ODE Visualization Using MATLAB

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Parameters

```
clear all;
clc;

f = input('Enter your function: ');

x0 = input('Enter initial value of independent variable (x0): ');
y0 = input('Enter initial value of dependent variable (y0): ');
h = input('Enter step size (h): ');
xn = input('Enter point at which you want to evaluate the solution (xn): ');

n = round((xn - x0) / h);
```

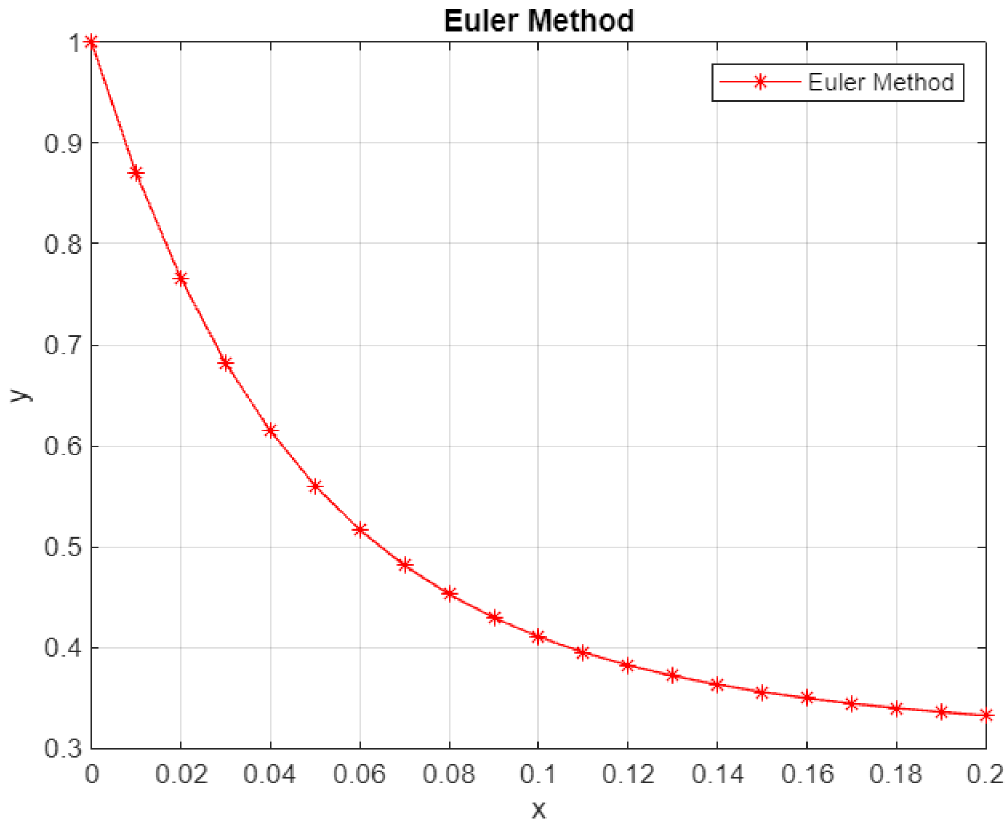
Euler Method

$$y_{n+1} = y_n + h\, f(x_n, y_n)$$

```
function [x, y] = euler_method(f, x0, y0, h, n)
    x = zeros(1, n+1);
    y = zeros(1, n+1);
    x(1) = x0;
    y(1) = y0;
    for i = 1:n
        y(i + 1) = y(i) + h * f(x(i), y(i));
        x(i + 1) = x0 + i * h;

        fprintf('y(%.4f) = %.6f\n',x(i+1),y(i+1))
    end
end

[x_euler, y_euler] = euler_method(f, x0, y0, h, n);
```

```
y(0.0100) = 0.870000
y(0.0200) = 0.765651
y(0.0300) = 0.681824
y(0.0400) = 0.614417
y(0.0500) = 0.560148
y(0.0600) = 0.516390
y(0.0700) = 0.481043
y(0.0800) = 0.452427
y(0.0900) = 0.429197
y(0.1000) = 0.410277
y(0.1100) = 0.394808
y(0.1200) = 0.382100
y(0.1300) = 0.371604
y(0.1400) = 0.362878
y(0.1500) = 0.355570
y(0.1600) = 0.349398
y(0.1700) = 0.344136
y(0.1800) = 0.339605
y(0.1900) = 0.335659
y(0.2000) = 0.332183
```

```
figure;

plot(x_euler, y_euler, 'r-*', 'DisplayName', 'Euler Method');
grid on;
legend('show');
xlabel('x');
ylabel('y');
title('Euler Method');
```



