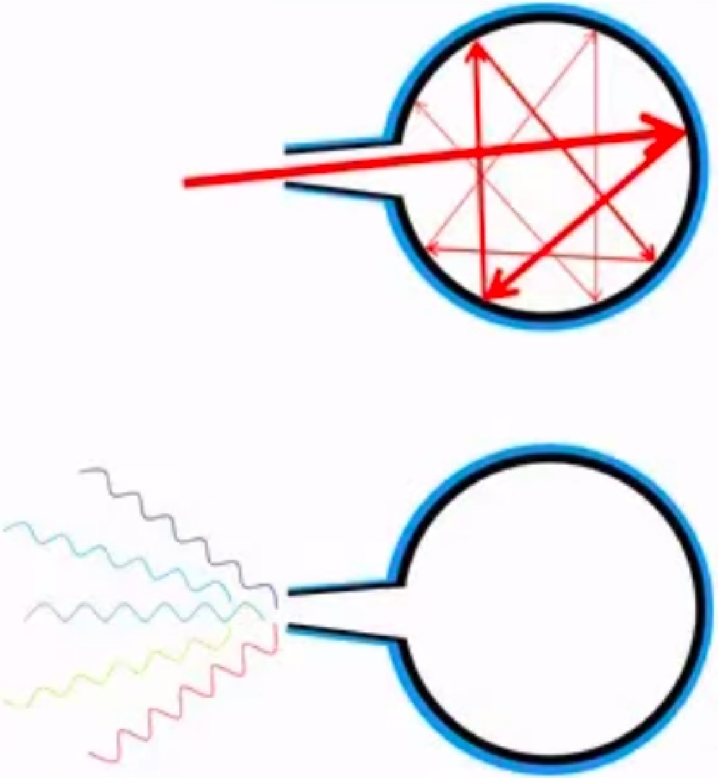
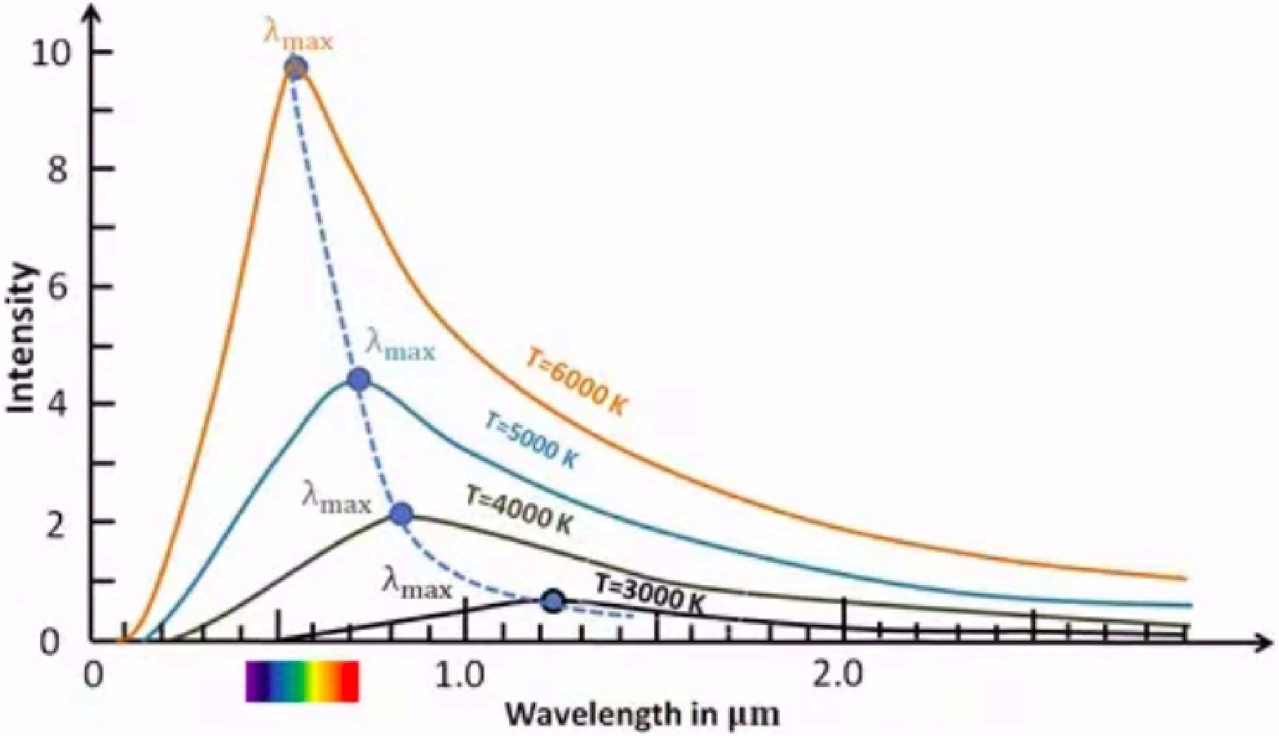
Black Body Radiation





