Black Scholes Calculator

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January 4, 2022

1 What is a Black Scholes Calculator?

Our Black Shcoles Calculator determines the fair market price of a call and/or put option based on the Black Scholes pricing model. In addition to calculating the price of the option, it also calculates the *greeks* that are useful measures indicating the sensitivity of the option price with respect to other factors.

When would you want to use it? Let's consider a call option for an underlying stock. This gives the option holder the right to buy a specific number of shares of a specific stock on a specific date in the future at a set strike price. If all these quantities are fixed, the question becomes: what is a fair price to charge for the option? The Black-Scholes calculator uses the Black Scholes model to calculate the **price of the option**, in terms of other quantities, which are assumed known. These include the strike price, time to maturity and the current price of the stock, risk free interest rate, dividend and a constant volatility.

Underlying principle: The calculator is based one the Black Scholes Model. Even though the model is based on EU options, it is the state-of-the-art standard model for valuing all options. Brokers, investors and option holders use the Black Scholes model for determining the cost of options.

2 How to calculate black scholes option pricing model?

Mathematical equations are what estimates the theoretical value of the call and put options of other investment instruments, taking into account the

impact of time and other risk factors.
BSM for the value of Call Option: The Black-Scholes call option formula is calculated by multiplying the stock price by the cumulative standard normal probability distribution function. Thereafter, the strike price multiplied by the cumulative standard normal distribution is subtracted from the resulting value of the previous calculation:

$$C = SN(d_1) - Ke^{-r(T-t)}N(d_2)$$

BSM for the value Put Option price: The Black Scholes put option formula is calculated by multiplying the strike price by the cumaltive standard normal distribution. Thereafter the stock price multiplied by the cumalitive standard distribution is subtracted from the resulting value of the previous caclulation:

$$P = Ke^{-r(T-t)}N(-d_2) - SN(-d_1)$$

where $N(d_i)$ is a cumalitive standard normal distribution

$$N(d_i) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{d_i} e^{\frac{-x^3}{2}} dx$$

$$d_1 = \frac{ln(\frac{S}{K}) + (r - d + \frac{\sigma^2}{2})(T-t)}{\sigma\sqrt{(T-t)}}$$

$$d_2 = d_1 - \sigma_s\sqrt{(T-t)}$$

where C and P is the price for an European call and put option. Moreover ln is the natural logarithm, and e is the exponential constant.
Does BS model work for real? The model and therefore the Black Scholes calculator is based on certain *assumptions:*

- stock prices follow a lognormal probability distribution
- stock prices are random and follow a Wiener process (denoted by W)
- the option is European and can only be exercised at expiration
- no dividends are paid out during the life of the option
- market movements cannot be predicted
- that no transaction costs in buying the option

- that risk-free rate and volatility of the underlying are known and constant
- there is no arbitrage

One may question the applicability of Black Scholes under the above mentioned assumptions as it simplifies reality.The model assumes that the dividends, volatility, and risk-free rates remain constant over the option's life. Moreover, it does not take into account taxes, commissions or trading costs. It can be rightly argued that the model, based on the assumptions, can lead to valuations that deviate from real-world results. Despite all of the practical concerns, Black Scholes Mode is widely used and the Black Scholes calculator makes it easy to get the best estimate for the options price.
Good news: Fortunately, you don't need to know or even understand the math to use Black-Scholes modeling in your own strategies. We will do the work for you. All you need to do is provide the right inputs.

3 How to use the Hardbacon Black Scholes Calculator

Our Black Scholes calculator will perform the calculations and output the options pricing values. All you need to provide are the **six input variables** to calculate the price of call/put options:

- S :=current price at time t of the underlying
- K := the strike price of the option,
- r := the risk free intereste rate,
- $T - t$:=time to expiration of the option
- d :=dividend yield
- σ :=volatility of the underlying

The first five input variables are totally observable and can be directly obtained from the market. In the list of the above input parameters, only volatility is not observable in the market. For more details, look into Section 6.

4 Understanding the results of the Black Scholes Calculator

Once you've provided the input, you will see a number of output results from the calculator. Let's see what they are. The direct output is the estimated price of the option. Alongside, our Hardcabon Black Scholes calculator also calculates the greeks for you - oh yes, all of them: not just the delta, theta, vega but also gamma, rho, phi, charm, vanna). The greeks let you understand the risk exposures related to an option. As financial investors and option holders, your overall objective is to reduce the risk in your portfolio. Fo this, you might be taking a long (short) position in an option contract while simultaneously taking a short (long) position in the underlying. The greeks are useful indicators for making that call by providing measures on how sensitive is the option value to other factors such as underlying price, time, and volatility. The most widely used greeks are delta, theta, vega:

- Delta is the measure of sensitivity of option price with respect to the change in the price of the underlying
- Theta is the measure of sensitivity of option price with respect to the change in time
- Vega is the measure of sensitivity of option price with respect to the change in volatility

There should be some information on the other greeks: gamma, rho, phi, charm, vanna. Not sure about these (need to know how the calculator is encoded at the backend).

