# Measuring linear drift rates in fast radio bursts with 2D auto-correlations and Monte Carlo resampling

*This text is adopted from §6.3 in Ziggy Pleunis's PhD thesis, submitted to McGill University.*

Many bursts from repeating sources of FRBs are comprised of downward-drifting sub-bursts. In order to gain more insight into what might be the cause of these downward drifts, we need a way to robustly determine if downward-drifting sub-bursts are present in any one burst envelope and we have to find a way to quantify drift rates. Only then can we start to compare drift rates in different sources and look for correlations with other burst parameters, such as burst width, luminosity or repeater activity.

By eye, the observable drift rate within a telescope receiver seems to be sufficiently well described as a linear change of frequency as time progresses, $\mathrm{d}f/\mathrm{d}t$. The high-S/N downward-drifting sub-bursts, then, drift through the CHIME band at rates of a few to a few tens of MHz $\mathrm{ms}^{-1}$. For lower S/N bursts, in which burst structure can be unresolved, it is near impossible to decide whether a burst is drifting[1]. Besides burst S/N, there are obvious limits to the detectability of drifts set by the telescope receiver: if sub-bursts drift too fast in frequency and/or in time the drifting behaviour goes unnoticed, and if sub-bursts drift too slowly in frequency and/or in time it would not be called drifting. We develop a method below to account for these detectability limits.

We define the linear drift rate on the basis of a tilted ellipse in the delay space of a dynamic spectrum, as illustrated in Fig. 1. The linear drift rate $\mathrm{d}f/\mathrm{d}t$ is related to the tilt $\theta$ as $\mathrm{d}f/\mathrm{d}t = \tan^{-1}(-\theta)$, where we have defined $\theta$ such that the ellipse rotates in the counter-clockwise direction for increasing $\theta$ with $\theta = 0$ on the vertical axis. The measured $\theta$ is a continuous variable, but the derived $\mathrm{d}f/\mathrm{d}t$ has an asymptote for $\theta \to 0$ from either side. For $\theta = \frac{\pi}{2} + n\pi$, with $n \in \mathbb{Z}$, the drift rate is 0 MHz $\mathrm{ms}^{-1}$, i.e., there is no drift. For $\theta = 0 + n\pi$, with $n \in \mathbb{Z}$, the drift rate is $\pm\infty$ MHz $\mathrm{ms}^{-1}$, i.e, the drift rate is unconstrained/there is no drift. Approaching $\theta = 0$ corresponds to measuring ever increasing negative/positive drift (i.e., downward/upward-drifting sub-bursts). There is a practical limit to the maximum drift rate that can be measured that is set by the bandwidth and sampling time of the receiver with which the data were collected.

Hessels et al. (2019) successfully applied this technique on a sample of high S/N bursts from FRB 121102 detected by the Arecibo telescope, where the reality of the drift is evident. Low S/N bursts, however, can appear to be drifting. This *apparent* drift can be due to a noise fluctuation. In terms of the ellipse in delay space: if a burst really has no drift, one would expect to measure $\theta = 0$ or $\pm\frac{\pi}{2}$. Yet, in real data one might measure a positive $\theta$ close to 0, which corresponds to a large *negative* drift rate. Whereas, if the noise fluctuations in the data would have been slightly different a negative $\theta$ close to 0 might have been measured, now corresponding to a large *positive* drift rate. Moreover, DM and drift rate are correlated, and dedispersion to a slightly different DM might lead to a different or no drift rate measurement. It should be clear that simulating multiple random instances of the data will be beneficial

[1] See Gourdji et al. (2019) for a study of such dim, *smudgy* bursts from FRB 121102 detected near the detection threshold of the Arecibo telescope.

for exploring the range of measured drift rates in a given data set. `dfdt` thus uses a Monte Carlo resampling algorithm for the robust determination of burst drift rates.

