Tuesday, July 26, 2022 10:27 AM

Properties of absorbing markov chain :-

(1) In a finite no. of steps, the chain will enter an absorbing state and remain there.

(2) The powers of the transition matrix get closer and closer to some particular matrix.

(3) The long-term trend depends on the initial state.

(4) Let P be the transition matrix for an absorbing markov chain.
Rearrange the rows and columns of P so that the absorbing states come first

$$P = \left[\begin{array}{c|c} I_m & O \\ \hline R & Q \end{array}\right]$$

where

I_m = identity matrix, with m equal to the no. of absorbing states.

O = Matrix of all zeros.

I_m $\quad$ O

$$\begin{array}{cccc} & 1 & 2 & 3 \\ P = & [\ 1 & 0 & |\ 0\ ] \end{array}$$

$$
\begin{array}{c} \\ P = 1 \\ 2 \\ 3 \end{array}
\begin{array}{c}
\begin{array}{ccc} 1 & 2 & 3 \end{array} \\
\left[\begin{array}{cc|c} 1 & 0 & 0 \\ 0 & 1 & 0 \\ \hline \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \end{array}\right]
\end{array}
$$

$I_m = I_2$ R. Q

The fundamental matrix is defined as

$$F = (I_n - Q)^{-1}$$

where, I_n has the same size as Q.

The element in row i, column j of the fundamental matrix (F) gives the number of visits to state j that are expected to occur before absorbing, given that the current state is state i,

(5) The Product FR gives the matrix of probs. that a Particular initial non-absorbing state will lead to a Particular absorbing state.

Example-1

Find the long-term trend for the Transition matrix.

$$
\begin{array}{c} \\ P = 1 \\ 2 \end{array}
\begin{array}{c}
\begin{array}{ccc} 1 & 2 & 3 \end{array} \\
\left[\begin{array}{ccc} 0.3 & 0.2 & 0.5 \\ 0 & 1 & 0 \end{array}\right.
\end{array}
$$

$$\begin{array}{c} \\ 2 \\ 3 \end{array}\begin{bmatrix} \cdots & \cdots & 0.3 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Solution:-

2 and 3 are absorbing states.

Re-arrange P matrix.

$$P = \left[\begin{array}{c|c} I_m & 0 \\ \hline R & Q \end{array}\right]$$

I_m = identity matrix.
m = No. of absorbing state (2)

I_m $\qquad$ I_2

$$\begin{array}{c|ccc} & 2 & 3 & 1 \\ \hline 2 & 1 & 0 & 0 \\ 3 & 0 & 1 & 0 \\ 1 & 0.2 & 0.5 & 0.3 \end{array}$$

O

R. $\qquad$ Q

$R = [0.2 \quad 0.5]$.

$Q = [0.3]$

$F = (I_n - Q)^{-1}$.

identity matrix

I_n

I_1

$[1]$

I_n is same as Q.

$F = [1 - 0.3]^{-1}$

$F = [0.7]^{-1}$

$F = \left[\frac{1}{0.7}\right] = \left[\frac{10}{7}\right]$

The product FR is.

$$\begin{bmatrix} \frac{10}{7} \end{bmatrix} \begin{bmatrix} 0.2 & 0.5 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{2}{7} & \frac{5}{7} \end{bmatrix} = \begin{bmatrix} 0.286 & 0.714 \end{bmatrix}$$

If the system starts in the non-absorbing state 1, there is a ($\frac{2}{7}$ or 0.286) chance of ending up in the absorbing state 2 and a ($\frac{5}{7}$ or 0.714) chance of ending up in the absorbing state 3.

<u>Example-2</u>

Find the long-term trend.

$$P = \begin{array}{c c} & \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \end{array} & \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \end{bmatrix} \end{array}$$

<u>Ans</u>

$$\left[\begin{array}{cc|c} 1 & 0 & 0 \\ 0 & 1 & 0 \\ \hline \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \end{array}\right]$$

R Q

$F = (I_n - Q)^{-1}$

$(1 - \frac{1}{\ })^{-1}$

$$\left(1-\frac{1}{3}\right)^{-1}$$

$$\left(\frac{2}{3}\right)^{-1} = \frac{3}{2}$$

$$FR = \frac{3}{2}\begin{bmatrix} \frac{1}{3} & \frac{1}{3} \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} \end{bmatrix}$$

50% chances of non-absorbing to absorbing state

<u>Example 3</u>

$$P = \begin{array}{c} \\ 1 \\ 2 \\ 3 \end{array} \begin{array}{c} \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ \begin{bmatrix} 0.15 & 0.05 & 0.8 \\ 0 & 1 & 0 \\ 0.4 & 0.6 & 0 \end{bmatrix} \end{array}$$

<u>Soln:-</u>

Im

$$\begin{array}{c} \\ 2 \\ 3 \\ 1 \end{array} \begin{array}{c} \begin{array}{c|cc} 2 & 3 & 1 \end{array} \\ \left[\begin{array}{c|cc} 1 & 0 & 0 \\ \hline 0.6 & 0 & 0.4 \\ 0.05 & 0.8 & 0.15 \end{array}\right] \end{array} \rightarrow 0$$

R. $\quad$ Q

$$F = (I_n - Q)^{-1}$$

$$\left\{\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 0 & 0.4 \\ 0.8 & 0.15 \end{bmatrix}\right\}^{-1}$$

$$F = \begin{bmatrix} 1.604 & 0.754 \\ 1.509 & 1.886 \end{bmatrix}$$

$$FR = \begin{bmatrix} 1.604 & 0.754 \end{bmatrix} \begin{bmatrix} 0.6 \end{bmatrix}$$

$$FR = \begin{bmatrix} 1.604 & 0.754 \\ 1.509 & 1.886 \end{bmatrix}_{2\times 2} \begin{bmatrix} 0.6 \\ 0.05 \end{bmatrix}_{2\times 1}$$

$$FR = \begin{array}{c} 3 \\ 1 \end{array} \begin{bmatrix} \sim 1 \\ \sim 1 \end{bmatrix}$$

<u>Find the equilibrium vector.</u>

(i) $$P = \begin{array}{cc} & \begin{array}{cc} 1 & 2 \end{array} \\ \begin{array}{c} 1 \\ 2 \end{array} & \begin{bmatrix} 1/4 & 3/4 \\ 1/2 & 1/2 \end{bmatrix} \end{array}$$

$$\boxed{VP^n \approx V}$$

P = (regular matrix)

$VP \approx V$.

$v_1 = 2/5$

$v_2 = 3/5$