LIN380M Semantics I Homeworks

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EID:zh4378 Spring 2018

1 HW2

1. **Translate the following sentences into Predicate Logic. (You may choose your own letters to serve as non-logical constants,)**

   (a) **If one person is taller than another person and that second person is taller than a third one, then the first is taller than the third one.**

   $(\forall x)(\forall y)(\forall z)(\text{person}(x) \wedge \text{person}(y) \wedge \text{person}(z) \wedge x \neq y \wedge x \neq z \wedge y \neq z \wedge \text{taller}(x,y) \wedge \text{taller}(y,z)) \rightarrow (\text{taller}(x,z))$

   (b) **If one number is between two other numbers, then neither of the two others is between it and the third one.**

   $(\forall a)(\forall b)(\forall c)(\text{number}(a) \wedge \text{number}(b) \wedge \text{number}(c) \wedge a < b \wedge b < c) \rightarrow (\neg(b < a \wedge a < c) \wedge \neg(b < c \wedge c < a))$

   (c) **If you move a wolf, a goat and a cabbage across a river and you have a boat that can hold two but no more than two of the four of you, then there is exactly one strategy (for getting all of you safely across) . (N.B. you do not need to translate the part in parentheses.)**

   $(\forall x)(\forall y)(\forall z)(\forall k)(\forall w)(\text{wolf}(x)\wedge\text{goat}(y)\wedge\text{cabbage}(z)\wedge\text{you}(k)\wedge\text{boat}(w)\wedge(\text{canHold}(w,\{x\})\wedge \text{canHold}(w,\{y\})\wedge\text{canHold}(w,\{z\})\wedge\text{canHold}(w,\{k\})\wedge\text{canHold}(w,\{x,y\})\wedge\text{canHold}(w,\{x,z\})\wedge \text{canHold}(w,\{x,k\})\wedge\text{canHold}(w,\{y,z\})\wedge\text{canHold}(w,\{y,k\})\wedge\text{canHold}(w,\{z,k\})\wedge \neg\text{canHold}(w,\{x,y,z\})\wedge\neg\text{canHold}(w,\{x,y,k\})\wedge\neg\text{canHold}(w,\{x,z,k\})\wedge\neg\text{canHold}(w,\{y,z,k\})\wedge \neg\text{canHold}(w,\{x,y,z,k\})\wedge\text{moveAcrossRiver}(x)\wedge\text{moveAcrossRiver}(y)\wedge\text{moveAcrossRiver}(z)\wedge \text{moveAcrossRiver}(k)) \rightarrow (\exists x)(\text{strategy}(x) \wedge \text{True}(x)) \wedge (\forall y)((\text{strategy}(y) \wedge y \neq x) \rightarrow (\neg\text{True}(y)))$

2. **Give as many non-equivalent translations of sentence (a) into Predicate Logic as you can think of.**

   (a) **Three girls met two boys.**

- $(\forall x)(\forall y)(\forall z)(\forall a)(\forall b)(\text{girl}(x) \wedge \text{girl}(y) \wedge \text{girl}(z) \wedge x \neq y \wedge y \neq z \wedge x \neq z \wedge \text{boy}(a) \wedge \text{boy}(b) \wedge a \neq b \wedge \text{met}(\{x, y, z\}, \{a, b\})$ [1]
- $(\forall x)(\forall y)(\forall z)(\forall a)(\forall b)(\text{girl}(x) \wedge \text{girl}(y) \wedge \text{girl}(z) \wedge x \neq y \wedge y \neq z \wedge x \neq z \wedge \text{boy}(a) \wedge \text{boy}(b) \wedge a \neq b \wedge \text{met}(x, a) \wedge \text{met}(x, b) \wedge \text{met}(y, a) \wedge \text{met}(y, b) \wedge \text{met}(z, a) \wedge \text{met}(z, b))$

3. (Note well: this exercise consists of 4 parts: a, b, c and d.)

(a) Translate the following sentences into Predicate Logic. Use the 1-place predicate P for person and the 2-place predicate R for 'met.

i. No one has met everyone.

$(\forall x)(\exists y)(P(x) \wedge P(y) \wedge x \neq y \wedge \neg R(x, y))$

ii. No one has met anyone.

$\neg((\exists x)(\exists y)(P(x) \wedge P(y) \wedge x \neq y \wedge R(x, y)))$

iii. No one has met no one.

$(\forall x)(\exists y)(P(x) \wedge P(y) \wedge x \neq y \wedge R(x, y))$

iv. If someone has met no one, then no one has met everyone.

$(\exists x)(\forall y)((P(x) \wedge P(y) \wedge x \neq y \wedge \neg R(x, y)) \rightarrow (\neg(\exists z)(P(z) \wedge z \neq y \wedge R(z, y)))$

v. If someone has met someone, then it is not the case that no one has met anyone.

