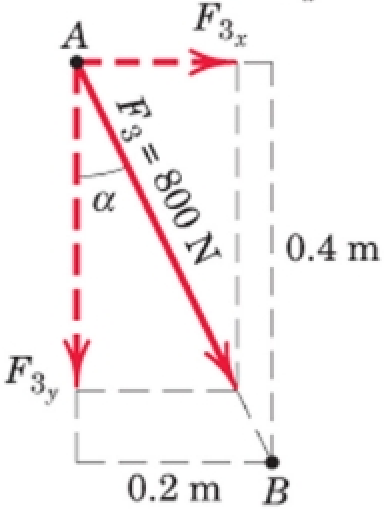


Components of Force









$$F_{1_x} = 600 \cos 35^\circ = 491 \text{ N}$$

$$F_{1_y} = 600 \sin 35^\circ = 344 \text{ N}$$

$$F_{2_x} = -500(\tfrac{4}{5}) = -400 \text{ N}$$

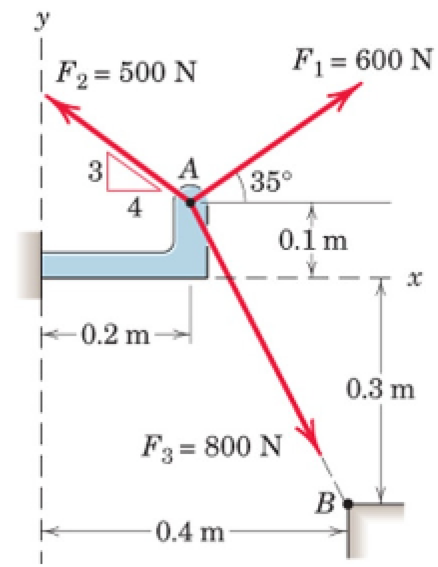
$$F_{2_y} = 500(\tfrac{3}{5}) = 300 \text{ N}$$

$$\alpha = \tan^{-1}\left[\frac{0.2}{0.4}\right] = 26.6^\circ$$

$$F_{3_x} = F_3 \sin \alpha = 800 \sin 26.6^\circ = 358 \text{ N}$$

$$F_{3_y} = -F_3 \cos \alpha = -800 \cos 26.6^\circ = -716 \text{ N}$$

Components of Force



Alternative Solution

$$\boldsymbol{F}_1 = F_1\boldsymbol{n}_1 = F_1\frac{cos(35^o)\boldsymbol{i} + sin(35^o)\boldsymbol{j}}{\sqrt{\left(cos(35^o)\right)^2 + \left(sin(35^o)\right)^2}}$$

$$= 600[0.819\boldsymbol{i} + 0.5735\boldsymbol{j}]$$

$$= 491\boldsymbol{i} + 344\boldsymbol{j}$$

$F_{1x} = 491\ N$ $\quad F_{1y} = 344\ N$

$$\boldsymbol{F}_2 = F_2\boldsymbol{n}_2 = F_2\frac{-4\boldsymbol{i} + 3\boldsymbol{j}}{\sqrt{(-4)^2 + (3)^2}}$$

$$= 500[-0.8\boldsymbol{i} + 0.6\boldsymbol{j}] \quad = 400\boldsymbol{i} + 300\boldsymbol{j}$$

$F_{2x} = 400\ N$ $\quad F_{2y} = 300\ N$

Components of Force

Alternative Solution

$$\overrightarrow{AB} = 0.2\boldsymbol{i} - 0.4\boldsymbol{j}$$

$$\overline{AB} = \sqrt{(0.2)^2 + (-0.4)^2}$$

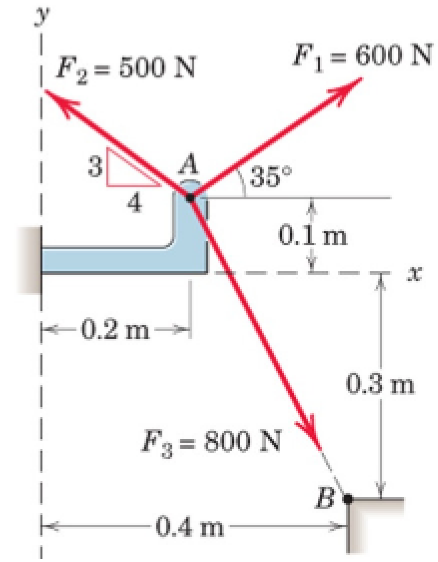
$$\boldsymbol{F_3} = F_3\boldsymbol{n}_3 = F_3\frac{\overrightarrow{AB}}{\overline{AB}}$$

