

# On the Properties of Solenoid Originated Magnetic Fields

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## 1 Introduction

### 1.1 Aim

This small project aims to investigate some of the properties of solenoid originated magnetic fields in a Helmholtz Pair configuration investigating magnetic field neutralisation along with investigation of the magnitudes of the magnetic field on a non symmetrical axis to a single solenoid.

This will be accomplished with a combination of theoretical calculations along with experimental evidence in order to prove that Biot-Savart Law is valid for all axis relative to a solenoid of origin.

### 1.2 Methods

#### 1.2.1 Neutralising Magnetic Fields

1. Zero hall monitor against earths magnetic fields.
2. Position two solenoids 20cm apart. Such as in **Figure 1**.
3. Reverse current connections on one of the solenoids in order to reverse the direction of the magnetic fields.
4. Set power supply to 1.5A.
5. In a systematic manner, vary the hall probe along the common axis of the solenoids and record the magnetic field in Gauss and the distance from a solenoid of one's choice.
6. Plot Gauss against distance  $z$  (cm).

#### 1.2.2 Alternate Axis Measurements from a Single Solenoid

1. Zero hall monitor against earth's magnetic fields.
2. Set up a single solenoid at 1.5A.

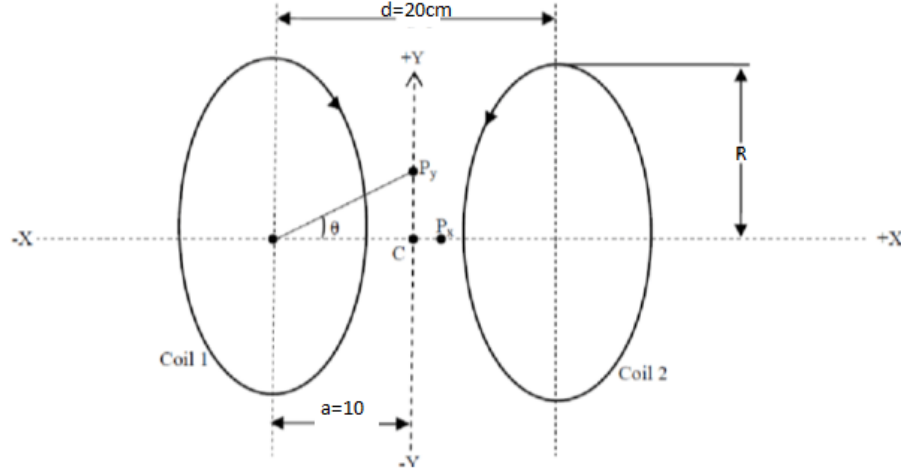


Figure 1: A Helmholtz pair

3. Vary the hall probe's distance along the x axis 90° from C (C=13cm on the z axis for our measurements) as seen in **Figure 2**.
4. Record hall effect in Gauss and distance x in cm.
5. Plot Gauss against distance x (cm).

## 2 Results

### 2.1 Neutralising Magnetic Fields

The magnetic fields generated by the opposed solenoids can be seen in **Figure 3** creating a system of magnetic fields between the 2 solenoids that oppose each other. The theoretical values can be calculated with the **Biot-Savart Law**;

$$d\mathbf{B} = \frac{\mu_0}{4\pi} \frac{Id\boldsymbol{\ell} \times \hat{\mathbf{r}}}{r^2} \quad (1)$$

$\mathbf{B}$  in (1) is measured in Tesla, so in order to compare it to our experimental values, we converted it to Gauss by multiplying by  $10^4$ .

In order for a calculation to be done on a coil like those found in a Helmholtz pair in order to find magnetic field strength at distance z from the centre of one of the coils, the Biot-Savart Law is derived into equation (2) where I is the current,  $\mu_0$  is the permeability constant and N are the number of loops in the solenoid.

$$B_z = \frac{\mu_0 NI}{A\ell} \left( \frac{z + \ell}{\sqrt{(z + \ell)^2 + R^2}} - \frac{z - \ell}{\sqrt{(z - \ell)^2 + R^2}} \right) \quad (2)$$

