

Chapter 4: Quantization of Light

Modern Physics Study Notes

1 Quantization (Section 4.1)

Quantization means a physical quantity can only take on certain discrete (separate, fixed) values, rather than any continuous value.

Examples:

- Electric charge is quantized: $q = ne$, where $e = 1.60 \times 10^{-19}$ C.
- Matter is quantized: everything is made of atoms.
- Radiation (light) is quantized: it comes in discrete packets called **photons**.

Photons

A **photon** is a single quantum (packet) of electromagnetic radiation. Each photon has:

$$E = hf \tag{1}$$

$$p = \frac{h}{\lambda} \tag{2}$$

Where:

- E is the energy of *one photon* of the light beam (units: J).
- p is the momentum carried by *one photon* (units: kg·m/s).
- h is Planck's constant, $h = 6.626 \times 10^{-34}$ J · s.
- f is the frequency of the electromagnetic wave / light (Hz = s⁻¹).
- λ is the wavelength of that same light wave (m).

Key idea: photons have both particle-like properties (energy E , momentum p) and wave-like properties (frequency f , wavelength λ). This is the first sign of **particle-wave duality**.

2 Blackbody Radiation (Section 4.2)

A **blackbody** is an ideal object that:

- absorbs *all* incoming radiation,
- emits radiation that depends *only* on its temperature T .

Examples: glowing metal, a hot stove filament, the Sun's surface, the inside of a very small hole in an oven-like cavity.

Experimentally observed:

- Hotter objects emit *more* radiation overall.
- As temperature T increases, the peak of the emitted spectrum shifts to *shorter* wavelengths (more blue).

The classical prediction (Rayleigh–Jeans)

Classical physics treated the radiation inside the cavity as standing electromagnetic waves. Each standing wave mode acts like a harmonic oscillator that can have any (continuous) energy. The Rayleigh–Jeans law predicts the energy density per wavelength goes like

$$u(\lambda, T) \propto \frac{T}{\lambda^4}.$$

This works at long wavelengths (radio / infrared), but at short wavelengths it blows up \rightarrow the “ultraviolet catastrophe.”

Planck's hypothesis (1900)

Planck proposed something radical:

$$E_n = nhf, \quad n = 0, 1, 2, \dots$$

Where:

- E_n is the allowed energy of one electromagnetic mode (one “oscillator”) in the cavity,
- f is the frequency of that mode,
- n is an integer,
- h is Planck's constant.

Meaning:

- Energy is not continuous. It comes in *quanta* of size hf .

- When the blackbody emits radiation of frequency f , it does so by emitting photons of energy $E = hf$.

Planck's idea successfully fits *all* blackbody data at *all* wavelengths and temperatures. This marks the birth of quantum theory. The constant h was first extracted from this fit.

3 The Photoelectric Effect (Section 4.3)

Photoelectric effect: shining light on a metal surface can eject electrons from that surface.

Discovered by Hertz (1887). Explained by Einstein (1905).

Experimental setup

Light of frequency f hits a metal cathode. Electrons are emitted and can be collected by an anode to make a current. If we apply a **stopping potential** V_s (a reverse voltage) large enough, even the fastest electrons will be stopped.

The maximum kinetic energy of the emitted electrons is measured via:

$$eV_s = K_{\max}. \quad (3)$$

Where:

- $e = 1.60 \times 10^{-19}$ C is the magnitude of the electron charge,
- V_s is the stopping potential (volts),
- K_{\max} is the maximum kinetic energy of the emitted (photo)electrons (units: J or eV).

Classical expectations (wrong)

Classical wave theory said:

- Increasing light intensity should make electrons come off with *more* kinetic energy.
- *Any* light, if intense enough, should eject electrons, eventually.

Observed instead:

- Below some threshold frequency f_0 , *no electrons are ejected at all*, no matter how intense the light is.
- Increasing intensity increases the *number* of emitted electrons, not their maximum kinetic energy.

Einstein's photon explanation

Einstein said: Treat light as photons. Each photon of frequency f carries energy $E = hf$. One photon interacts with one electron.

Energy conservation for that one electron:

$$hf = \phi + K_{\max} \quad (4)$$

Where:

- hf is the energy of a *single incoming photon* (frequency f),
- ϕ is the **work function** of the metal (the minimum energy required to free an electron from the surface),
- K_{\max} is the maximum kinetic energy of an emitted electron.

Combining with $K_{\max} = eV_s$ gives:

$$hf = \phi + eV_s. \quad (5)$$

Rearrange (4) to get:

$$K_{\max} = hf - \phi. \quad (6)$$

This predicts a **threshold frequency**:

$$f_0 = \frac{\phi}{h} \quad (7)$$

Below f_0 no electrons can be ejected, because each photon simply doesn't carry enough energy to overcome ϕ .

