$|0\rangle, |1\rangle$ find orthogonal basis set

$\langle 0|0\rangle = \langle 1|1\rangle = 1$

$\langle \psi_1|\psi_2\rangle = [\langle 0|(-\tfrac{1}{2}) + \langle 1|(-\tfrac{\sqrt{3}}{2})] [-\tfrac{1}{2}|0\rangle + \tfrac{\sqrt{3}}{2}|1\rangle]$

$(\tfrac{1}{2})^2 \langle 0|0\rangle - \tfrac{3}{4}\langle 1|1\rangle$

$= -\tfrac{1}{2}$

Let $|\psi\rangle = \begin{bmatrix} e^{i\phi}\cos\theta \\ \sin\theta \end{bmatrix}$

$\begin{bmatrix} e^{i\phi}\cos\theta \\ \sin\theta \end{bmatrix}$

$\rho = |\psi\rangle\langle\psi|$

$\mathrm{Tr}\,\rho$ $\qquad \theta, \phi \in \mathbb{R}$

$\rho^2 = ?$

$\rho = |\psi\rangle\langle\psi| = \quad \langle\psi| = [e^{-i\phi}\cos\theta \quad \sin\theta]$

$\begin{bmatrix} e^{i\phi}\cos\theta \\ \sin\theta \end{bmatrix}_{2\times 1} [e^{-i\phi}\cos\theta \quad \sin\theta]_{1\times 2} = \begin{bmatrix} \cos^2\theta & e^{i\phi}\cos\theta\sin\theta \\ e^{-i\phi}\cos\theta\sin\theta & \sin^2\theta \end{bmatrix}$

$\mathrm{Tr} = 1$

$$\rho^2 = \begin{bmatrix} \cos^2\theta & e^{i\phi}\cos\theta\sin\theta \\ e^{-i\phi}\cos\theta\sin\theta & \sin^2\theta \end{bmatrix} \begin{bmatrix} \cos^2\theta & e^{i\phi}\cos\theta\sin\theta \\ e^{-i\phi}\cos\theta\sin\theta & \sin^2\theta \end{bmatrix}$$

$$= \begin{matrix} \cos^4\theta + \cos^2\theta\sin^2\theta & e^{i\phi}\cos^3\theta\sin\theta + e^{i\phi}\sin^3\theta\cos\theta \\ e^{-i\phi}\cos^3\theta\sin\theta + e^{-i\phi}\cos\theta\sin^3\theta & \sin^4\theta + \cancel{e^{i\phi}}\cos^2\theta\sin^2\theta \end{matrix}$$

$$= \begin{bmatrix} \cos^2\theta & \cos\theta\sin\theta\, e^{i\phi} \\ e^{-i\phi}\cos\theta\sin\theta & \sin\theta \end{bmatrix} = \rho$$

given Hamiltonian operator $\hat{H} = \hbar w \sigma_x$

find $|\psi(t)\rangle = e^{-iHt/\hbar} |\psi(t=0)\rangle$

$$i\hbar \frac{d}{dt}|\psi\rangle = \hat{H}|\psi\rangle$$

~~individual~~ init cond. $\} \Rightarrow |\psi(t=0)\rangle \begin{bmatrix} 1 \\ 0 \end{bmatrix}$

ψ is a solution for

$$e^{i\frac{Ht}{\hbar}} = \exp\left[\frac{i\hbar w \sigma_x t}{\hbar}\right]$$

Taylor expansion of e^x

$$\frac{(i\omega t\sigma_x)^0}{0!} + \frac{i\omega t\sigma_x}{1!} + \frac{(i\omega t\sigma_x)^2}{2!} + \frac{(i\omega t\sigma_x)^3}{3!} + \frac{(i\omega t\sigma_x)^4}{4!}$$

$$I_2 + i\omega t\sigma_x + -\omega t\sigma_x^2$$

$$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$ (all pauli matrices)

$$I_2 + i\omega t\sigma_x - \frac{\omega^2 t^2 I_2}{2!} + \frac{i\omega^3 t^3\sigma_x}{3!} + \frac{\omega^4 t^4 I_2}{4!} \dots$$

$$= \left[1 - \frac{\omega^2 t^2}{2!} + \frac{\omega^4 t^4}{4!} - \frac{\omega^6 t^6}{6!}\right] \mathbb{I}_2$$

$$+ i\left[\frac{\omega t}{1!} - \frac{\omega^3 t^3}{3!} + \frac{\omega^5 t^5}{5!} \dots\right]\sigma_x$$

$$= \cos\omega t\,\mathbb{I}_2 + i\sin\omega t\,\sigma_x$$

$$= \begin{bmatrix} \cos\omega t & +i\sin\omega t \\ -i\sin\omega t & \cos\omega t \end{bmatrix} \overset{\psi(0)}{\begin{bmatrix} 1 \\ 0 \end{bmatrix}} = \overset{\psi(t)}{\begin{bmatrix} \cos\omega t \\ -i\sin\omega t \end{bmatrix}}$$

i think(?)