Heun's Method

$$k_1 = h\ f(x_n, y_n)$$

$$k_2 = h\ f(x_n + h, y_n + k_1)$$

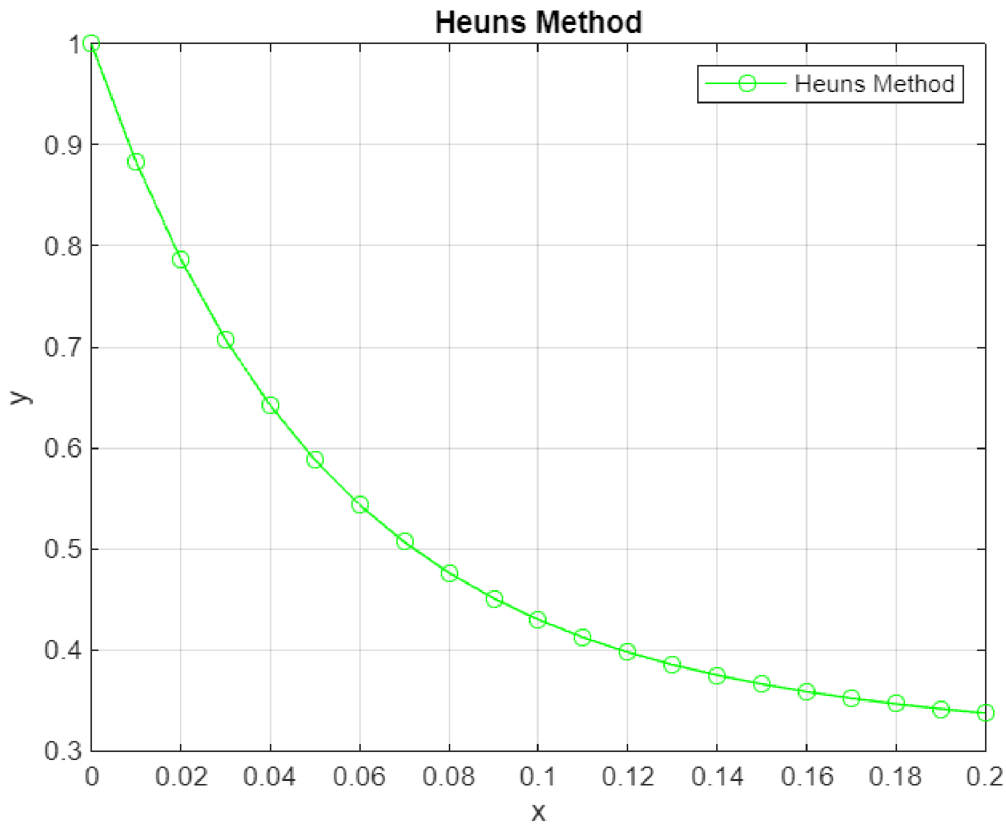
$$y_{n+1} = y_n + \frac{1}{2}\ (k_1 + k_2)$$

```
function [x, y] = heuns_method(f, x0, y0, h, n)
    x = zeros(1, n+1);
    y = zeros(1, n+1);
    x(1) = x0;
    y(1) = y0;
    for i = 1:n
        k1 = h * f(x(i), y(i));
        k2 = h * f(x(i) + h, y(i) + k1);

        y(i + 1) = y(i) + (1/2) * (k1 + k2);
        x(i + 1) = x0 + i * h;

        fprintf('y(%.4f) = %.6f\n',x(i+1),y(i+1))
    end
end

[x_heun, y_heun] = heuns_method(f, x0, y0, h, n);
```

```
y(0.0100) = 0.882825
y(0.0200) = 0.786429
y(0.0300) = 0.707072
y(0.0400) = 0.641689
y(0.0500) = 0.587767
y(0.0600) = 0.543243
y(0.0700) = 0.506428
y(0.0800) = 0.475935
y(0.0900) = 0.450629
y(0.1000) = 0.429577
y(0.1100) = 0.412014
y(0.1200) = 0.397315
y(0.1300) = 0.384965
y(0.1400) = 0.374543
y(0.1500) = 0.365703
y(0.1600) = 0.358163
y(0.1700) = 0.351689
y(0.1800) = 0.346091
y(0.1900) = 0.341212
y(0.2000) = 0.336926
```

```
figure;

plot(x_heun, y_heun, 'g-o', 'DisplayName', 'Heuns Method');
grid on;
```

```
legend('show');
xlabel('x');
ylabel('y');
title('Heuns Method');
```



