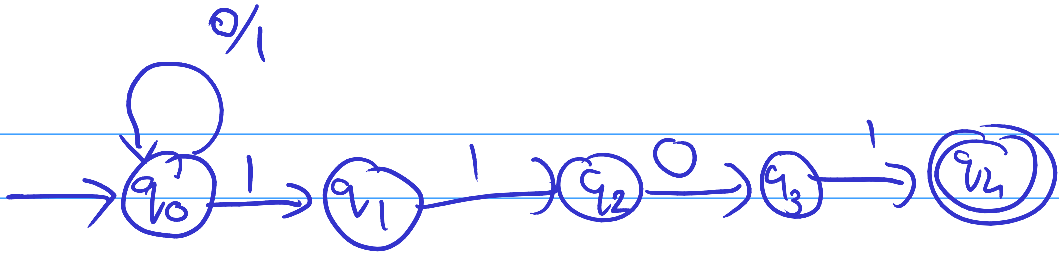
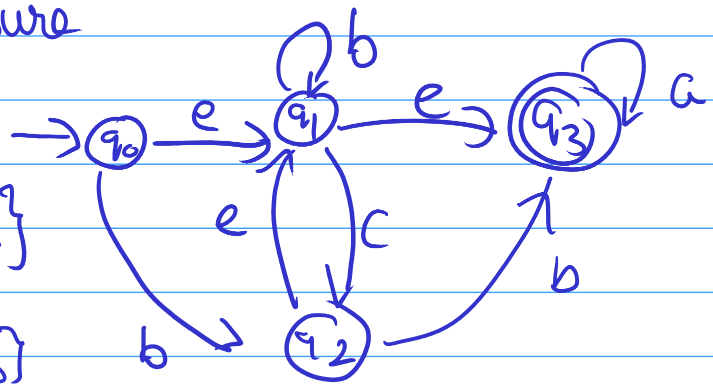


Subset construct method

		0	1	
	q_0	q_0	$[q_0\ q_1]$	
×	q_1	ϕ	q_2	q_1 not reachable
×	q_2	q_3	ϕ	$\Rightarrow q_2, q_3, q_4$ not reachable
×	q_3	ϕ	q_4	
×	q_4	ϕ	ϕ	
	$[q_0\ q_1]$	q_0	$[q_0\ q_1\ q_2]$	
	$[q_0\ q_1\ q_2]$	$[q_0, q_3]$	$[q_0\ q_1\ q_2]$	
	$[q_0\ q_3]$	$[q_0]$	$[q_0\ q_1\ q_4]$	
×	ϕ	ϕ	ϕ	
	$[q_0\ q_1\ q_4]$	q_0	$q_0\ q_1\ q_2$	

Epsilon closure

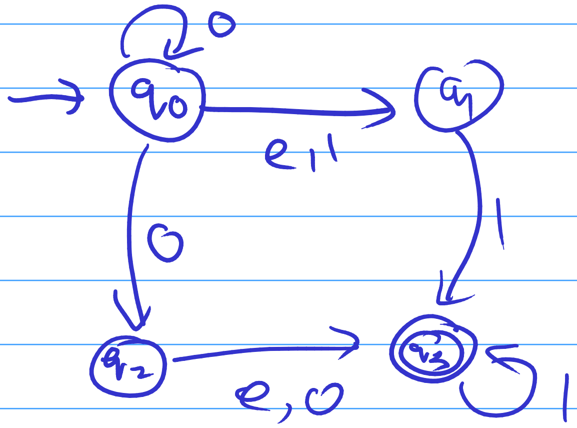


$E(q_0) = \{q_0\, q_1\, q_3\}$

$E(q_2) = \{q_1\, q_2\, q_3\}$

$E(q_1) = \{q_1\, q_3\}$ ⟩ consider these as new states

$E(q_3) = \{q_3\}$



$E(q_0)$

$= \{q_0, q_1\}$

$E(q_1) = \{q_1\}$

$E(q_2) = \{q_2, q_3\}$

$E(q_3) = \{q_3\}$

	0	1
$[q_0 q_1]$	$[E(q_0) \cup E(q_2)]$ $= [q_0 q_1, q_2 q_3]$	$E(q_1) \cup E(q_1) = [q_1 q_3]$
X q_1	ϕ	q_3
X $E(q_2) = [q_2\ q_3]$	q_3	q_3
q_3	ϕ	q_3
$[q_1\ q_3]$	ϕ	q_3
$[q_0, q_1, q_2 q_3]$ ϕ	$[q_0 q_1, q_2 q_3]$ ϕ	$[q_1 q_3]$ ϕ

