Unit – II

Push Down Automata (PDA)

A push down automaton is a way to implement a Context Free grammar in a similar way we design DFA for a regular grammar.

→ A DFA can remember a finite amount of information.

→ A PDA can remember an infinite amount of information.

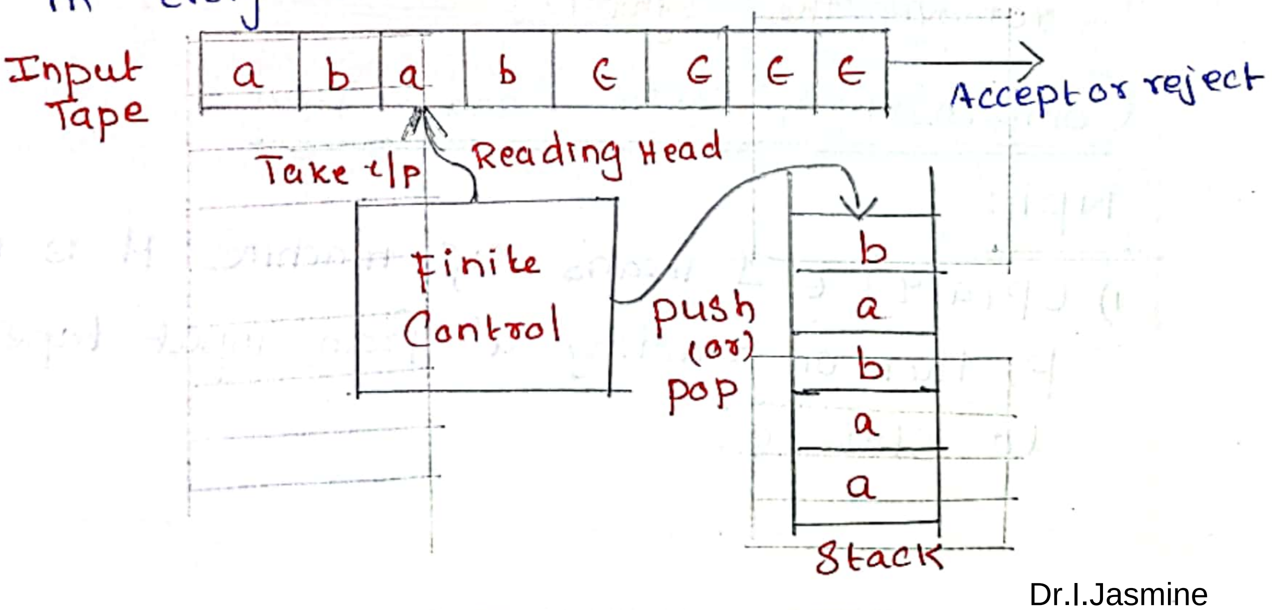
Basically a PDA is

$\boxed{\text{Finite State Machine + A Stack.}}$

PDA has three Components:

1. An input tape
2. A control unit
3. A Stack with infinite Size

→ A PDA may or may not read input Symbol, but it has to read the top of the Stack in every transaction.



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→ The input tape is divided in many cells.

→ At each cell only one input symbol is placed thus certain input string is placed on tape.

→ The finite control has some pointer which point the current symbol which is to be read.

→ At the end of the input $ or Δ or ϵ (blank) symbol is placed in which indicates end of input.

→ Stack is a Last In First out. Stack is used for storing the items temporarily.

Stack having two operations

1. PUSH
2. POP

* push operation which is used to insert or add the symbols
* pop operation which is used to delete (or) remove the symbol.

Comparison of NFA and PDA:-

NFA:-

1) $(p, a, q) \in \Delta$ means if machine M is in state p, then on reading 'a' from input tape go to state q.

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2) $(p, \epsilon, q) \in \Delta$ means if machine M is in State p, goes to State q, without consuming input.

PDA:

State, input, Stack top i/p, State, Replace Stack value y

1) $((p, a, \beta), (q, y))$ → If machine M is in State p, the Symbol read from input tape is 'a' and β is on top of Stack, goes to State q, and replace 'β' by y on top of Stack.

