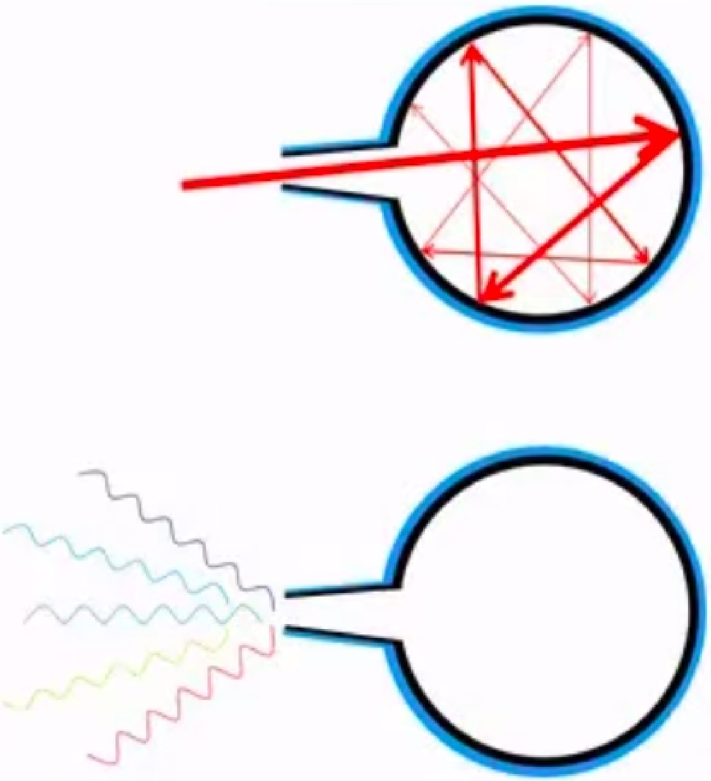
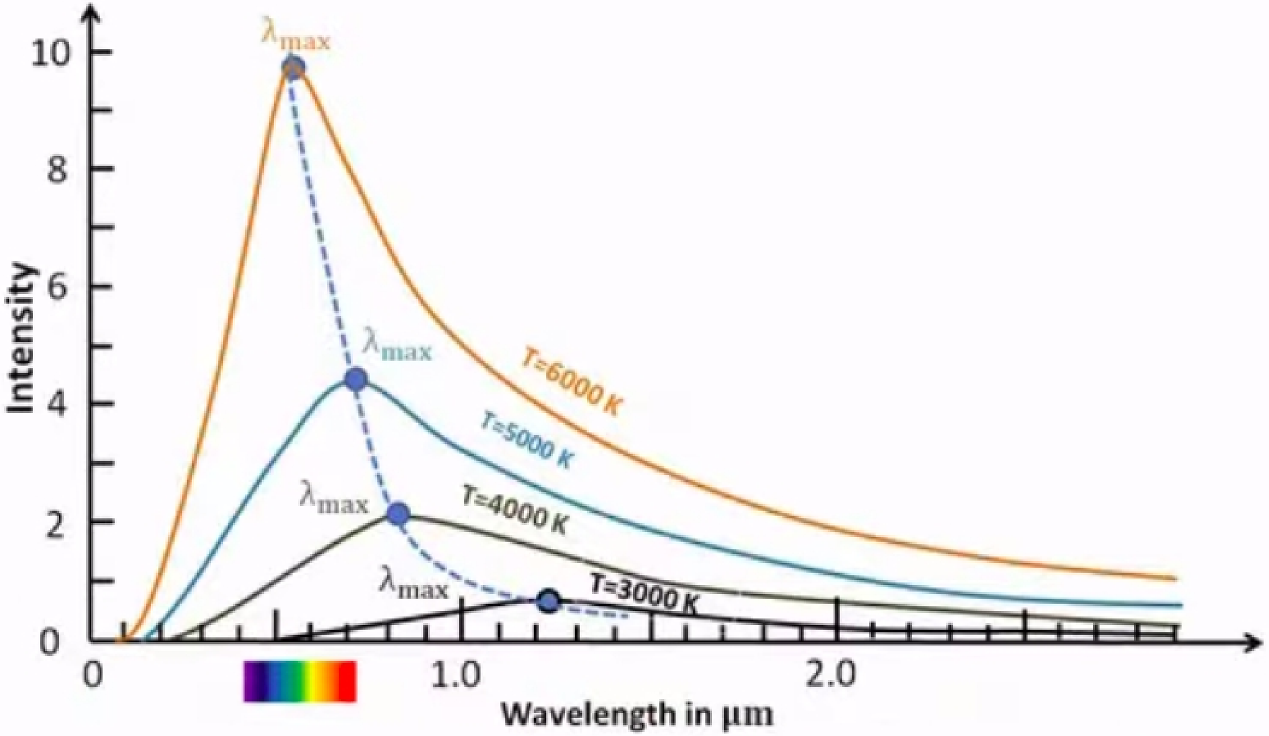
Black Body Radiation





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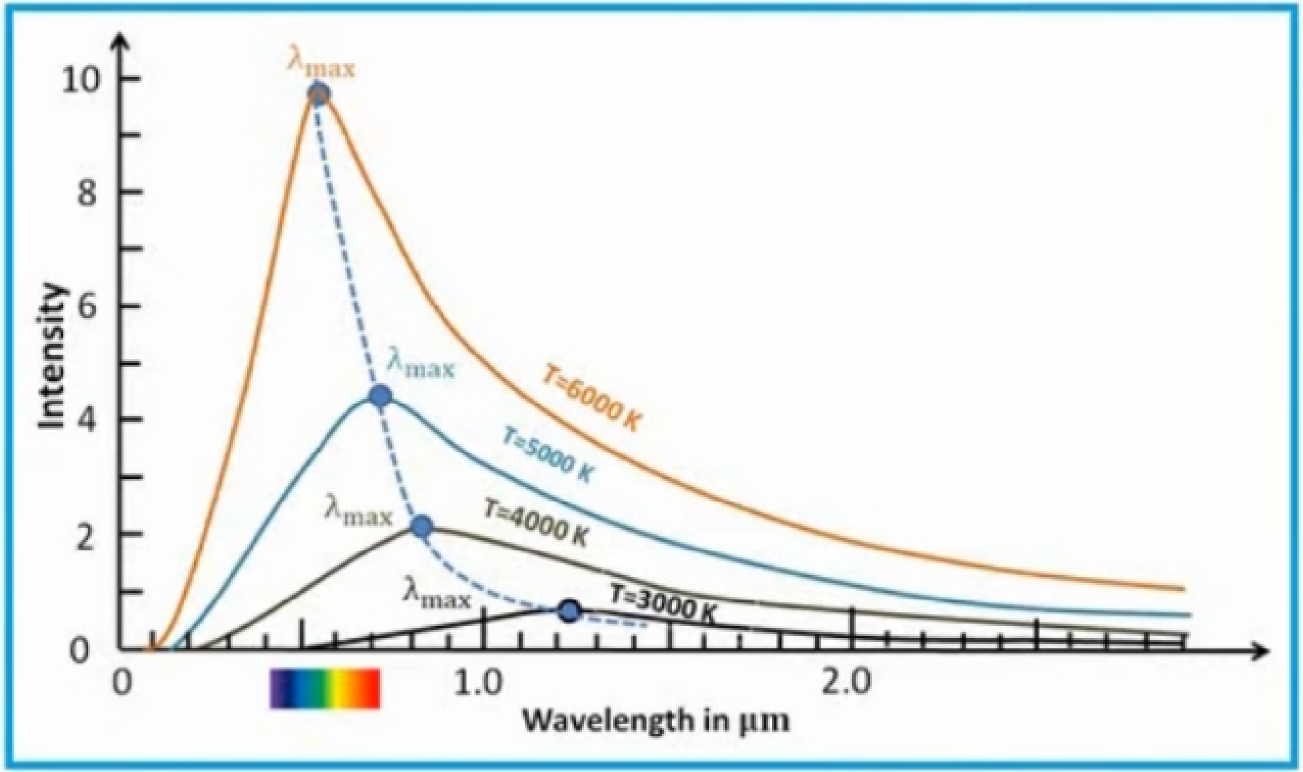


