Materiais Elétricos e Magnéticos para Engenharia

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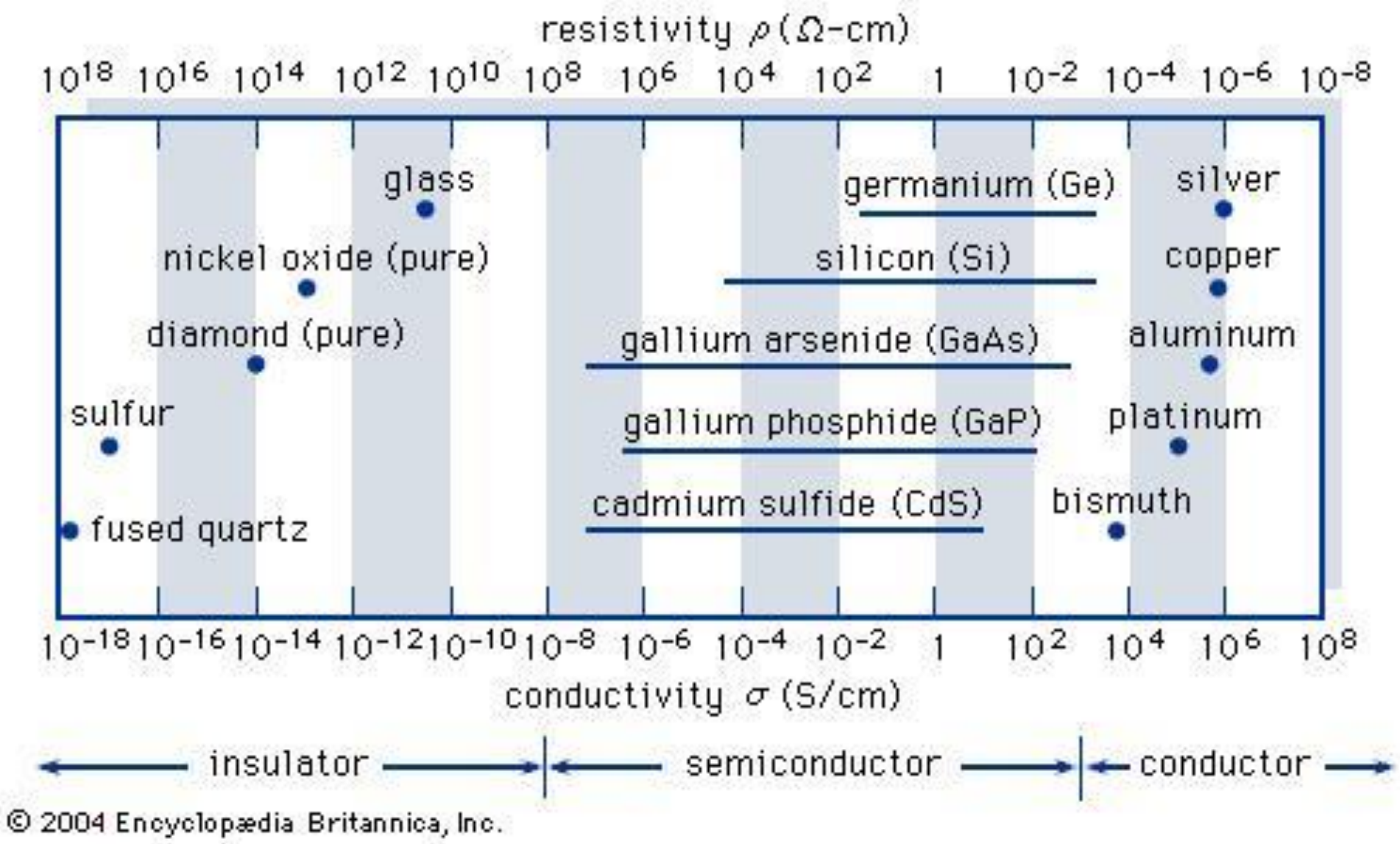
Laboratório #1