2) $((S, a, \epsilon), (S, a))$ → If machine M is in State S, read 'a' remains in State 'S' and push 'a' onto Stack (ϵ empty Stack).

3) $((S, c, \epsilon), (f, \epsilon))$ → If read 'c' in State S and Stack is empty, goes to final State f and nothing to push onto Stack.

4. $((S, \epsilon, \epsilon), (f, \epsilon))$ → If in State S, go to State 'f'.

5) $((f, a, a), (f, \phi))$ → If read a in State f, remain in State f and pop a from Stack.

6) PDA's are non-deterministic.

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Definition of PDA:-

A push down automaton M is defined by $\boxed{M = (Q, \Sigma, \Gamma, \delta, q_0, z_0, F),}$ where

Q – is a finite set of States.

Σ – is an alphabet called as input alphapet – is a finite alphabet of tape symbols.

Γ – is an alphabet called stack alphabet – is a finite alphabet of Stack Symbols.

$q_0 \in Q$ – is the Start State (or) Initial State.

Z_0 – in Γ is a particular stack symbol called Start symbol. (Initial Stack Symbol)

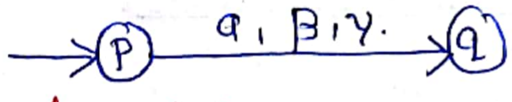
$F \subseteq Q$ is the set of final States.

δ is the transition function.

ie. δ is a subset of $(Q \times (\Sigma \cup \{\epsilon\}) \times \Gamma^*) \rightarrow (2^Q \times \Gamma^*)$

Transitions and Graphical Notation:-

Let $((p, a, \overset{\text{pop}}{\beta}), (q, \overset{\text{push}}{\gamma})) \in \delta$. It means that



$\Rightarrow$ Equivalent Transition Diagram.

1) read 'a' from the tape
2) pop the String β from the Stack
3) move from State p to State q.
4) push string 'γ' on to stack.

Instantaneous Description (ID):

The instantaneous description (ID) of a PDA is represented by a triple (q, w, S) where

→ q is the State

→ w is unconsumed i/p

→ S is the Stack contents.

ID is an informal notation of how a PDA compute an input String and make a decision that String is accepted or rejected.

Push Down Automata Acceptance:

There are two different ways to define acceptabity

1. Final State Acceptability:

A PDA accepts a String when, after reading the entire String, the PDA in final State.

→ From the Starting State, we can make moves that end up in a final State with any stack values.

→ The Stack values are irrelevant as long as we end up in a final State.

2) Empty Stack Acceptability:

A PDA accepts a string when, after reading the entire string the PDA has emptied its stack.

$$L(PDA) = \{w \mid (q_0, w, I) \vdash^{*} (q, \epsilon, \epsilon), q \in Q\}$$

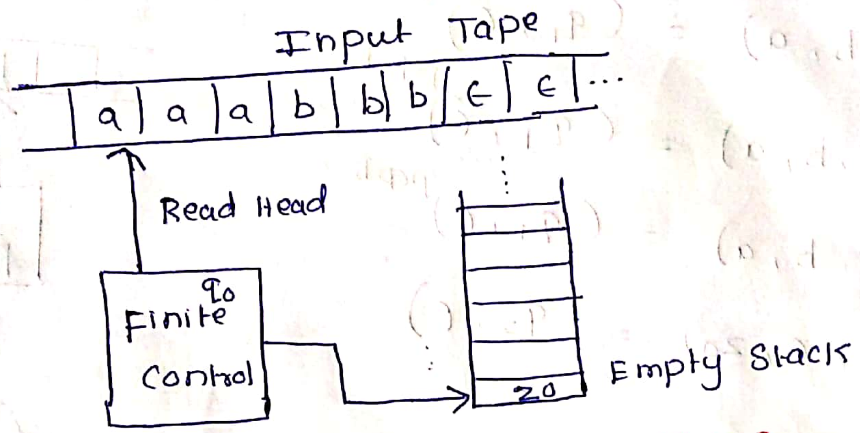
Construct a PDA for the language $L = \{a^n b^n \mid n \geq 1\}$