Wien's displacement Law

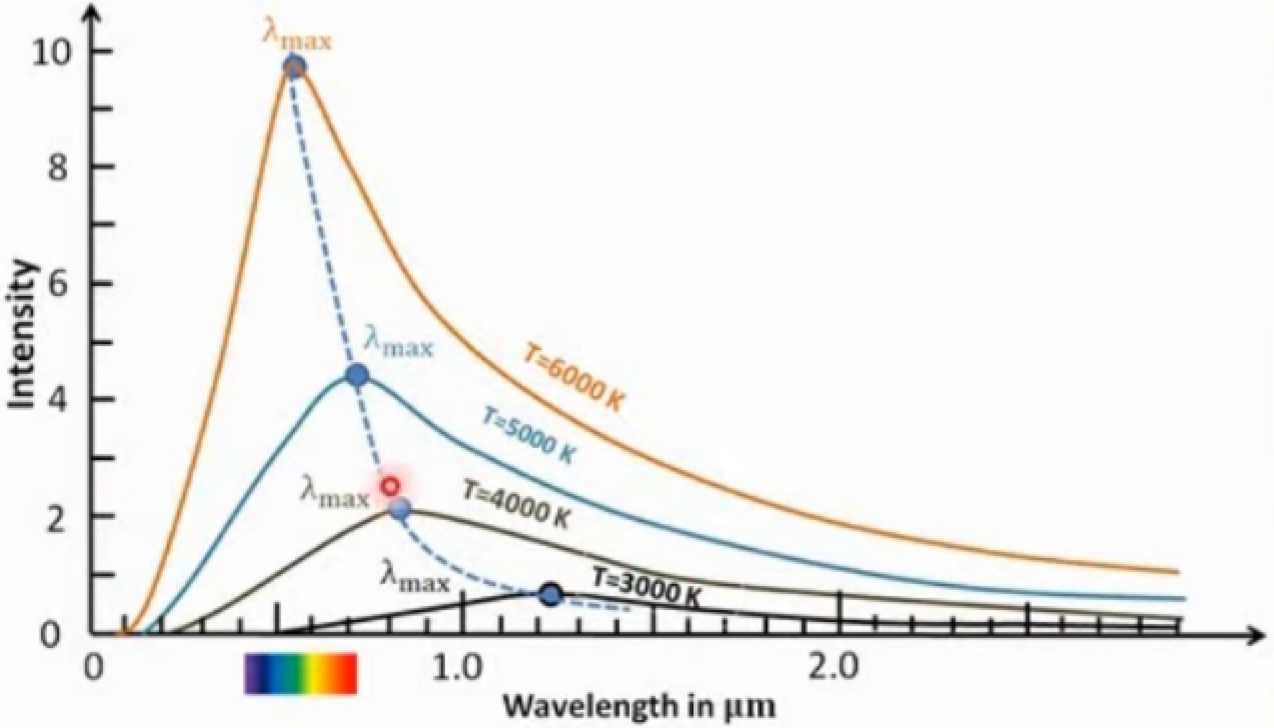
$$\lambda_{max} \propto \frac{1}{T}$$

Rayleigh Jean's Radiation Law gives Energy density of the radiation as -

$$\rho(v) = \frac{8\pi v^2}{c^3} kT$$

Black Body Radiation

$$\rho(v) = \frac{8\pi\nu^2}{c^3}\,kT$$



Max Planck suggested that radiated energy must be depending on frequency of radiation and represented this energy $E = h\nu$ and replaced the average energy **kT by** $\frac{h\nu}{e^{\frac{h\nu}{kT}}-1}$

$$\therefore\ \rho(v) = \frac{8\pi\nu^2}{c^3}\frac{h\nu}{e^{\frac{h\nu}{kT}}-1}$$

$$\therefore\ \rho(v) = \frac{8\pi h\nu^3}{c^3}\left(\frac{1}{e^{\frac{h\nu}{kT}}-1}\right)$$

- Einstein used idea of Planck to explain his photoelectric effect.
- He confirmed that light consists of discrete units of energy known as photons carrying energy - $E = h\nu$.
- **This dual nature of light is known as wave-particle duality**.

- Motion of macroscopic particles can be explained by classical theory of Mechanics.
- But it fails to explain the motion of microscopic particles like electron, proton etc.
- **Quantum mechanics** was developed from Quantum theory to explain the properties associated with such particles.

de Broglie Hypothesis

- It states that –

There is a wave associated with every moving particle moving with velocity v, and the wavelength of this wave is given by –

$$\boxed{\lambda = \frac{h}{p} = \frac{h}{mv}}$$

$h = 6.626 \times 10^{-34}$ J-s is Plank's constant

$p = m\,v$ is momentum

de Broglie Hypothesis

Let us consider the case of the photon. Energy of the photon, according to Plank's theory of radiation is given by –

$$\boxed{E = h\nu} \tag{1}$$

where h is Plank's constant and ν is frequency of radiation

If we consider a photon as a particle of mass m, its energy is given by Einstein Mass Energy relation as –

$$\boxed{E = mc^2} \tag{2}$$

From equation (1) and (2), we get,

$$\boxed{h\nu = mc^2} \tag{3}$$

As photon travels with velocity of light 'c' in free space, its momentum 'p' is given by –

$$\boxed{p = mc} \tag{4}$$

de Broglie Hypothesis

$$E = h\nu \quad (1)$$

$$E = mc^2 \quad (2)$$

$$h\nu = mc^2 \quad (3)$$

$$p = mc \quad (4)$$

Dividing equation 3 by 4 we get –

$$\frac{h\nu}{p} = \frac{mc^2}{mc} = c$$

$$\therefore \frac{h}{p} = \frac{c}{\nu} = \lambda \qquad \left(\because \frac{c}{\nu} = \lambda\right)$$

$$\boxed{\therefore \lambda = \frac{h}{p}}$$

de Broglie assumed that this relation holds good for all material particles like electrons, neutrons etc.

