$$U^\dagger U = 1$$

$$\begin{bmatrix} a^* & c^* \\ b^* & d^* \end{bmatrix} \begin{bmatrix} a & c \\ b & d \end{bmatrix} = I$$

$$|a|^2 + |c|^2 = 1$$

$$a = e^{-ia'} \cos\frac{\gamma}{2} \qquad a^* = e^{ia'} \cos\frac{\gamma}{2}$$

$$|c|^2 = 1 + |a|^2 = 1 - \cos^2\frac{\gamma}{2} = \sin^2\frac{\gamma}{2}$$

$$c = e^{-ic'} \sin^2\frac{\gamma}{2}$$

$$a^*a + c^*c = 1$$
$$b^*b + d^*d = 1$$
$$a^*b + c^*d = 0$$
$$b^*a + d^*c = 0$$

$$\left(e^{ia} c \cos\frac{\gamma}{2}\right) b + \left(e^{ic'} \sin\frac{\gamma}{2}\right) d = 0$$

$$b^* \left(e^{-ia} \cos\frac{\gamma}{2}\right) + d^* \left(e^{-ic'} \sin\frac{\gamma}{2}\right) = 0$$

$$|b|^2 + |d|^2 = 1$$

$$\Rightarrow \quad b = -e^{-ib'} \sin\frac{\gamma}{2} \qquad d = e^{-id'} \cos\frac{\gamma}{2}$$

$$|b|^2 + |d|^2 = 1$$

$$(b^*a + d^*c = 0)$$

$$-e^{i(a'-b')} \sin\frac{\gamma}{2} \cos\frac{\gamma}{2} + e^{i(c'-d')} \cos\frac{\gamma}{2} \sin\frac{\gamma}{2} = 0$$

$$-e^{i(b'-a')} \sin\frac{\gamma}{2} \cos\frac{\gamma}{2} + e^{i(d'-c')} \cos\frac{\gamma}{2} \sin\frac{\gamma}{2} = 0$$

(We are assuming same $\frac{\gamma}{2}$ as these equations are interdependent,
If we choose diff gamma then you'll get them to be equal anyway

$$e^{i(a'-b')} \sin\frac{\gamma}{2}\cos\frac{\gamma}{2} = e^{i(c'-d')}\cos\frac{\gamma}{2}\sin\frac{\gamma}{2}$$

$$e^{i(a'-b')} = e^{i(c'-d')}$$

$$a'-b'-c' = -d' \qquad (i)\ e^{i(a'-b'-c'+d')} = 1$$

$$e^{i(b'-a'-d'+c')} = 1$$

Lot of solutions. People chose:

$$a = \left(\frac{-\delta-\beta}{2}\right) - \alpha$$

$$b = \left(\frac{+\delta-\beta}{2}\right) - \alpha$$

$$c = \frac{-\delta+\beta}{2} - \alpha$$

$$d = \frac{\delta+\beta}{2} - \alpha$$

$$U = e^{-i\alpha}\begin{bmatrix} e^{-i\frac{\delta}{2}} e^{-i\frac{\beta}{2}} \cos\frac{\gamma}{2} & -e^{+i\frac{\delta}{2}} e^{-i\frac{\beta}{2}} \sin\frac{\gamma}{2} \\ e^{-i\frac{\delta}{2}} e^{+i\frac{\beta}{2}} \sin\frac{\gamma}{2} & e^{i\frac{\delta}{2}} e^{i\frac{\beta}{2}} \cos\frac{\gamma}{2} \end{bmatrix}$$

$$e^{-i\alpha}\begin{bmatrix} e^{-i\beta/2} & 0 \\ 0 & e^{i\beta/2} \end{bmatrix}\begin{bmatrix} e^{-i\delta/2}\cos\frac{\gamma}{2} & -e^{i\delta/2}\sin\frac{\gamma}{2} \\ e^{-i\delta/2}\sin\frac{\gamma}{2} & e^{i\delta/2}\cos\frac{\gamma}{2} \end{bmatrix}$$

$$e^{-i\alpha}\begin{bmatrix} e^{-i\beta/2} & 0 \\ 0 & e^{+i\beta/2} \end{bmatrix}\begin{bmatrix} \cos\frac{\gamma}{2} & -\sin\frac{\gamma}{2} \\ \sin\frac{\gamma}{2} & \cos\frac{\gamma}{2} \end{bmatrix}\begin{bmatrix} e^{-i\frac{\delta}{2}} & 0 \\ 0 & e^{i\delta/2} \end{bmatrix}$$

$$U = e^{-i\alpha} R_z(\beta) R_y(\gamma) R_z(\delta)$$

$$= e^{-i\alpha} R_z(\beta)\, R_y(\gamma)\, R_z(\delta)$$

$$\mathcal{H} \simeq \quad R_z\left(\tfrac{\pi}{2}\right) R_y\left(\tfrac{\pi}{2}\right) R_z\left(\tfrac{\pi}{2}\right)$$

$$\begin{pmatrix} e^{-i\pi/4} & 0 \\ 0 & e^{i\pi/4} \end{pmatrix} = e^{-i\pi/4} \begin{bmatrix} 1 & 0 \\ 0 & e^{i\pi/2} \end{bmatrix}$$

$$= e^{-i\pi/4} \begin{bmatrix} 1 & 0 \\ 0 & i \end{bmatrix}$$

$$R_y = \begin{bmatrix} 1 & -i \\ -i & 1 \end{bmatrix} \qquad \Rightarrow \mathcal{H} = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$

(multiply with $e^{-i\alpha}$)