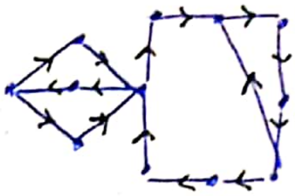
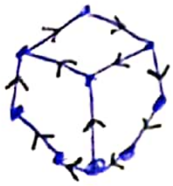
CHAPTER: 07

7.3



G_1



G_2

CHAPTER: 08

Ex: 8.1

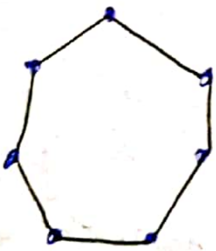
Q81

Q_1: u-u path in a tree = 0

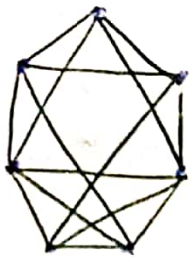
Q_2: Hamiltonian cycles in transitive tournament = 0.

Q_3: Bridges in tree = $n-1$

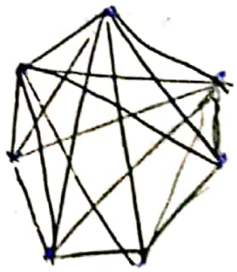
Q_4:



2 reg



4 reg

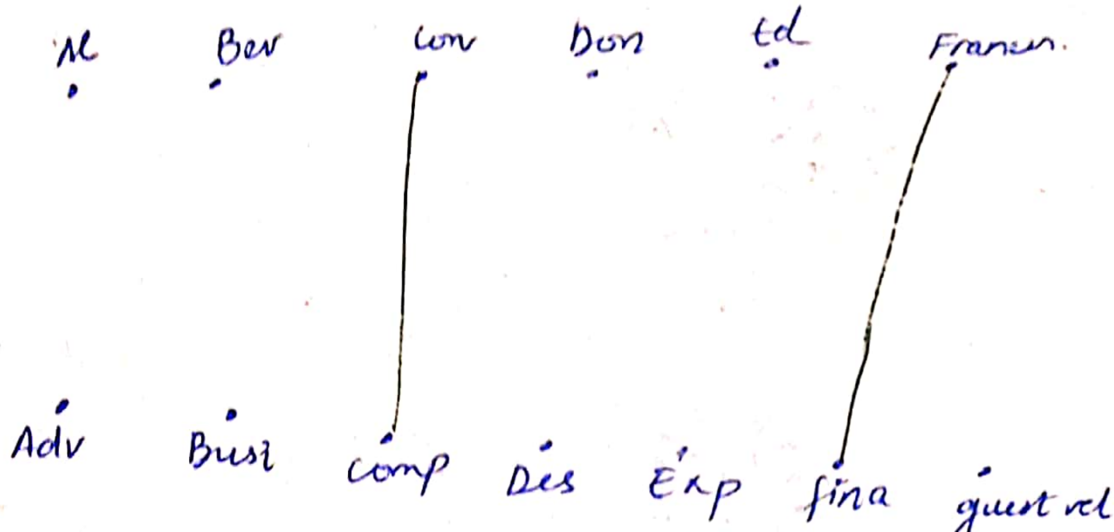


6 reg

Q_5: maximum degree of tree = $n-1$

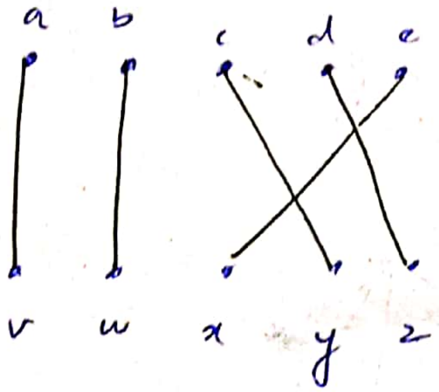
Q_6: max number of cut vertices = $n-2$

8.2

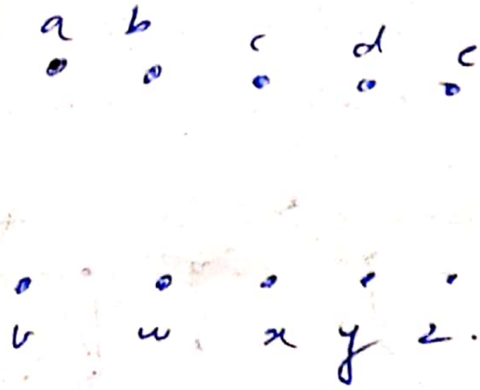


· Not possible because union of 4 sets
Alvin, Connie, Edward and Frances = {a, c, f} = S
which contains only 3 values
$k \leq |S|$.

