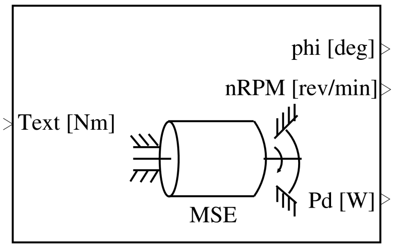
Rotatory Mass with Friction and End Stops (MERMASS11MSEREF) Documentation

February 1, 2021



Description

This is a model of a mass in rotary motion with friction effects and inertia. The displacement of the mass may be limited by lower and upper values. One detailed model of friction is implemented, as well as one model for the phenomenon of end stop.

Features

- Inertia Optional friction model:
  - Mixed form: Stribeck and stick/Coulomb
- Optional end stop model:
  - Elastic with spring and damper (the damping coefficient is indirectly defined by a coefficient of restitution)
- Dissipated power

Application area

This model computes the rotational speed and the angular displacement of a rotating mass when external torques are known.

Optionally, it is possible to consider frictional effects. These are implemented according to the Stribeck and the stick/Coulomb friction models, and with viscous damping effect (here linear) for high speed values (see the background documentation).
Furthermore the displacement of the mass may be limited. The physical phenomenon of end-stop is modelled as *spring-damping* system.

This element may be connected with the following elements:

- Elements that compute torques.
- Elements that need information about rotational speed or/and angular displacement.

Model assumptions and limits

Note that the following equations hold for SI units. Since most of the parameters, inputs and outputs of this model are not in SI units a unit conversion is done before and after the calculation.

The following conversions are performed:

$$\varphi = \frac{\pi}{180}\varphi_{Out}\frac{rad}{1^\circ} \tag{1}$$

$$\omega = \frac{\pi}{30}n_{RPM}\frac{rad}{s}\frac{min}{rev} \tag{2}$$

$$\omega_\epsilon = \frac{\pi}{30}\epsilon_n\frac{rad}{s}\frac{min}{rev} \tag{3}$$

$$\omega_{Stri} = \frac{\pi}{30}n_{Stri}\frac{rad}{s}\frac{min}{rev} \tag{4}$$

$$d_{visc\omega} = \frac{30}{\pi}d_{visc}\frac{s}{rad}\frac{rev}{min} \tag{5}$$

$$c_{stop\varphi} = \frac{180}{\pi}c_{stop}\frac{1^\circ}{rad} \tag{6}$$

The angular displacement should not be used in adjacent blocks for example to calculate the difference of two angular displacement inputs. This is because the value of the angular displacement may become very large and hence the difference becomes inaccurate.

- Stribeck + slip/Coulomb friction mode:
  It is recommended to cautiously use step signals for the sum of the external torques T_{ext}. Please be sure that the step is correctly detected by an appropriate zero-crossing procedure outside of this block (e.g. "enabling zero crossing detection" in the *Simulink* Step-signal block).

- End-stop modelled as a *spring-damping* system:
  At the maximum penetration, the spring torque has been assumed to be greater than the sum of external torques T_{ext}[1]. To ensure that this assumption is correct during a simulation, the parameter *spring coefficient at the angular displacement limit* c_{stop} should be chosen such that at a displacement limit, $\frac{|T_{ext}+T_{fr}|}{c_{stop\varphi}} \ll 1\ [rad]$.

- In the range $\varphi \geq \varphi_{max}$ or $\varphi \leq \varphi_{min}$, the viscous damping $d_{visc\omega} \cdot \omega$ and the viscous losses $d_{visc\omega} \cdot \omega^2$ are not considered. The damping torque is then only a function[2] of the inertia, the spring coefficient and the coefficient of restitution.

Solver settings

With the Stribeck and stick/Coulomb friction mode, it is recommended to use a stiff solver, since a high damping torque is used in stiction mode.

With the elastic end-stop mode, it is recommended to use a stiff solver, since the spring-damping system introduces oscillations at one displacement limit. Moreover, the tolerances should be chosen low enough, so that the solver correctly computes these oscillations that have mostly small amplitudes and high frequencies.