$$= 800\frac{0.2\boldsymbol{i} - 0.4\boldsymbol{j}}{\sqrt{(0.2)^2 + (-0.4)^2}}$$

$$= 800[0.447\boldsymbol{i} - 0.894\boldsymbol{j}]$$

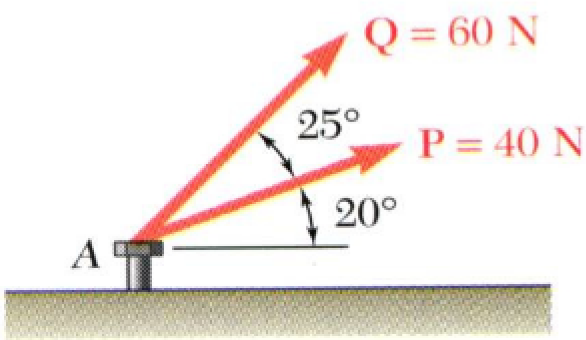
$$= 358\boldsymbol{i} - 716\boldsymbol{j}$$

$F_{3x} = 358\ N$ $\qquad$ $F_{3y} = 716\ N$



Components of Force

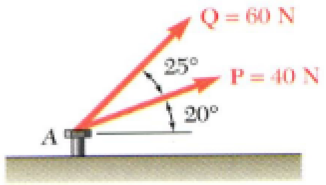
Example 2: The two forces act on a bolt at A. Determine their resultant.



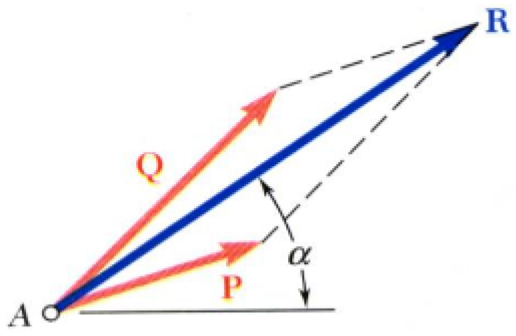
Graphical solution - construct a parallelogram with sides in the same direction as P and Q and lengths in proportion. Graphically evaluate the resultant which is equivalent in direction and proportional in magnitude to the diagonal.

Trigonometric solution - use the triangle rule for vector addition in conjunction with the law of cosines and law of sines to find the resultant.

Components of Force

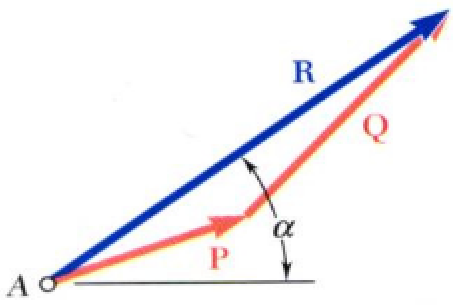


Solution:



- Graphical solution - A parallelogram with sides equal to **P** and **Q** is drawn to scale. The magnitude and direction of the resultant or of the diagonal to the parallelogram are measured,

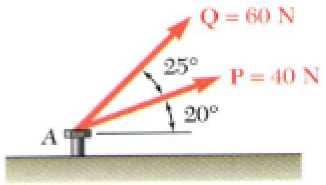
$$\boxed{\mathbf{R} = 98\ \mathrm{N} \quad \alpha = 35°}$$



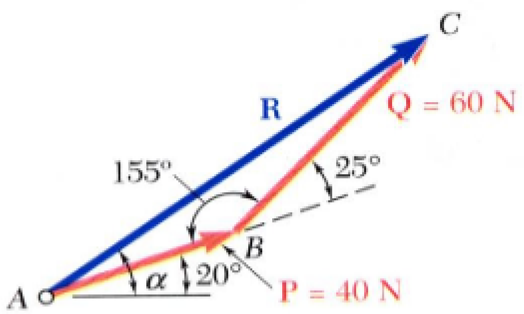
- Graphical solution - A triangle is drawn with **P** and **Q** head-to-tail and to scale. The magnitude and direction of the resultant or of the third side of the triangle are measured,

$$\boxed{\mathbf{R} = 98\ \mathrm{N} \quad \alpha = 35°}$$

Components of Force



Trigonometric Solution: Apply the triangle rule.



