

**Heuristic Rule 2:** Let  $A \rightarrow A\alpha_1, \dots, A \rightarrow A\alpha_n$  and  $A \rightarrow \beta_1, \dots, A \rightarrow \beta_m$  be all rules with  $A$  on the left-hand side, where the  $\beta_i$ 's do not start with an  $A$  and  $n > 0$  (that is, there is at least one left-recursive rule). Then replace these rules by  $A \rightarrow \beta_1 A', \dots, A \rightarrow \beta_m A'$  and  $A' \rightarrow \alpha_1 A', \dots, A' \rightarrow \alpha_n A'$ , and  $A' \rightarrow \epsilon$ , where  $A'$  is a new nonterminal.

$E \rightarrow T$   
 $E \rightarrow E + T$   
 $T \rightarrow T * F$   
 $T \rightarrow F$   
 $F \rightarrow (E)$   
 $F \rightarrow id$   
 $F \rightarrow id(E)$

$E \rightarrow TE'$   
 $E' \rightarrow TE' \mid \epsilon$   
 $T \rightarrow FT'$   
 $T' \rightarrow *FT' \mid \epsilon$   
 $F \rightarrow (E)$   
 $F \rightarrow idA$   
 $A \rightarrow \epsilon$   
 $A \rightarrow (E)$

	first	follows
$E$	$(, id$	$\epsilon, \$$
$E'$	$+, \epsilon$	$\epsilon, \$$
$T$	$(, id$	$\epsilon, +, \$$

$S \rightarrow bBA$   
 $A \rightarrow ab$   
 $B \rightarrow b$

$(p, \epsilon, \epsilon) (q, S)$   
 $(q, \epsilon, S) (q, bBA)$   
 $(q, \epsilon, A) (q, ab)$   
 $(q, \epsilon, B) (q, b)$   
 $(q, a, a) (q, \epsilon)$   
 $(q, b, b) (q, \epsilon)$

$S \rightarrow AX/xB$   
 $X \rightarrow axb\epsilon$   
 $A \rightarrow aA\epsilon$   
 $B \rightarrow bB\epsilon$

$(p, \epsilon, \epsilon) (q, S)$   
 $(q, \epsilon, S) (q, AX)$   
 $(q, \epsilon, S) (q, xB)$   
 $(q, \epsilon, A) (q, aA)$

$w c w^r$

$q_0$

$a^m b^n$

$m \leq n \leq 2m$

$a a \underline{a} b b b$   
 $a a \underline{b} b b$

$a a a a a$

$(b, e, e), (q_0, e)$

$(q_0, a, e) (q_0, a)$

$(q_0, a, a) (q_0, a)$

$(q_0, b, a) (q_1, b a)$

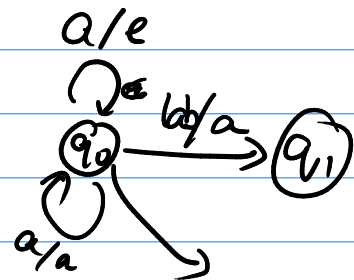
$(q_1, b, b a) (q_1, e)$

$(q_1, e,$

$S \rightarrow a S b / a S b b / e$

$a a \underline{a} b b b b b$

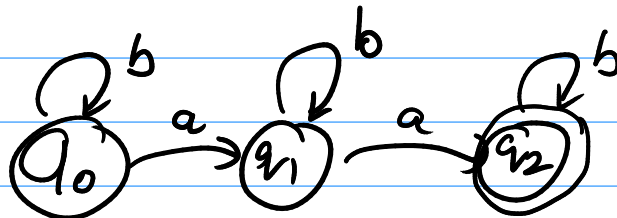
~~$a a a b b$~~   
 $a a b$   
 $a a a b b$



$a a a \underline{b} b b$

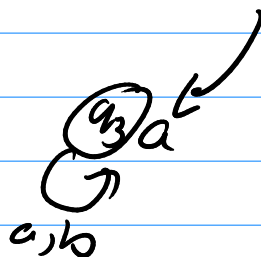
$(b, e, e) (q, S)$   
 $q$

$a$



$b^* a b^* e b^*$

$(q_0, b, e) (q_0, e)$   
 $(q_0, a, e) (q_1, e)$   
 $(q_1, b, e) (q_1, e)$   
 $(q_1, a, e) (q_2, e)$   
 $(q_2, b, e) (q_2, e)$



$S \rightarrow B a B a B$   
 $B \Rightarrow b B / e$

$S \rightarrow aA$   
 $S \rightarrow bC$   
 $A \rightarrow aC$   
 $A \rightarrow bB$   
 $B \rightarrow aC$   
 $B \rightarrow bC$   
 $C \rightarrow aC$   
 $C \rightarrow bC$

