$$\psi(\theta+\phi) = R_y(\theta)\,\psi(\phi)$$

Mapping a qubit on a bloch sphere

$$R_y\theta = \begin{bmatrix} \cos\frac{\theta}{2} & -\sin\frac{\theta}{2} \\ \sin\frac{\theta}{2} & \cos\frac{\theta}{2} \end{bmatrix} \begin{bmatrix} \cos\frac{\phi}{2} \\ \sin\phi/2 \end{bmatrix}$$

$$= \begin{bmatrix} \cos\frac{\theta}{2}\cos\phi/2 - \sin\frac{\theta}{2}\sin\frac{\phi}{2} \\ \sin\theta/2\cos\phi/2 + \cos\theta/2\sin\phi/2 \end{bmatrix}$$

$$= \begin{bmatrix} \cos\left(\frac{\theta}{2}+\frac{\phi}{2}\right) \\ \sin\left(\frac{\theta}{2}+\frac{\phi}{2}\right) \end{bmatrix}$$

$$R_y(\theta) = \begin{bmatrix} \cos\theta/2 & -\sin\theta/2 \\ \sin\theta/2 & \cos\theta/2 \end{bmatrix}$$

$$X = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

$$Y = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}$$

$$Y = -i\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} \cos\theta/2 & 0 \\ 0 & \cos\theta/2 \end{bmatrix} + \begin{bmatrix} 0 & -\sin\theta/2 \\ \sin\theta/2 & 0 \end{bmatrix}$$

$$= \cos\theta/2 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \sin\theta/2 \begin{bmatrix} 0 & +1 \\ -1 & 0 \end{bmatrix}$$

$$= \cos\frac{\theta}{2}\,\mathbb{I} - i\sin\theta/2\, Y = e^{-i\frac{\theta Y}{2}}$$

$$R_x(\theta) = e^{\frac{-i\theta x}{2}}$$

$$= 1 - \frac{i\theta x}{2} + \frac{\left(\frac{i\theta x}{2}\right)^2}{2!} - \frac{\left(\frac{i\theta x}{2}\right)^3}{3!}$$

$$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \quad x^2 = 1$$

$$\begin{matrix} 1 & 0 \\ 0 & 1 \end{matrix}$$

$$= \mathbb{I} - \frac{i\theta x}{2} - \frac{\frac{\theta^2 x^2}{4}}{2!} + \frac{\frac{i\theta^3 x^3}{8}}{8!} \quad x^2 \rightarrow \mathbb{I}$$

$$= \cos\frac{\theta}{2}\,\mathbb{I} - i\sin\frac{\theta}{2}\,x$$

$$\begin{bmatrix} \cos\frac{\theta}{2} - i\sin\frac{\theta}{2} & 0 \\ 0 & \cos\frac{\theta}{2} - i\sin\frac{\theta}{2} \end{bmatrix} \quad \begin{bmatrix} -i\sin\frac{\theta}{2} & \\ & -i\sin\frac{\theta}{2} \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$R_z[\theta] = e^{\frac{-i\theta z}{2}}.$$

$$1 - \frac{i\theta z}{2} + \left(\frac{i^2\theta^2 z^2}{2}\right)$$

$$\cos\frac{\theta}{2} I - i\sin\frac{\theta}{2} z$$

$$\begin{bmatrix} \cos\frac{\theta}{2} & 0 \\ 0 & \cos\frac{\theta}{2} \end{bmatrix}\begin{bmatrix} -i\sin\frac{\theta}{2} & 0 \\ 0 & +i\sin\frac{\theta}{2} \end{bmatrix} = \begin{bmatrix} \cos\frac{\theta}{2} - i\sin\frac{\theta}{2} & 0 \\ 0 & \cos\frac{\theta}{2} + i\sin\frac{\theta}{2} \end{bmatrix}$$

Unitary transform on Bloch sphere

$$U = e^{-i\alpha} R_z(\beta) R_y(\gamma) R_z(\delta)$$

$\alpha \quad \beta \quad \gamma \quad \delta$ $\longrightarrow$ z transform