

Problem Set 3

$$T(n) = 2T(n/2) + c$$

$$T(n) = 2T(n/2) + c$$

$$= 2[2T(n/4) + c] + c$$

$$= 4T(n/4) + 3c$$

$$= 4[2T(n/8) + 3c] + 3c$$

$$= 8T(n/8) + 7c$$

$$= 8[2T(n/16) + c] + 7c$$

$$= 16T(n/16) + 15c$$

$$= 2^k T(n/2^k) + (2^k - 1)c$$

$$\text{for } \frac{n}{2^k} = 1$$

$$\frac{n}{2^k} = 1 \quad \therefore n = 2^k \Rightarrow k = \log n$$

$$T(n) = 2 \log n T(n/2 \log n) + (2 \log n - 1)c$$

$$= n T\left(\frac{n}{n}\right) + (n - 1)c$$

$$= nT + (n - 1)c$$

$$= O(n)$$

$$T(n) = O(n)$$