Midpoint Method

$$k_1 = h\ f(x_n, y_n)$$

$$k_2 = h\ f\left(x_n + \frac{h}{2}, y_n + \frac{k_1}{2}\right)$$

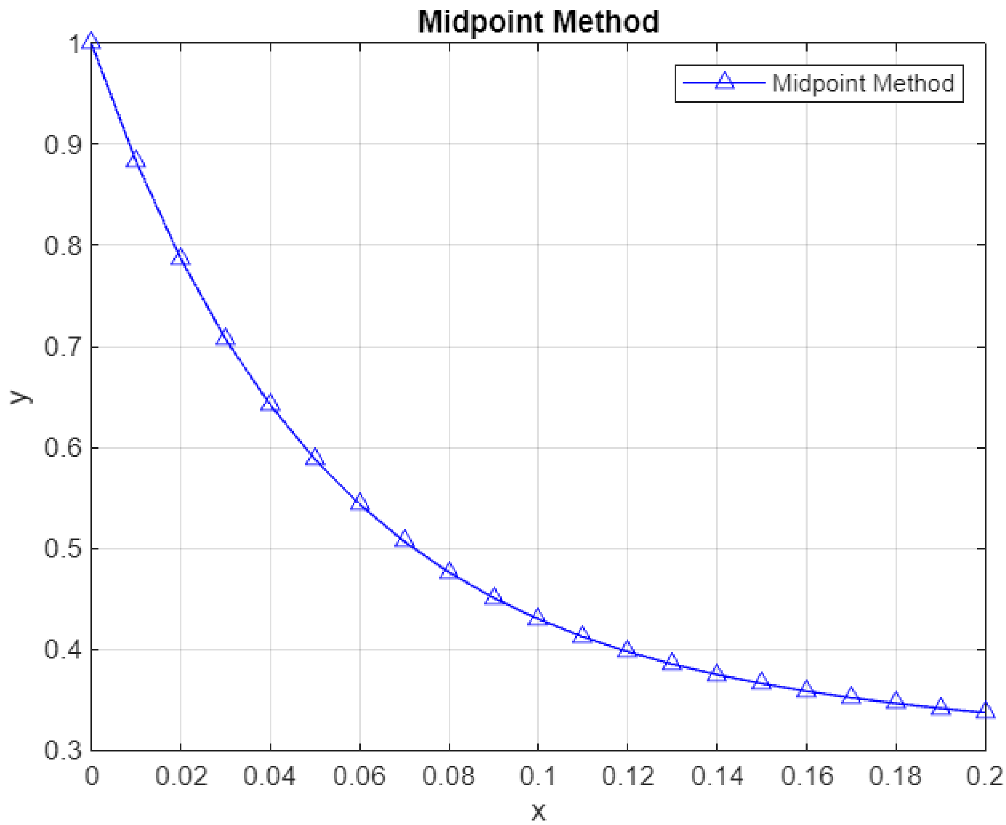
$$y_{n+1} = y_n + k_2$$

```
function [x, y] = midpoint_method(f, x0, y0, h, n)
    x = zeros(1, n+1);
    y = zeros(1, n+1);
    x(1) = x0;
    y(1) = y0;
    for i = 1:n
```

```
        k1 = f(x(i), y(i));
        k2 = f(x(i) + (h / 2), y(i) + (h / 2) * k1);

        y(i + 1) = y(i) + h * k2;
        x(i + 1) = x0 + i * h;

        fprintf('y(%.4f) = %.6f\n',x(i+1),y(i+1))
    end
end

[x_mid, y_mid] = midpoint_method(f, x0, y0, h, n);
```

```
y(0.0100) = 0.882825
y(0.0200) = 0.786429
y(0.0300) = 0.707072
y(0.0400) = 0.641689
y(0.0500) = 0.587766
y(0.0600) = 0.543242
y(0.0700) = 0.506427
y(0.0800) = 0.475934
y(0.0900) = 0.450628
y(0.1000) = 0.429576
y(0.1100) = 0.412013
y(0.1200) = 0.397314
y(0.1300) = 0.384964
y(0.1400) = 0.374542
y(0.1500) = 0.365702
y(0.1600) = 0.358162
y(0.1700) = 0.351687
y(0.1800) = 0.346089
y(0.1900) = 0.341211
y(0.2000) = 0.336925
```

```
figure;

plot(x_mid, y_mid, 'b-^', 'DisplayName', 'Midpoint Method');
grid on;
legend('show');
xlabel('x');
ylabel('y');
title('Midpoint Method');
```



