Prelab 2

1) The continuous plant:

$$G(s)=\frac{K_a K_t K_e}{(J_e s+B_e)s}$$

$$G(s)=\frac{K_a K_t K_e}{J_e}\left(\frac{1}{\left(s+\frac{B_e}{J_e}\right)s}\right)$$

Then the discrete version is:

$$G(z)=\frac{K_a K_t K_e}{J_e} Z\left\{\frac{1}{s^2\left(s+\frac{B_e}{J_e}\right)}\right\}(1-z^{-1})K_d$$

$$=\frac{K_a K_t K_e}{J_e}(1-z^{-1})\, Z\left(\frac{J_e}{B_e}\frac{1}{s^2}+\frac{-J_e^2}{B_e^2}\frac{1}{s}+\frac{J_e^2}{B_e^2}\frac{1}{s+\frac{B_e}{J_e}}\right)K_d$$

$$=\frac{K_a K_t K_e}{J_e}(1-z^{-1})\left(\frac{J_e}{B_e}\frac{Tz^{-1}}{(1-z^{-1})^2}-\left(\frac{J_e}{B_e}\right)^2\frac{1}{(1-z^{-1})}+\left(\frac{J_e}{B_e}\right)^2\frac{1}{1-z^{-1}e^{-\frac{B_e}{J_e}T}}\right)K_d$$

plug in number:

$$=\boxed{\frac{5.8017E\text{-}5-5.805E\text{-}5\,z^{-1}}{1-1.998253z^{-1}+0.998253z^{-2}}K_d}$$

