

Q1: Show that  $\mathbb{Z}_{12}/\{0,4,8\} \cong \mathbb{Z}_4$

$$\mathbb{Z}_{12}/I \cong \mathbb{Z}_4$$

$$I = \{0,4,8\}$$

We defined a mapping

$$\psi: \mathbb{Z}_{12} \rightarrow \mathbb{Z}_4$$

$$\psi(\bar{0}) = 0 = \psi(\bar{4}) = \psi(\bar{8})$$

$$\psi(\bar{1}) = 1 = \psi(\bar{5}) = \psi(\bar{9})$$

$$\psi(\bar{2}) = 2 = \psi(\bar{6}) = \psi(\bar{10})$$

$$\psi(\bar{3}) = 3 = \psi(\bar{7}) = \psi(\bar{11})$$

$$\mathbb{Z}_4 = \{0,1,2,3\}$$

Now we show  $\psi$  is Ring Homo, Onto.

$$\text{and } \ker \psi = I = \{\bar{0}, \bar{4}, \bar{8}\}$$

i) To show  $\psi$  is a Ring Homomorphism

$$\psi(\bar{a} + \bar{b}) = \psi(\bar{a}) + \psi(\bar{b})$$

$$\psi(\bar{a} \cdot \bar{b}) = \psi(\bar{a}) \cdot \psi(\bar{b})$$

$$\text{Let } a = \bar{1} \in \mathbb{Z}_{12} \quad b = \bar{2} \in \mathbb{Z}_{12}$$

$$\psi(a+b) = \psi(\bar{1} + \bar{2})$$

$$= \psi(\bar{3})$$

$$= 3$$

$$\psi(a) + \psi(b) = \psi(\bar{1}) + \psi(\bar{2})$$

$$= \bar{1} + \bar{2} = 3$$