From the Law of Cosines,

$$R^2 = P^2 + Q^2 - 2PQ\cos B$$
$$= (40\,\mathrm{N})^2 + (60\,\mathrm{N})^2 - 2(40\,\mathrm{N})(60\,\mathrm{N})\cos 155°$$

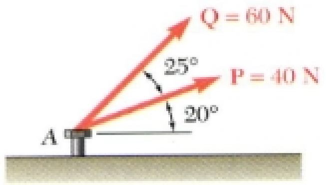
$$\boxed{R = 97.73\,\mathrm{N}}$$

From the Law of Sines,

$$\frac{\sin A}{Q} = \frac{\sin B}{R}$$

$$\sin A = \sin B\,\frac{Q}{R}$$
$$= \sin 155°\,\frac{60\,\mathrm{N}}{97.73\,\mathrm{N}}$$
$$A = 15.04°$$
$$\alpha = 20° + A$$
$$\boxed{\alpha = 35.04°}$$

Components of Force



$\boldsymbol{R} = \boldsymbol{P} + \boldsymbol{Q}$

$\boldsymbol{P} = 40[\cos(20)\boldsymbol{i} + \sin(20)\boldsymbol{j}]$

$= 37.58\mathbf{i} + 13.68\mathbf{j}$

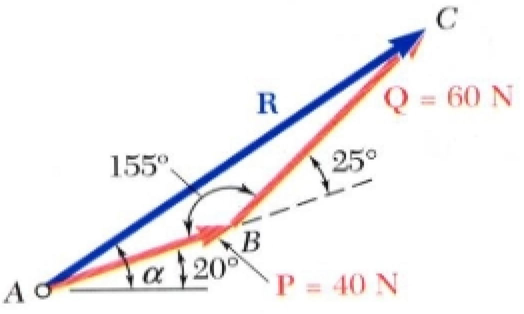
$\boldsymbol{Q} = 60[\cos(45)\,\boldsymbol{i} + \sin(45)\boldsymbol{j}]$

$= 42.43\mathbf{i} + 42.43\mathbf{j}$

$\boldsymbol{R} = 80.01\boldsymbol{i} + 56.10\boldsymbol{j}$

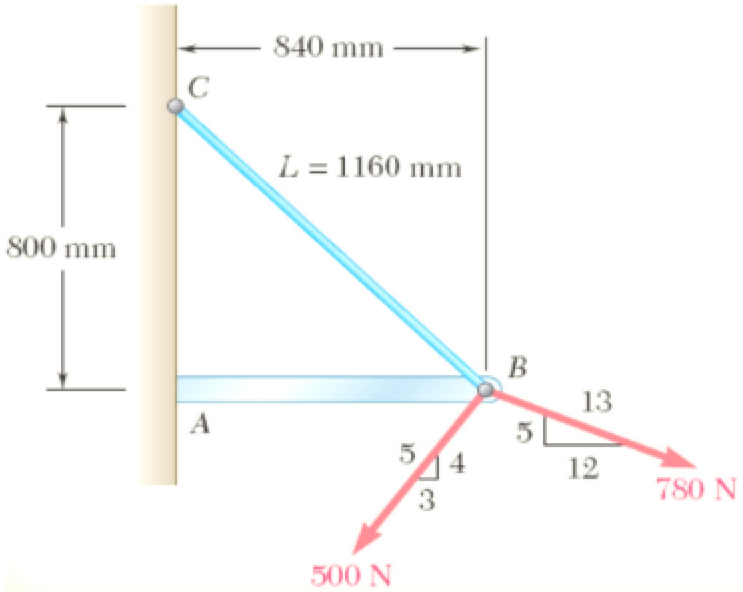
$R = 97.72$

$\alpha = 35.03^o$



Components of Force

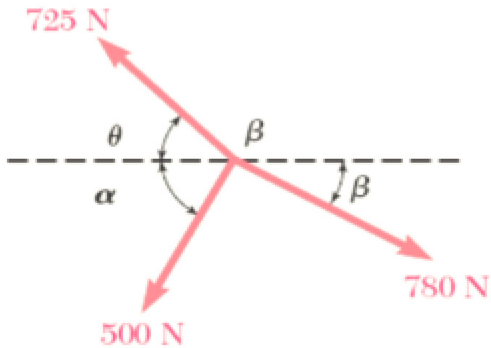
Example 3:Tension in cable *BC* is 725-N, determine the resultant of the three forces exerted at point *B* of beam *AB*.