The algorithm is illustrated in Fig. 2 for a burst detected by CHIME/FRB and I will go over all the steps of the algorithm here. Starting with a structure-optimizing DM, one obtains a burst dynamic spectrum (or "waterfall") centred on the emission region and a noise waterfall away from the emission region (top row). We then calculate the 2D auto-correlations of both waterfalls and we remove the peak of the auto-correlation signal at zero lag (second row from the top). A 2D Gaussian fixed at zero lag is fitted to the 2D auto-correlation of the burst (third row from the top, on the left). We develop a model for the 2D auto-correlation of the noise waterfall, by calculating the per-$\Delta$f-channel mean and standard deviation of the image (third row from the top, on the right, from the non-hatched region in the panel right above it). We then resample the data by drawing a random instance of the noise 2D auto-correlation model and adding it to the burst 2D auto-correlation model to then refit a 2D Gaussian and remeasure $\theta$. The residuals between the two 2D auto-correlation models and the data are plotted as a diagnostic tool (bottom row).

This procedure gives a good handle on how the noise of the waterfall affects the drift rate measurements. On top of this, we can repeat the analysis for different DMs drawn from the uncertainty on DM from the structure-optimizing algorithm, where for each randomly drawn DM we dedisperse the burst to that DM, and repeat the procedure described in the paragraph above. After calculating $n_{\rm noise} \times n_{\rm DM}$ trials (typically 100 $\times$ 100 = 10,000), we require that 99.73% ($3\sigma$) of all $\theta$s fall into the same quadrant, i.e., that in 99.73% of the random instances a positive (negative) drift rate was measured. In all other cases we call the drift rate of the burst "unconstrained." Examples of an unconstrained drift rate and a drift rate measurement are shown in Figs. 3 and 4.

In the analysis it is important to only convert $\theta$ to $\mathrm{d}f/\mathrm{d}t$ after a best-fit $\theta$ and confidence interval have been determined, because of the asymptote in linear drift rate close to no drift (where $\theta \sim 0$) that has been mentioned before. Note also that the marginalized drift rate distributions are non-Gaussian due to the covariance between DM and drift and that the 68% containment regions thus do not fully describe the underlying distribution. This can be seen already in Fig. 4 and is further exemplified by Fig. 5.

As described, noise is normally added in the auto-correlation space (as this is less computationally intensive than adding noise to the burst model in frequency-time and calculating the 2D auto-correlation for each iteration). However, a high S/N burst with sharp components makes a tilted 2D Gaussian an insufficient model for describing the 2D auto-correlation of the burst, as shown in Fig. 6. One way to quantify this is by comparing the magnitude of the burst 2D auto-correlation residuals with the noise 2D auto-correlation residuals. For a burst like this we can instead resample the data in time-frequency space. In that case, we take a multi-component model for the burst waterfall and add noise drawn from the distribution of an off-burst region to that burst model before dedispersing the data to a random DM value from the DM uncertainty distribution and proceeding with the 2D auto-correlation.

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## References


CHIME/FRB Collaboration, Bandura, K., Bhardwaj, M., et al. 2019, The Astrophysical Journal, 885, L24

Gourdji, K., Michilli, D., Spitler, L. G., et al. 2019, The Astrophysical Journal Letters, 877, L19

Hessels, J. W. T., Spitler, L. G., Seymour, A. D., et al. 2019, The Astrophysical Journal Letters, 876, L23


