$$z^2 \cdot \left( \pi \frac{z+1}{z} + \sum (-1)^n \left( \pi \frac{z+1}{z} \right)^{2n+1} \cdot \frac{1}{(2n+3)!} \right) =$$

$$= \pi z^2 + \pi z + \sum_1^{\infty} (-1)^n \pi^{2n+1} (z^2+z)^{2n+1} / (2n+3)!$$

3) $z^3 \cos \frac{1}{z-2}$ $\quad a=2, \; 0 < |z-2| < \infty$

$\cancel{f(z) = z^3 - \frac{z^3}{2z-4} + \sum \frac{(-1)^n z^3}{}}$

$w = \frac{1}{z-2} \qquad z^3 = \left(\frac{1}{w} - 2\right)^3$

$$\left(\frac{1}{w} - 2\right)^3 \cdot \cos w = \left(\frac{1}{w} - 2\right)^3 \cdot \left(1 - \frac{w^2}{2!} + \frac{w^4}{4!} \ldots\right)$$

$\cancel{\frac{1}{w} - \frac{w}{2!} + \frac{w^3}{4!} \ldots}$

5) $\frac{e^{\frac{1}{z-1}}}{z(1+z)}$ $\qquad a=1 \quad 1 < |z-1| < 2.$

$t = \frac{1}{z-1} \qquad z = \frac{1}{t} + 1$

$\cancel{\frac{e^t (\frac{1}{t}+1)}{(1+\frac{1}{t+1})}} \qquad \frac{e^{\frac{1}{t}}}{(t+1)(t+2)}$

$$\frac{1}{(t+1)(t+2)} \cdot \left(1 + t^{-1} + \frac{t^{-2}}{2!} + \ldots\right)$$

$\cancel{f(z) = \frac{1}{z+1}}$ $f(z) = \frac{z^2}{(1+\frac{1}{z})(1-\frac{2}{z})}$ $t = \frac{1}{z}$ $f(z) = z\, g(t)$

$f(t) = \frac{1}{t^2(1+t)(1-2t)} = \frac{1}{t} + \cancel{\frac{1}{t^2}} + \frac{4}{3}\cdot\frac{1}{1-2t} + \frac{(-1)}{3}\frac{1}{t+1}$

$\frac{1}{(t+1)-1} = \frac{1}{(t+1)}\cdot\frac{1}{1-\frac{1}{(t+1)}} = \sum \frac{1}{(t+1)^n}$

$\frac{1}{1-2t} = \frac{1}{2-(t+1)}$

3) $\frac{1}{(z^2-1)(z^2+4)}$ $a=0$ $|z|>2$

$f(z) = \frac{1}{10}\left(\frac{1}{z-1} - \frac{1}{z+1}\right) + \left(-\frac{1}{5}\right)\frac{1}{z^2+4}$

$\frac{1}{z-1} = \frac{1}{z}\,\frac{1}{1-\frac{1}{z}} = \sum \frac{1}{z^{n+1}}$

$\frac{1}{z+1} = -\frac{1}{z}\,\frac{1}{1-\frac{1}{z}} = -\sum \frac{1}{z^{n+1}}$

$\frac{1}{z^2+4} = \frac{1}{z^2}\,\frac{1}{1+\frac{4}{z^2}} = \sum \frac{4^n(-1)^n}{z^{2n+2}}$

$f(z) = \frac{1}{5} + \sum_{-\infty}^{-1} \frac{(-1)^{n+1}\, z^{2n+2}}{4^n\cdot 5}$

20.16.

1) $z^3 e^{1/z}$ $a=0$ $0<|z|<\infty$

$z^3\left(1 + \frac{1}{z} + \frac{1}{2!\,z^2} + \frac{1}{3!\,z^3} + \dots\right) = z^3 + z^2 + \frac{z}{2} + \frac{1}{6} +$
$+ \sum_{1}^{\infty} \frac{z^{-n}}{(n+3)!}$

2) $z^2 \sin \pi \frac{z+1}{z}$ $a=0$ $0<|z|<\infty$

20.09.

