Tuesday, July 5, 2022 10:06 AM

1. How to calculate the prob. of the states.
2. How to calculate the prob. after n steps.
3. How to find the prob. of the chain.

Calculation of the prob. of the states.

$$q_n = q_0 P^n.$$

q_0 = Initial prob. of the state.

P = TPM.

n = time (n steps) $\quad n = 1, 2, 3, \dots$

Example

$$P = \begin{array}{c} \\ C \\ B \\ T \end{array} \begin{array}{c} \begin{array}{ccc} C & B & T \end{array} \\ \begin{bmatrix} 0.1 & 0.5 & 0.4 \\ 0.6 & 0.2 & 0.2 \\ 0.3 & 0.4 & 0.3 \end{bmatrix} \end{array}$$

C = car
B = Bus.
T = Train.

Initial Prob. $(q_0) = (0.7 \quad 0.2 \quad 0.1)$.

$\boxed{P(\underbrace{X_2}_{\text{time}} = \underbrace{3}_{\text{state}}) = ?}$ $\qquad P(X_2 = T)$

$$q_n = q_0 P^n. \qquad n = 2.$$

$$q_2 = q_0 P^2.$$

$$P^2 = P \cdot P = \begin{bmatrix} 0.1 & 0.5 & 0.4 \\ 0.6 & 0.2 & 0.2 \\ 0.3 & 0.4 & 0.3 \end{bmatrix} \begin{bmatrix} 0.1 & 0.5 & 0.4 \\ 0.6 & 0.2 & 0.2 \\ 0.3 & 0.4 & 0.3 \end{bmatrix}$$

$$\begin{bmatrix} \cdots \\ 0.6 & 0.2 & 0.2 \\ 0.3 & 0.4 & 0.3 \end{bmatrix} \begin{bmatrix} \cdots \\ 0.3 & 0.4 & 0.3 \end{bmatrix}$$

$$= \begin{array}{c c} & \begin{array}{ccc} C & B & T. \end{array} \\ \begin{array}{c} C \\ B \\ T \end{array} & \begin{bmatrix} 0.43 & 0.31 & 0.26 \\ 0.24 & 0.42 & 0.34 \\ 0.36 & 0.35 & 0.29 \end{bmatrix} \end{array}$$

$$\therefore q_2 = \begin{bmatrix} 0.7 & 0.2 & 0.1 \end{bmatrix} \begin{bmatrix} 0.43 & 0.31 & 0.26 \\ 0.24 & 0.42 & 0.34 \\ 0.36 & 0.35 & 0.29 \end{bmatrix}$$

$$q_2 = \begin{array}{c} \begin{array}{ccc} C & B & T. \end{array} \\ \begin{bmatrix} 0.385 & 0.336 & \underline{0.279} \end{bmatrix} \end{array}$$

$P(x_2 = \underline{3}) = ?$

Ans $\underline{\underline{0.279}}$

<u>Example</u>

$$P = \begin{array}{c c} & \begin{array}{ccc} A & B & C \end{array} \\ \begin{array}{c} A \\ B \\ C \end{array} & \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1/2 & 1/2 & 0 \end{bmatrix} \end{array} \qquad q_0 = \begin{bmatrix} 0.3 & 0.4 & 0.3 \end{bmatrix}$$

$P(x_2 = B) = ?$

$$\boxed{q_2 = q_0 P^2} \rightarrow \begin{bmatrix} 0.3 & 0.4 & 0.3 \end{bmatrix} \begin{bmatrix} 0 & 0 & 1 \\ 0.5 & 0.5 & 0 \\ 0 & 0.5 & 0.5 \end{bmatrix}$$

$$\begin{array}{c} \begin{array}{ccc} A & B & C \end{array} \\ \begin{bmatrix} 0.20 & 0.35 & 0.45 \end{bmatrix} \end{array}$$

<u>Ans :-</u> $P(x_2 = \underline{B}) = \underline{\underline{0.35}}$

$\downarrow$

$P(x_2 = 2)$

Type - 2 :- How to calculate the prob. after n steps.

$[3-1=2]$

after n steps.

$[3-1=2]$

ie $P(X_3 = 3 \mid X_1 = 1)$

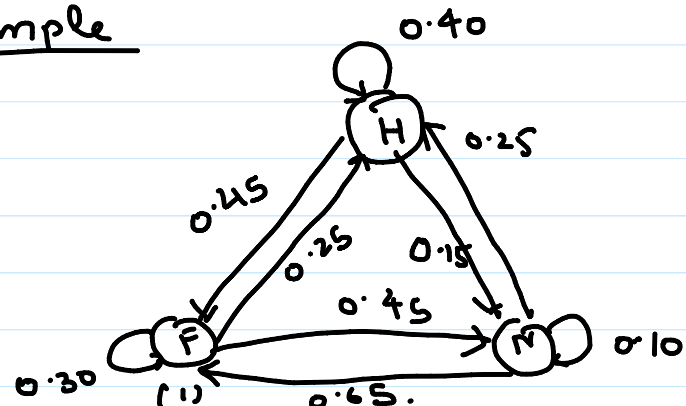
state

$\downarrow$

$P_{13}^{(2)}$

The Prob. of movement of state 1 to state 3 after two time period | steps.