} useless nonterminal string

$a s b s a b$

~~$a b$~~

$(a+b)^*(a+b)$

$S \rightarrow aC | S$  [

$A \rightarrow b s c a l a d ;$

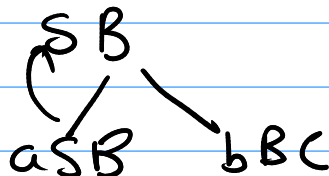
~~$B \rightarrow a s B | b B ;$~~

~~$C \rightarrow a s c l a d ;$~~

~~$a b c l a d$~~

~~$a c$~~   $a c B$   $a c B B$

$a c$



$S \rightarrow aC$   
 ~~$A \rightarrow b s c a l a d$~~

$C \rightarrow a d$

$S \rightarrow a a d$

$a^n b^{2n} a^m$

$(q, e, e) (q, s)$

$S \rightarrow aAbbbB$

$A \rightarrow aAbbb|e$

$B \rightarrow aB|e$

ab

$(q, e, a) (q, a)$

$(q, b, a) (q, e)$

abba

$S \rightarrow aBa$

$B \rightarrow aBa|abba$

$(q, b, e) (q, b)$

$(q, a, b) (q, e)$

$S \rightarrow aAa$

$S \rightarrow aAbbbB$

$A \rightarrow aAbbb|e$

$B \rightarrow aB|e$

$p, e, e \quad q, s$

$(q, e, s)$

$q, aAbbbB$

$q, e, A$

$q, aAbbb$

$q, e, B$

$q, aB$

$q, e, B$

$q, e$

$q, a, a$

$q, e$

$q, b, b$

$q, e$

$q, e, A$

$q, e$

State	Input	stack	Transition
$p$	abba	$e$	
$q$	abba	$S$	1
$q$	abba	$aAbbbB$	2
$q$	bb a	$A bbbB$	3
$q$	bb a	$bbB$	4
$q$	b a	$bB$	5
$q$	a	$B$	6
$q$	a	$aB$	7
$q$	e	$B$	8
$q$	e	$e$	9

11th Nov

$S \rightarrow AB$

$A \rightarrow aAb$

$B \rightarrow cA$

$C \rightarrow cC | e$

first

follow

S

a, b

c, a

A

a, b

c, a, b

B

c, b, a

~~S~~

C

c, e

a, b

$q_{a,e} q_S$   
 $q_{a,e} q_a e$   
 $q_{b,e} q_b e$   
 $q_{c,e} q_c e$   
 $q_{\$,e} q_{\$} e$   
 $(q_a, e, S), (q, AB)$   
 $(q_b, e, S), (q, AB)$   
 $(q_a, e, A), (q, aA)$   
 $(q_b, e, A), (q, b)$   
 $C$

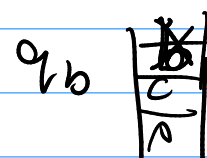
$(q, e, e) (q, S)$   
 $(q, e, S) (q, AB)$   
 $(q, e, A) (q, aA)$   
 $(q, e, A) (q, e)$   
 ~~$(q, e, B) (q, e)$~~   
 $(q, e, B) (q, cA)$   
 $(q, e, C) (q, cC)$   
 $(q, e, C) (q, e)$   
 $(q, a, a) (q, e)$   
 $\begin{matrix} b \\ c \end{matrix}$

b c c a a b

$q_b S AB$



$q_b B$



### Deterministic Look-ahead PDA transitions:

1.  $((p, e, e), (q, S))$

2.  $((q, a, e), (q_a, e))$

3.  $((q, b, e), (q_b, e))$

4.  $((q, c, e), (q_c, e))$

5.  $((q, \$, e), (q_{\$}, e))$

6.  $((q_a, e, S), (q_a, AB))$

// since a is in FIRST of S

7.  $((q_b, e, S), (q_b, AB))$

// since b is in FIRST of S

8.  $((q_a, e, A), (q_a, aA))$

// since a is in FIRST of A

9.  $((q_b, e, A), (q_b, b))$

// since b is in FIRST of A

10.  $((q_b, e, B), (q_b, cA))$

// since b is in FIRST of B

11.  $((q_a, e, B), (q_a, cA))$

// since a is in FIRST of B

12.  $((q_c, e, B), (q_c, cA))$

// since c is in FIRST of B

13.  $((q_a, e, C), (q_a, e))$

// since a is in FOLLOWS of C and  $C \rightarrow e$

14.  $((q_b, e, C), (q_b, e))$

// since b is in FOLLOWS of C and  $C \rightarrow e$

15.  $((q_c, e, C), (q_c, cC))$

// since c is in FIRST of C

16.  $((q_x, e, x), (q, e))$

// for each x in  $\Sigma$