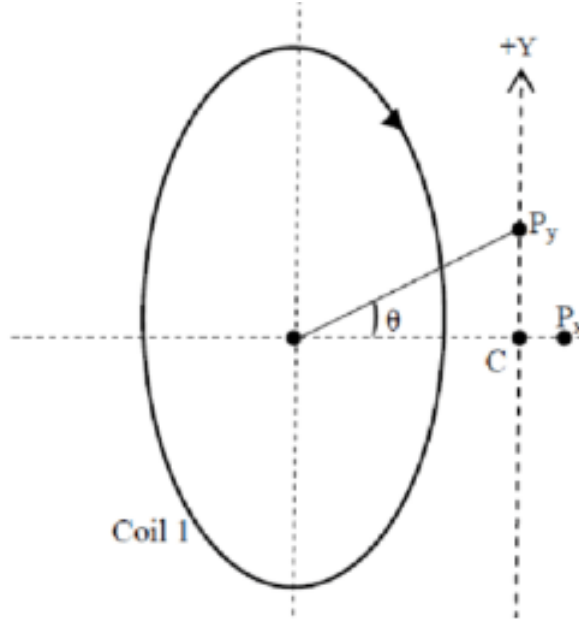


Figure 2: Alternate Axis Setup

Using known values of  $N = 397$ ,  $I = 1.5\text{A}$ ,  $\ell = 2.15\text{cm}$  and  $R = 15.15\text{cm}$  the theoretical values can be calculated for all values of  $z$ , allowing us to plot the same values as those we measure experimentally, these results can be seen in **Figure 3** and **Figure 4**

### 2.1.1 Expected Results

The results expected from this experiment are as follows

- The same magnitude of magnetic field at distances 0cm and 20cm.
- Magnetic field 0 Gauss at 10cm.
- Opposing quadratic relationships "added" together to create a cubic relationship since the magnetic field strength varies like  $\frac{1}{r^3}$ .

These critical features can clearly be seen in theoretical values calculated seen graphed in **Figure 3**.

### 2.1.2 Observed Results

From experimental values shown in **Figure 4**, these features can clearly be identified in relation to the expected model, however, accounting for standard uncertainty represented in the curve fit, some aspects are still far from theoretical values. These aspects and possible explanations include;

