Causal Inference - HW1

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November 17, 2019

Question 1

Using potential outcomes notation, give an example of a data generating process (a joint distribution) which includes a hidden confounder H, a binary treatment T, and two potential outcomes Y_0 and Y_1, such that:

Ignorability does not hold, and

$$E(Y_1 - Y_0) \neq E[Y|T = 1] - E[Y|T = 0]$$

where

$$Y = T * Y_1 + (1 - T) * T_0$$

Solution

Let

Y_0	Y_1	H	T	Y
1	1	1	1	1
0	1	0	0	0

We can see that ignorability does not hold. e,g $(Y_0, Y_1) \not\perp T|H$ because $(Y = T|H)$.

$$E[Y|T = 1] - E[Y|T = 0] = 1 - 0 = 1 \neq E(Y_1 - Y_0) = 0.5$$

Question 2

Let $(x_1, t_1, y_1).....(x_n, t_n, y_n)$ be a sample from a randomized con- trolled trial, where for each $i = 1...n, x_i \in R$ is a covariate measured before treatment assignment, $t_i \in \{0, 1\}$ is a binary treatment, and $y_i \in R$ is an outcome measured after the treatment. Let $\pi : R \to \{0, 1\}$ be a policy: a function which for each value of the covariate x assigns a treatment 0 or 1. Let $V(\pi) = E(y|t = \pi(x))$ be the value of the policy $\pi(x)$: what we expect the outcome to be if treatment were assigned according to $\pi(x)$, as opposed to randomly. Give an unbiased estimator of $V(\pi(x))$ for the sample above.

Solution

Let

$$V(\pi) = E(y|t = \pi(x)) = E(y|\pi(x), T = 1) \cdot p(T = 1) + E(y|\pi(x), T = 0) \cdot p(T = 0)$$

(Law of total expectation)
This is already enough as an unbiased estimator since:

$$E(E(y|\pi(x), T = 1) \cdot p(T = 1) + E(y|\pi(x), T = 0) \cdot p(T = 0)) = E(y|\pi(x), T = 1) \cdot p(T = 1) + E(y|\pi(x), T = 0) \cdot p(T = 0)$$

If needed, can further develop:

$$E(y|\pi(x), T = 1) \cdot p(T = 1) = E(y|\pi(x) = 0, T = 1) \cdot p(T = 1, \pi(x) = 0) + E(y|\pi(x) = 1, T = 1) \cdot p(T = 1, \pi(x) = 1)$$

And vice versa for $E(y|\pi(x), T = 1) \cdot p(T = 0)$

Question 3

Give an example of a dataset with features X and one or more observed outcome variables $Y_1 .... Y_k$. For this dataset give:

1. Two examples of interesting causal questions relating one of the features and one of the outcomes. Explain what would be the treatment and what would be the potential outcomes in this case.
2. Two examples of interesting prediction questions which do not require causal reasoning.

Examples can come from the fields of politics, biology, sports, economics, entertainment, medicine, transportation and so on - use your imagination.

Solution