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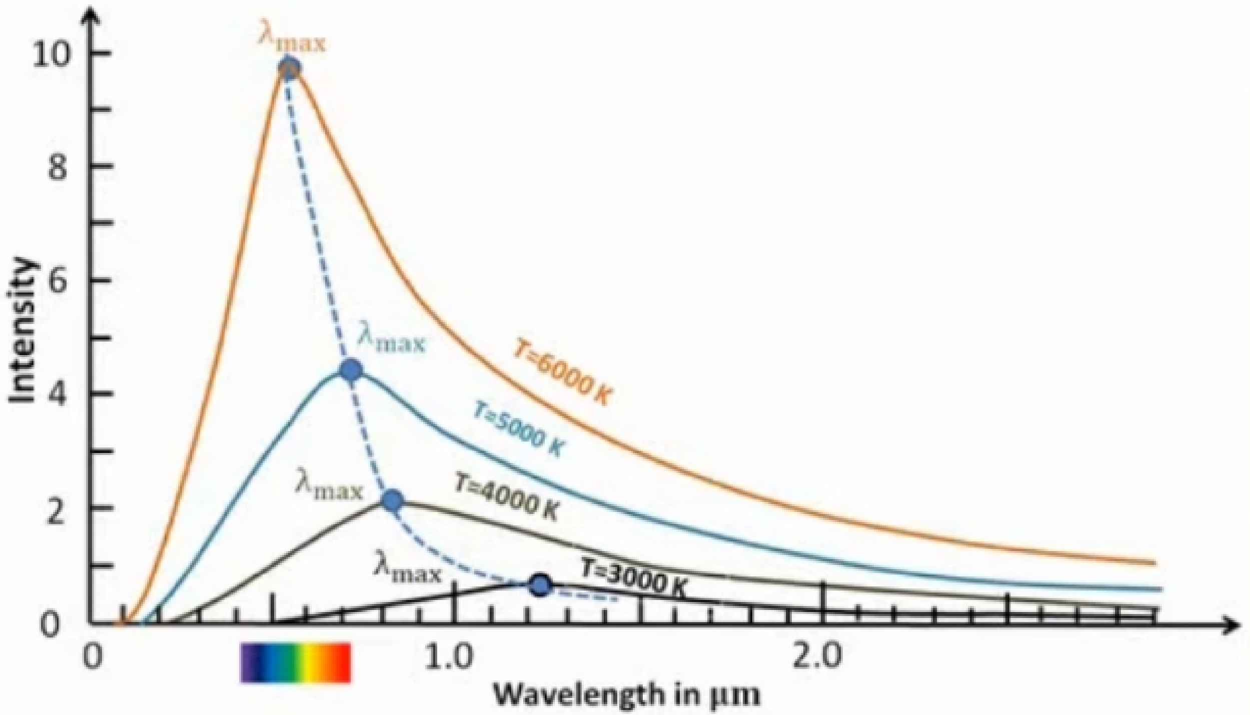