Resistência de Folha

1º Semestre de 2018

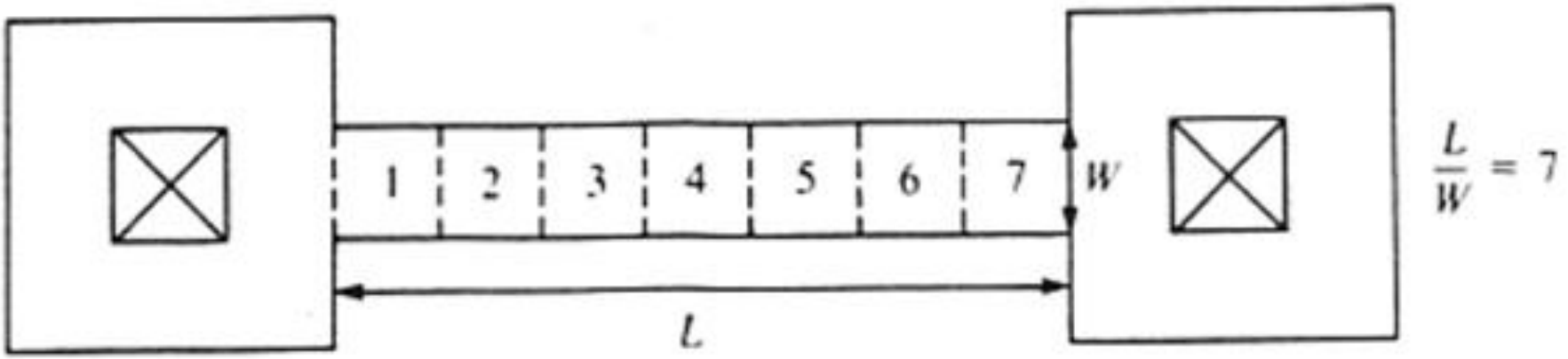
FGA - Universidade de Brasília

Condutividade e Resistividade



https://www.britannica.com/science/electrical-conductivity

Resistência de Folha

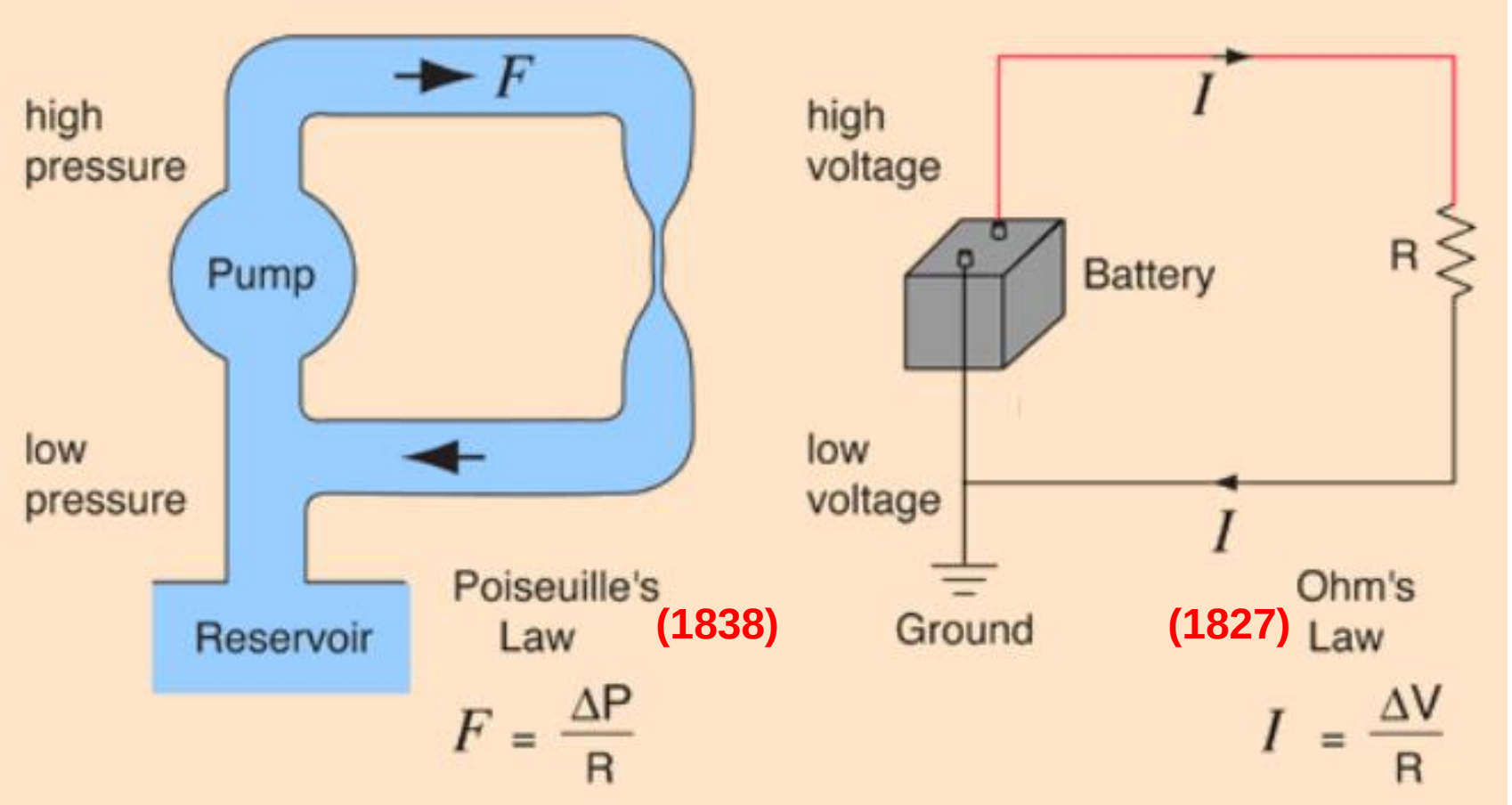


$$R = \rho \frac{L}{A} = \rho \frac{L}{t.W} = R_S \frac{L}{W}$$

$$R_S = \frac{\rho}{t} \ [\Omega/\square]$$

$$t = \frac{\rho}{R_S} \ [\mathrm{m}]$$

Analogia Hidráulica



http://hyperphysics.phy-astr.gsu.edu/hbase/electric/watcir.html

$$\Delta P = \frac{8\mu L F}{\pi r^4} = RF$$

$$R = \rho \frac{L}{A}$$

Fluidos Newtonianos, Incompressíveis, Não-turbulentos.

ΔP is the pressure difference between the two ends,

L is the length of pipe,

μ is the dynamic viscosity,

F is the volumetric flow rate,

r is the pipe radius.

Campo Estático resultante de Fluxo de Corrente Constante

$$\vec{J} = \sigma\vec{E} = -\sigma\nabla V$$

$$\nabla.\vec{J} = -\frac{\partial\rho}{\partial t} = 0$$

Fluxo Contínuo (Modelo Clássico)

$$\nabla.\vec{J} = \nabla.(\sigma\vec{E}) = \nabla.(-\sigma\nabla V) = 0$$

Constante

$$\nabla.(-\sigma\nabla V) = -\sigma\nabla.(\nabla V) = 0$$

$$\nabla.(\nabla V) = \nabla^2 V = 0$$

Equação de Laplace

Condutividade e Densidade de Corrente Elétrica

$$\vec{J} = \sigma \vec{E}$$

$$\frac{I}{A\sigma} = \frac{J}{\sigma} = E$$

Campo elétrico constante
ao longo de um condutor uniforme

$$\vec{E} = -\nabla V$$

$$\rho = \frac{1}{\sigma}$$

$$E = -\frac{\partial V}{\partial x}$$

$$V(x_2) = -\int_{x1}^{x_2} E dx = -Ex\Big|_{x_1}^{x_2} = -Ex_2 + Ex_1 = -Ex_2 + V(x_1)$$

Equação de Laplace

$$\nabla^2 f(x,y,z) = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} + \frac{\partial^2 f}{\partial z^2} = 0$$

Caso 1D: $$\frac{\partial^2 f}{\partial x^2} = 0$$

$$Lim_{\Delta x \to 0}\left[\frac{f(x+\Delta x) - 2f(x) + f(x-\Delta x)}{\Delta x^2}\right] = \frac{\partial^2 f}{\partial x^2}$$

$$\frac{\partial^2 f}{\partial x^2} \approx \left[\frac{f(x+\Delta x) - 2f(x) + f(x-\Delta x)}{\Delta x^2}\right] = 0$$