Closure properties of regular languages

Th: The class of regular languages is closed w.r.t complement

$$L \subseteq \Sigma^* \qquad L^c = \Sigma^* \setminus L$$

Proof: Let L be some regular language accepted by DFA $A = (Q, \Sigma, \delta, q_0, F)$

Construct the DFA $A' = (Q, \Sigma, \delta, q_0, Q - F)$ by interchanging the roles of final & nonfinal states

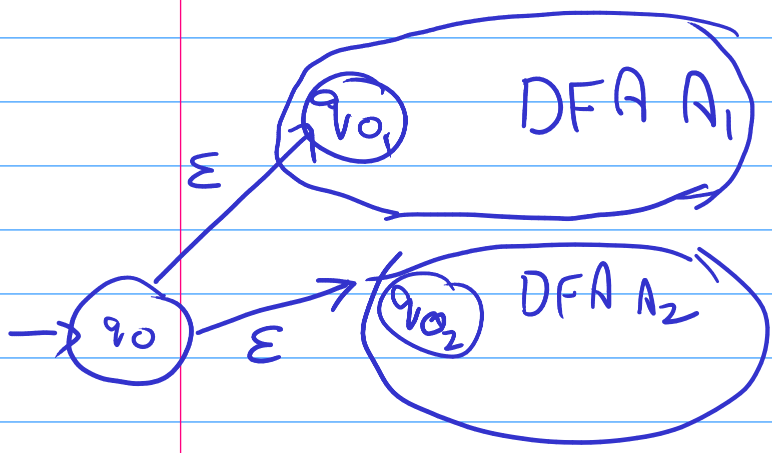
We claim $L(A') = L^c$ so that L^c is regular

$$\forall x \in \Sigma^* \quad x \in L^c \iff x \notin L$$
$$\iff \hat{\delta}(q_0, x) \notin F$$
$$\iff \hat{\delta}(q_0, x) \in Q - F$$
$$\iff x \in L(A') \quad \square$$

Corollary: Class of RL is closed w.r.t intersection

Proof: L_1 & L_2 are RL

L_1^c & L_2^c are also RL, their union $L_1^c \cup L_2^c$ are also regular



Hence $(L_1^c \cup L_2^c)^c$ is also regular

Demorgan's
$L_1 \cap L_2 = (L_1^c \cup L_2^c)^c$ □

For $i = 1,2$ $\quad A_i(Q_i, \Sigma, \delta_i, F_i)$
be 2 DFA accepting L_i

$L(A_1) = L_1 \qquad L(A_2) = L_2$

$$A = (Q_1 \times Q_2, \Sigma, \delta, (q_1, q_2), F_1 \times F_2)$$

where δ is defined as

$$\delta((p,q), a) = (\delta_1(p,a), \delta_2(q,a))$$

$\forall (p,q) \in Q_1 \times Q_2$

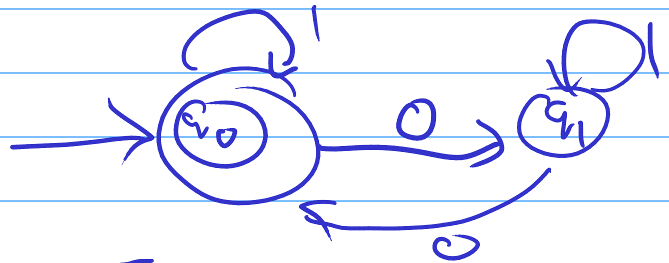
Claim is $L(A) = L_1 \cap L_2$ (string in both L_1 & L_2)

