Myhill Nerode Theorem

an equivalence relation $\sim$ on Σ^* is said to be right invariant

$x, y \in \Sigma$

$$x \sim y \Rightarrow \forall z (xz \sim yz)$$

eg: $\sim L$ on Σ^* $\qquad x \sim_L y \iff \forall z (\cancel{z} z \in L \iff yz \in L)$

(basically x & y can be final state)

$$A = (Q, \Sigma, \delta, q_0, F)$$

$\sim_A$ Σ^* $x \sim_A y$ iff $\hat{\delta}(q_0, x) = \hat{\delta}(q_0, y)$

for every z in Σ^*

$$\hat{\delta}(q_0, xz) = \hat{\delta}(\hat{\delta}(q_0, x), z)$$
$$= \hat{\delta}(\hat{\delta}(q_0, y), z)$$
$$= \hat{\delta}(q_0, yz)$$

$xz \sim_A yz \Rightarrow \sim_A$ is right invariant

Theorem: Let L be a language over Σ

(1) L is accepted by DFA

(2) $\exists$ a right invariant eq. rel $\sim$

L is union of some of class of $\sim$

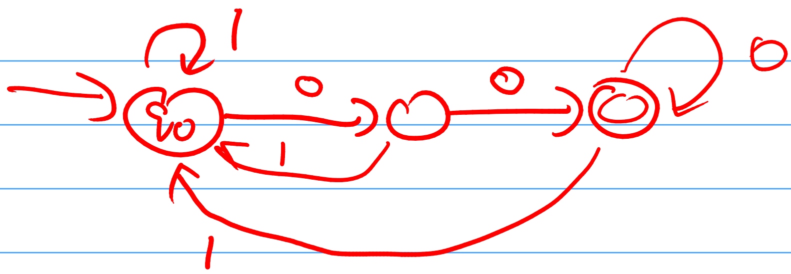
The equivalence relation $\sim_L$ is of finite index

② ⇒ ③

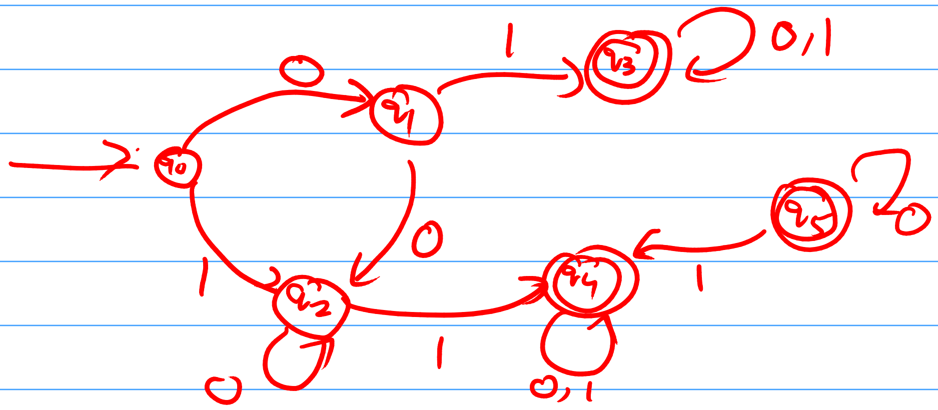
$L = \{x \in \{a,b\}^* \mid ab \text{ is a substring of } t\}$