Equação de Laplace 1D

$$\frac{\partial^2 f(x)}{\partial x^2} = 0$$

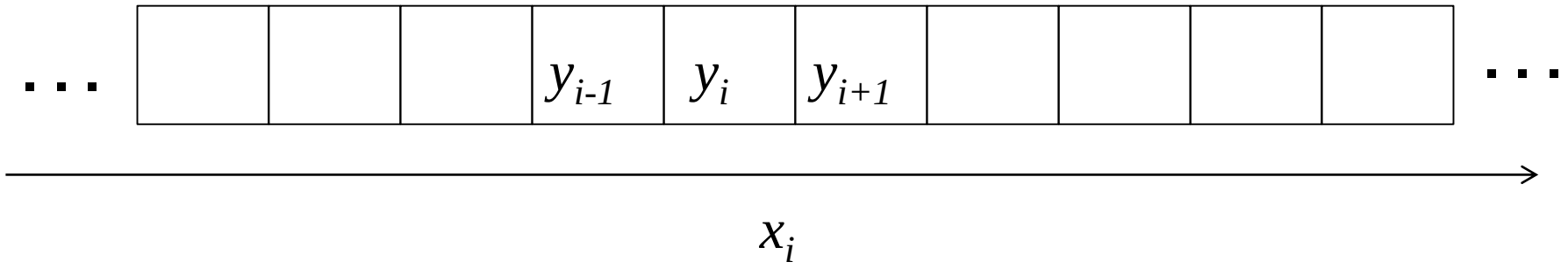
Forma da Solução da Equação de Laplace

$$f(x) = Kx + C$$ Equação da Reta

$$\frac{\partial f(x)}{\partial x} = K$$

$$\frac{\partial^2 f(x)}{\partial x^2} = \frac{\partial}{\partial x}\left(\frac{\partial f(x)}{\partial x}\right) = \frac{\partial}{\partial x}(K) = 0$$

Diferenças Finitas



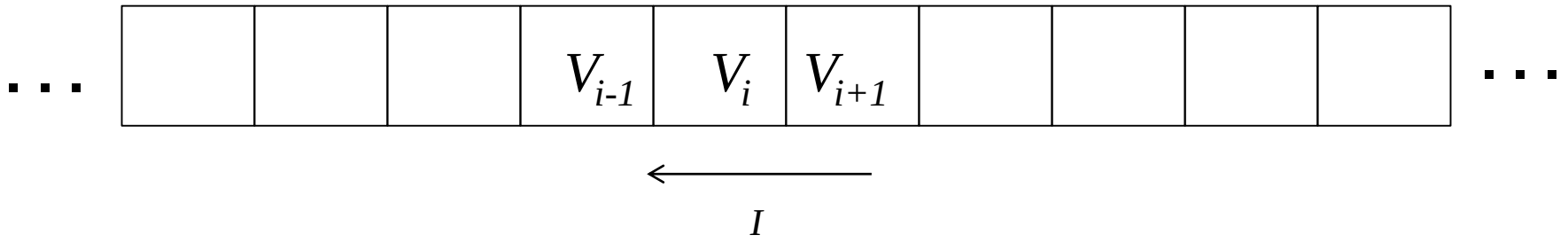
$$\frac{\partial^2 f}{\partial x^2} \approx \left[\frac{f(x_{i+1}) - 2f(x_i) + f(x_{i-1})}{\Delta x^2} \right] = 0$$

$$\frac{\partial^2 y}{\partial x^2} \approx \left[\frac{y_{i+1} - 2y_i + y_{i-1})}{\Delta x^2} \right] = 0$$

$$y_i = \frac{y_{i+1} + y_{i-1}}{2}$$

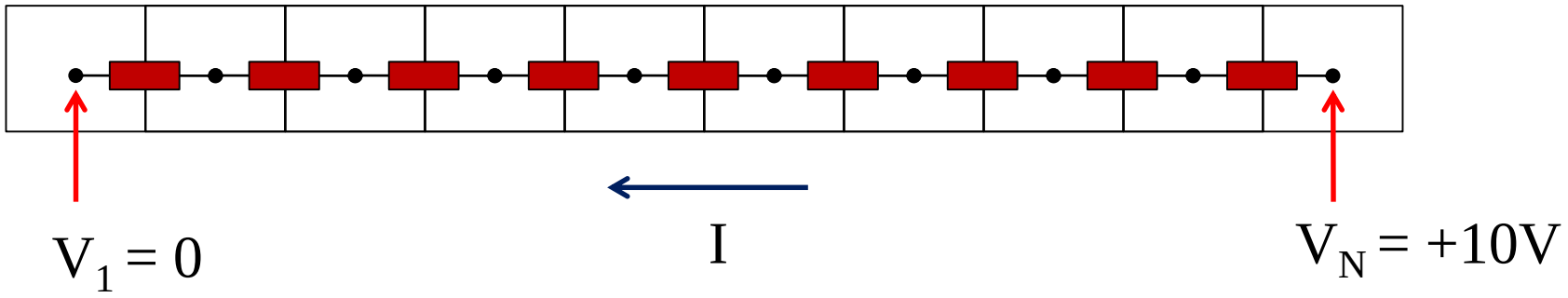
Diferenças Finitas – Potencial Elétrico

Trilha Resistiva



$$V_i = \frac{V_{i+1} + V_{i-1}}{2}$$

Potencial (Volt)