(Q) Using the construction given in the above proof, design a DFA that accepts the language

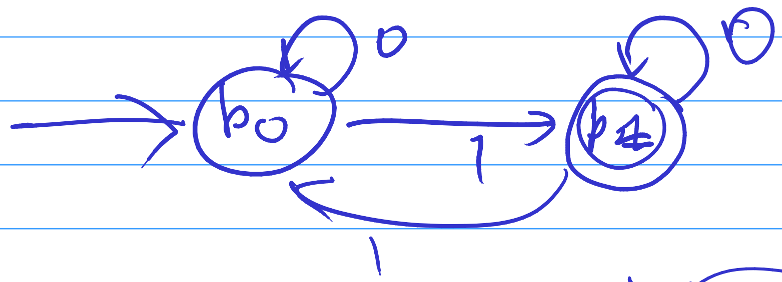
$L = \{x \in (0+1)^* \mid |x|_0 \text{ is even and } |x|_1 \text{ is odd}\}$

so that L is regular

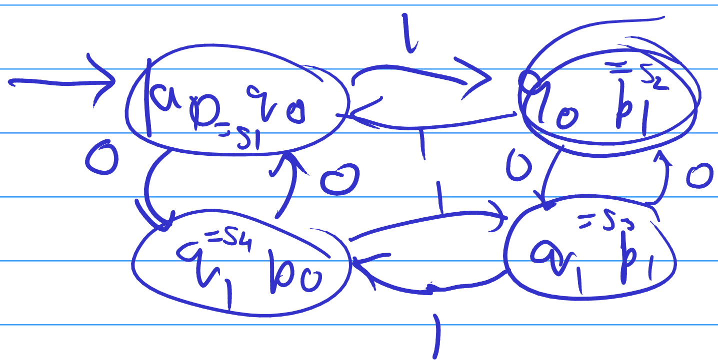
$L_1 = \{ x \in (0+1)^* \mid |x|_0 \text{ is even}\}$ → no. of zeroes is even