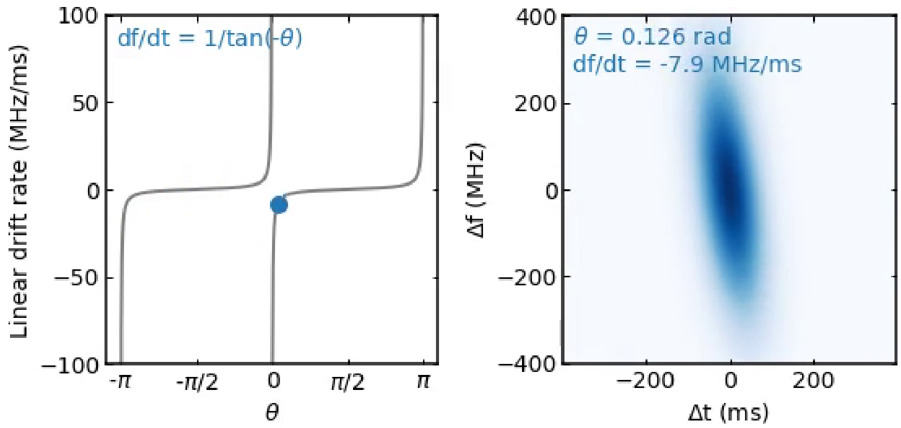


Figure 1: Definition of the linear drift rate as derived from the tilt $\theta$ of an ellipse in the delay space of a dynamic spectrum.