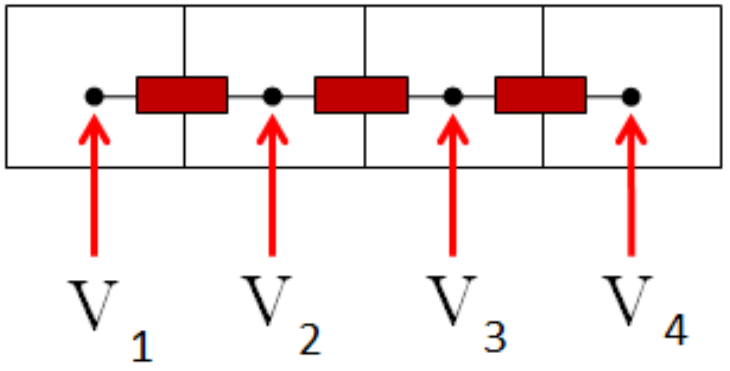


Diferenças Finitas – Potencial Elétrico

Trilha Resistiva com 4 nós

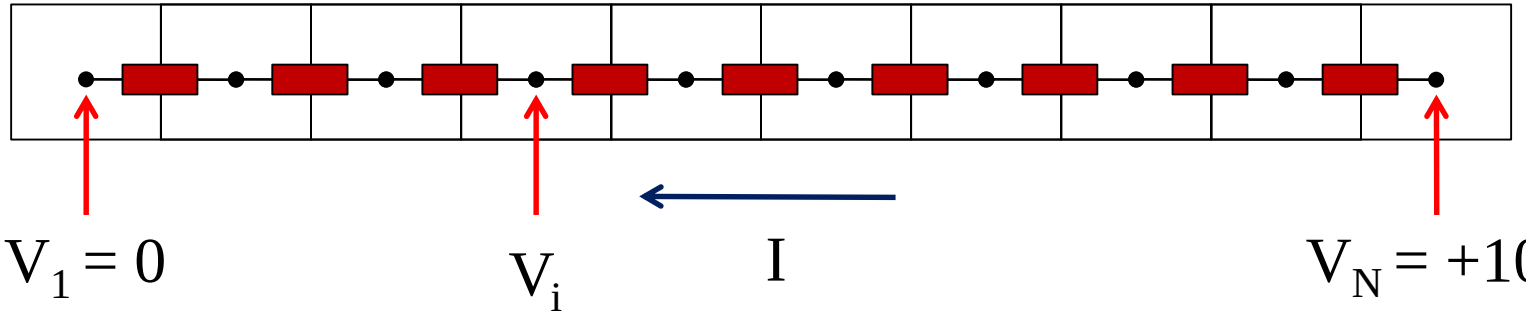
$$V_i = \frac{V_{i+1} + V_{i-1}}{2}$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ -1/2 & 1 & -1/2 & 0 \\ 0 & -1/2 & 1 & -1/2 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 10 \end{bmatrix}$$



Diferenças Finitas – Potencial Elétrico

Trilha Resistiva – Solução Analítica Discreta



$$I = \frac{V_i - V_1}{(i-1)R_S} = \frac{V_N - V_i}{(N-i)R_S}$$

$$\frac{V_i - V_1}{(i-1)} = \frac{V_N - V_i}{(N-i)}$$

$$\frac{V_i}{(i-1)} - \frac{V_1}{(i-1)} = \frac{V_N}{(N-i)} - \frac{V_i}{(N-i)}$$

Diferenças Finitas – Potencial Elétrico

Trilha Resistiva

$$\frac{V_i}{(i-1)} - \frac{V_1}{(i-1)} = \frac{V_N}{(N-i)} - \frac{V_i}{(N-i)}$$

$$(N-i)V_i - (N-i)V_1 = (i-1)V_N - (i-1)V_i$$

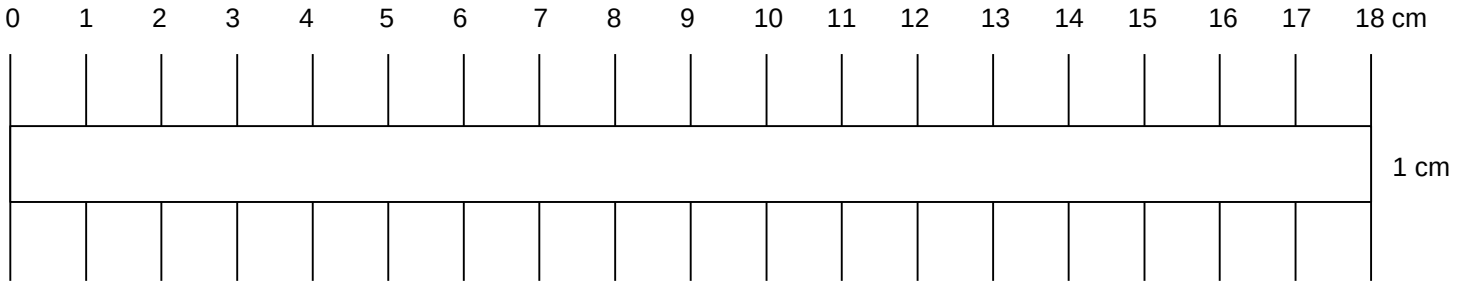
$$(N-i)V_i + (i-1)V_i = (i-1)V_N + (N-i)V_1$$

$$(N-1)V_i = (i-1)V_N + (N-i)V_1$$

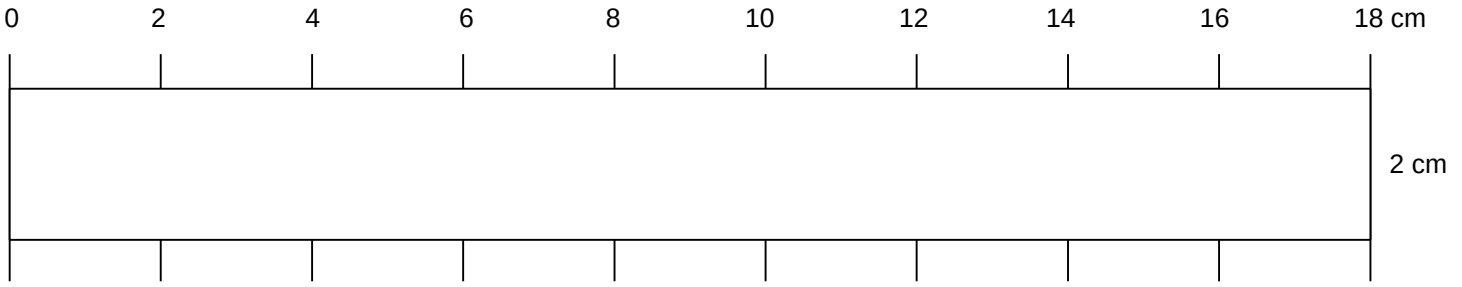
$$\boxed{V_i = \frac{(i-1)}{(N-1)}V_N + \frac{(N-i)}{(N-1)}V_1}$$