8.3



q1
Yes.



Not possible because
v, x and y need a, b
means that sets of v, x, y
produces {a, c} which has
a lower cardinality than
number of selected sets.

8.13

$\alpha(H) = 5 \quad \{w, x, y, z, u\}$

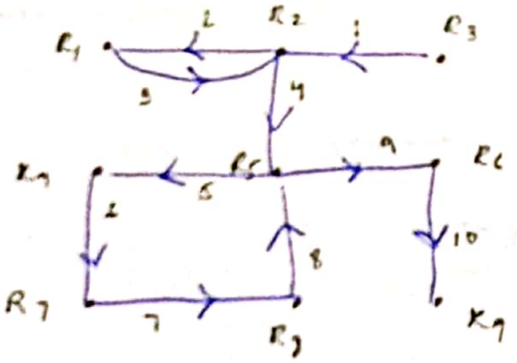
$\beta(H) = 2 \quad \{t, u\}$

$\alpha'(H) = 2 \quad \cancel{\{xt, wt, yt, zt, uv\}} \quad \{wt, uv\}$

$\beta(H) = 5 \quad \{xt, wt, yt, zt, uv\}$

CHAPTER: 06

6.1:



6.2 The graph would be Eulerian because both number of vertices of graph G_1 and G_2 is even. So G_1 & G_2 would be even because no new vertice is added.

6.3 G_1 and n ~~degree~~ order. n is even

G_2 ~~have~~ and G_3 has odd order. But a graph with odd number of vertices can only be connected if degree r_1 and r_2 is even.

Number of vertices

6.2 : G_1 vertices has even degree
G_2 vertices has even degree

add an edge u, v means 1 vertices in G_1 and other in G_2 has odd degree.

Non Eulerian

6.3 : G_1 is eulerian hence $\boxed{r_1 = \text{even}}$
G_1' is eulerian hence $n_1 - r_1 - 1$ = even

so $\boxed{n_1 = \text{odd}}$

G_2 and G_3 are non eulerian so $\boxed{r_2, r_3 \text{ is } \cancel{\text{odd}}.}$

If degree of each vertex in connected graph is odd then number of vertices is even

so n_2 and n_3 is even.

G is $G_1 + G_2 + G_3$.

i) Every vertex of G_1 in G has degree

$\cancel{r_1}\ r_1 + n_2 + n_3 \longrightarrow$ even.

ii) Every vertex of G_2 in G has degree

$r_2 + n_1 + n_3 \longrightarrow$ even

iii) Every vertex of G_3 in G has degree

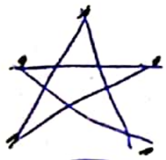
$r_3 + n_1 + n_2 \longrightarrow$ even

So every vertice has even degree and $G_1 + G_2 + G_3$ is eulerian.

6.4:

a) Both G and $\bar{G}$ are Eulerian.

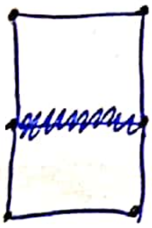
G



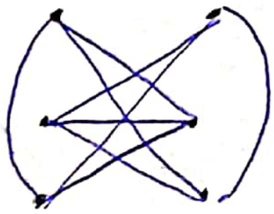
$\bar{G}$

Any odd cycle.

b)



G



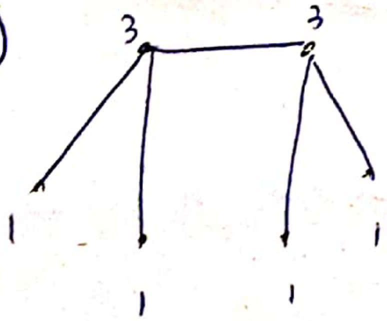
3 reg

$\bar{G}$

Any even cycle.

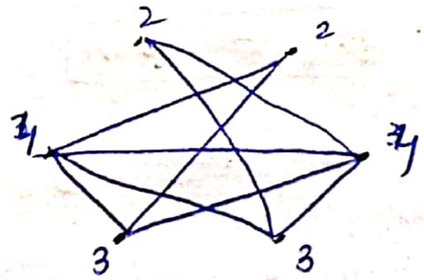
c)

d)



G

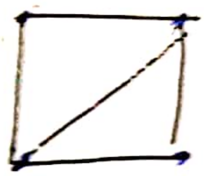
Non Eulerian



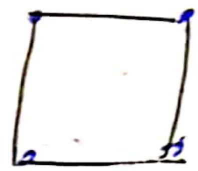
$\bar{G}$

Eulerian trail.

e



G

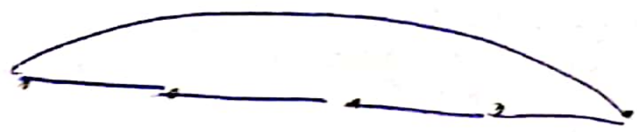


G-e.

