→ $\Delta Q = \Delta U + \Delta W$

→ State of entropy ↑

given 'n'

Relation Between Entropy and Probabilities

- " The Boltzmann formula shows the relationship between entropy and the number of ways the atoms or molecules of a thermodynamic system can be arranged."

$$S = k \log W$$

- "At equilibrium, the system will be in its most probable state and the entropy will be maximum."

Ludwig Boltzmann (1844 - 1906)

Statistical entropy:

$$S = -k \sum_i P_i \ln P_i$$

What is classical info:

→ Sun will rise on the east today

→ We may have rainfall tonight at Chennai.

→ Statement 1 is a certain event (doesn't add to knowledge)

Information is all about probabilities

- Statement 3: One-third of the world's population is India+ China population.
- Statement 4: Two-third of the earth's surface is covered by water.
- Is there any difference between the information contents of these two statements?.
- No! Given any person, our ignorance about his/her nationality is same as our ignorance about the character (viz. water or solid) of any given surface area of the Earth.
- So information is all about the probabilities of occurrences of different events: probability that a person (from all over the world) is (India + China) is 1 / 3, probability that a surface area of the Earth is solid is equal to 1 / 3, etc.

- Thus the amount of information about an event is the amount of ignorance (or, uncertainty) about that event.
- Ignorance increases with increase of the inverse of the probability p of occurrence of the event.
- Total amount of ignorance of two independent events is sum of the ignorance.
- So the amount of ignorance H(p) should be an additive function of p.

Shannon entropy $H = -\, g p_i \Sigma p_i$

$\equiv$ We define $0 \log 0 = 0$

- **Example of Shannon's noiseless coding theorem**

  Code 4 symbols {1, 2, 3, 4} with probabilities {1/2, 1/4, 1/8, 1/8}.

  Code without compression:

$$[1,2,3,4] \xrightarrow{\text{withoutcompr.}} [00,01,10,11]$$

  But, what happens if we use this code instead?

$$[1,2,3,4] \xrightarrow{\text{with compr.}} [0,10,110,111]$$

  Average string length for the second code:

$$\langle \text{lenght} \rangle = \frac{1}{2}1 + \frac{1}{4}2 + \frac{1}{8}3 + \frac{1}{8}3 = \frac{7}{4} < 2$$

Note: $H\left(\frac{1}{2},\frac{1}{4},\frac{1}{8},\frac{1}{8}\right) = -\frac{1}{2}\log_2\frac{1}{2} - \frac{1}{4}\log_2\frac{1}{4} - \frac{2}{8}\log_2\frac{1}{8} = \frac{7}{4}$!!!

Classical encodings don't work necessarily work in quantum

0-2	2-4	4-5	10-11

2-5	

5-10	

10-13	