Experimento #1 – Resistência de Folha

Trilha-1:

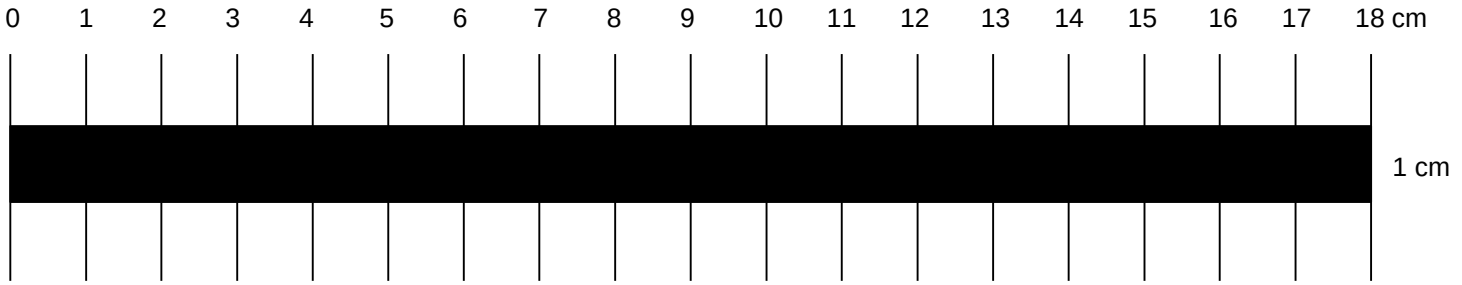


Trilha-2:



Preencher com lápis de grafite

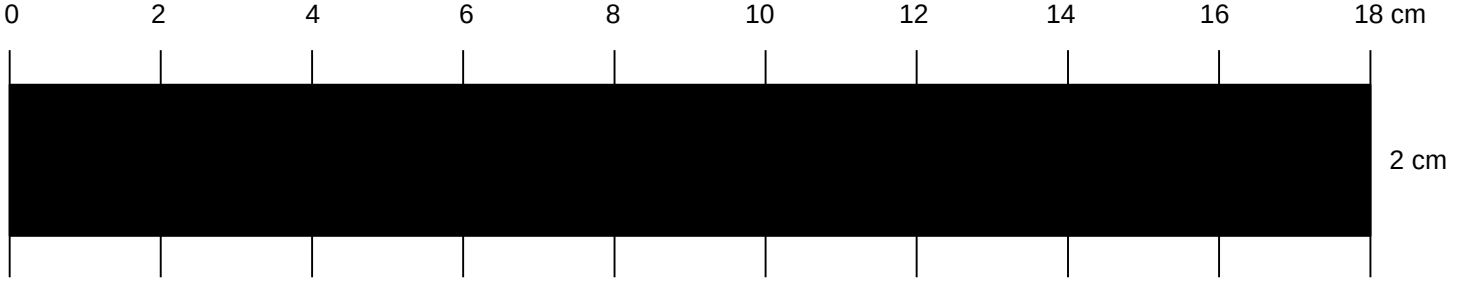
Trilha-1:



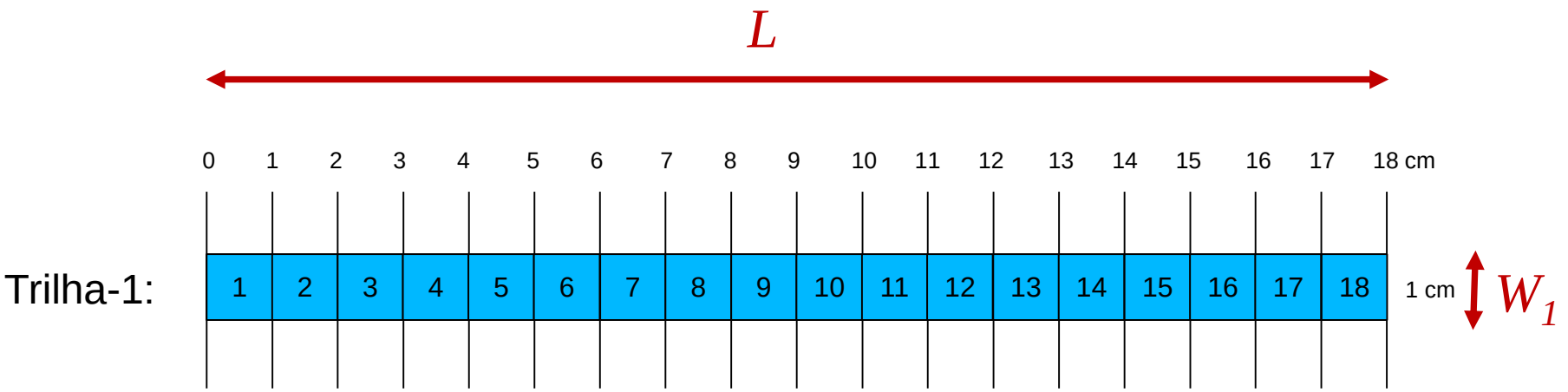
Não use borracha!

Faça o teste contra a luz!
(A trilha deve ser opaca)

Trilha-2:

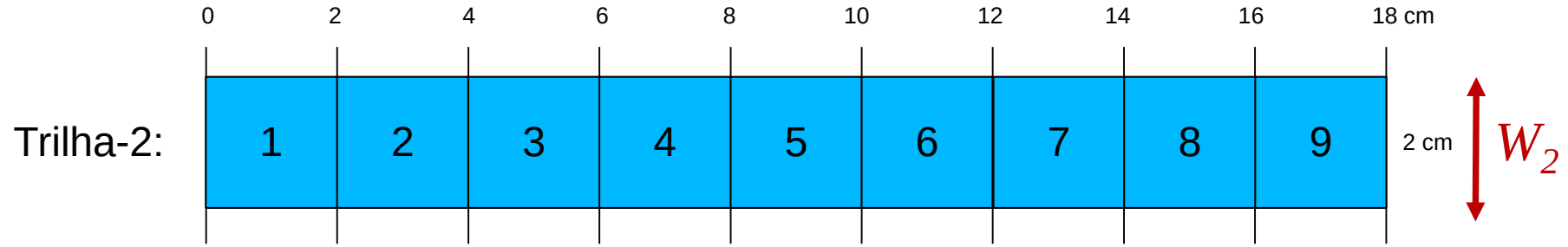


Experimento #1 – Resistência de Folha



Número de Quadrados:

$$N_1 = \frac{L}{W_1} \qquad N_2 = \frac{L}{W_2}$$



Posição dos Centros:

(i = 1, 2, 3, ..., N)

$$x_i = W(i-1) + W/2$$

Medida de Potencial na Trilha

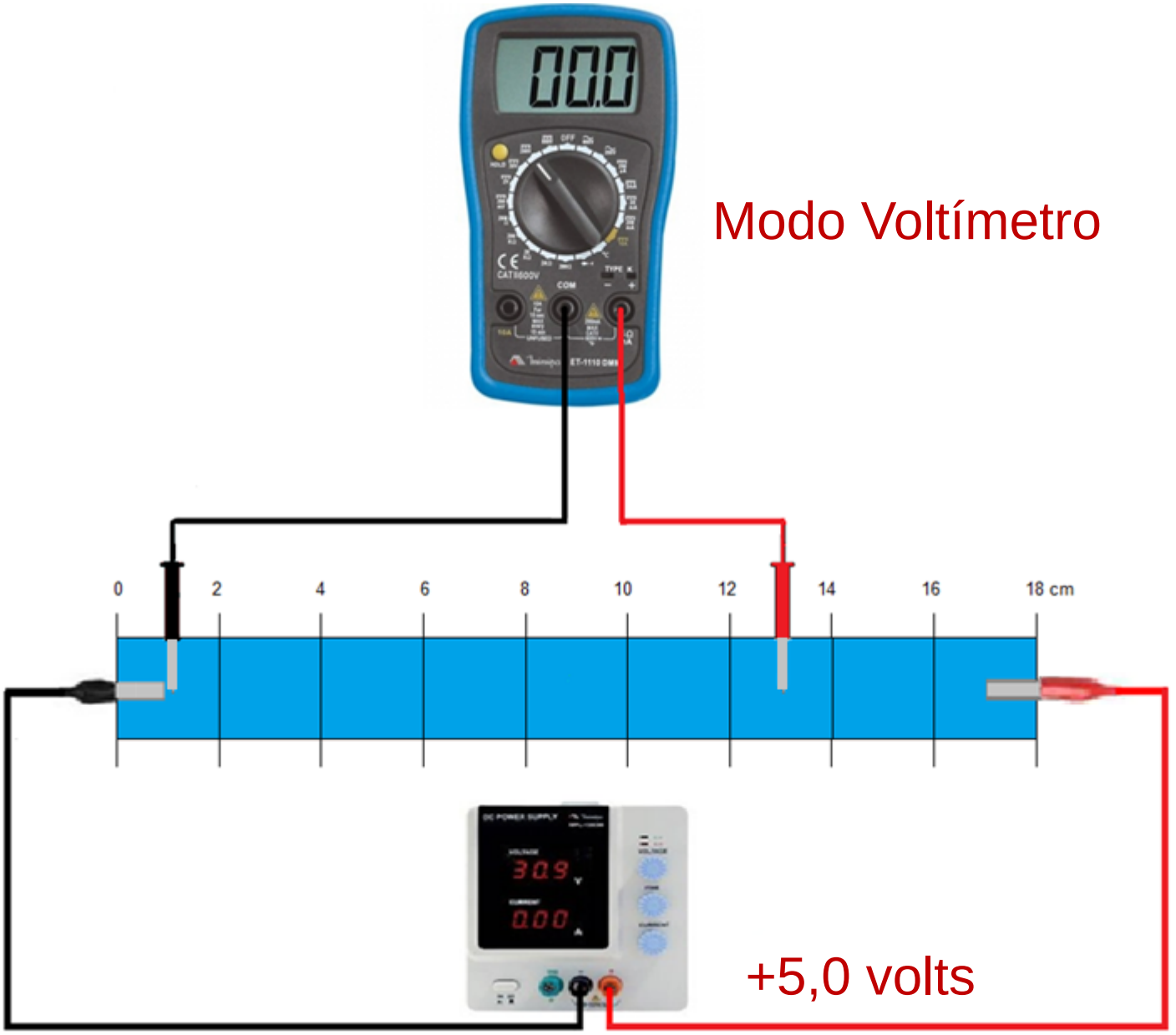
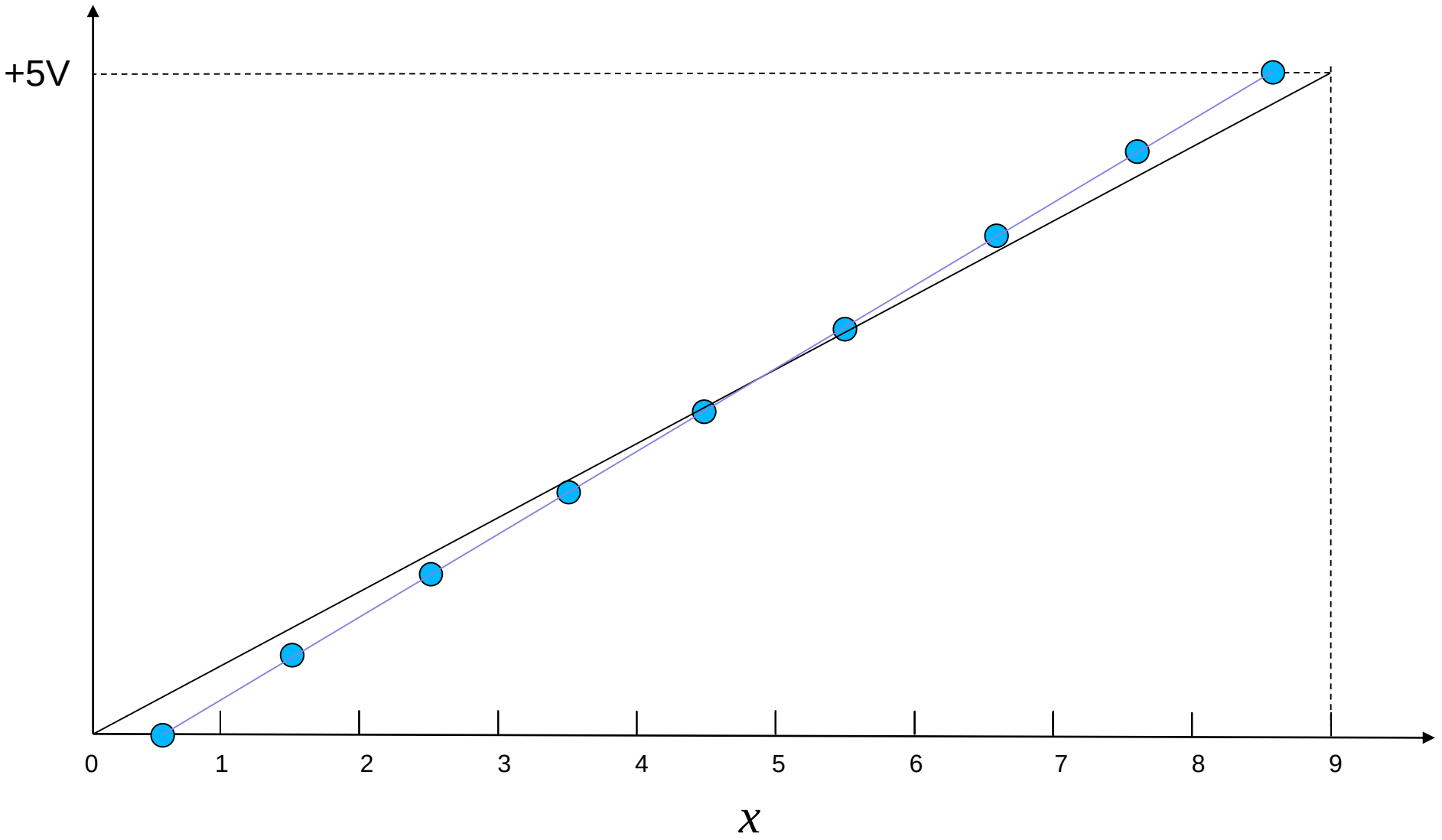


