

A vector in a complex vector space with inner product.

$$\begin{array}{c}
 \underbrace{\quad}_{\text{bra}} \quad \xleftrightarrow{\text{dual}} \quad \underbrace{\quad}_{\text{ket}} \\
 \text{bracket } \langle \psi | = [ \quad ] \\
 \langle \psi | \psi \rangle
 \end{array}$$

$$\boxed{(\psi, \psi)}_{\text{Maths}} = \langle \psi | \psi \rangle_{\text{physics}}$$

$$(\psi, B\psi) = \langle \psi | B | \psi \rangle$$

$$[ \psi ] [ B ] [ \psi ]$$

$$|\psi\rangle = \frac{\alpha}{\sqrt{2}} |\uparrow\rangle + \frac{\beta}{\sqrt{2}} |\downarrow\rangle$$

$$\xrightarrow{\text{Probability}} \frac{\alpha}{\sqrt{2}} \begin{bmatrix} 1 \\ 0 \end{bmatrix} + \frac{\beta}{\sqrt{2}} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \frac{1}{\sqrt{2}} \begin{bmatrix} \alpha \\ \beta \end{bmatrix}$$

$|\psi\rangle$  Wave function [Prob. distribution]

$$\langle \psi | \psi \rangle = \int_a^b \psi^* \psi d^3x = 1$$

Wave function doesn't mean anything

$$\begin{bmatrix} \quad \end{bmatrix}_{1 \times n} \begin{bmatrix} \quad \end{bmatrix}_{n \times 1} \quad \psi \text{ is normalized}$$

eg:  $|\psi\rangle = \alpha_1^* |H\rangle + \alpha_2^* |C\rangle + \alpha_3^* |Sc\rangle$

$$\langle\psi|\psi\rangle = \sum |\alpha_i|^2 = 1$$

$$\frac{1}{\sqrt{2}} |H\rangle + \frac{1}{\sqrt{2}} |T\rangle = |\psi\rangle$$

$$|\psi\rangle = \begin{bmatrix} \alpha \\ \beta \end{bmatrix} \text{ On time evolution?}$$

$$|\psi(t_0)\rangle \xrightarrow{t} |\psi(t)\rangle$$

$$|\psi(t)\rangle = U |\psi(t_0)\rangle$$

$\begin{bmatrix} \end{bmatrix}_{1 \times n}$   $\begin{bmatrix} \end{bmatrix}_{n \times n}$   $\begin{bmatrix} \end{bmatrix}_{1 \times n} \rightarrow n \text{ events, sample space is fixed.}$

$$\langle\psi(t)|\psi(t)\rangle = \langle\psi(0)|\psi(0)\rangle$$

$$\langle\psi(0)|U^\dagger U|\psi(0)\rangle \quad U^\dagger U =$$

(adjoint) complex conj + transpose