I $x=t$ $y=0$ $t\in[0;1]$

$u=t^2$ $v=0$

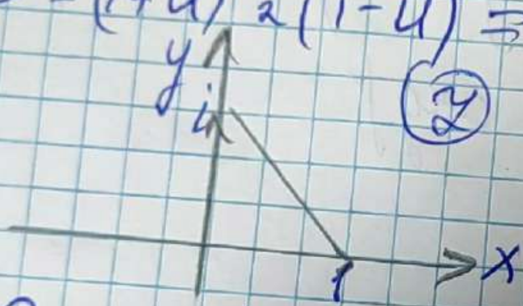
II $x=0$ $y=t$

$u=-t^2$ $v=0$

III $y=1-t$ $x=t$

$u=2t-1$ $v=2t(1-t) \to t=\frac{u+1}{2}$

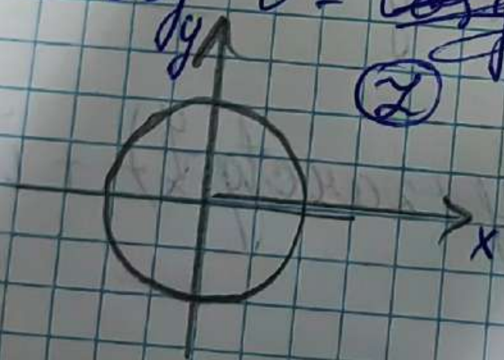
$v=(1+u)\frac{1}{2}(1-u)=\frac{1}{2}-\frac{1}{2}u^2$

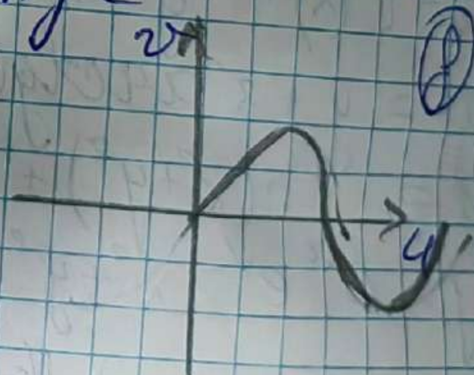


2) $C: z=it$ $0\le t\le 2\pi$ $w=e^z$

$f(z)=e^z=e^{x+iy}=e^x(\cos y+i\sin y)$

$u=e^x\cos y$ $v=\sin y\, e^x$





$\cancel{\text{I } x=t \quad y=0 \quad u=e^t \quad v=0 \quad t=\ln u}$

II $x=0$ $y=t$

$u=\cos t$ $v=\sin t$

$\cancel{t=\arcsin v}$

$u=\cos\arcsin v=\sqrt{1-v^2}$

$$\frac{\partial v}{\partial x} = e^x(\cancel{x\sin y} + \cancel{y\cos y} + \cancel{2\sin y}) + e^x \sin y + c'(x) =$$
$$= -e^x(-\cancel{x\sin y} - \cancel{\sin y} + y\cos y)$$
$$c(x) = -\int 2e^x \sin y = -2e^x \sin y + C \qquad C = 0$$
$$v = e^x(x\sin y + y\cos y)$$
$$f = xe^x e^{yi} + yie^x e^{yi} = ze^z$$

5) $|f| = (x^2+y^2)e^x \qquad \omega = \ln f = \ln f e^{i\arg f} = \ln|f| + i\arg f$

$$\operatorname{Re}\omega = \ln|f| \qquad \operatorname{Im}\omega = \arg f$$
$$u = \ln(x^2+y^2) + x$$
$$\frac{\partial u}{\partial x} = \frac{2x}{x^2+y^2} + 1 = \frac{\partial v}{\partial y}$$
$$v = y + \cancel{\frac{2\operatorname{arctg}(\frac{y}{x})}{\partial x}} + c(x) + 2\operatorname{arctg}\left(\frac{y}{x}\right)$$
$$\frac{\partial v}{\partial x} = c'(x) - \frac{2y}{y^2+x^2} = -\frac{2y}{x^2+y^2} \qquad \underline{c'(x) = 0}$$
$$v = y + 2\operatorname{arctg}\left(\frac{y}{x}\right) + c$$
$$f = \ln(x^2+y^2) + x + i\left(y + 2\operatorname{arctg}\frac{y}{x}\right) + ic$$
$$f = e^{ic} e^z \quad x^2+y^2 \quad ?$$

9.16.

