Trignometry

Graphs and Properties

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Topics : *Trignometry, Functions.*

> "It is impossible for a man to learn what he thinks he already knows".
> –Epictetus

1. Shift the graph of the function $y=\sin(-x)$ to the right by $\frac{\pi}{3}$ units. The analytical expression of the resulting function graph is _____. Shift the graph of the function $y=\cos(-2x)$ to the left by $\frac{\pi}{6}$ units. The analytical expression of the resulting function graph is _____.

2. The domain of the function $y=\sqrt{\cos x-2\cos^2 x}$ is _____, and the range is _____.

3. The minimum positive period of the function $y=\sin x\left(1+\tan\frac{x}{2}\right)$ is _____.

4. Given $f(x)=a\sin^3(x)+b\sqrt[3]{x}\cos^3(x)+4(a,b\in\mathbb{R})$, and $f(\sin 10^\circ)=5$, then $f(\cos 100^\circ)=$ _____.

5. The domain of the function $f(x)=\sqrt{\sin 2x+\sqrt{3}\cos 2x-1}$ is _____.

6. The axis of symmetry of the graph of the function $y=\sin\left(2x+\frac{\pi}{3}\right)$ is _____.

7. The center of symmetry of the graph of the function $y=\tan\left(\frac{1}{2}x+\frac{\pi}{6}\right)$ is _____.

8. Given the function $f(x)=\sin(\omega x+\varphi)$ $(\omega>0,|\varphi|<\frac{\pi}{2})$. Use any two of the judgments as conditions and consider the other two judgments as conclusions. The two correct conclusions are _____.

   a. Its graph is symmetric about the line $x=\frac{\pi}{12}$;
   b. Its graph is symmetric about the point $\left(\frac{\pi}{3},0\right)$;
   c. Its smallest positive period is π ;
   d. It is an increasing function in the interval $\left[-\frac{\pi}{6},0\right]$.

9. There are four functions :

   a. $y=\sin^2 x$;
   b. $y=\sin|x|$;
   c. $y=\tan\frac{x}{2}-\cot\frac{x}{2}$;
   d. $y=|\sin x|$.

The function with period π and increasing in the interval $\left(0, \frac{\pi}{2}\right)$ is _____.

10. The equation $\sin x = \frac{x}{100}$ has _____ real roots.

11. Given real numbers x and y satisfying $x^2 + 2\cos y = 1$, the range of values of $x - \cos y$ is _____.

12. For all positive integers n satisfying $\frac{1}{4} < \sin\frac{\pi}{n} < \frac{1}{3}$, the sum of n is _____.

13. Given $\alpha, \beta \in [0, \pi]$, the maximum value of $(\sin\alpha + \sin(\alpha+\beta)) \cdot \sin\beta$ is _____.

14. Given the function $y = \sin x + a\cos x$ is symmetric about the line $x = \frac{5\pi}{3}$, then the equation of the axis of symmetry for the function $y = a\sin x + \cos x$ is _____.

15. Plot the function $y = \sqrt{3}\sin 2x - \cos 2x - 1$ over one period, and indicate its relationship with the graph of $y = \sin x$.

16. Given $f(x) = 2\sin\left(x + \frac{\theta}{2}\right)\cos\left(x + \frac{\theta}{2}\right) + 2\sqrt{3}\cos^2\left(x + \frac{\theta}{2}\right) - \sqrt{3}$:

i. Simplify the expression of $f(x)$;

ii. If $0 \le \theta \le \pi$, find the value of θ such that the function $f(x)$ is an even function ;

iii. Under the condition in ii., find the set of x in $[-\pi, \pi]$ that satisfies $f(x) = 1$.

17. Given that $x \in [0, 1]$, the inequality $x^2\cos\theta - x(1-x) + (1-x)^2\sin\theta \ge 0$ always holds. Find the range of θ.

18. Given the function $f(x) = \sin(\omega x + \varphi)$ $(\omega > 0, 0 \le \varphi \le \pi)$, which is an even function on $\mathbb{R}$, its graph is symmetric about the point $M\left(\frac{3\pi}{4}, 0\right)$, and it is increasing in the interval $\left[0, \frac{\pi}{2}\right]$. Find the values of φ and ω.

19. Given real numbers α, β, a, b satisfying $\alpha < \beta$, $\alpha + \beta < \pi$, $a + b < \pi$ and $\frac{\sin a}{\sin b} \le \frac{\sin\alpha}{\sin\beta}$, prove that $a < b$.

20. Given $f(x) = \frac{\sqrt{2}\sin x}{\sqrt{1+\cos 2x}}$:

i. Find the domain, range, and smallest positive period of the function $f(x)$;

ii. Determine whether the function $f(x)$ is odd.

21. If the equation $\sin^2 x + \cos x + a = 0$ has a solution, find the range of the real number a.

22. Determine whether there exists a real number x that makes $\tan x + \sqrt{3}$ and $\cot x + \sqrt{3}$ real.

23.

i. Find the maximum value of $f(\theta) = \cos\frac{\theta}{2}\sin\theta$, where $\theta \in \left(0, \frac{\pi}{2}\right)$;

ii. Find the maximum value of $g(\theta) = \sin\frac{\theta}{2}\cos\theta$, where $\theta \in \left(0, \frac{\pi}{2}\right)$.

24. A classroom wall has a blackboard with its top and bottom edges at a meters and b meters above the students' horizontal line of sight, respectively. At what distance from the wall will the students' viewing angle of the blackboard be maximized ?

25. Given the function $f(x) = \tan x$, where $x \in \left(0, \frac{\pi}{2}\right)$, if $x_1, x_2 \in \left(0, \frac{\pi}{2}\right)$ and $x_1 \ne x_2$, prove that $\frac{1}{2}[f(x_1) + f(x_2)] > f\left(\frac{x_1+x_2}{2}\right)$.

26. Find the positive integer k such that $f(x) = \sin kx \cdot \sin^k x + \cos kx \cdot \cos^k x - \cos^k 2x$ does not depend on x.