MATLAB has similar result

```
ans =

  5.805e-05 z + 5.801e-05
  -----------------------
  z^2 - 1.998 z + 0.9983
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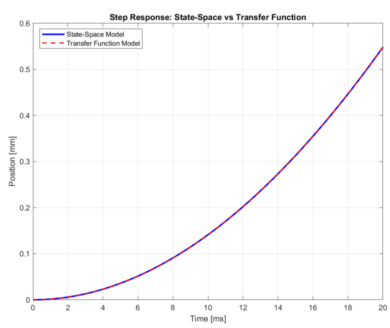



2) $2^{nd}$ older system, let's choose $\omega$ and $x_a$

$$\begin{bmatrix}\dot{\omega}\\ \dot{x}_a\end{bmatrix}=A\underbrace{\begin{bmatrix}\omega\\ x_a\end{bmatrix}}_{x}+B\underbrace{\begin{bmatrix}V_{in}\\ T_d\end{bmatrix}}_{u}$$

$$x_a=C\,x+Du$$

$$X_a = C\,x$$

$$J_e\dot{\omega}(t) + B_e\,\omega(t) = K_a K_t V_{in} - T_d$$

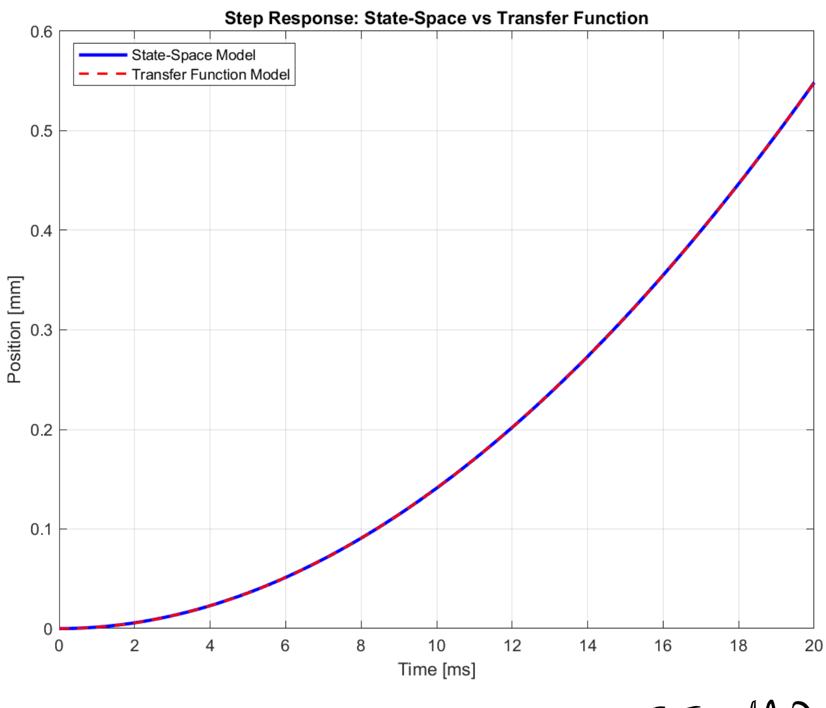
$$\dot{\omega}(t) = \frac{K_a K_t}{J_e} V_{in} - \frac{1}{J_e} T_d - \frac{B_e}{J_e}\omega$$

$$\dot{X}_a = K_e\,\omega$$

$$\begin{bmatrix}\dot{\omega}\\ \dot{X}_a\end{bmatrix} = \begin{bmatrix}-\frac{B_e}{J_e} & 0\\ K_e & 0\end{bmatrix}\begin{bmatrix}\omega\\ X_a\end{bmatrix} + \begin{bmatrix}\frac{K_a K_t}{J_e} & -\frac{1}{J_e}\\ 0 & 0\end{bmatrix}\begin{bmatrix}V_{in}\\ T_d\end{bmatrix}$$

$$X_a = \begin{bmatrix}0 & 1\end{bmatrix}\begin{bmatrix}\omega\\ X_a\end{bmatrix}$$

The following is the MATLAB result