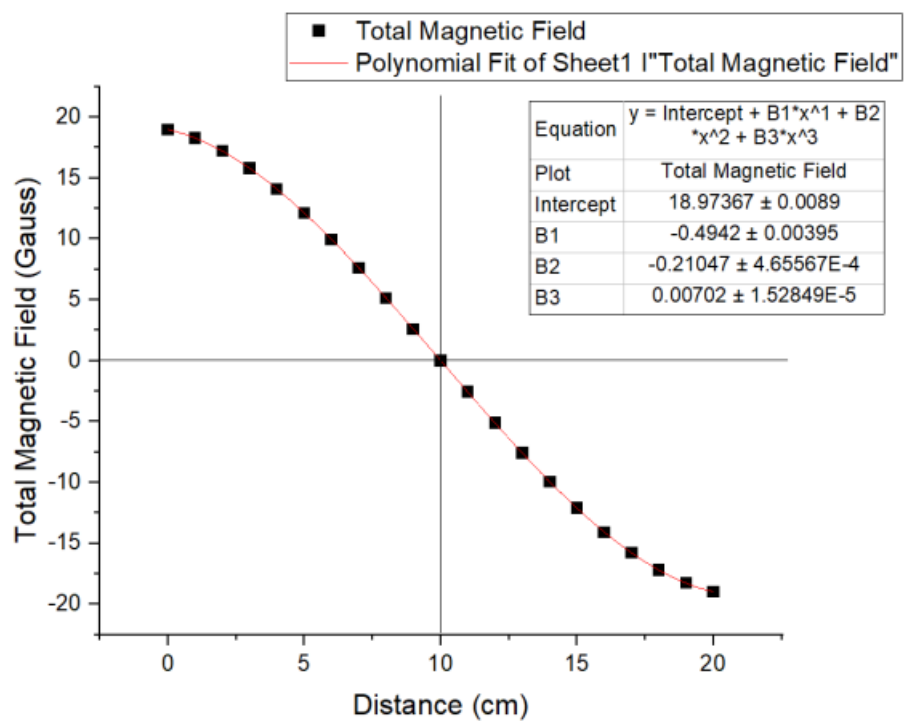


Figure 3: Graphing Theoretical Magnetic Field in Gauss against z distance in cm

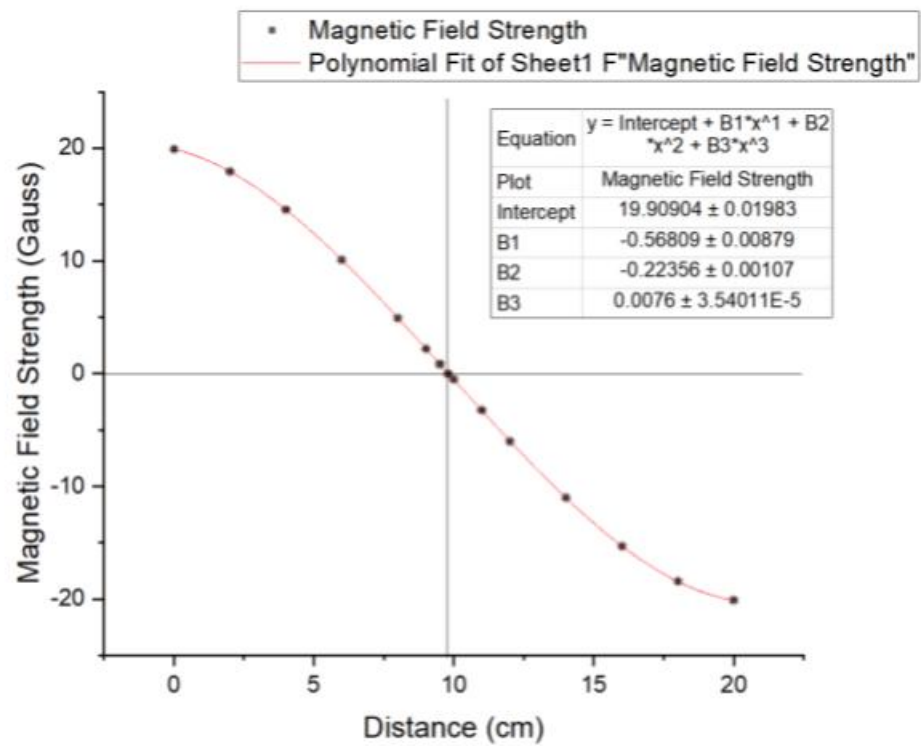


Figure 4: Graphing Experimental Magnetic Field in Gauss against z distance in cm

- Different values at 0cm and 20cm → This arises from the two solenoids generating different magnetic fields from the same current supply, as seen from **Figure 3**, a difference of 0.2 Gauss in magnitude can be observed, with the solenoid generating a more powerful magnetic field being the one located at 20cm. This may be for a variety of reasons such as different number of loops, higher density of loops or lower resistance.
- The experimental difference referenced above also affects a non 0 Gauss measurement at 10cm → Due to the difference in generated magnetic fields and the solenoid located at 20cm being stronger at the same current of 1.5A, instead of the magnitudes of magnetic field being equal at equal radius, they are instead different by 0.5 Gauss bias toward the solenoid located at 20cm. This means that the actual location of equilibrium of the magnetic fields is at 9.8cm instead of 10cm.
- Another inconsistency is in the absolute values when compared to theoretical values, most easily seen at 0cm and 20cm, where comparing theoretical against experimental yields differences from the expected 18.97 Gauss where the solenoids at 0cm generate a magnitude of 19.9 Gauss and at 20cm generate a magnitude of 20.1 Gauss. → If you consider an average of the two experimental values at 20 Gauss, this is roughly 1 Gauss higher than the expected theoretical value. While we are unsure as to what definitively caused this difference since 1 Gauss difference is much larger than the uncertainty maximum of  $\pm 0.11$  Gauss one can speculate that this difference is perhaps due to other factors such as greater unmeasurable inaccuracies in the hall probe or miscalculations in the hall voltage to gauss automated calculations not under our control.

Focusing now on the similarities;

- Experimental results yielded values and relationships extremely similar to theoretical values (**Figure 3**, **Figure 4**), both boast a negative cubic relationship formed from the difference of two  $\frac{1}{r^2}$  relationships.
- The polynomial curve fits of **Figure 3** and **Figure 4** have similar coefficients, the largest difference being the coefficient of x at 0.08, however this is only 6.625% of the average of the two coefficients. Considering other inaccuracies that cannot be measured due to our setup, this is a reasonable difference of fits such that their similarity can be noted.

## 2.2 Alternate Axis Measurements from a Single Solenoid

This experiment aims to prove that the Biot-Savart Law is valid for not only the z axis but for other axis as well. The setup for this can be seen in **Figure 2** where the hall probe will be varied along the y axis in order to measure the magnetic force generated by a single coil.