Gráfico do Potencial Teórico na Trilha

N = 9



Precisão e Acurácia

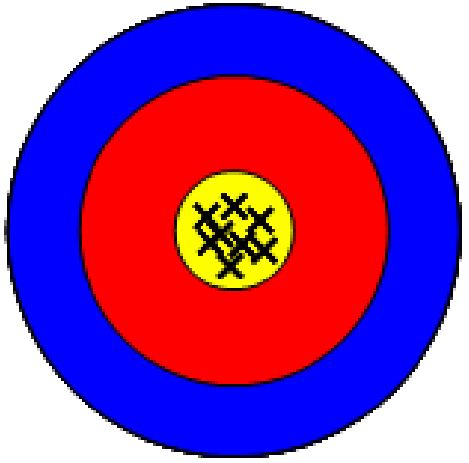
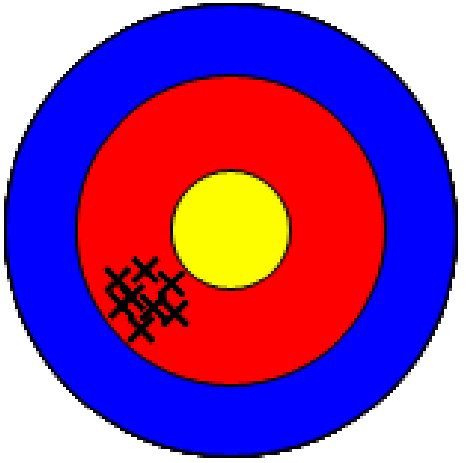
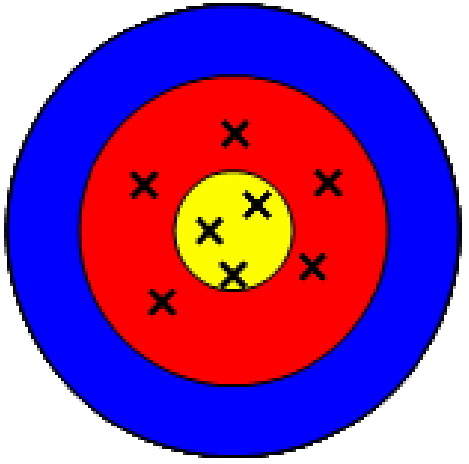
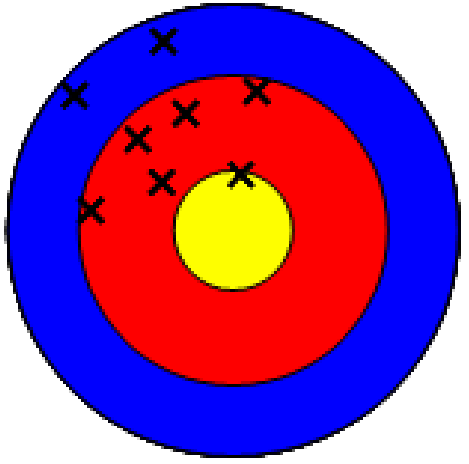
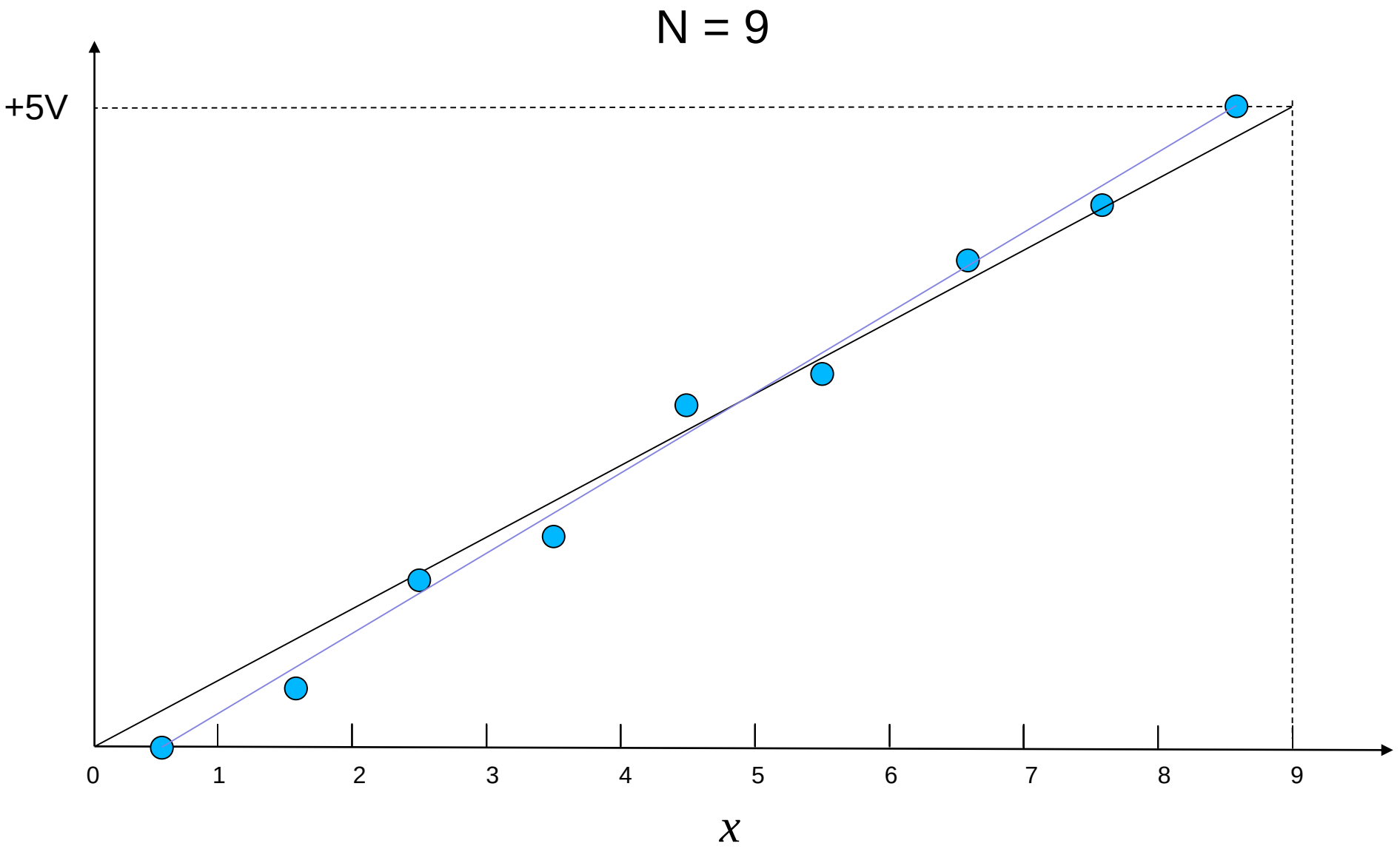
	Accurate	Inaccurate (systematic error)
Precise		
Imprecise (reproducibility error)		