Both the state space and TF Model are close

3)

a) The equation is:

$$G_c = \frac{K_p G_o}{1 + K_p G_o}$$

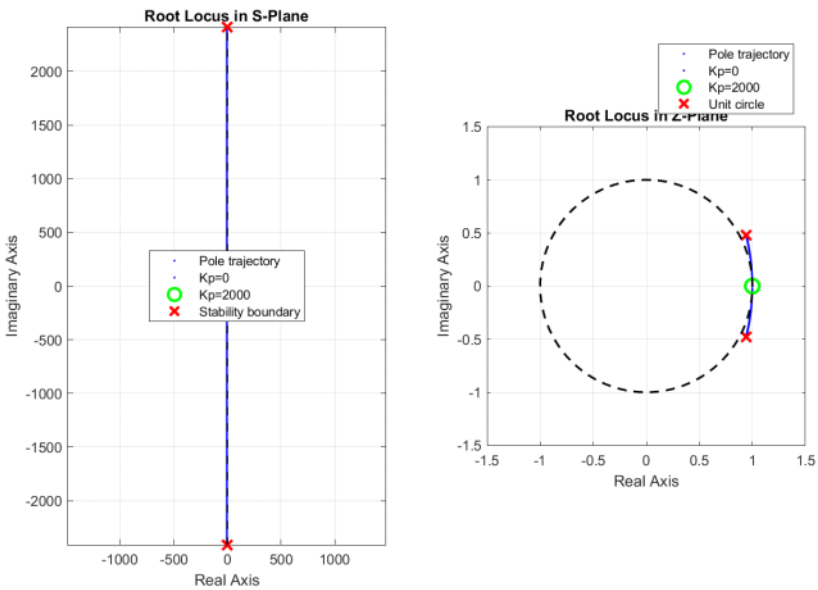
...lves for the root

$G_c = \dfrac{}{1+K_pG_0}$

So the root locus solves for the root

$1+K_pG_0 = 0$

Below are the figure from matlab:




In continuous domain, the poles begin in the negative real axis with kp=0, and as kp increases our poles begin to go to the centroid of the poles. Then after they meet, they will begin to diverge to infinite in the imaginary axis. If you look at this graph, the system is stable (goes to 0 as times goes to infinity). One thing to note is that if you keep increasing Kp, imaginary number increases, this means that there will be more oscillations, which is what we expect.

For the z plane, the root goes around the circle, and if Kp is high enough, we will see a pole that goes outside the unit circle. That means the system is unstable for high enough Kp, a very different behavior than the continuous system.

b) The bode plots are:

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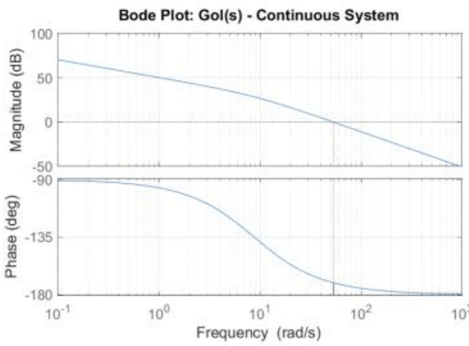