de Broglie wavelength in terms of Kinetic Energy

$$\boxed{\lambda = \frac{h}{p} = \frac{h}{mv}}$$ de Broglie Relation

The Kinetic energy of the particle is –

$$E = \frac{1}{2}mv^2 = \frac{1}{2m}m^2v^2 = \frac{1}{2m}p^2$$

$$\therefore p^2 = 2mE$$

$$\therefore p = \sqrt{2mE}$$

$$\boxed{\therefore \lambda = \frac{h}{p} = \frac{h}{\sqrt{2mE}}}$$

$$\boxed{\therefore \lambda = \frac{h}{p} = \frac{h}{\sqrt{2meV}}}$$

$$\boxed{\therefore \lambda = \frac{h}{p} = \frac{h}{\sqrt{2mkT}}}$$

Properties of Matter Waves

- Waves associated with moving particles are called matter waves.
- Wavelength of matter wave is give by $\lambda = \frac{h}{p} = \frac{h}{mv}$
- Matter waves are not electromagnetic waves and can be associated with any particle whether charged or uncharged
- Matter waves can propagate in a vacuum, hence they are not mechanical wave.
- Phase velocity of matter wave $v_p = \frac{c^2}{v} > c$

A bullet of mass 40 gm and an electron both travel with the velocity of 1100 m/s. What wavelengths can be associated with them? Why the wave nature of bullet can not be revealed using diffraction effect?

A bullet of mass 40 gm and an electron both travel with the velocity of 1100 m/s. What wavelengths can be associated with them? Why the wave nature of bullet can not be revealed using diffraction effect?

i) For electron, Given : $h = 6.63 \times 10^{-34}\ \text{J} - \text{s}$, $m = 9.1 \times 10^{-31}\text{kg}$,

$e = 1.6 \times 10^{-19}\text{C}$ $v = 1100\frac{m}{s}$, $\lambda = ?$

$$\lambda = \frac{h}{mv} = \frac{6.63 \times 10^{-34}}{9.1 \times 10^{-31} \times 1100} = \mathbf{6.623 \times 10^{-7} m = 6623\ A^0}$$

ii) For bullet, Given : $h = 6.63 \times 10^{-34}\ \text{J} - \text{s}$, $m = 40\ \text{gm} = 40 \times 10^{-3}\ \text{kg}$,

$v = 1100\frac{m}{s}$, $\lambda = ?$

$$\lambda = \frac{h}{mv} = \frac{6.63 \times 10^{-34}}{40 \times 10^{-3} \times 1100} = \mathbf{1.507 \times 10^{-35} m}$$

As the wavelength associated with bullet is of the order of 10^{-35}m, to reveal the wave nature of wave associated with bullet, a diffraction grating having width of the slit of the order of 10^{-35}m is needed. Such diffraction grating is not available. So the wave nature of the bullet can not be revealed.

Calculate the de Broglie wavelength of the proton moving with a velocity equal to $\frac{1}{20}$ th of velocity of light. Mass of proton is 1.6×10^{-27} kg.

Calculate the de Broglie wavelength of the proton moving with a velocity equal to $\frac{1}{20}$ th of velocity of light. Mass of proton is 1.6×10^{-27} kg.

Given : $h = 6.63 \times 10^{-34} \text{J} - \text{s}$, $m = 1.6 \times 10^{-27}\text{kg}$

$v = \frac{1}{20} \times 3 \times 10^{8}$ m/s, $\lambda = ?$

$$\lambda = \frac{h}{p} = \frac{h}{mv} = \frac{6.63 \times 10^{-34}}{1.6 \times 10^{-27} \times \frac{1}{20} \times 3 \times 10^{8}}$$

$$= \mathbf{2.763 \times 10^{-14}\,m}$$

Calculate the wavelength of the wave associated with a neutron moving with energy 0.025eV. Mass of neutron is 1.676×10^{-27} kg.

Calculate the wavelength of the wave associated with a neutron moving with energy 0.025eV. Mass of neutron is 1.676×10^{-27} kg.

Given : $h = 6.63 \times 10^{-34} \mathrm{J-s}$, $m = 1.676 \times 10^{-27} \mathrm{kg}$

$E = 0.025 \mathrm{eV} = 0.025 \times 1.6 \times 10^{-19} \mathrm{J}$

$\lambda = ?$

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2mE}} = \frac{6.63 \times 10^{-34}}{\sqrt{2 \times 1.676 \times 10^{-27} \times 0.025 \times 1.6 \times 10^{-19}}}$$

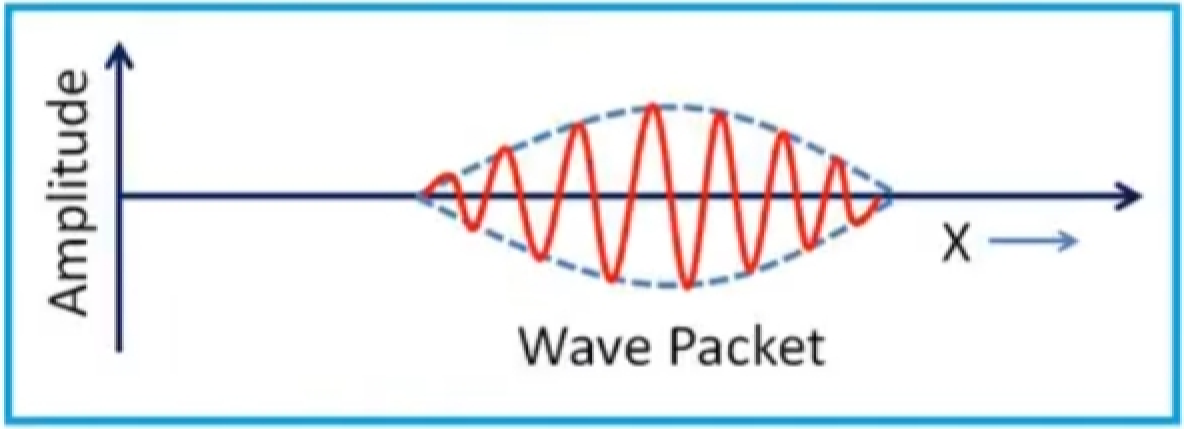
$$= \mathbf{1.811 \times 10^{-10} m}$$

Which type of wave associated with matter?

- Single Monochromatic wave?
  - Location of the particle?

  - Phase velocity $v_p = \nu\lambda = \frac{h\nu\lambda}{h} = \frac{h\nu}{h/\lambda} = \frac{E}{p} = \frac{mc^2}{mv} = \frac{c^2}{v}$

- Schrodinger's solution
  - Group of waves
  - Wave packet



The **phase velocity** is the velocity with which a particular phase of the wave propagates in the medium.