Wien's displacement Law

$$\lambda_{max} \propto \frac{1}{T}$$

Rayleigh Jean's Radiation Law gives Energy density of the radiation as -

$$\rho(v) = \frac{8\pi\nu^2}{c^3}\,kT$$

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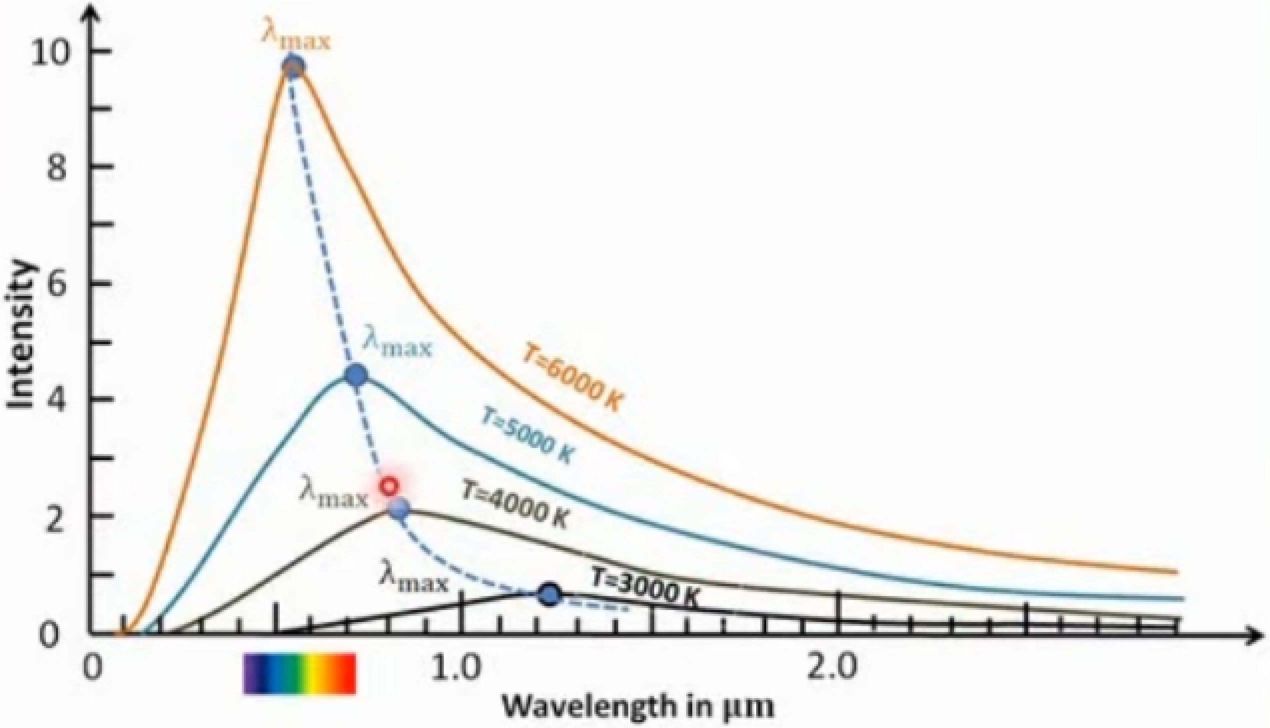

Max Planck suggested that radiated energy must be depending on frequency of radiation and represented this energy $E = h\nu$ and replaced the average energy **kT by** $\frac{h\nu}{e^{\frac{h\nu}{kT}}-1}$

$$\therefore\ \rho(v) = \frac{8\pi\nu^2}{c^3}\frac{h\nu}{e^{\frac{h\nu}{kT}}-1}$$

$$\therefore\ \rho(v) = \frac{8\pi h\nu^3}{c^3}\left(\frac{1}{e^{\frac{h\nu}{kT}}-1}\right)$$

- Einstein used idea of Planck to explain his photoelectric effect.
- He confirmed that light consists of discrete units of energy known as photons carrying energy - $E = h\nu$.
- **This dual nature of light is known as wave-particle duality**.

- Motion of macroscopic particles can be explained by classical theory of Mechanics.
- But it fails to explain the motion of microscopic particles like electron, proton etc.
- **Quantum mechanics** was developed from Quantum theory to explain the properties associated with such particles.

de Broglie Hypothesis

- It states that –

There is a wave associated with every moving particle moving with velocity v, and the wavelength of this wave is given by –

$$\boxed{\lambda = \frac{h}{p} = \frac{h}{mv}}$$

$h = 6.626 \times 10^{-34}$ J-s is Plank's constant

$p = m\,v$ is momentum

de Broglie Hypothesis

Let us consider the case of the photon. Energy of the photon, according to Plank's theory of radiation is given by – $\boxed{E = h\nu}$ (1)

where h is Plank's constant and ν is frequency of radiation

If we consider a photon as a particle of mass m, its energy is given by Einstein Mass Energy relation as – $\boxed{E = mc^2}$ (2)