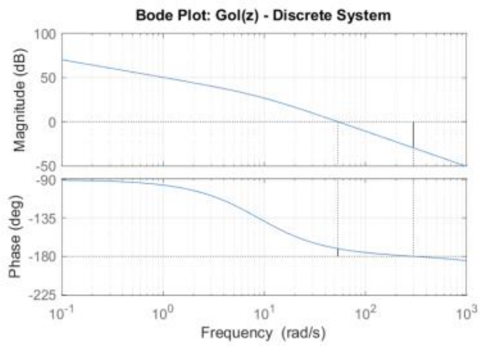


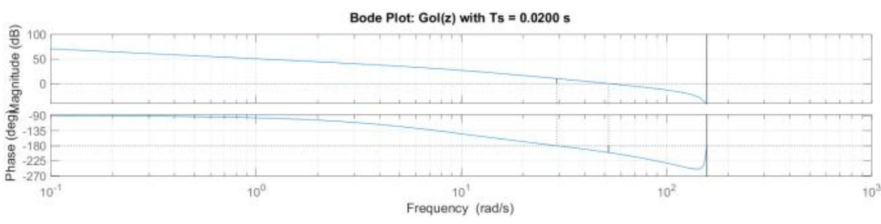



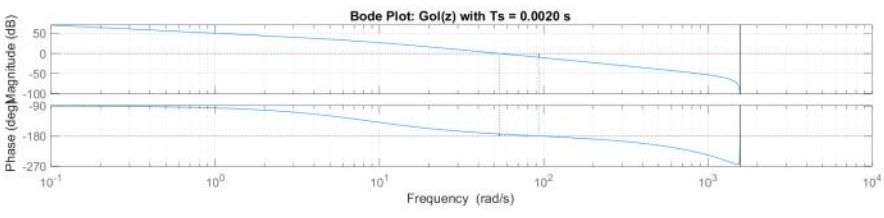

From ~~the~~ matlab command:

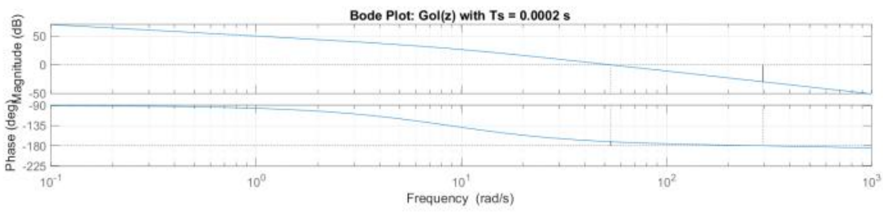
| | gain margin | Phase margin |
|---|---|---|
| continuous | $\infty$ | $9.28^\circ$ |
| discrete | 29.58 db | $8.97^\circ$ |

despite using a 2E-4 s, there is still a gain margin in discrete system, while it's infinite in continuous system. The phase margin is also different.

c)



As you can see from the bode plot, they even look like a different system, with different phase and gain margin. Note that for low sampling time, they get much more unstable at high frequency (makes sense, as the delay added by our discrete system will be much more noticeable if we give a high frequency input.

- Ts = 0.02 s: UNSTABLE (negative gain margin)
- Ts = 0.002 s: Marginally stable (9.60 dB gain margin)
- Ts = 0.0002 s: Most stable (29.58 dB gain margin)

The 0.02s system is even unstable.

4) $$G_{CL} = \frac{K_p G_0}{1 + K_p G_0}$$

$$1 + K_p G_0 = 0$$