Let the equation of the wave travelling in x-direction with vibrations in y-direction is –

$$y = A\sin(\omega t - kx) \quad (1)$$

Where A is amplitude of vibration,

$k = \frac{2\pi}{\lambda}$ is propagation constant,

$\omega = 2\pi\nu$ is the angular frequency

$$\therefore \lambda = \frac{2\pi}{k} \text{ and } \nu = \frac{\omega}{2\pi} \quad (2)$$

Phase velocity,

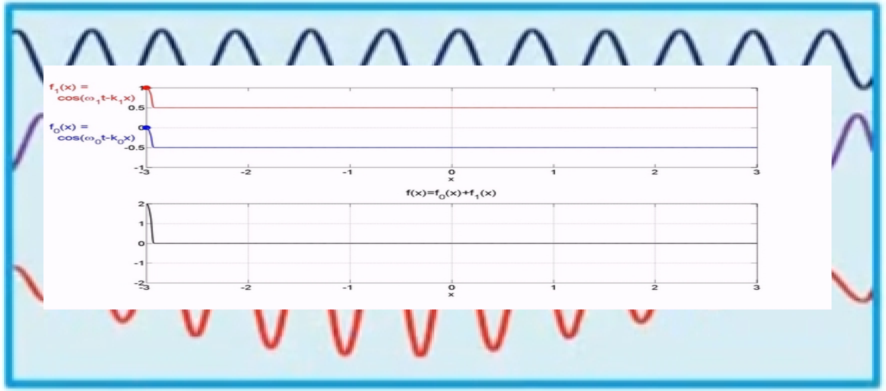
$$v_p = \nu\lambda = \frac{\omega}{2\pi} \times \frac{2\pi}{k} = \frac{\omega}{k} \quad (3)$$

From (2)

$$k = \frac{2\pi}{\lambda} = \frac{2\pi}{h/_{mv}} = \frac{2\pi mv}{h} \quad (4)$$

As from de Broglie relation says $\lambda = \frac{h}{mv}$

Group Velocity



Let us consider two waves represented by -

$$y_1 = A\sin(\omega t - kx)$$

$$y_2 = A\sin[(\omega + d\omega)t - (k + dk)x]$$

The resultant displacement y an any time t is $y = y_1 + y_2$

$$y = 2A\sin\left(\frac{2\omega + d\omega}{2}t - \frac{2k + dk}{2}x\right)\cos\left(\frac{d\omega}{2}t - \frac{dk}{2}x\right)$$

The sine term represents a wave of angular frequency ω and propagation constant k

The Cosine term modulates this wave with angular frequency $\frac{d\omega}{2}$ to produce wave-group travelling with velocity $v_g = \frac{d\omega}{dk}$

Group Velocity

We know - $\lambda = \dfrac{2\pi}{k}$ and $\nu = \dfrac{\omega}{2\pi}$ $\quad$ 1

Group velocity $\quad v_g = \dfrac{d\omega}{dk} = \dfrac{d\omega/dv}{dk/dv}$ $\quad$ 2

$$h\nu = mc^2 \quad \Rightarrow \quad \nu = \frac{mc^2}{h}$$

$$\omega = 2\pi\nu = \frac{2\pi mc^2}{h}$$

Putting $m = \dfrac{m_0}{\sqrt{1-\frac{v^2}{c^2}}}$ (Relativistic mass)

$$\omega = 2\pi\nu = \frac{2\pi c^2}{h}\frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$\therefore \frac{d\omega}{dv} = \frac{2\pi m_0}{h} v\left(1-\frac{v^2}{c^2}\right)^{-3/2} \tag{3}$$

$$k = \frac{2\pi}{\lambda} = \frac{2\pi mv}{h} = \frac{2\pi v}{h}\frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$\therefore \frac{dk}{dv} = \frac{2\pi m_0}{h}\left(1-\frac{v^2}{c^2}\right)^{-3/2} \tag{4}$$

Group Velocity

We know - $\lambda = \frac{2\pi}{k}$ and $\nu = \frac{\omega}{2\pi}$ 1

Group velocity $v_g = \frac{d\omega}{dk} = \frac{d\omega/dv}{dk/dv}$ 2

$$\therefore \frac{d\omega}{dv} = \frac{2\pi m_0}{h} v \left(1 - \frac{v^2}{c^2}\right)^{-3/2} \quad 3$$

$$\therefore \frac{dk}{dv} = \frac{2\pi m_0}{h} \left(1 - \frac{v^2}{c^2}\right)^{-3/2} \quad 4$$

From 2 3 and 4

Group velocity

$$v_g = \frac{d\omega/dv}{dk/dv} = \frac{\frac{2\pi m_0}{h} v \left(1 - \frac{v^2}{c^2}\right)^{-3/2}}{\frac{2\pi m_0}{h} \left(1 - \frac{v^2}{c^2}\right)^{-3/2}}$$

$$\boxed{\therefore v_g = v}$$

Thus, group velocity associated with the wave packet is equal to the velocity of the particle.

Group Velocity

We know - $\lambda = \frac{2\pi}{k}$ and $\nu = \frac{\omega}{2\pi}$ (1)

Group velocity $v_g = \frac{d\omega}{dk} = \frac{d\omega/dv}{dk/dv}$ (2)

$$\therefore \frac{d\omega}{dv} = \frac{2\pi m_0}{h} v \left(1 - \frac{v^2}{c^2}\right)^{-3/2} \quad (3)$$

$$\therefore \frac{dk}{dv} = \frac{2\pi m_0}{h} \left(1 - \frac{v^2}{c^2}\right)^{-3/2} \quad (4)$$

From 2 3 and 4

Group velocity

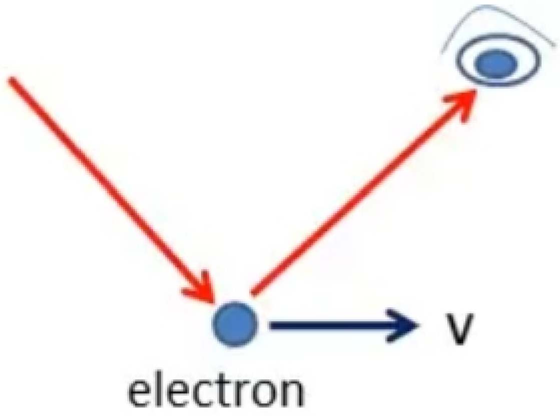
$$v_g = \frac{d\omega/dv}{dk/dv} = \frac{\frac{2\pi m_0}{h} v \left(1 - \frac{v^2}{c^2}\right)^{-3/2}}{\frac{2\pi m_0}{h} \left(1 - \frac{v^2}{c^2}\right)^{-3/2}}$$

$$\boxed{\therefore v_g = v}$$

Thus, group velocity associated with the wave packet is equal to the velocity of the particle.