1) $\frac{1}{z(z-3)^n}$ $(a=1;\ 1<|z-1|<2)$

$f(z) = \frac{1}{9}\left(\frac{1}{z} + \frac{1}{z-3}\right) + \frac{1}{3}\frac{1}{(z-3)^2}$

1) $\frac{1}{(z-1)+1} = \frac{1}{(z-1)}\frac{1}{1+\frac{1}{z-1}} = \frac{1}{(z-1)}\sum_{n=0}^{\infty}\frac{1}{(-1)^n(z-1)^n}$

2) $\frac{1}{(z-1)-2} = \frac{1}{-2}\cdot\frac{1}{1-\frac{z-1}{2}} = -\frac{1}{2}\sum_{n=0}^{\infty}\left(\frac{z-1}{2}\right)^n$

3) $\frac{1}{(z-1)^2-4} = -\frac{1}{4}\cdot\frac{1}{1-\frac{(z-1)^2}{2}} = -\frac{1}{4}\sum_{n=0}^{\infty}\left(\frac{z-1}{2}\right)^n\cdot(n+1)$

$f(z) = \sum_{-\infty}^{-1}\frac{(-1)^n(z-1)^{n+1}}{9} + \sum_{n=0}^{\infty}$

3) $\frac{1}{z(z-1)(z-2)}$ $(a=0,\ -\frac{3}{2}\in D)$

$1<|z|<2$

$f(z) = \frac{1}{2}\left(\frac{1}{z} + \frac{1}{z-2}\right) + \frac{(-1)}{z-1}$

1) $\frac{1}{z-2} = \frac{1}{-2}\cdot\frac{1}{1-\frac{z}{2}} = -\frac{1}{2}\sum\left(\frac{z}{2}\right)^n$

2) $\frac{1}{z-1} = \frac{1}{z}\cdot\frac{1}{1-\frac{1}{z}} = \frac{1}{z}\sum_{0}^{\infty}\left(\frac{1}{z}\right)^n$

$f(z) = z^{-1}\cdot 2^{-1} - \sum_{-\infty}^{-2} z^{n+1} - \sum_{0} 2^{-n-2} z^n$

2) $\frac{z^3}{(z+1)(z-2)}$ $(a=-1)$ $0<|z+1|<3$

20.09.

1) $\frac{1}{z(z-3)^2}$ $\quad (a=1,\ D: 1<|z-1|<2)$.

$\cancel{f(z)=\frac{1}{z}+\frac{1}{z-3}+\frac{1}{(z-3)^2}}$

$\cancel{c_n=\frac{1}{2\pi i}\int_\Gamma \frac{\frac{1}{z(z-3)^2}}{(z-1)^{n+1}}\,dz=\frac{1}{2\pi i}\int_\Gamma\frac{dz}{z(z-3)^2(z-1)^{n+1}}}$

$f(z)=\frac{1}{z(z-3)^2}$ $\quad z_1=0 \quad z_2=3$

$f(z)=\frac{1}{9}\cdot\frac{1}{z}+\left(-\frac{1}{9}\right)\frac{1}{z-3}+\frac{1}{3}\frac{1}{(z-3)^2}$

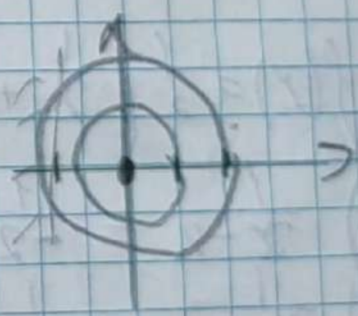
$\cancel{f(z)=\frac{1}{9}\sum\frac{1}{z}-\frac{1}{9z}\frac{1}{1-\frac{3}{z}}=}$

$\frac{1}{(z+1)-1}=\cancel{-\frac{1}{1-(z+1)}}=\frac{1}{(z+1)\left(1-\frac{1}{z+1}\right)}=\sum_{n=0}^{\infty}\frac{1}{(z+1)^{n+1}}$

$|z+1|>1$

$\frac{1}{(z+1)}$

5) $\frac{1}{z(z-1)(z-2)}$ $\quad \left(a=0,\ -\frac{3}{2}\in D\right)$.



$z_3=0 \quad z_1=1 \quad z_2=2. \quad a=0.$

$f(z)=\frac{1}{2}\left(\frac{1}{z}+\frac{1}{z-2}\right)-\frac{1}{z-1}.$

1) $\frac{1}{z+1-1}=-\frac{1}{1}\frac{1}{1-(z+1)}=-\sum_{0}^{\infty}(z+1)^n$

2) $\frac{1}{z+1-3}=-\frac{1}{3}\cdot\frac{1}{1-\left(\frac{z+1}{3}\right)}=-\sum_{0}^{\infty}\frac{(z+1)^n}{3^{n+1}}$

3) $\frac{1}{z+1-2}=-\frac{1}{2}\cdot\frac{1}{1-\left(\frac{z+1}{2}\right)}=-\sum\frac{(z+1)^n}{2^{n+1}}$

2) $\operatorname{ctg} z$   $z = 0, \pm\pi, \pm 2\pi \ldots$ – полюс I

$z = \pm\frac{\pi}{2}, \pm\frac{3\pi}{2}, \pm\frac{5\pi}{2} \ldots$ – нули I.