Solution:

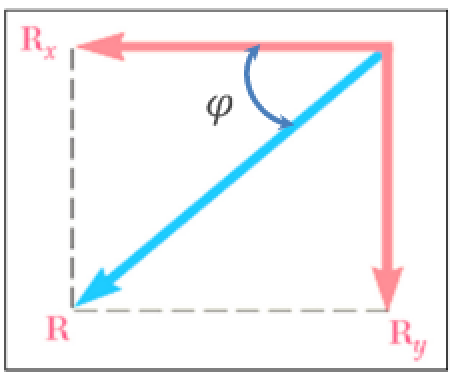
- Resolve each force into rectangular components.
- Determine the components of the resultant by adding the corresponding force components.
- Calculate the magnitude and direction of the resultant.

Components of Force



Resolve each force into rectangular components

Magnitude (N)	X-component (N)	Y-component (N)
725	-525	500
500	-300	-400
780	720	-300
	$R_x = -105$	$R_y = -200$



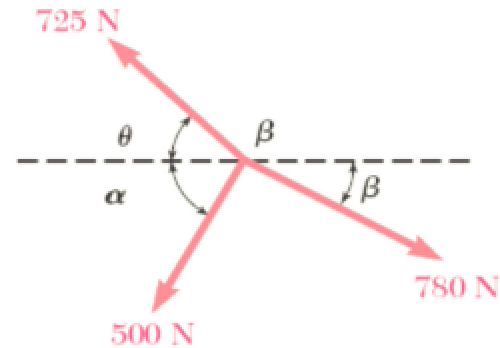
$\mathbf{R} = R_x i + R_y j$ $\mathbf{R} = (-105)i + (-200)j$

Calculate the magnitude and direction

$$tan\varphi = \frac{R_x}{R_y} = \frac{105}{200} \qquad \varphi = 62.3^o$$

$$\mathbf{R} = \sqrt{R_x^2 + R_y^2} = 225.9N$$

Components of Force



Alternate solution

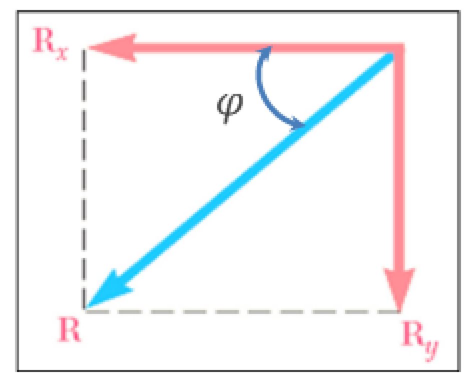
$$\boldsymbol{R} = \boldsymbol{F_1} + \boldsymbol{F_2} + \boldsymbol{F_3}$$