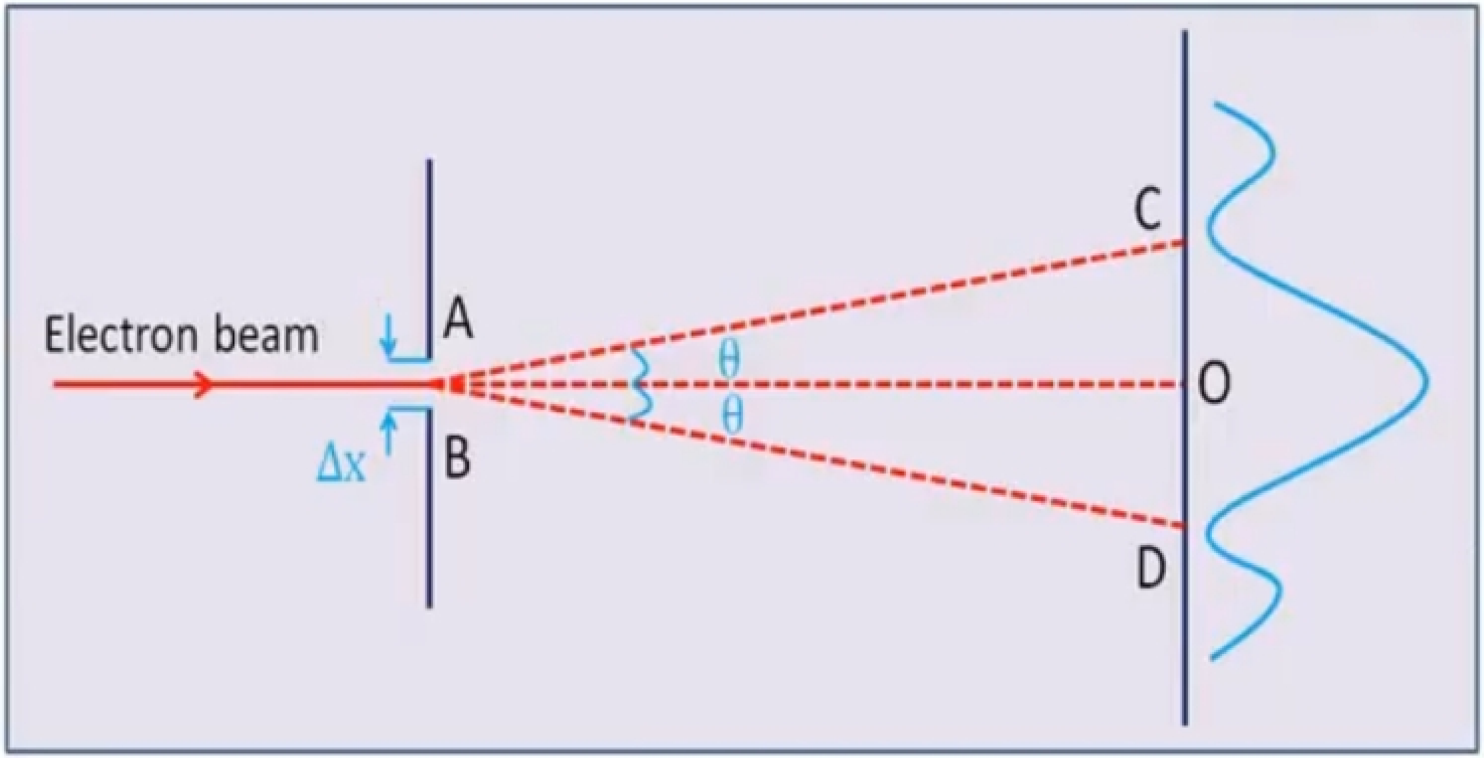
Heisenberg Uncertainty Principle

- If we are finding position of an moving electron, a photon must be seen by us after collision with the electron.
- But then the velocity and hence momentum of the electron will change due to collision.
- This will cause error or uncertainty in the measurement of momentum.
- If we try to determine momentum accurately, error or uncertainty in position is introduced.
- Thus, It is not possible to measure the position and momentum of an electron simultaneously and accurately.



(Experimental Verification of HUP)

Single Slit Diffraction Experiment



Condition for minimum -

$$d \sin\theta = n\lambda$$

Here $d = \Delta x$

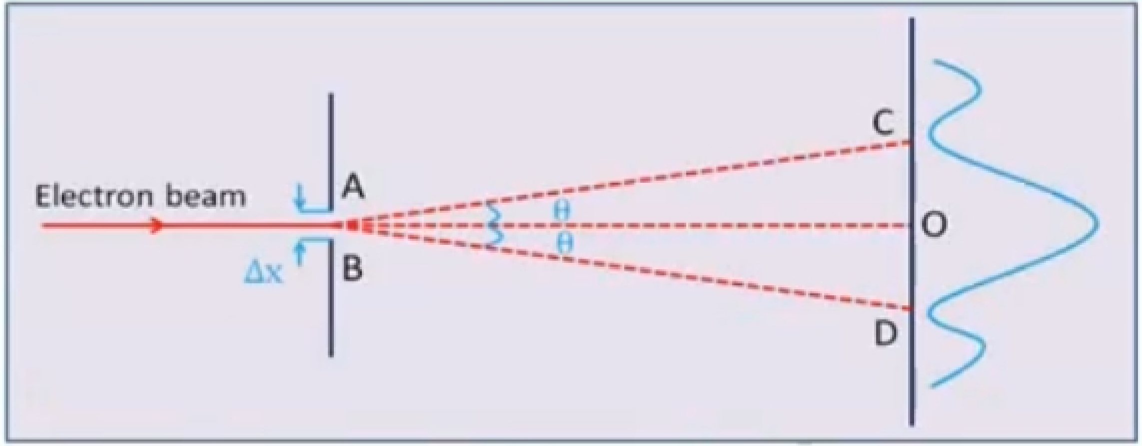
For 1st minimum, $n = 1$

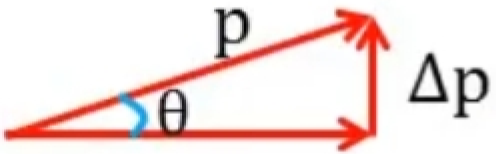
$$\therefore \Delta x \sin\theta = 1\,\lambda$$

$$\boxed{\Delta x = \frac{\lambda}{\sin\theta}} \quad (1)$$

(Experimental Verification of HUP)

Single Slit Diffraction Experiment





$$\boxed{\Delta x = \frac{\lambda}{\sin\theta}} \quad (1)$$

$$\boxed{\Delta p = p \sin\theta} \quad (2)$$

$$\therefore \Delta x \, \Delta p = \frac{\lambda}{\sin\theta} \, p \sin\theta$$

$$\therefore \Delta x \, \Delta p = \frac{\lambda}{\sin\theta} \frac{h}{\lambda} \sin\theta$$

$$\boxed{\therefore \Delta x \, \Delta p = h > \frac{h}{4\pi}}$$

Heisenberg Uncertainty Principle

Heisenberg's Uncertainty Principle (HUP) is applicable to all conjugate or complimentary pairs of physical variables whose product has the dimension of Planck's constant 'h'. Some common such pairs are

- Energy-Time,
- Position-Linear momentum,
- Angular momentum-Angular displacement etc.