6.5: P_5

Non eulerian

P_5



Eulerian

6.6: Since G is an r_1-regular Non eulerian graph all vertices have odd degree $\boxed{r_1 = odd}$

Since G is also connected hence it is necessary for G to have even number of vertices

Hence $\boxed{n_1 = even}$

we know that degree of each vertex in $\bar{G}$ is given as

$$n_1 - r_1 - 1$$

$$(even) - (odd) - (1) = even$$

Hence degree of each vertex in $\bar{G}$ is even so it is Eulerian. Proved.

EXCERCISE : 6.2

6.10 G is hamiltonian

G is a 6 regular graph. According to Theorm

$\deg v \geq n/2$ for every vertex

$n = 10$

$n/2 = 5$ since every vertex degree is

$6 \geq 5$

G - v is hamiltonian (9 vertices)

In G-v every vertex has degree 5 or 6.

deg $5 \geq 4$ → hamiltonian

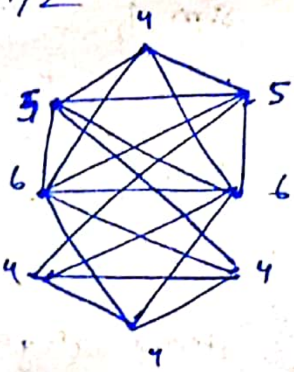
G-u-v is hamiltonian

Total number of vertices $n = 8$ $\boxed{n/2 = 4}$

in G-u-v some vertices will have degree 6, some will have 5 while others will have 4.

which is greater than or equal to $n/2$

Hence G-u-v is Hamiltonian



6.11 Cn is a 2 regular graph.

$\bar{C_n}$ each vertex's degree is given as $n-2-1$

$\boxed{= n-3}$

for $n = 5$

$5-3 \geq 5/2$

$\boxed{2 \geq 2}$ → Hence $C_{n=5}$ is Hamiltonian.

Similarly for $\boxed{n \geq 5}$

$6-3 \geq 6/2$

$3 \geq 3$ and so on.

<u>6.12</u> G is a 3 regular graph of order 12.
H is a 4 regular graph of order 11.

i) G in G+H, every vertex has degree

$n_2 + r_1 = 11 + 3$

$= 14$

ii) Every vertex of H in G+H degree given as

$n_1 + r_2 = 12 + 4$

$= 16$

a) Every vertex has even degree so Eulerian.

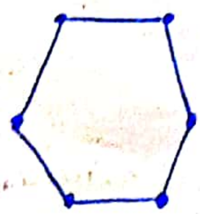
b) In G+H degree of each vertex is either 14 or 16.
number of vertexes in G+H $= n = 23$

so $n/2 = 23/2 \doteq 11$

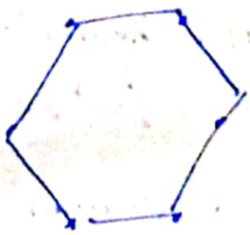
$\deg(v) \geq n/2$

$14 \text{ or } 16 \geq 11$

<u>6.13</u> a) C_6



Eulerian
because all vertex have even degree.

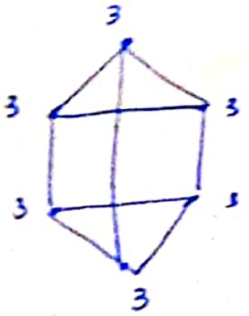


Non hamiltonian because degree of each vertex = 2

$2 \not\geq 6/2 = 3$

Hence non hamiltonian

b)

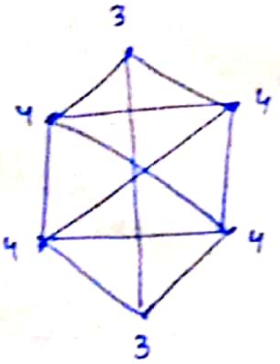


- Non-eulerian because every vertex has odd degree

- Hamitonian because degree of vertex = 3

$$3 \geq n/2 = 6/2 = 3$$

c)

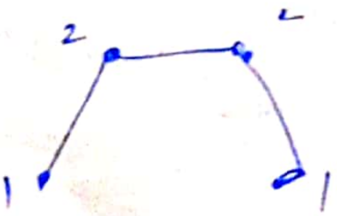


- Hamitonian becaus vertex have degree 3 or 4

$$3 \text{ or } 4 \geq n/2 = 6/2 = 3$$

- Eulerian Trail because 2 vertex have deg = 3 (odd), all other have deg = 4 (even)

d)



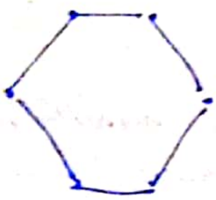
- Eulerian trail beeane 2 vertex have odd deg while all other have even.