$$\boldsymbol{F_1} = 725[-0.724\boldsymbol{i} + 0.689\boldsymbol{j}]$$

$$\boldsymbol{F_2} = 500[-0.6\boldsymbol{i} - 0.8\boldsymbol{j}]$$

$$\boldsymbol{F_2} = 780[0.923\boldsymbol{i} - 0.384\boldsymbol{j}]$$

$$\boldsymbol{R} = -105\boldsymbol{i} - 200\boldsymbol{j}$$

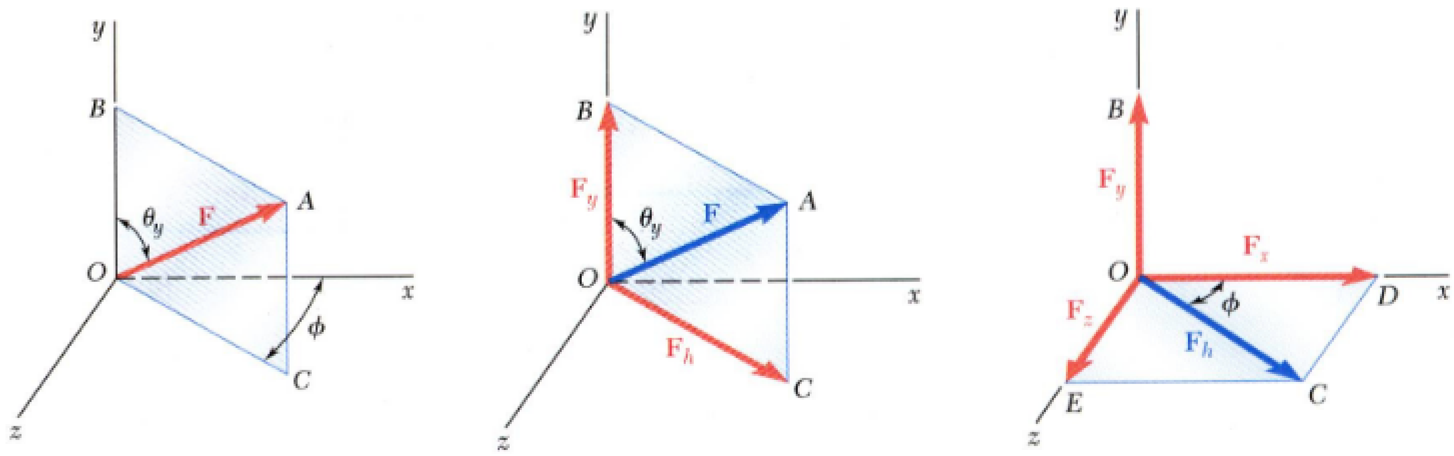


Calculate the magnitude and direction

$$tan\varphi = \frac{R_x}{R_y} = \frac{105}{200} \qquad \varphi = 62.3^o$$

$$\mathbf{R} = \sqrt{R_x^2 + R_y^2} = 225.9N$$

Rectangular Components in Space



- The vector $\vec{F}$ is contained in the plane *OBAC*.
- Resolve $\vec{F}$ into horizontal and vertical components.

$$F_y = F\cos\theta_y$$

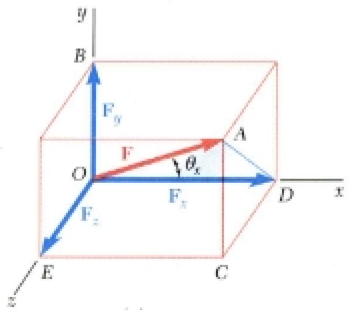
$$F_h = F\sin\theta_y$$

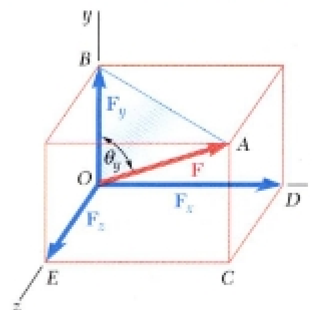
- Resolve F_h into rectangular components

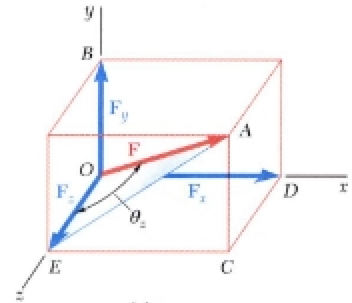
$$F_x = F_h\cos\phi = F\sin\theta_y\cos\phi$$

$$F_z = F_h\sin\phi = F\sin\theta_y\sin\phi$$

Rectangular Components in Space







$F_x = F cos\theta_x$ $\quad F_y = F cos\theta_y$ $\quad F_z = F cos\theta_z$

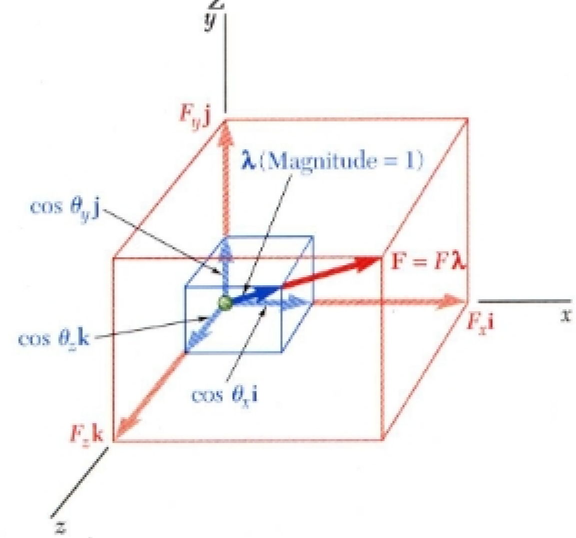
$$\boldsymbol{F} = F_x\boldsymbol{i} + F_y\boldsymbol{j} + F_z\boldsymbol{k}$$

$$\boldsymbol{F} = F cos\theta_x\boldsymbol{i} + F cos\theta_y\boldsymbol{j} + F cos\theta_z\boldsymbol{k}$$

$$\boldsymbol{F} = F\left(cos\theta_x\boldsymbol{i} + cos\theta_y\boldsymbol{j} + cos\theta_z\boldsymbol{k}\right)$$

$$\boldsymbol{F} = F\boldsymbol{\lambda}$$

Where $\boldsymbol{\lambda} = cos\theta_x\boldsymbol{i} + cos\theta_y\boldsymbol{j} + cos\theta_z\boldsymbol{k}$

