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Module 1

1. The random sequence $(X_n, n \in N)$ is a Markov Chain for all $X_0, X_1, \ldots X_n \in I$:

$$P(X_n = j_n \mid X_0 = j_0, X_1 = j_1 \ldots X_{n-1} = j_{n-1})$$
$$= P(X_n = j_n \mid X_{n-1} = j_{n-1})$$

provided this probability has meaning.
i.e.

A markov chain $(J_n, n \geq 0)$ is homogenous if the probabilities do not depend on n & for this case, $P(X_n = j \mid X_{n-1} = i) = P_{ij}$

$$\text{Matrix } P = |P_{ij}|$$

2. It provides a method for computing the n-step transition probabilities.

$$P_{ij}^{(n)} = \sum_{K=0}^{M} P_{iK}^{(m)} P_{ij}^{(n-m)}$$ for all i, j & any
$m = 1, 2, \ldots n-1$
$n = m+1, m+2, \ldots$

These eq^n point out that in going from state i to state j in n steps, the process will be some state k after exactly m steps. $P(X_{t+2} = j \mid X_t = i) = \sum_k P(X_{t+2} = j \mid X_{t+1} = K)$
$P(X_{t+1} = K \mid X_t = i)$

i.e.

$$Q\left(x(t_f)\,|\,x(t_1)\right) = \int dx(t')\, Q\left(x(t_f)\,|\,x(t')\right) Q\left(x(t')\,|\,x(t_f)\right)$$

<u>Tutorial</u>

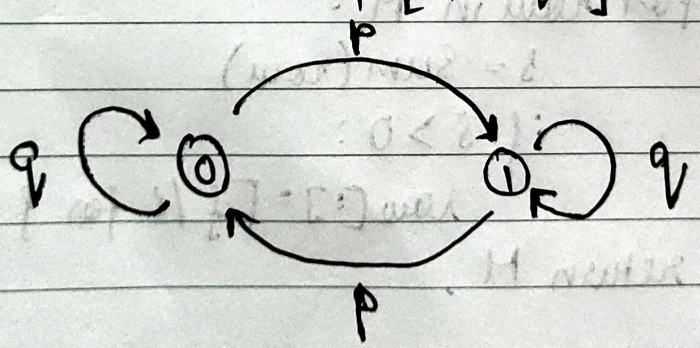
Sol.) Given, stage $\longrightarrow$ 0 & 1

- $P_{01} = P_{10} = p$
- $P_{00} = P_{11} = q$
- $q = 1 - p$

Taking as states, the digits 0 & 1, we can identify Markov chain by specifying & transition probability matrix.

$$TPM = \begin{array}{c|cc} & 0 & 1 \\ \hline 0 & q & p \\ 1 & p & q \end{array}$$

Tree :



now,

probability of starting from 0 & after two stages producing the digit '0' can be:

$$P_{00}^{(2)}.$$

$$P^{(2)} = \begin{pmatrix} q & p \\ p & q \end{pmatrix} \cdot \begin{pmatrix} q & p \\ p & q \end{pmatrix} = \begin{pmatrix} q^2+p^2 & 2pq \\ 2pq & q^2+p^2 \end{pmatrix}$$

Hence,

required probability $= p^2+q^2$.

It can also be rewritten as : $p^2 + (1-p)^2$

$p^2 + 1 - 2p + p^2$

$= 2p^2 - 2p + 1$.

Assignment

1. Markov Chain containing values in set

$S = \{i : i = 0, 1, 2, 3, 4\}$ where i = no. of umbrellas

If $i = 1$ & it rains, then I take the ~~two~~ umbrella, move to other place where there are already 3 umbrellas other than the one I bring. So, total = 4.

