

Statement form: $p \rightarrow (q \vee r)$

Substitution instance: If the room is cold then either the window is open or the door is open.

Can be written as : If C then either W or D

Can be written as : $C \rightarrow (W \vee D)$

Here, $C \rightarrow (W \vee D)$ is the substitution instance of the statement form $p \rightarrow (q \vee r)$.

Statement form is like saying there is a relationship like father-son relationship. Substitution instance is giving a concrete example of such a relationship, like saying, "Mr. John and his son Mr. David", or "Mr. David and his son Mr. Peter". Form is a general format. Instance is a specific example.

Tautology = A statement form which has only true substitution instances.

E.g.: The following statement form is tautologous/tautology:

$p \vee \sim p$

because it is always true. Its truth does not depend on finding the truth value of 'p'. Its truth value is formally fixed (i.e. decided because of its form).

Now, all statements, which are of the form $(p \vee \sim p)$ shall be tautologous.

E.g.: John is angry or not angry. Mary wants to go home or does not want to go home.

We identify the $(p \vee \sim p)$ structure, and hence we can declare that the statements are tautologous.

Contradiction = A statement form which has only false substitution instances.

E.g.: The following statement form is contradictory/contradiction:

$p * \sim p$

because it is always false. Its truth does not depend on finding the truth value of 'p'. Its truth value is also formally fixed. *

Statements that are neither tautologous nor contradictory are called contingent/contingencies.

This is because their truth value is not formally fixed, but depends on what happens to be the case.

Are the following tautology or contingency?

1. $p \rightarrow (q \rightarrow p)$
2. $\sim p \rightarrow (p \rightarrow q)$

		Tautology	Contradiction
p	$\sim p$	$p \vee \sim p$	$p \wedge \sim p$
1	0	1	0
1	0	1	0
0	1	1	0
0	1	1	0

$p \rightarrow (q \rightarrow p)$ tautology why?