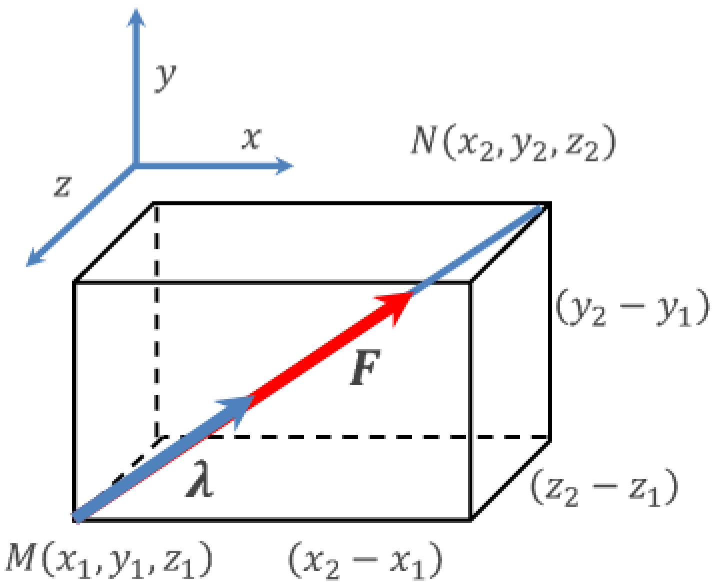


$\boldsymbol{\lambda}$ is a unit vector along the line of action of $\boldsymbol{F}$ and $cos\theta_x$, $cos\theta_y$ and $cos\theta_z$ are the direction cosine for $\boldsymbol{F}$

Rectangular Components in Space

Direction of the force is defined by the location of two points

$M(x_1, y_1, z_1)$ and $N(x_2, y_2, z_2)$



$\boldsymbol{d}$ is the vector joining M and N

$$\boldsymbol{d} = d_x\boldsymbol{i} + d_y\boldsymbol{j} + d_z\boldsymbol{k}$$

$$d_x = (x_2 - x_1) \qquad d_y = (y_2 - y_1)$$

$$d_z = (z_2 - z_1)$$

$$\boldsymbol{F} = F\boldsymbol{\lambda}$$

$$= F\left(\frac{d_x\boldsymbol{i} + d_y\boldsymbol{j} + d_z\boldsymbol{k}}{d}\right)$$

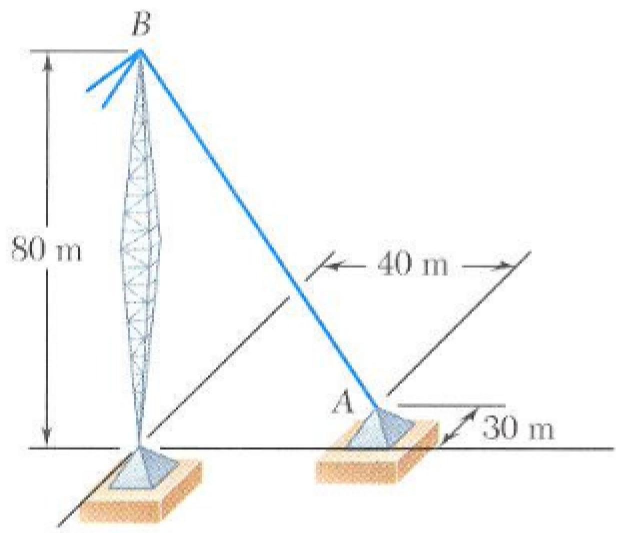
$$F_x = F\frac{d_x}{d} \qquad F_y = F\frac{d_y}{d} \qquad F_z = F\frac{d_z}{d}$$