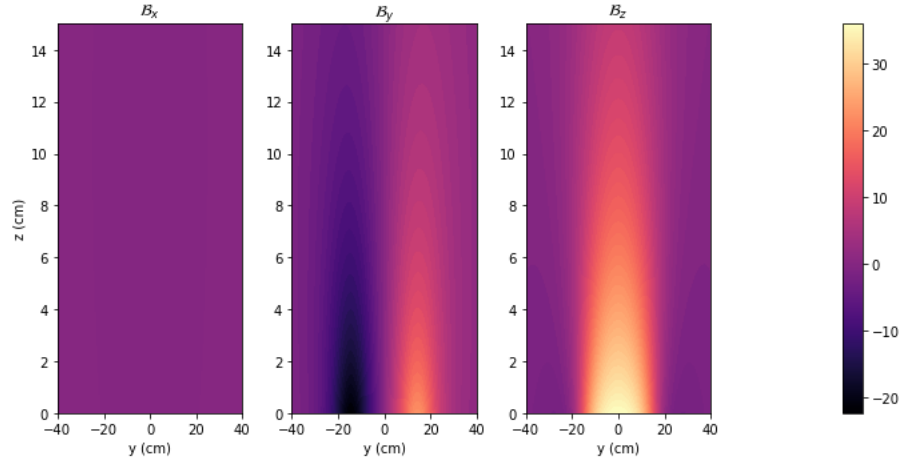


Figure 5: Heatmap of components of the theoretical magnetic field strength (Gauss) of a single coil in the y-z plane.

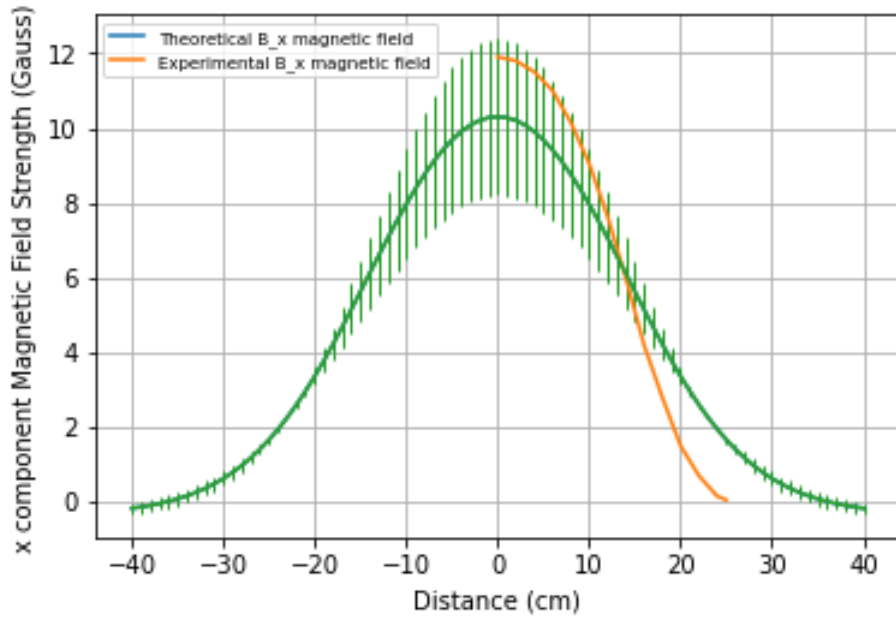


Figure 6: Plot of theoretical vs experimental values at  $z = 13$  cm, theoretical values have error bars of  $\pm 1$  cm in the z axis.

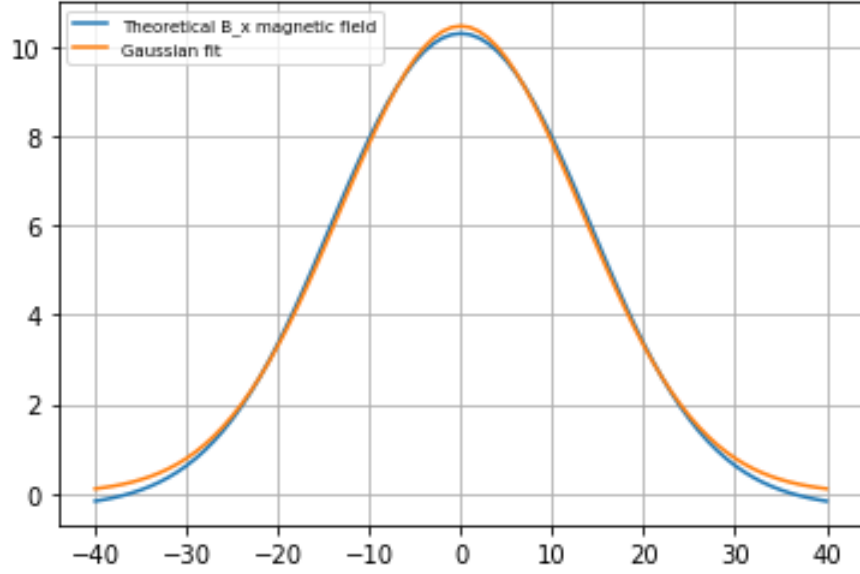


Figure 7: Theoretical values at  $z = 13\text{cm}$  plotted with a gaussian fit of the same data set.