Solution:

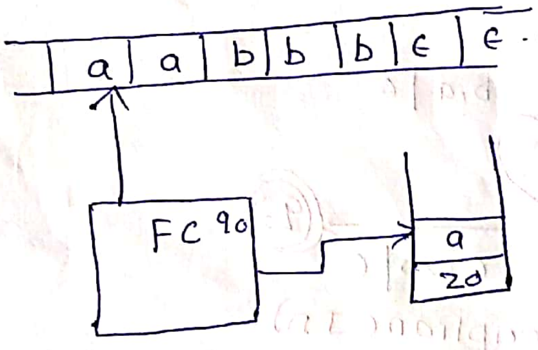
$$L = \{a^n b^n \mid n \geq 1\}$$

$$\therefore \quad L = \{ab, aabb, aaabbb, \ldots\}$$

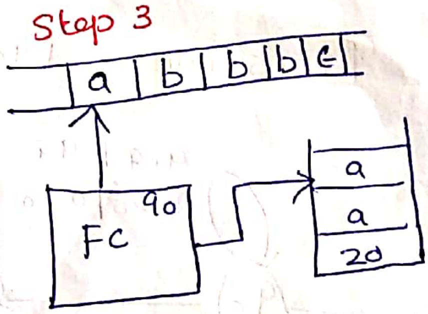
Step 1:



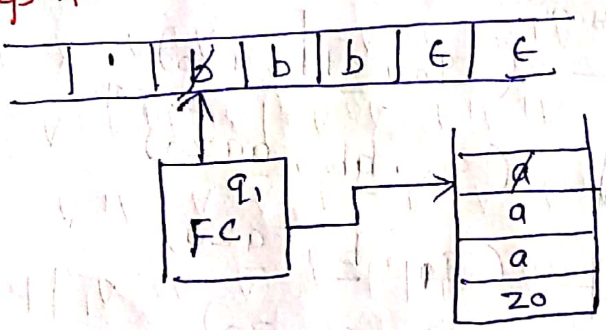
Step: 2



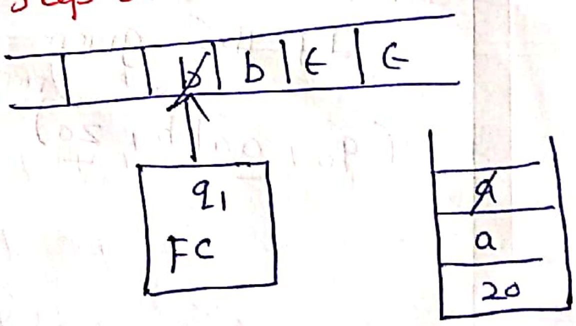
Step 3



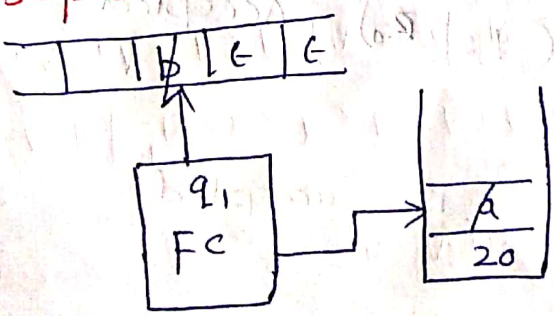
Step 4:



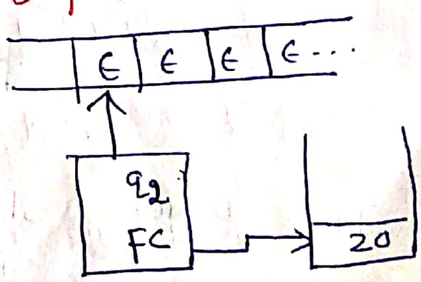
Step 5:



Step 6:



Step 7:



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It is accepted.

