Entangled State is A & B

$|\psi_A\rangle \quad |\psi_B\rangle$

$|\psi_{AB}\rangle = |\psi_A\rangle \otimes |\psi_B\rangle$ separable

otherwise entangled

$A \rightarrow \mathcal{H}_A \rightarrow |\psi_A\rangle \qquad B \rightarrow H_B \rightarrow |\psi_B\rangle$

$\mathcal{H}_{AB} = \mathcal{H}_A \otimes \mathcal{H}_B$ separable

$\neq \mathcal{H}_A \otimes \mathcal{H}_B$ entangled

$|\psi_A\rangle = \alpha_1 |0_A\rangle + \beta_1 |1_A\rangle$

$|\psi_B\rangle = \alpha_2 |0_B\rangle + \beta_2 |1_B\rangle$

The state $|0\rangle$ to spin up state

$|1\rangle$ to spin down state

$|\psi_{AB}\rangle = \alpha_1\alpha_2 |0_A 0_B\rangle + \alpha_1 \beta_2 |0_A 1_B\rangle + \beta_1 \alpha_2 |1_A 0_B\rangle + \beta_1 \beta_2 |1_A 1_B\rangle$

$|\uparrow_C\rangle = |0_A\rangle \qquad |\uparrow_B\rangle = |0_B\rangle$

$|\downarrow_C\rangle = |1_A\rangle \qquad |\downarrow_B\rangle = |1_B\rangle$

$|\psi_{CB}\rangle = \frac{1}{2} |\uparrow_C \uparrow_B\rangle + \frac{1}{2} |\downarrow_C \downarrow_B\rangle$

$\neq |\psi_C\rangle \otimes |\psi_B\rangle$

<u>Entangled state</u> $\quad |\downarrow_C \uparrow_B\rangle \quad |\downarrow_C \uparrow_B\rangle$

$\langle \uparrow | \psi \rangle = \frac{1}{2} \langle \uparrow | \uparrow \rangle + \frac{1}{2} \langle \uparrow | \downarrow \rangle$

$= \frac{1}{2} [1 \; 0] \begin{bmatrix} 1 \\ 0 \end{bmatrix} + \frac{1}{2} [1 \; 0] \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

$= \frac{1}{2}$

$\langle \downarrow | \psi \rangle = \frac{1}{2}$

$$|0_A\rangle \equiv |0_B\rangle \equiv \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$|1_A\rangle \equiv |1_B\rangle \equiv \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$\frac{1}{\sqrt{2}}$ $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$ & $\begin{bmatrix} 1 \\ -1 \end{bmatrix}$ are also basis vectors

definition of entangled state :

$$|\psi_{AB}\rangle \neq |\psi_A\rangle \otimes |\psi_B\rangle$$

in <u>ANY</u> basis

→ This is a hard problem

<u>Bell Basis</u>

$$|\phi^+\rangle = \frac{1}{\sqrt{2}}\left[|0\rangle_A \otimes |0\rangle_B + |1\rangle_A \otimes |1\rangle_B\right]$$

$$|\phi^-\rangle = \frac{1}{\sqrt{2}}\left[|0\rangle_A \otimes |0\rangle_B - |1\rangle_A \otimes |1\rangle_B\right]$$

$$|\psi^+\rangle = \frac{1}{\sqrt{2}}\left[|0\rangle_A \otimes |1\rangle_B + |1\rangle_A \otimes |0\rangle_B\right]$$

$$|\psi^-\rangle = \frac{1}{\sqrt{2}}\left[|0_A\rangle \otimes |1\rangle_B - |1_A\rangle \otimes |0_B\rangle\right]$$

<u>Cat State</u>

$$|\psi_{c3}\rangle = \frac{1}{2}|0_A 0_B\rangle + \frac{1}{2}|0_A 1_B\rangle + \frac{1}{2}|1_A 0_B\rangle + \frac{1}{2}|1_A 1_B\rangle$$

$|\uparrow_c\rangle = |0_A\rangle =$ cat alive

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Phase change : $|\psi_1\rangle_A = e^{i\theta}|\psi\rangle_A = e^{i\theta_1}|\psi_A\rangle$

not q.m'ally ← classically all are different

$$\langle\psi_A|\psi_A\rangle = \langle\psi_A|e^{-i\theta}e^{i\theta}|\psi_A\rangle = \langle\psi_A|\psi_A\rangle$$

But intermediate phases cannot
be removed

⇒ the 4 bell basis vectors
have different intermediate