From equation (1) and (2), we get,

$$\boxed{h\nu = mc^2} \quad (3)$$

As photon travels with velocity of light 'c' in free space, its momentum 'p' is given by –

$$\boxed{p = mc} \quad (4)$$

de Broglie Hypothesis

$$\boxed{E = h\nu} \quad (1)$$

$$\boxed{E = mc^2} \quad (2)$$

$$\boxed{h\nu = mc^2} \quad (3)$$

$$\boxed{p = mc} \quad (4)$$

Dividing equation (3) by (4) we get –

$$\frac{h\nu}{p} = \frac{mc^2}{mc} = c$$

$$\therefore \frac{h}{p} = \frac{c}{\nu} = \lambda \qquad \left(\because \frac{c}{\nu} = \lambda\right)$$

$$\boxed{\therefore \lambda = \frac{h}{p}}$$

de Broglie assumed that this relation holds good for all material particles like electrons, neutrons etc.

de Broglie wavelength in terms of Kinetic Energy

$$\boxed{\lambda = \frac{h}{p} = \frac{h}{mv}}$$ de Broglie Relation

The Kinetic energy of the particle is –

$$E = \frac{1}{2}mv^2 = \frac{1}{2m}m^2v^2 = \frac{1}{2m}p^2$$

$$\therefore p^2 = 2mE$$

$$\therefore p = \sqrt{2mE}$$

$$\boxed{\therefore \lambda = \frac{h}{p} = \frac{h}{\sqrt{2mE}}}$$

$$\boxed{\therefore \lambda = \frac{h}{p} = \frac{h}{\sqrt{2meV}}}$$

$$\boxed{\therefore \lambda = \frac{h}{p} = \frac{h}{\sqrt{2mkT}}}$$

Properties of Matter Waves

- Waves associated with moving particles are called matter waves.
- Wavelength of matter wave is give by $\lambda = \frac{h}{p} = \frac{h}{mv}$
- Matter waves are not electromagnetic waves and can be associated with any particle whether charged or uncharged
- Matter waves can propagate in a vacuum, hence they are not mechanical wave.
- Phase velocity of matter wave $v_p = \frac{c^2}{v} > c$

A bullet of mass 40 gm and an electron both travel with the velocity of 1100 m/s. What wavelengths can be associated with them? Why the wave nature of bullet can not be revealed using diffraction effect?

A bullet of mass 40 gm and an electron both travel with the velocity of 1100 m/s. What wavelengths can be associated with them? Why the wave nature of bullet can not be revealed using diffraction effect?

i) For electron, Given : $h = 6.63 \times 10^{-34}\ J - s,$ $m = 9.1 \times 10^{-31} kg,$

$e = 1.6 \times 10^{-19} C$ $v = 1100 \frac{m}{s},$ $\lambda = ?$

$$\lambda = \frac{h}{mv} = \frac{6.63 \times 10^{-34}}{9.1 \times 10^{-31} \times 1100} = 6.623 \times 10^{-7} m = 6623\ A^0$$

ii) For bullet, Given : $h = 6.63 \times 10^{-34}\ J - s,$ $m = 40\ gm = 40 \times 10^{-3}\ kg,$

$v = 1100 \frac{m}{s},$ $\lambda = ?$

$$\lambda = \frac{h}{mv} = \frac{6.63 \times 10^{-34}}{40 \times 10^{-3} \times 1100} = 1.507 \times 10^{-35} m$$

As the wavelength associated with bullet is of the order of 10^{-35}m, to reveal the wave nature of wave associated with bullet, a diffraction grating having width of the slit of the order of 10^{-35}m is needed. Such diffraction grating is not available. So the wave nature of the bullet can not be revealed.

Calculate the de Broglie wavelength of the proton moving with a velocity equal to $\frac{1}{20}$ th of velocity of light. Mass of proton is 1.6×10^{-27} kg.

