Programming and Data Analysis Assignment 2: The Hubble Constant

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1 Step 1

In order to obtain the Cepheid period-luminosity (PL) relation, two constants, α and β needs to be determined to form the equation;

$$M = \alpha \log P + \beta \tag{1}$$

Where M is absolute magnitude and P is period.
Therefore, in order to find these two constants, a fit must be determined relating M and $\log P$ in the form of $y = mx + c$.

Finding $\log P$ is trivial, simply take logarithms (in this case of base 10) of all values of P.
Finding M is slightly more in depth however. In order to convert our measured values of $m_{Apparent}$ to $M_{Absolute}$.
To convert between absolute and apparent magnitudes we can use;

$$M = m - 5 \log d_{pc} + 5 - A \tag{2}$$

Where d_{pc} distance from Earth in parsecs and A is extinction.
While we have data for extinction values A, we don't have any values for distance from Earth d_{pc}. These can be derived using parallax of the measured stars;

$$d_{pc} = 1000/p_{mas} \tag{3}$$

Where p_{mas} is parallax of a star measured in milli-arcseconds.

Fig 1 shows relationship (1) along with propagated errors and a best fit. The fit has gradient $\alpha = -2.419 \pm 0.112$ and intercept $\beta = -1.550 \pm 0.104$

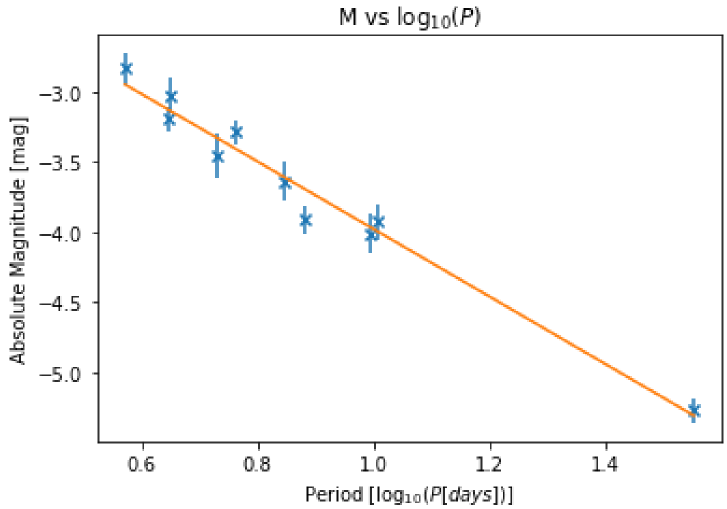

Figure 1: The relationship between Absolute Magnitude M and logarithmic Period $\log P$

Performing a χ^2 fit test on the data returns 10.66, with a χ^2_{red} of 1.18 which is acceptable as a good fit as it lies in $1 \pm \sqrt{\frac{2}{9}}$ This suggests that the α and β values obtained are valid for this data set.

2 Step 2

To calculate the distance of the nearby galaxy NGC4527 we can use our new equation with constants;

$$M = -2.419 \pm 0.112 \log P - 1.550 \pm 0.104 \tag{4}$$

along with (2) to find values of d_{pc} for all data on NGC4527.
In order to gain a more precise approximation of NGC4527's distance from Earth, the data for body C1-V11 was removed as a statistical outlier since it was more than two times the standard deviation away from the average.
Since $\log P$ is already known, corresponding values of M can be calculated. With these values and rearraging (2) to find d_{pc} with $A = 0.0682$ (The extinction of NGC4527), values for d_{pc} can be found for each data point in the galaxy of NGC4527.

Calculating an average from both the values and the errors on each value returns a distance from Earth to NGC4527 of $13.76 \pm 1.01 Mpc$.
This value however, is calculated from only one extinction value, in reality, each data point should have its own distinct extinction. This may skew the results in either positve or negative direction.

