Bloch Sphere Rep of a qubit

Bits 0, 1 $\quad |\psi\rangle = \alpha|0\rangle + \beta|1\rangle$

Qun $|0\rangle, |1\rangle \quad |\psi|^2 = \langle\psi|\psi\rangle = 1$

$|\alpha|^2 + |\beta|^2 = 1$ (complex)

$x^2 + y^2 = 1 \rightarrow$ Real, 2D

α, β if they are real

$\alpha = r\cos\theta$

$\beta = r\sin\theta$

but α & β are not Real

$z = x + iy \quad |z|^2 = |x|^2 + |y|^2 \quad$ if $x = \cos\theta . r$

$y = r\sin\theta$

$z = re^{i\theta} \quad \bar{z} = re^{-i\theta}$

So represent α & β as

$\alpha = e^{i\gamma}\cos\frac{\theta}{2}, \quad \beta = e^{i(\gamma+\phi)}\sin\frac{\theta}{2} \quad \boxed{r=1}$

γ???

$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$

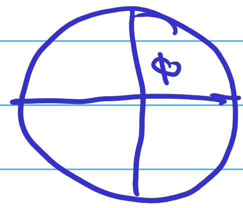
$= e^{i\gamma}[\cdots] \quad e^{i\gamma}(\cos\frac{\theta}{2}|0\rangle + e^{i\phi}\sin\frac{\theta}{2}|1\rangle)$

$|\chi\rangle = e^{i\gamma}|\chi_0\rangle$ (since $\langle\psi|\psi\rangle$ must be 1)

$x = r \sin\theta \cos\phi$ $\qquad$ $\phi \Rightarrow 0$ to 2π

$y = r \sin\theta \sin\phi$ $\qquad$ $\theta \Rightarrow 0$ to π

$z = r\cos\theta$ $\qquad$ $r \Rightarrow 0$ to R



$$Y = x + iy = r\sin\theta\,[\cos\phi + i\sin\phi]$$
$$= r e^{i\phi}\sin\theta$$

$X = z$ $\qquad$ $Z = X + iY = r[\cos\theta + \sin\theta\cos\phi + \sin\theta\, i\sin\phi]$

$$= r[\cos\theta + \sin\theta\, e^{i\phi}]$$

$|\alpha|^2 + |\beta^2| = 1$ $\qquad$ $\alpha, \beta \in C$

$$r e^{i\gamma}\cos\frac{\theta}{2} + r e^{i(\gamma+\phi)}\cos\frac{\theta}{2}$$

$\alpha = r e^{i\gamma}\cos\frac{\theta}{2}$ $\qquad$ $|\alpha|^2 = r^2 e^{-i\gamma} e^{+i\gamma}\cos^2\frac{\theta}{2}$

$|\beta|^2 = r^2 e^{-i(\gamma+\phi)} e^{i(\gamma+\phi)}\sin^2\frac{\theta}{2}$

$|\alpha|^2 = r^2\cos^2\frac{\theta}{2}$

$|\beta|^2 = r^2\sin^2\frac{\theta}{2}$

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$$

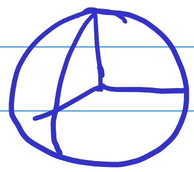
(R=1)
we see why

$$= e^{i\gamma}\cos\frac{\theta}{2}|0\rangle + e^{i(\gamma+\phi)}\sin\frac{\theta}{2}|1\rangle$$

$$|\psi\rangle = e^{i\gamma}\left[\cos\frac{\theta}{2}|0\rangle + e^{i\phi}\sin\frac{\theta}{2}|1\rangle\right]$$

$|\psi\rangle = \cos\frac{\theta}{2}|0\rangle + e^{i\phi}\sin\frac{\theta}{2}|1\rangle$

$|0\rangle$



$|1\rangle$

$|\alpha|^2 + |\beta|^2 = 1$

Value of $|\alpha|^2$ & $|\beta|^2$ are points surface of the sphere

$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$

$|\psi'\rangle = \alpha'|0\rangle + \beta'|1\rangle$

if $|\psi\rangle$ & $|\psi'\rangle$ are points on the surface of the same Bloch's ~~sphere~~

then $|\psi'\rangle = R(\theta)|\psi\rangle$ (very important The particle must & should exist in the given space)

Why r=1??

$\alpha|0\rangle + \beta|1\rangle$

$\langle\psi|\psi\rangle = 1$ provided $|\psi\rangle$ is normalized $\Rightarrow$ it should be (0,1) as ψ^2 is probability

$|\alpha|^2 + |\beta|^2 = R^2 \quad |\psi'\rangle = \frac{1}{R}|\psi\rangle$

$\langle\psi'|\psi'\rangle = \frac{1}{R^2}\langle\psi|\psi\rangle = \frac{1}{R^2}\cdot R^2 = 1$

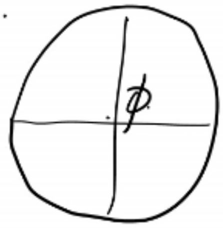
Note: 2 vectors of the same Hilbert space will lie on the bloch sphere!!

$$\boxed{\begin{aligned} x &= r\sin\theta\cos\phi \\ y &= r\sin\theta\sin\phi \\ z &= r\cos\theta \end{aligned}}$$

$\phi \to 0$ to 2π

$\theta \to 0$ to π

$r \to 0$ to R



$$z = x + iy, \quad \bar{z} = x - iy$$

$$|z|^2 = 1 = z\bar{z} = \boxed{x^2 + y^2 = 1}$$

$4 - 1$

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