Time-Energy Uncertainty Principle

Let us consider a particle of mass 'm' moving with a velocity 'v' so that its K.E. is –

$$E = \frac{1}{2} m v^2$$

$$\therefore \Delta E = \frac{1}{2} m\, 2\, v\, \Delta v$$

As $m\Delta v = \Delta p$

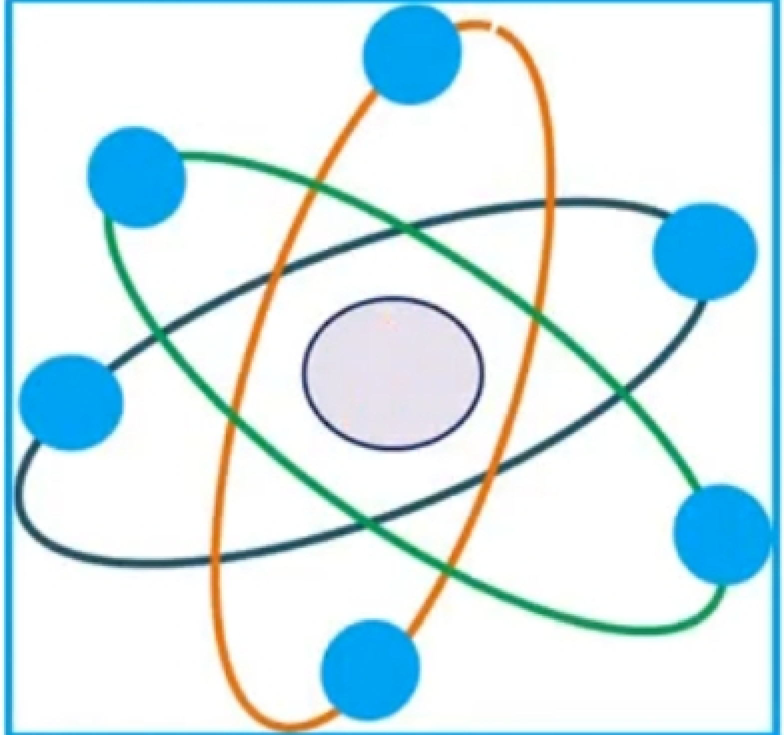
$$= v \Delta p$$

As $\frac{\Delta x}{\Delta t} = v$

$$= \frac{\Delta x}{\Delta t} \Delta p$$

$$\boxed{\therefore \Delta E\, \Delta t = \Delta x\, \Delta p \geq \frac{h}{4\pi}}$$

Why electron can not exist in nucleus?



Approximate radius of an nucleus $= r \approx 5 \times 10^{-15}$m

∴Uncertainty in position if electron exist in nucleus $= \Delta x = 2r$

$$\therefore \ \Delta x = 2r \approx 10^{-14} m$$

According to HUP $\boxed{\Delta x \, \Delta p \geq \frac{h}{4\pi}}$

$$\therefore \Delta x \, m\Delta v \geq \frac{h}{4\pi}$$

$$\therefore \Delta v \geq \frac{h}{4\pi \, m \, \Delta x} = \frac{6.63 \times 10^{-34}}{4 \times 3.142 \times 9.1 \times 10^{-31} \times 10^{-14}}$$

$$\therefore \Delta v \geq 5.797 \times 10^{9} \ m/s$$

From this equation, the uncertainty of velocity is more than c (3×10^{8} m/s). For this to happen, velocity of an electron must be greater that c. which is not possible. So the position of electron can't be in nucleus.

A position and momentum of 1 keV electron are simultaneously measured. If position is located within 10 nm then what is the percentage uncertainty in its momentum?

Given : $E = 1\ \text{keV} = 1000 \times 1.6 \times 10^{-19}\ \text{J}$, $\Delta x = 10 \times 10^{-9}\text{m}$

$$\frac{\Delta p}{p} \times 100 = ?$$

$$p = \sqrt{2mE} = \sqrt{2 \times 9.1 \times 10^{-31} \times 1000 \times 1.6 \times 10^{-19}} = 1.706 \times 10^{-23}\ \frac{\text{kg. m}}{\text{s}}$$

According to HUP, $\Delta x . \Delta p \geq \frac{h}{4\pi}$

$$\therefore \Delta p \geq \frac{h}{4\pi} \times \frac{1}{\Delta x} = \frac{6.63 \times 10^{-34}}{4 \times 3.142 \times 10 \times 10^{-9}}$$

$$\therefore \Delta p = 5.275 \times 10^{-27}\ \frac{\text{kg.m}}{\text{s}}$$

$$\therefore \frac{\Delta p}{p} \times 100 = \frac{5.275 \times 10^{-27}}{1.706 \times 10^{-23}} \times 100 = 0.0309\ \%$$

An electron has a speed of 400 m/s with uncertainty of 0.01%. Find the accuracy in its position.

Given : $v = 400\frac{m}{s}$, $\frac{\Delta v}{v} = \frac{0.01}{100}$, $m = 9.1 \times 10^{-31}$ kg,

$h = 6.63 \times 10^{-34}$, J – s $\Delta x = ?$

$$p = m\,v = 9.1 \times 10^{-31} \times 400 = 3.64 \times 10^{-28}\,\frac{kg.m}{s}$$

$$\Delta p = m\,\Delta v = m\,v\,\frac{\Delta v}{v}$$

$$= p \times \frac{\Delta v}{v} = 3.64 \times 10^{-28} \times \frac{0.01}{100} = 3.64 \times 10^{-32}\,\frac{kg.m}{s}$$

According to HUP, $\Delta x.\Delta p \geq \frac{h}{4\pi}$

$$\therefore \Delta x \geq \frac{h}{4\pi} \times \frac{1}{\Delta p} = \frac{6.63 \times 10^{-34}}{4 \times 3.142 \times 3.64 \times 10^{-32}} = 1.449 \times 10^{-3}\,m$$

An electron has a speed of 900 m/s with an accuracy of 0.001%. Calculate the uncertainty in the position of the electron.

Given : $v = 900\frac{m}{s}$, $\frac{\Delta v}{v} = \frac{0.001}{100}$, $m = 9.1 \times 10^{-31}$ kg,

$h = 6.63 \times 10^{-34}\ J - s$, $\Delta x = ?$

$$p = m\,v = 9.1 \times 10^{-31} \times 900 = 8.19 \times 10^{-28}\ \frac{kg.m}{s}$$

$$\Delta p = m\,\Delta v = m\,v\,\frac{\Delta v}{v}$$

$$= p \times \frac{\Delta v}{v} = 8.19 \times 10^{-28} \times \frac{0.001}{100} = 8.19 \times 10^{-33}\ \frac{kg.m}{s}$$

According to HUP, $\Delta x.\Delta p \geq \frac{h}{4\pi}$

$$\therefore \Delta x \geq \frac{h}{4\pi} \times \frac{1}{\Delta p} = \frac{6.63 \times 10^{-34}}{4 \times 3.142 \times 8.19 \times 10^{-33}} = 6.44 \times 10^{-3} m$$

The speed of an electron is measured to within an uncertainty of 2 x 10^4 m/s. What is the minimum space required by the electron to be confined to an atom?

