Exercise Round 1

The deadline of this exercise round is **Thursday January 16th, 2025**. The solutions will be discussed during the exercise session in the T2 lecture hall of Computer Science building starting at 14:15.

The problems should be *solved before the exercise session.* During the session those who have completed the exercises will be asked to present their solutions on the board/screen.

Exercise 1. (Mean as the Minimum Mean Square Estimator)

Find the optimal point estimate $\mathbf{a}$ that minimizes the expected value of the loss function

$$C(\boldsymbol{\theta}, \mathbf{a}) = (\boldsymbol{\theta} - \mathbf{a})^\mathsf{T} \, \mathbf{R} \, (\boldsymbol{\theta} - \mathbf{a}), \tag{1}$$

where $\mathbf{R}$ is a positive definite matrix, and the distribution of the parameter is $\boldsymbol{\theta} \sim p(\boldsymbol{\theta} \mid \mathbf{y}_{1:T})$.

Exercise 2. (Linear Least Squares Estimation)

Assume that we have obtained T measurement pairs (x_k, y_k) from the linear regression model

$$y_k = \theta_1 \, x_k + \theta_2, \qquad k = 1, 2, \ldots, T. \tag{2}$$

The purpose is now to derive estimates of the parameters θ_1 and θ_2 such that the following error is minimized (least squares estimate):

$$E(\theta_1, \theta_2) = \sum_{k=1}^{T} (y_k - \theta_1 \, x_k - \theta_2)^2. \tag{3}$$

(a) Define $\mathbf{y} = (y_1 \; \ldots \; y_T)^\mathsf{T}$ and $\boldsymbol{\theta} = (\theta_1 \; \theta_2)^\mathsf{T}$. Show that the set of Equations (2) can be written in matrix form as

$$\mathbf{y} = \mathbf{X} \, \boldsymbol{\theta},$$

with a suitably defined matrix $\mathbf{X}$.

(b) Write the error function in Equation (3) in matrix form in terms of $\mathbf{y}$, $\mathbf{X}$, and $\boldsymbol{\theta}$.

(c) Compute the gradient of the matrix form error function, and solve the least squares estimate of the parameter $\boldsymbol{\theta}$ by finding the point where the gradient is zero.

Exercise 3. (Kalman filtering illustration using provided code)

The example code for this exercise is provided as material on MyCourses. In the exercise session you task is to introduce the environment that you installed or took into use.

(a) Setup a Python environment of your choice. Typically we recommend using

```
Locally:
Anaconda distribution + Jupyter
or
Anaconda distribution + IDE with notebook support
(Pycharm or VS Studio is recommended)
Remotely:
Google Colab https://colab.research.google.com/
```

(b) Run the first section of the notebook consisting of the noisy sinusoidal example

(c) After running them, read the content of the notebook and try to understand how it has been implemented. Additionally, read the documentation of the functions `kf_predict`, `kf_update`, and `rts_update`.

(d) Follow the instructions in the rest of the notebook.