

$$p(X=x_1) = \mu^{x_1} (1-\mu)^{n-x_1} \binom{n}{x_1}$$

Likelihood  $\mu^{x_1} (1-\mu)^{n-x_1} \times \mu^{x_2} (1-\mu)^{n-x_2} \times \dots \times \mu^{x_N} (1-\mu)^{n-x_N}$

$$L(\mu) = \mu^{x_1+x_2+\dots+x_N} (1-\mu)^{(1-x_1)(1-x_2)+\dots+(1-x_N)} = \mu^m (1-\mu)^{n-m}$$

$\max_{\mu} L(\mu)$        $m$  heads  $n-m$  tails

$$= \mu^m (1-\mu)^{n-m} \Rightarrow \max_{\mu} m \log \mu + (n-m) \log (1-\mu)$$

$$\frac{m}{\mu} - \frac{(n-m)}{1-\mu} = 0 \Rightarrow m(1-\mu) = \mu(n-m)$$

$$\Rightarrow m - \mu m = \mu n - \mu^2 m$$

$$\boxed{\mu = \frac{m}{n}}$$

$\mu$  makes the ' $m$ ' heads maximally happen

$\Rightarrow \mu = \text{max likelihood estimator}$

$\mu$  : probability of getting head  
one expt  $\Rightarrow$  ' $N$ '  
' $m$ '

$m = 0, 1, 2, \dots, N$

$$E(m) = \sum_{k=0}^N k \binom{N}{k} \mu^k (1-\mu)^{N-k}$$

$$P(m=k) = \mu^k (1-\mu)^{n-k} \binom{n}{k}$$

$$= N\mu$$

$$\text{Var}(m) = \sum_{k=0}^N \frac{(k-E(m))^2 \binom{N}{k} \mu^k (1-\mu)^{N-k}}{\mu^k (1-\mu)^{N-k}}$$

$$< N\mu(1-\mu)$$

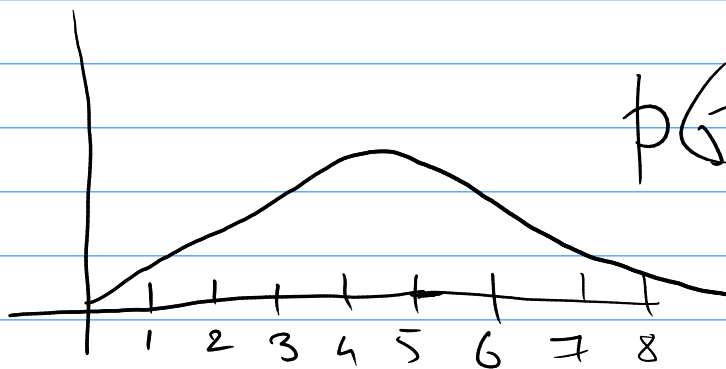
P1) for a continuous random var:

$P_x(x = 5.637219583 \dots)$   
doesn't make sense

Probability Density function (PDF)

$P: \mathcal{D} \rightarrow \mathbb{R}$  } only many

①  $p(x) \geq 0$     ②  $\int_{\mathcal{D}} p(x) dx = 1$



$$p(5.2 \leq x \leq 5.4) = \int_{5.2}^{5.4} p(x) dx$$

Pareto    Normal    beta    gamma

$$p(x | \mu, \sigma^2) = \frac{1}{\sqrt{2\pi} \sigma} e^{-\frac{(x-\mu)^2}{2\sigma^2}} \quad \left. \vphantom{\frac{1}{\sqrt{2\pi} \sigma}} \right\} \text{normal (gaussian)}$$

$$p(x) \geq 0$$

$$\int_{-\infty}^{\infty} p(x) dx = 1$$