Photon energy and wavelength

Useful alternate form for the photon energy:

$$E_{\text{photon}} = hf = \frac{hc}{\lambda}. \quad (8)$$

Where:

- E_{photon} is the energy of one photon,
- $c = 3.00 \times 10^8 \text{ m/s}$ is the speed of light,
- λ is the photon's wavelength.

A common shortcut (when using electronvolts and nanometers):

$$hc \approx 1240 \text{ eV} \cdot \text{nm}.$$

So, for example,

$$E_{\text{photon}} (\text{in eV}) \approx \frac{1240 \text{ eV} \cdot \text{nm}}{\lambda (\text{in nm})}.$$

4 X-Rays and Bragg Diffraction (Section 4.4)

What are X-rays?

X-rays are high-frequency electromagnetic waves with very short wavelengths:

$$0.001 \text{ nm} \lesssim \lambda \lesssim 1 \text{ nm}.$$

Shorter wavelength \Rightarrow higher photon energy. Typical X-ray photon energies are $\sim \text{keV}$ (kiloelectronvolts).

They are commonly generated in an X-ray tube:

- Electrons are accelerated through a high voltage V_0 (thousands of volts).
- These high-energy electrons smash into a metal anode and decelerate suddenly.
- That rapid deceleration produces X-ray photons.

Two mechanisms:

1. **Bremsstrahlung** (“braking radiation”): a continuous spectrum from electrons being slowed near nuclei.
2. **Characteristic X-rays**: sharp lines at specific energies that depend on the target material, caused by inner-shell electrons being knocked out and higher-shell electrons dropping down to fill the hole.

For characteristic X-rays, the emitted photon energy is roughly

$$E_\gamma = E_{\text{outer shell}} - E_{\text{inner shell}}.$$

Wave nature of X-rays: Bragg diffraction

Experiments by Laue (1912) and the Braggs (1913) proved that X-rays behave as waves and diffract from crystals.

A crystal can be seen as many parallel atomic planes separated by distance d . Constructive interference (a strong reflected intensity) occurs when **Bragg’s law** is satisfied:

$$2d \sin \theta = n\lambda. \tag{9}$$

Where:

- d is the spacing between adjacent crystal planes (m),
- θ is the “glancing angle” between the incident X-ray beam and the crystal plane (this is the angle used in Bragg scattering, not necessarily the same as the usual angle from the surface normal),

- $n = 1, 2, 3, \dots$ is the diffraction order,
- λ is the X-ray wavelength (m).

Bragg diffraction lets us:

- measure unknown X-ray wavelengths λ ,
- determine crystal structure (X-ray crystallography),
- famously, determine the double-helix structure of DNA.

5 X-Ray Spectra and the Duane–Hunt Law (Section 4.5)

When you measure the intensity of X-rays from an X-ray tube as a function of frequency f (or wavelength λ), you observe:

- a smooth “continuous” background from bremsstrahlung,
- sharp peaks at specific frequencies (the characteristic X-rays),
- a hard cutoff: no X-rays above some maximum frequency f_{\max} .

This maximum frequency is set only by the accelerating voltage, not by the anode material. This is summarized by the Duane–Hunt law:

$$hf_{\max} = eV_0. \quad (10)$$

Where:

- f_{\max} is the highest photon frequency produced in the tube,
- h is Planck’s constant,
- e is the electron charge,
- V_0 is the accelerating voltage applied to the electrons in the tube.

We can rewrite this as

$$f_{\max} = \frac{eV_0}{h} \quad \text{and} \quad \lambda_{\min} = \frac{hc}{eV_0}. \quad (11)$$

Meaning: an electron accelerated through V_0 can give *at most* all its kinetic energy to a single photon. That limits the smallest possible wavelength / highest possible frequency of the emitted X-ray.

6 The Compton Effect (Section 4.6)

Observation

When high-energy electromagnetic radiation (like X-rays) scatters off (approximately) free electrons, the scattered radiation is found to have *lower frequency* and therefore *longer wavelength* than the incident radiation.

Classically, you'd expect the scattered wave to come off with the same frequency (no change), $f = f_0$. But in reality, $f < f_0$.

Compton's interpretation

Arthur Compton (1923) treated the collision between a photon and an electron like a two-body collision obeying conservation of energy and momentum.

- Before: a photon with wavelength λ_0 , energy $E_0 = hf_0 = \frac{hc}{\lambda_0}$ and momentum $p_0 = \frac{h}{\lambda_0}$, hits an electron (initially at rest).
- After: a photon scatters at an angle θ with new wavelength λ (so new energy $E = hf = \frac{hc}{\lambda}$), and the electron recoils with some kinetic energy.

