Let's say

$|\chi\rangle = \alpha|0_3\rangle + \beta|1_3\rangle$ Quantum info is stored in qubits

State $|\chi\rangle$, Alice wants to teleport to Bob
Alice is in possession of the qubit $|\chi\rangle$ and her entangled state

$$|\psi\rangle = |\phi^+\rangle \otimes |\chi\rangle$$

$$= \frac{1}{\sqrt{2}}\left[|0_A 0_B\rangle + |1_A 1_B\rangle\right] \otimes \alpha|0_3\rangle + \beta|1_3\rangle$$

$$= \frac{1}{\sqrt{2}}\left[|0_A\rangle \otimes |0_B\rangle \otimes \alpha|0_3\rangle + |1_A\rangle \otimes |1_B\rangle \otimes \alpha|0_3\rangle\right.$$
$$\left. + |0_A\rangle \otimes |0_B\rangle \otimes \beta|1_3\rangle + |1_A\rangle \otimes |1_B\rangle \otimes \beta|1_3\rangle\right]$$

χ in terms of Alice & third state basis

$$|\psi\rangle = \left[\frac{1}{\sqrt{2}}|0_A\rangle \otimes |0_3\rangle \otimes \alpha|0_B\rangle\right.$$
$$\frac{1}{\sqrt{2}}|1_A\rangle \otimes |0_3\rangle \otimes \alpha|1_B\rangle$$
$$+ \frac{1}{\sqrt{2}}|0_A\rangle \otimes |1_3\rangle \otimes \beta|0_B\rangle$$
$$\left.+ \frac{1}{\sqrt{2}}|1_A\rangle \otimes |1_3\rangle \otimes \beta|1_B\rangle\right]$$

Bell Basis

$$|\phi^+\rangle = \frac{1}{\sqrt{2}}\left[|0\rangle_A \otimes |0\rangle_B + |1\rangle_A \otimes |1\rangle_B\right]$$

$$|\phi^-\rangle = \frac{1}{\sqrt{2}}\left[|0\rangle_A \otimes |0\rangle_B - |1\rangle_A \otimes |1\rangle_B\right]$$

$$|\psi^+\rangle = \frac{1}{\sqrt{2}}\left[|0\rangle_A \otimes |1\rangle_B + |1\rangle_A \otimes |0\rangle_B\right]$$

$$|\psi^-\rangle = \frac{1}{\sqrt{2}}\left[|0_A\rangle \otimes |1\rangle_B - |1_A\rangle \otimes |0_B\rangle\right]$$

$$|0_A\rangle \otimes |0_B\rangle = \frac{1}{\sqrt{2}}\left(|\phi^+\rangle + |\phi^-\rangle\right]$$

$$|1_A\rangle \otimes |1_B\rangle = \frac{1}{\sqrt{2}}\left[|\phi^+\rangle - |\phi^-\rangle\right]$$

$$|0_A\rangle \otimes |1_B\rangle = \frac{1}{\sqrt{2}}\left[|\psi^+\rangle + |\psi^-\rangle\right]$$

$$|1_A\rangle \otimes |0_B\rangle = \frac{1}{\sqrt{2}}\left[|\psi^+\rangle - |\psi^-\rangle\right]$$

$|\Psi\rangle$ in terms of Alice & third state basis

$$|\Psi\rangle = \Big[\frac{1}{\sqrt{2}}|0_A\rangle \otimes |0_3\rangle \otimes \alpha|0_B\rangle + \frac{1}{\sqrt{2}}|1_A\rangle \otimes |0_3\rangle \otimes \alpha|1_B\rangle + \frac{1}{\sqrt{2}}|0_A\rangle \otimes |1_3\rangle \otimes \beta|0_B\rangle + \frac{1}{\sqrt{2}}|1_A\rangle \otimes |1_3\rangle \otimes \beta|1_B\rangle\Big]$$

