Fog Computing in Multi-Tier Data Center Networks: A Hierarchical Game Approach

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***Abstract*—With the increasing popularity of data services and applications, data center networks have been introduced to serve users in a centralized fashion. Furthermore, the flexibility of the data service subscribers' (DSSs') requirements motivates data center virtualization so as to optimize the resource allocation among all DSSs. However, as the massive data centers are usually far away from the DSSs, the quality of services are severely affected for delay-sensitive DSSs. Accordingly, fog computing is considered to solve the problem, where some virtualized edge data centers acting as fog nodes (FNs) are added in the network and help massive data center operators (MDCOs) serve DSSs. In this paper, we analyze the resource management problem in the multi-FN multi-MDCO, and multi-DSS networks. We model the network architecture with 3-layer model, where the FNs are in the upper layer, MDCOs in the middle layer, the DSSs in the bottom layer. The FNs first share computing resource with the MDCOs, and thus the MDCOs are able to serve their DSSs with low delay. Based on the model, we propose a hierarchical game, where the interaction between FNSs and MDCOs is regarded as a multi-leader multi-follower Stackelberg game, and the interactions between MDCOs and DSSs are regarded as the single-leader single-follower Stackelberg games. By making decisions distributively, all FNs, MDCOs, and DSSs receive high utilities. Simulation results show the correctness of the analysis and the performance improvement of the proposed strategies in the fog computing networks.**

Index Terms — **Data center networks, fog computing, Stackelberg game, resource management.**

I. Introduction

The rapid developments of the data services and applications have brought increasing investments on data centers. In order to improve flexibility and efficiency of the computing resources, the concept of data center virtualization is recently adopted [1]–[3], where the massive data center operators (MDCOs) are able to create virtualized servers based on different data requirements of data service subscribers (DSSs). Accordingly, with data center virtualization, one server in the data center is able to serve multiple DSSs with little computing requirement, and some servers can be shut down when few DSSs request services, and thus the capacity utilization and energy efficiency of the massive data center networks can be significantly improved.

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As the massive data centers are normally constructed far away from DSSs, the transmission delay and communication cost during the service are unsatisfying. In order to solve the problem, fog computing is proposed to add multiple virtualized edge data centers, known as FNs, in the data center network, and offload the services from the massive data centers. FNs can be edge data centers owned by the MDCOs or any collaborative computing devices in the network. As the FNs are in a small scale and require a low construction cost, they are normally distributed around the DSSs, thus providing low-delay services and economical network communication by reducing the traffic routed across the network [4].

Recent work has advocated to the adoption of FNs in the massive data center network. The proposals of mist [5], EdgeCloud [6], micro-data centers [7], and nano data centers [8] have been introduced in order to offload the data services from massive data centers, and provide DSSs with better services. Furthermore, for many existing telecommunication and Internet service providers, the authors in [9] have shown that leveraging the existing infrastructure and providing value-added services with FNs are beneficial. In [10], the vision of fog computing was outlined and key characteristics of fog computing was illustrated. In [11], the web optimization was considered with fog computing. With network edge knowledge about DSSs' locations, dynamic adaptation of computing resources to the SSs' conditions can be accomplished. Accordingly, the data center networks are developing into a multi-tier network, where massive data centers serve DSSs requiring high computing resources, while FNs serve DSSs that are delay-sensitive.

However, most existing work does not consider the competition among multiple FNs and multiple MDCOs. In a multi-FN multi-MDCO, and multi-DSS data center network, how to efficiently allocate computing resources to all DSSs in both virtualized and physical networks is still an open problem. In this paper, we model the network architecture as a 3-layer structure, where the FNs are in the upper layer, MDCOs in the middle layer, and DSSs in the bottom layer. The FNs first share computing resource with the MDCOs, and thus the MDCOs are able to serve their DSSs with low delay. Accordingly, we propose a hierarchical game, where the interactions between FNs and MDCOs is a multi-leader multi-follower Stackelberg game, and the interactions between MDCOs and DSSs are the single-leader single-follower Stackelberg games.

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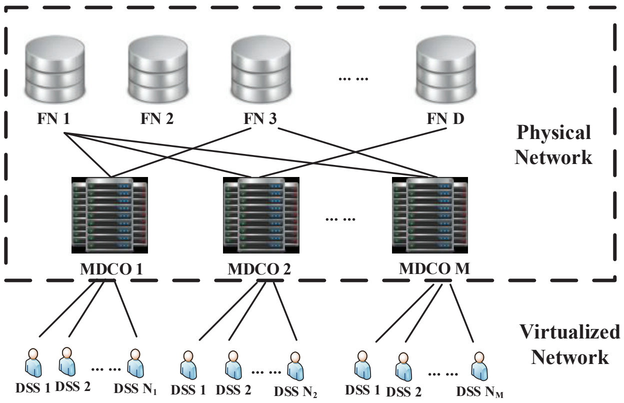

Fig. 1: System architecture

By making distributed decisions, all FNs, MDCOs, and DSSs receive stable and high utilities. Simulation results show the correctness of the analysis, and the performance improvement of the proposed strategies in the virtualized edge data center networks.

The rest of this paper is organized as follows. We describe the system model and formulate problems in Section II. According to the formulated problem, we analyze the interactions between MDCOs and DSSs in Section III, and the interactions between FNs and MDCOs in Section IV. Finally, we evaluate our work using simulation results in Section V, and summarize the work in Section VI.

II. System Model and Problem Formulation

A. System Model

We consider a multi-tier data center network in which there are M MDCOs, labeled as $l_1, l_2, \ldots, l_M$. For each MDCO l_m, $\forall m \in \{1, 2, \ldots, M\}$, it serves N_m DSSs, labeled as $u_1^m, u_2^m, \ldots, u_N^m$, in the virtualized data center network. We define λ_{mj} as the workload arrival rate for each user u_j^m. Correspondingly, when λ_{mj} arrives in DSS u_j^m, the DSS is served by virtualized servers in MDCO l_m. We define the unit amount of computing resources that can be used in the virtualized server as "computing resource block (CRB)", which has the service rate of μ. We assume that different MDCOs offer different special services, and each DSS is required to subscribe to a certain MDCO. Each MDCO serves its DSSs with SecondNet topology [12], where the network resources can be guaranteed for the data center services. Furthermore, in the physical network, in order to decrease the service delay, each MDCO offloads its service to the FNs close to the its serving DSSs. We assume that there are D FNs in the network, labeled as $s_1, s_2, \ldots, s_D$. The network architecture is illustrated in Fig. 1.

In this paper, we assume DSSs are delay-sensitive. Therefore, service delay is measured to evaluate the quality of service of each DSS. For each DSS u_j^m, $\forall m \in \{1, 2, \ldots M\}$, $\forall j \in \{1, 2, \ldots N\}$, we consider the cost of the delay as

$$t_{mj} = q_{mj} + d_{mj}, \tag{1}$$

which consists of the cost due to queuing delay at the data center q_{mj} and the cost due to network delay from the physical server to DSS u_j^m.

Following the queuing delay model in [13], [14], which can be easily extended to other models, the cost of queuing delay for each DSS u_j^m is defined as

$$q_{mj} = \frac{\lambda_{mj}}{\mu_m - \frac{\lambda_{mj}}{b_{mj}}}, \tag{2}$$

where b_{mj} is the number of CRBs allocated to serve the DSS u_j^m. As the network resource is guaranteed in the SecondNet topology, we assume the cost incurred by the network delay d_{mj} can be regarded as a linear function of the distance w_{mj} between the physical server and DSS u_j^m. Thus $d_{mj} = \xi w_{mj}$, where ξ is the weight factor.

Accordingly, in the virtualized network, the utility of each DSS equals the total revenue from the workload data minus the cost of both delay and service from MDCO, i.e.,

$$W_{mj}^u = \alpha_{mj}\lambda_{mj} - \beta_{mj} b_{mj}^p p_m - \gamma_{mj} t_{mj}, \tag{3}$$

where p_m is the unit price of the MDCO l_m, b_{mj}^{th} is the minimum amount of CRBs required to guarantee the service delay larger than the threshold t_{th}. Thus $b_{mj}^p = \max\left(b_{mj} - b_{mj}^{th}, 0\right)$ is the amount of CRBs the DSS u_j^m need to pay to the MDCO l_m. α_{mj}, β_{mj}, and γ_{mj} are weight factors.

In the physical data center network, if the MDCO wants to offload its service to FNs, we assume that FN s_i charges price a_i for each CRB. Moreover, when the FN serves DSSs offloaded by the MDCOs, we denote the increment of energy cost in the FN s_i is

$$e_i^s = b_{mj} E_{i,\mu}^s \frac{\lambda_{mj}}{\mu}, \tag{4}$$

where $E_{i,\mu}^s$ is the power consumed by the service when the FN s_i serves the DSS u_j^m. We can therefore write the utility of each FN s_i, $\forall i \in \{1, 2, \ldots, D\}$ as

$$W_i^s = \sum_{m=1}^{M} \sum_{j=1}^{N_m} \theta_{imj}(a_i b_{mj} - e_i^s), \tag{5}$$

where θ_{imj} is a binary variable to determine whether or not the data service from DSS u_j^m is offloaded from MDCO l_m to FN s_i.

If MDCO l_m chooses to serve the DSSs by itself, the increment of the energy cost in the MDCO is

$$e_m^l = b_{mj} E_{m,\mu}^l \frac{\lambda_{mj}}{\mu}, \tag{6}$$

where $E_{m,\mu}^l$ is the power requirement of the service when the MDCO l_m serves DSS u_j^m. Therefore, the utility of the MDCO l_m equals the total revenues received from the DSSs minus the cost c_m, i.e.,

$$W_m^l = \sum_{j=1}^{N_m} \left(p_{mj} b_{mj}^p - c_{mj}\right), \tag{7}$$

where

$$c_{mj} = \min\{e_m^l + kz_{mj}, a_i b_{mj} + kz_{ij}\}, \ \forall i \in \{1, 2, \ldots, D\}, \tag{8}$$

The cost c_{mj} is the minimum value of the energy and transmission cost served by the MDCO itself or the payment and transmission cost served by any FN s_j. z_{mj} is the distance between MDCO l_m and DSS u_j^m. z_{ij} is the distance between FN s_i and DSS u_j^m. k is the transmission cost for each kilometer in the transmission network.

B. Problem Formulation

According to the heterogeneous architecture of the data center network, it is generally impossible to simultaneously achieve satisfying utilities for all FNs, MDCOs, and DSSs. We consider a sequential decision making process for all FNs, MDCOs, and DSSs. Each FN s_i sets the price of its idle servers and sells the computing resources to MDCOs based on the CRB requirement of the DSSs in MDCOs $\mathbf{b} = [b_{mj}, \forall m \in \{1, 2, \ldots, M\}, \forall j \in \{1, 2, \ldots, N\}]$ and the setting prices of other FNs s_{-i}, i.e.,

$$\max_{a_i} \; W_i^s(a_i|\boldsymbol{\theta}_i, \mathbf{b}, \mathbf{a}_{-i}), \qquad \forall i \in \{1, 2, \ldots, D\},$$
$$s.t. \begin{cases} a_i \geqslant 0, \\ a_i b_{mj} \geqslant e_i^s, \\ \sum_{m=1}^{M} \sum_{j=1}^{N_m} \theta_{imj} b_{mj} \leqslant R_i, \end{cases} \tag{9}$$

where $\boldsymbol{\theta}_i = [\theta_{imj}, \forall m \in \{1, 2, \ldots, M\}, \forall j \in \{1, 2, \ldots, N\}]$, reflecting the selected DSSs physically served by FN s_i. The third constraint shows that the amount of CRBs sold to MDCOs cannot exceed the total amount of available computing resources R_i in idle servers of FN s_i.

Receiving the price declared by FN a_i, each MDCO firstly determines whether to serve a DSS with the servers in the massive data center or to offload the service to an FN. Then it determines service price $\mathbf{p}_m = [p_{mj}, \forall j \in \{1, 2, \ldots, N\}]$ to its serving DSSs in order to improve its revenues. Accordingly, the optimization problem for each MDCO l_m can be written as,

$$\max_{\mathbf{p}_m} \; W_m^l(p_m|a_i, \mathbf{b}), \qquad \forall m \in \{1, 2, \ldots, M\},$$
$$s.t. \begin{cases} \mathbf{p}_m \geqslant 0, \\ p_{mj} b_{mj} \geqslant c_{mj}, \qquad \forall j \in \{1, 2, \ldots, N\}. \end{cases} \tag{10}$$

In a virtualized network, each DSS determines the amount of CRBs b_{mj} that it would like to purchase from the MDCO by observing the prices set by all MDCOs. Therefore, the problem for each DSS is given by

$$\max_{b_{mj}} \; W_{mj}^u(b_{mj}|p_m), \qquad \forall j \in \{1, 2, \ldots, N\},$$
$$s.t. \; b_{mj} \geqslant 0, \; t_{mj} \geqslant 0, \; and \; t_{mj} \leqslant t_{th}. \tag{11}$$

where t_{th} is the upper bound of the time delay during the service. When $t_{mj} \geqslant t_{th}$, the DSS refuses the service from MDCO l_m.

As all FNs, MDCOs, and DSSs are rational and autonomous and can make decisions in a distributed fashion. In order to achieve the optimal and stable solution, we model the data center system pricing as a hierarchical game which consists the interactions between FNs and MDCOs and the interactions between MDCOs and DSSs as shown in Fig. 2. In the remainder of this paper, we analyze the problems in (9), (10), and (11), and derive solutions for all FNs, MDCOs, and DSSs.

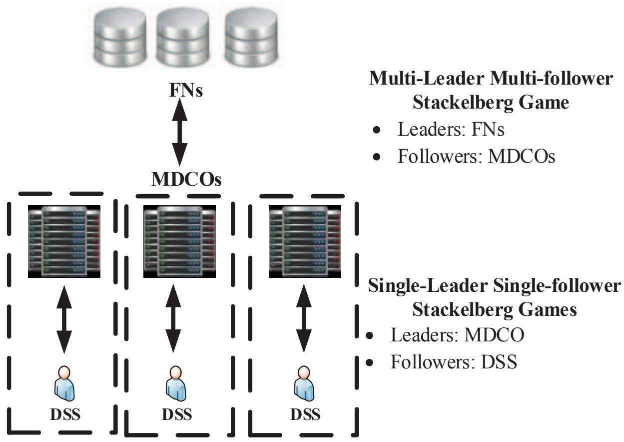

Fig. 2: Hierarchical game model

III. Game Analysis on the Interaction between MDCOs and DSSs

The interactions between MDCOs and DSSs are considered in the virtualized network, where the MDCOs firstly set the service price, and all DSSs make optimal decisions based on the service price imposed by the MDCOs. We can regard the MDCOs as leaders, and the DSSs as followers. As the interaction between each MDCO and DSS is independent with each other, we model each interaction as a single-leader single-follower Stackelberg game. In this section, without loss of generality, we consider the game between MDCO l_m, where $m \in \{1, 2, \ldots, M\}$, and DSS u_j^m, where $j \in \{1, 2, \ldots, N\}$.

Lemma 1. *In a single-leader single-follower Stackelberg game, given service price p_{mj} by MDCO l_m, the optimal amount of CRBs b_{mj} purchased by the DSS satisfies*

$$b_{mj}^* = \frac{\lambda_{mj}}{\mu\sqrt{p_{mj}}} + \frac{\lambda_{mj}}{\mu}. \tag{12}$$

Proof: The proof is given in Appendix A. ∎

Furthermore, according to the constraint of problem (11), the service delay cannot surpass t_{th} for the DSSs. Thus, the CRB purchased by DSSs has the following low bound

$$b_{mj}^{th} \geqslant \frac{\lambda_{mj}(t_{th} - d_{mj})}{\mu(r_{th} - d_{mj}) - \lambda_{mj}}. \tag{13}$$

Otherwise, the connection between the MDCO and the DSS is unsuccessful, so the DSS refuses to accept data service from the MDCO.

Based on the interaction between each MDCO l_m and each DSS u_j^m (12), we derive a one-to-one correspondence between service price p_{mj} from the MDCO and the amount of purchasing CRBs b_{mj} from the DSS as

$$p_{mj} = \left(\frac{\lambda_{mj}}{\mu b_{mj} - \lambda_{mj}}\right)^2. \tag{14}$$

Accordingly, considering the corresponding behaviors of the DSS, the problem (10) for each MDCO can be written as,

$$\forall m \in \{1,2,\ldots,M\}, \forall j \in \{1,2,\ldots,N\},$$

$$\max_{b_{mj}} \ \widetilde{W}^{l}_{mj}(b_{mj}|a_i),$$
$$s.t.\ b_{mj}\left(\frac{\lambda_{mj}}{\mu b_{mj}-\lambda_{mj}}\right)^2 \geqslant c_{mj}, \tag{15}$$

where

$$\widetilde{W}^{l}_{mj} = \delta_{mj} b_{mj}\left(\frac{\lambda_{mj}}{\mu b_{mj}-\lambda_{mj}}\right)^2 - \delta_{mj} c_{mj}. \tag{16}$$

IV. Game Analysis on the Interactions between FNs and MDCOs

Based on the system architecture in Fig. 1, the interactions between FNs and MDCOs are considered in the physical network, where the FNs set their prices of the CRBs from their servers first. Based on the behaviors of all FNs, all MDCOs purchase the appropriate amounts of CRBs for their corresponding DSSs.

When all FNs have already set their prices of CRBs, each MDCO l_m first chooses the servers from FNs or the MDCO itself for the DSSs, which achieves the lowest cost during the service. The MDCO then determines the amount of CRBs from the servers so as to achieve high utilities. Accordingly, we have the following lemma:

Lemma 2. *The optimal amount of CRBs allocated to the DSS u^m_j is*

$$b^*_{mj} = \begin{cases} \widetilde{b}_{mj}, & b^{th}_{mj} < \widetilde{b}_{mj} < 3\mu b^{th}_{mj} - 2\lambda, \\ b^{th}_{mj}, & otherwise, \end{cases} \tag{17}$$

where

$$\widetilde{b}_{mj} = \frac{1}{\mu}(H+\lambda), \tag{18}$$

$$H_{mj} = \sqrt[3]{-\frac{D_{mj}}{A_{mj}} + \sqrt{\eta}_{mj}} + \sqrt[3]{-\frac{D_{mj}}{A_{mj}} - \sqrt{\eta}_{mj}}, \tag{19}$$

$$\eta_{mj} = \left(\frac{D_{mj}}{A_{mj}}\right)^2 + \left(\frac{C_{mj}}{3A_{mj}}\right)^3, \tag{20}$$

$$A_{mj} = -c_{mj}, \tag{21}$$

$$C_{mj} = -\lambda^2, \tag{22}$$

$$D_{mj} = -2\lambda^3 + 2\lambda^2 \mu b^{th}_{mj}. \tag{23}$$

Proof: The proof is shown in Appendix B. ■

In order to receive satisfying utilities, all FNs adopt sub-gradient strategies and determine their pricing strategies based on Algorithm 1,

In Algorithm 1, all FNs firstly set the highest value a^{max}_i of their prices, where no MDCO will offload their data services to the FN s_i because of the high cost. Thus, based on the (8),

$$a^{max}_i = \max\{\frac{1}{b_{mj}}(e^l_m + kz_{mj} - kz_{ij}),$$
$$\forall m \in \{1,2,\ldots,M\}, \forall j \in \{1,2,\ldots,N\}\}. \tag{24}$$

In this case, no MDCOs would choose the servers from FNs because of the intolerably high cost. Then in order to attract MDCOs, all FNs start to sequentially adjust their prices in each iteration of the circulation. If increasing or decreasing the price increases the utilities of the FNs, in the next round, the corresponding FNs increase or decrease their prices with Δ_i, and reconsider the CRB requirements from all MDCOs. The above process continues until all FNs stop changing their prices.

Algorithm 1 Strategy of FNs in multi-FN multi-MDCO scenario.

Initially, each FN s_i sets price $a_i = a^{max}_i$, $\forall i \in \{1,2,\ldots,D\}$, which is a high price that no DSSs choose to be served by the FN s_i.
while At least one FN changes its price **do**
  for MDCO m **do**
    Based on the prices set by all FNs, each MDCO chooses its servers from all FNs or the MDCO itself with the lowest cost, and determine the amount of CRBs b_{mj} purchased from the corresponding servers.
  end for
  for DCO i **do**
    Each FN tries to increase or decrease its price a_i with a small step Δ_i, and calculates its own utility based on the behaviors of all other FNs a_{-i}.
    if $W^s_i(a_i, a_{-i}) < W^s_i(a_i + \Delta_i, a_{-i})$ **then**
      $a^{new}_i = \min\{a^{max}_i, a_i + \Delta_i\}$; % Increase the price
    else
      if $W^s_i(a_i, a_{-i}) < W^s_i(a_i - \Delta_i, a_{-i})$ **then**
        $a_i = \max\{0, a_i - \Delta_i\}$; % Reduce the price
      else
        $a^{new}_i = a_i$; % Keep the price unchanged
      end if
    end if
  end for
  for DCO i **do**
    Each FN i update the service prices. $a_i = a^{new}_i$;
  end for
end while

Lemma 3. *When the starting price a^{max}_i and step size Δ of FNs are fixed, the sub-gradient algorithm can converge to a unique outcome, and all FNs achieve Nash Equilibrium solutions, where each FN is unable to deviate its strategy unilaterally in order to increase its utilities.*

Proof: The proof is shown in Appendix C. ■

V. Simulation Results

In this section, we present the simulation results to show the performances of the proposed hierarchical game in the virtualized edge data center network. In the simulated scenario, we apply 100 DSSs and 10 FNs allocated randomly in a district with a diameter of 200 meters. Far away from both the DSSs and FNs, there are 3 massive data centers, which are at $(1000, 0)$, $(570, 700)$, and $(-669, -420)$, respectively. Because of the small distance between the FNs and DSSs, the transmission delay in FNs is small, but because of the high cost in FNs, the energy cost of each CRB in each FN

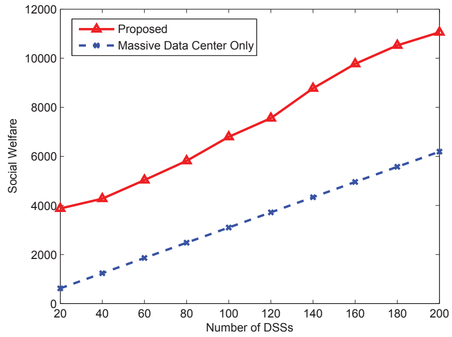

Fig. 3: Social welfare versus number of DSSs in the data center networks

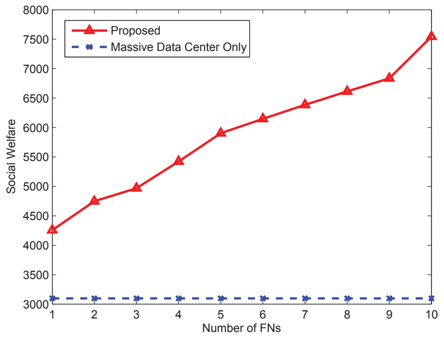

Fig. 4: Social welfare versus number of FNs in the data center networks

is 3. Although the massive data centers are located far away from DSSs, the energy cost of each CRB in the massive data center is 1. Without loss of generality, we set the processing speed μ of each CRB to be 1 MB/s, each user u_j^m has the workload λ_{mj} of 8 MB/s, and each FN has 50 available CRBs. Furthermore, the threshold of the time delay is 100 ms, and the weight factor of α, β, and γ are set as 6, 0.05, and 0.15, respectively. In order to show the performance improvement of the proposed hierarchical game in the virtualized edge data center network, we compare the performance of our work with the traditional method in the data center networks, where all the users are served by the massive data center only.

As shown in Fig. 3, with the number of DSSs increasing, more data services are handled in the data center networks. Accordingly, the social welfare of both the proposed strategies in the virtualized edge data center network and the traditional method in the massive data center network increases. However, as data services of some DSSs can be offloaded to CRBs of FNs nearby, and many DSSs are able to enjoy the low-delay service with satisfying costs, the social welfare in our proposed virtualized edge data center network is higher than the social welfare where DSSs can only be served by the MDCOs only.

Furthermore, we compare the social welfare when the number of FNs increases while the number of DSSs keep unchanged. With more FNs in the neighborhood, the DSSs are more likely to be served by a FN in a lower distance, thus low transmission delay for the DSS and low communication cost for the MDCO. Moreover, the increasing number of FNs

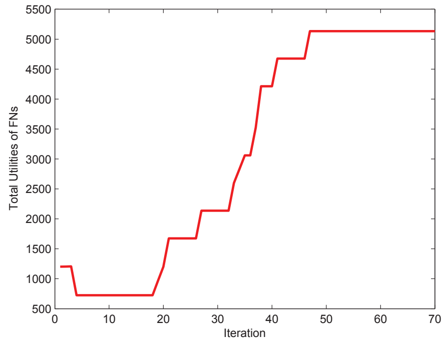

Fig. 5: Utility of FNs versus number of iterations in Algorithm 1

also aggravates the competition among DSSs, and reducing the service prices of each FN. As we can see in Fig. 4, with the number of FNs increasing, the social welfare of the proposed virtualized edge data center network improves and continuously enlarges the gap to the traditional massive data center networks.

In Fig. 5, we evaluate the performances of all FNs in the proposed hierarchical game. According to Algorithm 1, all FNs decrease their prices in order to serve more DSSs in the physical layer. In the figure, with the number of iterations increases in the circulation of Algorithm 1, the setting price of each FN gradually decreases, while the utility of all the FNs generally increases and achieves satisfying results when all FNs reach equilibrium results.

VI. Conclusions

When the FNs are added into the traditional massive data center networks, low delay and transmission costs can be achieved during the data services. However, as all FNs, MDCOs, and DSSs are rational and autonomous individuals, it is challenging to optimize the resource management in the multi-tier network. In this paper, we have modeled the data center network in 3 layers, and proposed a hierarchical game. In the game, the interaction between all FNs and MDCOs are regarded as a multi-leader multi-follower Stackelberg game, and the interaction between each MDCO and DSS is regarded as a single-leader single-follower Stacekelberg game. Simulation results show the performance improvement of the proposed resource management strategies, and all FNs, MDCOs, and DSSs are able to receive satisfying utilities.

Appendix A: Proof of Lemma 1

Proof: According to the utility function of DSS u_j^m 3, the first and second derivative of W_{mj}^u with respect to b_{mj} are, respectively,

$$\frac{\partial W_{mj}^u}{\partial b_{mj}} = \left(\frac{\lambda_{mj}}{\mu b_{mj} - \lambda_{mj}}\right)^2 - p_{mj}, \tag{25}$$

$$\frac{\partial^2 W_{mj}^u}{\partial b_{mj}^2} = -\frac{2\lambda_{mj}^2 \mu}{(\mu b_{mj} - \lambda_{mj})^3}. \tag{26}$$

Since $\frac{\partial^2 W_{mj}^u}{\partial b_{mj}^2} < 0$, W_{mj}^u is quasi-concave with respect to b_{mj}. Accordingly, we set the first derivative of W_{mj}^u equal to zero and obtain

$$b_{mj} = \frac{\lambda_{mj}}{\mu\sqrt{p_{mj}}} + \frac{\lambda_{mj}}{\mu}. \tag{27}$$

■

Appendix B: Proof of Lemma 2

Proof: According to the problem (15), we take the first and second derivative of $\widetilde{W}_{mj}^l$ with respect to b_{mj},

$$\frac{\partial \tilde{W}_{mj}^l}{\partial b_{mj}} = \frac{\lambda_{mj}^2}{H^2} - 2\frac{\lambda_{mj}^2 \mu b_{mj}}{H^3} + 2\frac{\lambda_{mj}^2 \mu b_{mj}^{th}}{H^3} - c_{mj}, \tag{28}$$

$$\frac{\partial^2 \tilde{W}_{mj}^l}{\partial b_{mj}^2} = \frac{2\lambda_{mj}^2 \mu^2 b_{mj} + 4\lambda_{mj}^3 \mu - 6\lambda_{mj}^2 \mu^2 b_{mj}^{th}}{H^4}, \tag{29}$$

where

$$H = \mu b_{mj} - \lambda_{mj}. \tag{30}$$

When $\frac{\partial^2 \tilde{W}_{mj}^l}{\partial b_{mj}^2} < 0$, we get

$$b_{mj} < 3\mu b_{mj}^{th} - 2\lambda_{mj}. \tag{31}$$

Based on (13), we know,

$$\mu b_{mj}^{th} > \lambda, \tag{32}$$

thus when $b_{mj}^{th} < b_{mj} < 3\mu b_{mj}^{th} - 2\lambda_{mj}$, the utility function of MDCO is convex. When $b_{mj} > 3\mu b_{mj}^{th} - 2\lambda_{mj}$, the utility function is concave. Furthermore, we discover,

$$\lim_{b_{mj} \to +\infty} \tilde{W}_{mj}^l \to -\infty. \tag{33}$$

Accordingly, we know the optimal solution $b_{mj}^* < 3\mu b_{mj}^{th} - 2\lambda_{mj}$, we set $\frac{\partial \tilde{W}_{mj}^l}{\partial b_{mj}} = 0$, and suppose

$$H_{mj} = \mu b_{mj} - \lambda_{mj}, \tag{34}$$

and thus we have

$$-c_{mj} H_{mj}^3 - \lambda_{mj}^2 H_{mj} + 2\lambda_{mj}^2 \left(\mu b_{mj}^{th} - \lambda_{mj}\right) = 0, \tag{35}$$

which is a one variable cubic equation. We set

$$A_{mj} = -c_{mj}, \tag{36}$$

$$C_{mj} = -\lambda^2, \tag{37}$$

$$D_{mj} = -2\lambda^3 + 2\lambda^2 \mu b_{mj}^{th}, \tag{38}$$

and get

$$\eta_{mj} = \left(\frac{D_{mj}}{A_{mj}}\right)^2 + \left(\frac{C_{mj}}{3A_{mj}}\right)^3 > 0. \tag{39}$$

Accordingly, there is only one real solution in the equation (35), which is

$$H_{mj} = \sqrt[3]{-\frac{D_{mj}}{A_{mj}} + \sqrt{\eta}_{mj}} + \sqrt[3]{-\frac{D_{mj}}{A_{mj}} - \sqrt{\eta}_{mj}}. \tag{40}$$

Thus we get

$$\widetilde{b}_{mj} = \frac{1}{\mu}(H + \lambda). \tag{41}$$

Moreover, as b_{mj} is constrained to be no smaller than b_{mj}^{th}, when the solution $\widetilde{b}_{mj} < b_{mj}^{th}$, the optimal solution $b_{mj}^* = b_{mj}^{th}$.

Above all, the optimal solution b_{mj}^* is

$$b_{mj}^* = \begin{cases} \widetilde{b}_{mj}, & b_{mj}^{th} < \widetilde{b}_{mj} < 3\mu b_{mj}^{th} - 2\lambda, \\ b_{mj}^{th}, & otherwise. \end{cases} \tag{42}$$

■

Appendix C: Proof of Lemma 3

Proof: The convergence properties of the sub-gradient algorithm has been proved in [15] and [16].

When Algorithm 1 converges, no FN is able to change its prices unilaterally to serve more DSSs or increase its revenues from the current serving DSSs. Accordingly, all FNs reach the Nash equilibrium solutions eventually. ■

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