1 Proofs

What is a proof:

In CSC165, we consider proofs as a "convincing argument"

A proof is

- Relative to an audience. Audiences can include:
  - Yourself (checking yourself - convince yourself)
  - Peers
- A way to save someone time by the level of detail and organization of the exploration

When writing a proof, you must remember **why**. It is a structured presentation, often in a different order than the initial exploration of the material.

1.1 Some general forms

The Direct Proof of an existential.

Proof: Prove: $\exists x \in D, P(x)$
The **Structure** often follows:
Let $x_0 =$
Check / show / prove
...
Show that $x_0 \in D$ (for the chosen x_0)
Check/ show / prove (again)
Show that $P(0)$ □
The Direct proof of universal:

Proof: Prove: $\forall x \in D, P(x)$
The **Structure** often follows:
Let $x \in D$ - Think in terms of individual values that doesn't' depend on which particular value is chosen.
Then prove $P(x)$ □

Notice that for the existential, we introduce an individual value and write a proof **for that value.**

To prove:

- $\exists x \in D, P(x)$: We choose the x
- $\forall x \in D, P(x)$: Someone else chooses $x \in D$. Our proof must work without knowing which x.

1.2 Further Exploration

"Particular" vs "Arbitrary"
Explore "Prove $P \Rightarrow Q$

Proof: Structure follows:
Assume: P is `True`
Show that □

1.3 Sample Proofs

Prove that if a natural number is larger than 20 then its square minus 165 is at least four

First, we make this statement precise $\forall n \in \mathbb{N}, n > 20 \Rightarrow n^2 - 165 \geq 4$
Second, We do some rough work...

Try a goal:

$$\begin{aligned} n^2 - 165 &\geq 4 \\ n^2 &\geq 169 \\ n &\geq 13 \end{aligned}$$

We might have found an "equivalent condition" above.
Once we have completed the rough work, we can begin with the formal proof.

Proof: Let $n \in \mathbb{N}$.
Suppose $n > 20$
Then, $n \geq 13$, so $n^2 \geq 169$
Hence $n^2 - 165 \geq 169 - 165$
Thus $n^2 - 165 \geq 4$ □
Another way to prove the same thing

Proof: Let $n \in \mathbb{N}$
Assume $n > 20$
Then:

$$n^2 - 165 > 400 - 165$$
$$n^2 - 165 > 235$$
$$n^2 - 165 \geq 4$$

□