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Given : $h = 6.63 \times 10^{-34} \text{J} - \text{s}$, $m = 1.6 \times 10^{-27} \text{kg}$

$v = \frac{1}{20} \times 3 \times 10^{8}$ m/s, $\lambda = ?$

$$\lambda = \frac{h}{p} = \frac{h}{mv} = \frac{6.63 \times 10^{-34}}{1.6 \times 10^{-27} \times \frac{1}{20} \times 3 \times 10^{8}}$$

$$= \mathbf{2.763 \times 10^{-14} m}$$

Calculate the wavelength of the wave associated with a neutron moving with energy 0.025eV. Mass of neutron is 1.676×10^{-27} kg.

Calculate the wavelength of the wave associated with a neutron moving with energy 0.025eV. Mass of neutron is 1.676 x 10^{-27} kg.

Given : $h = 6.63 \times 10^{-34} \text{J} - \text{s}$, $m = 1.676 \times 10^{-27} \text{kg}$

$E = 0.025 \text{ eV} = 0.025 \times 1.6 \times 10^{-19} \text{ J}$

$\lambda = ?$

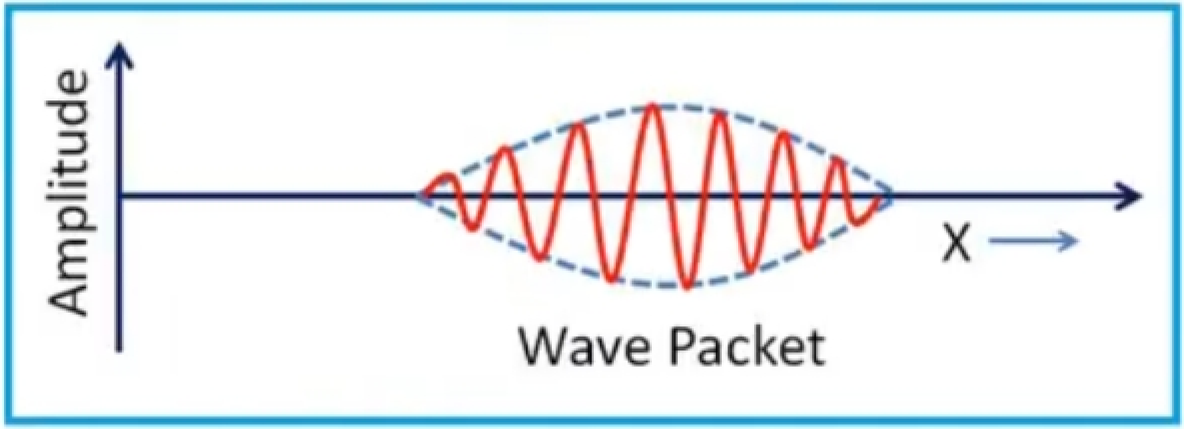
$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2mE}} = \frac{6.63\times10^{-34}}{\sqrt{2\times1.676\times10^{-27}\times0.025\times1.6\times10^{-19}}}$$

$$= \mathbf{1.811 \times 10^{-10} m}$$

Which type of wave associated with matter?

- Single Monochromatic wave?
  - Location of the particle?
  - Phase velocity $v_p = \nu\lambda = \frac{h\nu\lambda}{h} = \frac{h\nu}{h/\lambda} = \frac{E}{p} = \frac{mc^2}{mv} = \frac{c^2}{v}$

- Schrodinger's solution
  - Group of waves
  - Wave packet



The **phase velocity** is the velocity with which a particular phase of the wave propagates in the medium.

Let the equation of the wave travelling in x-direction with vibrations in y-direction is –

$$y = A\sin(\omega t - kx) \quad (1)$$

Where A is amplitude of vibration,

$k = \frac{2\pi}{\lambda}$ is propagation constant,

$\omega = 2\pi\nu$ is the angular frequency

$$\therefore \lambda = \frac{2\pi}{k} \text{ and } \nu = \frac{\omega}{2\pi} \quad (2)$$

