Thm: The class of regular lang is closed wrt complement

$$L \subseteq \Sigma^* \qquad L^c = \Sigma^* \setminus L$$

Pf: Let L be a regular language accepted by a DFA we obtain L^c by interchanging the roles of final & non final states

we claim that $L(A') = L^c$ so that L^c is regular

For $x \in \Sigma^*$, $x \in L^c \iff x \notin L$

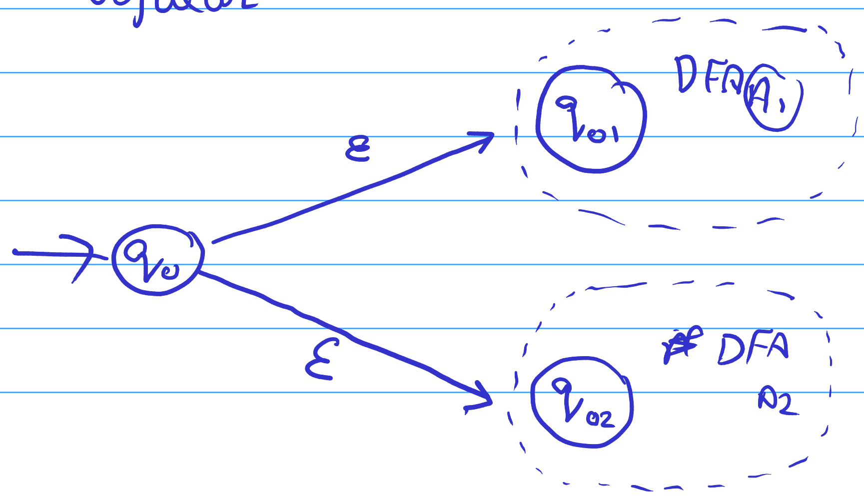
$\iff \hat{\delta}(q_0, x) \notin F$

$\iff \hat{\delta}(q_0, x) \in Q - F$

$\iff x \in L(A')$ $\square$

Corollary: The class of regular languages is closed with respect to intersection

Proof: If L_1 & L_2 are regular lang, and so are L_1^c & L_2^c. Their union is $L_1^c \cup L_2^c$ is also regular.



Hence from theorem $(L_1^c \cup L_2^c)^c$ is regular

But deMorgann's law $L_1 \cap L_2 = (L_1^c \cup L_2^c)^c$ $\square$

for $i=1,2$ let $A_i = (Q_i, \Sigma, \delta_i, F_i)$ be two DFA accepting L_i

$L(A_1) = L \quad L(A_2) = L_2$

if $A = (Q_1 \times Q_2, \Sigma, \delta(q_1, q_2), F_1 \times F_2)$

Where δ defined by

$$\delta((p,q), a) = (\delta_1(p,a), \delta_2(q,a))$$

for all $(p,q) \in Q_1 \times Q_2$

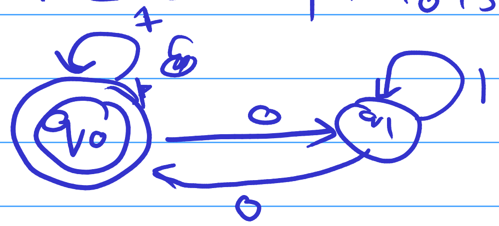
Claim is $L(A) = L_1 \cap L_2$

Using the construction given in the above proof we design a DFA that accepts the language

$L = \{ x \in (0+1)^* / |x|_0$ is even and $|x|_1$ is odd$\}$

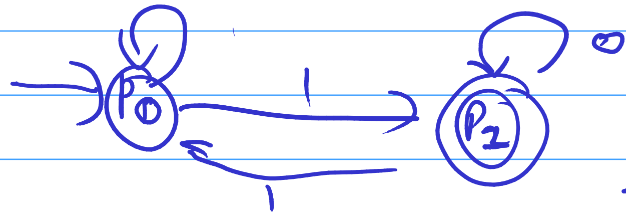
so that L is regular

$L_1 = \{ x \in (0+1)^* \mid |x|_0$ is even$\}$ # of zeros in x is even



$L_2 \{ x \in (0+1)^* \mid |x|_1 \text{ is odd} \}$

↳ # of 1s is odd



The product DFA that accepts $L_1 \cap L_2$ is the foll: