Question 1

Your ant colony has worker ants that reside next to n different food sources, with an infinite quantity of each food source present. The worker ants regularly go out to collect food, collecting up to F grams of food each day. A worker ant will lose e_i "energy points" per gram of food they collect for each type of food due to how far away different types of food are. Each food source has an assigned "yum" score of y_i, representing the deliciousness factor as assigned by the queen ant. As the queen of the ant colony, you must decide how much of each food item is to be collected to maximize the yum score on a particular day while ensuring that the total energy expended stays under the total collective energy E, a colony has each day.

1.1 Determine whether the question defined above should be formulated as a Linear Programming (LP) or an Integer Linear Programming (ILP) problem and justify how your choice might impact whether a solution can be derived in polynomial time.

Both ILP and LP seek to find optimal values (maximum "yum" score in this question) by building an objective function of a set of decision variables that represent all possible actions within the system. In this problem, the amount of each food collected is not constrained to take integer values. The ants may collect 1g or 2.6g, or any number of grams, therefore this should be formulated using Linear Programming. LP problems are Polynomial (P). If we had chosen Integer Linear Programming, then there is no polynomial solution unless the assumption that $P \neq NP$ does not hold true.

- [**1 marks**] Correctly identifies the type of problem
- [**1 marks**] Justifies why it is LP
- [**1 marks**] Justifies how their choice impacts whether the solution is polynomial time

1.2 Define the variables, constraints and objective function for the defined problem?

Variables defined as:

$$\text{let } x_i = \text{the total weight of food i collected by the colony in grams}$$

Constraints defined as:

$$\sum_{i=1}^{n} x_i <= F$$

$$\sum_{i=1}^{n} x_i e_i <= E$$

Objective function defined as:

$$maximize \sum_{i=1}^{n} x_i u_i$$

- [**1 marks**] Correctly defines the variables and objective function
- [**1 marks**] Correctly identifies the constraints

1.3 An additional new food source has grown next to the colony, but can only be collected in 20 gram packets by each worker ant. Your colony decides to also start collecting from this food source. F, e_i, and y_i remain unchanged but you now also have e_{new} and y_{new}. Can this problem be formulated with Linear Programming? Justify your answer. Can a solution be derived in polynomial time?

This problem can no longer be formulated directly with a linear programming (LP) solution. We could try, by setting a value for x_{new} that is a multiple of 20, and then formulating the rest of the question as an LP algorithm around this by changing F to F - x_{new} , and changing E to E - $(x_{new} * e_{new})$. However, the number of different ways we could formulate this question is F/x_{new} so our time complexity becomes multiplied by F/x_{new}. This means that the time complexity depends on the magnitude of the input so this algorithm runs in pseudo-polynomial time, which makes it NP.

- [**1 mark**] Identifies that it is no longer LP
- [**1 mark**] Identifies that it cannot be solved in polynomial time