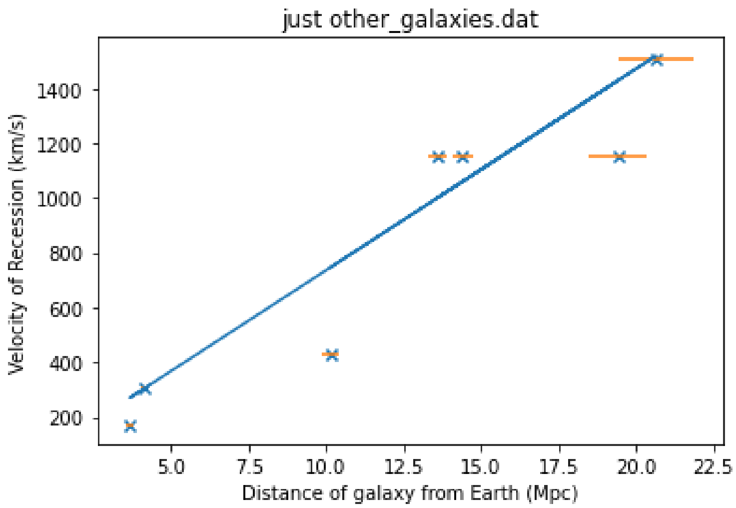

Figure 2: Recession Velocity (v_{rec} in km/s) against galaxy's distance from Earth (D_{gal} in Mpc) only including other_galaxies.dat

3 Step 3

In order to estimate the Hubble Constant and therefore the Expansion Rate of the Universe, we can use the simple relationship of Hubble's Law;

$$v_{rec} = H_0 D_{gal} \tag{5}$$

Where v_{rec} is the recession velocity of a given galaxy, H_0 is the Hubble Constant and D_{gal} is the distance from a given galaxy.

In the case of NGC4527, $v_{rec} = 1152kms^{-1}$. Given this, and using d_{pc} of NGC4527 from step 2 as D_{gal}, the Hubble Constant can be estimated to be $83.72 \pm 12.33kms^{-1}Mpc^{-1}$

This, of course, is a wildly inaccurate estimation due to the errors involved in it's calculation. To reduce these errors we can use more data to get a more acceptable Hubble Constant with a reduced error.

Finding a fit of $y = mx$, with y as v_{rec}, m as H_0 and x as D_{gal} returns H_0 as $73.63 \pm 0.0001518kms^{-1}Mpc^{-1}$ (see **Figure 2**)

Additionally, if we combine this data with our data on NGC4527 from step 2 we can gain greater accuracy. Performing the same fitting on the new data set (see **Figure 3**) shows a slightly shallower gradient and H_0 of $71.07 \pm 0.0257kms^{-1}Mpc^{-1}$.

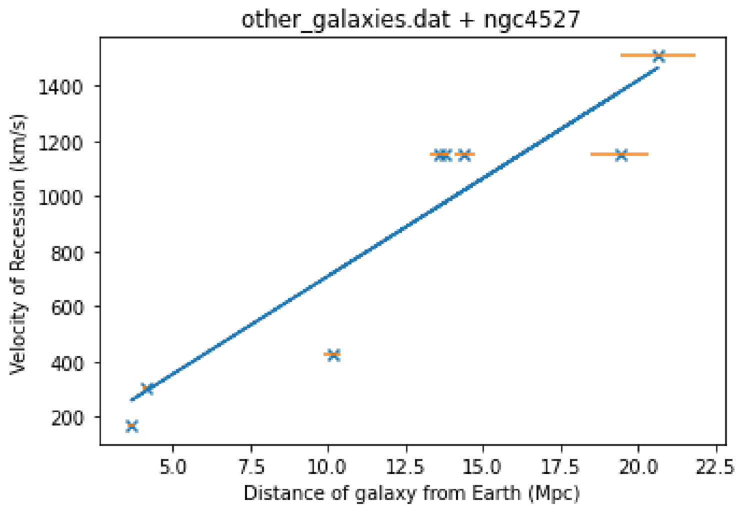

Figure 3: Recession Velocity (v_{rec} in km/s) against galaxy's distance from Earth (D_{gal} in Mpc) from other_galaxies.dat and data from NGC4527

Performing a χ^2 fit test on **Figure 3** data produces a χ^2 value of 429.3 with a χ^2_{red} of 61.33 While this seems large, it should be accepted since the data clearly shows the trend of a positive correlation. We can add an intrinsic dispersion to the error of the calculated velocities equal to 147. This reduces χ^2 to 7.029 and χ^2_{red} to 1.004. As this is within the range of $1 \pm \sqrt{\frac{2}{7}}$ we can accept this model.