Solutions for Exercise Round 4

Exercise 1. (Extended Kalman Filter)

See the notebook.

Exercise 2. (Single Iteration of IEKF)

The prediction steps are the same, so if the results of the update step match, then so do the results of the prediction step. In EKF we have the update step:

$$
\begin{aligned}
\mathbf{v}_k &= \mathbf{y}_k - \mathbf{h}(\mathbf{m}_k^-), \\
\mathbf{S}_k &= \mathbf{H_x}(\mathbf{m}_k^-)\,\mathbf{P}_k^-\,\mathbf{H_x^T}(\mathbf{m}_k^-) + \mathbf{R}_k, \\
\mathbf{K}_k &= \mathbf{P}_k^-\,\mathbf{H_x^T}(\mathbf{m}_k^-)\,\mathbf{S}_k^{-1}, \\
\mathbf{m}_k &= \mathbf{m}_k^- + \mathbf{K}_k\,\mathbf{v}_k, \\
\mathbf{P}_k &= \mathbf{P}_k^- - \mathbf{K}_k\,\mathbf{S}_k\,\mathbf{K}_k^\mathsf{T}.
\end{aligned}
\tag{1}
$$

In IEKF we start from $\mathbf{x}_k^{(0)} = \mathbf{m}_k^-$. The iteration is

$$
\begin{aligned}
\mathbf{v}_k^{(i)} &= \mathbf{y}_k - \mathbf{h}(\mathbf{x}_k^{(i-1)}) - \mathbf{H_x}(\mathbf{x}_k^{(i-1)})\,(\mathbf{m}_k^- - \mathbf{x}_k^{(i-1)}), \\
\mathbf{S}_k^{(i)} &= \mathbf{H_x}(\mathbf{x}_k^{(i-1)})\,\mathbf{P}_k^-\,\mathbf{H_x^T}(\mathbf{x}_k^{(i-1)}) + \mathbf{R}_k, \\
\mathbf{K}_k^{(i)} &= \mathbf{P}_k^-\,\mathbf{H_x^T}(\mathbf{x}_k^{(i-1)})\,\left[\mathbf{S}_k^{(i)}\right]^{-1}, \\
\mathbf{x}_k^{(i)} &= \mathbf{m}_k^- + \mathbf{K}_k^{(i)}\,\mathbf{v}_k^{(i)},
\end{aligned}
\tag{2}
$$

where we substitute $\mathbf{x}_k^{(i-1)} = \mathbf{x}_k^{(0)} = \mathbf{m}_k^-$, these become

$$
\begin{aligned}
\mathbf{v}_k^{(1)} &= \mathbf{y}_k - \mathbf{h}(\mathbf{m}_k^-), \\
\mathbf{S}_k^{(1)} &= \mathbf{H_x}(\mathbf{m}_k^-)\,\mathbf{P}_k^-\,\mathbf{H_x^T}(\mathbf{m}_k^-) + \mathbf{R}_k, \\
\mathbf{K}_k^{(1)} &= \mathbf{P}_k^-\,\mathbf{H_x^T}(\mathbf{m}_k^-)\,\left[\mathbf{S}_k^{(1)}\right]^{-1}, \\
\mathbf{x}_k^{(1)} &= \mathbf{m}_k^- + \mathbf{K}_k^{(1)}\,\mathbf{v}_k^{(1)},
\end{aligned}
\tag{3}
$$

Then the final mean and covariance are computed as

$$
\begin{aligned}
\mathbf{m}_k &= \mathbf{x}_k^{(1)} = \mathbf{m}_k^- + \mathbf{K}_k^{(1)}\,\mathbf{v}_k^{(1)}, \\
\mathbf{P}_k &= \mathbf{P}_k^- - \mathbf{K}_k^{(1)}\,\mathbf{S}_k^{(1)}\,\left[\mathbf{K}_k^{(1)}\right]^\mathsf{T}.
\end{aligned}
\tag{4}
$$

which are exactly the same as the EKF equations. Please note that the equations match only if we start from $\mathbf{x}_k^{(0)} = \mathbf{m}_k^-$ – with other initial guesses the equations would not match.

Exercise 3. (Bearings Only Target Tracking with EKF)

See the notebook.