Runge-Kutta Method

$$k_1 = h\ f(x_n, y_n)$$

$$k_2 = h\ f\left(x_n + \frac{h}{2}, y_n + \frac{k_1}{2}\right)$$

$$k_3 = h\ f\left(x_n + \frac{h}{2}, y_n + \frac{k_2}{2}\right)$$

$$k_4 = h\ f(x_n + h, y_n + k_3)$$

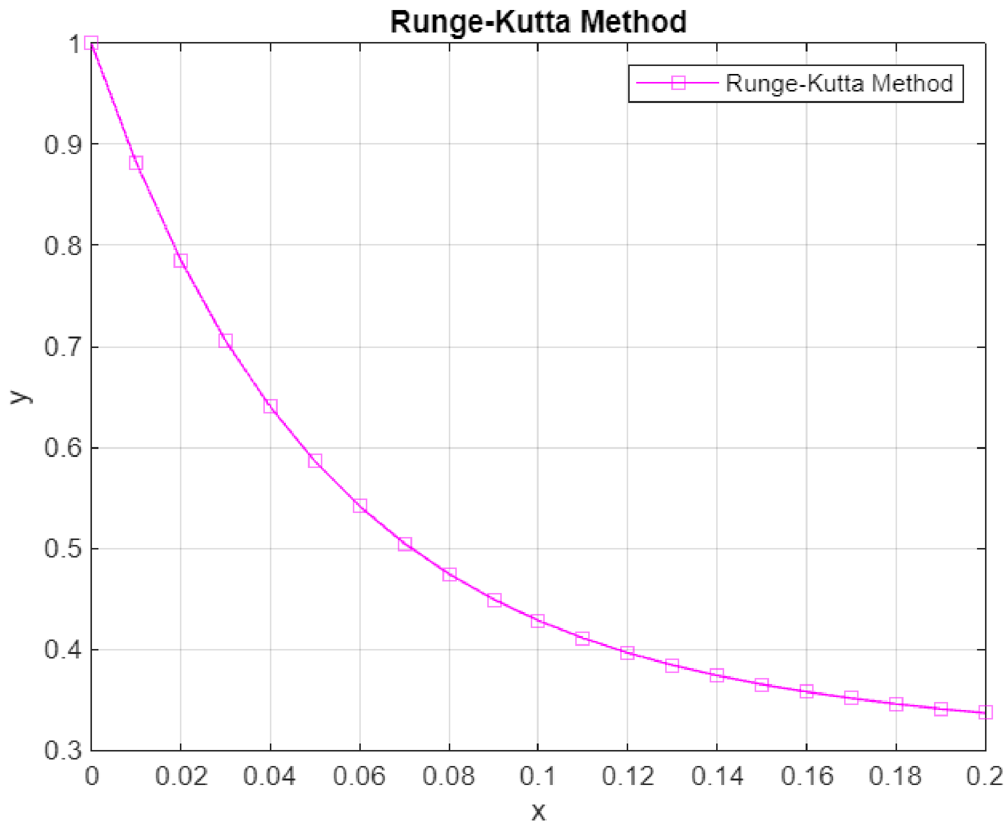
$$y_{n+1} = y_n + \frac{1}{6}\ [k_1 + 2\,k_2 + 2\,k_3 + k_4]$$

```
function [x, y] = rk_method(f, x0, y0, h, n)
    x = zeros(1, n+1);
    y = zeros(1, n+1);
```

```
    x(1) = x0;
    y(1) = y0;
    for i = 1:n
        k1 = h * f(x(i), y(i));
        k2 = h * f(x(i) + (h / 2), y(i) + (k1 / 2));
        k3 = h * f(x(i) + (h / 2), y(i) + (k2 / 2));
        k4 = h * f(x(i) + h, y(i) + k3);

        y(i + 1) = y(i) + (1/6) * (k1 + 2 * k2 + 2 * k3 + k4);
        x(i + 1) = x0 + i * h;

        fprintf('y(%.4f) = %.6f\n',x(i+1),y(i+1))
    end
end

[x_rk, y_rk] = rk_method(f, x0, y0, h, n);
```