Model equations

The equations of motion are given by

$$\boxed{J\dot{\omega} = T_{ext} + T_{friction} + T_{stop}}, \text{with } \omega(0) = \frac{\pi}{30} n_{RPM0} \frac{rad}{s} \frac{min}{rev} \tag{7}$$

$$\boxed{\dot{\varphi} = \omega}, \text{with } \varphi(0) = \frac{\pi}{180} \varphi_0 \frac{rad}{1^\circ} \tag{8}$$

T_{ext} is the sum of external torques. The sum may be realized with the *summation block* of the *Simulink* library. Due to the sign convention, a positive torque leads to a positive

[1] With this assumption, it is possible to formulate the damping coefficient of the *spring-damping* system as a simple function of the coefficient of restitution. For further details, see the background documentation

[2] To know how the spring-damping system is parametrized, see equation 9. For its derivation, see report [1]

acceleration.

The optional friction torque is computed as follows (for more information refer to the background documentation):

$$T_{friction} = \begin{cases} -T_{ext} - T_{stop} - J\tau^{-1}\omega & \text{(case } DS_{fr} = 0) \\ -\text{sign}(DS_{fr}) \left[T_{Coulomb} + (T_{Stick} - T_{Coulomb}) \cdot e^{\left(-\text{sign}(DS_{fr}) \frac{\omega}{3\omega_{Stri}}\right)} \right] - d_{visc\omega} \cdot \omega & \text{(case } DS_{fr} \neq 0) \end{cases}$$

For the optional phenomenon of end-stop [3], the end-stop torque T_{stop} is implemented as follows (for more information refer to the background documentation):

$$T_{stop} = -c_{stop\varphi} \cdot (\varphi - \varphi_{max}) - \sqrt{\frac{4 \cdot c_{stop\varphi} \cdot J}{1 + \left[\frac{\pi}{ln(k_{impact})}\right]^2}} \cdot \omega \tag{9}$$

The dissipated power P_d contains two terms, the friction losses $P_{d,fr}$ and the end stop losses $P_{d,stop}$, $\boxed{P_d = P_{d,fr} + P_{d,stop}}$, that are computed as follows:

- $P_{d,fr}$:
  - No Stribeck + slip/Coulomb friction:

$$P_{d,fr} = d_{visc\omega} \cdot \omega^2 \tag{10a}$$

  - Stribeck + slip/Coulomb friction:
    * Motion

$$P_{d,fr} = |T_{fric} \cdot \omega| \tag{11a}$$

    * Stiction

$$P_{d,fr} = 0 \tag{12a}$$

- $P_{d,stop}$:
  - No end stop or $\varphi \in]\varphi_{min}; \varphi_{max}[$:

$$P_{d,stop} = 0 \tag{13a}$$

[3]The phenomenon of end-stop is documented here for $\varphi \geq \varphi_{max}$. Similar results may be obtained for $\varphi \leq \varphi_{min}$.

– End stop with $\varphi \leq \varphi_{min}$ or $\varphi \geq \varphi_{max}$:

$$P_{d,stop} = \sqrt{\frac{4 \cdot c_{stop\varphi} \cdot J}{1 + \left[\frac{\pi}{ln(k_{impact})}\right]^2}} \cdot \omega^2 \tag{14a}$$

This model is implemented in the same way as METMASS11MSEREF (For more information refer to the METMASS11MSEREF documentation.).

Parameter specifics

Parameter transformation

There exists a relationship between the spring coefficient $c_{stop\varphi}$, the coefficient of restitution k_{impact} and the time of contact [4] ΔT: $c_{stop\varphi} = \frac{J}{\Delta T^2}\left[\pi^2 + (ln(k_{impact}))^2\right]$.

Model validation

No experiment is performed to validate this model. The motion of the mass without detailed friction and without displacement limitation is just compared with an analytical model. Since the results are the same, it is validated.

To validate the Stribeck and slip/Coulomb friction model, this mass model is used in combination with a DC-motor model and a gear model (see [2][5]).

Code Generation

To inquire if it is principally possible to generate code from this block, please consult the related *Code generation* section in the overview documentation.

[4] Bouncing duration, i.e. while $\varphi \leq \varphi_{min}$ or $\varphi \geq \varphi_{max}$. For further information refer to [1].

[5] In this report, the current is the most important variable for the validation of the friction model. Indeed, the current is proportional to the torque delivered by this DC-motor. The simulated current peak at the beginning of a screwing or a drilling indicates that the motor torque has to overcome a given value (T_{Stick}) before it begins to rotate.

