1 Predicates

1.1 Some Rules about Predicates

Let S be the set of non-empty strings over alphabet a, b, c

Define binary predicate P with domain $S \times S$ by $P(x, y)$:

"x and y have the same first character"

Another option for this definition without announcing the domain in advance:

Define $P(x, y)$ by/as: "x and y have the same first character", where $x, y \in S$

Both of the above statements are valid. You should not, however, put quantification in the meaning of predicate. eg.

$x, y \in S$ for each $P(x, y) : \ldots$

$P(x, y)$: "x and y have ... where $x, y \in S$ is also invalid as is assumes x, and y are in S.

1.2 Example

$$\forall x \in X, \forall y \in S, P(x, y)$$

This could be translated to:

"For each x in S and for each y in S, x and y have the same first character.

However, To simplify it and make it more natural english:

"For all pair of string over S have the same first character"

1.3 Grouping Quantifiers

Note that variables with the same quantifier can be grouped together so:

$$\forall x \in S, \forall y \in S, P(x, y) \equiv \forall x, y \in S, P(x, y)$$

1.4 Example

Say we wanted to say that there are a pair on non-equal strings to start with the same character, we can combine propositions with and:

$$\exists x, y \in S, x \neq y \land P(x, y)$$

Which is true: eg. $x = aaa, y = baab$"

1.5 Example

Say we wanted to say *if* a pair of strings are different, they start with the same first letter

$$\exists x, y \in S, x \neq y \Rightarrow P(x, y)$$

1.6 Example

Given the statement:

In each pair of distinct strings, the strings start with the same character

$$\forall x, y \in S, x \neq y \Rightarrow P(x, y)$$

This statement is **FALSE**

Consider counter-example x = aaa, y = baa

Note that to say that a statement is false is the same as saying that the negation is true so:

$$\neg\big(\forall x, y \in S, x \neq y \Rightarrow P(x, y)\big)$$
$$\equiv$$
$$\exists x, y \in S, x \neq y \wedge \neg P(x, y)$$

Where in the negation, the counter-example of the original statement is an example for the negation.

1.7 Examining Commas

Consider the question:

$$\forall x, y \in S, x \neq y, P(x, y)$$

what does the comma mean?
Some possibilities are:

- and
- implication
- such that

The convention is that:

- After universal, comma *often* means implication ($\Rightarrow$)
- After existential, comma *often* means and ($\wedge$)

However, **DO NOT RELY ON COMMAS**. Just use precise language.

2 Examine Divisibility

Definition:
d devides n (also written as $d|m$)
if $n = dk$ for some $k \in \mathbb{Z}$

Note that in definitions, "if" almost certainly means "iff"