We saw that Venn Diagram and truth tables are very cumbersome and clunky ways to work with.

➔ Another problem: Truth tables only show validity. They do not prove validity.

Truth tables can show that an argument is valid or not.

But why is the argument valid?

Logic

Syllogistic

What is a valid argument?

The one in which the conclusion is incontrovertibly supported by the premises. If premises are assumed to be true, then conclusion has to be true.

So which logical connective do you think exists between premises and conclusion in an argument?

Conditional.

premises→conclusion

Premises imply conclusion.

A valid argument has this condition:

- when premises are true the conclusion is also true

BUT... premises can be made up of multiple propositions. What logical connective should there be between these propositions? We need a connective that results in 'true' only when all of its propositions are 'true'.

How do we test the validity of an argument?

1. Venn Diagrams = Draw the circles, shade them, put x-mark.

messy   no member   at least 1 member

2. Truth tables = Fill up the rows and columns with all the possible scenarios of truth values.

2^n, verbose

Premise 1 = $A \vee (B \rightarrow D)$

2 = $\sim C \rightarrow (D \rightarrow E)$

3 = $A \rightarrow C$

Conclusion = $B \rightarrow E$

$B \rightarrow D$

$D \rightarrow E$

C $D \rightarrow E$ false

$B \rightarrow D$ false

Recall what we learnt about validity. If an argument is valid, the conjunction of the premises will imply the conclusion. Therefore, we can simplify the argument as:

(1) Modus Ponens (MP)

$p \to q$
p
$\therefore q$

(2) Modus Tollens (MT)

$p \to q$
$\sim q$
$\therefore \sim p$

[since $p \to q$ is a kinda strong relation, MT can rule out stuff]

(3) Hypothetical syllogism (HS)

$p \to q$
$q \to r$
$\therefore p \to r$

(4) Disjunctive syllogism (DS)

$p \vee q$
$\sim p$
$\therefore q$

(5) Constructive dilemma (CD)

$(p \to q) * (r \to s)$
$p \vee r$
$\therefore q \vee s$

Dilemma
$(a \to c) * (b \to c)$
either a or b T
regardless, c is true

(6) Destructive dilemma (DD)

$(p \to q) * (r \to s)$
$\sim q \vee \sim s$
$\therefore \sim p \vee \sim r$

4. Disjunctive syllogism (DS)

p v q

~p

therefore q

5. Constructive dilemma (CD)

(p→q) * (r→s)

p v r

therefore q v s

6. Destructive Dilemma (DD)

(p→q) * (r→s)

~q v ~s

therefore ~p v ~r

7. Simplification (Simp)

p*q

therefore p

8. Conjunction (Conj.)

p

q

therefore p*q

9. Addition (Add)

p

therefore p v q

We saw that Venn Diagram and truth tables are very cumbersome and clunky ways to work with.

→ Another problem: Truth tables only show validity. They do not prove validity.

Truth tables can show that an argument is valid or not.

But why is the argument valid?

Can we show it step-by-step?

Can we derive the validity?

Can we prove the validity?

Yes. Through formal proof of validity.

Burden of Proof?

Formal proof of validity is a step-by-step demonstration of the validity of an argument.

1. We start with the premises, and assume them to be true.
2. On the basis of the premises, we derive the next statement, which has to be true if the premises were true.
3. On the basis of this step, we derive the next statement, which has to be true if the premises and the previous step was true.
4. And we repeat step 3 until we arrive at the statement of conclusion.

If the premises are true, then you can be sure that the intermediate steps were true, and hence the conclusion that you arrived at the end would also be true. Thus, you establish the validity of the conclusion (i.e. conclusion shall be true if the premises were true).

This whole chain of deducing one step from the previous step, and thus arriving at the conclusion, that's called the formal proof of validity.

FORMAL PROOF = (i) There is a proper procedure and the rules have to be followed. (ii) the validity is proven through the form (and not the content).

$A \vee (B \to D)$

$\sim C \to (D \to E)$

$A \to C$

$\sim C$

$B \to E$

1) $A \vee (B \to D)$
2) $\sim C \to (D \to E)$
3) $A \to C$
4) $\sim C$

(5) $\sim A$ (3,4 MT)

(6) $D \to E$ (2,4 MP)

(7) $B \to D$ (1,5 DS)

(8) $B \to E$ (7,6 HS)