Parameters

- It is assumed that $T_{Stick} >= T_{Coulomb}$.
- It is assumed that $\varphi_{min} < \varphi_{max}$.
- With the *elastic* end-stop mode, a warning message is displayed when $\varphi_0 < \varphi_{min}$ or $\varphi_0 > \varphi_{max}$.
- With the *elastic* end-stop mode, a warning message is displayed when $c_{stop} >= 10^9$.

Type[6]	Name	Description	Symbol	Unit	Default	Values
Initial values						
F	`phiinit`	Initial displacement	φ_0	deg	0	$[-10^7, 10^7]$
F	`nRPMinit`	Initial rotational speed	n_{RPM0}	rev min^{-1}	0	$[-10^7, 10^7]$
Parameters						
F	`J`	Inertia	J	kg m^2	0.1	$[10^{-8}, 10^9]$
F	`dvisc`	Viscous damping	d_{visc}	Nm/(rev/min)	10^{-3}	$[0, 10^6]$
E	`frictionmode`	Friction mode			no friction	no friction stribeck + slip/ Coulomb friction
F	`epsn`[7]	Numerical parameter for friction treatment	ϵ_n	rev min^{-1}	10^{-3}	$[10^{-6}, 10^6]$
F	`tauinv`	Inverse of time constant at stiction (variable step solvers only)	τ^{-1}	1/s	10^3	$[0, 10^6]$
F	`TStick`	Torque at stick friction	T_{Stick}	N m	1	$[10^{-6}, 10^6]$
F	`TCoulomb`	Torque at Coulomb friction	$T_{Coulomb}$	N m	0.9	$[0, 10^6]$
F	`nStri`	Stribeck rotational speed	n_{Stri}	rev min^{-1}	0.01	$[10^{-6}, 10^6]$
E	`endstopmode`	End stop mode			no end stop	no end stop elastic
F	`phimin`	Lower angular displacement limit	φ_{min}	deg	-10^6	$[-10^6, 10^6]$
F	`phimax`	Upper angular displacement limit	φ_{max}	deg	10^6	$[-10^6, 10^6]$
F	`cstop`	Spring coefficient at displacement limit	c_{stop}	N m deg^{-1}	10^6	$[0, 10^{12}]$
F	`kimpact`	Coefficient of restitution	k_{impact}	-	0.5	$[0, 1]$

[6] E: Enumerated values parameter, F: Float parameter

[7] For fixed step solvers, ϵ_n should be not too small, as the sign of n cannot be trusted for n approximately zero. Select $\epsilon_n > \tau_{sum}\Delta$ t/J, e.g. $\epsilon_n = 10$ rev min^{-1}.

Ports

The angular displacement output variable is implemented using φ_{Out}, because φ is already used for the state variable.

Inputs

Direction	Type	Name	Symbol	Description	Unit
input	Float	`Text`	T_{ext}	Sum of external torques	N m

Outputs

Direction	Type	Name	Symbol	Description	Unit
output	Float	`phi`	φ_{Out}	Angular displacement	deg
output	Float	`nRPM`	n_{RPM}	Rotational speed	rev min^{-1}
output	Float	`Pd`	P_d	Dissipated power	W

Internals

Direction	Type	Name	Symbol	Description	Unit
output	Vector/Bus	`Internals`[8]		Internals output vector	

States

- Possible integer values for DS_{fr} are: [-99, 99, -1, 0, 1] (for more information refer to the background documentation).
- Possible integer values for $DS_{endstop}$ are: [-99, 99, -1, 0, 1] (for more information refer to the background documentation).

Type[9]	Name	Symbol	Description	Unit	Initial Value
CS	`phi`	φ	Angular displacement	rad	$\pi/180\ \varphi\ 0$
CS	`omega`	ω	Angular velocity	rad s^{-1}	$\pi/30$ nRPM0
EDS	`DSfr`	DS_{fr}	Discrete state of friction		99
EDS	`DSendstop`	$DS_{endstop}$	Discrete state of end stop		99

[8] Contains the states `DSfr` and `DSendstop` after the internal variables given in the table below.
[9] CS: Continuous state, EDS: Event-discrete state

References

[1] Mark Nagurka and Shuguang Huang. A mass-spring-damper model of a bouncing ball. *American Control Conference, 2004. Proceedings of the 2004*, 1(0-7803-8335-4):499–504, 2004.

[2] A. Vandamme. Functional modelling & parametrization of a drill-driver as an example for systems simulations of pt-tools. Report 05/2444, Robert Bosch GmbH, 2005.