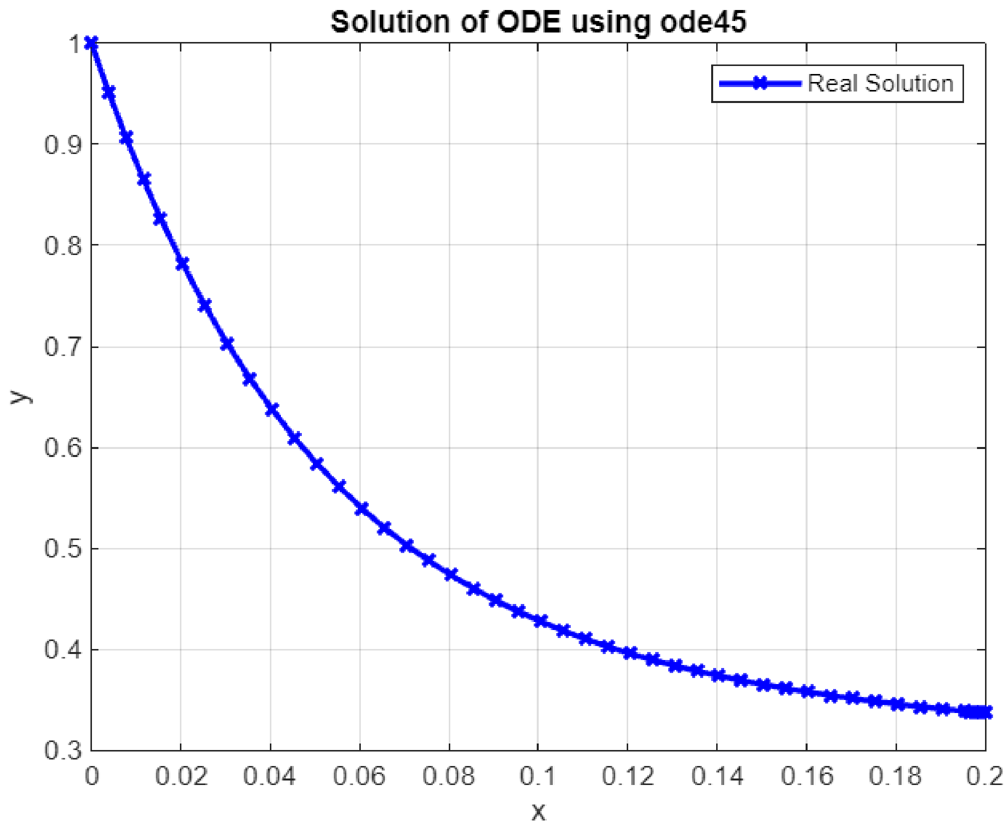
```
y(0.0100) = 0.882013
y(0.0200) = 0.785098
y(0.0300) = 0.705436
y(0.0400) = 0.639901
y(0.0500) = 0.585935
y(0.0600) = 0.541442
y(0.0700) = 0.504706
y(0.0800) = 0.474323
y(0.0900) = 0.449142
y(0.1000) = 0.428223
y(0.1100) = 0.410794
y(0.1200) = 0.396224
y(0.1300) = 0.383996
y(0.1400) = 0.373688
y(0.1500) = 0.364952
y(0.1600) = 0.357506
y(0.1700) = 0.351117
y(0.1800) = 0.345594
y(0.1900) = 0.340783
y(0.2000) = 0.336555
```

```
figure;

plot(x_rk, y_rk, 'm-s', 'DisplayName', 'Runge-Kutta Method');
grid on;
legend('show');
xlabel('x');
ylabel('y');
title('Runge-Kutta Method');
```



