



$$y = \Omega(\theta) \quad (\text{regularized})$$

$$\min L(y, \hat{y}) + \lambda \Omega(\Theta)$$

\Downarrow
 w, b

$$\Omega(\Theta) = w_1^2 + w_2^2 + \dots + w_{10}^2 + w_{\hat{n}}^2$$

$$\nabla_{w_1} J$$

$$\nabla_{b_1} J$$

$$\nabla_{w_2} J$$

$$\nabla_{b_2} J$$

$$\nabla_{w_3} J$$

$$\nabla_{b_3} J$$

A matrix of weights can be treated as a vector of vectors

How can we find the derivative of a scalar with a vector

3-layered NN

$$\nabla_{a_3} (L(\hat{y}, y)) = \nabla_{a_3} (h_3) \nabla_{h_3} (L(\hat{y}, y)) \quad \textcircled{1}$$

$$= \nabla_{a_3} (f(a_3)) \odot g = f'(a_3) \odot g$$

↙ element-wise mul

$$\frac{\partial L}{\partial a_3} = \frac{\partial L}{\partial h_3} \cdot \frac{\partial h_3}{\partial a_3}$$

Δ

$$\nabla_{w_3} (J) = \nabla_{a_3} (L(\hat{y}, y) + \lambda \Omega(\Theta))$$

$$= \nabla_{\omega_3} (L(\hat{y}, y)) + \lambda \nabla_{\omega_3} (\Omega(\theta))$$

$$= \nabla_{a_3} (L(\hat{y}, y)) \nabla_{\omega_3} (a_3)$$

$$\frac{\partial \mathcal{L}}{\partial \omega_3} = \frac{\partial L}{\partial a_3} \cdot \frac{\partial a_3}{\partial \omega_3} + \lambda \nabla_{\omega_3} (\Omega(\theta))$$

$$\left[\nabla_{a_3} (L(\hat{y}, y)) = f'(a_3) \odot g \right] \dots (1)$$

$$f'(a_3) \odot g \nabla_{\omega_3} (\omega_3^T h_2 + b_3)$$

$$\frac{\partial \mathcal{J}}{\partial \omega_3} \Big|_{\omega_3 = \omega_3^{(k)}}$$

$$\underline{\underline{1114}} \quad \frac{\partial \mathcal{L}}{\partial b_3} = \frac{\partial \mathcal{L}}{\partial a_3} \cdot \frac{\partial a_3}{\partial b_3} = f'(a_3) \odot g \nabla_{b_3} (\omega_3^T h_2 + b_3)$$

⋮
keep applying
Chain rule