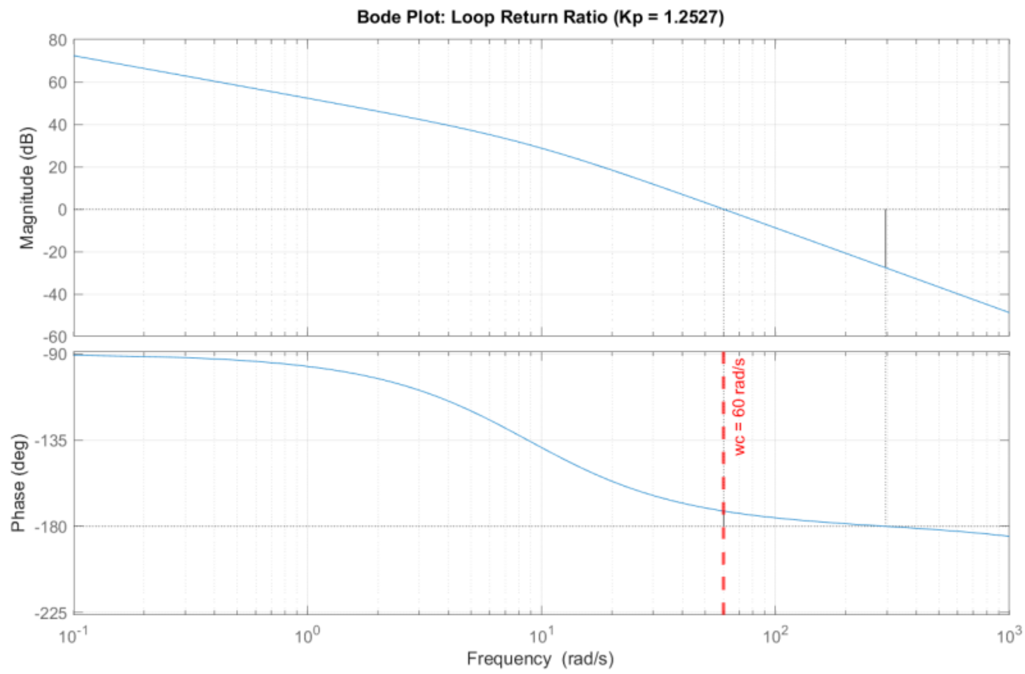
$$K_p = \frac{-1}{G_0}$$

So we just plug in $G_0(60j)$, and get the magnitude

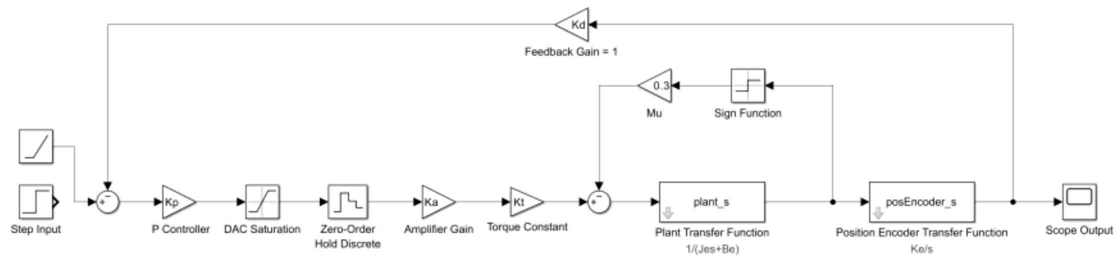
$$K_p = \left| \frac{1}{G_0} \right|$$

$$\boxed{K_p = 1.2527 \ \frac{V}{mm}}$$

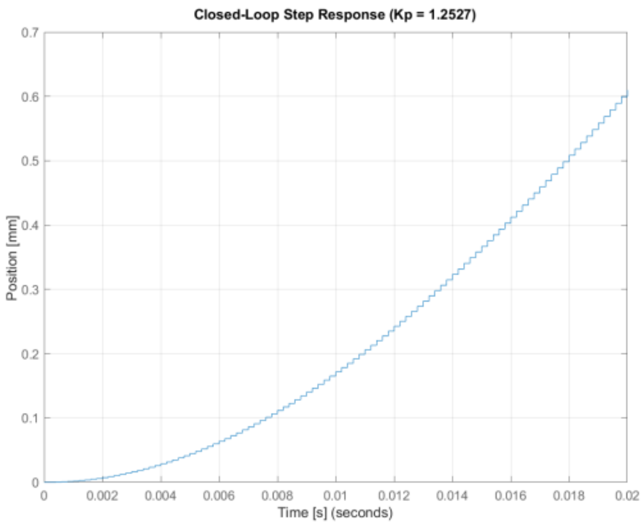
Here is the bode plot:

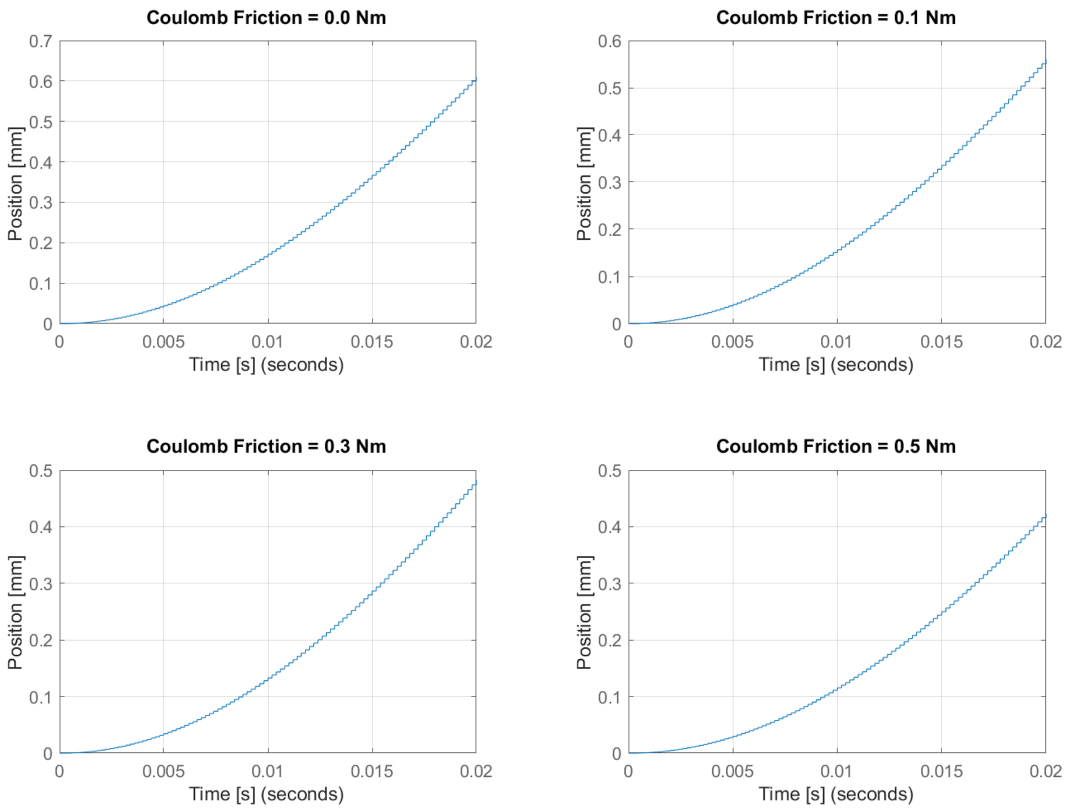


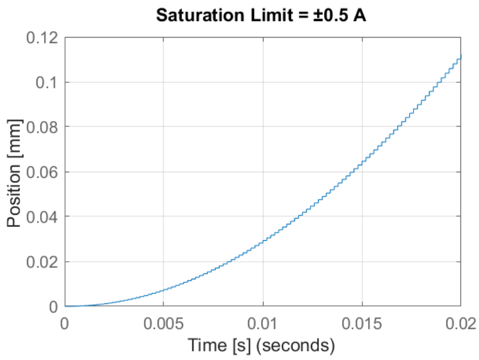

Below is the simulink model:

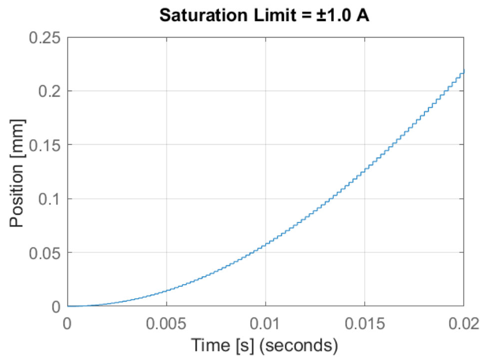


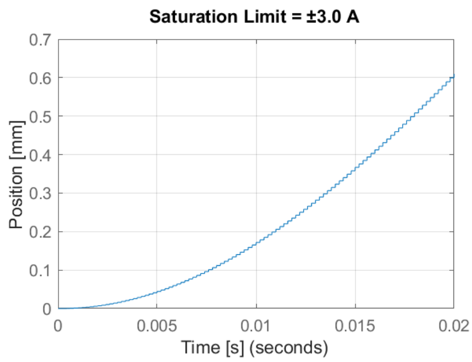

And the resulting graph:


Closed-Loop Step Response (Kp = 1.2527)
Position [mm]
Time [s] (seconds)


Coulomb Friction = 0.0 Nm
Coulomb Friction = 0.1 Nm
Coulomb Friction = 0.3 Nm
Coulomb Friction = 0.5 Nm
Position [mm]
Time [s] (seconds)


Saturation Limit = ±0.5 A
Position [mm]
Time [s] (seconds)
0.12
0.1
0.08
0.06
0.04
0.02
0
0.005
0.01
0.015
0.02


Saturation Limit = ±1.0 A
Position [mm]
Time [s] (seconds)
0.25
0.2
0.15
0.1
0.05
0
0.005
0.01
0.015
0.02


Saturation Limit = ±3.0 A
Position [mm]
Time [s] (seconds)
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0.005
0.01
0.015
0.02