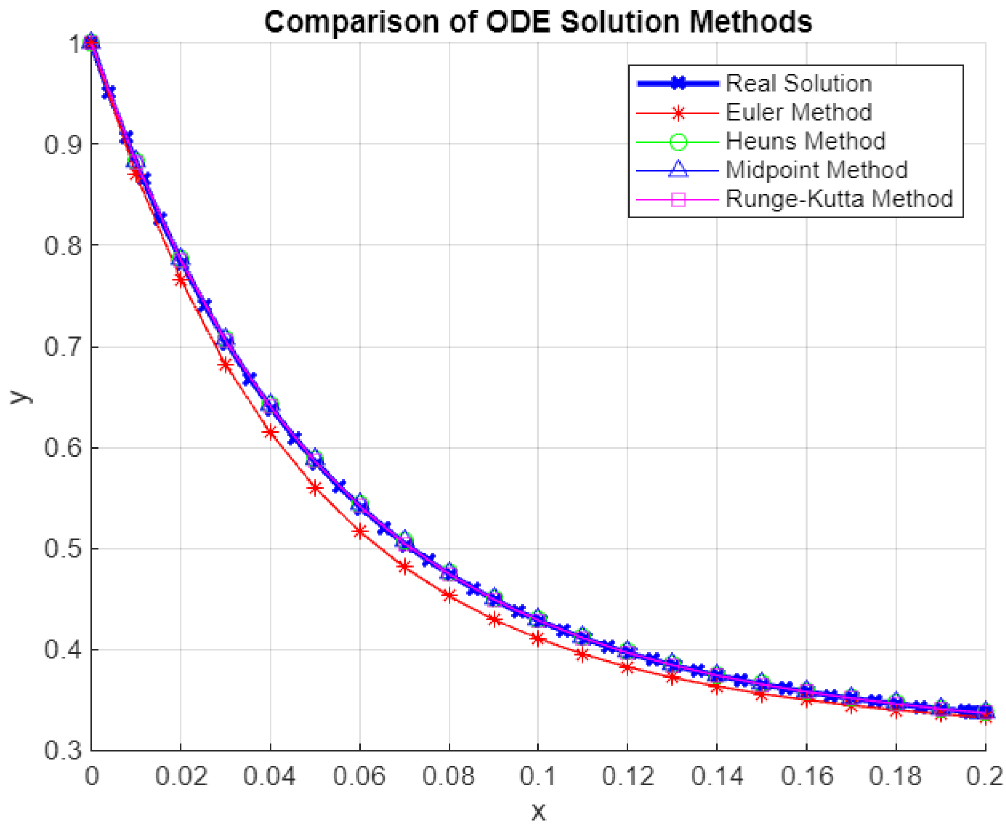
Real solution of ODE

```
[x, y] = ode45(f, [x0 xn], y0);

figure;
plot(x, y, 'b-x', 'LineWidth', 2);
grid on;
xlabel('x');
ylabel('y');
title('Solution of ODE using ode45');
legend('Real Solution');
```



Comparison of ODE Solution Methods

```
figure;
hold on
plot(x, y, 'b-x', 'LineWidth', 2, 'DisplayName', 'Real Solution');
plot(x_euler, y_euler, 'r-*', 'DisplayName', 'Euler Method');
plot(x_heun, y_heun, 'g-o', 'DisplayName', 'Heuns Method');
plot(x_mid, y_mid, 'b-^', 'DisplayName', 'Midpoint Method');
plot(x_rk, y_rk, 'm-s', 'DisplayName', 'Runge-Kutta Method');
grid on;
legend('show');
xlabel('x');
ylabel('y');
title('Comparison of ODE Solution Methods');
hold off;
```



Accuracy

$$\text{Relative Error } = \left|\frac{y_{\text{real}} - y_{\text{method}}}{y_{\text{real}}}\right| * 100\%$$

```
relative_error_euler = abs((y_euler - interp1(x, y, x_euler)) ./ interp1(x,
y, x_euler)) * 100;
relative_error_heun = abs((y_heun - interp1(x, y, x_heun)) ./ interp1(x, y,
x_heun)) * 100;
relative_error_mid = abs((y_mid - interp1(x, y, x_mid)) ./ interp1(x, y,
x_mid)) * 100;
relative_error_rk = abs((y_rk - interp1(x, y, x_rk)) ./ interp1(x, y, x_rk))
* 100;
```

```
fprintf('Accuracy of Euler Method: %.6f%%\n', 100 -
mean(relative_error_euler));
```

```
Accuracy of Euler Method: 96.986856%
```

```
fprintf('Accuracy of Heun''s Method: %.6f%%\n', 100 -
mean(relative_error_heun));
```

```
Accuracy of Heun's Method: 99.784741%
```

```
fprintf('Accuracy of Midpoint Method: %.6f%%\n', 100 -
mean(relative_error_mid));
```

```
Accuracy of Midpoint Method: 99.784948%
```

```
fprintf('Accuracy of Runge-Kutta Method: %.6f%%\n', 100 -
mean(relative_error_rk));
```

```
Accuracy of Runge-Kutta Method: 99.989922%
```