3) $z \operatorname{tg}^2 z$   $z = 0$ – нуль III   $z = \pm\pi, \pm 2\pi$ – нули II

$z = \pm\frac{\pi}{2}, \pm\frac{3\pi}{2}$ – полюс II.

20.1.

1) $\sum\limits_{n=-\infty}^{\infty} 2^{-|n|} z^n = \sum\limits_{n=-\infty}^{0} 2^{-|n|} z^n + \sum\limits_{n=1}^{\infty} 2^{-|n|} z^n$

$\sum\limits_{n=-\infty}^{0} c_n (z_0 - a)^n + \sum\limits_{n=1}^{\infty} c_n (z_0 - a)^n$   $c_n = 2^{-|n|}$

$R = \lim\limits_{n\to\infty} \left|\frac{c_n}{c_{n+1}}\right| = \lim\limits_{n\to\infty} \frac{2^{-|n|}}{2^{-|n+1|}} = 2.$

$r = \lim\limits_{n\to\infty} \sqrt[n]{|c_n|} = \lim\limits_{n\to\infty} \sqrt[n]{\frac{1}{2^{|n|}}} = \frac{1}{2}.$

$\frac{1}{2} < |z| < 2.$

2) $\sum\limits_{n=-\infty}^{\infty} \frac{z^n}{3^n+1}$   $c_n = c_{-n} = \frac{1}{3^n+1}$

$R = \lim\limits_{n\to\infty} \frac{3\cdot 3^n+1}{3^n+1} = 3.$

$r = \cancel{\lim\limits_{n\to\infty} \sqrt[n]{\frac{1}{3^n+1}}} = \lim \frac{3^n+1}{3\cdot 3^n+1} = \frac{1}{3}$

$\frac{1}{3} < |z| < 3.$

1) $\sum\limits_{n=-\infty}^{\infty} 2^{-n^2} (z+1)^n$   $c_n = c_{-n} = 2^{-n^2}$

$R = \frac{2^{-n^2}}{2^{-n^2-2n-1}} = \lim\limits_{n\to\infty} 2^{2n+1} = \infty$

$r = \lim\limits_{n\to\infty} \sqrt[n]{\frac{1}{2^{n^2}}} = 0$   $0 < |z+1| < \infty$

√19.07

2) $e^{-z^2}$ $(a=\infty)$

$z=x$ $\lim\limits_{z\to a} e^{-z^2}=\underline{0}$

$z=y$ $\lim\limits_{z\to a} e^{-z^2}=e^{y^2}=\underline{\infty}$

3) $\sin\frac{\pi}{z^2}$ $(a=0)$

$z=x$ $\lim\limits_{z\to a}\sin\frac{\pi}{z^2}=\sin\frac{\pi}{a^2}=\sin\infty=A$

$z=y$

19.15

1) $\frac{(1+z^2)^2}{1-z^2}$ $z=\frac{1}{\xi}$

$\varphi(\xi)=f\left(\frac{1}{\xi}\right)=\frac{\left(1+\frac{1}{\xi^2}\right)^2}{1-\frac{1}{\xi^2}}=1$

$z\neq\pm1,\ z=\infty$

$f(z)=\frac{(1+z^2)^2}{(1-z^2)}=\frac{(z+i)^2(z-i)^2}{1-z^2}=\frac{(z+i)^2(z-i)^2}{(-z-i)(z+i)}$

$z=\pm i$ 2 порядка

$f(z)=(z+i)^2\cdot\varphi(z)$ $\varphi(z)=(z-i)^2$

$\varphi(-i)=(-2i)^2\neq0$ $\cancel{\varphi(i)=(2i)^2\neq0}$ $\varphi(i)\neq0.$

$\cancel{f(i)=\frac{(1+i^2)^2}{(1-i^2)}=0}$ $z=\pm i$ - нули II порядка

$\cancel{f'(i)}$ $z=\pm1$ - полюс I порядка

$z=\infty$ - полюс II порядка.