1) $C: z = it + 1 \quad 0 \le t \le 1; \quad \omega = z^2$

$$\omega = z^2 = u + iv = x^2 - y^2 + 2ixy$$
$$u = x^2 - y^2, \quad v = 2xy$$

Диффиренц. ~~8.51~~: $\frac{((n+1)\varepsilon^n - 1)(\varepsilon - 1) - 1\cdot(\varepsilon^{n+1} - \varepsilon)}{(\varepsilon-1)^2}$

Подставляя сюда $\varepsilon^n = 1$ получаем:

$$\frac{((n+1)-1)(\varepsilon-1) - 1(\varepsilon - \varepsilon)}{(\varepsilon-1)^2}$$

$$\frac{n(\varepsilon-1)}{(\varepsilon-1)^2} = \frac{n}{\varepsilon - 1}$$

8.51.

1) $\operatorname{Re} f = x^3 + 6x^2y - 3xy^2 - 2y^3 \qquad f(0) = 0$

$$\frac{\partial u}{\partial x} = 3x^2 + 12xy - 3y^2 = \frac{\partial v}{\partial y}$$

$$v(x,y) = 3x^2y + 6xy^2 - y^3 + c(x)$$

$$\frac{\partial v}{\partial x} = \cancel{6xy} + \cancel{6y^2} + c'(x) = -6x^2 + \cancel{6xy} + \cancel{6y^2}$$

$$c(x) = -\int 6x^2\,dx = -2x^3 + C \qquad C = 0$$

$$v = 3x^2y + 6xy^2 - y^3 - 2x^3$$

$$f = u + iv = (x+iy)^3 - 2i(x+iy)^3 = z^3(1-2i)$$

2) $\operatorname{Re} f = e^x(x\cos y - y\sin y) \qquad f(0) = 0$

$$\frac{\partial u}{\partial x} = e^x(x\cos y - y\sin y) + e^x\cos y = \frac{\partial v}{\partial y}$$

$$v(x,y) = e^x(x\sin y + y\cos y + \sin y + \sin y) + c(x)$$

$$=(-1)^{n+1}\frac{i(\sin n\Theta\cos\frac{\Theta}{2}-\cos n\Theta\sin\Theta/2)}{-\cos\Theta}\quad \operatorname{Im} e^{i\Theta+\frac{(2n-1)}{2}i\Theta - i\Theta}$$

$$=(-1)^{n+1}\frac{\cos\frac{(2n-1)\Theta}{2}}{\cos\Theta}\cdot\sin\frac{(2n-1)\Theta}{2}=(-1)^{n+1}\frac{\sin(2n-1)\Theta}{2\cos\Theta}$$

?

1.53.

$z^n=1$

$|z|^n\cdot(\cos n\varphi+i\sin n\varphi)=1(\cos 0+i\sin 0)$

$$\begin{cases}|z|^n=1\\ n\varphi=0+2\pi k\end{cases}$$

в случае $|z|^n=1$ $z=1$ и в таком случае n может быть любым.

в случае $n\varphi=2\pi k$ означает также что существует n различных решений для φ. $\varphi=\frac{2\pi k}{n}$

$\varphi_1=\frac{2\pi k}{n}$ $\varphi_2=\frac{2\pi(k+1)}{n}$ ... $\varphi_n=\frac{2\pi(k+n-1)}{n}$

1.60.