Reached Empty Stack or Reached the Final State

Transition Function

Tape, Stack

$\delta(q_0, a, Z_0) = (q_0, aZ_0)$

push, stack top

$\delta(q_0, a, a) = (q_0, aa)$

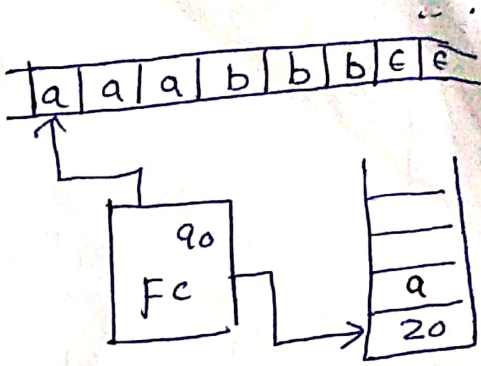
$\delta(q_0, a, a) = (q_0, aa)$ Recursive

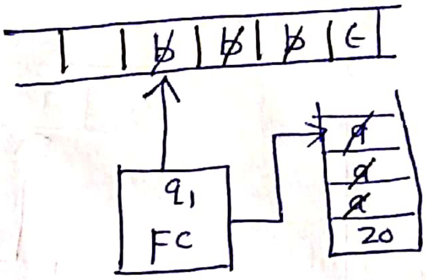
$\delta(q_0, b, a) = (q_1, \epsilon)$ pop a

$\delta(q_1, b, a) = (q_1, \epsilon)$ pop a

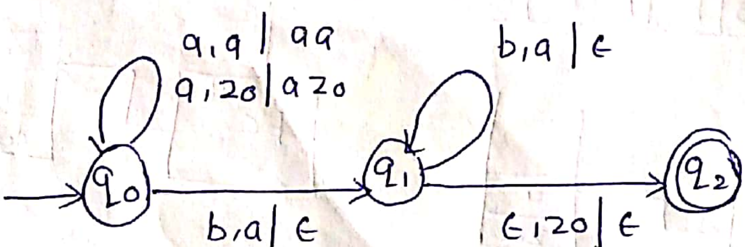
$\delta(q_1, b, a) = (q_1, \epsilon)$ pop a

$\delta(q_1, \epsilon, Z_0) = (q_2, \epsilon)$





Transition Diagram:



Instantaneous Description (ID):

If the given String is accepted or not

sample, Empty stack

$(q_0, aabb, Z_0) \vdash (q_0, abb, aZ_0)$

$\vdash (q_0, bb, aaZ_0)$ pop

$\vdash (q_1, b, aZ_0)$

$\vdash (q_1, \epsilon, Z_0)$

$\vdash (q_2, \epsilon)$ Accepted.

∴ The given String is accepted.

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Construct a PDA for the Languages $L = \{wcw^R \mid 'w'$ in $(a+b)^*\}$. For the given PDA write the instantaneous description for the string 'aabcbaa'.

Solution

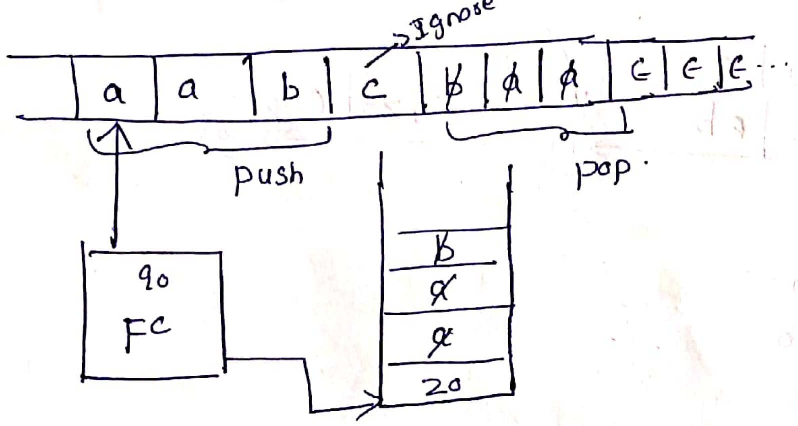
$L = \{ wcw^R \mid w \in (a+b)^* \}$

$w = \{ \epsilon, a, b, aa, ab, bb, aba, bba, aab, \ldots \}$

Given String $w = aab$

$w^R = baa$

$\therefore$ The String $wcw^R = aabcbaa$



Transition Function:

push, stack, push, Top

$\delta(q_0, a, Z_0) = (q_0, aZ_0)$
$\delta(q_0, b, Z_0) = (q_0, bZ_0)$ } Initial condition.

$\delta(q_0, a, a) = (q_0, aa)$
$\delta(q_0, b, a) = (q_0, ba)$
$\delta(q_0, a, b) = (q_0, ab)$
$\delta(q_0, b, b) = (q_0, bb)$ } Recursive condition Pushed

$\delta(q_0, c, b) = (q_1, b)$ → Ignore Top of the stack.
$\delta(q_0, c, a) = (q_1, a)$

$\delta(q_1, \cancel{b}) =$

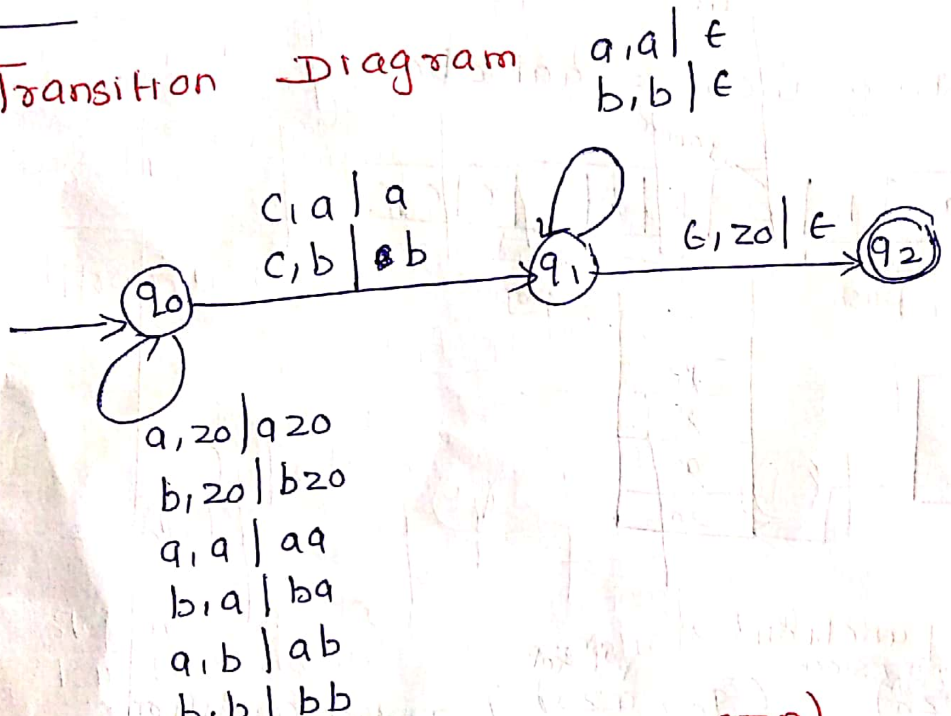
$\delta(q_1, a) =$

$\delta(q_1, b, b) = (q_1, \epsilon)$ } Recursive condition

$\delta(q_1, a, a) = (q_1, \epsilon)$ } pop

$\delta(q_1, \epsilon, z_0) = (q_2, \epsilon)$

Transition Diagram



Instantaneous Description (ID)

The given String 'aabcbaa' accepted or not

Empty stack

$(q_0, \underline{a}abcbaa, z_0) \vdash (q_0, \underline{a}bcbaa, az_0)$

$\vdash (q_0, \underline{b}cbaa, aaz_0)$

$\vdash (q_0, \underline{c}baa, baaz_0)$

$\vdash (q_1, \underline{b}aa, baaz_0)$

$\vdash (q_1, \underline{a}a, aaz_0)$

$\vdash (q_1, a, az_0)$

$\vdash (q_1, \epsilon, z_0)$

$\vdash (q_2, \epsilon)$ Accepted.