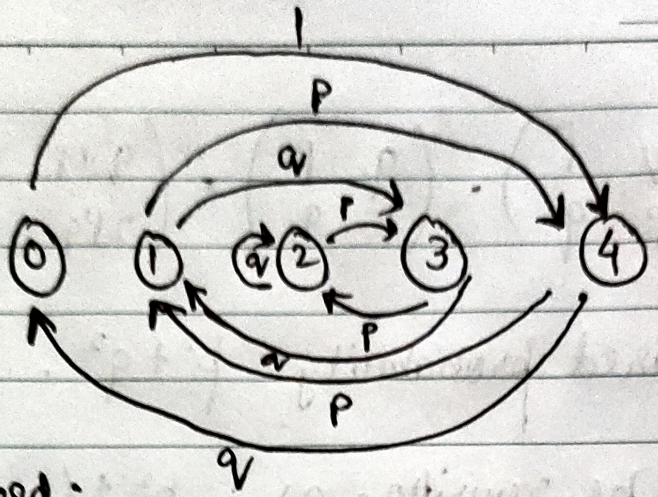
$$\boxed{P_{14} = P}$$

where, P = prob. of rain.

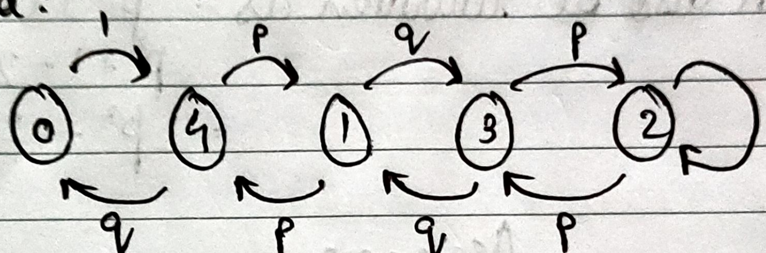
If $i = 1$ but does not rain then I don't take umbrella i.e. other place has 3 umbrellas.

$$\boxed{P_{13} = 1 - P \equiv q}$$

Tree :-



Re-arranged :



Stationary distribution :

$$\pi(2) = \pi(3) = \pi(1) = \pi(4)$$

$$\pi(0) = \pi(4)\ q$$

So, $\sum_{i=0}^{4} \pi(i) = 1$

when $i=1$, it rains & I take umbrella & move.

expressing all prob. in terms of $\pi(4)$:-

$$\pi(4)q + 4\pi(4) = 1$$

$$\Rightarrow \pi(4) = \frac{1}{q+4} = \pi(1) = \pi(2) = \pi(3)$$

$$\pi(0) = \frac{q}{q+4}$$

when,

I am is state 0 and it rains, I get wet.

Chance in state $0 = \pi(0)$

Chance it rains $= P$

$$P(\text{wet}) = \pi(0) \cdot P = \frac{qP}{q+4} \quad \text{with, } P = 0.6$$

$$P(\text{wet}) \approx \frac{0.4 \times 0.6}{0.4+4} \quad \text{i.e. } q = 0.4$$

$$\boxed{P(\text{wet}) \approx 0.0545}$$

(ii) P (wet) less than 6% currently

If we want chance less than 1% we need to move umbrellas.

$$\pi(N) = \pi(N-1) = \cdots = \pi(1)$$

Probability:

$$\pi(0) = \pi(N)\, q$$

where, N = no. of umbrellas,

Insert $\sum_{i=0}^{N} \pi(i)$

$$\pi(N) = \frac{1}{q+N} = \pi(N-1) = \cdots = N(1)$$

$$\pi(0) = \frac{q}{q+N}$$

so, $$P(\text{wet}) = \frac{Pq}{q+N}$$

we want, $P/\text{wet} = \frac{1}{100}$ or $q+N > 100\,pq$

or $N > 100\,pq - q$

$= 100 \times 0.4 \times 0.6 - 0.4$

$= 23.6 \simeq 24.$

So, ~~we~~

we need to reduce approx. 24 umbrellas to have less than 1%.

2. def trans_mat(trans):

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def trans_mat(trans):
        n=10
        M = [[0]*n for _ in range (n)]
        for (i, j) in zip (trans, trans[1:]):
                M[i][j] += 1

        for row in M:
                s = sum(row)
                if s > 0:
                        row[:] = [f/s for f in row]
        return M.
```