Given : $\Delta v = 2 \times 10^4 \frac{m}{s}$, $m = 9.1 \times 10^{-31}$ kg,

$h = 6.63 \times 10^{-34} \, J - s$, $\Delta x = ?$

According to HUP, $\Delta x \, \Delta p = \Delta x \, m \, \Delta v \geq \frac{h}{4\pi}$

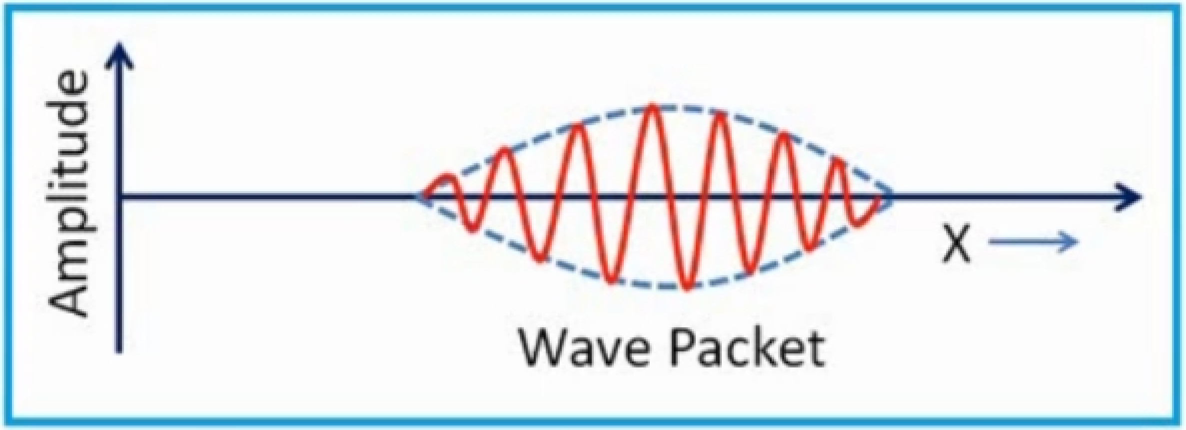
$$\therefore \Delta x \geq \frac{h}{4\pi} \times \frac{1}{m \, \Delta v} = \frac{6.63 \times 10^{-34}}{4 \times 3.142 \times 9.1 \times 10^{-31} \times 2 \times 10^4}$$

$$\therefore \Delta x = 2.898 \times 10^{-9} m$$

Schrodinger's solution

– Group of waves

– Wave packet



- ✓ The phases and amplitude of the waves in group are such that they undergo constructive interference only over a small region where the particle may be located.
- ✓ Outside this region, destructive interference occurs and amplitude is zero.
- ✓ Velocity with which this wave packet moves is called as group velocity. This group velocity is equal to the velocity of particle.

The wave function Ψ

- If particle exists, probability of finding the particle somewhere in the space must be unity.

$$\boxed{\iiint_{-\infty}^{\infty} \Psi^{*}\Psi \, dx \, dy \, dz = 1}$$

- This condition is called as normalization condition. The wave function is normalized. It means, it satisfies this condition.

The wave function Ψ must fulfill the following conditions -

- ψ must be finite, continuous and single valued everywhere.
- Its derivative, $\frac{\partial \Psi}{\partial x}$ or $\frac{\partial \Psi}{\partial t}$ must also be finite, continuous and single valued everywhere.
- ψ must have atleast some physically acceptable solutions.
- ψ must obey the principle of linear superposition i.e. ψ can anytime be expressed as a linear combination of two wave functions say

$$\Psi(x,t) = A\phi_1(x,t) + B\phi_2(x,t)$$

Schrodinger's Time Independent Wave Equation (STIE)

According to the De Broglie theory, a particle of mass 'm' moving with velocity 'v' is associated with a wave of wavelength-

$$\boxed{\lambda = \frac{h}{mv}} \quad (1)$$

The wave equation of stationary wave associated with the particle in terms of Cartesian co-ordinate system at any instant is given by – $\Psi = \Psi_0 \sin\omega t$

$$\boxed{\therefore \Psi = \Psi_0 \sin 2\pi\nu t} \quad (2)$$

Where ψ_0 is the amplitude at the point under consideration

One dimensional classical differenti equation of wave motion can be writte as –

$$\boxed{\frac{\partial^2\Psi}{\partial t^2} = v^2 \frac{\partial^2\Psi}{\partial x^2}} \quad (3)$$

Differentiating equation (2)

$$\frac{\partial\Psi}{\partial t} = \Psi_0\, 2\pi\nu \cos 2\pi\nu t$$

Differentiating it further –

$$\frac{\partial^2\Psi}{\partial t^2} = -\Psi_0\, 4\pi^2\nu^2 \sin 2\pi\nu t$$

$$\boxed{\therefore \frac{\partial^2\Psi}{\partial t^2} = -\,4\pi^2\nu^2\,\Psi} \quad (4)$$

Schrodinger's Time Independent Wave Equation (STIE)

$$\boxed{\lambda = \frac{h}{mv}} \quad (1)$$

$$\boxed{\therefore \Psi = \Psi_0 \sin 2\pi\nu t} \quad (2)$$

$$\boxed{\frac{\partial^2 \Psi}{\partial t^2} = v^2 \frac{\partial^2 \Psi}{\partial x^2}} \quad (3)$$

$$\boxed{\therefore \frac{\partial^2 \Psi}{\partial t^2} = -\,4\pi^2 \nu^2 \,\Psi} \quad (4)$$

As, frequency $(\nu) = \dfrac{\text{velocity } (v)}{\text{wavelength } (\lambda)}$

Equation (4) Becomes-

$$\frac{\partial^2 \Psi}{\partial t^2} = -\frac{4\pi^2 v^2}{\lambda^2}\Psi \quad (5)$$

From equation (3) and (5) we get

$$v^2 \frac{\partial^2 \Psi}{\partial x^2} = -\frac{4\pi^2 v^2}{\lambda^2}\Psi$$

$$\boxed{\frac{\partial^2 \Psi}{\partial x^2} + \frac{4\pi^2}{\lambda^2}\Psi = 0} \quad (6)$$

Schrodinger's Time Independent Wave Equation (STIE)

$$\lambda = \frac{h}{mv} \quad (1)$$

$$\frac{\partial^2\Psi}{\partial x^2} + \frac{4\pi^2}{\lambda^2}\Psi = 0 \quad (6)$$

$$\frac{\partial^2\Psi}{\partial x^2} + \frac{4\pi^2 m^2 v^2}{h^2}\Psi = 0 \quad (7)$$

Energy $E = K.E. + P.E.$

$$\therefore E = \frac{1}{2}mv^2 + V$$

$$\therefore m^2v^2 = 2m\,(E - V) \quad (8)$$

Substituting (8) in (7) we get

$$\frac{\partial^2\Psi}{\partial x^2} + \frac{4\pi^2}{h^2}2m(E - V)\,\Psi = 0$$

$$\frac{\partial^2\Psi}{\partial x^2} + \frac{2m}{\hbar^2}(E - V)\,\Psi = 0 \qquad \because \hbar = \frac{h}{2\pi}$$

$$-\frac{\hbar^2}{2m}\frac{\partial^2\Psi}{\partial x^2} + V\Psi = E\Psi \quad (9)$$

This is Schrodinger's Time Independent Wave Equation (STIE)

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- This equation is independent of time and gives a steady value.
- It is particularly useful when the energy of the particle is very small as compared to its rest energy.
- In most atomic problems, energy of the particle is very small when compared to rest energy.