Solving conservation of energy and momentum gives the **Compton shift formula**:

$$\Delta\lambda = \lambda - \lambda_0 = \frac{h}{mc}(1 - \cos\theta). \quad (12)$$

Where:

- λ_0 is the initial photon wavelength,
- λ is the scattered photon wavelength,
- $\Delta\lambda = \lambda - \lambda_0$ is the increase in wavelength,
- θ is the scattering angle of the photon (angle between the incident and scattered photon directions),
- m is the electron rest mass ($m = 9.11 \times 10^{-31}$ kg),
- c is the speed of light in vacuum.

Important features:

- $\Delta\lambda = 0$ if $\theta = 0^\circ$ (no change in direction, no change in wavelength).
- $\Delta\lambda$ is largest at $\theta = 180^\circ$ (backscattering).

- The prefactor $\frac{h}{mc}$ is called the **Compton wavelength of the electron**:

$$\frac{h}{mc} = 2.43 \times 10^{-12} \text{ m} = 0.00243 \text{ nm.}$$

Why you don't notice this for visible light: visible light has wavelengths ~ 500 nm, so adding ~ 0.002 nm is a tiny fractional change. For X-rays ($\lambda \sim 0.1$ nm), the fractional change is large enough to measure.

This was direct proof that photons carry momentum and behave like particles in collisions.

7 Particle-Wave Duality (Section 4.7)

Light has:

- **Wave-like behavior:** interference, diffraction, Bragg scattering.
- **Particle-like behavior:** photoelectric effect (photons eject electrons), Compton scattering (photon-electron collisions), discrete energy packets $E = hf$.

We summarize photon behavior with:

$$E = hf, \tag{13}$$

$$p = \frac{h}{\lambda}. \tag{14}$$

This duality is not just for light. In later chapters you'll see that *matter* (electrons, neutrons, etc.) also shows wave-like behavior.

Key Equations + Variable Definitions

This section is for fast exam review.

Photon energy

$$E = hf \quad (15)$$

E : energy of one photon of light

f : frequency of that light wave

h : Planck's constant

Photon momentum

$$p = \frac{h}{\lambda} \quad (16)$$

p : momentum carried by a single photon

λ : wavelength of that photon

Blackbody quantization

$$E_n = nhf, \quad n = 0, 1, 2, \dots \quad (17)$$

E_n : allowed energy level of one electromagnetic mode in a blackbody cavity

f : frequency of that mode

n : integer (quantum number)

Photoelectric effect energy balance

$$hf = \phi + K_{\max} = \phi + eV_s \quad (18)$$

f : frequency of the incoming light

ϕ : work function of the metal (minimum energy to free an electron)

K_{\max} : max kinetic energy of emitted electron

e : electron charge

V_s : stopping potential (voltage needed to stop even the fastest emitted electrons)

Equivalent form:

$$K_{\max} = hf - \phi \quad (19)$$

Threshold frequency

$$f_0 = \frac{\phi}{h} \quad (20)$$

f_0 : minimum light frequency needed to eject electrons

ϕ : work function

h : Planck's constant

Photon energy vs wavelength

$$E_{\text{photon}} = \frac{hc}{\lambda} \quad (21)$$

E_{photon} : energy of one photon

c : speed of light in vacuum

λ : photon wavelength

Numerical shortcut: $hc \approx 1240 \text{ eV} \cdot \text{nm}$.

Bragg diffraction (X-ray crystallography)

$$2d \sin \theta = n\lambda \quad (22)$$

d : spacing between atomic planes in a crystal

θ : glancing angle between incoming beam and the crystal plane

n : diffraction order (1,2,3,...)

λ : X-ray wavelength

This tells you when reflected/scattered X-rays from different planes will interfere constructively.

Duane–Hunt law (X-ray cutoff)

$$hf_{\max} = eV_0 \quad (23)$$

f_{\max} : maximum frequency of X-rays produced in the tube

V_0 : accelerating voltage applied to the electrons

e : electron charge

h : Planck's constant

Also:

$$\lambda_{\min} = \frac{hc}{eV_0} \quad (24)$$

λ_{\min} : shortest wavelength X-ray that can be produced at that voltage

Compton scattering / Compton shift

$$\Delta\lambda = \lambda - \lambda_0 = \frac{h}{mc}(1 - \cos \theta) \quad (25)$$

$\Delta\lambda$: increase in wavelength of the scattered photon

λ_0 : initial photon wavelength

λ : scattered photon wavelength

θ : scattering angle of the photon

m : electron rest mass

c : speed of light

The constant $\frac{h}{mc} = 2.43 \times 10^{-12} \text{ m}$ is the **Compton wavelength of the electron**.

Wave-particle duality summary

$$E = hf \quad (26)$$

$$p = \frac{h}{\lambda} \quad (27)$$

These two equations link the wave description (frequency f , wavelength λ) to the particle description (energy E , momentum p) of light.