1) $1+2\varepsilon+3\varepsilon^2+\ldots+n\varepsilon^{n-1}=\frac{n}{\varepsilon-1}$ $\varepsilon^n=1$ $\cancel{\varepsilon=e^{\frac{2\pi k}{n}}}$

данная сумма является производной суммы $\varepsilon+\varepsilon^2+\varepsilon^3+\ldots+\varepsilon^n$

$S_n=\frac{\varepsilon(-1+\varepsilon^n)}{-1+\varepsilon}=\frac{\varepsilon^{n+1}-\varepsilon}{\varepsilon-1}$

$$\operatorname{Im} e^{i\theta}\frac{e^{in\theta}-1}{e^{i\theta}-1}=\operatorname{Im} e^{i\theta}\frac{e^{in\theta}\cdot 2i\sin\frac{n\theta}{2}}{e^{i\theta/2}\cdot 2i\sin\frac{\theta}{2}}=$$

$$=\frac{\sin\frac{n\theta}{2}}{\sin\frac{\theta}{2}}\cdot\operatorname{Im} e^{i\theta+in\theta-i\theta/2}=\frac{\sin\frac{n\theta}{2}}{\sin\frac{\theta}{2}}\sin\frac{\theta(n+1)}{2}$$

1.52.

1) $\cos\theta+\cos 3\theta+\ldots+\cos(2n-1)\theta=\operatorname{Re}(e^{i\theta}+e^{3i\theta}+\ldots+e^{(2n-1)i\theta})$

$$\operatorname{Re} e^{i\theta}\frac{e^{i(2n-1)\theta}-1}{e^{2i\theta}-1}=\operatorname{Re} e^{i\theta}\frac{e^{(2n-1)\frac{\theta}{2}}\,2i\sin\frac{(2n-1)\theta}{2}}{e^{i\frac{\theta}{\cancel{2}}}\,2i\sin\frac{\theta}{\cancel{2}}}=$$

$$=\frac{\sin\frac{(2n-1)\theta}{2}}{\sin\frac{\theta}{\cancel{2}}}\operatorname{Re} e^{i\theta+(2n-1)i\frac{\theta}{2}-i\frac{\theta}{\cancel{2}}}=\frac{\sin\frac{(2n-1)\theta}{2}}{\sin\frac{\theta}{\cancel{2}}}\cdot$$

$$\cdot\frac{\cos}{\cancel{\sin}}\frac{2n\theta}{2}=\frac{\sin(n-\frac{1}{2})\theta}{\sin\frac{\theta}{\cancel{2}}}\cdot\cos(n-\tfrac{1}{2})\cancel{\sin n}\theta=$$

$$=\cancel{\frac{\cos n\theta\cdot(\sin n\theta\cos\frac{\theta}{2}-\cos n\theta\sin\frac{\theta}{2})}{\sin\frac{\theta}{2}}}=$$

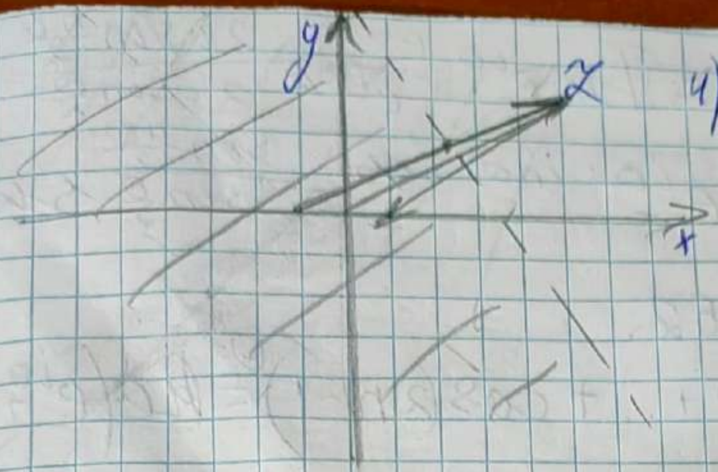
$$=\frac{1}{2}\frac{\sin 2n\theta\cdot\cancel{\cos\frac{\theta}{2}}}{\sin\frac{\theta}{\cancel{2}}}\cancel{-\cos^2 n\theta}$$

