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Note: Problem Set 0 Available

LECTURE 2 (January 12, 2022)

## 1 Functions:

 $f: A \to B \text{ means } f \text{ is a function from set } A \text{ to set } B$ 

This means for each element of a  $(a \in A)$  there's a corresponding  $f(a) \in B$ In the above definition, A is the **domain** where B is the **codomain** so we **MAP** elements from the domain to the codomain

Example:

Take the sets:

$$A = \{0, 2, 4\}$$
$$B = \{1, 2, 3\}$$

Define  $f: A \to B$  by  $f(x) = \frac{x}{2} + 1$ 

Some notes about Functions:

- ullet You cannot have one value in A corresponding to multiple values of B
- $\bullet$  You CAN have multiple values in A corresponding to a single value of B
- We also say that f(x) is called the image of x
  - $-f(x) \in B$ . It is **NOT** the function since we do not know what x is.
  - We must distinguish between the function f and the element of B, f(x)
- Say we define f by  $f(x) = \frac{x^2 3x}{x 3} \dots$  well for what f
  - We must define a domain
  - Possible example could be  $f: \mathbb{R} 3 \to \mathbb{R}$
  - Another option is "Define  $f: \mathbb{R} \to \mathbb{R}$  by g(x)" which means "for each  $x \in \mathbb{R}$ , let g(x) = x"
  - We must be clear about "what x" when we define functions

## 2 Predicates:

Given:  $P:A\to \{\text{True, False}\}$  so P maps to the set of boolean values

Example:

Define:  $P : \mathbb{R} \to \{T, F\}$  by P(x) : x > 165 With this we can say the set of values in the domain where P(x) = T which is:

$$\{x \in \mathbb{R} : P(x) = T\}$$

So we're interested in the "Set in the predicate where the given condition is true"

## 3 Notation:

 $\sum$  is for adding up numbers where there is a sequential pattern. Take:

$$4 + \frac{9}{2} + \frac{16}{3} + \dots + \frac{164^2}{164}$$

which can be notated as

$$\sum_{i=1}^{164} \frac{(n+1)^2}{n}$$

or

$$\sum_{i=2}^{165} \frac{n^2}{n-1}$$

Note: A method for breaking down the above summation is puting it in a uniform form so rewrite it as:

$$\frac{2^2}{1} + \frac{3^2}{2} + \frac{4^2}{3} + \dots + \frac{165^2}{164}$$

There is a convention that if we take a sum from on value to a value that is one fewer than the originating value, the sum is 0.

Example:

$$\sum_{i=165}^{164} \frac{i}{i-1} = 0$$

Similarly,  $\Pi$  is  $f(a) * f(a+1) * \cdots * f(b)$  so:

## 4 Propositional Logic

A proposition is a statement that is true ... or it's false

x < 165 is **not** a proposition since it is dependent on x Some Propositional statements could be:

- (valid) It's sunny in Los Angeles right now
- (valid) There is life on Europa
- (invalid) She likes cauliflower well who is "she"?