### 2.2.1 Expected Results

Expected results for this experiment are slightly harder to pin down, as multiple values are now being varied. This is because while the increase in radius would suggest the magnitude will fall like  $\frac{1}{r^2}$  the magnetic flux now passes through the hall probe at angle  $\tan^{-1}(\frac{y}{+Cz})$ . So expected results seem likely to follow the inverse square law along with a correction with a  $\tan\theta$  factor to find the magnitude of components.

### 2.2.2 Observed Results

**Figure 5** shows a heat map across the  $z$ - $y$  plane with the intensity of the heat map being equal to the magnitude of the magnetic field in Gauss for all three components. These were calculated using python and matplotlib plotting libraries along with the Biot-Savart Law [Equation (1)].

Using this large matrix of values, any cross section can be taken to compare against experimentally found data.

This is what **Figure 6** shows, along with error bars generated from  $z = 13 \pm 1\text{cm}$  since the true position of the hall monitor within it's encasement is unknown and may vary by up to 2cm.

**Figure 6** shows a clear similarity between observed experimental values and theoretical ones with a similar relationship. Despite this some clear differences



are present such as the peak magnitude at 0cm and the rate of decrease in the magnetic field strength along with the point where 0 Gauss is reached. Even though there are some differences in the curves, the similarity in the relationship is shown as when the theoretical values are calculated closer to  $z = 14\text{cm}$ , the first half of the curves, up to around 15cm (incidentally, the radius of the solenoid is 15.15cm) where the two curves diverge as they approach 0. Some possibilities to the differences between experimental values and theoretical ones are as follows;

- Larger inaccuracies in experimental results than expected, possibly due to factors such as, the measurements not being precisely  $90^\circ$  to the  $z$  axis causing a greater drop at lower  $y$  distances as the true radius would be larger than expected.
- There is a possibility that the values of the properties of the coil provided are incorrect or inaccurate.
- Incorrect simulated theoretical results.

**Figure 7** shows the same theoretical values as in **Figure 6** in addition to a gaussian curve represented as;

$$X \sim \mathcal{N}(\mu, \sigma^2). \quad (3)$$

and calculated from;

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} \quad (4)$$

The theoretical values for the  $x$  component of magnetic field strength generated from a single coil, form a closely approximated gaussian distribution. Performing a  $\chi^2$  goodness of fit test using;

$$\chi^2 = \sum \frac{(O - E)^2}{E} \quad (5)$$

gives a result of  $\chi^2 \approx 0.73$  indicating that the distribution in **Figure 7** is indeed extremely close to a true gaussian distribution.

I believe this relationship to be formed due to the hall probes orientation. As the probe continues to increase its distance in the  $y$  direction, while maintaining the same orientation and  $z$  value, the angle between the radius vector and the  $z$  axis continues to increase at a decreasing rate. This will result in the rate of change of magnetic flux with respect to distance to decrease for larger values of  $y$ .

### 3 Conclusions

Following the aims of the experiments, the Helmholtz Pair acted as expected, following expected theoretical values in relationship very closely with a maximum

deviation of 6.625% from theoretical values. Further experiments to extend this would be along the lines of investigating at different coil separation distances along with varying the current in one or both of the coils.

Using the results from the Alternate Axis Measurements from a Single Solenoid experiment. We can conclude that the Biot-Savart Law is valid for all axis relative to a solenoid of origin. This is because, the relationship shown experimentally matches that which was calculated from the Biot-Savart Law.

In addition, it was shown that along the y axis the magnetic field varies as gaussian distribution with a  $\chi^2 \approx 0.73$  when compared to a true gaussian distribution.

## Index

### Figure 1 & 2:

Yousad, Adnan & Khan, Fatimah & Reindl, Leo. (2011). Wireless sensing of open loop micro inductors using Helmholtz coil. International Journal on Smart Sensing and Intelligent Systems. 4. 527-546. 10.21307/ijssis-2017-455

### Figure 5:

Elliott Ashby. (2021). Biot-Savart python library rewritten for python 3.9.9, Plotting from matplotlib