

Solutions for Exercise Round 8

See the notebook for an implementation of a generic particle filter which can be used for all 3 exercises.

Exercise 2 (a)

We need to compute the optimal importance proposal

$$\pi(x_k) = p(x_k \mid x_{k-1}, y_{1:k}) = p(x_k \mid x_{k-1}, y_k). \quad (1)$$

$p(x_k, y_k \mid x_{k-1})$ has joint distribution

$$p(x_k, y_k \mid x_{k-1}) \sim \mathcal{N} \left(\begin{bmatrix} Ax_{k-1} \\ HAx_{k-1} \end{bmatrix}, \begin{bmatrix} Q & QH^\top \\ HQ & HQH^\top + R \end{bmatrix} \right). \quad (2)$$

Then from the conditioning formula from problem 3 of Exercise Round 3, the posterior $p(x_k \mid x_{k-1}, y_k) \sim \mathcal{N}(\mu_k, \Sigma_k)$ where

$$\mu_k = Ax_{k-1} + K_k(y_k - HAx_{k-1}), \quad (3)$$

$$\Sigma_k = (I - K_kH)Q, \quad (4)$$

$$K_k = QH^\top (HQH^\top + R)^{-1}. \quad (5)$$