$$E(w_0, w_1) = \frac{1}{2}\sum_{n=1}^{N} \left((w_0 + w_1 x_n) - t_n \right)^2$$

$$\frac{\partial E}{\partial w_0} = \frac{1}{2} \sum_{n=1}^{N} 2\left((w_0 + w_1 x_n) - t_n\right)^2$$

$$\frac{\partial E}{\partial w_0} = \sum_{n=1}^{N} \left((w_0 + w_1 x_n) - t_n\right)$$

$$\frac{\partial E}{\partial w_1} = \sum_{n=1}^{n} \left((w_0 + w_1 x_n) - t_n\right) x_n$$

$$\frac{\partial^2 E}{\partial w_0^2} = \sum_{n=1}^{N} 1 = n \qquad \frac{\partial^2 E}{\partial w_1 \partial w_0} = \sum_{i=1}^{n} x_i$$

$$\frac{\partial^2 E}{\partial w_0 \partial w_1} = \sum_{i=1}^{n} x_i \qquad \frac{\partial^2 E}{\partial w_1^2} = \sum_{i=1}^{n} x_n^2$$

det. of leading principle minor

$$\begin{bmatrix} \boxed{\sum 1} & \sum x_n \\ \sum x_n & \sum x_n^2 \end{bmatrix} \rightarrow \text{positive semidefinite}$$

<u>determinant:</u> $n \sum x_n^2 - \left(\sum x_n\right)^2$

$$= (n-1) \sum x_n^2 - 2 \sum_{i=1} \sum_{j=i+1} x_i \cdot x_j$$

$$= \sum_{i=1} \sum_{j=i+1} (x_i - x_j)^2 \geq 0$$

$$\lambda_1 + \lambda_2 = \text{trace } N + \sum_{x=1}^{N} x_n^2 \geq 0$$

$\lambda_1 \lambda_2 \geq 0$
(det)

$\Rightarrow \lambda_1 \geq 0 \quad \lambda_2 \geq 0 \Rightarrow$ Eigenvalues are non negative

So the error function is convex for $w_0 + w_1 x$

$$\frac{\partial E}{\partial w_0} = 0 \Rightarrow \sum_{n=1}^{N} \left((w_0 + w_1 x_n) - t_n \right) = 0 \Rightarrow \left(\sum_{n=1}^{N} 1 \right) w_0 + \left(\sum_{x=1}^{N} x_n \right) w_1$$

$$= \sum_{n=1}^{N} t_n$$

$$= N w_0 + \left(\sum x_n \right) w_1 = \sum t_n$$

$$\frac{\partial E}{\partial w_1} = 0 \Rightarrow \sum \left((w_0 + w_1 x_n) - t_n \right) x_n = 0 \Rightarrow \sum t_n x_n = 0$$

$$\begin{bmatrix} N & \sum x_n \\ \sum x_n & \sum x_n^2 \end{bmatrix} \begin{bmatrix} w_0 \\ w_1 \end{bmatrix} = \begin{bmatrix} \sum t_n \\ \sum t_n x_n \end{bmatrix}$$

$Aw = b \Rightarrow w = A^{-1} b$

if you have A as 100×100

99 91 none wp

This process is called solving by normal equations.