Rectangular Components in Space

Example: The tension in the guy wire is 2500 N. Determine:

a) components F_x, F_y, F_z of the force acting on the bolt at A,

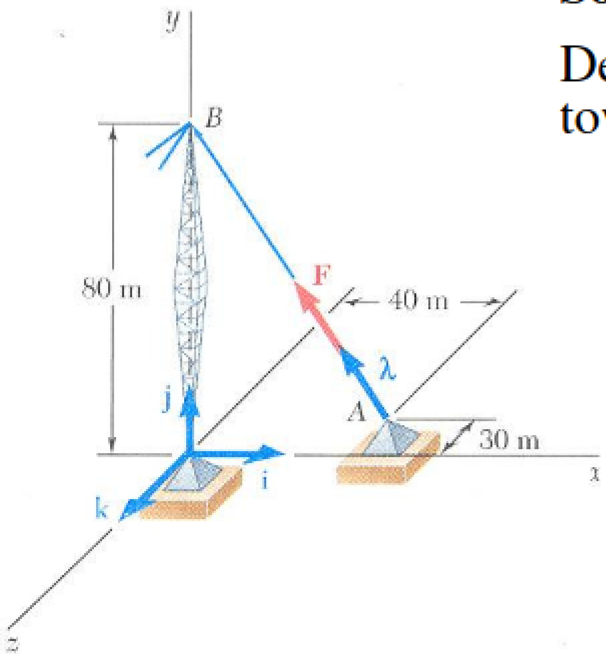
b) the angles q_x, q_y, q_z defining the direction of the force



SOLUTION:

- Based on the relative locations of the points A and B, determine the unit vector pointing from A towards B.
- Apply the unit vector to determine the components of the force acting on A.
- Noting that the components of the unit vector are the direction cosines for the vector, calculate the corresponding angles.

Rectangular Components in Space



Solution

Determine the unit vector pointing from *A* towards *B*.

$$\boldsymbol{AB} = -40\boldsymbol{i} + 80\boldsymbol{j} + 30\boldsymbol{k}$$

$$AB = \sqrt{(-40)^2 + (80)^2 + (30)^2} = 94.3$$

$$\boldsymbol{\lambda} = \frac{\boldsymbol{AB}}{AB} = \frac{-40\boldsymbol{i} + 80\boldsymbol{j} + 30\boldsymbol{k}}{94.3}$$

$$= -0.424\boldsymbol{i} + 0.848\boldsymbol{j} + 0.318\boldsymbol{k}$$

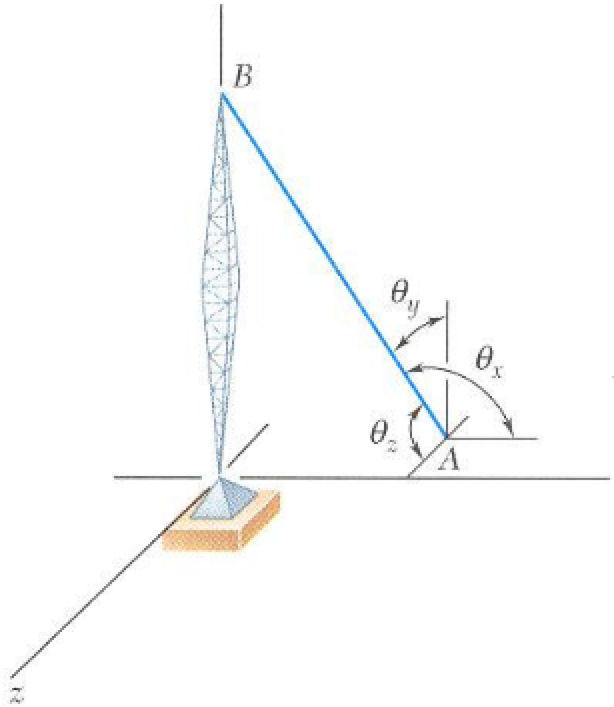
Determine the components of the force.

$$\boldsymbol{F} = F\boldsymbol{\lambda} \qquad = 2500(-0.424\boldsymbol{i} + 0.848\boldsymbol{j} + 0.318\boldsymbol{k})$$

$$= -1060\boldsymbol{i} + 2120\boldsymbol{j} + 795\boldsymbol{k}$$

$$\boxed{\begin{aligned} F_x &= -1060\ N \\ F_y &= 2120\ N \\ F_z &= 795\ N \end{aligned}}$$

Rectangular Components in Space



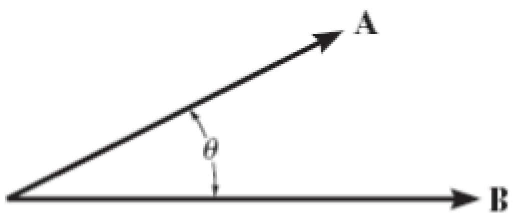
Solution

Noting that the components of the unit vector are the direction cosines for the vector, calculate the corresponding angles.

$$\boldsymbol{\lambda} = cos\theta_x \boldsymbol{i} + cos\theta_y \boldsymbol{j} + cos\theta_z \boldsymbol{k}$$
$$= -0.424\boldsymbol{i} + 0.848\boldsymbol{j} + 0.318\boldsymbol{k}$$

$$\boxed{\begin{aligned} \theta_x &= 115.1^\circ \\ \theta_y &= 32.0^\circ \\ \theta_z &= 71.5^\circ \end{aligned}}$$

Vector Products



Dot Product $\boldsymbol{A}.\boldsymbol{B} = ABcos\theta$

Applications:

to determine the angle between two vectors

to determine the projection of a vector in a specified direction

A*.*B* = *B*.*A (commutative)

A*.(*B*+*C*) = *A*.*B*+*A*.*C (distributive operation)

$$\boldsymbol{A}.\boldsymbol{B} = \left(A_x\boldsymbol{i} + A_y\boldsymbol{j} + A_z\boldsymbol{k}\right).\left(B_x\boldsymbol{i} + B_y\boldsymbol{j} + B_z\boldsymbol{k}\right)$$

$$= A_xB_x + A_yB_y + A_zB_z$$

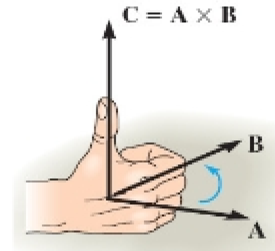
$$\boldsymbol{i}.\boldsymbol{i} = 1$$

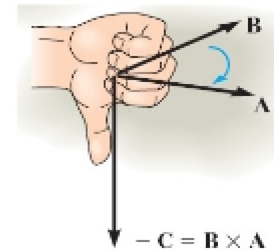
$$\boldsymbol{i}.\boldsymbol{j} = 0$$

Vector Products

Cross Product: $\boldsymbol{A} \times \boldsymbol{B} = \boldsymbol{C} = ABsin\theta$

$\boldsymbol{A} \times \boldsymbol{B} = -(\boldsymbol{B} \times \boldsymbol{A}\,)$

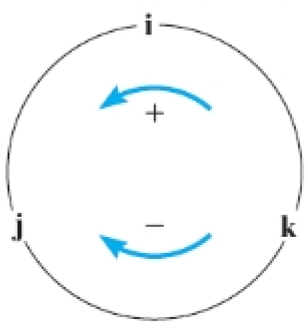


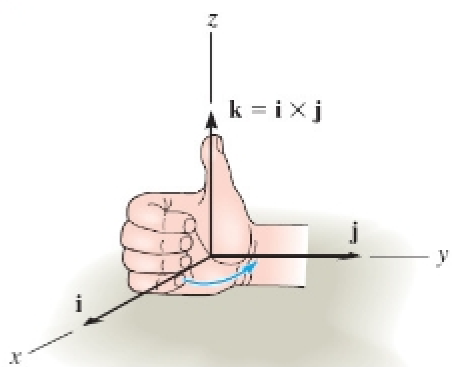


$$\boldsymbol{A} \times \boldsymbol{B} = \left(A_x\boldsymbol{i} + A_y\boldsymbol{j} + A_z\boldsymbol{k}\right) \times \left(B_x\boldsymbol{i} + B_y\boldsymbol{j} + B_z\boldsymbol{k}\right)$$

$$= \begin{vmatrix} \boldsymbol{i} & \boldsymbol{j} & \boldsymbol{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix} \left(A_yB_z - A_zB_y\right)\boldsymbol{i} + \left(A_zB_x - A_xB_z\right)\boldsymbol{j} + \left(A_xB_y - A_yB_x\right)\boldsymbol{k}$$

Cartesian Vector





$\mathbf{i} \times \mathbf{j} = \mathbf{k}$ $\quad \mathbf{i} \times \mathbf{k} = -\mathbf{j}$ $\quad \mathbf{i} \times \mathbf{i} = \mathbf{0}$

$\mathbf{j} \times \mathbf{k} = \mathbf{i}$ $\quad \mathbf{j} \times \mathbf{i} = -\mathbf{k}$ $\quad \mathbf{j} \times \mathbf{j} = \mathbf{0}$

$\mathbf{k} \times \mathbf{i} = \mathbf{j}$ $\quad \mathbf{k} \times \mathbf{j} = -\mathbf{i}$ $\quad \mathbf{k} \times \mathbf{k} = \mathbf{0}$