Домашнее задания по математическому анализу

abcdw

Интегралы.

1634. $\int \frac{\sqrt{x}-2\sqrt[3]{x^2}+1}{\sqrt[4]{x}}dx = \int \left(x^{\frac{1}{4}} - 2x^{\frac{5}{12}} + x^{-\frac{1}{4}}\right)dx = \frac{4x^{\frac{5}{4}}}{5} - \frac{24x^{\frac{17}{12}}}{17} + \frac{4x^{\frac{3}{4}}}{3} + C$

1744. $\int \sin 3x \sin 5x dx = \frac{1}{2}\int(\cos 2x - \cos 8x)dx = \frac{1}{4}\sin 2x - \frac{1}{16}\sin 8x + C$

1750. $\int \cos^4 x dx = \int \frac{\cos 4x+4\cos 2x+3}{8}dx = \frac{1}{32}\sin 4x + \frac{1}{4}\sin 2x + \frac{3}{8}x + C$

1756. $\int \frac{dx}{\sin x \cos^3 x} = \int \frac{\sin^2 x+\cos^2 x}{\sin x \cos^3 x}dx = \int \frac{\operatorname{tg} x dx}{\cos^2 x} + \int \frac{dx}{\sin x \cos x} = {}^1\int t dt + \int \left(\frac{\sin x}{\cos x} + \frac{\cos x}{\sin x}\right)dx =$

$= \frac{1}{2}\operatorname{tg}^2 x - \int \frac{d\cos x}{\cos x} + \int \frac{d\sin x}{\sin x} = \frac{1}{2}\operatorname{tg}^2 x - \ln|\cos x| + \ln|\sin x| + C =$
$= \frac{1}{2}\operatorname{tg}^2 x + \ln|\operatorname{tg} x| + C$

1763. $\int \operatorname{sh} x \operatorname{sh} 2x dx = \int \frac{(e^x-e^{-x})(e^{2x}-e^{-2x})}{4}dx = \frac{1}{4}\int(e^{3x}+e^{-3x}-e^x-e^{-x})dx =$
$= \frac{1}{12}e^{3x} - \frac{1}{12}e^{-3x} - e^x + e^{-x} + C = \frac{1}{6}\operatorname{sh} 3x - \frac{1}{2}\operatorname{sh} x + C$

1775. $\int \frac{dx}{e^{\frac{x}{2}}+e^x} = \int \frac{dx}{e^{\frac{x}{2}}(1+e^{\frac{x}{2}})} = \int \frac{2dt}{t^2(1+t)} = 2\int(\frac{1}{t^2} + \frac{1}{1+t} - \frac{1}{t})dt =$
$= -2e^{\frac{x}{2}} + 2\ln|e^{\frac{x}{2}}+1| - x$

1787. $\int \frac{x^2 dx}{\sqrt{a^2+x^2}} = \int \frac{a^2 \operatorname{sh}^2 t \ a \operatorname{ch} t dt}{a\sqrt{1+sh^2 t}}$

1700(a). $\frac{\sin x \ d\sin x}{\sqrt{(a^2-b^2)\sin^2 x+b^2}}, t = sinx, u = \sqrt{(a^2-b^2)t^2+b^2}, du = \frac{2(a^2-b^2)t \ dt}{2\sqrt{(a^2-b^2)t^2+b^2}}$

1710. $t = \arcsin x, dt = \frac{dx}{\sqrt{1-x^2}}$

1760. $\int \frac{(1+e^x)^2}{1+e^{2x}}dx = \int \frac{(1+t)^2 t dt}{1+t^2} = t - \operatorname{arctg} t + \frac{1}{2}\log|1+t^2| + \frac{t^3}{3} + C$

1718. $t = \sin x, dt = \cos x dx.\ u = t^2, du = tdt.\ du = \sqrt{2}dv.$

1730. $x = -\frac{1}{5}(2-5x) + \frac{2}{5}$

1736. $A = \frac{1}{5}, B = -\frac{1}{5}.\ \int(\frac{A}{x^2-2} + \frac{B}{x^2+3})dx$

1741. $\sin^2 x = \frac{1-\cos 2x}{2}$

1743. $\sin x \sin(x+\alpha) = \frac{\cos\alpha - \cos(2x+\alpha)}{2}$

1749. $\sin^4 x = \frac{\cos 4x+4\cos 2x+3}{8}$

1761. $\operatorname{sh}^2 x = \frac{\operatorname{ch} 2x-1}{2}$

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[1] $t = \operatorname{tg} x,\ dt = \frac{dx}{\cos^2 x}$

1768. $\sqrt{2-x} = t, dt = \frac{dx}{\sqrt{2-x}}, x = 2 - t^2$

1770. $t = (2 - 5x^3)^{\frac{5}{3}}, dt = -25(2 - 5x^3)^{\frac{2}{3}}x^2dx, x^3 = \frac{2-t^{\frac{3}{5}}}{5}$

1774. $t = \sqrt{1 + \ln x}, dt = \frac{dx}{2x\sqrt{1+\ln x}}, \ln x = t^2 - 1$

1776. $t = \sqrt{1 + e^x}, dt = \frac{e^x dx}{2\sqrt{1+e^x}}, e^x = t^2 - 1$