Phase velocity,

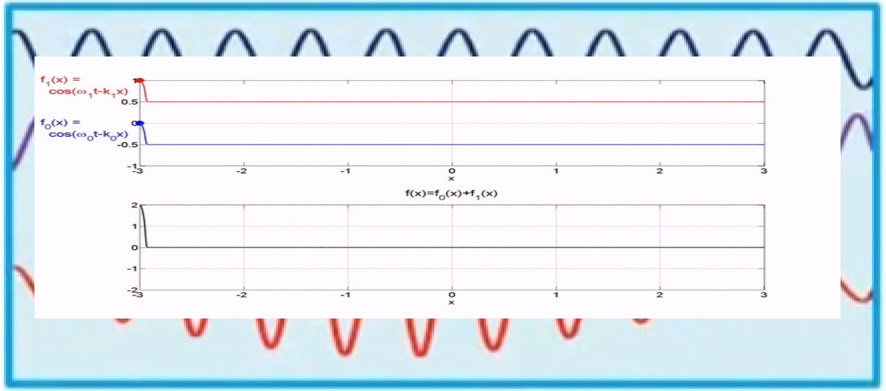
$$v_p = \nu\lambda = \frac{\omega}{2\pi} \times \frac{2\pi}{k} = \frac{\omega}{k} \quad (3)$$

From (2)

$$k = \frac{2\pi}{\lambda} = \frac{2\pi}{h/_{mv}} = \frac{2\pi mv}{h} \quad (4)$$

As from de Broglie relation says $\lambda = \frac{h}{mv}$

Group Velocity



Let us consider two waves represented by -

$$y_1 = A\sin(\omega t - kx)$$

$$y_2 = A\sin[(\omega + d\omega)t - (k + dk)x]$$

The resultant displacement y an any time t is $y = y_1 + y_2$

$$y = 2A\sin\left(\frac{2\omega + d\omega}{2}t - \frac{2k + dk}{2}x\right)\cos\left(\frac{d\omega}{2}t - \frac{dk}{2}x\right)$$

The sine term represents a wave of angular frequency ω and propagation constant k

The Cosine term modulates this wave with angular frequency $\frac{d\omega}{2}$ to produce wave-group travelling with velocity $v_g = \frac{d\omega}{dk}$

Group Velocity

We know - $\lambda = \frac{2\pi}{k}$ and $\nu = \frac{\omega}{2\pi}$ (1)

Group velocity $v_g = \frac{d\omega}{dk} = \frac{d\omega/dv}{dk/dv}$ (2)

$h\nu = mc^2 \Rightarrow \nu = \frac{mc^2}{h}$

$\omega = 2\pi\nu = \frac{2\pi mc^2}{h}$

Putting $m = \frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}}$ (Relativistic mass)

$$\omega = 2\pi\nu = \frac{2\pi c^2}{h}\frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$\therefore \frac{d\omega}{dv} = \frac{2\pi m_0}{h} v\left(1-\frac{v^2}{c^2}\right)^{-3/2} \quad (3)$$

$$k = \frac{2\pi}{\lambda} = \frac{2\pi mv}{h} = \frac{2\pi v}{h}\frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$\therefore \frac{dk}{dv} = \frac{2\pi m_0}{h}\left(1-\frac{v^2}{c^2}\right)^{-3/2} \quad (4)$$

Group Velocity

We know - $\lambda = \frac{2\pi}{k}$ and $\nu = \frac{\omega}{2\pi}$ 1

Group velocity $v_g = \frac{d\omega}{dk} = \frac{d\omega/dv}{dk/dv}$ 2

$$\therefore \frac{d\omega}{dv} = \frac{2\pi m_0}{h} v \left(1 - \frac{v^2}{c^2}\right)^{-3/2} \quad 3$$

$$\therefore \frac{dk}{dv} = \frac{2\pi m_0}{h} \left(1 - \frac{v^2}{c^2}\right)^{-3/2} \quad 4$$

From 2 3 and 4

Group velocity

$$v_g = \frac{d\omega/dv}{dk/dv} = \frac{\frac{2\pi m_0}{h} v \left(1 - \frac{v^2}{c^2}\right)^{-3/2}}{\frac{2\pi m_0}{h} \left(1 - \frac{v^2}{c^2}\right)^{-3/2}}$$

$$\boxed{\therefore v_g = v}$$

Thus, group velocity associated with the wave packet is equal to the velocity of the particle.

Heisenberg Uncertainty Principle

- If we are finding position of an moving electron, a photon must be seen by us after collision with the electron.
- But then the velocity and hence momentum of the electron will change due to collision.
- This will cause error or uncertainty in the measurement of momentum.
- If we try to determine momentum accurately, error or uncertainty in position is introduced.
- Thus, It is not possible to measure the position and momentum of an electron simultaneously and accurately.