Gráfico com Dispersão do Potencial na Trilha



Métodos dos Mínimos Quadrados

Ajuste de Funções usando Bases Não-ortogonais (N=3)

$$\varphi(x) = \alpha_1 g_1(x) + \alpha_2 g_2(x) + \alpha_3 g_3(x)$$

$$\begin{bmatrix} a_{11} & a_{21} & a_{31} \\ a_{12} & a_{22} & a_{32} \\ a_{13} & a_{23} & a_{33} \end{bmatrix} \begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$$

$$a_{ij} = \sum_{k=1}^{m} g_j(x_k) g_i(x_k) = a_{ji}$$

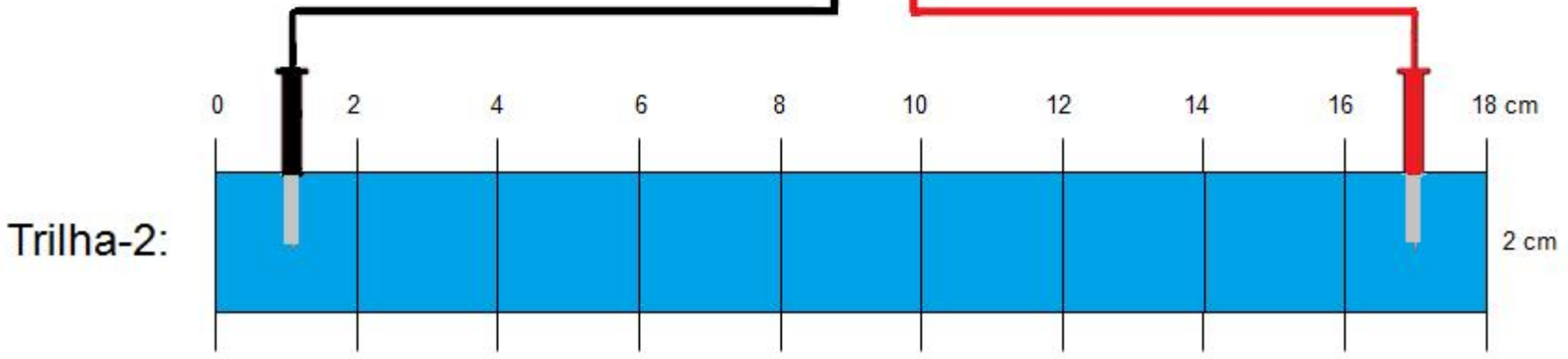
$$b_i = \sum_{k=1}^{m} f(x_k) g_i(x_k)$$

Medida da Resistência da Trilha

$$R_{T1} = R_S\left(\frac{L}{W_1} - 1\right) = R_S(18-1)$$

$$R_{T2} = R_S\left(\frac{L}{W_2} - 1\right) = R_S(9-1)$$

Modo Ohmímetro



Resistividade

Resistividade do Grafite: $\rho = 7{,}8 \times 10^{-6}$ [Ω.m]

Resistividade do Carbono (Amorfo): $\rho = 3{,}5 \times 10^{-5}$ [Ω.m]

Resistividade do Grafeno: $\rho \sim 10^{-8}$ [Ω.m]

Resistividade do Cobre: $\rho = 1{,}72 \times 10^{-8}$ [Ω.m]

Resistividade da Prata: $\rho = 1{,}59 \times 10^{-8}$ [Ω.m]

Resistividade da Níquel: $\rho = 6{,}99 \times 10^{-8}$ [Ω.m]

Estrutura do Grafite