CSC165 **Arthur Gao**
LECTURE 12 (*May 17, 2022*)

# 1 Algorithm Analysis

Algorithm analysis has two components:

1. Correctness:
   - why does my program work?
   - Does it produce the corre t results for valid inputs
   - *left to CSC236*
2. Complexity - aka "efficiency"
   - How much resources does a program use
     - time
     - space (memory)
     - bandwidth (communication)
   - It is not cow concise / elegant the code itself is.

Reviewing past ideas (csc108)

Say we have program:

We measures resources relative to a size measure of the input

- If the input is a string, we might use the length
- If the input is a natural number, we might us it or maybe $log_2$ of it.

We'll assume the size is a natural number and what we measure is a non-negative number therefore the measure is some function: $\mathbb{N} \to \mathbb{R}^{\geq 0}$

Define:
For $f, g : \mathbb{N} \to \mathbb{R}^{\geq 0}$, $f$ absolutely dominates $g$ if:

$$\forall n \in \mathbb{N}, g(n) \leq f(n)$$

Given that, we can define $h, j : N \to \mathbb{R}^{\geq 0}$ by $h(n) = 2n + 3, j(n) = \frac{n}{2} + 5$ Examining this, we note that if we say $100h$

$$\begin{aligned} j(n) &= \frac{n}{2} + 5 \\ &\leq 200n + 5 \\ &\leq 200n + 300 \\ &= 100h(n) \end{aligned}$$

therefore, $n \geq 0$

Formalizing this, we can say that:

Define:
h dominates j up to a control factor if:

$$\exists c \in \mathbb{R}^+, \forall n \in \mathbb{N}, j(n) \leq ch(n)$$

**Proof:**

Let $c = 4 \in \mathbb{R}^+$. Let $n \in \mathbb{N}$
Then,

$$\begin{aligned} h(n) &= 2n + 3 \\ &\leq 2n + 20 \\ &= c(\frac{n}{2} + 5) \\ &= cj(n) \end{aligned}$$

■

Example:

- Define $k : \mathbb{N} \to \mathbb{R}^{\geq 0}$ by $k(n) = n^2$
- Define $h : \mathbb{N} \to \mathbb{R}^{\geq 0}$ by $h(n) = 2n + 3$

We note that it is impossible to make it so that either $h$ dominates $k$ or $k$ dominates $g$ even up to a control factor, therefore we have a third situation:

Define:
For $f, g : \mathbb{N} \to \mathbb{R}^{\geq 0}$, $f$ eventually dominates $g$ up to a control factor if:

$$\exists c, n_0 \in \mathbb{R}^+, \forall n \in \mathbb{N}, n \geq n_0 \to g(n) \leq cf(n)$$

This is also written as $g \in \mathcal{O}(f)$

Formally proving the given example that $h \in \mathcal{O}(k)$ would yield:
**Proof:**

Let $c = 5\mathbb{N}R^+$
Let $n_0 = 1 \in \mathbb{R}^+$

$$\begin{aligned} h(n) &= 2n + 3 \\ &\leq 2n + 3n && \text{since } n \geq 1 \\ &= 5n \\ &\leq 5n^2 \\ &= cn^2 && \text{since } n \geq 1 \\ &= ck(n) && \text{since } c = 5 \end{aligned}$$

$\blacksquare$

We can also say that theo opposite is of the above statement is :

$$k \notin \mathcal{O}(h) : \forall c, n_0 \in \mathbb{R}^+, \exists n \in \mathbb{N}, n \geq n_0 \wedge k(n) > ch(n)$$