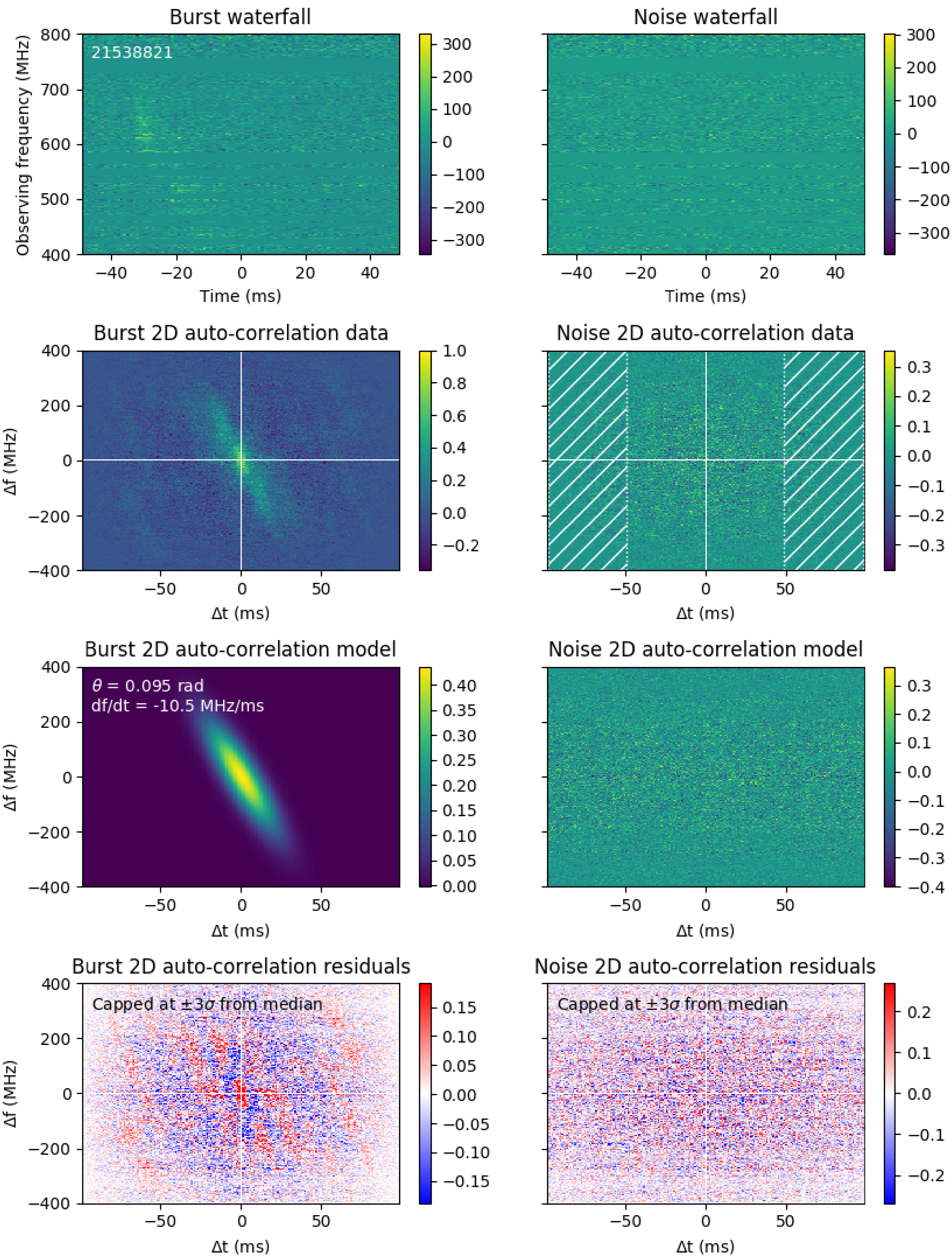


Figure 2: Illustration of a linear drift rate measurement with the Monte Carlo auto-correlation method described in the text.

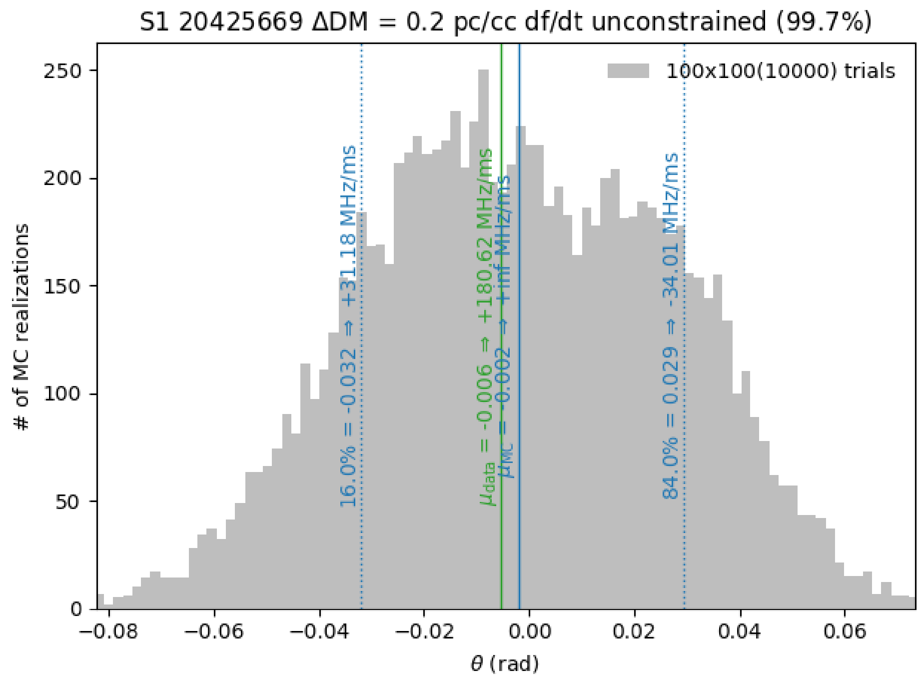


Figure 3: Linear drift rates measured for a 2018 November 4 burst of repeating source of FRBs 180916.J0158+65 (Source 1 in CHIME/FRB Collaboration et al., 2019). The green vertical line and text shows the drift rate measured from the data, and the blue vertical line and text shows the drift rate measured from the Monte Carlo resampling along with the 68% confidence interval in blue dashed lines. As $\theta$ peaks around 0 (i.e., in about half the random instance a positive linear drift and in the other half a negative drift rate is measured), the drift rate is unconstrained and the burst morphology in consistent with showing no drift.

