

1 Worst-Case, Best-Case, Average-Case, Runtime

Recall: for algorithm A with domain of input I , we choose a natural number size (measure for input)

Then for $n \in \mathbb{N}$ let $\mathcal{I}_n = \{z \in I : z \text{ has size } n\}$

and $WC_A(n) = \max\{RT_A(z), z \in \mathcal{I}_n\}$

and $BC_A(n) = \min\{RT_A(z), z \in \mathcal{I}_n\}$

So the upper bound and lower bounds can be obtained by:

$$WC \in \mathcal{O}(f) \text{ iff } \exists c, n_0, \in \mathbb{R}^{\geq 0}, \forall n \in \mathbb{N}, n \geq n_0 \Rightarrow \forall z \in \mathcal{I}_n, RT(z) \leq cf(n)$$

$$WC \in \Omega(f) \text{ iff } \exists c, n_0, \in \mathbb{R}^{\geq 0}, \forall n \in \mathbb{N}, n \geq n_0 \Rightarrow \exists z \in \mathcal{I}_n, RT(z) \geq cf(n)$$

$$BC \in \mathcal{O}(f) \text{ iff } \exists c, n_0, \in \mathbb{R}^{\geq 0}, \forall n \in \mathbb{N}, n \geq n_0 \Rightarrow \exists z \in \mathcal{I}_n, RT(z) \leq cf(n)$$

$$BC \in \Omega(f) \text{ iff } \exists c, n_0, \in \mathbb{R}^{\geq 0}, \forall n \in \mathbb{N}, n \geq n_0 \Rightarrow \forall z \in \mathcal{I}_n, RT(z) \geq cf(n)$$

Given the code

```
1 def is_in(x: int l: List) -> bool:
2     for item in l:
3         if x == item:
4             return True
5     return False
```

Notice that for all conditions, the number of iterations is less than or equal to n