<u>Example</u>



$$\begin{array}{c} \\ F \\ H \\ N \end{array} \begin{array}{c} \begin{array}{ccc} F & H & N \end{array} \\ \begin{bmatrix} 0.30 & 0.25 & 0.45 \\ 0.45 & 0.40 & 0.15 \\ 0.65 & 0.25 & 0.10 \end{bmatrix} \end{array} \begin{array}{c} \\ =1 \\ =1 \\ =1 \end{array}$$

(i) $P(X_3 = F \mid X_2 = N) \Rightarrow P_{NF}^{(1)} = 0.65$

(ii) $P(X_2 = N \mid X_1 = H) = P_{HN}^{(1)} = 0.15$

(iii) $P(X_4 = H \mid X_2 = F) = P_{FH}^{(2)} = ? = \underline{\underline{0.2876}}$

$p^2 = P.P = 0.2876$

<u>Type-3</u>

How to calculate the Prob. of Markov chain.

Eg:- $P(\underbrace{X_3=2, \quad X_1}_{2}=\underbrace{3, \quad X_0}_{1}=2)$

$P(\underbrace{X_0=2, X_1}_{1}=\underbrace{3, X_3}_{2}=2)$

We started from state 2.

After 1 time period, it moves to state 3.

Then after 2 time period, it moves to state 2

<u>Example</u>

$$P = \begin{array}{c} \\ 1 \\ 2 \\ 3 \end{array} \begin{array}{c} \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ \begin{bmatrix} 0.1 & 0.5 & 0.4 \\ 0.6 & 0.2 & 0.2 \\ 0.3 & 0.4 & 0.3 \end{bmatrix} \end{array}$$

$$q_0 = \begin{bmatrix} 0.7 & 0.2 & 0.1 \end{bmatrix}.$$

$P(X_3=2, X_2=3, X_1=3, X_0=2) = ?$

Initial Prob.

$$\text{- - -} \xrightarrow{q_0} 2 \xrightarrow{(1)} 3 \xrightarrow{(1)} 3 \xrightarrow{(1)} 2$$

$$= q_0(2) \times P_{23}^{(1)} \times P_{33}^{(1)} \times P_{32}^{(1)}$$

$$= 0.2 \times 0.2 \times 0.3 \times 0.4$$

$$= 0.00\underline{\underline{48}}$$

<u>Example</u>

$$P = \begin{array}{c} \\ A \end{array} \begin{array}{c} \begin{array}{ccc} A & B & C \end{array} \\ \begin{bmatrix} 0 & 1 & 0 \end{bmatrix} \end{array} \qquad q_0 = \begin{bmatrix} 0.3 & 0.4 & 0.3 \end{bmatrix}$$

$$
P = \begin{array}{c} \\ A \\ B \\ C \end{array}
\begin{array}{c}
\begin{array}{ccc} A & B & C \end{array} \\
\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1/2 & 1/2 & 0 \end{bmatrix}
\end{array}
\qquad q_0 = \begin{bmatrix} 0.3 & 0.4 & 0.3 \end{bmatrix}
$$

$$P(X_3 = B,\ X_2 = C,\ X_1 = B,\ X_0 = A) = ?$$

$$q_0(A) \times P_{AB}^{(1)} \times P_{BC}^{(1)} \times P_{CB}^{(1)}$$

$$0.3 \times 1 \times 1 \times \frac{1}{2} = \underline{\underline{0.15}}$$

Example: -

$$
P = \begin{array}{c} \\ 0 \\ 1 \\ 2 \end{array}
\begin{array}{c}
\begin{array}{ccc} 0 & 1 & 2 \end{array} \\
\begin{bmatrix} 0.2 & 0.3 & \underline{0.5} \\ \underline{0.1} & 0.6 & 0.3 \\ 0.4 & \underline{0.3} & 0.3 \end{bmatrix}
\end{array}
\begin{array}{c} \\ =1 \\ =1 \\ =1 \end{array}
$$

$$q_0 = (0.5 \quad 0.3 \quad 0.2)$$

$$P(X_3 = 2\ X_1 = 0,\ X_0 = 2) = ?$$

$$\cdots \xrightarrow{q_0} 2 \xrightarrow{(1)} 0 \xrightarrow{(2)} 2$$

$$q_0(2) \times P_{20}^{(1)} \times P_{02}^{(2)}$$

$$\downarrow$$

$$?$$

$$
P^2 = \begin{array}{c} \\ 0 \\ 1 \\ 2 \end{array}
\begin{array}{c}
\begin{array}{ccc} 0 & 1 & 2 \end{array} \\
\begin{bmatrix} 0.27 & 0.39 & 0.34 \\ 0.20 & 0.48 & 0.32 \\ 0.23 & 0.39 & 0.38 \end{bmatrix}
\end{array}
$$

$$= 0.2 \times 0.4 \times 0.34 = 0.0272.$$

Example.

$$P = \begin{array}{c} \\ 1 \\ 2 \\ 3 \end{array} \begin{array}{c} \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ \begin{bmatrix} 1/2 & 1/4 & 1/4 \\ 1/3 & 0 & 2/3 \\ 1/2 & 1/2 & 0 \end{bmatrix} \end{array}$$

$P(x_1 = 1) = P(x_1 = 2) = \frac{1}{4}$ $\quad q_0 = \left[\frac{1}{4}, \frac{1}{4}, \frac{1}{2}\right]$.

Find $P(x_1 = 3, x_2 = 2, x_3 = 1) = ?$

$\longrightarrow$ (1) $\quad (1)$

$\xrightarrow{q_0} 3 \xrightarrow{(1)} 2 \xrightarrow{(1)} 1$

$q_0(3) \times P_{32}^{(1)} \times P_{21}^{(1)}$

$\frac{1}{2} \times \frac{1}{2} \times \frac{1}{3} = \frac{1}{12}$