2) $\sin\theta-\sin 3\theta+\ldots+(-1)^{n+1}\sin(2n-1)\theta=(-1)^{n+1}\cdot\frac{\sin 2n\theta}{2\cos\theta}$

$$\operatorname{Im}(\sin\theta-\sin 3\theta+\ldots+(-1)^{n+1}\sin(2n-1))=$$

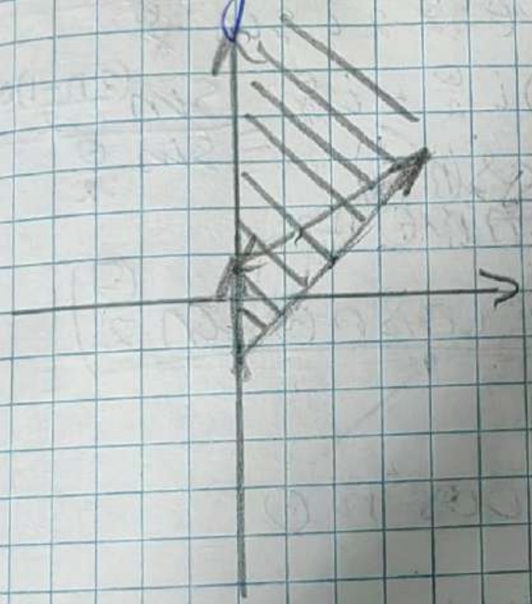
$$=\operatorname{Im}(e^{i\theta}-e^{3i\theta}+\ldots+(-1)^{n+1}e^{(2n-1)i\theta})$$

$$\operatorname{Im} e^{i\theta}\frac{(-1)^{n+1}e^{(2n-1)i\theta}-1}{-(e^{2i\theta}+1)}=\operatorname{Im} e^{i\theta}\frac{(-1)^{n+1}e^{\frac{(2n-1)i\theta}{2}}\cdot 2\cos}{+e^{i\theta}\cdot 2\cos\theta}=$$

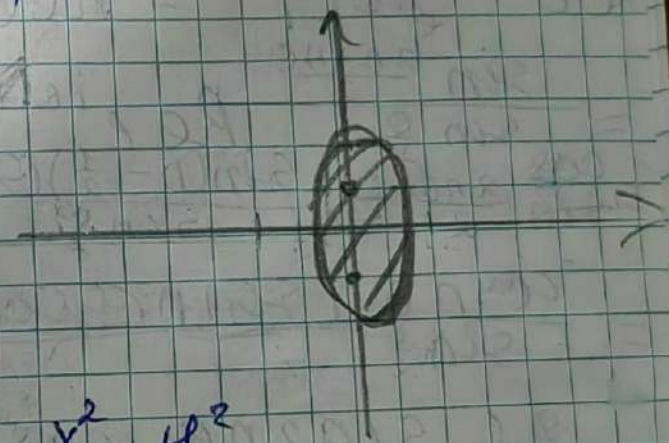


4) $|1+z| < |1-z|$

5) $0 < \arg \frac{i-z}{z+i} < \frac{\pi}{2}$



1) $|z-i|+|z+i| < 4$



$$\frac{x^2}{3}+\frac{y^2}{4}=1$$

1.51.

1) $\sin\theta + \sin 2\theta + \dots + \sin n\theta = \frac{\sin\frac{n+1}{2}\theta}{\sin\frac{\theta}{2}}\cdot\sin\frac{n\theta}{2}$

$\mathrm{Im}\left(e^{i\theta}+e^{2i\theta}+\dots+e^{ni\theta}\right)$

$$q+q^2+\dots+q^n = q\,\frac{q^n-1}{q-1}$$

7) $(-4+3i)^3 = -27i + 108 + 144i - 64 = 44 + 117i$

$|z| = \sqrt{161}$ $\quad \varphi = \operatorname{arctg}\left(\frac{117}{44}\right)$

8) $(1+i)^8 (1-i\sqrt{3})^{-6} = \frac{(1+i)^8 \cdot (1+i\sqrt{3})^6}{4^6} = \frac{16 \cdot 16 (1-i\ldots}{16 \cdot 64}$