Schrodinger's Time Dependent Wave Equation (STDE)

- Let us consider a free particle of mass 'm' moving with velocity 'v' in one dimension.
- Let 'p' be the momentum and 'E' be the energy of the particle.
- By the term free particle, it means that no forces are acting on it and its total energy E is entirely kinetic energy.

$$\boxed{\text{Kinetic Energy} = \frac{1}{2}\,mv^2 = \frac{1}{2m}\,m^2v^2 = \frac{p^2}{2m}} \quad (1)$$

This moving particle is associated with De Broglie waves which have wavelength λ and frequency ν. These are related as -

$$\boxed{p = \frac{h}{\lambda} = \frac{h}{2\pi} \times \frac{2\pi}{\lambda} = \hbar k} \quad (2)$$

$k = \frac{2\pi}{\lambda}$ is propagation constant

$$\boxed{E = h\nu = \frac{h}{2\pi} \times 2\pi\nu = \hbar\omega} \quad (3)$$

$\omega = 2\pi\nu$ is angular frequency

Schrodinger's Time Dependent Wave Equation (STDE)

$$E = \frac{p^2}{2m} \quad (1)$$

$$p = \frac{h}{\lambda} = \frac{h}{2\pi} \times \frac{2\pi}{\lambda} = \hbar k \quad (2)$$

$$E = h\nu = \frac{h}{2\pi} \times 2\pi\nu = \hbar\omega \quad (3)$$

From equation 1 2 and 3 we get

$$\hbar\omega = \frac{\hbar^2 k^2}{2m} \quad (4)$$

The wave equation describing the travelling waves must satisfy equation 4

The wave function associated with the particle may by a sine, cosine or exponential function of $(kx - \omega t)$. Such functions are harmonic and can be superimposed, thereby giving a wave packet.

Let, $\Psi = \Psi_0 \, e^{i(kx-\omega t)}$ (5)

Schrodinger's Time Dependent Wave Equation (STDE)

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$$\hbar\omega = \frac{\hbar^2 k^2}{2m} \qquad (4)$$

Let, $\Psi = \Psi_0\, e^{i(kx-\omega t)}$ (5)

Let us consider

$$i\hbar \frac{\partial \Psi}{\partial t} = i\hbar \frac{\partial\left(\Psi_0\, e^{i(kx-\omega t)}\right)}{\partial t}$$

$$\therefore i\hbar \frac{\partial \Psi}{\partial t} = i\hbar\,(-i\omega)\,\Psi_0\, e^{i(kx-\omega t)}$$

$$\boxed{\therefore i\hbar \frac{\partial \Psi}{\partial t} = \hbar\omega\Psi = E\,\Psi} \qquad (6)$$

Schrodinger's Time Dependent Wave Equation (STDE)

$$p = \frac{h}{\lambda} = \frac{h}{2\pi} \times \frac{2\pi}{\lambda} = \hbar k \qquad (2)$$

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Let, $\Psi = \Psi_0\, e^{i(kx-\omega t)}$ (5)

$$\therefore i\hbar \frac{\partial \Psi}{\partial t} = \hbar\omega\Psi = E\,\Psi \qquad (6)$$

Let us consider

$$\frac{-\hbar^2}{2m}\frac{\partial^2 \Psi}{\partial x^2} = \frac{-\hbar^2}{2m}\frac{\partial^2\left(\Psi_0\, e^{i(kx-\omega t)}\right)}{\partial^2 x}$$

$$\therefore \frac{-\hbar^2}{2m}\frac{\partial^2 \Psi}{\partial x^2} = \frac{-\hbar^2}{2m}(ik)^2\Psi_0\, e^{i(kx-\omega t)}$$

$$\therefore \frac{-\hbar^2}{2m}\frac{\partial^2 \Psi}{\partial x^2} = \frac{\hbar^2 k^2}{2m}\Psi = \frac{p^2}{2m}\Psi \qquad (7)$$

From equation (6) and (7) we get

$$E = i\hbar\frac{\partial}{\partial t} \qquad p = -i\hbar\frac{\partial}{\partial x} \qquad (8)$$

Schrodinger's Time Dependent Wave Equation (STDE)

$$p = \hbar k \quad (2)$$

$$E = \hbar\omega \quad (3)$$

$$\hbar\omega = \frac{\hbar^2 k^2}{2m} \quad (4)$$

Let, $\Psi = \Psi_0\, e^{i(kx-\omega t)}$ (5)

$$\therefore i\hbar\frac{\partial\Psi}{\partial t} = \hbar\omega\Psi = E\,\Psi \quad (6)$$

$$E = i\hbar\frac{\partial}{\partial t} \qquad p = -i\hbar\frac{\partial}{\partial x}$$

$$\therefore \frac{p^2}{2m} = \frac{-\hbar^2}{2m}\frac{\partial^2}{\partial x^2} \quad (8)$$

If the particle is not free particle, the total energy of the particle is -

$$E = \frac{p^2}{2m} + V$$

Let us write these as operators operating on wave function Ψ

$$\therefore E\,\Psi = \frac{p^2}{2m}\Psi + V\,\Psi$$

Using (8)

$$i\hbar\frac{\partial\Psi}{\partial t} = \frac{-\hbar^2}{2m}\frac{\partial^2\Psi}{\partial x^2} + V\,\Psi$$

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Schrodinger's Time Dependent Wave Equation (STDE)

$p = \hbar k$ (2)

$E = \hbar\omega$ (3)

$$\hbar\omega = \frac{\hbar^2 k^2}{2m}$$ (4)

Let, $\Psi = \Psi_0\, e^{i(kx-\omega t)}$ (5)

$$\therefore i\hbar\frac{\partial\Psi}{\partial t} = \hbar\omega\Psi = E\,\Psi$$ (6)

$$E = i\hbar\frac{\partial}{\partial t}$$

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(8)

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Schrodinger's Time Dependent Wave Equation (STDE)

$$i\hbar \frac{\partial \Psi}{\partial t} = \frac{-\hbar^2}{2m} \frac{\partial^2 \Psi}{\partial x^2} + V\Psi$$

- The solution of time dependent Schrodinger equation results in states which are not stationary.
- The corresponding probability densities are not constant and they vary with time.
- Let free electron in a box. If we apply an electric field on the box periodically, the original wave functions and the corresponding probability densities of the electron are not constant and they do depend on the time.

STIE

$$-\frac{\hbar^2}{2m}\frac{\partial^2\Psi}{\partial x^2} + V\Psi = E\Psi$$

The solution of time independent Schrodinger equation results in stationary states, where the probability density is independent of time.

STDE

$$i\hbar\frac{\partial\Psi}{\partial t} = \frac{-\hbar^2}{2m}\ \frac{\partial^2\Psi}{\partial x^2} + V\Psi$$

The solution of time dependent Schrodinger equation results in states which are not stationary. The corresponding probability densities vary with time.

As Newton's laws predict the future behavior of a dynamic system in classical mechanics, Schrodinger's equations are used to predict future behavior in quantum mechanics.