$$i\hbar \frac{\partial}{\partial t}|\psi\rangle = H\psi$$

$H(x, P)$ independent of t

$$|\psi(t)\rangle = e^{\frac{iHt}{\hbar}} \text{ state } |\psi\rangle$$

$$H|\psi\rangle = E|\psi\rangle$$

solve for psi

$$\langle\psi|x|\psi\rangle = [\quad]_{1\times n}[\quad]_{n\times n}[\quad]_{n\times 1} = \#$$

Prob dist function

$$f(x) = \int_a^b \psi\, x\psi\, dx$$

$$\langle\psi|\psi\rangle = \int_a^b x f(x)\, dx$$

$$\sqrt{\langle x^2\rangle - \langle x\rangle^2} = SD(x)$$

$$\langle x|P|\psi\rangle \qquad \begin{bmatrix} x \\ y \end{bmatrix} = x\begin{bmatrix} 1 \\ 0 \end{bmatrix} + y\begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$\langle\psi| = [1\ 0]x^* + [0\ 1]y^*$$

$$\langle\psi|P|\psi\rangle = [x^*\ y^*]\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}$$

$$= x^*y + yx^*$$

$$|\psi_{HO}\rangle = \sqrt{\ } e^{-x^2} \qquad \langle\psi|x|\psi\rangle = \int_{\infty}^{-\infty} x e^{-x^2} dx = 0$$

$$\langle \psi | x^2 | \psi \rangle = \int_{-\infty}^{\infty} x^2 e^{-x^2} dx$$

Measurement : ~~$\langle \psi |$~~ $a|\uparrow\rangle + b|\downarrow\rangle$

↳ Calculate Expectation value of $\hat{a}$ or $\hat{b}$

Probability of $\boxed{\uparrow = a^2}$ $\boxed{\downarrow = b^2}$

Otherwise state is an LC of $a|\uparrow\rangle$ & $b|\downarrow\rangle$

Entanglement
Two partics should exist

$|\psi_A\rangle$ & $|\psi_B\rangle$

$|\psi_{AB}\rangle = |\psi_A\rangle \otimes |\psi_B\rangle$ separable

$\neq |\psi_A\rangle \otimes |\psi_B\rangle$ entangled

$\#$ → rank 0 tensor

$[\ ]$ → rank 1 tensor

$[\ \ ]$ → rank 2 tensor

$\langle \psi | \psi \rangle = \#$

$[\ \ ]_{1\times n} \begin{bmatrix} n \\ 1 \end{bmatrix}_{n\times 1} = [\ ]_{1\times 1}$

↳ a scalar

$\vec{a} \cdot \vec{b}$ :: inner pdt

$\vec{a} \times \vec{b}$ :: outer pdt (but not the same)

↳ a vector

$|\psi \times \psi| = [\ ]_{n\times 1} [\ \ ]_{1\times n} = [\ \ ]_{n\times n}$

$$|\psi\rangle = \begin{bmatrix} x \\ y \end{bmatrix} \qquad \langle\psi| = \begin{bmatrix} x^* & y^* \end{bmatrix}$$

$$\langle\psi|\psi\rangle = \begin{bmatrix} x^* & y^* \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = |x|^2 + |y|^2$$

$$|\psi_{ket} \times \psi_{bra}| = \begin{bmatrix} x x^* & x y^* \\ x^* y & y^* x \end{bmatrix} = \begin{bmatrix} x \\ y \end{bmatrix}\begin{bmatrix} x^* & y^* \end{bmatrix}$$

$$\underset{2\times 2}{\begin{bmatrix} a & b \\ c & d \end{bmatrix}} \otimes \underset{2\times 2}{\begin{bmatrix} e & f \\ g & h \end{bmatrix}} = \underset{4\times 4}{\begin{bmatrix} a\begin{bmatrix} e & f \\ g & h \end{bmatrix} & b\begin{bmatrix} e & f \\ g & h \end{bmatrix} \\ c\begin{bmatrix} e & f \\ g & h \end{bmatrix} & d\begin{bmatrix} e & f \\ g & h \end{bmatrix} \end{bmatrix}}$$

$$= \underset{4\times 4}{\begin{bmatrix} ae & af & be & bf \\ ag & ah & bg & bh \\ ce & cf & de & df \\ cg & ch & dg & dh \end{bmatrix}}$$

$$B \otimes A \neq A \otimes B$$

$$\begin{bmatrix} 1 \\ 0 \end{bmatrix} \otimes \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1\begin{bmatrix} 0 \\ 1 \end{bmatrix} \\ 0\begin{bmatrix} 0 \\ 1 \end{bmatrix} \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 \\ 1 \end{bmatrix} \otimes \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0\begin{bmatrix} 1 \\ 0 \end{bmatrix} \\ 1\begin{bmatrix} 1 \\ 0 \end{bmatrix} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}$$