- Non Hamiltonian becauaure degree of each vertex

$$1 \text{ or } 2 \ngeq n/2 = 4/2 = 2.$$

6.14

a) C_6

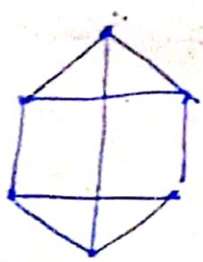


→ no cut vertices

- Euleran
- Non hamiltonian

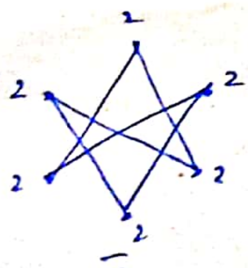
$$2 \ngeq n/2 = 3/2 = 6.$$

b)



G

- Non Eulerian
- Hamitonian



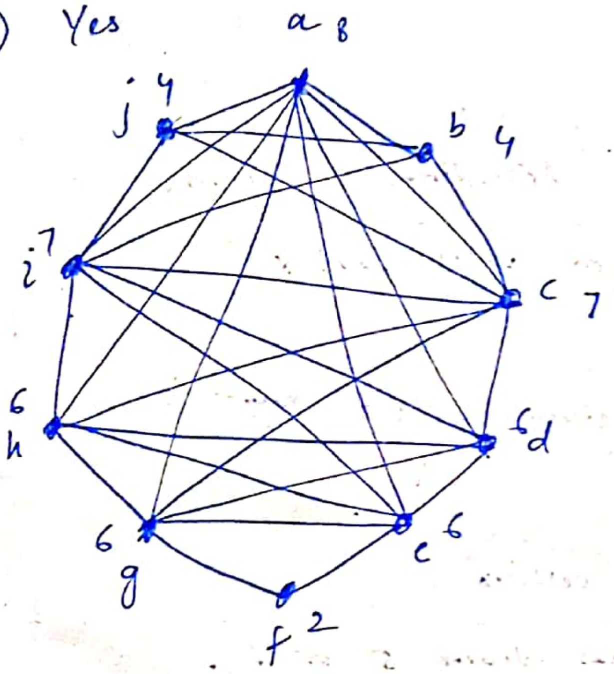
$\bar{G}$

- Eulerian.

Scanned with CamScanner

6.22

a) Yes



. Non hamiltonian
because vertex f has deg = 2

$2 \ngeq n/2 = 10/2$

$2 \ngeq 5$

Hence non-hamiltonian.

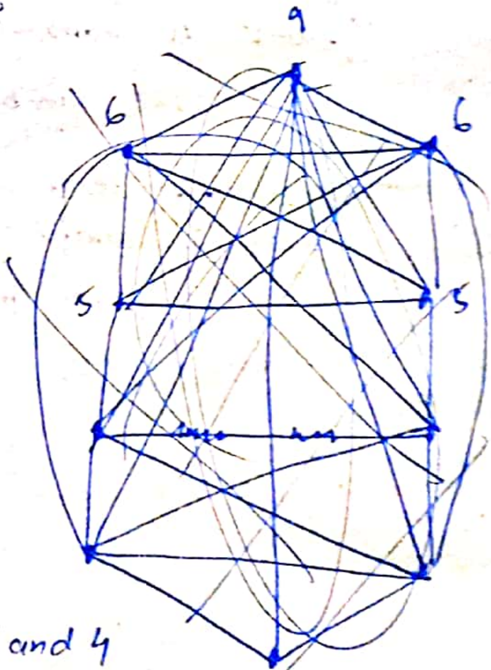
b) Yes

we know that for a connected graph

sum of degree of all vertex = 2m

m = 28

$\boxed{2m = 56}$



Sum of degrees of given 8 vertices =

$5+5+5+5+5+6+6+6 = 43$

Degree of remaining 2 vertices = $56 - 43 = 13$

Hence degree of remaing vertices can be 9 and 4

if degree of a vertex is 4

$4 \ngeq n/2 = 10/2 = 5$