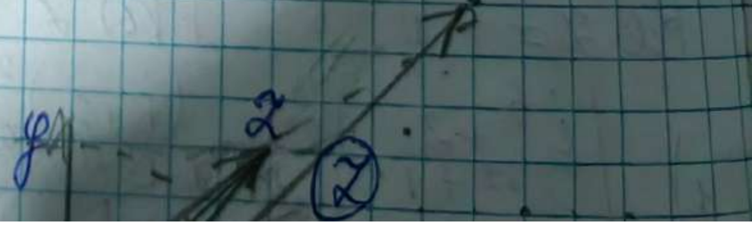
$= \frac{1}{4}(1-i\sqrt{3})$ $\quad |z| = \frac{1}{4}$ $\quad \varphi = \frac{5\pi}{3}$

9) $1 + \cos\frac{\pi}{7} + i\sin\frac{\pi}{7} = 2\cos^2\frac{\pi}{14} + 2i\cos\frac{\pi}{14}\sin\frac{\pi}{14} =$

$= 2\cos\frac{\pi}{14}\left(\cos\frac{\pi}{14} + i\sin\frac{\pi}{14}\right)$ $\quad |z| = 2\cos\frac{\pi}{14}$ $\quad \varphi = \frac{\pi}{14}$

1.21.

1) $|z-i| + |z+i| < 4$



1.04.

1) $\frac{1}{1-i} = \frac{1+i}{(1-i)(1+i)} = \frac{1+i}{2} = \frac{1}{2}(1+i)$ $\{ Re\, z = \frac{1}{2}; Im\, z = \frac{1}{2}$

2) $\left(\frac{1-i}{1+i}\right)^3 = \left(\frac{(1-i)^2}{(1+i)(1-i)}\right)^3 = \left(\frac{1-2i-1}{2}\right)^3 = (-i)^3 = i$

$\cancel{= 1-3i-3+i = -2i-2}$ $Re\, z = 0$, $Im\, z = 1$.

3) $\left(\frac{1}{2} - i\frac{\sqrt{3}}{2}\right)^3 = \left(\frac{1}{2}\right)^3 - 3\left(\frac{1}{2}\right)^2 \cdot i\frac{\sqrt{3}}{2} + 3\left(\frac{1}{2}\right)\left(i\frac{\sqrt{3}}{2}\right)^2 - \left(i\frac{\sqrt{3}}{2}\right)^3 = \frac{1}{8} - \frac{3\sqrt{3}i}{8} - \frac{9}{8} + \frac{3\sqrt{3}i}{8} = -1$

$Re\, z = -1$ $Im\, z = 0$.

4) $\left(\frac{i^5+2}{i^{19}+1}\right)^2 = \frac{(i+2)^2}{(1-i)^2} = \frac{3+4i}{-2i} = \frac{6i-8}{2} = 3i-4$

$Re\, z = -4$ $Im\, z = 3$

5) $\frac{(1+i)^5}{(1-i)^3} = \frac{(1+i)^8}{8} = \frac{(2i)^4}{8} = \frac{(-4)^2}{8} = \frac{16}{8} = 2$

$Re\, z = 2$ $Im\, z = 0$.

1.06.

1) i. $|z| = 1$ $\varphi = \frac{\pi}{2}$

2) -3 $|z| = 3$ $\varphi = 0$

3) $1 + i^{123}$ $|z| = \sqrt{2}$ $\varphi = -\frac{\pi}{4}$

4) $-\frac{1}{2} + i\frac{\sqrt{3}}{2}$ $|z| = 1$ $\varphi = \frac{2\pi}{3}$

5) $\frac{1-i}{1+i} = -i$ $|z| = 1$ $\varphi = \frac{3\pi}{2}$

6) $-\cos\frac{\pi}{7} + i\sin\frac{\pi}{7} = \cos\left(\pi - \frac{\pi}{7}\right) + i\sin\left(\pi - \frac{\pi}{7}\right)$ $|z| = 1$ $\varphi = \frac{6\pi}{7}$