$((\exists x)(\exists y)(P(x) \wedge P(y) \wedge x \neq y \wedge R(x, y)) \rightarrow (\neg(\forall x)(\forall y)(P(x) \wedge P(y) \wedge x \neq y \wedge \neg R(x, y)))$

(b) Let $M = \langle U, I \rangle$ be the following model for the language P, R:

$U = \{a, b, c, d\}$

$I(P) = U$(**so every individual in the universe of M is a person**)

$I(R) = \{< a, b >, < a, c >, < a, d >, < b, a >, < c, a >, < d, a >, < c, d >, < d, c >, \}$

i. Which of the sentences (i)-(v) are true in M and which false, assuming that the words no one, everyone, anyone, someone range over persons and that R translates the transitive verb meet?

(i), (ii), (iv) are false and (iii), (v) are true.

[1] "met" in $\text{met}(\{x, y, z\}, \{a, b\})$ has meaning of $\{\alpha | (\forall \alpha)(\forall \beta)((\text{function}(\alpha) \wedge \text{function}(\beta) \wedge \alpha \neq \beta) \rightarrow (\alpha \in \{a, b\}^{\{x,y,z\}} \wedge \beta \notin \{a, b\}^{\{x,y,z\}}))\}$ (we use PC loosely here as). In words, "met" is a set of functions that maps set $\{x, y, z\}$ to set $\{a, b\}$. $\{a, b\}^{\{x,y,z\}}$ represents all possible combinations of "met" relation that can happen between boys and girls.

ii. Go through the calculation of the truth value in M of (i) by applying the truth definition for PC to the translation of (i) into PC you have given under (a).

$[\![(\forall x)(\exists y)(P(x) \wedge P(y) \wedge x \neq y \wedge \neg R(x,y))]\!]_{M,a} = 1$

iff for every $a' \approx_x a \quad [\![(\exists y)(P(x) \wedge P(y) \wedge x \neq y \wedge \neg R(x,y))]\!]_{M,a'} = 1$

iff for every $a' \approx_x a$ for some $a'' \approx_y a' \quad [\![(P(x) \wedge P(y) \wedge x \neq y \wedge \neg R(x,y))]\!]_{M,a''} = 1$

iff for every $a' \approx_x a$ for some $a'' \approx_y a' \quad \langle [\![x]\!]_{M,a''} \rangle \in I_M(P)$ *and*

$\langle [\![y]\!]_{M,a''} \rangle \in I_M(P)$ *and* $[\![x]\!]_{M,a''} \neq [\![y]\!]_{M,a''}$ *and* $[\![R(x,y)]\!]_{M,a''} = 0$

iff for every $a' \approx_x a$ for some $a'' \approx_y a' \quad \langle [\![x]\!]_{M,a''} \rangle \in I_M(P)$ *and*

$\langle [\![y]\!]_{M,a''} \rangle \in I_M(P)$ *and* $[\![x]\!]_{M,a''} \neq [\![y]\!]_{M,a''}$ *and* $\langle [\![x]\!]_{M,a''}, [\![y]\!]_{M,a''} \rangle \notin I_M(R)$

iff for every $a' \approx_x a$ for some $a'' \approx_y a' \quad \langle a''(x) \rangle \in I_M(P)$ *and*

$\langle a''(y) \rangle \in I_M(P)$ *and* $a''(x) \neq a''(y)$ *and* $\langle a''(x), a''(y) \rangle \notin I_M(R)$

The above statement is false because if we let $a''(x) = a$ and there is no such $a''(y)$ that can make whole PC to true. If $a''(y) = a$, then $a''(x) \neq a''(y)$ is false. If $a''(y) = b$, then $\langle a, b \rangle \in I_M(R)$, which makes $\langle a''(x), a''(y) \rangle \notin I_M(R)$ false. If $a''(y) = c$, then $\langle a, c \rangle \in I_M(R)$, which makes $\langle a''(x), a''(y) \rangle \notin I_M(R)$ false.

(c) Which of the sentences (i)-(v), if any is/are true in all models for PC?

(v)

(d) For each of the sentences (i)-(v) that is not true in M modify the interpretation of $I(M)$ in such a way that this sentence comes out true in the modified model. (But leave the Universe U as it is!)

- (i) We modify the model by removing $\langle a, b \rangle$ from $I(R)$
- (ii) We modify the model by making $I(R) = \{\langle a, a \rangle\}$
- (iv) We modify the model by removing $\langle a, b \rangle, \langle a, c \rangle, \langle a, d \rangle$ from $I(R)$