Calculate equivalence classes of L

$L = \{w \in \{0,1\}^* : w \text{ ends with } 00\}$



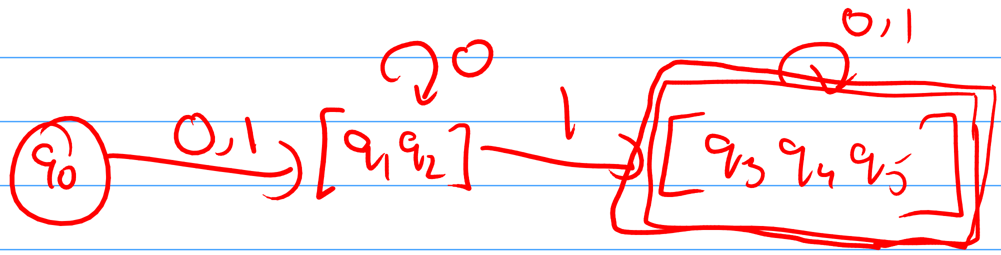
minimize the following DFA



$\Pi_0 = \{\{q_0\ q_1\ q_2\}\ \{q_3\ q_4\ q_5\}\}$

$\Pi_1 = \{\{q_0\}\ \{q_1, q_2\}\ \{q_3\ q_4\ q_5\}\}$

$\Pi_2 = \{\ \{q_0\}\ \{q_1\ q_2\}\ \{q_3\ q_4\ q_5\}\ \}$

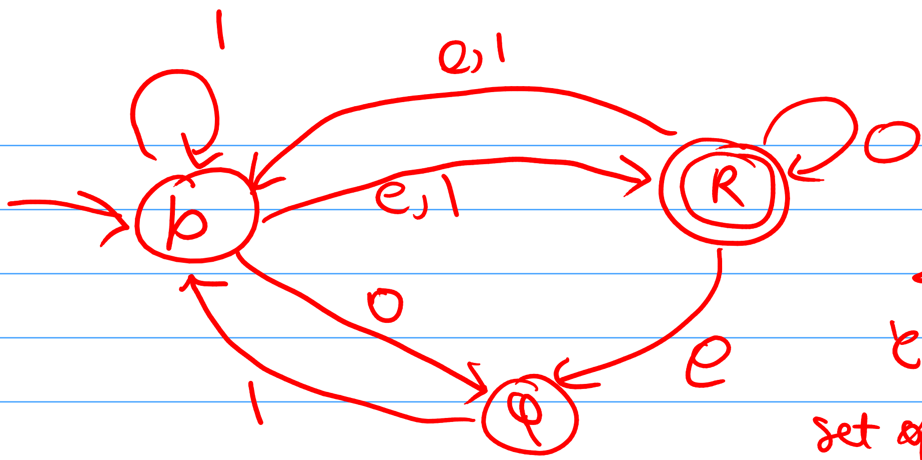


q_0	q_3	q_2
q_1	q_6	q_2
q_2	q_8	q_5
q_3	q_0	q_1
q_4	q_2	q_5
q_5	q_4	q_3
q_6	q_1	q_0
q_7	q_4	q_6
q_8	q_2	q_7
q_9	q_7	q_{10}
q_{10}	q_5	q_9

$\Pi_0 = \{\{q_0\ q_1\ q_2\ q_5\ q_7\ q_9\ q_{10}\}, \{q_3\ q_4\ q_6\ q_8\}\}$

$\Pi_1 = \{\{q_0\ q_1, q_2\}, \{q_5, q_7\}, \{q_9, q_{10}\}, \{q_3, q_4, q_6, q_8\}\}$

$\Pi_2 = \{\{q_0, q_1\}, \{q_2\}, \{q_5, q_7\}, \{q_9, q_{10}\}, \{q_3\ q_6\}, \{q_4, q_8\}\}$



$\Sigma = \{0,1\}$

ε ' stands for null

set of states $= \{P, Q, R\}$

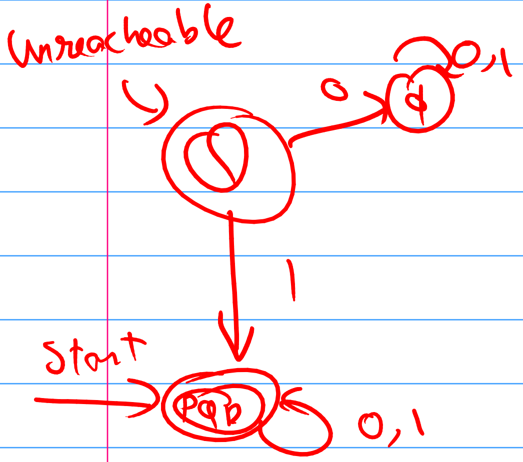
final state $= \{R\}$

start state $= P$

$E(P) = \{P, Q, R\}$

$E(R) = \{P, R, Q\}$

$E(Q) = \{Q\}$



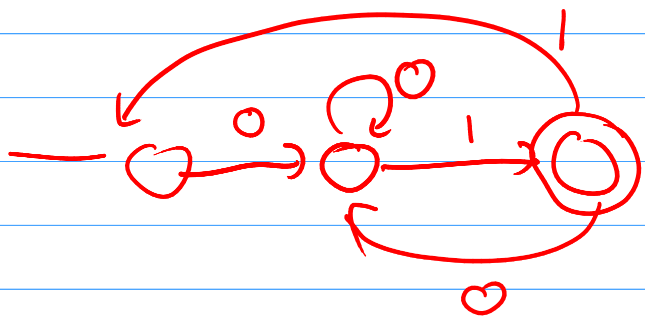
{PQR} = {PQR} {PQR}

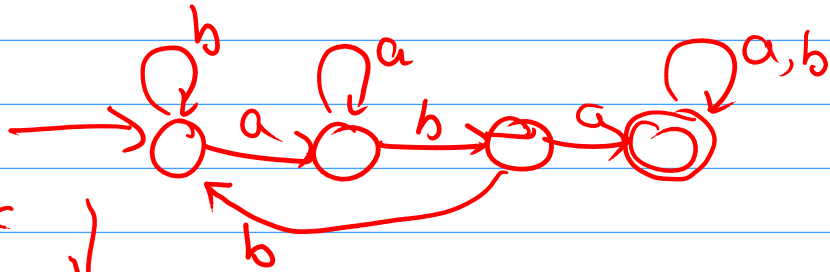
{PQR}

X Q Q

Q Q Q

(a) DFA $\{0,1\}^*$ 01





L^c