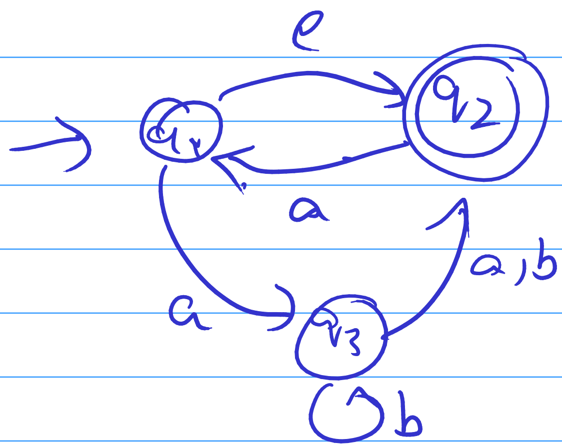


$L_2 = \{x \in (0+1)^* \mid |x|_1 \text{ is odd}\}$



Compute product





$E q_1 = q_1 q_2$

$E q_2 = q_2$

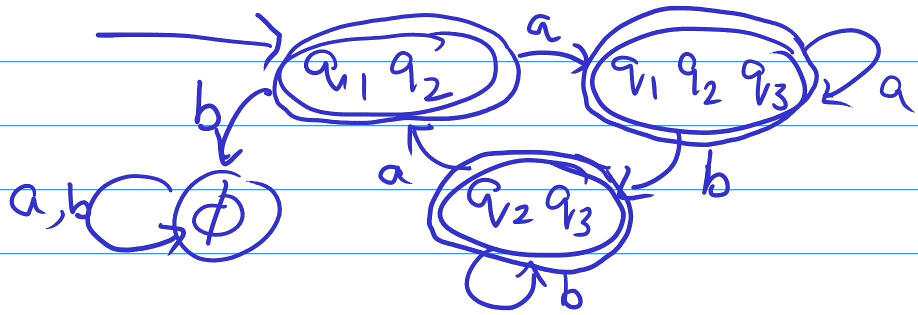
$E q_3 = q_3$

q_1, q_2 $q_1 q_2 q_3, \phi$

$q_1 q_2 q_3$ $q_1 q_2 q_3$ $q_2 q_3$

$q_2 q_3$ $q_1 q_2$ $q_3 q_2$

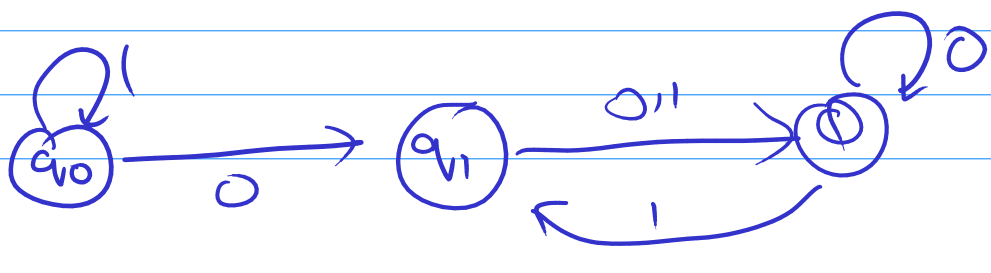
ϕ ϕ ϕ

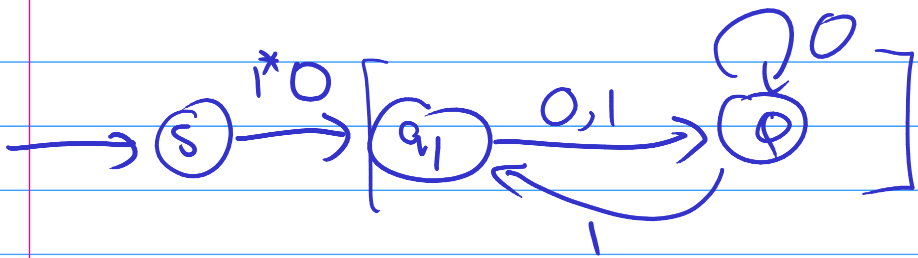


{01, 1, 100} then which of the following are in kleene* of L

101011001101 yes!

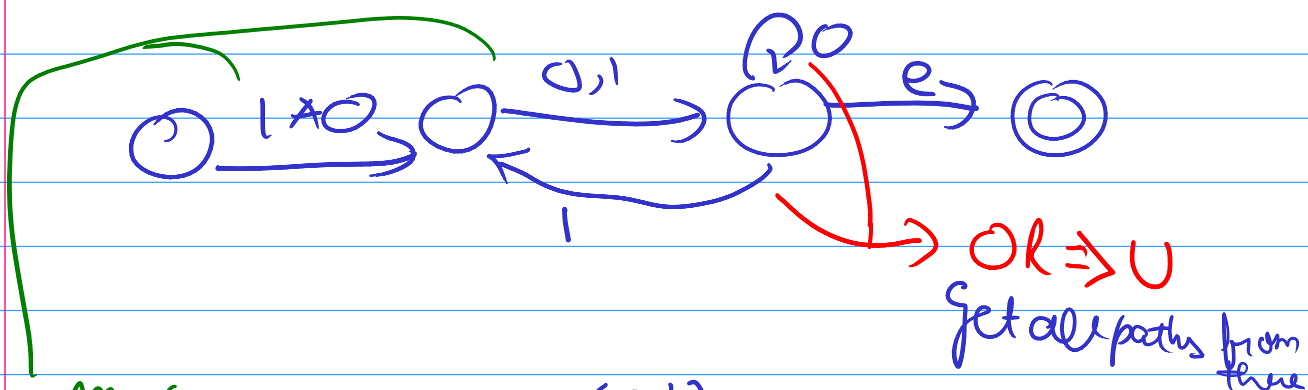
01111100100010 × No!





$$S \xrightarrow{1^*0} \left(\left[(0 \cup 1)\, 0^*\right] 1\right)^* \left[(0 \cup 1)\, 0^*\right]$$

$$1^*0\left(\left[[0+1]\,0^*\right]1\right)^* \left[(0+1)\,0^*\right]$$



merge .