# Discrete case

$$\max\left(-\sum_{i=1}^{n} p_i \log p_i\right)$$

Subject $\sum_{i=1}^{n} p_i = 1$

& $p_i \geq 0$

$\rightarrow p_i = \frac{1}{n} \forall i$

$$\max\left(-\sum_{i=1}^{n} p_i \log p_i\right) + \lambda\left(\sum_{i=1}^{n} p_i - 1\right)$$

$$H(x) = -\sum_{i=1}^{n} \frac{1}{n} \log \frac{1}{n} = -\frac{1}{n} \sum_{i=1}^{n} \log \frac{1}{n}$$

entropy (for a uniform distribution) $= + \log n$

$x \sim p(x)$

## Continuous random var $H(x)$

$H(x) = -\sum p_i \log p_i$ for discrete

$p(x) \geq 0 \qquad \int_D p(x) = 1$

$$H(x) = -\int_D p(x) \log(p(x))\, dx$$

$x \sim p(x)$

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$p(x) \geq 0$

$\int_{-\infty}^{\infty} p(x) = 1 \qquad E(x) = \mu \qquad Var(x) = \sigma^2$

$$\max -\int p(x) \log(p(x))\, dx$$

Such that

$$\int_{-\infty}^{\infty} p(x)\,dx = 1$$

$$\int x\,p(x)\,dx = \mu$$

$$\int (x-\mu)^2 p(x)\,dx = \sigma^2$$

find the pdf WITH THE MEAN & VARIANCE

You need to find a function! instead of $n$ real nos.

$$\tilde{p} \quad -\int p(x)\log(p(x))\,dx + \lambda_1\left(\int p(x)\,dx - 1\right) + \lambda_2\left(\int x\,p(x)\,dx - \mu\right) + \lambda_3\left(\int (x-\mu)^2 p(x)\,dx - \sigma^2\right)$$

solve using Calculation of variations!

This gives $N(x/\mu, \sigma^2)$

$$p(x) = \frac{1}{\sqrt{2\pi}\,\sigma} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

That's why normal distribution!

$$H(x) = -\int_D \left(\frac{1}{\sqrt{2\pi}\,\sigma} e^{\frac{-(x-\mu)^2}{2\sigma^2}}\right)\left(\log\frac{1}{\sqrt{2\pi}\,\sigma^2}\right) + \left(-\frac{(x-\mu)^2}{2\sigma^2}\right)dx$$

$$= \frac{1}{2}\left\{1 + \log(2\pi\sigma^2)\right\}$$