∴ The given String is accepted.

part B

4 Let G be the Grammar $S \to 0B \mid 1A$

$A \to 0 \mid 0S \mid 1AA$

$B \to 1 \mid 1S \mid 0BB$

Convert this grammar into PDA and check the String "00110101" is accepted or not.

Solution:

Step: 1

The given productions are exist in GNF form only, therefore the PDA can be constructed directly.

Step 2:

The PDA

$P = (Q, \Sigma, \Gamma, \delta, q_0, Z_0, F)$ where

$Q = \{q_0, q_1, q_2\}$

$\Sigma = \{0, 1\}$

$\Gamma = \{S, A, B, Z_0\}$

$F = \{q_2\}$

$q_0 \to$ Start State.

1. First the start symbol S is put on the stack.

$\delta(q_0, \epsilon, Z_0) = \{(q_1, SZ_0)\}$

2. The productions $S \to 0B \mid 1A$ is simulated by PDA as per $A \to \alpha$ rule.

$\delta(q_1, \epsilon, S) = \{(q_1, 0B), (q_1, 1A)\}$

3. In a similar manner, the other productions are:

$\delta(q_1, \epsilon, A) = \{(q_1, 0)(q_1, 0S), (q_1, 1AA)\}$

$\delta(q_1, \epsilon, B) = \{(q_1, 1)(q_1, 1S), (q_1, 0BB)\}$

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4. For each $a \in \Sigma$ the corresponding productions are:

$$\delta(q_1, 0, 0) = \delta(q_1, \epsilon)$$

$$\delta(q_1, 1, 1) = (q_1, \epsilon)$$

5. Then the end of the derivations is identified by the stack symbol.

$$\delta(q_1, \epsilon, z_0) = (q_2, \epsilon)$$

Check whether the given string is "0011 0101" is accepted or not.

$(q, 00110101, S) \vdash (q, \cancel{0}0110101, \cancel{0}\underline{B})$ $S \to 0B$

$\vdash (q, 0110101, B)$ $B \to 0BB$

$\vdash (q, \cancel{0}110101, \cancel{0}\underline{B}B)$ $B \to 1SB$

$\vdash (q, \cancel{1}10101, \cancel{1}\underline{S}B)$ $S \to 1A$

$\vdash (q, \cancel{1}0101, \cancel{1}\underline{A}B)$ $A \to 1AA$

$\vdash (q, \cancel{0}101, \cancel{1}\underline{A}AB)$ $A \to 0$

$\vdash (q, \cancel{1}01, \cancel{0}\cancel{A}B)$ $A \to 0$

$\vdash (q, \cancel{0}1, \cancel{0}\underline{B})$ $B \to 1$

$\vdash (q, 1, 1)$

$\vdash (q, \epsilon)$ Accepted.

$\therefore$ The given string is accepted.

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part B
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Construct the grammar for the following PDA.

$M = (\{q_0, q_1\}, \{0,1\}, \{X, Z_0\}, \delta, q_0, Z_0, \phi)$ and

where δ is given by.

$\delta(q_0, 0, Z_0) = \{(q_0, XZ_0)\}$

$\delta(q_0, 0, X) = \{(q_0, XX)\}$

$\delta(q_0, 1, X) = \{(q_1, \epsilon)\}$

$\delta(q_1, 1, X) = \{(q_1, \epsilon)\}$

$\delta(q_1, \epsilon, X) = \{q_1, \epsilon)\}$

$\delta(q_1, \epsilon\ Z_0) = \{(q_1, \epsilon)\}$

Solution:

Step: 1

$T = \{0,1\}$

$V = \{S, [q_0, X, q_0], [q_0, X, q_1], [q_0, Z_0, q_0], [q_0, Z_0, q_1], [q_1, Z_0, q_1], [q_1, X, q_0], [q_1, X, q_1]$

$[q_1, Z_0, q_0]\}$

Here 'V' is a start symbol, whose first & third elements are states and the second element is a push down symbol.