Hedging the greeks for options portfolio: One develops a hedging methodology for making a portfolio of options greeks (delta, vega and gamma) neutral by taking positions in other available options, and simultaneously minimizing the net premium to be paid for the hedging. A hedging portfolio becomes the backbone in calculating the Black Scholes option pricing model. The greeks define the hedging strategy.

For example, if we consider delta (Δ) hedging strategy, the δ of the option is defined as the ratio of change in option price (∂V) and change in the price of the underlying (∂S)

$$\Delta = \frac{\partial V}{\partial S}.$$

A risk free delta-hedged portfolio, denoted by Π, can be constructed as

$$\Pi = V - \frac{\partial V}{\partial S} S$$

and a change in the portfolio value is

$$d\Pi = dV - \frac{\partial V}{\partial S} dS$$

More details are provided in on this in Section 5.1.

5 Learn more about the Black Scholes Calculator Inputs

Black Scholes models assumes constant volatility hoewever the observations of market prices of identical options with different strike prices and expiration time indicates otherwise.
Before we discuss the different ways of what volatility assumes as a constant, lets look closely into the Black Scholes Option Pricing model.

5.1 How to calculate black scholes?

The Black Scholes model is derived under the assumption that the time interval between observations is very small, and that the log prices of the underlying asset follow a random walk that has an underlying normally distribution. The model for the stock prices themselves is called geometric Brownian motion or Wiener process.
Let's assume the movement of price of the underlying is described by the stochastic differential equation of small change in S

$$dS = Srdt + S\sigma dW$$

and the squared stochastic equation of small change in S is

$$dS^2 = S^2 r^2 dt^2 + 2S^2 r\sigma dt dW + S^2 \sigma^2 dW^2.$$

We can simplify this as

$$dS^2 = S^2 \sigma^2 dt$$

Using Stochastic calculus, Ito's lemma and Taylor's expansion, one can come to

$$dV = S\sigma\frac{\partial V}{\partial S}dW + (Sr\frac{\partial V}{\partial S} + \frac{1}{2}S^2\sigma^2\frac{\partial^2 V}{\partial S^2} + \frac{\partial V}{\partial t})dt$$

If we substitute this into $d\Pi = dV - \frac{\partial V}{\partial S}dS$ and substituting the above we end up with

$$d\Pi = \frac{1}{2}S^2\sigma^2\frac{\partial^2 V}{\partial S^2} + \frac{\partial V}{\partial t})dt.$$

The assumption of a risk free portfolio and no arbitrage, we have $d\Pi = r\Pi dt$. By some algebraic manipulations we end up with a Black Scholes partial differential equation (PDE)

$$\frac{dV}{dt} + \frac{1}{2}S^2\sigma^2\frac{\partial^2 V}{\partial S^2} + (rS\frac{\partial V}{\partial S} - rV = 0$$

The Black Scholes model is infact the exact solution of this PDE using the payoffs as boundary conditions. For the call option, the boundary condition takes the form

$$V(S,T) == C(S,T) = max(S-K,0)$$

and

$$V(S,T) = P(S,T) = max(K-S,0)$$

for the put option respectively.

6 How to compute volatility?

A key input to the Black-Scholes model is the volatility, σ. The price of an option depends on how volatile the price of the underlying is. There are mainly two ways of viewing volatility: Implied and historical volatility. Investors and option holders can price the derivatives based on underlying by computing the volatility of the underlying. Moreover, the premium for an underlying at expiration time is likely to increase with increasing volatility.

- **Implied Volatility:** One way of viewing volatility is forward looking: to view it as the volatility of the underlying asset given its market's option price. This particular kind is referred to as implied or local volatility. This is a theoretical value representing future volatility of the underlying for an option based on current market price of the option.

Under the conditions of Wiener process and no-arbitrage market prices for European call options, the volatility can be extracted by using the market price in the Black Scholes PDE in the form:

$$\sigma^2 = \sqrt{\frac{\frac{\partial C}{\partial T} + rK\frac{\partial C}{\partial K} + C}{0.5K^2} \frac{\partial^2 C}{\partial K^2}}$$

Another way of computing implied volatility from empirical observations is by using the Heston model, whereby the value is not a constant. It is more commonly referred to as stochastic volatility (provide reference).

- **Historical Volatility:** Another way to view volatility is in the form of a historical volatility. It is derived through historical data of the annualized squared log returns of the market option prices observed in the past. Here, one could view volatility as the standard deviation of the stock's continuously compounded rate of return. If the time interval between observations is sufficiently small, σ is just the standard deviation of the innovations in the random walk, so σ can be viewed as a measure of the volatility of the stock price. According to the random walk model, σ must remain constant over time. Its value will not be known, however, so it is usually estimated from the available historical data.

  - Close-Close Volatility Estimator which is also known as the "classical" estimator.
  - High-Low Volatility Estimator from Parkinson - it incorporates the intraday high and low prices of the financial asset into its estimation of volatility.
  - High-Low-Open-Close Volatility Estimator from Garman and Klass.

There is no one way to compute volatility. Different financial investors and option holders use various ways for computing volatility. No matter how you computing the volatility, implied or historical, the Black Scholes pricing calculator will determine the best fair option price for you.

7 How to calculate black schole outputs from historical data?

I am not sure what this question means.