$$|\psi\rangle = \begin{bmatrix} x \\ y \end{bmatrix} = x\begin{bmatrix} 1 \\ 0 \end{bmatrix} + y\begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$|\psi\rangle = x|e_1\rangle + y\,|e_2\rangle \cdots\cdots \infty$$

$\langle\psi|\psi\rangle$

Bra Ket

$[\;\cdots\cdots\;]_{1\times n}$ $\xleftrightarrow{\text{dual}}$ $\begin{bmatrix} \vdots \end{bmatrix}_{n\times 1}$

dual vector space

Inner product (defined only for vector spaces with a dual space)

↳ get scalar from vector

→ $|\psi'\rangle_{n\times 1} = [A]_{n\times n}\,|\psi\rangle_{n\times 1}$

Complex vector space

$[\;] = [\quad\;]^T$ in non complex

$[\;] = [\text{complex conjugate}]^\dagger$

$\langle\psi|^\dagger = |\psi\rangle$

$$[e_1^* \; e_2^* \; e_3^*]^\dagger = \begin{bmatrix} e \\ e \\ e \end{bmatrix}$$

$$\langle\psi|\psi\rangle = |\psi|^2$$

inner product

$$[e_1^* \; e_2^*]\begin{bmatrix} e_1 \\ e_2 \end{bmatrix} = |e_1|^2 + |e_2|^2 = \sum_i |e_i|^2$$

basis

$\langle e_2 | e_1 \rangle = \begin{bmatrix} 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = 0$

$\langle e_1 | e_2 \rangle = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = 0$

orthogonality condition

$\langle e_1 | e_2 \rangle = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = 1$

For any isolated system, associated is a complex vector space with inner product (i.e. a Hilbert space). The system is completely described by its state vector, which is a unit vector in the system's state space.

Hilbert Space – guarantees inner product:

$\langle \psi | \phi \rangle = 0$

$|\chi\rangle$ & $|\phi\rangle$ are orthogonal

$\langle \chi | \chi \rangle = k \not\to \infty$ (not for us)

using hilberts $\langle \chi_\infty | \chi_\infty \rangle = k$

$[\ ]$ & diff equations : L^2 functions

Bijection for [ ] to diff. equations through L^2 function

State is a vector in a complex vector space
$\boxed{\text{column vector}}$

$\vec{A} \times \vec{B} = \vec{C}$

$$[\ ]_{n\times 1}[\ \ \ ]_{1\times n} = [\ \ \ ]_{n\times n}$$

Tensors (outer product)

Second Postulate

Let $\langle \psi | \psi \rangle = n$

$$|\phi\rangle = \frac{1}{\sqrt{n}} |\psi\rangle$$

$$\langle \phi | \phi \rangle = \frac{1}{n} \langle \psi | \psi \rangle = 1$$

$|\phi\rangle$ & $\langle\phi|$ normalized vectors

The evolution of a closed quantum system is described by a unitary transformation i.e. the state $|\psi\rangle$ of the system @ time t_1 → is related to $|\psi_0\rangle$ @ t_2 by a unitary Operator U $\boxed{|\psi\rangle = U|\psi_0\rangle}$ which only depends on time t_1 & t_2

$U =$ unitary

time evolution is also done using matrices

$U^{\dagger} U = 1$

$(\chi | \psi) \rightarrow$ math $\langle \chi | \psi \rangle$ physics

$(\chi, B\psi) \rightarrow \langle \chi | B | \psi \rangle$

$[\langle \chi |]_{1\times n} [B]_{n\times n} [|\psi\rangle]_{n\times 1} = [\ ]_{1\times n} [\ ]_{n\times n} [\ ]_{n\times 1}$

$= \#_{1\times 1}$

3rd postulation

time evolution of a state of a closed quantum system is described by the schrodinger equation

$$i\hbar \frac{\partial}{\partial t} |\psi\rangle = H |\psi\rangle$$