Step: 2

The production in 'p' are:

The production are given by $S \to [q_0, Z_0, q]$ for

R1: S - productions are given by every q in Q.

The productions for S

$P_1: S \to [q_0, Z_0, q_0]$

$P_2: S \to [q_0, Z_0, q_1]$

where q_0 is the start state and Z_0 is the initial stack symbol.

Step: 3

R_2: Each move, not erasing a pushdown symbol, given by. for each state (q_0, q_1)

$\delta(q_0, 0, Z_0) = \{(q_0, XZ_0)\}$

For q_0

$P_3 : [(q_0, Z_0, q_0] \rightarrow 0[q_0, X, q_0][q_0, Z_0, q_0]$

$P_4 : [q_0, Z_0, q_0] \rightarrow 0[q_0, X, q_1][q_1, Z_0, q_0]$

For q_1

$P_5 : [q_0, Z_0, q_1] \rightarrow 0[q_0, X, q_0][q_0, Z_0, q_1]$

$P_6 : [q_0, Z_0, q_1] \rightarrow 0[q_0, X, q_1][q_1, Z_0, q_1]$

Similarly:

$\delta(q_0, 0, X) = \{(q_0, XX)\}$ yields.

For q_0

$P_7 : (q_0, X, q_0] = 0[q_0, X, q_0][q_0, X, q_0]$

$P_8 : (q_0, X, q_0] = 0[q_0, X, q_1][q_1, X, q_0]$

For q_1

$P_9 : (q_0, X, q_1] = 0[q_0, X, q_0][q_0, X, q_1]$

$P_{10} : (q_0, X, q_1] = 0[q_0, X, q_1][q_1, X, q_1]$

Step 4: R_3: Each move erasing pushdown symbol given by $(q', \epsilon) \in \delta(q, a, z)$ produce the production $[q, z, q'] = q$

$\therefore$ 1P/n $\delta(q_0, 1, X) = \{(q_1, \epsilon)\}$ gives

$P_{11} : [q_0, X, q_1] \rightarrow 1$

Similarly $\delta(q_1, 1, X) = \{(q_1, \epsilon)\}$ gives

$P_{12} : [q_1, X, q_1] \rightarrow 1$

Similarly. $\delta(q_1, \epsilon, X) = \{(q_1, \epsilon)\}$

$P_{15} : [q_1, X, q_1] \rightarrow \epsilon$

Similarly $\delta(q_1, \epsilon, z_0) = \{(q_1, \epsilon\}$

P_{16}: $[q_1, z_0, q_1] \to \epsilon$

After analyzing all the productions, it is clear that there are no productions for the variables.

$[q_1, X, q_0]$, $[q_1, z_0, q_0]$

And the variables $[q_0, X, q_0]$ $[q_1, z_0, q_0]$ have $[q_1, X, q_0]$ or $[q_1, z_0, q_0]$ on the right, which do not derive any terminal string. Like so for the variables $[q_0, X, q_0]$, $[q_1, z_0, q_0]$.

$\therefore$ Deleting all the productions which involves these variables.

$\therefore$ The final productions are.

$S \to [q_0, z_0, q_1]$

$[q_0, z_0, q_1] \to 0\,[q_0, X, q_1]\,[q_1, z_0, q_1]$

$[q_1, X, q_1] \to 0\,[q_0, X, q_1]\,[q_1, X, q_1]$

$[q_0, X, q_1] \to 1$

$[q_1, z_0, q_1] \to \epsilon$

$[q_1, X, q_1] \to \epsilon$

$[q_1, X, q_1] \to 1$

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