0.1 Example Proofs

Example:
Prove that every integer x that divides $x + 5$ also divides 5. Formalized that is:

$$\forall x \in \mathbb{Z}, (x|x + 5) \Rightarrow (x|5)$$

Remember that $a|b : \exists k \in \mathbb{Z}, b = ka$. That means that the initial definition expanded is:

$$\forall x \in \mathbb{Z}, (\exists k \in \mathbb{Z}, x + 5 = kx) \Rightarrow (\exists k \in \mathbb{Z}, 5 = kx)$$

Note that the first k is not the same as the second k in the above equation

This means we have a universal that contains an implication, which in turn means we follow the structure of universal proofs shown below:

Proof: Let $x \in \mathbb{Z}$
Assume $(\exists k \in \mathbb{Z}, x + 5 = kx)$
...
Therefore $(\exists k \in \mathbb{Z}, 5 = kx)$ □

For the proof, we will try examples in the universal as rough work to work out the universal proof:

Proof: Let $x \in \mathbb{Z}$
Assume $(\exists k_1 \in \mathbb{Z}, x + 5 = k_1 x)$
Let $= k_1 - 1$
W.T.S. $k_2 \in \mathbb{Z} \equiv 5 = k_2 x$
Then, $k_2 \in \mathbb{Z}$ since $k_1 \in \mathbb{Z}$
Also,

$$\begin{aligned} k_2 x &= (k_1 - 1)x \\ &= k_1 x - x \\ &= x + 5 - x \\ &= 5 \end{aligned}$$

□

Exercise:
Prove that $\forall d, x \in \mathbb{Z}, x|x + dx|d$