Density Matrix

$\rho = |\psi\rangle\langle\psi|$

$|\psi\rangle$ – no physical significance

$\langle x|\psi\rangle$ – particle state for

$\langle\psi|\psi'\rangle$ – physical state

This gives the following problem $|\psi\rangle$ and $e^{i\gamma}|\psi\rangle$

$e^{i\alpha}|\psi\rangle$ multiply $|\psi\rangle$ with any glob phase

$\langle\psi|\psi\rangle$ is unchanged

$|\psi\rangle, e^{i\gamma}|\psi\rangle, e^{i\alpha}|\psi\rangle, e^{i\beta}|\psi\rangle \rightarrow$ same state

A vector has both phase & magnitude

Several copies of Same state

Density matrix

$$\rho = |\psi\rangle\langle\psi| \qquad |\psi\rangle = \begin{bmatrix} \\ \\ \end{bmatrix}_{n\times 1}$$

$$= \begin{bmatrix} & \\ & \end{bmatrix}_{n\times n} \qquad \langle\psi| = \begin{bmatrix} & & \end{bmatrix}_{1\times n}$$

$\Rightarrow \rho$ is a square matrix.

ρ properties

H, $|\psi\rangle$ is column vector

$\hookrightarrow$ are matrices

physical quantities/observables

$\rho = |\psi\rangle\langle\psi|$ (1) $\rho = \rho^{\dagger}$ (hermiticity)

(2)

observables

$\underline{\underline{\rho}}$ Properties:

H, $|\psi\rangle$

$\hookrightarrow$ are the matrices.

physical quant. as are

1). $\rho = \rho^{\dagger}$ hermiticity.

(2) $\mathrm{Tr}\,\rho = 1$

(3) $\rho \geq 0$

(4) $\rho^2 = \rho$ (sometimes)

(projection)

$\rho = |\psi\rangle\langle\psi|$

$|\psi\rangle$ is normalized.

$|\psi\rangle = \sum_n c_n |e_n\rangle$

$\langle\psi|\psi\rangle = 1.$

$\downarrow$

$\underline{|c_n|^2 = 1}.$

1) $\rho = \rho^{\dagger}$ hermiticity.

2) $\mathrm{Tr}\,\rho = 1$. nor.

3) $\rho \geq 0$. Pos.

4) $\rho^2 = \rho$. Projector

$$|\psi\rangle = \sum_n c_n |e_n\rangle$$

$$= c_0 |e_0\rangle + c_1 |e_1\rangle + \dots$$

$$c_n |e_n\rangle = \begin{pmatrix} c_0 \\ c_1 \\ c_2 \\ \vdots \\ c_n \end{pmatrix}$$

$$\langle\psi| = \sum_m \langle e_m| c_m^* = [c_0^* \; c_1^* \; \dots \; c_n^*].$$

$$\rho = |\psi\rangle\langle\psi| = \begin{bmatrix} c_0 \\ c_1 \\ c_2 \\ \vdots \\ c_n \end{bmatrix} [c_0^* \; c_1^* \; \dots \; c_n^*]$$

$$= \begin{bmatrix} |c_0|^2 & c_0 c_1^* & c_2 c_0^* & \\ c_1 c_0 & |c_1|^2 & c_2 c_1 & \\ c_2 c_0 & c_2 c_1 & c_2^2 & \\ \vdots & \vdots & \vdots & \ddots \\ c_n c_0 & & & c_n^2 \end{bmatrix}$$

$$\langle\psi|\psi\rangle = \sum_n \sum_m \langle e_n| c_n c_m^* |e_m\rangle.$$

$$= \sum_{nm} c_n c_m \langle e_n|e_m\rangle.$$

1 $n = m$

0 $m \neq n$

$$= \sum_n |c_n|^2 = 1.$$

$$\mathrm{Tr}\,\rho = \sum_n |c_n|^2 = 1.$$

[redacted]

$\rho^2 = \rho$ (not always true)

Quantum Computing

Pure state $\rightarrow \rho^2 = \rho$ | Thermodynamics system

Mixed state $\rightarrow \rho^2 \neq \rho$ | System + bath

$$\mathcal{H}|\psi\rangle = \mathcal{E}|\psi\rangle$$

are pure states, unitary evolution

The real systems $(\mathcal{H}_S + \mathcal{H}_B)|\psi\rangle = \mathcal{E}|\psi\rangle$

$\downarrow$

Trace out $\leftarrow$ pure state

Bath vars

$\mathcal{H}_S \rightarrow$ mixed state

[no clue lol]

$$i\frac{d}{dt}|\psi\rangle = \mathcal{H}|\psi\rangle$$

$$-i\hbar\frac{\partial}{\partial t}\langle\psi| = \langle\psi|\mathcal{H}$$

$$\rho = |\psi\rangle\langle\psi| \qquad i\hbar\frac{\partial\rho}{\partial t}$$

$$= i\hbar\left[\left(\frac{\partial}{\partial t}|\psi\rangle\right)\langle\psi| + |\psi\rangle\frac{\partial}{\partial t}\langle\psi|\right]$$

$$= \mathcal{H}|\psi\rangle\langle\psi| - |\psi\rangle\langle\psi|\mathcal{H} = \mathcal{H}\rho - \rho\mathcal{H}$$

$$\Rightarrow i\hbar\frac{\partial\rho}{\partial t} = [\mathcal{H}, \rho]$$