$$|\Psi\rangle = \frac{1}{\sqrt{2}}\big[|\phi^+\rangle + |\phi^-\rangle\big] \otimes \alpha|0_B\rangle + \frac{1}{\sqrt{2}}\big[|\psi^+\rangle - |\psi^-\rangle\big] \otimes \alpha|\cancel{0}_B\rangle + \frac{1}{\sqrt{2}}\big[|\psi^+\rangle + |\psi^-\rangle\big] \otimes \beta|0_B\rangle\big] + \frac{1}{\sqrt{2}}\big[|\phi^+\rangle - |\phi^-\rangle \otimes \beta|1_B\rangle\big]$$

$$|\chi\rangle = \frac{1}{\sqrt{2}}|\phi^+\rangle \otimes \alpha|0_B\rangle + \frac{1}{\sqrt{2}}|\phi^+\rangle \otimes \beta|1_B\rangle$$

$$+ \frac{1}{\sqrt{2}}|\phi^-\rangle \otimes \alpha|0_B\rangle - \frac{1}{\sqrt{2}}|\phi^-\rangle \otimes \beta|1_B\rangle$$

$$+ \frac{1}{2}|\chi^+\rangle \otimes \alpha|1_B\rangle + \frac{1}{\sqrt{2}}|\psi^+\rangle \otimes \beta|0_B\rangle$$

$$- \frac{1}{\sqrt{2}}|\psi^-\rangle \otimes \beta|0_B\rangle + \frac{1}{\sqrt{2}}|\chi^-\rangle \otimes \alpha|1_B\rangle$$

Alice, Bob & new spin State

$$|\psi\rangle = \frac{1}{\sqrt{2}}|\phi^+\rangle \otimes [\alpha|0_B\rangle + \beta|1_B\rangle]$$

$$+ \frac{1}{\sqrt{2}}|\phi^-\rangle \otimes [\alpha|0_B\rangle - \beta|1_B\rangle]$$

$$+ \frac{1}{\sqrt{2}}|\psi^+\rangle \otimes [\alpha|1_B\rangle + \beta|0_B\rangle]$$

$$+ \frac{1}{\sqrt{2}}|\chi^-\rangle \otimes [\alpha|1_B\rangle - \beta|0_B\rangle]$$

If alice gets: Bob operates:

Alice's measurement

$|\phi^+\rangle = 00$: $\mathbb{I}$

$|\phi^-\rangle = 10$: σ_x

$|\chi^+\rangle = 01$: $i\sigma_y$

$|\chi^-\rangle = 11$: σ_z

$$\boxed{\begin{array}{l} \mathbb{I} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \\ \sigma_x = \frac{1}{\sqrt{2}}\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \\ i\sigma_y = \frac{1}{\sqrt{2}}\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \\ \sigma_z = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \end{array}}$$

2 classical bits are sent to bob via a classical channel

$00 = \mathbb{I}|\chi_B\rangle$

$$10 : \sigma_z|\chi_B\rangle = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}\begin{bmatrix} \alpha \\ \beta \end{bmatrix} = \begin{bmatrix} \alpha \\ -\beta \end{bmatrix} = \alpha\begin{bmatrix} 1 \\ 0 \end{bmatrix} - \beta\begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$= \alpha|0_B\rangle - \beta|1_B\rangle$$

01

$$\sigma_x |\psi_B\rangle = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} \alpha \\ \beta \end{bmatrix} = \begin{bmatrix} \beta \\ \alpha \end{bmatrix}$$

$$= \beta \begin{bmatrix} 1 \\ 0 \end{bmatrix} + \alpha \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \beta|0_B\rangle + \alpha|1_B\rangle$$

11

$$i\sigma_y|\psi_B\rangle = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} \alpha \\ \beta \end{bmatrix} = \begin{bmatrix} -\beta \\ \alpha \end{bmatrix}$$

$$= -\beta \begin{bmatrix} 1 \\ 0 \end{bmatrix} + \alpha \begin{bmatrix} 0 \\ 1 \end{bmatrix}.$$

$$= \left[\alpha|1_B\rangle - \beta|0_B\rangle\right]$$

$\Rightarrow$ Bob reconstructs the state from Alice
But alice has to relay the info classically

Alice has a α/β flipped/ phase shifted /same state