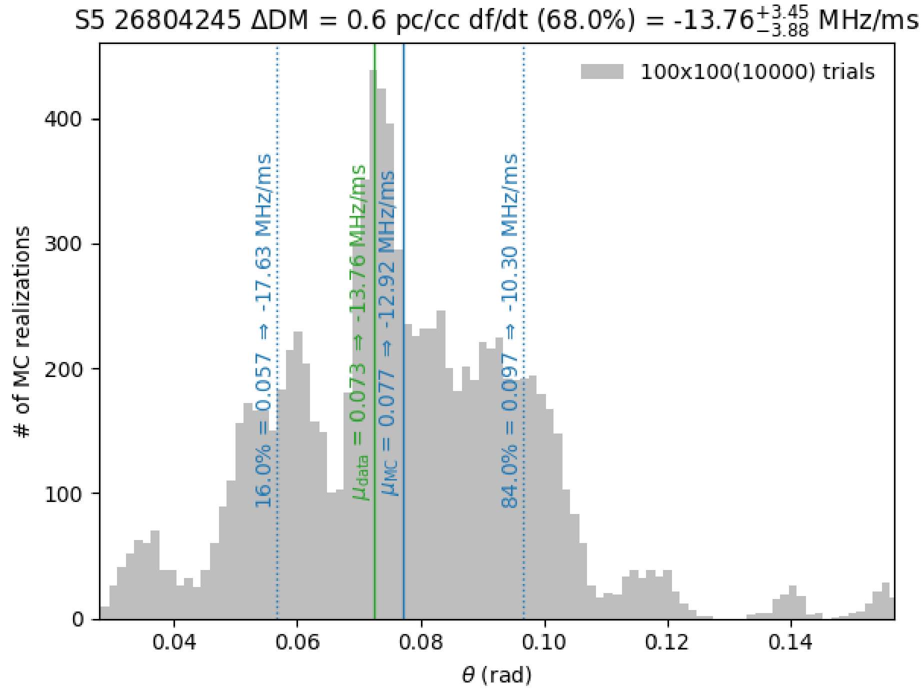


Figure 4: Linear drift rates measured for a 2019 January 16 burst of repeating source of FRBs 190116.J1249+27 (Source 5 in CHIME/FRB Collaboration et al., 2019). The green vertical line and text shows the drift rate measured from the data, and the blue vertical line and text shows the drift rate measured from the Monte Carlo resampling along with the 68% confidence interval in blue dashed lines. 99.7% of the $\theta$ measurements fall in the same quadrant and the linear drift rate is constrained to be $\mathrm{d}f/\mathrm{d}t = -14^{+3}_{-4}$ MHz ms$^{-1}$. Note the non-Gaussian distribution due to the covariance between DM and drift rate and that the 68% containment regions thus do not fully describe the underlying distribution.

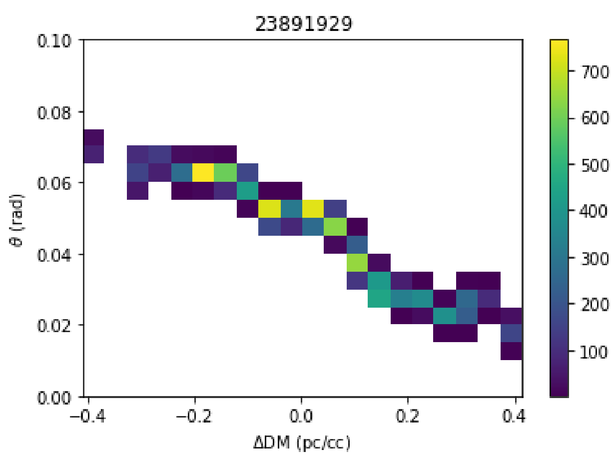


Figure 5: 2D histogram of Monte Carlo trials shows how the DM and linear drift rate are covariant in a burst comprised of drifting sub-bursts, which leads to multiple peaks (as in Fig. 4) in the distribution of linear drift rates after marginalizing over the DM uncertainty.

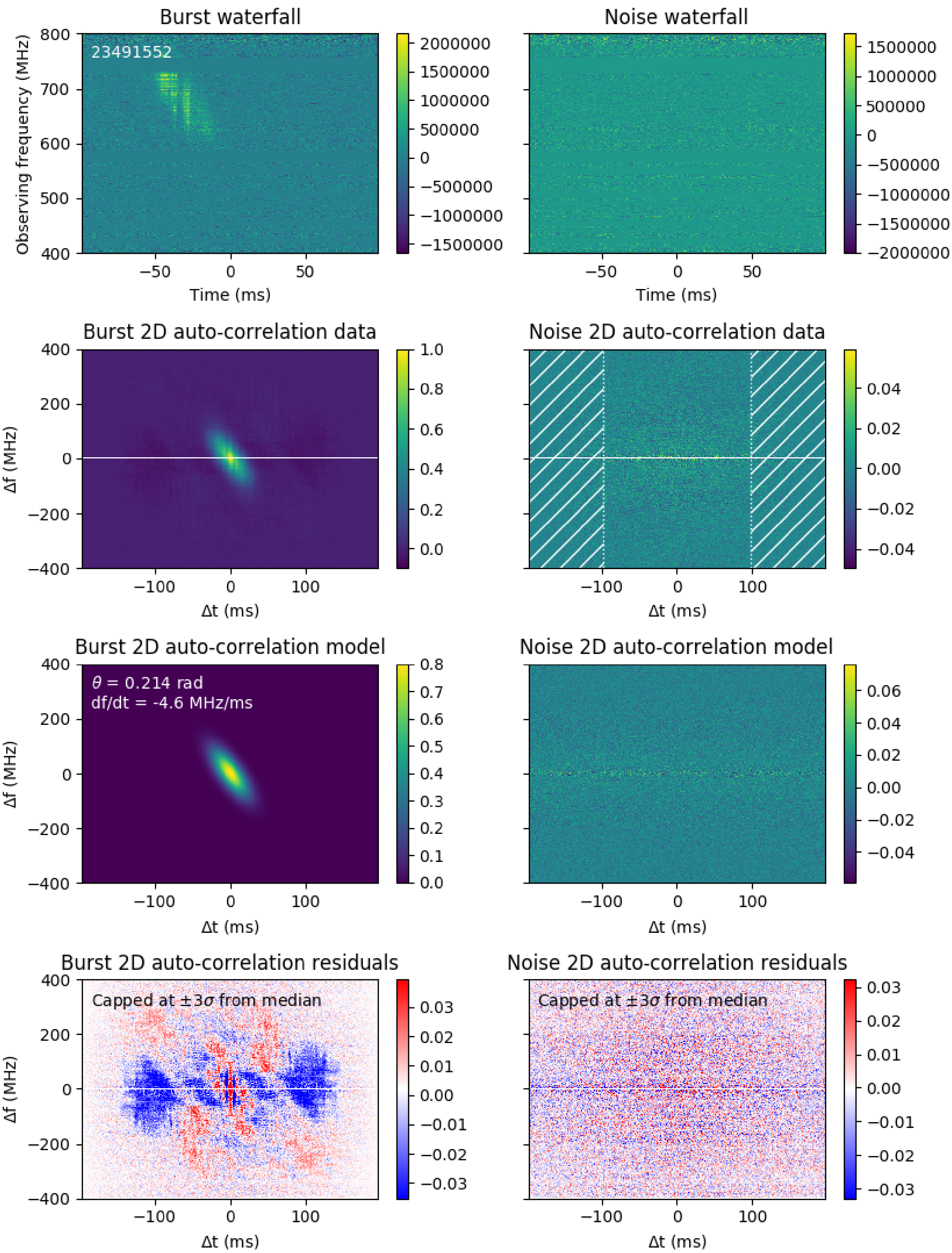


Figure 6: For a high S/N burst, a 2D Gaussian is not a good model for the 2D auto-correlation of the burst, as is most apparent from the residual structure in the bottom left corner.